Bayesian Forecasting of Stock Prices
Via the Ohlson Model

By

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A Thesis

Submitted to the Faculty

of

WORCESTER POLYTECHNIC INSTITUTE

in partial fulfillment of the requirements for the

Degree of Master of Science

in

Applied Statistics

by

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May 2005

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Abstract

Over the past decade of accounting and finance research, the Ohlson (1995) model has been widely adopted as a framework for stock price prediction. While using the accounting data of 391 companies from SP500 in this paper, Bayesian statistical techniques are adopted to enhance both the estimative and predictive qualities of the Ohlson model comparing to the classical approaches. Specifically, the classical methods are used for the exploratory data analysis and then the Bayesian strategies are applied using Markov chain Monte Carlo method in three stages: individual analysis for each company, grouping analysis for each group and adaptive analysis by pooling information across companies. The base data, which consist of 20 quarters' observations starting from the first quarter of 1998, are used to make inferences for the regression coefficients (or parameters), evaluate the model adequacy and predict the stock price for the first quarter of 2004, when the real observations are set as the test data to evaluate the predictive ability of the Ohlson model. The results are averaged within each specified group categorized via the general industrial classification (GIC). The empirical results show that classical models result in larger stock price prediction errors, more positively-biased predictions and have much smaller explanatory powers than Bayesian models. A few transformations of both classical and Bayesian models are also performed in this paper, however, transformations of the classical models do not outweigh the usefulness of applying Bayesian statistics.

Acknowledgements

I would like to express my deep gratitude to the following people for their academic, financial or spiritual support in finishing this thesis.

Advisors: Balgobin Nandram and Huong Higgins. Without their help and guidance, there would not have been such a piece of work. There are no words that can express the appreciation from the bottom of my heart to them. And I promise this is not the end of my devotion to the research on this topic. I will keep in touch with them wherever I am.

Professors in the Math Department of WPI: Joseph D. Petruccelli, Jayson Wilbur, Andrew Swift, Carlos Morales and Christopher J. Larsen. Their courses have strengthened my knowledge in Applied Statistics and Applied Math. Also thank to Mayer Humi, William W. Farr, John Goulet, Dalin Tang, Peter R. Christopher and William J. Martin for their guidance in my two years of working experience as a teaching assistant.

Secretaries in the Math Department of WPI: Colleen Lewis, Ellen Mackin, Deborah Riel. They are the nicest, most patient and helpful secretaries that I've ever seen. Best wishes to them forever!

My parents and brother: Zhongxiao Lu, Rongzhi Liu, Hanliang Lu. They are always there when I'm in need; they accept me when I'm rejected; they are still proud of me even when I do not feel any good about a single part of myself.

My fiancé and his family: John Cacka, Bethlynne Vanella and Jennifer Graham etc. They gave me a warm home when I was in the hardest time in USA, and have been supporting me even since then. They like Chinese, I love them.

WPI Ballroom Dance Team: Boris Mosis and Miles Schofield etc. They helped me develop a terrific hobby --- dancing, which is my life, my soul and my best joy in the free time. Thanks for all those exciting practice lessons and dance parties.

Colleagues and classmates: Fang Huang, Yan Bai, Guochun Liu, Alina Ursan, Shinji Uemura, Gregory Matthews, Shawn Hallinan, Ashley Moraski, Rajesh Kondapaneni, Jasraj Kohli, Daniel Onofrei, Bijaya Padhy, Elizabeth Teixeira and Scott Laine etc. I had fun discussing homework problems with them. Sometimes we also talked about life, worries and future dreams. They are nice, smart and very friendly. I will keep them in my memory.

Friends: Jeani Lu, Mark Bertolina, Alin Sirbu, Congmei Ma, Chi Hu, Pong Pang, Fang Liu, Yucong Huang, Qunwei Ai, Chao Ku and Xiaolin Nan etc. They are not many, but enough to make me a rich and confident woman.

Last but not least, to those who doubt my intelligence, willpower and potential. I will fan the flame of dissatisfaction and make a miracle to them, to myself and to this wonderful world.

Chapter 1

The Ohlson (1995) Model and Data of S&P 500

1.1 The Ohlson (1995) Model

Over the past two decades in finance and accounting area, considerable attention has been paid to the relationship between accounting numbers (book values, earnings, etc.) and the firm value. The Ohlson (1995) approach to the problem of stock valuation relates securities prices to accounting data, and provides a structure for applicable modeling. The Ohlson (1995) Valuation Model has been widely adopted by researchers and practitioners on profitability analysis as a framework for the fundamental valuation of equities. It also has been developed into several versions, e.g., Feltham-Ohlson (1995) Valuation Model, Bernard's (1995) Ohlson Approximation Model, Liu-Ohlson (2000) Valuation Model and Callen's (2001) Ohlson AR(2) Valuation Model. For a historical development process of the Ohlson model, see Appendix A.

This paper evaluates the Ohlson (1995) Forecasting Model (OFM), or briefly the Ohlson (1995) model, and uses it to forecast stock prices. OFM is a practicable case of Bernard's (1995) Ohlson Approximation Model (see Appendix C). For a single firm, OFM states: the stock price per share is a linear function of the company's book value per share and abnormal earnings per share for the following four periods with normally distributed innovation terms, which represents "other information" whose source is uncorrelated with abnormal earnings. In mathematical form, it can be expressed as

$$\begin{aligned} &y_t = \beta_1 + \beta_2 bv_t + \sum_{k=1}^{4} \beta_{k+2} x_{t+k}^a + v_t = \underset{\sim}{x}_t' \underset{\sim}{\beta} + v_t, \\ &k = 1, 2, 3, 4, \; t = 1, \cdots, T, \end{aligned} \tag{1.1.1}$$

where y_t denotes the stock price per share at time t, bv_t is the book value per share at time t, x_t^a represents the abnormal earning at time t, $\underset{\sim}{\beta} = (\beta_1, \cdots \beta_6)'$ is the vector of intercept and slope coefficients of the predictors, $\underset{\sim}{x}_t^{'} = (1, bv_t, x_{t+1}^a, x_{t+2}^a, x_{t+3}^a, x_{t+4}^a)'$ is the vector of intercept and predictors, and v_t is the innovation (error or residual) term.

To understand the "abnormal earning" term, we can view it as a contraction of "above normal earning". Ohlson (1995) proposes the abnormal earning as

$$x_t^a = x_t - r_t bv_{t-1}, \tag{1.1.2}$$

where x_t is the earning per share at time t for a company, r_t is the discount rate at time t.

Since the values of the following four periods ($x_{t+1}^a, x_{t+2}^a, x_{t+3}^a, x_{t+4}^a$) are used to forecast the stock price, this paper uses the expected earnings to replace x_t in (1.1.2). That is,

$$x_t^a = E[x_t] - r_t bv_{t-1} \quad . \tag{1.1.3}$$

For the innovation term, Ohlson (1995) assumes it has a first order autoregressive structure (AR(1)). This assumption can be described as

$$\begin{aligned} v_t &= \rho v_{t-1} + \varepsilon_t, \\ \varepsilon_t &\sim N(0, \sigma^2), \end{aligned} \tag{1.1.4}$$

where ρ is the correlation coefficient of time series v_t, ε_t is the white noise, σ^2 is the variance of the white noise. Note that if $|\rho| < 1$, the AR(1) process is stationary.

From (1.1.1) and (1.1.4) we can get

$$\begin{aligned} v_{t-1} &= y_{t-1} - \underset{\sim}{x}_{t-1}^{'} \underset{\sim}{\beta}, \\ v_t &= \rho v_{t-1} + \varepsilon_t = \rho\left(y_{t-1} - \underset{\sim}{x}_{t-1}^{'} \underset{\sim}{\beta} \right) + \varepsilon_t, \\ y_t &= \underset{\sim}{x}_t^{'} \underset{\sim}{\beta} + \rho\left(y_{t-1} - \underset{\sim}{x}_{t-1}^{'} \underset{\sim}{\beta} \right) + \varepsilon_t. \end{aligned}$$

But when $t=1$, $y_1 = \underset{\sim 1}{x}' \underset{\sim}{\beta} + \rho\left(y_0 - \underset{\sim 0}{x}' \underset{\sim}{\beta}\right) + \varepsilon_1$, there exists unobserved values y_0 and $\underset{\sim 0}{x}'$.

This paper lets $\mu = \rho\left(y_0 - \underset{\sim 0}{x}' \underset{\sim}{\beta}\right)$. Therefore, expressions (1.1.1) and (1.1.4) can be combined as

$$
\begin{aligned}
& y_1 = x_1' \underset{\sim}{\beta} + \mu + \varepsilon_1, \\
& y_t = x_t' \underset{\sim}{\beta} + \rho(y_{t-1} - x_{t-1}' \underset{\sim}{\beta}) + \varepsilon_t, t = 2, \cdots, T, \\
& \varepsilon_t \overset{iid}{\sim} N(0, \sigma^2), \quad t = 1, \cdots, T,
\end{aligned}
\tag{1.1.5}
$$

where μ , ρ and σ^2 are three unknown parameters besides the intercept and regression coefficients in $\underset{\sim}{\beta}$.

Expression (1.1.5) is the complete form of the Ohlson (1995) Forecasting Model, hereinafter the Ohlson model, that is used in this paper.

1.2 Retrieving Data of S&P 500 from Thomson ONE Analytics

Yearly or quarterly data from various sources have been applied to test the Ohlson model. For instances, besides many tests that use US data, Bao & Chow (1999) test the usefulness of the Ohlson model using data from listed companies in the People's Republic of China; McCrave & Nilsson (2001) compare the difference between Swedish and US firms by using data from a Swedish business magazine, Bonnier-Findata database and I/B/E/S database; Ota (2002) uses empirical evidence from Japan, etc. This paper applies quarterly data of S&P 500 from Thosmon ONE Analytics to the Ohlson model.

S&P 500 is one of the most widely used measures of U.S. stock market performance and is considered to be a bellwether for the U.S. economy. S&P refers to Standard & Poor's, which is a division of the McGraw-Hill Companies, Inc. 500 companies are selected among the leaders in the major industries driving U.S. economy by the S&P Index

Committee for market size, liquidity and sector representation. A small number of international companies that are widely traded in the U.S are included.

The needed data of S&P 500 can be retrieved from Thomson ONE Analytics by its Excel Add-in software, provided by the Thomson Corporation, which is a global leader in providing value-added information, software applications and tools in the fields of law, tax, accounting, financial services and corporate training and assessment etc. Thomson ONE Analytics is a web based application that allows users to research information about different companies and markets, including current stock prices, volume traded, EPS (expected earning per share) and so on. The “Thomson ONE Analytics Excel Add-in” is one of the most valuable features that Thomson ONE Analytics offers its users. Using the Add-in, financial analysts can pull data directly into Excel from a wealth of financial databases such as “Worldscope”, “Compustat”, “U.S. Pricing”, “I/B/E/S and I/B/E/S History” and “Extel” by using the powerful PFDL (Premier Financial Database Language).

Items in the retrieved data are: Total Assets, Total Liabilities, Preferred Stock, Common Shares Outstanding from database “Worldscope”; “EPSmeanQTR1-4” and “EPSConsensusForecastPeriodQTR1-4” from database “I/B/E/S History” (note that these are monthly data); Dow Jones Industry Group (DJIC), General Industry Classification (GIC), Dow Jones Market Sector (DJMS) and GICSSECTOR from “Thomson Financial”; PriceClose and 3-month T-bill (treasury bill) rate from “Datastream”. Book value per common share (BPS) can be calculated by the first four items in following formula:

BPS = (Total Assets – Total Liabilities – Preferred Stock)/(Common Shares Outstanding).

Companies forecast their expected earnings every month for the following four fiscal quarters. This paper uses the latest forecast value for each quarter to represent the corresponding quarter value. The quarterly EPS are extracted from the monthly data of “EPSmeanQTR1-4” and “EPSConsensusForecastPeriodQTR1-4”. For easier use, values of Dow Jones Industry Group, Dow Jones Market Sector and the Company Identity Keys are transformed into integers. (For example, use 3104 instead of the original value C000003104.) To understand these financial / accounting terms, please see Appendix D. Appendix E explains how to use Excel Add-in. Appendix F explains how to extract quarterly data out of monthly data.

After deleting all the missing and incomplete data points and the data points that cause programming errors, 391 companies are selected. This final quarterly data set have 21 points for each company, covering 25 quarters from the first quarter of 1998 and the first quarter of 2004. It is formatted into 16 items which are all numerical values and contain 4 sectors (DJIC, GIC, DJMS and GICSSECTOR), company identity key (ID), Time, PriceClose, BPCS0-3, EPS1-4 and R (3-month T-bill rate).

1.3 Exploratory Data Analysis by Classical Approaches

While considering the ideas in various versions of the Ohlson model, this paper sticks to the main frame of the Ohlson model in (1.1.5), and sets up 11 different models which are described in Table 1.3.1 for the exploratory analysis.

These 11 models can be classified into three groups by distinguishing the assumption of the innovation term: independent errors among time periods which belongs to the ordinary linear regression structure (OLR), AR(1) structure for the error and AR(2) structure for the error. The main point of this classification is to check whether the AR(1) assumption is proper for the innovation term of the Ohlson model. Besides this, four kinds of transformation to the term of stock price per share are applied to the model under either AR(1) or AR(2) assumption for the innovation term: logarithmic transformation (log trans), square root transformation (sqrt trans), cubic root transformation (curt trans) and inverse transformation (inv trans). Two relatively better transformations are to be selected by the classical statistical analysis. This paper assumes their priorities to be adopted in further research by the innovative methods, for the reason of their being more fit to the data. The purpose of using transformations is to improve both the estimative and predictive qualities of the Ohlson model.

Table 1.3.1 --- Various Models

Name	Equation
OFM --- OLR	$y_t = \beta_0 + \beta_1 bv_t + \sum_{k=1}^{4} \beta_{k+1} x^a_{t+k} + \varepsilon_t, \varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$
OFM --- AR(1)	$y_t = \beta_0 + \beta_1 bv_t + \sum_{k=1}^{4} \beta_{k+1} x^a_{t+k} + v_t, v_t = \rho v_{t-1} + \varepsilon_t, \varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$
OFM --- AR(2)	$y_t = \beta_0 + \beta_1 bv_t + \sum_{k=1}^{4} \beta_{k+1} x^a_{t+k} + v_t, v_t = \rho_1 v_{t-1} + \rho_2 v_{t-2} + \varepsilon_t, \varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$
log trans of OFM AR(1)	$\log(y_t) = \beta_0 + \beta_1 bv_t + \sum_{k=1}^{4} \beta_{k+1} x^a_{t+k} + v_t, v_t = \rho v_{t-1} + \varepsilon_t, \varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$
sqrt trans of OFM --- AR(1)	$\sqrt{y_t} = \beta_0 + \beta_1 bv_t + \sum_{k=1}^{4} \beta_{k+1} x^a_{t+k} + v_t, v_t = \rho v_{t-1} + \varepsilon_t, \varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$
curt trans of OFM --- AR(1)	$\sqrt[3]{y_t} = \beta_0 + \beta_1 bv_t + \sum_{k=1}^{4} \beta_{k+1} x^a_{t+k} + v_t, v_t = \rho v_{t-1} + \varepsilon_t, \varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$
inv trans of OFM --- AR(1)	$1/y_t = \beta_0 + \beta_1 bv_t + \sum_{k=1}^{4} \beta_{k+1} x^a_{t+k} + v_t, v_t = \rho v_{t-1} + \varepsilon_t, \varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$
log trans of OFM AR(2)	$\log(y_t) = \beta_0 + \beta_1 bv_t + \sum_{k=1}^{4} \beta_{k+1} x^a_{t+k} + v_t, v_t = \rho_1 v_{t-1} + \rho_2 v_{t-2} + \varepsilon_t, \varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$
sqrt trans of OFM --- AR(2)	$\sqrt{y_t} = \beta_0 + \beta_1 bv_t + \sum_{k=1}^{4} \beta_{k+1} x^a_{t+k} + v_t, v_t = \rho_1 v_{t-1} + \rho_2 v_{t-2} + \varepsilon_t, \varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$
curt trans of OFM --- AR(2)	$\sqrt[3]{y_t} = \beta_0 + \beta_1 bv_t + \sum_{k=1}^{4} \beta_{k+1} x^a_{t+k} + v_t, v_t = \rho_1 v_{t-1} + \rho_2 v_{t-2} + \varepsilon_t, \varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$
inv trans of OFM --- AR(2)	$1/y_t = \beta_0 + \beta_1 bv_t + \sum_{k=1}^{4} \beta_{k+1} x^a_{t+k} + v_t, v_t = \rho_1 v_{t-1} + \rho_2 v_{t-2} + \varepsilon_t, \varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$

Two procedures, PROC REG and PROC AUTOREG in SAS, are the classical methods that are used to the whole data set to test the estimative ability of the 11 models. Specifically, PROC REG is only used to model OFM---OLR and PROC AUTOREG is used to the models with AR(p) structures for the innovation term. Three kinds of criteria are used to compare their estimative abilities: R-squares (total R-square and Regress R-square), Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). See Table 1.3.2 for the empirical results. Note that R-square is the coefficient of determination (regression sum of squares divided by total sum of squares). Total R-square is R-square and regress R-square is R-square adjusted for additional covariates. They are nearly the same in PROC REG procedure, but can be very different in PROC AUTOREG procedure, especially when the innovation terms are highly correlated among time periods. BIC is a quantity proportional to the negative log likelihood after all parameters are integrated out. AIC is a deviance measure (i.e., difference between observed and fitted models). Models with small AIC and BIC values are preferred.

Table 1.3.2 --- Overall Estimative Ability Comparison of Various Models

Model	**Total R-square**	**Regress R-square**	**AIC**	**BIC**
OFM --- OLR	0.2184	0.2184		
OFM --- AR(1)	0.7526	0.0856	59112	59161
OFM --- AR(2)	0.7526	0.0857	59114	59170
log trans of OFM AR(1)	0.7680	0.1033	2653	2702
sqrt trans of OFM --- AR(1)	0.7716	0.1016	18013	18062
curt trans of OFM --- AR(1)	0.7733	0.1046	2026	2076
inv trans of OFM --- AR(1)	0.6230	0.0413	-35734	-35685
log trans of OFM AR(2)	0.7680	0.1034	2656	2712
sqrt trans of OFM --- AR(2)	0.7716	0.1021	18016	18072
curt trans of OFM --- AR(2)	0.7733	0.1048	2030	2086
inv trans of OFM --- AR(2)	0.6242	0.0413	-35756	-35701

The following conclusions can be drawn by comparing the R-squares, AIC's and BIC's in Table 1.3.2.

- Using PROC REG to model OFM---OLR, R-square turns out to be very small (0.2184). Using PROC AUTOREG to the other 10 models, the Total R-square values are over 0.75 to all except in the cases of using inverse transformation. For the models with AR(p) structure to the innovation term, the results show big difference between the Total R-square (>0.75) and the Regress R-square (<0.11). All these results indicate that the assumption of independence of the innovation terms among different time periods cannot stand. In other words, setting an AR (p) structure to the innovation term can be a sound assumption.
- AR(2) structure is no better than AR(1), for they have extremely close R-square values. This is in line with the conclusion drawn by Callen (2001).
- The Total R-square value (0.6230) under the inverse transformation is much less than without a transformation (0.7526), while the Total R-square values under the other three transformations are slightly bigger than without a transformation. This concludes that the inverse transformation cannot enhance the estimative ability, while the other three can slightly enhance the estimative ability.

- Based on Total R-square value, cubic root transformation enhances the estimative ability the most (0.7733), then the square root transformation (0.7716), and then the log transformation (0.7680). But the differences among them are very small. Based on AIC and BIC, cubic transformation has the smallest value (2026 and 2076), then the log transformation (2653 and 2702). The square root transformation has much larger AIC and BIC (18013 and 18062). Therefore, cubic root transformation and log transformation are relative better than the others.

After comparing the estimative abilities of the 11 models, this paper proceeds further exploratory analysis by concentrating on 3 models: OFM --- AR(1), log trans of OFM AR(1) and curt trans of OFM --- AR(1).

In order to test the predictive abilities of the models, the retrieved data of S&P 500 are divided into two parts for each company. The first part contains the first 20 periods of data which will be used as base data to estimate regression coefficients; the second part has the 21st period of data which will be used as test data to compare with the predictions for this period from the base data. (The same base data and test data as in this division are also used in the following chapters.)

After using PROC AUTOREG to the data in each GIC group (see Table 1.3.3 for the General Industrial Classification Distribution), the estimated regression coefficients for three models are collected in Table 1.3.4. The results show that the intercept, BPS and abnormal earnings per share of the first two following quarters are generally significant in the Ohlson Model. (The values in bold are significant, others are insignificant.)

Table 1.3.3 --- General Industrial Classification (GIC) Distribution

GIC Value	0	1	2	3	4	5	6
Class	Overall	Industrial	Utility	Transportation	Banks/Savings and Loan	Insurance	Other Financial
No. of Firms	391	284	42	5	29	18	13

Note: when GIC equals 0, it means this group includes all 391 firms.

The PROC AUTOREG procedure also gives the predicted values of the 21st period using the base data. To compare the predictive abilities among the three selected models for different GIC groups, the criterion is defined as

$$R = (\hat{y}_{21} - y_{21}) / y_{21} \tag{1.3.1}$$

where R is the relative difference of predicted stock price over real stock price for a company, $\hat{y}_{21}$ is the predicted stock price of a company for the 21st period, y_{21} is the real stock price of a company for the 21st period.

Table 1.3.4 --- Estimated Parameters from Base Data

GIC = 0						
Model	Beta1	Beta2	Beta3	Beta4	Beta5	Beta6
OFM --- AR(1)	**25.7102**	**0.8006**	**4.4180**	**3.4307**	-0.089	0.0456
LOG-trans of OFM AR(1)	**3.075**	**0.0288**	**0.1492**	**0.1047**	0.006134	-0.0111
CURT-trans of OFM --- AR(1)	**2.8435**	**0.0279**	**0.1471**	**0.1076**	0.003040	-0.007624
GIC = 1						
Model	Beta1	Beta2	Beta3	Beta4	Beta5	Beta6
OFM --- AR(1)	**25.2903**	**0.8929**	**6.1202**	**5.5564**	0.2602	0.1988
LOG-trans of OFM AR(1)	**3.0464**	**0.0329**	**0.2143**	**0.1678**	0.0279	-0.002011
CURT-trans of OFM --- AR(1)	**2.8201**	**0.0317**	**0.2094**	**0.1742**	0.0214	0.000176
GIC = 2						
Model	Beta1	Beta2	Beta3	Beta4	Beta5	Beta6
OFM --- AR(1)	**23.0625**	**0.6416**	**1.7959**	-0.3845	0.4846	0.2983
LOG-trans of OFM AR(1)	**2.9159**	**0.0290**	**0.0645**	0.000787	0.000355	-0.0129
CURT-trans of OFM --- AR(1)	**2.7138**	**0.0262**	**0.0608**	-0.00561 7	0.006043	-0.008238
GIC = 3						
Model	Beta1	Beta2	Beta3	Beta4	Beta5	Beta6
OFM --- AR(1)	**11.2044**	**0.9892**	3.4738	1.4118	1.0681	-1.3959
LOG-trans of OFM AR(1)	**2.6101**	**0.0342**	0.1238	0.0204	0.0220	-0.0415
CURT-trans of OFM --- AR(1)	**2.3632**	**0.0343**	0.1218	0.0292	0.0265	-0.0442
GIC = 4						
Model	Beta1	Beta2	Beta3	Beta4	Beta5	Beta6
OFM --- AR(1)	**18.6728**	**1.3250**	**8.7575**	-4.6200	-7.4562	0.7041
LOG-trans of OFM AR(1)	**3.0660**	**0.0316**	0.2109	-0.0500	-0.1922	-0.0129
CURT-trans of OFM --- AR(1)	**2.7515**	**0.0361**	**0.2419**	-0.0800	-0.2187	-0.002449
GIC = 5						
Model	Beta1	Beta2	Beta3	Beta4	Beta5	Beta6
OFM --- AR(1)	**31.9752**	**0.6065**	-3.0127	1.7879	-1.2500	0.0256
LOG-trans of OFM AR(1)	**3.4200**	**0.0147**	-0.0591	0.0356	-0.0168	-0.008506
CURT-trans of OFM --- AR(1)	**3.1393**	**0.0166**	-0.0739	0.0450	-0.0241	-0.006542
GIC = 6						
Model	Beta1	Beta2	Beta3	Beta4	Beta5	Beta6
OFM --- AR(1)	**30.5121**	**0.6323**	**15.9995**	5.2249	**-6.2083**	-1.3595
LOG-trans of OFM AR(1)	**3.2711**	**0.0194**	**0.4457**	0.1451	-0.1619	-0.0580
CURT-trans of OFM --- AR(1)	**3.0281**	**0.0199**	**0.4742**	0.1528	**0.1816**	-0.0536

The following conclusions can be drawn from Table 1.3.5 where the quantiles of R, the number of nonnegative R's (No.(+,0)) and the number of negative R's (No. (-)) are collected. The digital "1" and "2" after the names of transformations are to distinguish different scales of measurement. "1" denotes using the original scale, "2" denotes using the transformed scale.

Table 1.3.5 --- Quantiles of *R* and Number of Nonnegative/Negative R's
(1 --- Original Scale 2 --- Transformed Scale)

GIC	No. of Firms	Model	Min	Q1	Q2	Q3	Max	No. (+,0)	No. (-)
0	391	OFM --- AR(1)	-0.490	0.073	0.400	0.936	15.703	315	76
		log trans of OFM AR(1)-1	-0.564	-0.038	0.347	0.795	13.137	284	107
		log trans of OFM AR(1)-2	-0.196	-0.010	0.088	0.191	6.875	284	107
		curt trans of OFM AR(1)-1	-0.540	-0.004	0.360	0.804	13.992	292	99
		curt trans of OFM AR(1)-2	-0.227	0.000	0.109	0.219	1.466	293	98
1	284	OFM --- AR(1)	-0.458	0.058	0.440	1.059	15.277	225	59
		log trans of OFM AR(1)-1	-0.531	-0.029	0.365	0.879	12.626	208	76
		log trans of OFM AR(1)-2	-0.179	-0.008	0.098	0.214	6.780	208	76
		curt trans of OFM AR(1)-1	-0.507	-0.013	0.416	0.899	13.496	210	74
		curt trans of OFM AR(1)-2	-0.209	-0.003	0.124	0.240	1.439	210	74
2	42	OFM --- AR(1)	-0.197	0.068	0.281	1.411	9.235	35	7
		log trans of OFM AR(1)-1	-0.340	0.000	0.174	1.062	7.412	31	11
		log trans of OFM AR(1)-2	0.120	0.000	0.049	0.284	2.220	31	11
		curt trans of OFM AR(1)-1	-0.284	0.014	0.208	1.185	8.042	33	9
		curt trans of OFM AR(1)-2	-0.104	0.006	0.066	0.299	1.084	34	8
3	5	OFM --- AR(1)	0.041	0.195	0.359	0.474	0.574	5	0
		log trans of OFM AR(1)-1	0.162	0.180	0.181	0.357	0.368	5	0
		log trans of OFM AR(1)-2	0.042	0.052	0.056	0.093	0.105	5	0
		curt trans of OFM AR(1)-1	0.111	0.180	0.242	0.407	0.430	5	0
		curt trans of OFM AR(1)-2	0.037	0.058	0.076	0.122	0.128	5	0
4	29	OFM --- AR(1)	-0.251	0.171	0.306	0.480	1.394	25	4
		log trans of OFM AR(1)-1	-0.323	0.054	0.185	0.402	1.194	24	5
		log trans of OFM AR(1)-2	-0.100	0.015	0.051	0.113	0.299	24	5
		curt trans of OFM AR(1)-1	-0.302	0.089	0.220	0.424	1.252	24	5
		curt trans of OFM AR(1)-2	-0.112	0.030	0.070	0.126	0.312	24	5
5	18	OFM --- AR(1)	-0.313	0.120	0.242	0.390	3.663	15	3
		log trans of OFM AR(1)-1	-0.359	0.047	0.190	0.302	3.357	14	4
		log trans of OFM AR(1)-2	-0.109	0.013	0.047	0.076	0.645	14	4
		curt trans of OFM AR(1)-1	-0.347	0.066	0.197	0.324	3.435	14	4
		curt trans of OFM AR(1)-2	0.131	0.023	0.063	0.099	0.644	14	4
6	13	OFM --- AR(1)	-0.045	0.195	0.348	0.847	4.276	12	1
		log trans of OFM AR(1)-1	-0.105	0.143	0.266	0.810	3.562	11	2
		log trans of OFM AR(1)-2	-0.031	0.037	0.071	0.174	0.807	11	2
		curt trans of OFM AR(1)-1	0.075	0.370	0.466	1.266	4.128	13	0
		curt trans of OFM AR(1)-2	0.026	0.112	0.138	0.315	0.726	13	0

The empirical results show that the distributions of R are asymmetrical with long tails, which suggests the 50% quantile (Q2) of R as a major criterion. From Table 1.3.5, the following conclusions can be drawn.

- Based on Q2 values in original scale, the ratio value ranges from 24.2% (GIC = 5) to 44% (GIC = 1) and 40% overall (GIC = 0) under no transformation, from 17.4% (GIC = 2) to 36.5% (GIC = 1) and 34.7% overall (GIC = 0) under log transformation, and from 19.7% (GIC = 5) to 46.6% (GIC = 6) and 36% overall (GIC = 0) under cubic root transformation. Based on Q2 values in transformed scale, the ratio value ranges from 4.7% (GIC = 5) to 9.8% (GIC = 1) and 8.8% overall (GIC = 0) under log transformation, and from 6.3% (GIC = 5) to 13.8% (GIC = 6) and 10.9% overall (GIC = 0) under cubic root transformation. These conclude that the log transformation improves the predictive ability more than the cubic root transformation does while using the classical method.
- In all cases, the number nonnegative R 's is much larger than the number of negative R 's, which shows the high overestimation by the classical method.

The too large magnitude of R and the extremely high overestimation state that using the classical method (the PROC AUTOREG procedure) to interpret the Ohlson model is not efficient enough in forecasting stock prices. A better approach is desired to improve both the estimative and predictive abilities of the Ohlson model.

Summarily, the exploratory data analysis by PROC REG/AUTOREG confirms the AR(1) assumption of the innovation term in the Ohlson model, and the promising effect of adopting logarithmic transformation as well as cubic root transformation. It suggests that the remaining work focus on 3 models: OFM --- AR(1), log trans of OFM AR(1) and curt trans of OFM --- AR(1). Since the Ohlson model is not able to predict the stock price efficiently by the classical means, this paper applies an innovative statistical method, Bayesian statistical analysis, to the 3 chosen models in the remaining work.

1.4 An Outline of Bayesian Statistical Analysis

In the following three chapters of this paper, Bayesian approaches are used for the purpose of satisfying the requirement of improving both the estimative and predictive qualities of the Ohlson model, comparing to the classical methods. In detail, Chapter 2 uses the most basic Bayesian techniques to each company, which is the case that different companies have different regression coefficients; Chapter 3 applies the Bayesian method by letting all the companies in each group share the same regression coefficients. While Chapter 2 represents the individual analysis, Chapter 3 represents the grouping analysis. And Chapter 4 ends up to be the adaptive analysis by pooling information across companies. That is, different companies have different regression coefficients in Chapter 4, and in the mean time they are pooled together. Basically, Chapter 4 compromises the ideas in Chapter 2 and the ones in Chapter 3.

For each Bayesian approach in following three chapters, the main tasks are to make inferences for the regression coefficients (or parameters), evaluate the model adequacy and test the predictive ability of the Ohlson model. Chapter 5 concludes all the work in this paper, which includes the comparison among the three Bayesian approaches as well as the comparison of the best Bayesian approach to the classical method.

Chapter 2

Bayesian Statistical Analysis for Individual Firm

2.1 Bayesian Version of the Ohlson Model for a Single Firm

As an extreme case, this chapter assumes all the companies are independent of each other and have their own regression coefficients in the Ohlson model. At the very beginning of applying the Bayesian statistical analysis to each company, a Bayesian version of the Ohlson model is set up in the following three steps.

First, for a specific company, describe the observation $\underset{\sim}{y} = (y_1, y_2, \cdots, y_T)$ by the parameters $\{\underset{\sim}{\beta}, \mu, \rho, \sigma^2\}$. Under the assumption that the observations are conditionally independent among the time periods, we can get the likelihood function from expression (1.1.5):

$$p(\underset{\sim}{y} \mid \underset{\sim}{\beta}, \mu, \rho, \sigma^2) = N(y_1 \mid \underset{\sim 1}{x}' \underset{\sim}{\beta} + \mu, \sigma^2) \cdot \prod_{t=2}^{T} N\left(y_t \mid \underset{\sim t}{x}' \underset{\sim}{\beta} + \rho \left(y_{t-1} - \underset{\sim t-1}{x}' \underset{\sim}{\beta} \right), \sigma^2\right). \tag{2.1.1}$$

Second, assign a prior distribution to each unknown parameter. The prior distribution represents a population of possible parameter values, from which the parameter of current interest has been drawn. The guiding principle is to express the knowledge (and uncertainty) about the parameter as if its value could be thought of as a random realization from the prior distribution. In order to get the practical advantage of being interpretable as additional data and computational convenience, this paper assigns the conjugate prior distributions as follows:

$$\begin{aligned} \pi(\underset{\sim}{\beta}) &= N_K(\underset{\sim}{\beta} \mid \underset{\sim 0}{\theta}, \Delta_0), \\ \pi(\mu) &= N(\mu \mid \mu_0, \sigma_0^2), \\ \pi(\rho) &= U(\rho \mid -1, 1), \\ \pi(\sigma^2) &= I\Gamma(\sigma^2 \mid a, b). \end{aligned} \tag{2.1.2}$$

The hyperparameters $\{\underset{\sim}{\theta}_0, \Delta_0, \mu_0, \sigma_0^2\}$ in (2.1.2) are set as follows.

(P2.1) $\underset{\sim}{\theta}_0 = B = (X'X)^{-1} X' \underset{\sim}{y}$.

The idea of setting $\underset{\sim}{\theta}_0$ is to use the estimation of $\underset{\sim}{\beta}$ in the ordinary linear regression model

$$y_t = x_t' \underset{\sim}{\beta} + \varepsilon_t, \varepsilon_t \overset{iid}{\sim} N(0, \sigma^2),\ t = 1, \cdots, T, \qquad \text{(OLM-1)}$$

by the method of least squares. Note that $X = \left(\underset{\sim}{x}_1', \underset{\sim}{x}_2', \cdots, \underset{\sim}{x}_T'\right)$ is the matrix of all covariates together with an intercept, $\underset{\sim}{x}_t' = (1, bv_t, x_{t+1}^a, x_{t+2}^a, x_{t+3}^a, x_{t+4}^a)'$ $(t = 1, \cdots, T)$ is the regression coefficient vector consisting of the 1 and predictors, and T is the number of time periods.

(P2.2) $\Delta_0 = 100(X'X)^{-1} \dfrac{SS_E}{T-P}$, where $SS_E = y'y - B'X'y$ is the sum of squares of the errors of (OLM-1), P is the number of regression coefficients (including the intercept), $\dfrac{SS_E}{T-P}$ is the estimation of the σ^2 in (OLM-1) by the method of least squares, and $(X'X)^{-1} \dfrac{SS_E}{T-P}$ is the estimation of the covariance matrix of $\underset{\sim}{\beta}$ in (OLM-1). Multiplying the estimation of the covariance matrix by 100 is to add more variability.

(P2.3) $\mu_0 = T^{-1} \sum_{i=1}^{T} (y_i - \underset{\sim}{x}_i B)$.

From $y_1 = x_1' \underset{\sim}{\beta} + \mu + \varepsilon_1, \varepsilon_1 \sim N(0, \sigma^2)$ in expression (1.1.5), we can get the estimation of μ which is $\hat{\mu} = y_1 - x_1' B$. Taking each observation can as starting point, that is,

$$\mu = y_i - x_i' \underset{\sim}{\beta} + \varepsilon_i, \varepsilon_i \sim N(0, \sigma^2), i = 1, 2, \cdots, T. \qquad \text{(OLM-2)}$$

The idea of setting μ_0 is to use the averaged estimation of μ in (OLM-2).

(P2.4) $\sigma_0^2 = \dfrac{SS_E}{T-P}$ is the estimation of variance σ^2 in (OLM-2) by the method of least squares.

(P2.5) $a = b = 0.001$.

These two hyperparameters are chosen by convention or experience in Bayesian statistical analysis.

Assume that all the parameters are independent of each other, the joint prior distribution of the parameters can be expressed as

$$p(\underset{\sim}{\beta},\mu,\rho,\sigma^2) = N_K(\underset{\sim}{\beta} \mid \underset{\sim 0}{\theta},\Delta_0)\cdot N(\mu \mid \mu_0,\sigma_0^2)\cdot U(\rho \mid -1,1),\cdot I\Gamma(\sigma^2 \mid a,b). \qquad (2.1.3)$$

Finally, from the likelihood function in (2.1.1) and joint prior distribution in (2.1.3), we can get the posterior distribution of the parameters given the data using Bayes' rule:

$$\begin{aligned}
& p(\underset{\sim}{\beta},\mu,\rho,\sigma^2 \mid \underset{\sim}{y}) \\
& \propto p(\underset{\sim}{\beta},\mu,\rho,\sigma^2)p(\underset{\sim}{y} \mid \underset{\sim}{\beta},\mu,\rho,\sigma^2) \\
& \propto N_K(\underset{\sim}{\beta} \mid \underset{\sim 0}{\theta},\Delta_0)\cdot N(\mu \mid \mu_0,\sigma_0^2)\cdot U(\rho \mid -1,1),\cdot I\Gamma(\sigma^2 \mid a,b)\cdot N(y_1 \mid \underset{\sim 1}{x}' \underset{\sim}{\beta}+\mu,\sigma^2) \\
& \qquad \cdot \prod_{t=2}^{T} N(y_t \mid \underset{\sim t}{x}' \underset{\sim}{\beta}+\rho\left(y_{t-1}-\underset{\sim t-1}{x}' \underset{\sim}{\beta}\right),\sigma^2).
\end{aligned}$$

(2.1.4)

2.2 Gibbs Sampling

The Gibbs sampler is an iterative Monte Carlo algorithm designed to extract the posterior distribution from the tractable complete conditional distributions rather than directly from the intractable joint posterior distribution, which is difficult to acquire in explicit form.

In this chapter, the target is to make inferences on the parameters $\{\underset{\sim}{\beta},\mu,\rho,\sigma^2\}$ given the data. We consider the complete conditional distributions $\pi(\underset{\sim}{\beta} \mid .)$, $\pi(\mu \mid .)$, $\pi(\rho \mid .)$, and $\pi(\sigma^2 \mid .)$ respectively. Here, the conditioning argument "·" denotes the observation and the remaining parameters. From the posterior distribution in (2.1.4) we can derive the complete conditional distributions.

First, $\underset{\sim}{\beta} \mid \underset{\sim}{y}, \mu, \rho, \sigma^2 \sim N_K\left((I-\Lambda)\underset{\sim}{\theta} + \Lambda \underset{\sim\beta}{\mu}, \Lambda\Sigma_\beta \right)$, where

$$\underset{\sim\beta}{\mu} = \left[\underset{\sim 1}{x}\underset{\sim 1}{x}' + \sum_{t=2}^{T} \left(\underset{\sim t}{x} - \rho \underset{\sim t-1}{x} \right)\left(\underset{\sim t}{x} - \rho \underset{\sim t-1}{x} \right)' \right]^{-1} \left[(y_1 - \mu)\underset{\sim 1}{x}' - \sum_{t=2}^{T} 2(y_t - \rho y_{t-1})\left(\underset{\sim t}{x} - \rho \underset{\sim t-1}{x} \right)' \right],$$

$$\Sigma_\beta = \underset{\sim 1}{x}\underset{\sim 1}{x}' + \sum_{t=2}^{T} \left(\underset{\sim t}{x} - \rho \underset{\sim t-1}{x} \right)\left(\underset{\sim t}{x} - \rho \underset{\sim t-1}{x} \right)',$$

$$\Lambda = (\Delta^{-1} + \Sigma_\beta^{-1})^{-1}\Sigma_\beta^{-1} = (\Sigma_\beta \Delta^{-1} + \Sigma_\beta \Sigma_\beta^{-1})^{-1} = (\Sigma_\beta \Delta^{-1} + \Delta\Delta^{-1})^{-1} = \Delta(\Sigma_\beta + \Delta)^{-1} \cdot$$

Second, $\mu \mid \underset{\sim}{y}, \underset{\sim}{\beta}, \rho, \sigma^2 \sim N\left((1-\Phi)\mu_0 + \Phi\left(y_1 - \underset{\sim 1}{x}' \underset{\sim}{\beta} \right), \Phi\sigma^2 \right)$, where $\Phi = \dfrac{\sigma_0^2}{\sigma_0^2 + \sigma^2}$.

Third, $\rho \mid \underset{\sim}{y}, \underset{\sim}{\beta}, \mu, \sigma^2 \sim U(-1,1)N\left(\dfrac{\sum_{t=1}^{T-1}\left(y_t - \underset{\sim t}{x}' \underset{\sim}{\beta} \right)\left(y_{t+1} - \underset{\sim t+1}{x}' \underset{\sim}{\beta} \right)}{\sum_{t=1}^{T-1}\left(y_t - \underset{\sim t}{x}' \underset{\sim}{\beta} \right)^2}, \dfrac{\sigma^2}{\sum_{t=1}^{T-1}\left(y_t - \underset{\sim t}{x}' \underset{\sim}{\beta} \right)^2} \right)$.

Fourth,

$$\sigma^2 \mid \underset{\sim}{y}, \underset{\sim}{\beta}, \mu, \rho \sim Inv-gamma\left(a + \frac{T}{2}, b + \frac{\left(y_1 - \underset{\sim 1}{x}' \underset{\sim}{\beta} - \mu \right)^2 + \sum_{t=2}^{T}\left(y_t - \underset{\sim t}{x}' \underset{\sim}{\beta} - \rho\left(y_{t-1} - \underset{\sim t-1}{x}' \underset{\sim}{\beta} \right) \right)^2}{2} \right)$$

The Gibbs sampler is implemented using the following six steps.

Step 1, obtain starting values $\{\underset{\sim}{\beta}^0, \rho^0, \mu^0, \sigma^{2,0}\}$.

Step 2, draw $\underset{\sim}{\beta}^t$ from $\pi(\underset{\sim}{\beta} \mid \mu^{t-1}, \rho^{t-1}, \sigma^{2,t-1}, \underset{\sim}{y})$.

Step 3, draw μ^t from $\pi(\mu \mid \underset{\sim}{\beta}^t, \rho^{t-1}, \sigma^{2,t-1}, \underset{\sim}{y})$.

Step 4, draw ρ^t from $\pi(\rho \mid \underset{\sim}{\beta}^t, \mu^t, \sigma^{2,t-1}, \underset{\sim}{y})$.

Step 5, draw $\sigma^{2,t}$ from $\pi(\sigma^2 \mid \underset{\sim}{\beta}^t, \mu^t, \rho^t, \underset{\sim}{y})$.

Step 6, repeat many many many times.

This paper chooses the starting points $\{\underset{\sim}{\beta}^{0}, \rho^{0}, \mu^{0}, \sigma^{2,0}\}$ as follows.

First, $\underset{\sim}{\beta}^{0} = (X'X)^{-1}X'\underset{\sim}{y}$.

Second, $\rho^{0} = \dfrac{SS_{12}}{\sqrt{SS_{11}SS_{22}}}, SS_{12} = \sum_{t=1}^{T-1}(y_t - \underset{\sim t}{x} B - ave1)(y_{t+1} - \underset{\sim t+1}{x} B - ave2)$, where

$SS_{11} = \sum_{t=1}^{T-1}(y_t - \underset{\sim t}{x} B - ave1)^2, SS_{22} = \sum_{t=2}^{T}(y_t - \underset{\sim t}{x} B - ave2)^2$, and

$$ave1 = \frac{\sum_{t=1}^{T-1}(y_t - \underset{\sim t}{x} B)}{T-1}, ave2 = \frac{\sum_{t=2}^{T}(y_t - \underset{\sim t}{x} B)}{T-1}.$$

Third, $\mu^{0} = T^{-1}\sum_{i=1}^{T}(y_i - \underset{\sim i}{x} B)$.

Fourth, $\sigma^{2,0} = \dfrac{SS_E}{T-p}$.

The ideas of setting β^{0}, μ^{0} and $\sigma^{2,0}$ are the same as the ones of setting the hyper-parameters $\{\underset{\sim}{\theta}_0, \Delta_0, \mu_0, \sigma_0^2\}$ in (2.1.2), which is using the estimation of parameters from the (OLM-1) and (OLM-2). The idea of setting ρ^{0} is taking it as the autocorrelation of time series $v_t = y_t - \underset{\sim t}{x}' \underset{\sim}{\beta} = \rho v_{t-1} + \varepsilon_t$ in an AR(1) structure.

This chapter develops two algorithms using Markov-chain Monte Carlo methods, a restricted algorithm that enforces stationarity condition by letting $|\rho| < 1$ on the series and an unrestricted algorithm that does not.

2.3 Forecast

After getting the posterior distribution of the parameters, we can use it to predict the future stock prices. In this paper we wish to forecast the stock price at time period T + 1 denoted by y_{T+1}, given the data $y_{(T)} = (y_1, y_2, \cdots, y_T)$. Letting $\Omega = \{\underset{\sim}{\beta}, \mu, \rho, \sigma^2\}$, the prediction can be sampled from the posterior predictive distribution

$$f(y_{T+1} \mid y_{(T)}) = \int f(y_{T+1}, \Omega \mid y) \pi(\Omega \mid y) d\Omega. \qquad (2.3.1)$$

Letting $\Omega^{(1)}, \Omega^{(2)}, \ldots, \Omega^{(M)}$ be a sequence of range M from the Gibbs sampler, an estimator of $f(y_{T+1} \mid y_{(T)})$ is

$$\hat{f}(y_{T+1} \mid y_{(T)}) = M^{-1} \sum_{h=1}^{M} f(y_{T+1} \mid \Omega^{(h)}, y_{(T)}). \qquad (2.3.2)$$

To get samples of y_{T+1}, we use data argumentation to fill in y_{T+1} to each $\Omega^{(h)}$, $h = 1, 2, \cdots, M$, to get $y_{T+1}^{(h)}$, $h = 1, 2, \cdots, M$, from the normal distribution in (2.3.3).

$$y_{T+1} \mid \Omega, y_{(T)} \sim N(\underset{\sim T+1}{x'} \underset{\sim}{\beta} + \rho(y_T - \underset{\sim T}{x'} \underset{\sim}{\beta}), \sigma^2). \qquad (2.3.3)$$

The 95% predictive credible interval for y_{T+1} can be computed from the 2.5% and 97.5% empirical quantiles of the values $y_{T+1}^{(h)}$, $h = 1, 2, \cdots, M$.

2.4 Conditional Predictive Ordinate

We want to assess the goodness of fit of the Ohlson Model to the data. One procedure is to calculate the log conditional predictive ordinate $\log(p(y_{t+1} \mid y_{(t)}))$ with

$$p(y_{t+1} \mid y_{(t)}) \approx \sum_{h=1}^{M} \varpi_t^{(h)} p(y_{t+1} \mid \Omega^{(h)}, y_{(t)}), \qquad (2.4.1)$$

where y_{t+1} denotes the random future observation at period t + 1, $y_{(t)} = (y_1, y_2, \cdots, y_t)$ denotes the observations from period 1 to t, $\Omega^{(h)}$ denotes the h[th] draw of the parameters from the Gibbs sampler, and

$$\varpi_t^{(h)} = \frac{\dfrac{f(y_{(t)} \mid \Omega^{(h)})}{f(y \mid \Omega^{(h)})}}{\displaystyle\sum_{k=1}^{M} \dfrac{f(y_{(t)} \mid \Omega^{(h)})}{f(y \mid \Omega^{(h)})}}, \quad h = 1, \cdots, M.$$

(See Appendix G for the derivation of (2.4.1)).

2.5 Empirical Results of Individual Bayesian Analysis

After getting the Bayesian version of the Ohlson model for each firm, we fit it to the data corresponding to each company in the base data set. 11000 iterations are run in the Gibbs sampler, the first 1000 draws are thrown away, and finally 1000 draws are collected by picking one draw every 10 paces. Since there are too many companies (391), the results are averaged for each GIC group. Besides making conclusions from the empirical results, this chapter also tries to decide which models from {OFM --- AR(1), log trans of OFM AR(1), curt trans of OFM AR(1)} will be used for further Bayesian analysis, whether the stationary restriction is needed and which measurement scale to use, original one or the transformed one.

Four criteria are used for the model valuation: the relative difference of the predicted stock price over the real stock price (R), numbers of nonnegative ratios and negative ratios (No.(+,0) and No.(-,0)), length of 95% credible intervals, and log conditional predictive ordinate (CPO). The ratio of residual is defined in the same way as (1.3.1) in Chapter 1. But in this chapter and the following two chapters, No.(+,0) and No.(-,0) denote the rounded numbers of nonnegative and negative ratios divided by 1000 respectively.

The quantiles of R as well as No.(+,0) and No.(-,0) are collected in Table 2.5.1(a) while using the stationary restriction, and in Table 2.5.1 (b) for the case without the stationary restriction.

The LB and UB in these two tables are calculated from No.(+,0) and No.(-,0) by formulas:

$LB = \hat{p} - \sqrt{\hat{p}(1-\hat{p})/N}, UB = \hat{p} + \sqrt{\hat{p}(1-\hat{p})/N}$, where $\hat{p} = No.(+,0)/N$,

$N = No.(+,0) + No.(-)$. They are the lower bound (LB) and upper bound (UB) of the 95% confidence interval of $\hat{p}$ which are used to check the state of overestimation. If 0.5 is between LB and UB, then the method does not overestimate the stock prices, and vice versa.

Table 2.5.1(a) --- With Stationary Restriction (1-Original Scale, 2-Transformed Scale)

GIC	No. of Firms	Method	Min	Q1	Q2	Q3	Max	No. (+,0)	No. (-)	LB	UB
0	391	no trans	-25.141	-0.109	0.070	0.289	19.290	236	155	0.579	0.628
		log trans-1	-0.963	-0.101	0.069	0.274	23.080	237	154	0.581	0.631
		log trans-2	-7.275	-0.032	0.020	0.075	3.094	237	154	0.581	0.631
		curt trans-1	-1.094	-0.105	0.065	0.269	14.541	235	156	0.576	0.626
		curt trans-2	-1.455	-0.035	0.022	0.084	1.497	237	154	0.581	0.631
1	284	no trans	-25.141	-0.114	0.077	0.308	19.290	172	112	0.577	0.635
		log trans-1	-0.963	-0.105	0.075	0.289	23.080	173	111	0.580	0.638
		log trans-2	-7.275	-0.034	0.022	0.079	3.094	173	111	0.580	0.638
		curt trans-1	-1.094	-0.110	0.071	0.285	1.493	171	113	0.573	0.631
		curt trans-2	-1.455	-0.037	0.024	0.088	1.497	173	111	0.580	0.638
2	42	no trans	-12.948	-0.152	0.027	0.237	12.170	23	19	0.471	0.624
		log trans-1	-0.938	-0.147	0.017	0.211	15.154	22	20	0.447	0.601
		log trans-2	-2.891	-0.048	0.005	0.061	2.226	22	20	0.447	0.601
		curt trans-1	-1.018	-0.150	0.017	0.211	11.854	22	20	0.447	0.601
		curt trans-2	-1.264	-0.052	0.007	0.067	1.344	22	20	0.447	0.601
3	5	no trans	-1.292	-0.025	0.089	0.215	1.239	4	1	0.621	0.979
		log trans-1	-0.652	-0.042	0.072	0.207	2.099	3	2	0.381	0.819
		log trans-2	-0.315	-0.013	0.021	0.058	0.425	3	2	0.381	0.819
		curt trans-1	-0.752	-0.040	0.073	0.206	1.653	3	2	0.381	0.819
		curt trans-2	-0.371	-0.013	0.025	0.067	0.386	3	2	0.381	0.819
4	29	no trans	-2.004	-0.050	0.080	0.243	2.156	19	10	0.567	0.743
		log trans-1	-0.950	-0.043	0.088	0.251	4.336	19	10	0.567	0.743
		log trans-2	-0.754	-0.013	0.025	0.066	0.485	19	10	0.567	0.743
		curt trans-1	-0.897	-0.049	0.081	0.241	3.118	19	10	0.567	0.743
		curt trans-2	-0.531	-0.015	0.028	0.076	0.605	19	10	0.567	0.743
5	18	no trans	-1.587	-0.061	0.090	0.278	4.695	12	6	0.556	0.778
		log trans-1	-0.089	-0.048	0.089	0.272	5.143	12	6	0.556	0.778
		log trans-2	-0.437	-0.014	0.024	0.067	0.795	12	6	0.556	0.778
		curt trans-1	-0.759	-0.053	0.086	0.270	4.862	12	6	0.556	0.778
		curt trans-2	-0.377	-0.017	0.029	0.084	0.804	12	6	0.556	0.778
6	13	no trans	-8.584	-0.139	0.025	0.244	7.027	7	6	0.400	0.677
		log trans-1	-0.810	-0.119	0.037	0.222	5.635	7	6	0.400	0.677
		log trans-2	-0.882	-0.036	0.011	0.061	0.957	7	6	0.400	0.677
		curt trans-1	-0.994	-0.126	0.032	0.219	4.961	7	6	0.400	0.677
		curt trans-2	-0.818	-0.043	0.012	0.069	0.814	7	6	0.400	0.677

Table 2.5.1(b) --- Without Stationary Restriction (1-Original Scale, 2-Transformed Scale)

GIC	No. of Firms	Method	Min	Q1	Q2	Q3	Max	No. (+,0)	No. (-)	LB	UB
0	391	no trans	-26.845	-0.124	0.060	0.273	16.443	230	161	0.563	0.613
		log trans-1	-0.987	-0.113	0.059	0.260	43.221	231	160	0.566	0.616
		log trans-2	-5.938	-0.036	0.017	0.071	2.744	231	160	0.566	0.616
		curt trans-1	-1.834	-0.119	0.054	0.255	16.583	228	163	0.558	0.608
		curt trans-2	-1.942	-0.040	0.019	0.080	1.602	230	161	0.563	0.613
1	284	no trans	-26.715	-0.120	0.060	0.263	17.286	168	116	0.562	0.621
		log trans-1	-0.975	-0.107	0.063	0.259	33.557	170	114	0.570	0.628
		log trans-2	-5.938	-0.038	0.019	0.075	2.744	169	115	0.566	0.624
		curt trans-1	-1.834	-0.122	0.060	0.269	15.583	167	117	0.559	0.617
		curt trans-2	-1.942	-0.042	0.021	0.084	1.602	168	116	0.562	0.621
2	42	no trans	-4.539	-0.121	0.052	0.245	4.784	24	18	0.495	0.648
		log trans-1	-0.862	-0.111	0.061	0.264	9.311	25	17	0.519	0.671
		log trans-2	-3.640	-0.053	0.002	0.057	1.956	21	21	0.423	0.577
		curt trans-1	-1.039	-0.165	0.005	0.198	9.244	21	21	0.423	0.577
		curt trans-2	-1.339	-0.057	0.003	0.063	1.173	22	19	0.459	0.614
3	5	no trans	-16.924	-0.374	0.038	0.390	64.950	3	2	0.381	0.819
		log trans-1	-0.998	-0.199	0.035	0.280	58.857	3	2	0.381	0.819
		log trans-2	-0.266	-0.167	0.019	0.054	0.507	3	2	0.381	0.819
		curt trans-1	-0.638	-0.052	0.066	0.193	2.071	3	2	0.381	0.819
		curt trans-2	-0.286	-0.016	0.023	0.062	0.455	3	2	0.381	0.819
4	29	no trans	-4.497	-0.097	-0.085	0.313	4.389	18	11	0.531	0.711
		log trans-1	-0.860	-0.098	0.080	0.303	9.166	18	11	0.531	0.711
		log trans-2	-0.467	-0.016	0.024	0.067	0.795	19	10	0.567	0.743
		curt trans-1	-0.907	-0.057	0.078	0.238	2.799	19	10	0.567	0.743
		curt trans-2	-0.546	-0.018	0.026	0.075	0.562	19	10	0.567	0.743
5	18	no trans	-4.912	-0.137	0.083	0.322	4.025	11	7	0.496	0.726
		log trans-1	-0.994	-0.111	0.077	0.299	10.331	11	7	0.496	0.726
		log trans-2	-0.387	-0.018	0.021	0.065	0.631	12	6	0.556	0.778
		curt trans-1	-0.836	-0.069	0.074	0.258	3.297	11	7	0.496	0.726
		curt trans-2	-0.452	-0.022	0.025	0.081	0.627	11	7	0.496	0.726
6	13	no trans	-5.892	-0.199	0.044	0.286	4.361	7	6	0.400	0.677
		log trans-1	-0.851	-0.152	0.044	0.260	9.485	7	6	0.400	0.677
		log trans-2	-0.774	-0.047	0.003	0.056	0.773	7	6	0.400	0.677
		curt trans-1	-0.938	-0.164	0.001	0.192	3.719	7	6	0.400	0.677
		curt trans-2	-0.605	-0.057	0.002	0.062	0.678	7	6	0.400	0.677

Similar to Chapter 1, the empirical results show that the distributions of R are asymmetrical with long tails. This chapter also uses the 50% quantile (Q2) of R as a major criterion. The following conclusions can be drawn from Table 2.5.1(a).

- Based on Q2 values in original scale, the ratio value ranges from 2.5% (GIC = 6) to 9% (GIC = 5) and 7% overall (GIC = 0) under no transformation, from 1.7% (GIC = 2) to 8.9% (GIC = 5) and 6.9% overall (GIC = 0) under log transformation, and from 1.7% (GIC = 2) to 8.6% (GIC = 5) and 6.5% overall (GIC = 0) under cubic root transformation. Based on Q2 values in transformed scale, the ratio value ranges from 0.5% (GIC = 2) to 2.5% (GIC = 4) and 2% overall (GIC = 0) under log transformation, and from 0.7% (GIC = 2) to 2.9% (GIC = 5) and 2.2% overall (GIC = 0) under cubic root transformation. These conclude that with stationary restriction, both the log transformation and the cubic root transformation improve the predictive ability comparing to the method without using any transformation

- When GIC is 0, 1, or 4, 0.5 is not between LB and UB; when GIC is 3 or 6, 0.5 is not between LB and UB; when GIC is 2, 0.5 is between LB and UB except in the case of using log transformation under the original scale; when GIC is 5, 0.5 is between LB and UB except in the case of using log transformation under the transformed scale. Since 0.5 is not between LB and UB for large groups, we conclude that using Bayesian method to each company by restricting stationarity overestimates the stock prices.

Table 2.5.1(b) gives the following conclusions.

- Based on Q2 values in original scale, the ratio value ranges from 3.8% (GIC = 3) to -8.5% (GIC = 4) and 6% overall (GIC = 0) under no transformation, from 3.5% (GIC = 3) to 8% (GIC = 4) and 5.9% overall (GIC = 0) under log transformation, and from 0.1% (GIC = 6) to 7.8% (GIC = 4) and 5.4% overall (GIC = 0) under cubic root transformation. Based on Q2 values in transformed scale, the ratio value ranges from 0.2% (GIC = 2) to 2.4% (GIC = 4) and 1.7% overall (GIC = 0) under log transformation, and from 0.2% (GIC = 6) to 2.6% (GIC = 4) and 1.9% overall (GIC = 0) under cubic root transformation. These conclude that both the log transformation and the cubic root transformation also improve the predictive ability comparing to the method without using any transformation without stationary restriction.
- When GIC is 0, 1, or 4, 0.5 is not between LB and UB; when GIC is 3 or 6, 0.5 is not between LB and UB; when GIC is 2, 0.5 is between LB and UB except in the case of using log transformation under the original scale; when GIC is 5, 0.5 is between LB and UB except in the case of using log transformation under the transformed scale. Since 0.5 is not between LB and UB for large groups, we

conclude that using Bayesian method to each company by restricting stationarity overestimates the stock prices.

Comparing the conclusions from Table 2.5.1(a) to the ones from Table 2.5.1(b), there exist some slight differences between them, but this paper considers that those differences are minor. There are two things in common. First, using both transformations can enhance the predictive ability. Second, the Bayesian approach to each company overestimates the stock prices for most companies.

Table 2.5.2 --- Min, Max and Overall Values of R

Method	Transformation	min	max	overall
Classical	no trans	24.20%	44%	40%
Statistical	log trans	17.40%	36.50%	34.70%
Analysis	curt trans	19.70%	46.60%	36%
Individual	no trans	2.50%	9%	7%
Bayesian	log trans	1.70%	8.90%	6.90%
Analysis	curt trans	1.70%	8.60%	6.50%

This paper uses the minimum, maximum and overall values of R to compare the Bayesian approaches with the classical method. Table 2.5.2 gives those values under the original scale from both classical statistical analysis and individual Bayesian analysis. It shows the huge improvement of using individual Bayesian approach to the Ohlson model, compared to the classical method.

The average lengths of credible interval (CI) are in Table 2.5.3, from which it is easy to see that they are shorter under stationary restriction than without stationary restriction. In all case, GIC 3 has the longest length, GIC 0 and 1 have the shortest length. GIC 2, 4, 5 6 have similar length. It seems that the more companies a GIC group has, the shorter the CI is. This hints that pooling information across companies may improve the predictive ability of the Ohlson model. Generally, the average lengths of CI's are quite wide under the original scale, and extremely smaller under the transformed scale. The standard deviations are very big under the original scale and much smaller under the transformed scale. This implies that the results under the transformed scale make more sense, which can also been indicated by Table 2.5.1(a) and (b).

Table 2.5.3 --- Average Length of Credible Interval

GIC	No. of Firms	Method	With S-Restriction Ave. CI Length	With S-Restriction Std Dev	No S-Restriction Ave. CI Length	No S-Restriction Std Dev
0	391	no trans	4.699	13.321	4.786	13.573
		log trans-1	4.176	12.870	4.190	12.839
		log trans-2	0.949	0.460	0.966	0.472
		curt trans-1	26.528	13.521	26.690	13.702
		curt trans-2	0.958	0.425	0.975	0.434
1	284	no trans	5.017	13.269	5.073	13.547
		log trans-1	4.401	12.240	4.436	12.626
		log trans-2	1.003	0.486	1.020	0.499
		curt trans-1	27.455	14.295	27.577	14.496
		curt trans-2	1.006	0.456	1.023	0.466
2	42	no trans	17.537	20.441	17.762	20.614
		log trans-1	16.245	21.381	16.441	22.334
		log trans-2	0.900	0.464	0.908	0.469
		curt trans-1	20.679	8.734	20.737	9.150
		curt trans-2	0.866	0.317	0.878	0.326
3	5	no trans	112.275	120.773	117.649	125.112
		log trans-1	99.762	168.683	67.112	94.920
		log trans-2	0.699	0.195	0.732	0.203
		curt trans-1	19.146	6.832	19.944	7.135
		curt trans-2	0.682	0.134	0.715	0.147
4	29	no trans	25.364	18.994	25.895	19.231
		log trans-1	23.892	21.478	24.087	25.522
		log trans-2	0.652	0.184	0.674	0.197
		curt trans-1	22.107	9.685	22.554	9.876
		curt trans-2	0.686	0.223	0.707	0.237
5	18	no trans	34.002	12.296	34.225	12.120
		log trans-1	29.073	10.210	28.705	10.014
		log trans-2	0.764	0.173	0.803	0.205
		curt trans-1	31.286	10.909	32.207	11.431
		curt trans-2	0.883	0.226	0.923	0.249
6	13	no trans	35.106	14.488	35.659	14.415
		log trans-1	27.712	10.793	27.485	10.737
		log trans-2	0.941	0.295	0.950	0.320
		curt trans-1	31.271	13.805	30.732	12.817
		curt trans-2	1.012	0.326	1.030	0.329

The log conditional predictive ordinate (CPO) is to evaluate the model fitting adequacy. It always has negative values and is calculated under the original scale in this chapter. The bigger CPO is, the better the model fits the data.

Table 2.5.4 gives the quantiles and mean of CPO in each case and shows that the CPO values are smaller under stationary restriction than without stationary restriction.

Table 2.5.4 --- CPO

With Stationary Restriction								
GIC	No. of Firms	Method	Min	Q1	Q2	Q3	Max	Mean
0	391	no trans	-5.161	-3.800	-3.462	-3.066	-2.289	-3.444
		log trans	-5.550	-3.779	-3.449	-3.087	-2.243	-3.432
		curt trans	-4.760	-2.550	-2.272	-1.982	-1.309	-2.267
1	284	no trans	-5.578	-3.774	-3.442	-3.075	-2.260	-3.432
		log trans	-4.704	-3.769	-3.445	-3.079	-2.242	-3.417
		curt trans	-3.677	-2.583	-2.290	-1.995	-1.309	-2.285
2	42	no trans	-4.174	-3.789	-3.541	-3.056	-2.338	-3.424
		log trans	-4.121	-3.791	-3.578	-3.094	-2.291	-3.440
		curt trans	-2.995	-2.468	-2.202	-1.969	-1.373	-2.204
3	5	no trans	-8.450	-6.427	-6.077	-4.088	-3.907	-5.790
		log trans	-6.963	-6.625	-4.399	-3.976	-3.756	-5.144
		curt trans	-2.278	-2.208	-2.014	-1.715	-1.674	-1.978
4	29	no trans	-4.160	-3.778	-3.567	-3.330	-2.353	-3.494
		log trans	-5.618	-3.844	-3.598	-3.167	-2.592	-3.632
		curt trans	-2.924	-2.199	-2.040	-1.757	-1.392	-2.024
5	18	no trans	-5.832	-3.922	-3.730	-3.506	-2.977	-3.794
		log trans	-5.128	-4.304	-3.767	-3.412	-3.002	-3.874
		curt trans	-2.871	-2.527	-2.390	-2.125	-1.765	-2.339
6	13	no trans	-5.133	-3.995	-3.853	-3.430	-2.972	-3.832
		log trans	-5.318	-3.987	-3.736	-3.335	-3.016	-3.833
		curt trans	-4.760	-2.715	-2.488	-2.249	-1.801	-2.639
Without Stationary Restriction								
GIC	No. of Firms	Method	Min	Q1	Q2	Q3	Max	Mean
0	391	no trans	-5.272	-3.781	-3.489	-3.092	-2.244	-3.456
		log trans	-5.294	-3.800	-3.470	-3.096	-2.161	-3.435
		curt trans	-4.262	-2.606	-2.306	-2.027	-1.305	-2.311
1	284	no trans	-5.214	-3.753	-3.481	-3.100	-2.244	-3.440
		log trans	-5.146	-3.759	-3.474	-3.092	-2.244	-3.424
		curt trans	-2.715	-2.456	-2.246	-1.978	-1.361	-2.185
2	42	no trans	-4.286	-3.855	-3.517	-3.123	-2.247	-3.423
		log trans	-5.349	-3.909	-3.558	-3.108	-2.320	-3.521
		curt trans	-2.715	-2.456	-2.246	-1.978	-1.361	-2.185
3	5	no trans	-9.191	-6.379	-4.343	-4.173	-4.078	-5.632
		log trans	-7.410	-5.523	-4.514	-3.863	-3.595	-4.981
		curt trans	-2.317	-2.164	-1.930	-1.828	-1.622	-1.972
4	29	no trans	-4.268	-3.905	-3.625	-3.243	-2.382	-3.503
		log trans	-4.460	-3.964	-3.574	-3.253	-2.623	-3.546
		curt trans	-3.158	-2.253	-2.093	-1.869	-1.377	-2.077
5	18	no trans	-4.284	-4.005	-3.686	-3.451	-3.015	-3.696
		log trans	-5.453	-4.019	-3.799	-3.419	-3.013	-3.815
		curt trans	-2.887	-2.545	-2.367	-2.196	-1.806	-2.341
6	13	no trans	-4.815	-4.023	-3.903	-3.573	-3.108	-3.836
		log trans	-5.072	-4.046	-3.925	-3.275	-3.055	-3.765
		curt trans	-3.081	-2.669	-2.449	-2.278	-1.603	-2.446

Since the distributions of CPO are quite symmetrical without long tails, this chapter uses the mean value as a major criterion to analyze CPO. The following conclusions are for the case with stationary restriction.

- Under no transformation, the mean value of CPO ranges from -5.709 (GIC = 3) to -3.424 (GIC = 2) and -3.444 overall (GIC = 0).
- Under log transformation, the mean value of CPO ranges from -5.144 (GIC = 3) to -3.417 (GIC = 1) and -3.432 overall (GIC = 0).
- Under cubic root transformation, the mean value of CPO ranges from -2.639 (GIC = 6) to -1.978 (GIC = 3) and -2.267 overall (GIC = 0).
- For each GIC group, the mean values of CPO are much larger under cubic root transformation than under log transformation. Also, they have smaller values under no transformation than under either of the two transformations. These facts put a lot weight on using both log and cubic root transformations in the Bayesian analysis of the Ohlson model.

After analyzing the empirical results of the four criteria, this paper goes back to the decisions that need to make. Since all criteria show the improvement of using log transformation and cubic root transformation, this paper decides to use two models for further Bayesian analysis, which are log trans of OFM AR(1) and curt trans of OFM AR(1).

Since using the transformed scale shows more sensible results, this paper decide to keep it and stop using the original scale in the following two chapters.

About the restriction stationarity, Nandram & Petruccelli (1997) states that restricting stationary series to be stationary provides no new information, but restricting nonstationary series to be stationary leads to substantial differences from the unrestricted case. In this paper, the results of both average lengths of the credible intervals and CPO show the benefits of restricting stationarity. Besides, the time plots of the time series for the companies show that most series look stationary and a few do not. Therefore, this paper decides to use the stationary restriction in further Bayesian analysis.

In summary, the individual Bayesian analysis in this chapter strongly improves the predictive ability of the Ohlson model comparing to the classical analysis in Chapter 1. The grouping analysis in Chapter 3 and adaptive analysis by pooling information across companies in Chapter 4 will be applied to the two transformed models with stationary restriction. Most further results will be collected under the transformed scale.

Chapter 3

Bayesian Data Analysis within Each GIC Group

3.1 Bayesian Version of the Ohlson Model for a GIC Group

As another extreme case, this chapter assumes that all the companies in the same GIC group share the same parameters $\{\underset{\sim}{\beta}, \mu, \rho, \sigma^2\}$ and the GIC groups are independent of each other, having their own regression coefficients in the Ohlson model. The same structure as in Chapter 2 is used in this chapter. First of all, a Bayesian version of the Ohlson model for the grouping analysis is set up in the following three steps.

Step 1, describe the observation $\underset{\sim}{y} = (y_{11}, y_{12}, \dots, y_{1T}, y_{21}, y_{22}, \dots, y_{2T}, \dots, y_{11}, y_{N1}, \dots, y_{NT})$ by the parameters $\{\underset{\sim}{\beta}, \mu, \rho, \sigma^2\}$, where N is the number of companies in the GIC group, and T is the number of time periods. Under the assumption that the observations are independent among the companies, we can get the following likelihood function:

$$p(\underset{\sim}{y} \mid \underset{\sim}{\beta}, \mu, \rho, \sigma^2)$$

$$= \prod_{k=1}^{N} \left[N(y_{k1} \mid \underset{\sim k1}{x'} \underset{\sim}{\beta} + \mu, \sigma^2) \cdot \prod_{t=2}^{T} N(y_{kt} \mid \underset{\sim kt}{x'} \underset{\sim}{\beta} + \rho \left(y_{k,t-1} - \underset{\sim k,t-1}{x'} \underset{\sim}{\beta} \right), \sigma^2) \right]. \tag{3.1.1}$$

Step 2, assign a prior distribution to each parameter.

$$\begin{aligned} \pi(\underset{\sim}{\beta}) &= N_K(\underset{\sim}{\beta} \mid \underset{\sim 0}{\theta}, \Delta_0), \\ \pi(\mu) &= N(\mu \mid \mu_0, \sigma_0^2), \\ \pi(\rho) &= U(\rho \mid -1, 1), \\ \pi(\sigma^2) &= I\Gamma(\sigma^2 \mid a, b), \end{aligned} \tag{3.1.2}$$

where

(P3.1) $\underset{\sim 0}{\theta} = B = (X'X)^{-1} X' y$, where $X = \left(\underset{\sim 11}{x'}, \underset{\sim 12}{x'}, \cdots, \underset{\sim 1T}{x'}, \cdots, \underset{\sim N1}{x'}, \cdots, \underset{\sim NT}{x'} \right)$.

(P3.2) $\Delta_0 = 100(X'X)^{-1}\frac{SS_E}{NT-P}$, where $SS_E = \underset{\sim}{y}'\underset{\sim}{y} - B'X'\underset{\sim}{y}$, N is the number of companies, T is the number of observations and P is the number of regression coefficients (including the intercept).

(P3.3) $\mu_0 = (NT)^{-1}\sum_{j=1}^{N}\sum_{i=1}^{T}(y_{ij} - \underset{\sim}{x}_{ij}B)$.

(P3.4) $\sigma_0^2 = \frac{SS_E}{NT-p}$.

(P3.5) $a = b = 0.001$.

The ideas in choosing those hyperparameters $\{\underset{\sim}{\theta}_0, \Delta_0, \mu_0, \sigma_0^2\}$ are the same as the ideas in choosing the hyperparameters in Chapter 2. That is, use the estimation of parameters from two ordinary linear regression models: (OLM-3) and (OLM-4).

$$y_{it} = x_{it}'\underset{\sim}{\beta} + \varepsilon_{it}, \varepsilon_{it} \overset{iid}{\sim} N(0,\sigma^2), i = 1,\cdots,N; t = 1,\cdots,T. \qquad \text{(OLM-3)}$$

$$\mu = y_{it} - x_{it}'\underset{\sim}{\beta} + \varepsilon_{it}, \varepsilon_{it} \sim N(0,\sigma^2), i = 1,2,\cdots,N; t = 1,2,\cdots,T. \qquad \text{(OLM-4)}$$

Assume that all the parameters are independent of each other, the joint distribution of the parameters can be expressed as

$$p(\underset{\sim}{\beta},\mu,\rho,\sigma^2) = N_K(\underset{\sim}{\beta} \mid \underset{\sim}{\theta}_0, \Delta_0)\cdot N(\mu \mid \mu_0, \sigma_0^2)\cdot U(\rho \mid -1,1),\cdot I\Gamma(\sigma^2 \mid a,b). \qquad (3.1.3)$$

Step 3, from the likelihood function in (3.1.1) and joint prior distribution in (3.1.3), we can get the posterior distribution of the parameters given the data by Bayes' rule:

$$\begin{aligned} &p(\underset{\sim}{\beta},\mu,\rho,\sigma^2 \mid \underset{\sim}{y}) \\ &\propto \prod_{i=1}^{N}\left[N(y_{i1} \mid \underset{\sim}{x}_{i1}'\underset{\sim}{\beta} + \mu, \sigma^2)\cdot\prod_{t=2}^{T}N\left(y_{it} \mid \underset{\sim}{x}_{it}'\underset{\sim}{\beta} + \rho\left(y_{i,t-1} - \underset{\sim}{x}_{i,t-1}'\underset{\sim}{\beta}\right),\sigma^2\right)\right] \\ &\quad \cdot N_P(\underset{\sim}{\beta} \mid \underset{\sim}{\theta}, \Delta)N(\mu \mid \mu_0, \sigma_0^2)U(\rho \mid -1,1)I\Gamma(\sigma^2 \mid a,b) \end{aligned} \qquad (3.1.4)$$

3.2 Gibbs Sampling

The process of applying Gibbs sampler in this chapter is the same as in Chapter 2. Without repeating the steps, this section only specifies the complete conditional distributions for the parameters and the starting points for each parameter.

The complete conditional distributions for the parameters are as follows.

First, $\underset{\sim}{\beta} \mid \underset{\sim}{y}, \mu, \rho, \sigma^2 \sim N_P\left(\underset{\sim}{\beta} \mid (I-\Lambda)\underset{\sim}{\theta} + \Lambda \underset{\sim}{\mu}_{\beta}, \Lambda\Sigma_\beta\right)$, where

$$\underset{\sim}{\mu}_{\beta} = \left[\sum_{i=1}^{N}\left(\underset{\sim}{x}_{i1}\underset{\sim}{x}_{i1}^{'} + \sum_{t=2}^{T}\left(\underset{\sim}{x}_{it} - \rho\,\underset{\sim}{x}_{i,t-1}\right)\left(\underset{\sim}{x}_{kt} - \rho\,\underset{\sim}{x}_{k,t-1}\right)^{'}\right)\right]^{-1}$$

$$\cdot\left[\sum_{i=1}^{N}\left((y_{i1}-\mu)\underset{\sim}{x}_{i1}^{'} - \sum_{t=2}^{T}2(y_{it}-\rho y_{i,t-1})\left(y_{it}-\rho y_{i,t-1}\right)^{'}\right)\right],$$

$$\Sigma_\beta = \sum_{i=1}^{N}\left(\underset{\sim}{x}_{i1}\underset{\sim}{x}_{i1}^{'} + \sum_{t=2}^{T}\left(\underset{\sim}{x}_{it} - \rho\,\underset{\sim}{x}_{i,t-1}\right)\left(\underset{\sim}{x}_{it} - \rho\,\underset{\sim}{x}_{i,t-1}\right)^{'}\right),$$

$$\Lambda = (\Delta^{-1} + \Sigma_\beta^{-1})^{-1}\Sigma_\beta^{-1} = (\Sigma_\beta\Delta^{-1} + \Sigma_\beta\Sigma_\beta^{-1})^{-1} = (\Sigma_\beta\Delta^{-1} + \Delta\Delta^{-1})^{-1} = \Delta(\Sigma_\beta + \Delta)^{-1}.$$

Second, $\mu \mid \underset{\sim}{y}, \underset{\sim}{\beta}, \rho, \sigma^2 \sim N\left(\mu \,\middle|\, (1-\Phi)\mu_0 + \Phi\sum_{i=1}^{N}\left(y_{i1} - \underset{\sim}{x}_{i1}^{'}\underset{\sim}{\beta}\right), \Phi\sigma^2\right)$, where $\Phi = \dfrac{\sigma_0^2}{\sigma_0^2 + \sigma^2}$.

Third,

$$\rho \mid \underset{\sim}{y}, \underset{\sim}{\beta}, \mu, \sigma^2 \sim U(\rho \mid -1,1)N\left(\frac{\sum_{i=1}^{N}\sum_{t=2}^{T}\left(y_{it} - \underset{\sim}{x}_{it}^{'}\underset{\sim}{\beta}\right)\left(y_{i,t-1} - \underset{\sim}{x}_{i,t-1}^{'}\underset{\sim}{\beta}\right)}{\sum_{i=1}^{N}\sum_{t=2}^{T}\left(y_{i,t-1} - \underset{\sim}{x}_{i,t-1}^{'}\underset{\sim}{\beta}\right)^2}, \frac{\sigma^2}{\sum_{i=1}^{N}\sum_{t=2}^{T}\left(y_{i,t-1} - \underset{\sim}{x}_{i,t-1}^{'}\underset{\sim}{\beta}\right)^2}\right)$$

Fourth, $\sigma^2 \mid \underset{\sim}{y}, \underset{\sim}{\beta}, \mu, \rho \sim I\Gamma\left(a + \dfrac{TN}{2}, b + \dfrac{S}{2}\right)$, where

$$S = \sum_{i=1}^{N}\left(y_{i1} - \underset{\sim}{x}_{i1}^{'}\underset{\sim}{\beta} - \mu\right)^2 + \sum_{i=1}^{N}\sum_{t=2}^{T}\left(y_{it} - \underset{\sim}{x}_{it}^{'}\underset{\sim}{\beta} - \rho\left(y_{i,t-1} - \underset{\sim}{x}_{i,t-1}^{'}\underset{\sim}{\beta}\right)\right)^2.$$

This chapter chooses the starting points $(\beta^0, \rho^0, \mu^0, \sigma^{2,0})$ as follows.

First, $\underset{\sim}{\beta}^0 = (X'X)^{-1}X'\underset{\sim}{y}$.

Second, $\rho^0 = \dfrac{SS_{12}}{\sqrt{SS_{11}SS_{22}}}, SS_{12} = \sum_{i=1}^{N}\sum_{t=1}^{T-1}(y_{it} - \underset{\sim it}{x} B - ave1)(y_{i,t+1} - \underset{\sim i,t+1}{x} B - ave2)$, where

$$SS_{11} = \sum_{i=1}^{N}\sum_{t=1}^{T-1}(y_{it} - \underset{\sim it}{x} B - ave1)^2,$$

$$SS_{22} = \sum_{i-1}^{N}\sum_{t=2}^{T}(y_{it} - \underset{\sim it}{x} B - ave2)^2,$$

$$ave1 = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T-1}(y_t - \underset{\sim t}{x} B)}{N(T-1)}, ave2 = \frac{\sum_{i=1}^{N}\sum_{t=2}^{T}(y_t - \underset{\sim t}{x} B)}{N(T-1)}.$$

Fourth, $\mu^0 = (NT)^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it} - \underset{\sim it}{x} B)$.

Fifth, $\sigma^{2,0} = \dfrac{SS_E}{NT - p}$.

The ideas of setting β^0, μ^0 and $\sigma^{2,0}$ are the same as the ones of setting the hyper-parameters $\{\underset{\sim 0}{\theta}, \Delta_0, \mu_0, \sigma_0^2\}$ in (3.1.2), which is using the estimation of parameters from the (OLM-3) and (OLM-4). The idea of setting ρ^0 is taking it as the autocorrelation of time series $v_{it} = y_{it} - \underset{\sim it}{x}' \underset{\sim}{\beta} = \rho v_{i,t-1} + \varepsilon_{it}$ in an AR(1) structure.

3.3 Forecast

After getting the posterior distribution of the parameters, we can use it to predict the future stock prices at period T + 1 for each firm in the group $y_{i,T+1}$, $i = 1, 2, \ldots, N$, given the data $y_{(iT)} = (y_{i1}, y_{i2}, \cdots, y_{iT}),\ i = 1, 2, \ldots, N$. Letting $\Omega = \{\underset{\sim}{\beta}, \mu, \rho, \sigma^2\}$, the predictions can be sampled from the posterior predictive distribution

$$f(y_{i,T+1} \mid y_{(iT)}) = \int f(y_{i,T+1}, \Omega \mid y)\pi(\Omega \mid y)d\Omega. \qquad (3.3.1)$$

Letting $\Omega^{(1)}, \Omega^{(2)}, \ldots, \Omega^{(M)}$ be a sequence of range M from the Gibbs sampler, an estimator of $f(y_{i,T+1} \mid y_{(iT)})$ is

$$\hat{f}(y_{i,T+1} \mid y_{(iT)}) = M^{-1}\sum_{h=1}^{M} f(y_{i,T+1} \mid \Omega^{(h)}, y_{(iT)}). \tag{3.3.2}$$

To get samples of $y_{i,T+1}$, we use data argumentation to fill in $y_{i,T+1}$ to each $\Omega^{(h)}$, $h = 1, 2, \cdots, M$ to get $y_{i,T+1}^{(h)}$, $h = 1, 2, \cdots, M$ from the normal distribution described below.

$$y_{i,T+1} \mid \Omega, y_{(iT)} \sim N(\underset{\sim}{x}'_{i,T+1}\underset{\sim}{\beta} + \rho(y_{iT} - \underset{\sim}{x}'_{iT}\underset{\sim}{\beta}), \sigma^2) \tag{3.3.3}$$

The 95% predictive credible interval for $y_{i,T+1}$ can be computed from the 2.5% and 97.5% empirical quantiles of the values $y_{i,T+1}^{(h)}$, $h = 1, 2, \cdots, M$.

3.4 Conditional Predictive Ordinate

In this chapter, the conditional predictive ordinate is defined as

$$\begin{gathered} p(y_{i,t+1} \mid y_{(it)}) \approx \sum_{h=1}^{M} \varpi_{it}^{(h)} p(y_{i,t+1} \mid \Omega^{(h)}, y_{(it)}), \\ i = 1,2,\ldots,N, t = 1,2,\ldots T, \end{gathered} \tag{3.4.1}$$

where $y_{i,t+1}$ denotes the random future observation of company i at period t + 1, $y_{(it)} = (y_{i1}, y_{i2}, \cdots, y_{it})$ denotes the observations of company i from period 1 to t, $\Omega^{(h)}$ denotes the h$^{\text{th}}$ draw of the parameters from the Gibbs sampler, and

$$\varpi_{it}^{(h)} = \frac{\dfrac{f(y_{(it)} \mid \Omega^{(h)})}{f(y \mid \Omega^{(h)})}}{\sum_{k=1}^{M} \dfrac{f(y_{(it)} \mid \Omega^{(h)})}{f(y \mid \Omega^{(h)})}}, \quad h = 1, \cdots, M.$$

3.5 Empirical Results of Grouping Bayesian Analysis

The same criteria as in Chapter 2 are used for the model valuation in this chapter. Table 3.5.1 shows the quantiles of R under the transformed scale as well as numbers of positive ratios and negative ratios averaged in each GIC group with stationary restriction, from which we can draw the following conclusions. In order to be consistent with Chapter 2, Q2 is used as a major criterion in analyzing R .

- Based on Q2, the ratio value ranges from -0.7% (GIC = 1) to 2.3% (GIC = 5) and 0.4% over all (GIC = 0) under log transformation and from -0.7% (GIC = 1) to 3% (GIC = 5) and 0.8% over all (GIC = 0) under cubic root transformation.
- Based on Q2, for the same GIC group, the ratio under log transformation is no bigger than under cubic root transformation. This implies that log transformation is better for group analysis.
- Under both transformations, the numbers of positive ratios and negative ratios for each group are very close and 0.5 is between LB and UB, which indicates that both transformations do not overestimate the stock prices. The only exception is in the case with GIC equal to 0 under cubic root transformation.

Table 3.5.1 --- Quantiles of Ratio & Numbers of Nonnegative/Negative Ratios

With Stationary Restriction											
GIC	No. of Firms	Method	Min	Q1	Q2	Q3	Max	No. (+,0)	No. (-)	LB	UB
0	391	log trans	-2.728	-0.074	0.004	0.083	2.220	200	191	0.486	0.537
		curt trans	-1.082	-0.073	0.008	0.092	1.386	206	185	0.502	0.552
1	284	log trans	-2.678	-0.072	0.000	0.076	2.090	142	142	0.470	0.530
		curt trans	-0.899	-0.073	0.004	0.085	1.179	146	138	0.484	0.544
2	42	log trans	-0.520	-0.073	-0.007	0.061	0.689	20	22	0.399	0.553
		curt trans	-0.499	-0.078	-0.007	0.067	0.605	20	22	0.399	0.553
3	5	log trans	-2.217	-0.198	0.008	0.218	2.064	3	2	0.381	0.819
		curt trans	-2.078	-0.193	0.014	0.223	1.820	3	2	0.381	0.819
4	29	log trans	-0.831	-0.096	0.011	0.120	1.508	15	14	0.424	0.610
		curt trans	-0.882	-0.103	0.013	0.131	1.381	15	14	0.424	0.610
5	18	log trans	-0.804	-0.075	0.023	0.124	1.112	10	8	0.438	0.673
		curt trans	-0.911	-0.076	0.030	0.141	1.174	10	8	0.438	0.673
6	13	log trans	-1.617	-0.158	0.022	0.206	2.270	7	6	0.400	0.677
		curt trans	-1.678	-0.166	0.026	0.221	2.089	7	6	0.400	0.677

Table 3.5.2 gives minimum, maximum and overall values of R under the transformed scale from both classical statistical analysis and grouping Bayesian analysis. It shows the magnificent improvement of using grouping Bayesian approach to the Ohlson model compared to the classical method.

Table 3.5.2 --- Min, Max and Overall Values of R

Method	Transformation	min	max	overall
Classical Analysis	log trans	4.70%	9.80%	8.80%
	curt trans	6.30%	13.80%	10.90%
Grouping Analysis	log trans	-0.70%	2.30%	0.40%
	curt trans	-0.70%	3%	0.80%

Table 3.5.3 --- Average Length of Credible Interval

GIC	No. of Firms	Method	With S-Restriction	
			Ave Length	Std Dev
0	391	log trans	1.241	0.235
		curt trans	1.215	0.228
1	284	log trans	1.168	0.226
		curt trans	1.164	0.222
2	42	log trans	1.071	0.007
		curt trans	1.059	0.007
3	5	log trans	4.040	0.207
		curt trans	3.753	0.195
4	29	log trans	1.988	0.288
		curt trans	1.994	0.292
5	18	log trans	1.880	0.189
		curt trans	1.941	0.198
6	13	log trans	3.446	0.260
		curt trans	3.413	0.264

Table 3.5.3 gives the average length of credible intervals and the corresponding standard deviations for both log transformation and cubic root transformation under the transformed scale and with the stationary restriction, from which we can draw the following conclusions.

- Under log transformation, the average length of CI ranges from 1.071 (GIC = 2) to 4.040 (GIC = 3) and 1.241 overall (GIC = 0), the standard deviation ranges from 0.007 (GIC = 2) to 0.288 (GIC = 4) and 0.235 overall (GIC = 0).
- Under cubic root transformation, the average length of CI ranges from 1.059 (GIC = 2) to 3.753 (GIC = 3) and 1.215 overall (GIC = 0), the standard deviation ranges from 0.007 (GIC = 2) to 0.292 (GIC = 4) and 0.228 overall (GIC = 0).

- For each GIC group, the average length of CI is slightly smaller under cubic root transformation than under log transformation.

Table 3.5.4 --- Conditional Predictive Ordinate (each group has its own parameters)

With Stationary Restriction								
GIC	No. of Firms	Method	Min	Q1	Q2	Q3	Max	Mean
0	391	log trans	-16.440	-4.033	-3.641	-3.286	-2.326	-3.858
		curt trans	-15.338	-2.702	-2.387	-2.187	-1.794	-2.663
1	284	log trans	-15.146	-3.990	-3.630	-3.303	-2.306	-3.818
		curt trans	-14.061	-2.673	-2.358	-2.157	-1.798	-2.622
2	42	log trans	-6.135	-3.755	-3.498	-3.242	-2.372	-3.549
		curt trans	-4.889	-2.431	-2.267	-2.117	-1.713	-2.371
3	5	log trans	-7.589	-5.185	-4.806	-4.670	-4.046	-5.259
		curt trans	-6.127	-3.721	-3.606	-3.531	-3.340	-4.065
4	29	log trans	-8.444	-4.873	-4.485	-3.880	-3.444	-4.646
		curt trans	-7.153	-3.316	-3.044	-2.882	-2.783	-3.431
5	18	log trans	-6.974	-4.438	-4.370	-4.099	-3.348	-4.531
		curt trans	-5.704	-3.145	-2.986	-2.851	-2.679	-3.284
6	13	log trans	-9.195	-5.355	-4.866	-4.426	-3.952	-5.146
		curt trans	-7.768	-3.854	-3.587	-3.475	-3.299	-3.942

Table 3.5.4 gives the quantiles and mean of CPO for both log transformation and cubic root transformation under the original scale with stationary restriction, from which we can draw the following conclusions. As in Chapter 2, the mean value is used as a major criterion to analyze CPO in this chapter.

- Under log transformation, the mean value of CPO ranges from -5.146 (GIC = 3) to -3.284 (GIC = 2) and -3.858 overall (GIC = 0).
- Under cubic root transformation, the mean value of CPO ranges from -4.065 (GIC = 3) to -2.371 (GIC = 2) and -2.663 overall (GIC = 0).
- For each GIC group, the mean of CPO is much larger under cubic root transformation than under log transformation. This indicates that cubic root transformation does a better job for grouping analysis.

For a follow-up analysis, we gather the average CPO's for each group in the case that all companies share the same parameters. We call this "overall analysis". The results are in Table 3.5.5, from which the following conclusions can be drawn.

- Under log transformation, the mean value of CPO ranges from -4.429 (GIC = 2) to -3.401 (GIC = 4).
- Under cubic root transformation, the mean value of CPO ranges from -3.183 (GIC = 2) to -2.176 (GIC = 4).
- For each GIC group, the mean of CPO is much larger under cubic root transformation than under log transformation. This indicates that cubic root transformation does a better job than log transformation for the overall analysis.

Table 3.5.5 --- Conditional Predictive Ordinate (all companies have the same parameters)

With Stationary Restriction								
GIC	No. of Firms	Method	Min	Q1	Q2	Q3	Max	Mean
1	284	log trans	-16.442	-4.002	-3.625	-3.279	-2.326	-3.827
		curt trans	-15.338	-2.692	-2.383	-2.191	-1.794	-2.659
2	42	log trans	-9.295	-4.990	-3.998	-3.560	-2.428	-4.429
		curt trans	-7.924	-3.598	-2.630	-2.338	-1.852	-3.183
3	5	log trans	-4.657	-4.456	-3.812	-3.065	-2.579	-3.714
		curt trans	-3.323	-3.108	-2.568	-2.121	-1.875	-2.599
4	29	log trans	-4.033	-3.783	-3.360	-3.061	-2.495	-3.401
		curt trans	-2.473	-2.327	-2.152	-2.013	-1.915	-2.176
5	18	log trans	-4.437	-4.161	-3.739	-3.568	-3.139	-3.814
		curt trans	-2.943	-2.568	-2.412	-2.265	-2.053	-2.434
6	13	log trans	-4.738	-4.180	-3.855	-3.453	-2.950	-3.822
		curt trans	-3.236	-2.716	-2.498	-2.249	-2.013	-2.508

Table 3.5.6 provides the comparison of the mean of CPO from Table 3.5.4 and Table 3.5.5. This is the comparison between grouping analysis and overall analysis, from which we can conclude that: under both transformations, grouping analysis and overall analysis have almost the same predictive ability within GIC 1, which has the largest number of companies. Group analysis does a better job than overall analysis in GIC 2 and a worse job for the left GIC groups.

Table 3.5.6 --- Comparison of Table 3.5.3 to 3.5.4

log trans of OFM AR(1)-2				
GIC	Mean-3.5.3	Mean-3.5.4	dif	dif/Mean-3.5.3
1	-3.818	-3.827	-0.009	0.002
2	-3.549	-4.429	-0.88	0.248
3	-5.259	-3.714	1.545	0.294
4	-4.646	-3.401	1.245	0.268
5	-4.531	-3.814	0.717	0.158
6	-5.146	-3.822	1.324	0.257
curt trans of OFM AR(1)-2				
GIC	Mean-3.5.3	Mean-3.5.4	dif	dif/Mean-3.5.3
1	-2.622	-2.659	-0.037	0.014
2	-2.371	-3.183	-0.812	0.342
3	-4.065	-2.599	1.466	0.361
4	-3.431	-2.176	1.255	0.366
5	-3.284	-2.434	0.85	0.259
6	-3.942	-2.508	1.434	0.364

Summarily, the grouping Bayesian analysis in this chapter also greatly improves the predictive ability of the Ohlson model comparing to the classical analysis in Chapter 1. It does not overestimate the stock prices under both transformations with stationary restriction, and cubic root transformation is better than log transformation in this case. The cubic root transformation is especially applicable for those GIC groups which have large amount of companies.

Chapter 4

Bayesian Data Analysis by Adaptive Pooling Information Across Firms

4.1 Bayesian Hierarchical Model

The Bayesian approaches in Chapter 2 and Chapter 3 represents two extreme cases. Chapter 2 treats each company individually and does not borrow information across companies at all. Chapter 2 overestimates the stock prices. Chapter 3 supposes all the companies in the same GIC group follow the same rules of predicting stock prices, which brings more information in the investigation. The improvement in Chapter 3 is correcting the bias, but the model is too simple. Furthermore, these two extreme cases are barely seen in the real stock literature where on one side the companies have their own specific characteristics and on the other side they are affected by the same economic factors and therefore have some things in common. As a combination of Chapter 2 and Chapter 3, also in order to be closer to the reality, this chapter develops a hierarchical Bayesian approach is to simultaneously estimate the unknown coefficients for each company by adaptively pooling information across firms.

Considering that all the firms will be included in the Ohlson model, it is required to add the company index into expression (1.1.5) which turns out to be

$$\begin{aligned}
&y_{i1} = \underset{\sim i1}{x}' \underset{\sim i}{\beta} + u_i + \varepsilon_{i1}, i = 1, \cdots, n. \\
&y_{it} = \underset{\sim it}{x}' \underset{\sim i}{\beta} + \rho_i (y_{i,t-1} - \underset{\sim i,t-1}{x}' \underset{\sim i}{\beta}) + \varepsilon_{it}, i = 1, \cdots, n,\ t = 2, \cdots, T. \\
&\varepsilon_{it} \overset{iid}{\sim} N(0, \sigma_i^2),\ i = 1, \cdots, n,\ t = 1, \cdots, T.
\end{aligned} \tag{4.1.1}$$

The Bayesian version of the Ohlson Model in (4.1.1) for a GIC group is set up in four steps.

First, describe the observation $\underset{\sim}{y} = (y_{11}, y_{12}, \ldots, y_{1T}, y_{21}, y_{22}, \ldots, y_{2T}, \ldots, y_{11}, y_{N1}, \ldots, y_{NT})$ by the parameters $\{\underset{\sim}{\beta}_i, \mu_i, \rho_i, \sigma_i^2 : i = 1,2,\cdots,N\}$, where N is the number of companies, and T is the number of time periods. Under the assumption that the observations are independent among the companies, we can get the following likelihood function:

$$p(\underset{\sim}{y} \mid \underset{\sim}{\beta}_i, \mu_i, \rho_i, \sigma_i^2 : i = 1,2,\cdots N)$$
$$= \prod_{i=1}^{N} \left[N(y_{i1} \mid \underset{\sim}{x}_{i1}' \underset{\sim}{\beta}_i + \mu_i, \sigma_i^2) \cdot \prod_{t=2}^{T} N(y_{it} \mid \underset{\sim}{x}_{it}' \underset{\sim}{\beta}_i + \rho_i \left(y_{i,t-1} - \underset{\sim}{x}_{i,t-1}' \underset{\sim}{\beta}_i \right), \sigma_i^2) \right]. \tag{4.1.2}$$

Second, assuming independence at all levels, assign a prior distribution to each parameter,

$$\begin{aligned}
&\pi(\underset{\sim}{\beta}_i) = N_K(\underset{\sim}{\beta}_i \mid \underset{\sim}{\theta}, \Delta), \\
&\pi(\mu_i) = N(\mu_i \mid \psi_{0i}, \phi_{0i}^2), \\
&\pi(\rho_i) = N(\rho_i \mid \nu_{0i}, \tau_{0i}^2), \\
&\pi(\sigma_i^2) = I\Gamma(\sigma_i^2 \mid \alpha, \alpha/\gamma), \\
&i = 1,2,\cdots,N,
\end{aligned} \tag{4.1.3}$$

where $\{\underset{\sim}{\theta}, \Delta, \alpha, \gamma\}$ are unknown hyperparameters, and $\{\psi_{0i}, \phi_{0i}^2, \nu_{0i}, \tau_{0i}^2 : i = 1,2,\cdots,N\}$ are known hyperparameters which are set as follows.

(P4.1) $\psi_{0i} = \frac{1}{T}\sum_{j=1}^{T}(y_{ij} - X_i B_i), \phi_{0i}^2 = 0.25 \cdot \frac{1}{T-1}\sum_{j=1}^{T}(y_{ij} - X_i B_i - \psi_{0i})$.

(P4.2) $\nu_i = \frac{1}{T-1}\sum_{j=1}^{T-1}\frac{y_{ij} - X_i B_i}{y_{i,j+1} - X_i B_i}, \tau_i = 100 \cdot \frac{1}{T-2}\sum_{j=1}^{T-1}(\frac{y_{ij} - X_i B_i}{y_{i,j+1} - X_i B_i} - \nu_i)^2$, where

$B_i = (X_i' X_i)^{-1} X_i' y_i, X_i = (x_{i1}, x_{i2}, \cdots, x_{iT}), y_i = (y_{i1}, y_{i2}, \cdots, y_{iT})$, $i = 1,2,\cdots,N$.

This paper does not pool $\rho_i's$ because the time series of some companies are stationary while some are not. It does not pool $\mu_i's$ because the start values can be very different from the rest and it is difficult to pool them. In order to use the information across companies, we pool β_i's adaptively. As for σ_i^2's, we incorporate heterogeneity but we believe they come from the same population and allow pooling them.

Third, assign a hyper-prior distribution to each unknown hyperparameter in $\{\underset{\sim}{\theta}, \Delta, \alpha, \gamma\}$.

(P4.3) $\underset{\sim}{\theta} \sim N_P(\underset{\sim}{\theta} \mid \underset{\sim 0}{\theta}, \Delta_0)$, $\Delta^{-1} = Wishart_P(\Delta^{-1} \mid (\nu_0 \Delta_0)^{-1}, \nu_0)$, where

$$\underset{\sim 0}{\theta} = (X'X)^{-1} X' \underset{\sim}{y}, \Delta_0 = 100(X'X)^{-1} \frac{SS_E}{NT - p}, \nu_0 = P + 2,$$

$$X = \left(\underset{\sim 11}{x}', \underset{\sim 12}{x}', \cdots, \underset{\sim 1T}{x}', \cdots, \underset{\sim N1}{x}', \cdots, \underset{\sim NT}{x}'\right).$$

(P4.4) $p(\alpha) \propto \dfrac{1}{(1+\alpha)^2}, p(\gamma) \propto \dfrac{1}{\gamma}$.

Assume that all the parameters $\{\underset{\sim i}{\beta}, \mu_i, \rho_i, \sigma_i^2 : i = 1,2,\cdots,N\}$ are independent of each other, the joint distribution of the parameters can be expressed as

$$\begin{aligned} & p(\underset{\sim i}{\beta}, \mu_i, \rho_i, \sigma_i^2 : i = 1,2,\cdots N \mid \underset{\sim}{\theta}, \Delta, \alpha, \gamma : i = 1,2,\cdots N) \\ & = \prod_{i=1}^{N} [N_P(\underset{\sim i}{\beta} \mid \underset{\sim}{\theta}, \Delta) \cdot N(\mu_i \mid \psi_{0i}, \phi_{0i}^2) \cdot N(\rho_i \mid \nu_{0i}, \tau_{0i}^2) \cdot I\Gamma(\sigma_i^2 \mid \alpha, \alpha / \gamma)]. \end{aligned} \tag{4.1.4}$$

Since we have assumed that all the unknown hyperparameters $\{\underset{\sim}{\theta}, \Delta, \alpha, \gamma\}$ are independent of each other, the joint distribution for the unknown hyperparameters can be expressed as

$$p(\underset{\sim}{\theta}, \Delta^{-1}, \alpha, \gamma) \propto \frac{N_P(\underset{\sim}{\theta} \mid \underset{\sim 0}{\theta}, \Delta_0) Wishart(\Delta^{-1} \mid (\nu_0 \Delta_0)^{-1}, \nu_0)}{\gamma (1+\alpha)^2}. \tag{4.1.5}$$

Finally, from the likelihood function in (4.1.2) and joint prior distribution in (4.1.4) as well as in (4.1.5), we can get the posterior distribution of all the parameters by Bayes' rule:

$$\begin{aligned} & p(\underset{\sim i}{\beta}, \mu_i, \rho_i, \sigma_i^2, \underset{\sim}{\theta}, \Delta^{-1}, \alpha, \gamma : i = 1,2,\cdots,N \mid \underset{\sim}{y}) \\ \propto\ & p(\underset{\sim i}{\beta}, \mu_i, \rho_i, \sigma_i^2, \underset{\sim}{\theta}, \Delta^{-1}, \alpha, \gamma : i = 1,2,\cdots,N) p(\underset{\sim}{y} \mid \underset{\sim i}{\beta}, \mu_i, \rho_i, \sigma_i^2, \underset{\sim}{\theta}, \Delta^{-1}, \alpha, \gamma : i = 1,2,\cdots,N) \end{aligned}$$

$$\propto p(\underset{\sim}{\beta}_i,\mu_i,\rho_i,\sigma_i^2,\underset{\sim}{\theta},\Delta^{-1},\alpha,\gamma : i=1,2,\cdots,N)p(y\,|\,\underset{\sim}{\beta}_i,\mu_i,\rho_i,\sigma_i^2 : i=1,2,\cdots,N)$$

$$\propto p(\underset{\sim}{\beta}_i,\mu_i,\rho_i,\sigma_i^2 : i=1,2,\cdots,N\,|\,\underset{\sim}{\theta},\Delta^{-1},\alpha,\gamma)\cdot$$
$$p(\underset{\sim}{\theta},\Delta^{-1},\alpha,\gamma)p(y\,|\,\underset{\sim}{\beta}_i,\mu_i,\rho_i,\sigma_i^2 : i=1,2,\cdots,N)$$

$$\propto \prod_{i=1}^{N}[N_P(\underset{\sim}{\beta}_i\,|\,\underset{\sim}{\theta},\Delta)\cdot N(\mu_i\,|\,\psi_{0i},\phi_{0i}^2)\cdot N(\rho_i\,|\,\nu_{0i},\tau_{0i}^2)\cdot I\Gamma(\sigma_i^2\,|\,\alpha,\alpha/\gamma)]\cdot$$
$$\frac{N_P(\underset{\sim}{\theta}\,|\,\underset{\sim}{\theta}_0,\Delta_0)Wishart_P(\Delta^{-1}\,|\,(\nu_0\Delta_0)^{-1},\nu_0)}{\gamma(1+\alpha)^2}\cdot$$
$$\prod_{i=1}^{N}\left[N(y_{i1}\,|\,\underset{\sim}{x}_{i1}'\underset{\sim}{\beta}_i+\mu_i,\sigma_i^2)\cdot\prod_{t=2}^{T}N(y_{it}\,|\,\underset{\sim}{x}_{it}'\underset{\sim}{\beta}_i+\rho_i\left(y_{i,t-1}-\underset{\sim}{x}_{i,t-1}'\underset{\sim}{\beta}_i\right)\!,\sigma_i^2)\right].$$

(4.1.6)

4.2 Gibbs Sampling

The target here is to make inferences on the parameters $\{\underset{\sim}{\beta}_i,\mu_i,\rho_i,\sigma_i^2 : i=1,2,\cdots,N\}$ and the unknown hyperparameters $\{\underset{\sim}{\theta},\Delta,\alpha,\gamma\}$ given the data. The complete conditional distributions are as follows.

First, $\underset{\sim}{\beta}_i\,|\,\underset{\sim}{y},\mu_i,\rho_i,\sigma_i^2,\underset{\sim}{\theta},\Delta^{-1},\alpha,\gamma \sim N_P\left(\underset{\sim}{\beta}_i\,|\,(I-\Lambda)\underset{\sim}{\theta}+\Lambda\underset{\sim}{\mu}_{\beta_i},\Lambda\Sigma_{\beta_i}\right)$, where

$$\underset{\sim}{\mu}_{\beta_i}=\left[\underset{\sim}{x}_{i1}\underset{\sim}{x}_{i1}'+\sum_{t=2}^{T}\left(\underset{\sim}{x}_{it}-\rho_i\underset{\sim}{x}_{i,t-1}\right)\left(\underset{\sim}{x}_{kt}-\rho_i\underset{\sim}{x}_{k,t-1}\right)'\right]^{-1}$$
$$\cdot\left[(y_{i1}-\mu_i)\underset{\sim}{x}_{i1}'-\sum_{t=2}^{T}2(y_{it}-\rho_i y_{i,t-1})\left(y_{it}-\rho_i y_{i,t-1}\right)'\right],$$

$$\Sigma_{\beta_i}=\underset{\sim}{x}_{i1}\underset{\sim}{x}_{i1}'+\sum_{t=2}^{T}\left(\underset{\sim}{x}_{it}-\rho_i\underset{\sim}{x}_{i,t-1}\right)\left(\underset{\sim}{x}_{it}-\rho_i\underset{\sim}{x}_{i,t-1}\right)',$$

$$\Lambda=(\Delta^{-1}+\Sigma_\beta^{-1})^{-1}\Sigma_\beta^{-1}=(\Sigma_\beta\Delta^{-1}+\Sigma_\beta\Sigma_\beta^{-1})^{-1}=(\Sigma_\beta\Delta^{-1}+\Delta\Delta^{-1})^{-1}=\Delta(\Sigma_\beta+\Delta)^{-1},$$

$i=1,2,\cdots,N.$

Second, $\mu_i \mid \underset{\sim}{y}, \underset{\sim i}{\beta}, \rho_i, \sigma_i^2, \underset{\sim}{\theta}, \Delta^{-1}, \alpha, \gamma \sim N\left(\mu_i \,\middle|\, (1-\Phi_i)\psi_{0i} + \Phi_i \sum_{i=1}^{N}\left(y_{i1} - \underset{\sim i1}{x}' \underset{\sim}{\beta}\right), \Phi_i \sigma_i^2\right)$,

where $\Phi_i = \dfrac{\phi_{0i}^2}{\phi_{0i}^2 + \sigma_i^2}$, $i = 1,2,\cdots,N$.

Third, $\rho_i \mid \underset{\sim}{y}, \underset{\sim i}{\beta}, \mu_i, \sigma_i^2, \underset{\sim}{\theta}, \Delta^{-1}, \alpha, \gamma \sim N\left(\rho_i \,\middle|\, \dfrac{B_i \nu_{0i} + \tau_{0i}^2 A_i}{\tau_{0i}^2 + B_i}, \dfrac{\tau_{0i}^2 B_i}{\tau_{0i}^2 + B_i}\right)$, where

$$A_i = \frac{\sum_{t=2}^{T}\left(y_{it} - \underset{\sim it}{x}' \underset{\sim i}{\beta}\right)\left(y_{i,t-1} - \underset{\sim i,t-1}{x}' \underset{\sim i}{\beta}\right)}{\sum_{t=2}^{T}\left(y_{i,t-1} - \underset{\sim i,t-1}{x}' \underset{\sim i}{\beta}\right)^2},\quad B_i = \frac{\sigma_i^2}{\sum_{t=2}^{T}\left(y_{i,t-1} - \underset{\sim i,t-1}{x}' \underset{\sim i}{\beta}\right)^2},\quad i = 1,2,\cdots,N.$$

Fourth, $\sigma_i^2 \mid \underset{\sim}{y}, \underset{\sim i}{\beta}, \mu_i, \rho_i, \underset{\sim}{\theta}, \Delta^{-1}, \alpha, \gamma \sim I\Gamma\left(\alpha + \dfrac{T}{2}, \dfrac{\alpha}{\gamma} + \dfrac{S_i}{2}\right)$, where

$$S_i = \left(y_{i1} - \underset{\sim i1}{x}' \underset{\sim i}{\beta} - \mu_i\right)^2 + \sum_{t=2}^{T}\left(y_{it} - \underset{\sim it}{x}' \underset{\sim i}{\beta} - \rho_i\left(y_{i,t-1} - \underset{\sim i,t-1}{x}' \underset{\sim i}{\beta}\right)\right)^2,\quad i = 1,2,\cdots,N.$$

Fifth, $\underset{\sim}{\theta} \mid \underset{\sim}{y}, \underset{\sim i}{\beta}, \mu_i, \rho_i, \sigma_i^2, \Delta^{-1}, \alpha, \gamma : i = 1,2,\cdots,N \sim N_P(m, \Sigma)$, where

$$\Sigma = (N\Delta_0^{-1} + \Delta^{-1})^{-1}, m = \Sigma\left(\Delta^{-1}\sum_{i=1}^{N}\underset{\sim i}{\beta} + \Delta_0^{-1}\underset{\sim 0}{\theta}\right)$$

Sixth, $\Delta^{-1} \mid \underset{\sim}{y}, \underset{\sim i}{\beta}, \mu_i, \rho_i, \sigma_i^2, \underset{\sim}{\theta}, \alpha, \gamma : i = 1,2,\cdots,N \sim Wishart_P(h, H)$, where

$$h = \nu_0 + N, H = \nu_0\Delta_0 + \sum_{i=1}^{N}(\underset{\sim i}{\beta} - \underset{\sim}{\theta})(\underset{\sim i}{\beta} - \underset{\sim}{\theta})'.$$

Seventh, let $\tau = \dfrac{\alpha}{1+\alpha}$, then $\alpha = \dfrac{\tau}{1-\tau}$ $(0 < \tau < 1)$.

$$\tau \mid \underset{\sim}{y}, \underset{\sim i}{\beta}, \mu_i, \rho_i, \sigma_i^2, \underset{\sim}{\theta}, \Delta^{-1}, \gamma, \alpha : i = 1,2,\cdots,N$$
$$\sim \left\{\Gamma(\alpha)^{-N}\left(\frac{\alpha}{\gamma}\right)^{\alpha N}\left(\prod_{i=1}^{N}\sigma_i^{-2}\right)^{\alpha+1}\exp\left(-\frac{\alpha}{\gamma}\prod_{i=1}^{N}\sigma_i^{-2}\right)\right\}\Bigg|_{\alpha = \frac{\tau}{1-\tau}}.$$

Eighth, $\gamma \mid \underset{\sim}{y}, \underset{\sim i}{\beta}, \mu_i, \rho_i, \sigma_i^2, \underset{\sim}{\theta}, \Delta^{-1}, \alpha : i = 1,2,\cdots,N \sim I\Gamma\left(\alpha N, \alpha\sum_{i=1}^{N}\sigma_i^{-2}\right)$

The Gibbs sampling is to carry out the following steps (i) and (ii) iteratively. These two steps consist of sub-steps that are carried out sequentially for a single chain. The Markov chain can also be replicated by drawing independent initial values of the parameters and hyperparameters.

Step (i): Update the parameters $\{\underset{\sim}{\beta}_i, \mu_i, \rho_i, \sigma_i^2 : i = 1,2,\cdots,N\}$ given the hyperparameters and data. The following sub-steps (i-1) to (i-4) are executed independently for each $i = 1,2,\cdots,N$.

(i-1) Draw $\underset{\sim}{\beta}_i$ directly from the multivariate normal distribution described in (4a);

(i-2) Draw μ_i directly from the univariate normal distribution described in (4b);

(i-3) Use Devroye (1986) method to draw ρ_i from the univariate normal distribution described in (4c);

(i-4) Draw σ_i^2 directly from the inverse gamma distribution described in (4d).

Step (ii): Generate the hyperparameters $\{\underset{\sim}{\theta}, \Delta, \alpha, \gamma\}$ given the values of the parameters in step (i).

(ii-1) Draw $\underset{\sim}{\theta}$ from the multivariate normal distribution described in (4e);

(ii-2) Draw Δ from the P-dimensional Wishart distribution described in (4f);

(ii-3) Use a grid to draw α from the complete conditional distribution described in (4g);

(ii-4) Draw γ from the inverse Gamma distribution described in (4h).

4.3 Forecasting

After getting the posterior distribution of the parameters, we can use it to predict the future stock prices at period T + 1 for each firm in the group $y_{i,T+1}$, $i = 1, 2, \ldots, N$, given the data $y_{(iT)} = (y_{i1}, y_{i2}, \cdots, y_{iT})$, $i = 1, 2, \ldots, N$.

Letting $\Omega_i = \{\underset{\sim i}{\beta}, \mu_i, \rho_i, \sigma_i^2\}$, $i = 1,2,\cdots,N$, the predictions can be sampled from the posterior predictive distribution

$$f(y_{i,T+1} \mid y_{(iT)}) = \int f(y_{i,T+1}, \Omega_i \mid y)\pi(\Omega_i \mid y)d\Omega_i. \tag{4.3.1}$$

Letting $\Omega_i^{(1)}, \Omega_i^{(2)}, \ldots, \Omega_i^{(M)}$ be a sequence of range M from the Gibbs sampler, an estimator of $f(y_{i,T+1} \mid y_{(iT)})$ is

$$\hat{f}(y_{i,T+1} \mid y_{(iT)}) = M^{-1}\sum_{h=1}^{M} f(y_{i,T+1} \mid \Omega_i^{(h)}, y_{(iT)}). \tag{4.3.2}$$

To get samples of $y_{i,T+1}$, we use data argumentation to fill in $y_{i,T+1}$ to each $\Omega_i^{(h)}$, $h = 1, 2, \cdots, M$ to get $y_{i,T+1}^{(h)}$, $h = 1, 2, \cdots, M$ from the normal distribution described below:

$$y_{i,T+1} \mid \Omega_i, y_{(iT)} \sim N(\underset{\sim i,T+1}{x'}\underset{\sim i}{\beta} + \rho_i(y_{iT} - \underset{\sim iT}{x'}\underset{\sim i}{\beta}), \sigma_i^2). \tag{4.3.3}$$

The 95% predictive credible interval for $y_{i,T+1}$ can be computed from the 2.5% and 97.5% empirical quantiles of the values $y_{i,T+1}^{(h)}$, $h = 1, 2, \cdots, M$.

4.4 Conditional Predictive Ordinate

In this chapter, the conditional predictive ordinate is defined as

$$\begin{aligned} &p(y_{i,t+1} \mid y_{(it)}) \approx \sum_{h=1}^{M} \varpi_{it}^{(h)} p(y_{i,t+1} \mid \Omega_i^{(h)}, y_{(it)}), \\ &i = 1,2,\ldots,N,\ t = 1,2,\ldots T, \end{aligned} \tag{4.4.1}$$

where $y_{i,t+1}$ denotes the random future observation of company i at period t + 1, $y_{(it)} = (y_{i1}, y_{i2}, \cdots, y_{it})$ denotes the observations of company i from period 1 to t, $\Omega_i^{(h)}$ denotes the h^th^ draw of the parameters from the Gibbs Sampler, and

$$\varpi_{it}^{(h)} = \frac{\dfrac{f(y_{(it)} \mid \Omega_i^{(h)})}{f(y \mid \Omega_i^{(h)})}}{\displaystyle\sum_{k=1}^{M} \dfrac{f(y_{(it)} \mid \Omega_i^{(h)})}{f(y \mid \Omega_i^{(h)})}},\ h = 1,\cdots,M.$$

4.5 Empirical Results of Adaptive Bayesian Analysis by Pooling Information Across Firms

As in Chapter 2 or 3, the same criteria are used for the model valuation in this chapter.

Table 4.5.1 --- Quantiles of Ratio & Numbers of Nonnegative/Negative Ratios

With Stationary Restriction											
GIC	No. of Firms	Method	Min	Q1	Q2	Q3	Max	No. (+, 0)	No. (-)	LB	UB
0	391	log trans	-8.3252	-0.1449	0.0609	0.2805	9.1655	226	165	0.553	0.603
		curt trans	-3.2252	-0.1460	0.0641	0.2868	3.5102	227	164	0.556	0.606
1	284	log trans	-8.3252	-0.1489	0.0581	0.2791	9.1655	163	121	0.545	0.603
		curt trans	-3.2252	-0.1514	0.0600	0.2838	3.5102	163	121	0.545	0.603
2	42	log trans	-5.2459	-0.1490	0.0812	0.3287	4.2113	25	17	0.519	0.671
		curt trans	-3.0397	-0.1431	0.0880	0.3372	3.1222	25	17	0.519	0.671
3	5	log trans	-1.2484	-0.1413	0.0649	0.2749	2.8667	3	2	0.381	0.819
		curt trans	-1.2718	-0.1385	0.0741	0.2915	2.9801	3	2	0.381	0.819
4	29	log trans	-1.6030	-0.1239	0.0606	0.2475	1.6233	17	12	0.495	0.678
		curt trans	-1.6576	-0.1227	0.0669	0.2612	1.6809	17	12	0.495	0.678
5	18	log trans	-2.1939	-0.1253	0.0631	0.2636	2.1197	11	9	0.439	0.661
		curt trans	-2.2456	-0.1240	0.0707	0.2793	2.0704	11	9	0.439	0.661
6	13	log trans	-1.8886	-0.1332	0.0617	0.2735	2.4346	8	5	0.480	0.750
		curt trans	-1.9548	-0.1358	0.0630	0.2786	2.2311	8	5	0.480	0.750

Table 4.5.1 shows the quantiles of ratio under the transformed scale as well as numbers of positive ratios and negative ratios averaged in each GIC group with stationary restriction, from which we can draw the following conclusions.

- Based on Q2, the ratio value ranges from 5.81% (GIC = 1) to 8.12% (GIC = 2) and 6.09% overall (GIC = 0) under log transformation, and from 6% (GIC = 1) to 8.8% (GIC = 2) and 6.41% overall (GIC = 0)under cubic root transformation. Under both transformations, GIC 1 has the smallest ratio and GIC 5 has the largest ratio.
- Based on Q2, for the same GIC group, the ratio under log transformation is smaller than under cubic root transformation.
- When GIC is 0, 1 or 2, 0.5 is not between LB and UB; when GIC is 3, 4, 5, or 6, 0.5 is between LB and UB. This indicates that both transformations overestimate the stock price for large GIC groups and do not overestimate the stock price for small GIC groups.

Table 4.5.2 --- Min, Max and Overall Values of R

Method	Transformation	min	max	overall
Classical Analysis	log trans	4.70%	9.80%	8.80%
	curt trans	6.30%	13.80%	10.90%
Adaptive Pooling Analysis	log trans	5.81%	8.12%	6.09%
	curt trans	6%	8.80%	6.41%

Table 4.5.3 --- Average Length of Credible Interval

GIC	No. of Firms	Method	With S-Restriction Ave. CI Length	Std Dev
0	391	log trans	4.085	0.453
		curt trans	3.873	0.450
1	284	log trans	4.064	0.458
		curt trans	3.854	0.455
2	42	log trans	4.292	0.452
		curt trans	4.056	0.444
3	5	log trans	4.232	0.484
		curt trans	3.990	0.573
4	29	log trans	3.868	0.369
		curt trans	3.668	0.367
5	18	log trans	4.233	0.331
		curt trans	4.052	0.358
6	13	log trans	4.040	0.357
		curt trans	3.856	0.367

Table 4.5.2 gives minimum, maximum and overall values of R under the transformed scale from both classical statistical analysis and adaptive pooling Bayesian analysis. It shows the notable improvement of using adaptive pooling Bayesian approach to the Ohlson model compared to the classical method.

Table 4.5.3 gives the average length of credible intervals and the corresponding standard deviations for both log transformation and cubic root transformation under the transformed scale with stationary restriction, from which we can draw the following conclusions.

- Under log transformation, the average length of CI ranges from 3.868 (GIC = 4) to 4.292 (GIC = 2) and 4.085 overall (GIC = 0), the standard deviation ranges from 0.331 (GIC = 5) to 0.573 (GIC = 3) and 0.453 overall (GIC = 0).
- Under cubic root transformation, the average length of CI ranges from 3.668 (GIC = 4) to 4.056 (GIC = 2) and 3.873 overall (GIC = 0), the standard deviation ranges from 0.358 (GIC = 5) to 0.573 (GIC = 3).
- For each GIC group, the average length of CI is significantly smaller under cubic root transformation than under log transformation.

Table 4.5.4 --- Conditional Predictive Ordinate

With Stationary Restriction								
GIC	No. of Firms	Method	Min	Q1	Q2	Q3	Max	Mean
0	391	log trans	-2.558	-1.387	-1.274	-1.203	-1.054	-1.306
		cube root trans	-2.381	-1.337	-1.225	-1.142	-0.995	-1.250
1	284	log trans	-2.558	-1.358	-1.268	-1.202	-1.063	-1.296
		cube root trans	-2.381	-1.310	-1.216	-1.135	0.995	-1.240
2	42	log trans	-1.750	-1.497	-1.397	-1.242	-1.084	-1.384
		cube root trans	-1.687	-1.430	-1.341	-1.173	-1.016	-1.322
3	5	log trans	-1.638	-1.586	-1.478	-1.250	-1.162	-1.423
		cube root trans	-1.586	-1.526	-1.423	-1.164	-1.100	-1.360
4	29	log trans	-1.661	-1.275	-1.216	-1.144	-1.054	-1.228
		cube root trans	-1.610	-1.230	-1.164	-1.086	-0.995	-1.173
5	18	log trans	-1.675	-1.483	-1.378	-1.270	-1.160	-1.389
		cube root trans	-1.628	-1.426	-1.327	-1.212	-1.093	-1.338
6	13	log trans	-1.479	-1.353	-1.207	-1.212	-1.134	-1.282
		cube root trans	-1.440	-1.324	-1.197	-1.155	-1.059	-1.235

Table 4.5.4 gives the quantiles and mean of CPO for both log transformation and cubic root transformation under the transformed scale with stationary restriction, from which we can draw the following conclusions.

- Under log transformation, the mean value of CPO ranges from -1.423 (GIC = 3) to -1.228 (GIC = 4) and -1.306 overall (GIC = 0).
- Under cubic root transformation, the mean value of CPO ranges from -1.360 (GIC = 3) to -1.173 (GIC = 4) and -1.250 overall (GIC = 0).
- Under both transformations, group 4 has the largest CPO and group 3 has the smallest CPO.

- For each GIC group, the mean of CPO is quite larger under cubic root transformation than under log transformation. This indicates that cubic root transformation does a better job than log transformation for the approach of adaptive pooling information across companies.

In all, the adatpive Bayesian analysis by pooling information across companies in this chapter still improves the predictive ability of the Ohlson model comparing to the classical analysis in Chapter 1. It overestimates the stock price under both transformations with stationary restriction when the size of GIC group is large, and doesn't overestimate the stock price when the size is small. Cubic root transformation is better than log transformation in this case.

Chapter 5

Comparison of Three Bayesian Models and Overall Conclusions

In this last chapter, we present the conclusions of our research work on forecasting stock prices via the Ohlson model after comparing the three Bayesian models.

5.1 Comparison of Three Bayesian Models

This section first compares the three Bayesian models that are used the former three chapters based on the minimum, maximum and overall values of R, average length of credible interval and log conditional predictive ordinate (CPO), which are gathered in Table 5.1.1. All these values are on the transformed scale except that CPO's are calculated on the original scale.

The following conclusions can be drawn from Table 5.1.1.

- Based on R, grouping analysis has the smallest R values (less than 1%) and adaptive pooling analysis has the largest R values (around 6.5%). The individual analysis results in very small R values too (around 2%).
- Based on average length of credible interval, individual analysis has the shortest lengths (around 1) and adaptive pooling analysis has the longest lengths (around 4). The average length values got from grouping analysis are not large (around 1.2).
- Based on CPO, individual analysis has the largest values (greater than -1.5), and adaptive pooling analysis has comparable values with grouping analysis (mostly less than -2).

Theoretically, adaptive pooling analysis is supposed to achieve the best predictive results. But using the criteria above, adaptive pooling analysis does not show any advantages comparing to the other two methods.

Table 5.1.1

Method	R			
	Transformation	min	max	overall
Individual Analysis	log trans	0.50%	2.50%	2%
	curt trans	0.70%	2.90%	2.20%
Grouping Analysis	log trans	-0.70%	2.30%	0.40%
	curt trans	-0.70%	3%	0.80%
Adaptive Pooling Analysis	log trans	5.81%	8.12%	6.09%
	curt trans	6%	8.80%	6.41%
Method	Ave Length of CI			
	Transformation	min	max	overall
Individual Analysis	log trans	0.652	1.003	0.949
	curt trans	0.682	1.012	0.958
Grouping Analysis	log trans	1.071	4.04	1.241
	curt trans	1.059	3.753	1.215
Adaptive Pooling Analysis	log trans	3.868	4.292	4.085
	curt trans	3.668	4.056	3.873
Method	CPO			
	Transformation	min	max	overall
Individual Analysis	log trans	-5.144	-3.417	-3.432
	curt trans	-2.639	-1.978	-2.267
Grouping Analysis	log trans	-5.146	-3.284	-3.858
	curt trans	-4.065	-2.371	-2.663
Adaptive Pooling Analysis	log trans	-6.046	-3.245	-4.737
	curt trans	-4.233	-2.810	-3.429

For the adaptive Bayesian analysis, two transformations are also compared based on the same criteria as above. This paper concludes that log transformation and cubic transformation are comparable even though the results show that the cubic root transformation does a slightly better job.

In the former three chapters, LB and UB that are calculated by the number of nonnegative R's and the number of negative R's are used to check the state of overestimation. The conclusion is both individual analysis and adaptive pooling analysis overestimate the stock prices, but the grouping analysis does not. To have an overall look, see Table 5.1.2.

Table 5.1.2 --- Overestimation Check

Method	Transformation	LB	UB
Individual	log trans	0.581	0.631
Analysis	curt trans	0.581	0.631
Grouping	log trans	0.486	0.537
Analysis	curt trans	0.502	0.552
Adaptive Pooling	log trans	0.553	0.603
Analysis	curt trans	0.556	0.606

The histogram and QQ- plots of the residuals in the adaptive pooling analysis (see Figure 5.1.1 – Figure 5.1.4) are also used to check the model fitting. Using the log transformation under the transformed scale, the residual is normally distributed with mean -0.145 and standard deviation 0.385. Using the cubic root transformation under the transformed scale, the residual is normally distributed with mean -0.143 and standard deviation 0.370. Figure 5.1.5 shows the scatter plot of standardized residual versus prediction for all companies under log transformation in the adaptive pooling analysis Figure 5.1.6 shows the scatter plot for each GIC group. It is difficult to access these plots because the standard deviations are so large that the deleted standard residual plots are lost.

Figure 5.1.7 shows the lines plots of predicted stock prices versus real stock prices for all the companies corresponding to the three Bayesian methods. We still cannot see any advantages of the adaptive pooling method.

Figure 5.1.1

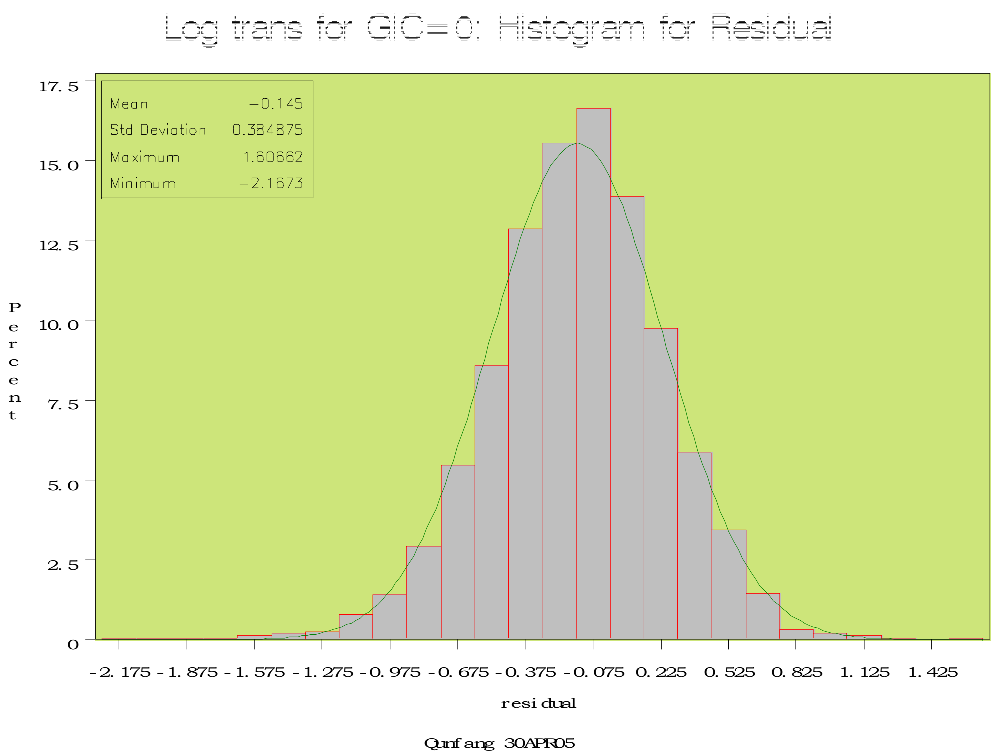


Figure 5.1.2

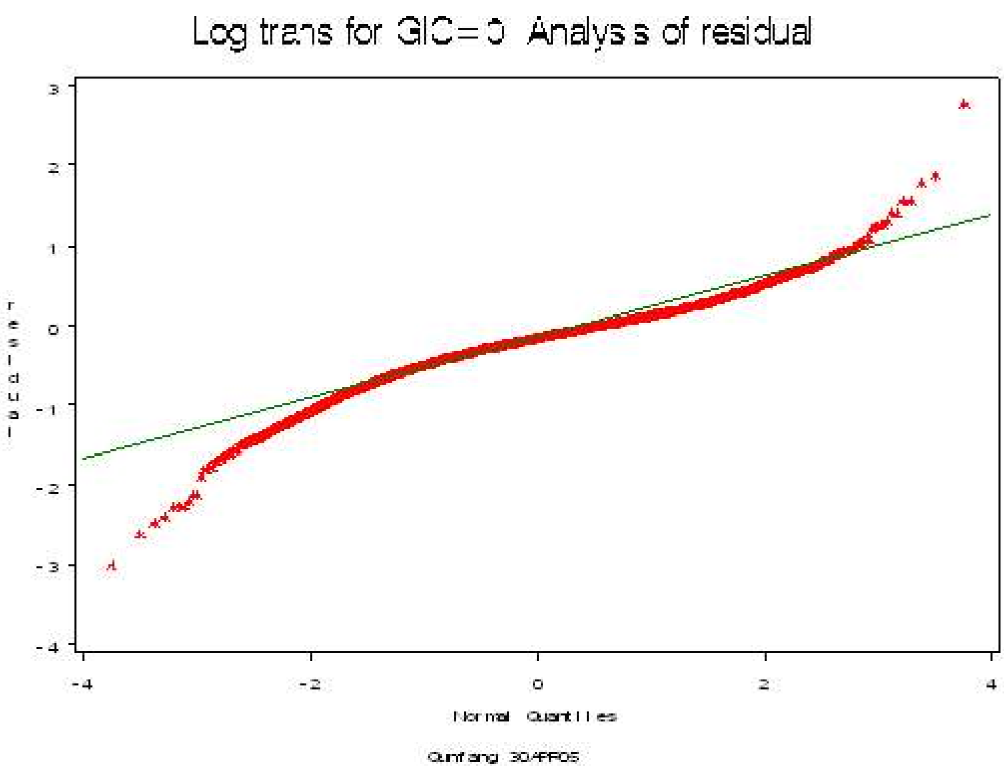


Figure 5.1.3

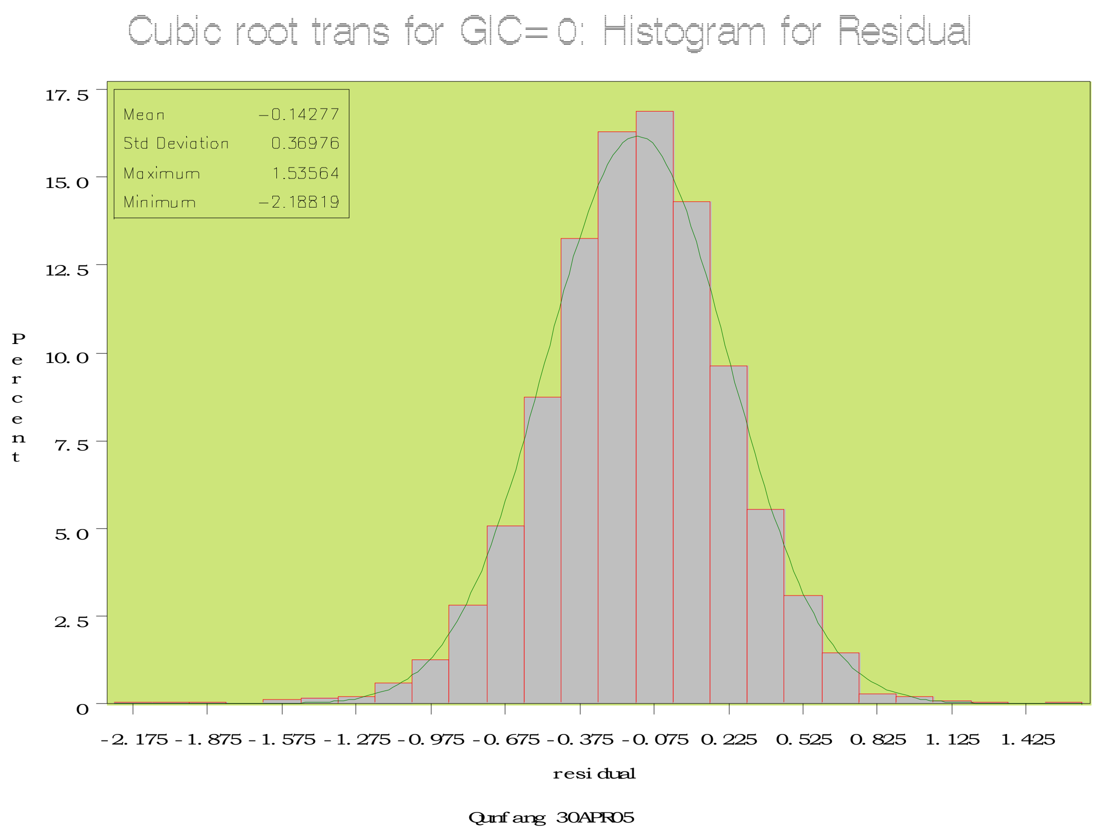


Figure 5.1.4

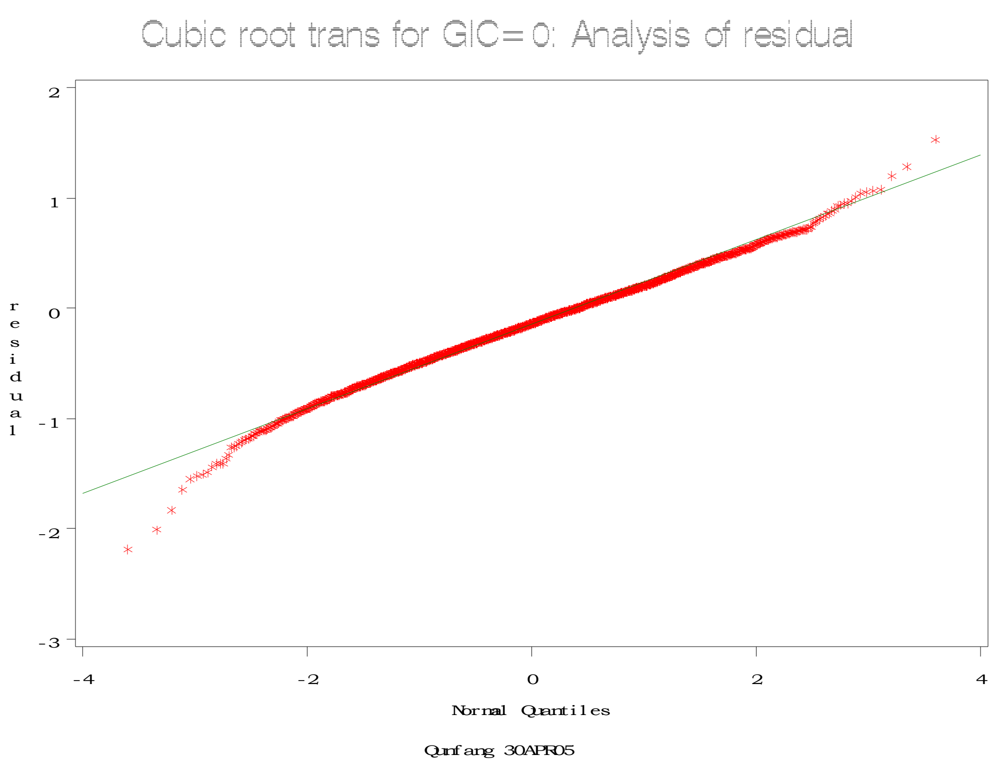


Figure 5.1.5

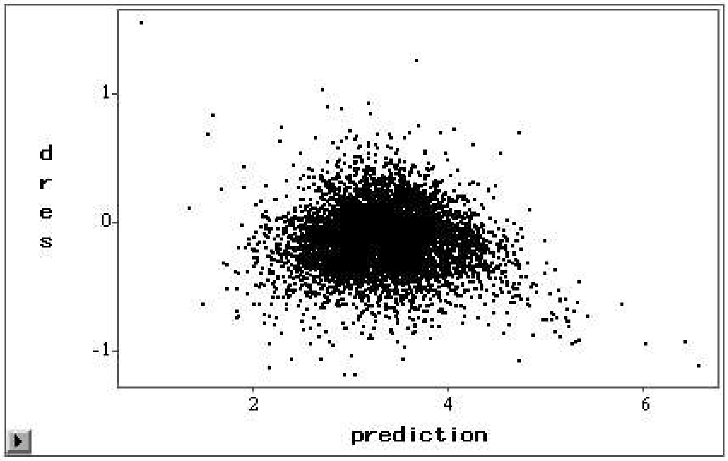


Figure 5.1.6

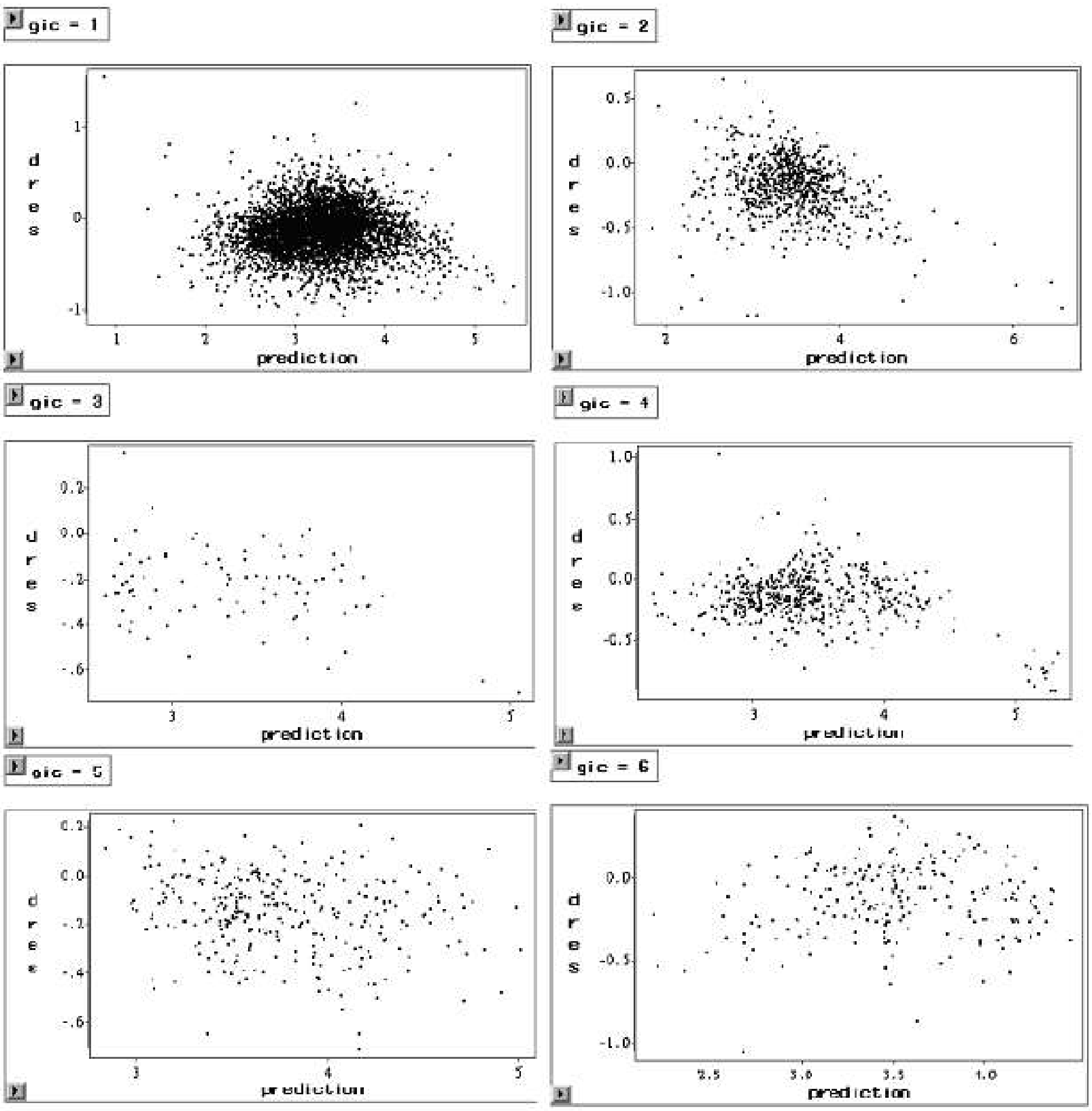
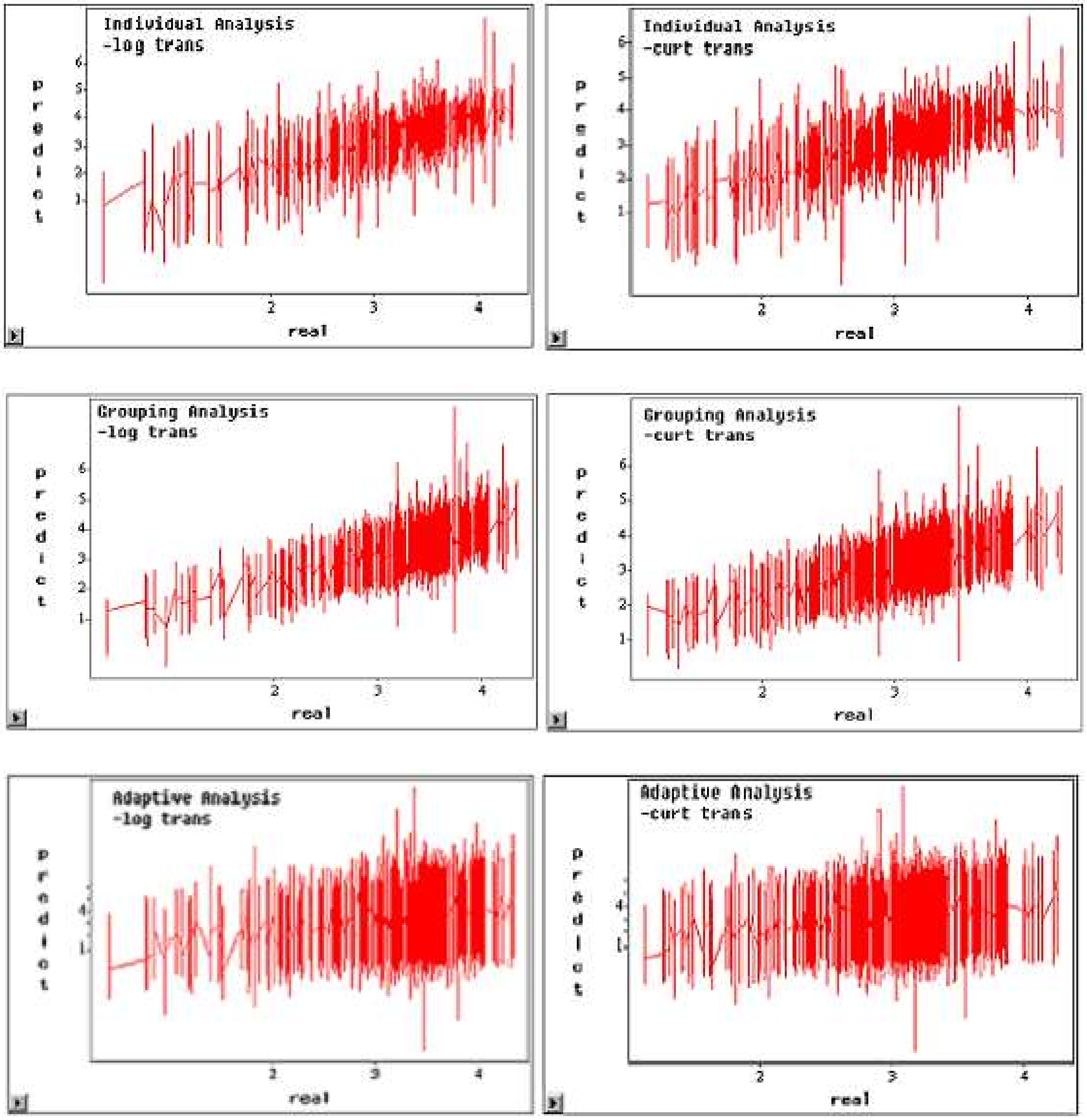


Figure 5.1.7


Individual Analysis
-log trans
Individual Analysis
-curt trans
Grouping Analysis
-log trans
Grouping Analysis
-curt trans
Adaptive Analysis
-log trans
Adaptive Analysis
-curt trans
predict
real

5.2 Estimated Regression Coefficients from Adaptive Bayesian Analysis

In Chapter 1, the classical approach gives the estimated regression coefficients for each GIC group and concludes that the intercept, BPS and the first two following quarters' abnormal earnings per share ($\beta_1 \sim \beta_4$) are generally significant in the Ohlson Model. To compare with this conclusion, the posterior distribution of $\underset{\sim}{\theta}$ in the Bayesian model for adaptive pooling analysis for each GIC group are collected in Table 5.2.1, which shows that only the intercept, BPS and the first following quarters' abnormal earnings per share ($\beta_1 \sim \beta_3$) are significant. A reduced model with only these three predictors is analyzed by the adaptive pooling Bayesian approach. Those three items turn out to be all significant in the reduced model (see Table 5.2.2).

Table 5.2.1 --- Summaries of the Posterior Distribution of $\underset{\sim}{\theta}$ for the Full Model

log trans with stationary restriction					
Parameters	Mean	STD	NSE	C025	C975
theta1	**3.20899**	0.07187	0.01513	3.0697	3.35517
theta2	**0.02597**	0.00682	0.00117	0.01221	0.03906
theta3	**0.1982**	0.09071	0.01886	0.02582	0.38316
theta4	0.1283	0.07958	0.01441	-0.01576	0.28409
theta5	0.02232	0.07982	0.0163	-0.14066	0.18115
theta6	-0.00458	0.07225	0.0149	-0.14558	0.14257
curt trans with stationary restriction					
Parameters	Mean	STD	NSE	C025	C975
theta1	**2.89697**	0.06865	0.01438	2.76542	3.0369
theta2	**0.02877**	0.00641	0.00111	0.01611	0.04133
theta3	**0.21271**	0.08863	0.01808	0.03906	0.39566
theta4	0.13214	0.07454	0.01371	-0.00397	0.28132
theta5	0.01312	0.07821	0.01577	-0.14944	0.16467
theta6	-0.0048	0.0693	0.01426	-0.14041	0.14078

Note that STD denotes the posterior standard deviation, NSE denotes the numerical standard error, C025 and C975 denote the lower bound and upper bound of the 95% credible interval separately.

Table 5.2.2 --- Summaries of the Posterior Distribution of $\underset{\sim}{\theta}$ for the Full Model

log trans with stationary restriction					
Parameters	Mean	STD	NSE	C025	C975
theta1	**3.20442**	0.06307	0.02058	3.07917	3.32686
theta2	**0.02736**	0.00539	0.00149	0.01648	0.03735
theta3	**0.23281**	0.05942	0.01453	0.11947	0.35121
curt trans with stationary restriction					
Parameters	Mean	STD	NSE	C025	C975
theta1	**2.88904**	0.061	0.02002	2.76801	3.00891
theta2	**0.03011**	0.00513	0.00143	0.01991	0.03958
theta3	**0.24232**	0.05713	0.01409	0.13454	0.3568

5.3 Overall Conclusions

Overall, four approaches are applied to interpret the Ohlson model in this paper. They are classical statistical analysis in Chapter 1, individual Bayesian analysis in Chapter 2, grouping Bayesian analysis in Chapter 3 and adaptive pooling Bayesian analysis in Chapter 4. The first two chapters prove that both the log transformation and cubic root transformation can enhance the predictive ability of the Ohlson model. The analysis in Chapter 2 results to use stationary restriction and transformed measurement scale. All three Bayesian approaches are compared to the classical method based on the minimum, maximum and overall values of R, and they are also compared with each other based on the same criteria. The conclusions are:

- Each Bayesian approach used in this paper does better job than the classical method
- Log transformation and cubic transformation have comparable efficiency in enhancing the predictive ability.
- Book value per share and the abnormal earning per share for the first following period affect the stock prices significantly, the last three abnormal earnings are not important.

This paper expects to reach the conclusion that the adaptive pooling Bayesian method is better than individual or grouping Bayesian approach. But all the four criteria, relative difference (R), average length of credible interval, lower bound (LB) and upper bound

(UB) of the 95% credible interval of the estimated probability of nonnegative R, and log conditional predictive ordinate (CPO), all fail to give this sensible conclusion. Further investigations are necessary.

Appendices

A. Origination of the Ohlson (1995) Valuation Model

For more accurate understanding, this paper splits the commonly called Ohlson (1995) model into two: the Ohlson (1995) Valution Model (OVM) and the Ohlson (1995) Approximation Model (OAM). The Ohlson (1995) Valution Model is a special case of the Residual Income Valuation Model (RIM), which was developed from the traditional Dividend Discount Model (DDM).

In economics and finance, the traditional approach to the problem of stock valuation based on a single firm has focused on the Dividend Discount Model (DDM) of Rubinstein (1976). It defines the value of a firm as the present value of the expected future dividends. That is, under the assumption of no arbitrage, there exists a pricing kernel $\pi_{t+i} = (1+r_t)^{-i}$ such that the price of a stock P_t is related to its dividends d_t by

$$P_t = E_t\left[\sum_{i=1}^{\infty} \pi_{t+i} d_{t+i}\right] = E_t\left[\sum_{i=1}^{\infty} (1+r_t)^{-i} d_{t+i}\right], \qquad \text{(A1)}$$

where r_t denotes the discount rate during time period t, $E_t[.]$ denotes the expectations operator conditioned on the date t information. Note that the "arbitrage" is a situation in which the company can make money by exploiting the efficiency of the market.

The idea of DDM implies that one should forecast dividends in order to estimate the stock prices. Since dividends are arbitrarily decided by management, it may be hard to estimate a dividend process in small samples (Ang & Liu, 1998). Moreover, market participants tend to focus on accounting information, especially earnings. The fundamental relation between book value of equity bv_t, earnings x_t and dividends d_t is described in the Clean Surplus Accounting Model by equation $bv_t = bv_{t-1} + x_t - d_t$, i.e., the change in book value between two dates equals earnings minus dividends. This relations is called the Clean Surplus Relation (CSR) which can also be written as

$$d_t = x_t - (bv_t - bv_{t-1}). \qquad \text{(A2)}$$

Substituting equation (A2) to equation (A1), thereby eliminating dividends, yields the Residual Income Valuation Model (RIM) described in Peasnell (1982). It is a function of only accounting variables, namely:

$$P_t = bv_t + \sum_{i=1}^{\infty}(1+r_t)^{-i}(x_{t+i} - r_t bv_{t+i-1}). \qquad \text{(A3)}$$

This principle shows that the theoretical value of the firm is equal to the opening book value of equity plus the present value of its residual income (or excess earning), and will not be affected by accounting choices.

The attention to the relationship between theoretical firm value and the residual income stream has attracted considerable practitioner interest and resulted in a number of proprietary models being marketed (Gregory, Saleh & Tucker, 2004). The Ohlson (1995) Valuation Model (OVM), one special case of the general class of RIM, is evaluated as a "major breakthrough" (Bernard, 1995) and "landmark works in financial accounting" (Lundholm, 1995). The particular innovation in the Ohlson model is the employment of an "AR(1) linear information dynamic" (LID) which is comprised of abnormal earnings and a variable v_t representing "other information" whose source is uncorrelated with accounting information. We can view the abnormal earnings as a contraction of "above normal earnings". The terminology is motivated by the concept that "normal" earnings should relate to the "normal" return on the capital invested at the beginning at the period, that is, net book value at date t – 1 multiplied by the interest rate r_t. Thus one interprets x_t^a as earnings (x_t) minus a charge for the use of capital ($r_t bv_{i,t-1}$) as in the following equation:

$$x_t^a = x_t - r_t bv_{t-1} \ . \qquad \text{(A4)}$$

The LID assumption can be written formally as

$$\begin{cases} x_{t+1}^a = v_t + \varpi_{11} x_t^a + \varepsilon_{1,t+1} \\ v_{t+1} = \gamma v_t + \varepsilon_{2,t+1} \end{cases} \text{--- AR(1) Linear Dynamic} \qquad \text{(A5)}$$

where ϖ_{11} is persistence parameter of abnormal earnings x_t^a, $\varepsilon_{1,t+1}, \varepsilon_{2,t+1}$ is white noise error terms with zero mean, γ is persistence parameter of other information v_t.

Substituting equations (A4) and (A5) to RIM, yields the Ohlson Valuation Model:

$$P_t = bv_t + \alpha_1 x_t^a + \beta_1 v_t,$$

(A6)

where

$$\alpha_1 = \frac{\varpi_{11}}{r_t - 1}, \beta_1 = \frac{1 + r_t}{r_t(1 + r_t - \varpi_{11})}.$$

(See Appendix B for the derivation process.)

The Ohlson model states that the stock price is a linear function of book value of the equity (bv_t), current abnormal earnings and an intercept term, which takes the following regression form:

$$P_t = \beta_0 + \beta_1 bv_t + \beta_2 x_t^a + \varepsilon_t \qquad \text{(A7)}$$

B. Derivation of the Ohlson Valuation Model

The following is the straightforward derivation process of equation (A7):

$$P_t = bv_t + \sum_{i=1}^{\infty}(1+r_t)^{-i}E_t[x_{t+i}^a]$$

$$= bv_t + \sum_{i=1}^{\infty}(1+r_t)^{-i}E_t[v_t + \varpi_{11}x_{t+i-1}^a + \varepsilon_{t+i}]$$

$$= bv_t + v_t\sum_{i=1}^{\infty}(1+r_t)^{-i} + \varpi_{11}\sum_{i=1}^{\infty}(1+r_t)^{-i}E_t[x_{t+i-1}^a] + \sum_{i=1}^{\infty}(1+r_t)^{-i}E_t[\varepsilon_{t+i}] \qquad (E_t[\varepsilon_{t+i}] = 0)$$

$$= bv_t + \frac{v_t}{r_t} + \varpi_{11}\sum_{i=1}^{\infty}(1+r_t)^{-i}E_t[x_{t+i-1}^a]$$

$$= bv_t + \frac{v_t}{r_t} + \frac{\varpi_{11}}{1+r_t}\sum_{i=1}^{\infty}(1+r_t)^{-(i-1)}E_t[x_{t+(i-1)}^a]$$

$$= bv_t + \frac{v_t}{r_t} + \frac{\varpi_{11}}{1+r_t}\sum_{i=0}^{\infty}(1+r_t)^{-i}E_t[x_{t+i}^a]$$

$$= bv_t + \frac{v_t}{r_t} + \frac{\varpi_{11}}{1+r_t}E_t[x_t^a] + \frac{\varpi_{11}}{1+r_t}\sum_{i=1}^{\infty}(1+r_t)^{-i}E_t[x_{t+i}^a]$$

$$= bv_t + \frac{v_t}{r_t} + \frac{\varpi_{11}}{1+r_t}x_t^a + \frac{\varpi_{11}}{1+r_t}(P_t - bv_t)$$

$$\Rightarrow (P_t - bv_t)\left(\frac{1+r_t-\varpi_{11}}{1+r_t}\right) = \frac{v_t}{r_t} + \frac{\varpi_{11}}{R_f}x_t^a$$

$$\Rightarrow P_t - bv_t = \frac{(1+r_t)v_t}{r_t(1+r_t-\varpi_{11})} + \frac{(1+r_t)\varpi_{11}}{(1+r_t-\varpi_{11})(1+r_t)}x_t^a$$

$$\Rightarrow P_t = bv_t + \frac{(1+r_t)v_t}{r_t(1+r_t-\varpi_1)} + \frac{\varpi_{11}}{1+r_t-\varpi_{11}}x_t^a$$

C. The Ohlson Approximation (1995) Model

Ohlson (1995) presents us with the license to break with the traditional focus on explaining price behavior and to shift that focus to predicting earnings. The key lies in the following approximation. It states that the value of the firm can be well approximated even over a finite horizon by a function of forecasted earnings, book value, and discount rates. The only assumption required is that these forecasts be consistent with clean surplus relation. We begin by defining a variable V_t^T as follows:

$$V_t^T = bv_t + \frac{(1+r_t)^T}{(1+r_t)^T - 1} \sum_{\tau=1}^{T} (1+r_t)^{-\tau} E_t[x_{t+\tau} - r_t bv_{t+\tau-1}]. \tag{C1}$$

Note that the amount V_t^T is a function of future earnings and book values measured over a finite horizon. However, despite the limited horizon, V_t^T approximates the value of the firm, so long as the horizon is "long enough", which can be described as the following equation:

$$\lim_{T \to \infty} V_t^T = P_t. \tag{C2}$$

Equations (C1) and (C2) imply that the ability to predict earnings and book value --- even over a finite horizon --- is tantamount to the ability to approximate current value. For a special empirical application of the Ohlson (1995) approximating model, we use the future earnings and book value forecasted over a horizon of 4 periods. That is, we let T = 4. Equation (C1) turns out to be

$$V_t^4 = bv_t + \frac{(1+r_t)^4}{(1+r_t)^4 - 1} \sum_{\tau=1}^{4} (1+r_t)^{-\tau} E_t[x_{t+\tau} - r_t bv_{t+\tau-1}]. \tag{C3}$$

If V_t^4 provides a good approximation of firm value, we should be able to explain a large fraction of the variation in stock prices with the variables on the right-hand side of equation (C3). This suggests the following regression model:

$$P_{it} = \beta_{i0} + \beta_{i1} bv_{it} + \sum_{\tau=1}^{4} \beta_{i,1+\tau} E_t[x_{i,t+\tau} - r_t bv_{i,t+\tau-1}] + v_{it}. \tag{C4}$$

Model (C4) is the Ohlson Model used in this paper.

D. Definitions of Data Items

(All the definitions are for industrials.)

DOW JONES INDUSTRY GROUP represents the industry classification assigned by Dow Jones based on the company's lines of business.

GENERAL INSTUSTY CLASSIFICATION represents a company's general industry classification. 01 Industrial; 02 Utility; 03 Transportation; 04 Banks/Savings and Loan; 05 Insurance; 06 Other Financial

DOW JOE MARKET SECTORS are a standardized series of digits that are used to categorize market segments issued by Dow Jones.

The Global Industry Classification Standard (GICS) was developed by Morgan Stanley Capital International (MSCI), a premier independent provider of global indices and benchmark-related products and services, and Standard & Poor's (S&P), an independent international financial data and investment services company and a leading provider of global equity indices.
The GICS classifications aim to enhance the investment research and asset management process for financial professionals worldwide. It is the result of numerous discussions with asset owners, portfolio managers and investment analysts around the world and is designed to respond to the global financial community's need for an accurate, complete and standard industry definition.

GICSSECTOR means GICS Codes. The following websites are about the sector definitions.
http://www.msci.com/equity/GICS_Sector_Definitions_2005.pdf

COMPANY IDENTITY KEY (ID)

PRICE CLOSE means the last price an issue traded at for that day. For the quarterly data set of PriceClose, all the values are the last price for the last day of corresponding quarter.

TOTAL ASSETS represent the sum of total current assets, long term receivables, investment in unconsolidated subsidiaries, other investments, net property plant and equipment and other assets.

TOTAL LIABILITIES represent all short and long term obligations expected to be satisfied by the company.

It includes:

(1) Current Liabilities
(2) Long Term Debt
(3) Provision for Risk and Charges (non-U.S. corporations)
(4) Deferred taxes
(5) Deferred income
(6) Other liabilities
(7) Deferred tax liability in untaxed reserves (non-U.S. corporations)
(8) Unrealized gain/loss on marketable securities (insurance companies)
(9) Pension/Post retirement benefits
(10) Securities purchased under resale agreements (banks)

It excludes:

(1) Minority Interest
(2) Preferred stock equity
(3) Common stock equity
(4) Non-Equity reserves

PREFERRED STOCK represents a claim prior to the common shareholders on the earnings of a company and on the assets in the event of liquidation.

For U.S. corporations, its value is shown at the total involuntary liquidation value of the number of preferred shares outstanding at year end. If preferred stock is redeemable at

anytime by the shareholder it is shown at redemption value, or if the company carries it at a higher value than the involuntary liquidation value, the stated value.
Preferred stock of subsidiary and premium on preferred stock is included in preferred stock. It excludes minority interest in preferred stock.
For Non-U.S. Corporations, the stated value of preferred stock is shown and it includes all preferred stock related accounts.
For Non-U.S. Corporations preference stock which participates with the common/ordinary shares in the profits of the company is included in common equity.

COMMON SHARES OUTSTANDING represent the number of shares outstanding at the company's year end. It is the difference between issued shares and treasury shares.
For companies with more than one type of common/ordinary share, common shares outstanding represents the combined shares adjusted to reflect the par value of the share type identified in field 6005 - Type of Share.

BOOK VALUE PER SHARE represents the book value (proportioned common equity divided by outstanding shares) at the company's fiscal year end for non-U.S. corporations and at the end of the last calendar quarter for U.S. corporations.
Preference stock has been included in equity and the calculation of book value per share where it participates with common/ordinary shares in the profits of the company. It is excluded in all other cases, deducted at liquidation value for U.S. companies and at par value for all others.

EARNINGS PER SHARE means the portion of a company's profit allocated to each outstanding share of common stock.

EPSMeanQTR1 is the mean value in a set of EPS estimates of the first fiscal quarter for a company; EPSMeanQTR2 is the mean value in a set of EPS estimates of the second fiscal quarter for a company. Same ideas can be adopted by EPSMeanQTR2 and EPSMeanQTR3. (See Table 2 for more details.)

EPSConsensusForecastPeriodQTR1 is the period (month) for which an EPS estimate is being forecasted. Same ideas can be adopted to EPSConsensusForecastPeriodQTR3-4. (See Table 3 for more details.)

Table 2 EPSMeanQTR1 (partly)

Key:	Feb-98	Mar-98	Apr-98	May-98	Jun-98	Jul-98
C000003104	0.5	0.5	0.5	0.53	0.46	0.46
C000000048	0.39	0.38	0.37	0.37	0.37	0.34
C000028595	0.54	0.54	0.54	0.56	0.56	0.56
C000000006	0.06	0.06	0.06	0.06	0.07	0.07
C000000085	0.12	0.12	0.11	0.11	0.1	0.14
C000001268	0.24	0.23	0.23	0.24	1.03	1.02
C000000104	-0.07	-0.09	-0.08	-0.08	-0.09	-0.08
C000000020	0.17	0.17	0.17	0.2	0.2	0.19
C000000110	0.93	0.92	0.84	0.94	0.94	0.92
C000029653	0.17	0.17	0.17	0.17	0.17	0.18
C000000015	0.19	0.19	0.19	0.19	0.19	0.18

Table 3 EPSConsensusForecastPeriodQTR1 (partly)

Key:	Feb-98	Mar-98	Apr-98	May-98	Jun-98	Jul-98
C000003104	Mar1998	Mar1998	Mar1998	Jun1998	Jun1998	Jun1998
C000000048	Mar1998	Mar1998	Jun1998	Jun1998	Jun1998	Sep1998
C000028595	Mar1998	Mar1998	Mar1998	Jun1998	Jun1998	Jun1998
C000000006	Apr1998	Apr1998	Apr1998	Apr1998	Jul1998	Jul1998
C000000085	Feb1998	Feb1998	May1998	May1998	May1998	Aug1998
C000001268	Mar1998	Mar1998	Mar1998	Mar1998	Jun1998	Jun1998
C000000104	Mar1998	Mar1998	Jun1998	Jun1998	Jun1998	Sep1998
C000000020	Mar1998	Mar1998	Mar1998	Jun1998	Jun1998	Jun1998
C000000110	Mar1998	Mar1998	Mar1998	Jun1998	Jun1998	Jun1998
C000029653	Mar1998	Mar1998	Mar1998	Mar1998	Mar1998	Jun1998
C000000015	Mar1998	Mar1998	Mar1998	Jun1998	Jun1998	Jun1998

Treasury bills are short-term debt instruments used by the U.S. Government to finance their debt. Commonly called T-bills they come in denominations of 3 months, 6 months and 1 year. Each treasury bill has a corresponding interest rate (i.e. 3-month T-bill rate, 1-year T-bill rate).

E. How to retrieve data by Thomsom ONE Analytics Excel Add-in

For instance, we want to retrieve the data set of PriceClose.

1. Make sure you have Microsoft Excel installed in your computer.
2. Download and install the software "Thomson ONE Analytics Excel Add-in".
   1) Open the Thomson ONE Banker Analytics Web page.
   2) Click on "Tools" at the top of the Thomson One page.
   3) Click on the "Office Tools" tab on the Tools page.
   4) See the section "Thomson ONE Analytics for Office Version 1.0".
   5) Click on the "Download" link in this section and follow the directions to save the add-in installation file to your computer.
   6) There are brief installation directions on this page, and step-by-step installation instructions also can be downloaded from this section. During the installation process, your Thomson Analytics username and password must be entered. Once installed, the Thomson Analytics Toolbar will be visible in Excel.
3. Upload S&P500 into Thomson Analytics, call 800-662-7878(-1-2) for help.
4. In a blank Excel sheet, choose the first cell (A1).
5. On the Thomson Analytics toolbar,
   1) Click on Wizards.
   2) Choose Report Wizards.
   3) In "Step 2", click "Add Entities".
   4) On the pop-up window, click "Download".
   5) Choose SP500 and click "OK".
   6) On the returned window, click "OK" again.
   7) In "Step 1", click "Add/Edit items".
   8) Under "Data Items For", click on the pull-down button, choose "Datastream".
   9) Under "Search for Item", input "priceclose", double click "priceclose" in the window below. Then click "OK".
   10) In "Step 3", under "Starting Period", click on the button with an icon of calendar. First choose "Quarterly", then choose "Q1" of "1998"; under "Ending Period", use the same way to choose "Q1" of "2004".

11) Click “Next”.
12) Click “Finish”.
13) On the pop-up window, choose “No”. You can get the PriceClose data set of SP500 in a spreadsheet.
14) Save the file.

F. How to extract quarterly data out of monthly data

Companies forecast their expected earnings every month for the following four fiscal quarters. Note that the fiscal quarters may have different starting and ending months from the calendar quarters. Different companies may have different definitions of their fiscal quarters.

This paper uses the latest forecast value for each quarter to represent the corresponding quarter value. Based on this idea, there are 4 steps to get quarterly data out of monthly data. For instance, we want to get the quarterly data of EPSMeanQTR1 in Table 2.

Table 4 Transformed EPSConsensusForecastPeriodQTR1 (partly)

ID	Feb-98	Mar-98	Apr-98	May-98	Jun-98	Jul-98
3104	2	2	2	3	3	3
48	2	2	3	3	3	4
28595	2	2	2	3	3	3
6	2	2	2	2	3	3
85	1	1	2	2	2	3
1268	2	2	2	2	3	3
104	2	2	3	3	3	4
20	2	2	2	3	3	3
110	2	2	2	3	3	3
29653	2	2	2	2	2	3
15	2	2	2	3	3	3

1. Replacing the values in each row in Table 3 (EPSConsensusForecastPeriodQTR1) with 1 to 4 by increasing 1 each time from the earliest date to the latest date, which results in Table 4.
2. Find out the last forecast month for each fiscal quarter and represent it with 1. All other forecast months are set to be 0. This can be done by the command "diff" in MatLab. The result can be shown in Matrix 1.
3. Construct a matrix (Matrix 2) containing the values of EPSMeanQTR1 in Table 3.
4. Find out the expected earning corresponding to each quarter. This can be done by dot multiplying (".*") Matrix 1 to Matrix2, which results in Matrix 3.

5. Extract the quarterly expected earning by deleting all the 0 from Matrix 3. This can be done by command “B=A(A>0)”, where A refers to the revised form Matrix 3, B is the quarterly data matrix. Note that this command only works when each row of A has the same amount of positive values. In application, A needs to be adjusted to satisfy this requirement. Also note that some expected earnings are negative and some are zeros. In order not to miss this values, we can add a relative large positive value, say 10, to each item in order to make them positive. After getting the quarterly data matrix, 10 can be subtracted to get the real values.

Matrix 1

0	0	1	0	0	1
0	1	0	0	1	0
0	0	1	0	0	1
0	0	0	1	0	0
0	1	0	0	1	0
0	0	0	1	0	0
0	1	0	0	1	0
0	0	1	0	0	0
0	0	1	0	0	1
0	0	0	0	1	1
0	0	1	0	0	1

Matrix 2

0.5	0.5	0.5	0.53	0.46	0.46
0.39	0.38	0.37	0.37	0.37	0.34
0.54	0.54	0.54	0.56	0.56	0.56
0.06	0.06	0.06	0.06	0.07	0.07
0.12	0.12	0.11	0.11	0.1	0.14
0.24	0.23	0.23	0.24	1.03	1.02
-0.07	-0.09	-0.08	-0.08	-0.09	-0.08
0.17	0.17	0.17	0.2	0.2	0.19
0.93	0.92	0.84	0.94	0.94	0.92
0.17	0.17	0.17	0.17	0.17	0.18
0.19	0.19	0.19	0.19	0.19	0.18

Matrix 3

0	0	0.5	0	0	0.46
0	0.38	0	0	0.37	0
0	0	0.54	0	0	0.56
0	0	0	0.06	0	0
0	0.12	0	0	0.1	0
0	0	0	0.24	0	0
0	-0.09	0	0	-0.09	0
0	0	0.17	0	0	0
0	0	0.84	0	0	0.92
0	0	0	0	0.17	0.18
0	0	0.19	0	0	0.18

G. Derivation of CPO

$$p(y_{t+1} \mid y_{(t)}) = \int p(y_{t+1}, \Omega \mid y_{(t)}) d\Omega = \int p(y_{t+1} \mid \Omega, y_{(t)}) \pi(\Omega \mid y_{(t)}) d\Omega$$

$$= \int p(y_{t+1} \mid y_{(t)}, \Omega) \frac{\pi(\Omega \mid y_{(t)})}{\pi(\Omega \mid y)} \pi(\Omega \mid y) d\Omega$$

$$= \frac{\int p(y_{t+1} \mid y_{(t)}, \Omega) \frac{\pi(\Omega \mid y_{(t)})}{\pi(\Omega \mid y)} \pi(\Omega \mid y) d\Omega}{\int \frac{\pi(\Omega \mid y_{(t)})}{\pi(\Omega \mid y)} \pi(\Omega \mid y) d\Omega}$$

$$\approx \frac{M^{-1} \sum_{k=1}^{M} p(y_{t+1} \mid y_{(t)}, \Omega^{(k)}) \frac{\pi(\Omega^{(k)} \mid y_{(t)})}{\pi(\Omega^{(k)} \mid y)}}{M^{-1} \sum_{k=1}^{M} \frac{\pi(\Omega^{(k)} \mid y_{(t)})}{\pi(\Omega^{(k)} \mid y)}}$$

$$= \sum_{k=1}^{M} \varpi_{(t)}^{k} p(y_{t+1} \mid y_{(t)}, \Omega^{(k)})$$

$$\varpi_{(t)}^{k} = \frac{\frac{\pi(\Omega^{(k)} \mid y_{(t)})}{\pi(\Omega^{(k)} \mid y)}}{\sum_{k=1}^{M} \frac{\pi(\Omega^{(k)} \mid y_{(t)})}{\pi(\Omega^{(k)} \mid y)}}$$

$$\frac{\pi(\Omega \mid y_{(t)})}{\pi(\Omega \mid y)} = \frac{p(y_{(t)}, \Omega) / f(y_{(t)})}{p(y, \Omega) / f(y)} = \frac{f(y_{(t)} \mid \Omega) \pi(\Omega) / f(y_{(t)})}{f(y \mid \Omega) \pi(\Omega) / f(y)} = \frac{f(y_{(t)} \mid \Omega)}{f(y \mid \Omega)} \cdot \frac{f(y)}{f(y_{(t)})}$$

$$\varpi_{(t)}^{k} = \frac{\frac{\pi(\Omega^{(k)} \mid y_{(t)})}{\pi(\Omega^{(k)} \mid y)}}{\sum_{k=1}^{M} \frac{\pi(\Omega^{(k)} \mid y_{(t)})}{\pi(\Omega^{(k)} \mid y)}} = \frac{\frac{f(y_{(t)} \mid \Omega^{(k)})}{f(y \mid \Omega^{(k)})} \cdot \frac{f(y)}{f(y_{(t)})}}{\sum_{k=1}^{M} \frac{f(y_{(t)} \mid \Omega^{(k)})}{f(y \mid \Omega^{(k)})} \cdot \frac{f(y)}{f(y_{(t)})}} = \frac{\frac{f(y_{(t)} \mid \Omega^{(k)})}{f(y \mid \Omega^{(k)})}}{\sum_{k=1}^{M} \frac{f(y_{(t)} \mid \Omega^{(k)})}{f(y \mid \Omega^{(k)})}}$$

$k = 1, \cdots, M$

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