Modeling Atmospheric distribution and Jeans loss on Mars.

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Abstract

In this thesis it is examined whether Mars, or a smaller body would have enough gravity to protect against Jeans escape if it had an Earth-like atmosphere. Other methods of atmospheric loss are ignored. Statistical mechanics were used to create a model of the atmosphere, and calculated how much gas would be able to escape. It was found that the under nearly Earth-like surface conditions, but with all particles uniform, and assuming equilibrium, the percentage of atmospheric particles at or above escape velocity would be $8.7946 \cdot 10^{-64}$. This suggests Mars would be able to sustain an Earth-like atmosphere. The Moon, and the dwarf planet Ceres were both shown to be too small to sustain an Earth-like atmosphere.

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1   Introduction

A popular idea in science fiction has always been that life could exist on other planets in the solar system, or that they could be terraformed so that they could support life. There are many obstacles to life that need to be accounted for when determining whether it would be possible, such as temperature, oxygen content, a magnetic field, and UV protection, all of which would require either unfeasible amounts of energy or technology and ingenuity well beyond what currently exists. For this thesis I will examine only the simplest of issues, which is whether or not a planet would have enough gravitational pull sustain an Earth like atmosphere without it escaping due to thermal velocity.

Mars's atmosphere as it currently exists is much too thin and much too cold for human life to survive. It also is almost entirely absent of molecules necessary for human biochemical processes, such as Oxygen and water. For this thesis, the problem of how one might go about altering the atmosphere will be ignored, but it has also been suggested that even if one could fix the atmosphere, it would simply all drift away again. Now there are several methods by which atmospheric gases can escape from a planet; the most significant in the case of Mars is stripping by solar winds. Some planets like Earth protect themselves from solar winds with magnetic fields, and Mars would need a similarly strong magnetic field of its own in order to protect its atmosphere from solar winds. For the purposes of this thesis though, it will be assumed that any problems such as that a magnetic field can be created and that most forms of atmospheric loss can be prevented, and instead the focus will be on the threat of the most basic form of atmospheric loss, and that is thermal Jeans loss. Any gaseous particle has an inherent velocity based on its temperature, and when this velocity exceeds the escape velocity for the planet on which it is located, this particle can escape the planet. This is known as Jeans loss. This thesis will examine whether Mars, and a couple other bodies in our solar system, have enough gravity to keep an Earth-like atmosphere from escaping.

2   Literature Review

Much research has been done for the purpose of modeling planetary atmospheres, and to gain insight in how they have evolved over the lifetime of the solar system.

Evidence exists, through studying patterns of erosion, that Mars once had a thick atmosphere with abundant water. Many theories exist regarding how and why it escaped[1]. Perhaps the most popular is what is known as sputtering. High speed ions escaping are constantly escaping from the Sun in all directions, and traveling

past the planets as part of what is known as the solar wind. Since these ions have a positive charge, on planets such as Earth with a strong global magnetic field, these particles are diverted away from the planet before they approach very close to the atmosphere. On planets such as Mars, with a weak magnetic field, particles in the solar wind penetrate far within the atmosphere and pick up atmospheric particles that have been ionized by ultraviolet rays from the sun. It has been speculated that Mars once had a strong global magnetic field which protected it and allowed for a dense atmosphere, and local magnetic fields still exist, but now the atmosphere is unprotected. Particles escaping in this manner have been measured directly by the Soviet Phobos probe in 1989, and if extrapolated over several billion ears, could explain most of the atmospheric loss on Mars[2].

Recent analysis of data from NASA's Mars Global Surveyor from 1998 suggests a different mechanism which involves solar wind. The present magnetic field of Mars is comprised of small pockets of magnetic charge which results in magnetic field lines that look like bubbles that extend beyond most of the atmosphere, and shield the area from solar wind like an umbrella. Data suggests that the current of charged particles in the solar wind passing nearby can cause the magnetic field lines to stretch, and then pinch off, almost like a droplet of water falling from a wet surface. The this can form magnetic capsules 1000 kilometers wide which can trap ionized particles and blow away into space.[3]

Another escape mechanism that has been suggested as an explanation for Mars's lack of atmosphere suggests that it may have been driven off by impacts from asteroids. When the asteroid hits the surface, solid and liquid matter from both the asteroid and the surface vaporize, create an explosive expansion of air, which can expand faster than escape velocity and force out the air above it. Mars's proximity to the asteroid belt make it especially vulnerable to collisions, and some suggest that with the expected size and frequency of impacts in Mars's early history, they should have blown away the atmosphere entirely[1].

Another important atmospheric escape mechanism is hydrodynamic escape. In "Thermally Driven Atmospheric Escape," Johnson gives an overview of several types of atmospheric escape, and goes into further detail of hydrodynamic escape. hydrodynamic escape is when hot light molecules colliding with and forcing up heavier molecules. Johnson does calculations involving Monte Carlo simulations in an effort to learn more about the evolution of planetary atmospheres.[4] It is generally accepted that both Mars and Earth experienced hydrodynamic escape earlier in their lifetime.[1]

In "Comparisons of Selected Atmospheric Escape Mechanisms on Venus, Mars and Titan", Hartle and Sittler identify the most prevalent atmospheric escape mechanisms for Venus, Mars and Titan[5]. It is determined that Jeans escape is the dominant mechanism on Mars, and determine the loss rate for hydrogen and deuterium. The thesis focuses on measured data utilizes direct measurements to calculate current loss rates.

Significant knowledge about Mars's atmosphere, including its atmospheric escape both know and in the past, as well as its potential inhabitability, should be gained in the near future with NASA's Mars Atmosphere and Volatile Evolution mission, or MAVEN. set to launch late in 2013, Maven will observe how the upper atmosphere and ionosphere interacts with the solar wind. The goal of the project is to determine how compounds such as carbon dioxide, nitrogen dioxide, and water may have been lost, and gain insight into the history of the Planet's atmosphere and climate.[6]

Highly advanced atmospheric models for Mars have been made in the past. The Mars Global Reference Atmosphere Model calculates wind structures and includes parameterizations of height, geographical temperature variations, and seasonal temperature changes to name a few. This is immensely more useful for most applications, but is unnecessarily and excessively complex for the calculations seen here.[7]

This thesis uses a simplified approach, setting aside measurements from space probes and data from highly specific atmospheric models, in order to draw conclusions about theoretical situation which deviates significantly from anything that can be measured directly. Rather than determine why Mars's atmosphere isn't hospitable or how it became so inhospitable, is assumed, or at least pretended, that actions could be taken to make Mars or other planet or planetary body hospitable, and that the atmosphere could be protected as by a magnetic field. A simplified model of what a Mars atmosphere may look like if the atmospheric density, temperature, and chemical composition were made more similar to those to Earth, and draws conclusions about the potential stability. Because so much about the situation cannot be defined, such as fluctuations in temperature and molecular composition with altitude, a complex model would gain little, and the prevalence of many lesser atmospheric escape mechanisms cannot be guessed at.

3   Methods

In order to create a working model for the particle density of a theoretical atmosphere, the canonical ensemble is used. In order to keep calculations simple a few

assumptions about the nature of the system must be made. First it is assumed that the only thing acting on each particle in the atmosphere is the force of gravity of the planet. Second it is assumed that the temperature is constant throughout. Third it is assumed that the atmosphere approximates a uniform ideal gas, where each particle is identical and there are no particle collisions. Given these assumptions the Hamiltonian can be written as

$$E(r) = \frac{\rho^2}{2m} - \frac{MmG}{r} \tag{1}$$

Where E is the total energy of the particle, is momentum, m is the mass of the particle, M is the mass of the planet, G is the gravitational constant, and r is the distance from the center of the planet. Now according to the canonical ensemble, the number density of particles is described by the Boltzmann distribution

$$n(r) = Ae^{-\beta E} = Ae^{\frac{-\beta \rho^2}{2m}} e^{\frac{\beta GmM}{r}} \tag{2}$$

Because the first exponential in this equation has no factor of r, it is a constant, and the expression can simplified as

$$n(r) = A'e^{\frac{\beta GmM}{r}} \tag{3}$$

Now this equation describes how an atmosphere will be distributed in a variable gravitational potential based on a ratio with the ground state and a constant. In order to obtain numerical results a normalization condition must be solved for A'. Fortunately the particle density at the surface is an easily measureable quantity and for Mars it has been recorded during various missions by NASA, and we will assume this quantity n(R), where R is the radius of the planet, is known for any planet one may wish to describe.

$$A' = \frac{n(R)}{e^{\frac{\beta GmM}{R}}} \tag{4}$$

Combining equations results in a solution for the number density of particles in an atmosphere.

$$n(r) = \frac{n(R)}{e^{\frac{\beta GmM}{R}}} e^{\frac{\beta gmM}{r}} \tag{5}$$

By altering input parameters this model can used to determine the number density of particles as a function of height for any planetary body and any conditions which may be desireable. Now with a working model of density to start with, it is necessary to determine how that effects the probability of thermal escape. The general formula for escape velocity from a planet is

$$v_e = \sqrt{\frac{2GM}{R+y}} \tag{6}$$

Where R is the radius of the planet and y is the altitude. Each gas molecule in the atmosphere is traveling with a thermal velocity dependent on the temperature of the gas. The average thermal velocity for particles in a gas of a given temperature is

$$V_t = \sqrt{\frac{3kT}{m}} \tag{7}$$

Where k is the Botzmann constant and T is the average temperature. By setting these two velocities equal to each other and using the model previously described the altitude at which the average thermal velocity is equal to the escape velocity can determined, as well as the density at that altitude.

$$H_c = \frac{\pi m M G}{4kt} - R \tag{8}$$

$$\eta H_c = \frac{n(R)m}{e^{\frac{\beta G m M}{R}}} e^{\frac{\beta G m M}{H_c + R}} \tag{9}$$

It should be understood, of course, that the thermal velocity shown here is only an average. Individual particles may have a velocity much less or much greater. The differential probability of a particle having a velocity dv is

$$P(v)dv = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{\frac{-mv^2}{2kT}} dv \tag{10}$$

This type of function has a long tail, so that there will always be a finite chance particle could reach escape velocity. This is a difficult function to integrate explicitly, but using Matlab the solution can approximated to find the probability that a particle will have at least a given velocity, which then makes it possible to calculate the probability that any particle will reach escape velocity, and the percentage of all particles at escape velocity.

Using all of the work above a detailed Matlab program was made, which is in a separate file included with this report.

4   Analysis

Fig. 1 shows a semilog plot of the atmospheric densities for both Mars and earth according to advaced models[8]. It can be seen the exponential model used here holds

for more than 100000 feet, and only falls apart at extremely high altitudes, at which point the deviation is likely caused by solar winds, which the initial assumptions say should be ignored. Fig. 2 shows a plot of the Mars atmosphere using the model described here. While the model is simplistic and imperfect, it is sufficiently accurate to use in the calculations here. Now by varying the planetary mass, radius, molecular weight, and temperature, the model can be made to resemple an atmosphere for any body with any atmospheric composition and temperature.
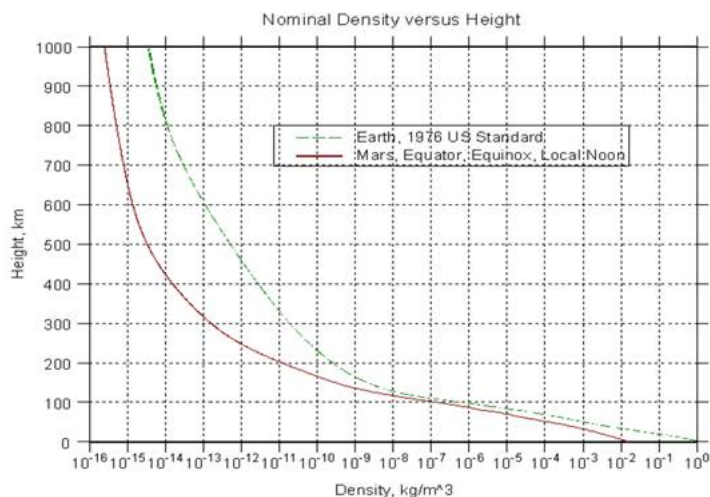

Figure 1: More advanced model of atmospheric density.

Let's start by analyzing thermal escape from mars as it is today. From NASA's Mars Fact Sheet, the average temperature on Mars is 210 K, and the average molecular mass of atmospheric particles is 43.34 g/mol[9], therefore from Eq. 7, the average thermal velocity is 347.65 m/s. Whereas the escape velocity is plotted versus height on Fig. 3. Using equations 8 and 9, it can be seen that the altitude at which the average thermal velocity is equal to the escape velocity is $7.0534 \cdot 10^8$ m, or more than 30 times the distance to its furthest moon. According to the model the atmospheric density that far away is only $5.4868 \cdot 10^{-138}$ kg/m^3.

Of course as has been mentioned, their some particles which will randomly achieve velocities much greater than the average. The distribution of particle velocities for a temperature of 210 K, found using Eq. 10, can be seen in Fig. 4. By making a cumulative sum of all the values in the graph it is possible to approximate the probability that a given particle will have a thermal velocity greater than any specified value as
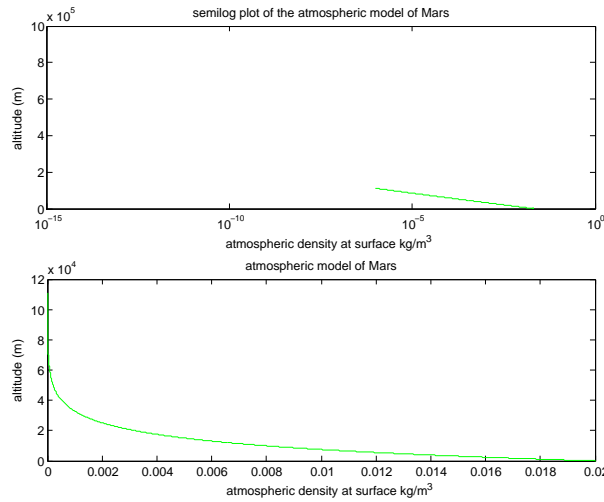
Figure 2: Our model of atmospheric density.

seen in Fig. 5. As you can see, anything over 1000 m/s is extremely unlikely.

By combining the calculations for the probability distribution with the calculations of escape velocity, a plot is made of the probability of a particle to achieve escape velocity by altitude as seen in Fig. 6. As can be seen from the scale of the graph it does not become a common occurrence until incredibly high altitudes.

What has been created to this point is a function of the percent chance of any particle being at or above escape velocity as a function of altitude, as well as a function of the atmospheric particle density as a function of altitude. By simply multiplying the functions together, the ratio of all particles in the atmosphere that are at escape velocity can be calculated. For Mars at its current average temperature of 210 K and assuming each particle has a molecular weight equal to the average of 43.34 g/mol, the ratio of all atmospheric particles that are at or about escape velocity is a miniscule $3.8302 \cdot 10^{-133}$. Of course this is skewed by the fact that some of the
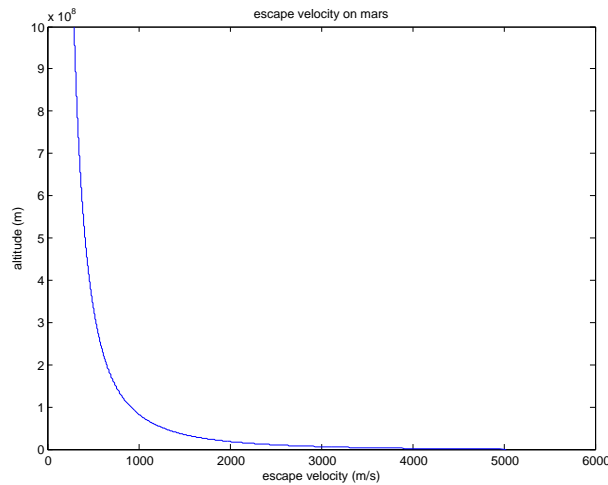
Figure 3: Escape velocity as a function of height on Mars assuming a temperature of 210 K.

molecules are lighter, it is clear that at least for heavy molecules, thermal escape is essentially non-existent. Performing the same series of calculations while assuming an atmosphere composed entirely of hydrogen produces very different results. While the critical height at which the average thermal velocity is equal to the escape velocity is still incredibly high at $1.5876 \cdot 10^7$ m, the percentage of particles with escape velocity skyrockets to very noteworthy 0.0184. So while most molecules on Mars cannot escape thermally, hydrogen most certainly can.

Now examine how things would change if it were somehow possible to magically set Mars's atmosphere to essentially earth-like, with an average temperature of a comfortable 288 K, with more human friendly molecules with an average molecular weight of 28.97 g/mol, and with a surface atmospheric density equal to the 1.217 kg/m^3 that we are used to[10], but maintaining the same planetary mass and radius. As you can see from Fig. 7, this constitutes a much more dense atmosphere which
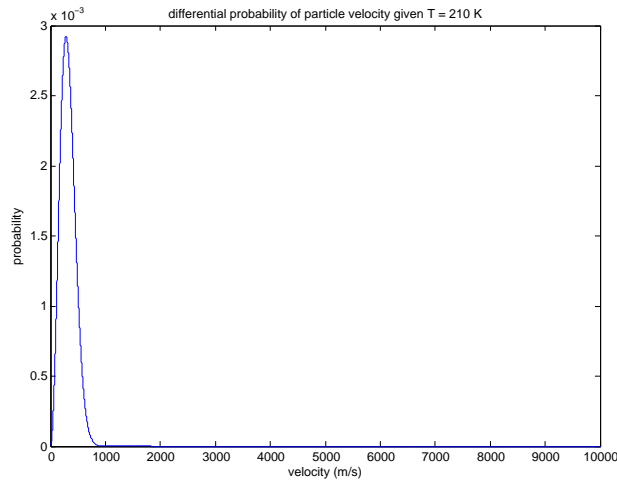
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Figure 4: The differential probability distribution fuction of thermal velocity for particles at T = 210 K.

some have suggested would escape.

As can seen in Fig. 8, the probability of a particle escaping is noticeably higher, but still extremely low at altitudes with a significant amount of atmosphere. The final calculation of the percentage of molecules at or above escape velocity gives us $8.7946 \cdot 10^{-64}$. As you can see, there are now enormously more particles escaping, as the figure increased by 69 orders of magnitude. Still though, this is an exceedingly small number. No matter how little is escaping at any given time, eventually it will all escape if there is no gas being put back into the atmosphere to replace it, but this is still a small enough number that it could be compensated for through various methods such as volcanic activity, and the capture of particles in space. looking at an atmosphere composed entirely of water with a molecular weight of 18.01528 g/mol, the percentage rises some to $4.0956 \cdot 10^{-54}$, and for oxygen it rises a bit more to $7.4461 \cdot 10^{-48}$. If mankind was somehow able to revitalize Mars's atmosphere, and if it were possible to neutralize atmospheric loss due to solar winds, then thermal
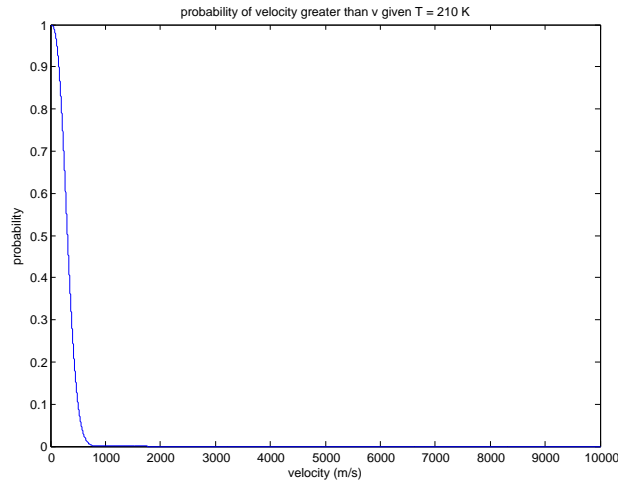
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Figure 5: The cumulative probability distribution fuction of thermal velocity for particles at T = 210 K.

atmospheric loss would not be overly problematic.

Now for fun let's look at how the model works for 2 much smaller bodies: our own moon, and the largest asteroid in the solar system, the dwarf planet Ceres. With earthlike surface conditions for each, the atmospheric models are shown for the Moon and Ceres together with the model for Mars in Fig. 9. As you can see the model for the Moon has at least the same shape as that of Mars, however the much smaller Ceres is not even close. Fig. 10 shows as plot the probability of particles reaching escape velocity with altitude, and demonstrates an even more stark difference. Running the calculation for the total percentage of particles at escape velocity gives us $3.5198 \cdot 10^{-13}$ for the Moon, and a startling 0.6742 for Ceres. This suggests that the Moon could potentially sustain an atmosphere of very heavy molecules for a little while, it would dissipate before long, while with Ceres it would dissipate incredibly fast.
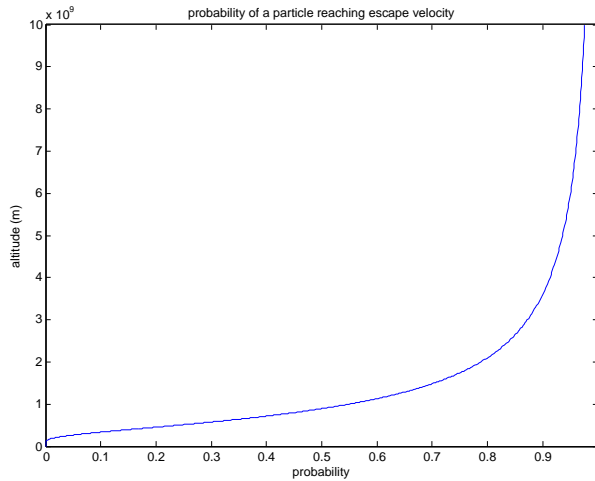
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Figure 6: Plot of the probability that a given particle at a given altitude will have escape velocity on Mars under the assumptions.

5 Conclusion

It has been shown in this thesis that the molecules necessary for human life would not escape via Jeans loss from Mars if it had an essentially Earth-like atmosphere. This conclusion does however come with caveats, because of the wealth of other escape processes that were ignored. It is also noteworthy that the model assumed uniform temperature throughout the atmosphere, whereas with Earth's atmosphere, temperatures above the exobase skyrocket to over 1000 K[11], which would make Jeans loss much more prevalent if those temperatures occurred on Mars. High temperatures above the exobase on earth are the result of the direct heating of particles by absorbing ultraviolet rays, which does not presently occur on Mars; current temperatures of Mars's exobase do not vary significantly from the rest of the atmosphere[12]. Still, the calculations were good, and represent significant evidence that Jeans escape would not occur.
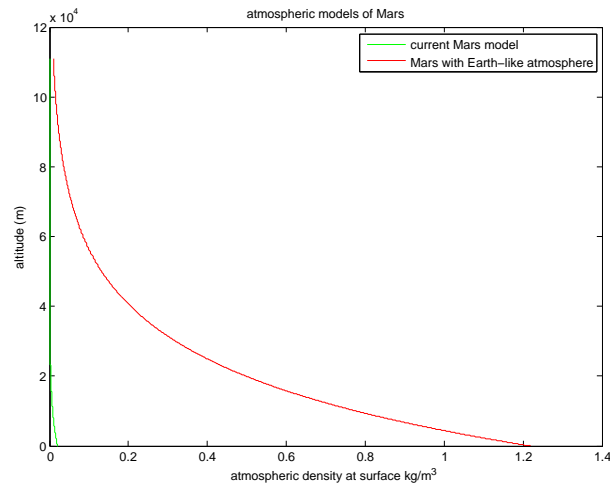
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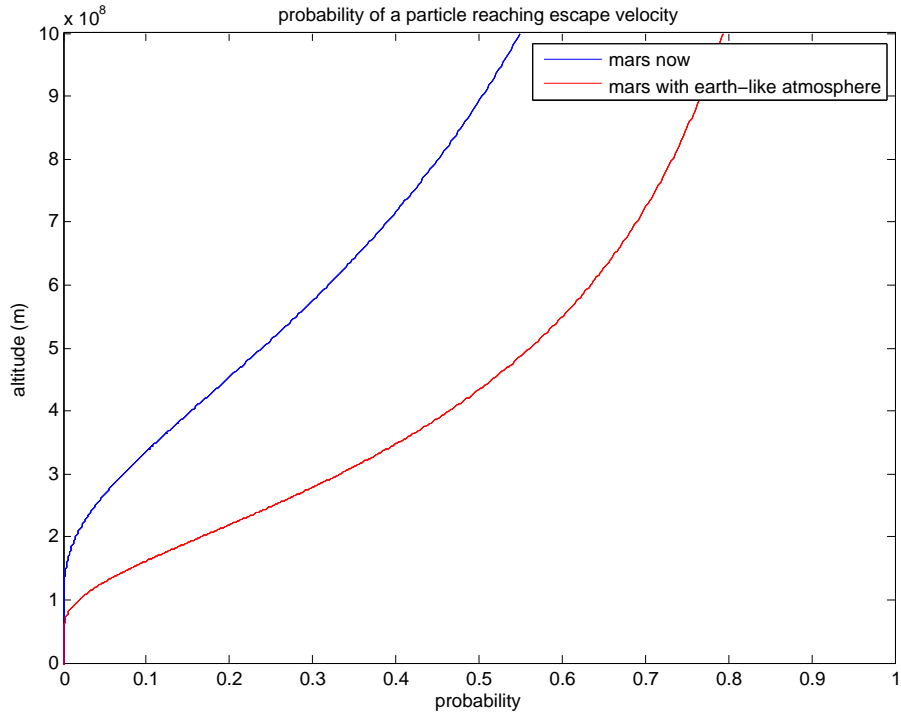
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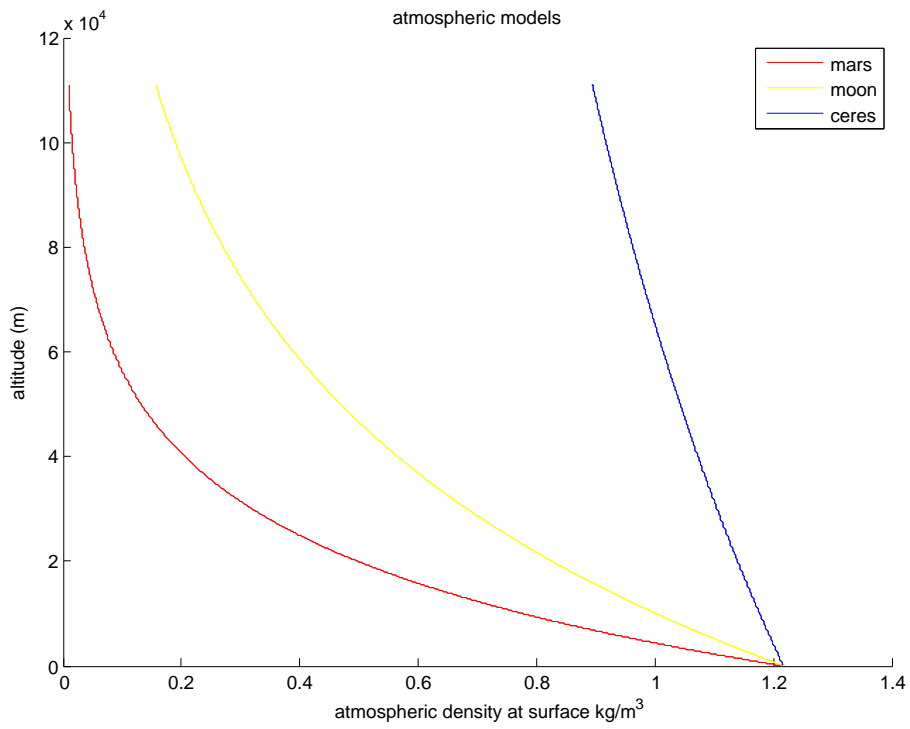
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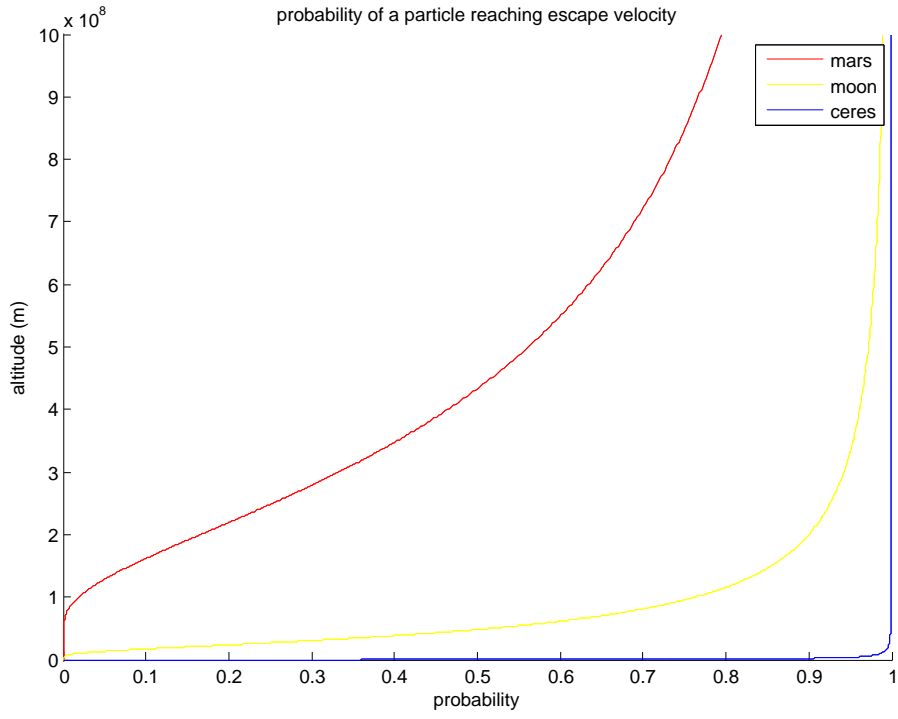
Figure 10: The probability of an individual particle being at escape velocity as a function of altitude on Mars, the Moon, and Ceres, each with Earth-like surface conditions and assuming equilibrium.

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