WASTE PAVILION

A Design for WPI's Center for Well-Being

A Major Qualifying Project Report
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ABSTRACT

Worcester Polytechnic Institute (WPI) has faced challenges in recent years with waste production and mental health. This project addresses these issues through the design of a pavilion made completely from recycled materials. Waste Polylactic Acid (PLA) and jute bast fibers were used to create a structurally viable composite material. Multiple PLA-jute composite ratios were tested, and results demonstrated that a 5% jute-by-mass PLA composite exhibits the highest tensile strength under various conditions. Principles of mathematical tilings were utilized in the pavilion's design for aesthetic and modularity. Collaborations with WPI's Center for Well-Being have led to the development of a new outdoor space on campus for student well-being that features a redesigned version of the pavilion.

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This project involved the interdisciplinary applications of architectural engineering and mathematical sciences in areas including structural design, aesthetic design, and material development. Completion of this project involved communication and collaboration within the Civil, Environmental, and Architectural Engineering Department. Computer-based programming, including Rhino, Grasshopper, and ANSYS Fluent, were utilized to achieve project objectives and test building performance and structural capabilities. Sustainable practices were considered in the material development of the project.

EXECUTIVE SUMMARY

The United States produces a significant amount of waste annually. In 2018, the Environmental Protection Agency reported that the US generated 292.4 million tons of solid waste, with only 32.1% of it being recycled or composted. Plastic has consistently been the least recycled of all waste products since the 1960s, accounting for only 9% of all recycling in 2018. WPI also struggles with waste production, as WPI's Annual Waste Audit reported that the university only recycled 19% of its total waste in 2019. A substantial portion of WPI's plastic waste is Polylactic Acid (PLA). It is the most common 3D printing material available and is used in nearly all the 3D printers on WPI campus.

WPI's challenges extend beyond waste production. During the fall of 2022, WPI conducted a campus-wide survey to better understand the state of student and faculty mental health. From the survey emerged major themes of overwork, a struggle with work-life balance, a general lack of appreciation, and specific mental health concerns. As a result of this study and findings from multiple national assessments, WPI created the Center for Well-Being and has been looking for additional ways to improve mental health on campus.

In conjunction with the 2023 Non Architecture International Waste Pavilion Competition, which challenged competitors to design a pavilion built entirely with reused materials and demonstrate the feasibility of integrating recyclable materials into architecture, this project had two primary objectives: to design a pavilion that addressed WPI's mental health crisis and the generation of plastic waste on campus, and to explore the feasibility of recycled PLA as a structural material.

The conceptual design for the pavilion was inspired by the ever-changing nature of mental health and featured a round structure with six curved walls. Each of the pavilion walls varied in height, length, visual density, curvature, angle, and path spacing to create six unique entry points. Interior paths converged at a central gathering point, symbolizing that although an individual's mental health journey is unique, experiences relating to mental health are universal and uniting. The pavilion also featured an interactive scale system to shield its interior from outside views and engage pavilion users.

Rhino, a design software, and Grasshopper, an object-oriented coding extension of Rhino, were utilized to form the pavilion in 3D space. The pavilion structure and substructure were created from tilings, otherwise known as tessellations. Periodic and aperiodic tilings were investigated for the design, including Escher and Penrose tilings. A rectangular tiling with a superimposed packing of fifteen, three-dimensional components was selected as the finalized form of the pavilion.

The finalized pavilion was submitted to the 2023 Non Architecture International Waste Pavilion Competition as a single render and competition brief explaining its inspiration from

WPI's waste production and mental health experiences. The pavilion was judged a finalist, earning 20th place and a publication in the Non Architecture Competitions Journal.

Per the requirements of the Non Architecture Waste Pavilion Competition, PLA was defined as the primary construction material based on its relevance to WPI's waste production. PLA is brittle and not applicable to large-scale structures, so a composite material of PLA and jute bast fibers was proposed to increase the material's structural capabilities. In the literature, jute-PLA composites with varying quantities of jute have been found to exceed the tensile capabilities of PLA alone.

To explore the structural potential of jute-PLA composites, three variations were created for tensile testing: a control made of 100% PLA, a 5% jute-by-mass PLA composite, and a 10% jute-by-mass PLA composite. 1.5 kilograms of PLA waste was collected from the Makerspace in WPI's Innovation Studio. The PLA waste was shredded into chips of less than 4 mm in diameter, washed for debris, and then dehydrated in a large oven at 40° C for six hours. The jute fibers were cut to 4 mm long strands, dehydrated, and weighed to produce the three composite variations. In accordance with ASTM standards, samples of the three composite variations were cast into dog bones. Tensile tests were conducted under three conditions: a controlled environment, a warm thermal environment of 32° C, and a high moisture environment. After determining the tensile strength of the PLA composite variations, the data was utilized in the engineering software ANSYS Fluent to model the construction of the pavilion and to simulate its performance under self-weight and under wind loads.

The 5% jute-by-mass PLA composite was determined to be the most effective, yielding the highest average tensile strength for both the high thermal and high moisture conditions. Additionally, the ANSYS Fluent simulations revealed minimal structural deformations under both self-weight and wind loading. Further experiments are needed to determine the true structural feasibility of jute-PLA composites and composites with other bast fibers or plastic types.

Connections were made with WPI's Center for Well-Being to implement a version of the pavilion on the WPI campus. The Center for Well-Being expressed a desire to expand their space outdoors to the open lawn in front of Daniels Residence Hall. Initial designs were produced in collaboration with the Director of the Center for Well-Being to create a new outdoor space promoting student well-being. WPI's Housing and Residential Experience Center joined the design process, as the proposed renovations would include a new entryway to Daniels Residence Hall and create a common outdoor space for future developments. As of Spring 2024, these lawn designs are being finalized, and construction is set to begin in Summer 2024. The new lawn space will include a redesigned and scaled version of the waste pavilion for future WPI students to enjoy.

INTRODUCTION

Architectural design significantly impacts the functionality of any structure, as the size, shape, and orientation of a structure determine its operational success for intended purposes. Beyond function though, architecture has the potential to signify and support a structure's greater meaning. Architectural design can tell stories through structures and create three-dimensional and occupiable works of art.

Balancing functionality and underlying aesthetic design goals in architecture can be challenging, and traditionally, one always outweighs the other. An office building may be built perfectly for its clerical purpose, but the exterior may be simple or uninviting, making work unappealing to employees. A school may have an angular, eye-catching floorplan and interior space, but the functionality of the school's classrooms may suffer from the irregular layout.

To find this balance between function and aesthetic, architectural design can rely on applied mathematical principles. Mathematical concepts and equations can dictate form development, whether it be through the optimization of usable space, the strategic curvature of a skyscraper wall, or the brick laying pattern used along an exterior walkway. Tiling, or tessellating, is a mathematical principle that is already common in two-dimensional design. The mathematically based, geometric patterns of tilings however are underutilized in three dimensions. This project examines the application of tilings in three-dimensional space and how mathematically based patterns can enhance both the functionality and greater meaning of architectural design.

Further carrying the theme of balancing aesthetic and function, this project involves the complete structural and artistic design of a pavilion, a building used as a shelter in outdoor spaces. The pavilion design takes inspiration from Worcester Polytechnic Institute's own challenges with mental health and aims to create a space that promotes the well-being of the campus community. Additionally, the pavilion takes inspiration from campus waste production, as the focus on innovation at WPI has led to an increase in discarded polylactic acid (PLA) from 3D printers. In attempts to combat this, the project investigates the ability to repurpose this waste into functional and structurally sound building materials.

Architectural design can be greatly enhanced by applications of mathematics and inspiration from current issues. This project supports that when multiple perspectives are considered in the design of a structure, not only are functionality and aesthetic balanced, but the structure is able to fulfill a purpose beyond what is initially possible.

BACKGROUND

Our pavilion design and consequent studies of polylactic acid (PLA) composites were motivated by WPI's mental health crisis, the rapid waste production the world is facing, and the ways in which architectural engineering and mathematics can help combat such issues. The following sections provide an overview of the current states of sustainability and mental health, both globally and at Worcester Polytechnic Institute (WPI), as well as the mathematical theories and principles that contributed to this project.

Waste Production on College Campuses

The United States produces a significant amount of waste annually. The 2018 report conducted by the Environmental Protection Agency found that the total municipal solid waste generated in 2018 was 292.4 million tons, which is equivalent to 4.9 pounds per day per person (United States Environmental Protection Agency, 2023). Of this generated waste, the largest contributors were paper and paperboard at 23.05%, food waste at 21.59%, and plastics at 12.20%. From 2017 to 2018, the average American produced an additional 0.5 pounds of waste per day, demonstrating a staggering national increase. Of the 292.4 million tons produced in 2018, 32.1% of it was either recycled or composted, with 68% of all recycled material being paper products. Since 1960, plastic has been the least recycled of all waste products, accounting for only 9% of all recycling in 2018 (United States Environmental Protection Agency, 2023).

WPI conducts an Annual Waste Audit to determine the makeup of waste in various locations on campus. The goal of the audit is to educate the WPI community on waste production and improve overall recycling efforts. During the audit, residential, academic, and administrative buildings are randomly selected, and all waste, including trash and recycling, is quantified and sorted. The proportions of waste found in recycling and trash bins are compared to the actual proportion of recyclable items, regardless of the bin they came from. The process is repeated in each building to understand how effective the WPI community is at recycling. In 2019, WPI was recorded to only recycle 19% of its total waste, which falls below the 2018 national average of 32.1% (WPI Green Team and Office of Sustainability, 2019). WPI's 8th Annual Waste Audit in a single day collected 477.2 pounds of waste from the Rubin Campus Center alone, 80.8% of which came from trash cans. Only 10.6% of the waste, regardless of its ability to be recycled, came from recycling bins, and the remaining 8.6% was cardboard. Of the waste collected from trash cans, only 55.9% of it was truly trash. The WPI Green Team, who assisted with the audit, reported that a portion of the trash could have been recycled had it not already been contaminated by other waste products. Additionally, 49.84% of all recyclable materials were placed in the wrong bin, demonstrating a consistent misplacement of recyclable materials on WPI's campus (WPI Green Team and Office of Sustainability, 2019).

Recycled Materials in Architecture

Outside of WPI, creative strategies in the field of architectural engineering are being implemented to combat the growing waste issue that the world is facing. Common household waste products, including paper, glass, and metal, can be repurposed to create building materials. Newspaper wood is created from the extreme pressing of stacked newspaper sheets. The end results closely match the appearance of standard wood and can be used for finishes (MaterialDistrict, 2013). Major support beams and building facades can be made of recycled metals such as steel and aluminum (Suharjanto, 2020). Other materials like glass, wood, and plastic have been repurposed and integrated into modern architectural designs (Patnaik, 2023).

Waste products have been used even more widely for aesthetic components in architectural design. Both the Zig-Zag House of the United States and the Plastic House of Dublin feature fully plastic facades made from recycled bottles and containers (ArchDaily, 2010). To create these facades, the plastic bottles were melted down and remolded and recolored into sheets to be used along the houses' exterior walls (ArchDaily, 2011). Pavilions have also become one of the most common ways for waste products to be recycled into architecture. Over the past few decades, pavilions have emerged as a prominent architectural form to convey urgency for recycling and waste reduction. The Head in the Clouds Pavilion, constructed in 2013 as the winner of the City of Dreams Pavilion contest in New York City, made a statement on the plastic waste crisis of the city (Studio KCA, 2013). The pavilion was constructed completely of plastic water bottles, specifically, the number of plastic bottles thrown away every hour in New York. The structure created a place that was not only beautiful, but forced the public to contemplate their own plastic consumption and truly understand the amount of waste the city produces (Studio KCA, 2013). The Governor's Cup Pavilion, also located in New York City and the winner of the 2014 City of Dreams Pavilion contest, chose to tackle the city's plastic consumption as their topic of interest as well (Architizer, 2014). However, they chose to convey their message about the city's plastic consumption in a hands-on approach; residents of New York City were rallied and encouraged to construct the pavilion, allowing those who participated to understand just how much plastic was being thrown away. The act of building the pavilion brought the community together and allowed those who participated to feel connected to both the pavilion and the cause itself (Architizer, 2014).

Mental Health on College Campuses

Challenges with mental health are consistently prevalent on college campuses. In the 2020-2021 school year alone, more than 60% of college students from 373 US campuses met the criteria of experiencing at least one mental health problem (Abrams, 2022). A 2021 study was conducted comparing the mental state of first- and second-year college students before and during the pandemic (Kim, et al., 2022). The study found a notable increase in clinical depression, alcoholism, bulimia, and binge-eating disorders among college students after one year into the pandemic, and most of these students are still in college today (Kim, et al., 2022).

At WPI, undergraduate and graduate students' mental health needs have been increasing in severity over the past several years. In the fall of 2022, WPI conducted a campus-wide survey and hosted multiple town-hall style meetings to better understand the state of student and faculty mental health (WPI Mental Health and Well-Being Task Force, 2022). From the survey emerged major themes and concerns relating to mental health, notably overwork, work-life balance, a general lack of appreciation, and specific mental health challenges. The compiled data revealed that much of WPI's faculty do not feel appreciated, nor do they feel as though they have control over their day-to-day schedules. As a result from this study and findings from multiple national assessments (Healthy Minds Network, ACHA, NCHA), WPI created the Center for Well-Being, a location on campus dedicated to improving student mental health (WPI Mental Health and Well-Being Task Force, 2022).

Designing Spaces to Promote Well-Being

Creating spaces on campuses designed for well-being has been shown to have positive impacts on students. A 2013 study conducted through the University of South Australia found that the architectural design of mental health facilities has strong implications on their effectiveness (Connellan, et al., 2013). 165 publications from 2005 to 2012 relating to the design of mental health spaces were reviewed for common themes of effectiveness. The most prevalent theme, safety and security, centered around having intentionally sized and dedicated spaces for certain mental health related activities. Spaces with specific purposes were found to relax occupants, and occupants experienced heightened benefits when they had multiple places to go to within larger facilities (Connellan, et al., 2013). Lighting was the second most significant theme, as the presence of natural light in a space was found to have significant positive implications on an occupant's mental health. The third and fourth themes, therapeutic milieu and gardens, discuss the benefits of integrating nature with architecture, as contact with nature and living things has been found to alleviate stress, restore attention, and promote relaxation (Connellan, et al., 2013).

Similarly, a study in 2016 from McMaster University examined the role of nature in improving mental health, as well as nature's underutilization to promote well-being on college campuses (Windhorst & Williams, 2016). The study was based on the biophilia hypothesis, a theory that suggests humans need a connection to nature beyond physical resources for psychological fulfillment (Windhorst & Williams, 2016). To better utilize nature on a college campus and fulfill the biophilia hypothesis, the study examines two scenarios: bringing nature indoors, and bringing working spaces outdoors. Interiorly, living walls, views of natural settings, and pictures of natural landscapes were all shown to boost mental health in indoor environments. Exteriorly, relocating therapy-based activities outdoors was shown to increase awareness and presence during reflection. This mindfulness-based connection of people to the outside world is known as ecotherapy (Windhorst & Williams, 2016).

Psychological Impacts of Patterns

The mind naturally creates patterns as a response to its surrounding environment, and these patterns typically mimic those found in nature or other human-made systems (Coburn, et al., 2016). Studies on human psychology have found that humans possess an innate need for repetition and order, as seen in many facets of day-to-day life (Salingaros, 1999). The calendar year is organized around weeks, months, and seasons that remain consistent as years progress. Annual holidays and events are expected and relied on as calendar year benchmarks. Even humans' everyday activities are structured in patterns: we wake up, eat our meals, engage in work or other activities, and go to sleep at approximately the same times every day. A diversion from these repetitive schedules could result in a person far removed and disconnected from society's rhythm (Salingaros, 1999). The natural inclination for humans to create repetition is expressed outwardly in art. In music, the mind enjoys songs with repeated chord loops, lyrics, and notes (de Clercq & Margulis, 2018). Further, the majority of mainstream pop and rock songs since the mid-1950s follow the same structure: a chorus repeated twice, followed by a song "bridge" that differs from the repeated chorus, concluded with a song "finale" with elements from the original chorus. The predictability of a song's structure tends to positively correlate to its mainstream success due to the psychological impact the repetition has on listeners (de Clercq & Margulis, 2018).

Visual patterns are noticeable in many pieces of traditional art, such as in the work of artists MC Escher, Andy Warhol, or Yayoi Kusama. Escher focused on the repetition of tiles, while Warhol was known to duplicate elements of pieces in every aspect but color to create a cohesive piece (Maddox Gallery, 2024). Kusama's most famous works feature complex shapes that are covered in repeated, recognizable patterns (Maddox Gallery, 2024).

(a) (Escher, Reptiles, 1943) (b) (Warhol, 1962) (c) (Kusama, 2023)

Figure 1: Patterns in Art by Escher (a), Warhol (b), and Kusama (c)

Patterns and repetition are noticeably prevalent in architectural forms. A 2018 study published in the Journal of Environmental Psychology conducted multiple experiments relating to humans' perceptions and psychological responses to patterns in architecture (Coburn, et al., 2016). One experiment tested people's ability to detect natural patterns in architecture by showing them images of different architectural forms and asking them to rate the forms' aesthetic appeal. The researchers concluded that over half of the variance in ratings between images was attributed to the patterns that were present. Additionally, the results supported a strong association between perception of natural patterns and aesthetic appeal (Coburn, et al., 2016). The study concluded that visual patterns positively impact peoples' aesthetic perceptions of architectural forms. Psychological responses to natural patterns in architecture These examples of noticeable patterns in art and architecture are all based in mathematics, and it is theorized that humans' ability to perform mathematics arose from the need to categorize and define observed patterns (Coburn, et al., 2016).

Tilings

Tilings, also referred to as tessellations, are a mathematical application of patterns. Tilings are groups of shapes that cover a plane without gaps or overlaps, meaning that the interiors of the shapes are disjoint (Barth, 2007). The shapes that form the patterns within tilings are called tiles, and they exist in the geometric Euclidean plane, a two-dimensional space in which two-number coordinates are required to determine point locations. An isometry is a mapping of the Euclidean plane to itself that is distance-preserving, meaning that any tile can be mapped to another tile by an isometry and a change in scale (Barth, 2007). A symmetry is an isometry that maps every tile to another tile within the same tiling. Tiles form patterns through symmetries, or geometric transformations, which include reflections, rotations, translations, and glide reflections (Hall, 1995). The set of all symmetries that maps a pattern onto itself for a particular tiling is referred to as the symmetry group (Hall, 1995). Examples of tilings can be seen in Figure 2.

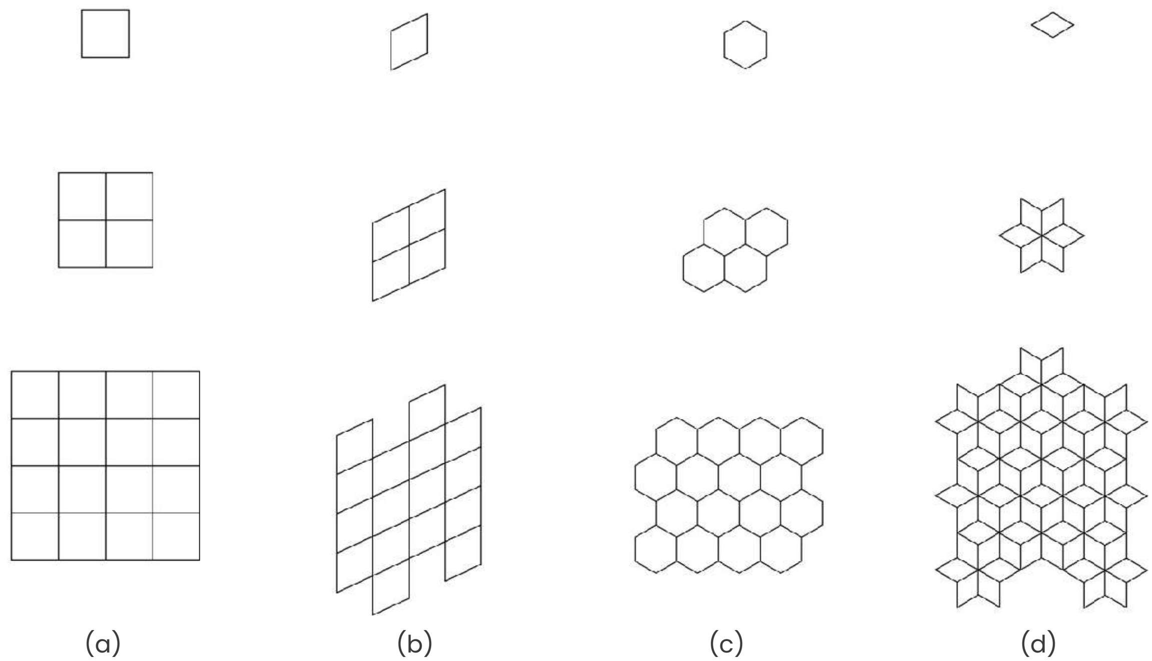


Figure 2: Examples of Periodic Tilings

Many patterns exist that are not considered tilings. A group of shapes in a plane with no overlaps is called a packing (Hall, 1995).

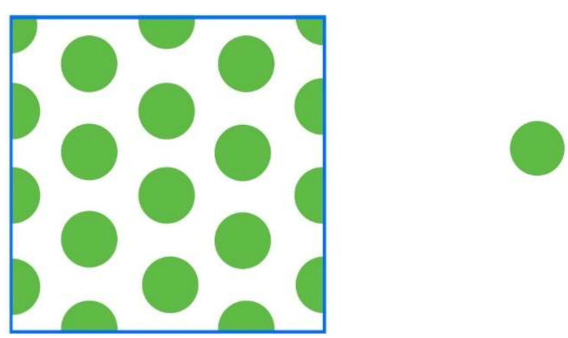


Figure 3: Packing of Circles

Not all packings are tilings, but they can be superimposed as a pattern or design on a tiling to fit the criteria. Consider the packing in Figure 3. Each can be superimposed on a geometric tiling grid to form "patterned" tiles as seen in Figure 4 (Gethner, 2002).

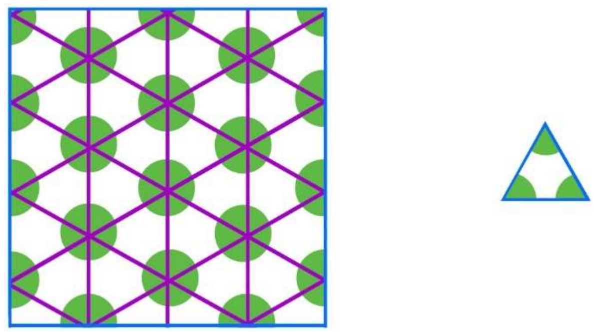


Figure 4: Packing of Circles Superimposed on Equilateral Triangle Tile

Periodic Tilings

The tilings in Figure 2 are all examples of periodic tilings. Periodic tilings can be completely described as a set of their translations (Chavey, 1984). In other words, a tiling is periodic if for any fixed point of the plane, the image of that point under all translations of the tiling will form a lattice, and this lattice of translated fixed points will in itself form a different periodic tiling. If all tiles in a given tiling are congruent, meaning that they coincide exactly when superimposed, then the tiling is monohedral (Chavey, 1984). For any periodic tiling, the edges and vertices of a tiling are also the edges and vertices of the graph of the tiling. The graph of the tiling is connected if any two vertices of the tiling on the graph are connected. Similarly, the tiles of a tiling are considered the vertices on the dual graph of the tiling, and any two vertices on the dual graph are connected by an edge of the dual graph if the corresponding tiles are adjacent (Chavey, 1984). To investigate the conditions of periodic tilings, we define $T(v, e, t)$, where T is a periodic tiling with a finite number of v vertices, e edges, and t tiles. We will specifically consider edge-to-edge periodic tilings by regular polygons, or instances where the edge of one tile connects to another to form the tiling pattern. The following theorems prove fundamental properties of periodic tilings.

Theorem 1

Let $T(v, e, t)$ be an edge to edge tiling by regular polygons. Then $v \leq e$ and $t \leq e + 1$.

As a condition of edge-to-edge tiling, each edge is shared by two adjacent polygons, excluding edges on the perimeter of the tiling. Additionally, every vertex is shared by at least two edges, excluding the outer vertices. Along the perimeter of the tiling, one edge coincides with one vertex. Therefore, the number of vertices v is less than or equal to the number of edges e, or $v \leq e$.

To prove $t \leq e + 1$, we can look at the dual graph of the tiling. The dual graph is bipartite if and only if every vertex in the original tiling has an even valence. In other words, if every tile in the tiling shares an even number of edges with neighboring tiles, the dual graph will be bipartite.

With this understanding, we can look at two cases, the tiles having either an even or odd valence:

For Case 1 where the tiles have even valence, the dual graph will be bipartite, and the inequality $t \leq e + 1$ will hold with equality.

For Case 2 where the tiles have odd valence, meaning they share an odd number of edges with neighboring tiles, the dual graph will not be bipartite, and the inequality $t < e + 1$ will hold, not with equality.

Therefore, the expression $t \leq e + 1$ holds for all tilings, regardless of whether the dual graph is bipartite. Because edge-to-edge tilings are a subset of all tilings, the expression must also hold true.

Theorem 2

Let $T(v, e, t)$ be an edge to edge tiling by regular polygons. Then $e \leq 5v$ and $t \leq 4v + 1$.

It is known that for $T(v, e, t)$ tilings, there exists a representative set V of v vertices whose induced subgraph is connected, meaning that we can select a subset of vertices such that the edges connecting them form a connected graph. Then, since every edge and tile in the tiling $T(v, e, t)$ is associated with a vertex in V, the total number of edges and tiles must be bounded by the number of edges and tiles incident to the representative set V.

When considering regular polygons, the maximum valence, or the number of edges that can connect to one vertex, is 6, which occurs in the case of a regular hexagon. In other words, valence ≤ 6 for each vertex in V. Therefore, without loss of generality, there exists some vertex in V with a valence ≤ 5. This means that the number of edges and tiles incident with V is at most $6v-1$.

Additionally, since the graph induced by V is connected, it contains a spanning tree, or a subset of a graph where all vertices are connected using the minimum possible number of edges, with $v - 1$ edges. Each edge of the spanning tree is counted twice, as it is incident with two vertices. Similarly, each edge in the spanning tree is incident with two tiles. By this method, each tile has been counted twice, once for each vertex it is incident with.

To correct the overcounting, we subtract $v - 1$ from the number of counted edges and $2(v - 1)$ from the number if counted tiles. Therefore, the number of edges is at most $6v - 1 - (v - 1) = 5v$, thus $e \leq 5v$. The number of tiles then is at most $6v - 1 - 2(v - 1) = 4v + 1$, thus $t \leq 4v + 1$.

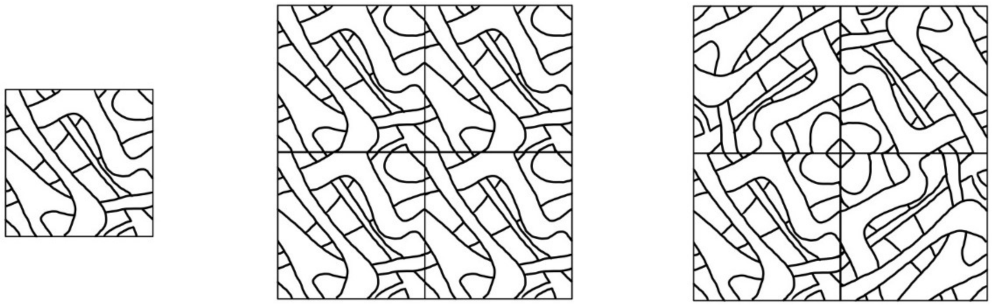
Escher Tilings

MC Escher was a Dutch graphic artist known for making mathematically based pieces of artwork (Kaplan, 2002). Famously, he created artwork from periodic tilings detailed to resemble familiar forms. These pieces of art became known as Escher Tilings and gained popularity due to their complex yet repetitive nature (Kaplan, 2002).

(a) (Escher, 1938) (b) (Escher, 1941) (c) (Escher, 1959)

Figure 5: Examples of Escher's Tilings

Escher's design process involved starting with a geometric tile known to be periodic, traditionally a square (Gethner, 2002). On the tile, he created intentional details called motifs that connected to themselves when the tile was rotated. The rotated forms of these motifs were called aspects, and the full rotational set of aspects formed a unique shape that was visually tessellated on a simple, periodic form (Gethner, 2002). Escher initially intended to sell his designs to a tiling company with the goal of creating more visually complex patterns with simple periodic forms (Gethner, 2002). An example of such tiling can be seen in Figure 6.



(a) (b) (c)

Figure 6: Escher-Styled Tile (a) in Translated (b) and Rotated (c) Tilings

As Escher's work developed, he began creating more complex periodic tiles (Gethner, 2002). It is unclear whether he followed a specific process in creating these tiles or whether he was able to conceptualize them in his mind, but all of his complex tiles derived from simple periodic tiles (Gethner, 2002). The process now known as Escherization involves starting with a common periodic tile, such as a square, and making identical changes to parallel edges until a desired form is achieved (Barth, 2007). The Escherization process can be seen in Figure 7. The Escherization process allows simple, geometric tiles to be transformed into more visually complex forms.

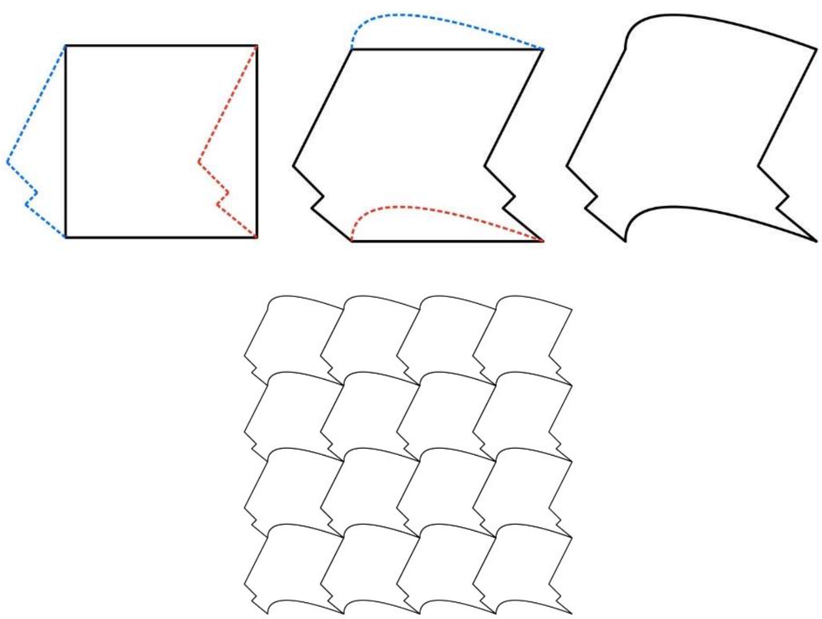


Figure 7: The Escherization Process and Resultant Tiling

Aperiodic Tilings

Unlike periodic tilings, aperiodic tilings have no translational symmetry (Cornell University, 2009). Further, aperiodic tilings do not have arbitrarily large regions with translational symmetry (Cornell University, 2009). In other words, for any fixed point of the plane, the image of that point under all translations of the tiling will never form a periodic tiling lattice, even for large regions of the tiling.

Penrose Tilings

Penrose tilings are one of the most well-known aperiodic tilings and are made of only two distinct tiles, typically the rhombus and the kite (Bruijn, 1981). Roger Penrose discovered the tilings in the early 1970s, setting the record for the lowest number of tiles necessary to create an aperiodic set (Cornell University, 2009). A Penrose tiling can be seen in Figure 8.

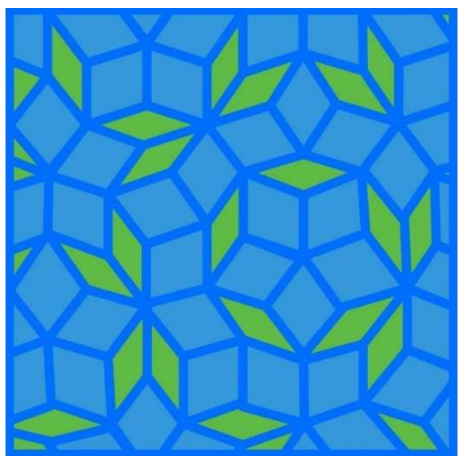


Figure 8: Penrose Tiling of Kites and Rhombuses

Penrose tilings are unique in the sense that they have no translational symmetry (Cornell University, 2009). Additionally, there are infinitely many distinct tile combinations of the rhombus and kite, and any finite region of a Penrose tiling occurs an infinite number of times within the tiling (Cornell University, 2009).

METHODOLOGY

The project had two primary goals: 1) to design a pavilion that addressed WPI's mental health crisis and the generation of plastic waste on campus, and 2) to understand the feasibility of recycled PLA as a structural material. These goals guided our methodology, from the initial stages of our pavilion concept to the design of our structural testing procedures. This section details the process of achieving our two primary project goals.

Architectural Pavilion Design

Our initial goal was to design a structure that addressed WPI's mental health crisis and the generation of plastic waste on campus. To guide our design process, we entered the Non Architecture International Waste Pavilion Competition, where participants were tasked with creating a pavilion made completely from recycled materials.

Our pavilion concept was based on the ever-changing nature of mental health. We designed a round structure with six curved walls. Each of the walls of the pavilion, varying in height, length, visual density, curvature, angle, and path spacing, create six unique entry points to the pavilion. The six entries each represent an emotion one may feel on their mental health journey: distracted, anxious, isolated, overwhelmed, resentful, and secure. The walls encompassing each path are designed to create conditions that resemble the emotions they are conveying. For example, 'isolated' has both walls curving inwards, creating a path that is shorter, darker, and enclosed. 'Secure', on the other hand, has both walls curving outward, opening the path to the most daylight to resemble a positive place in one's mental health journey. The outer wall spacing, wall thickness, rigidity, and color throughout the pavilion are consistent for structural and aesthetic purposes. Although the six emotions are innately different, their paths unite at the central gathering point. This is meant to demonstrate that although everyone's mental health journey is unique, the journey itself can be experienced by anyone. People can take any path along the pavilion, making it engaging for those experiencing it.

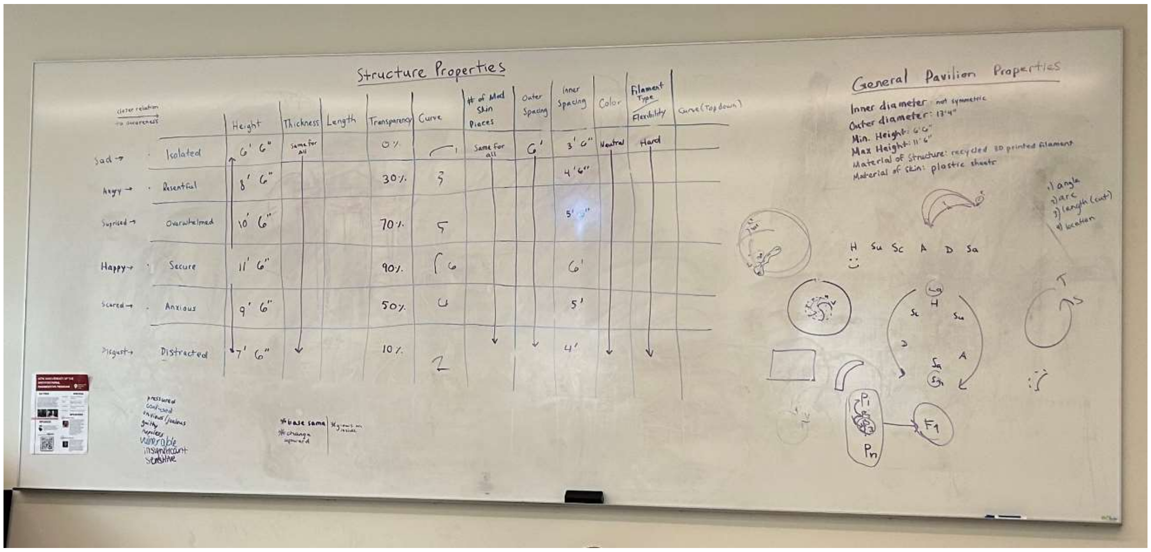


Figure 9: Initial Brainstorming for Form Design

Additionally, we wanted to create a way for people to interact with the pavilion. Between each of the structural components of the wall, we designed scale-like sheets that serve two purposes. Primarily, the scales shield the inside of the pavilion from outsiders looking in, helping create a centralized and semi-enclosed space. The scales also serve as an interaction point for those walking through, as they are designed to be written on. People engaging with the pavilion can write about their own mental health experiences on the scales or read the positive notes left by others. The goal of this feature is to unite the WPI community and promote discussion surrounding mental health on campus.

Rhino and Grasshopper

To create our complex form with so many variable parameters, we utilized Rhino, a design software, and Grasshopper, an object-oriented coding extension of Rhino. Grasshopper allowed us to input all our parameters, height, length, visual density, curvature, angle, and path spacing, to fully model our form in three-dimensional space. Each parameter was set uniquely for each of the six walls. We adjusted all parameters until the walls took our desired forms to represent each of the paths' emotions. The Grasshopper algorithm created to form the pavilion can be seen in Figure 10.

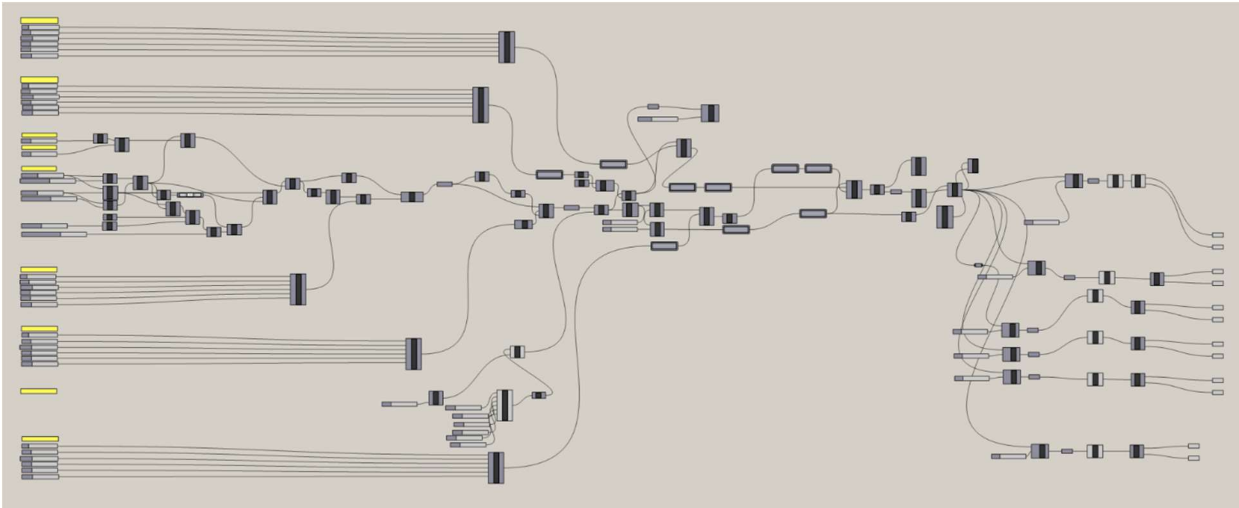


Figure 10: Grasshopper Algorithm for Pavilion

Mathematical Design Inspiration

The object-oriented structure of Grasshopper allowed the elements of the pavilion to be formed through mathematical principles and equations. With construction feasibility in mind, we wanted to make the pavilion semi-modular, meaning that sizable portions of the pavilion are identical. Initial designs of the pavilion featured hundreds of unique elements. Not only would such a design have been complicated to construct, but the randomness of the form would not have been psychologically pleasing to those engaging with the pavilion. In Grasshopper, we were able to simulate multiple periodic tilings in three-dimensional space by altering various parameters and components. Parameters and components can be customized and arranged in Grasshopper to create algorithms, making the modeling of tiling configurations in three-dimensional space easier to visualize. After modeling multiple configurations of tilings based on the designed general form of the pavilion, we decided on a rectangular periodic tiling for both the structure and substructure of the pavilion. The simplicity of the rectangular tiling allowed us greater flexibility on the design of each individual tile. We wanted the pavilion to be semi-open, meaning that there would still be partial visibility through and between the walls. Additionally, although the tiles were rectangular, we wanted to create the illusion of curves in the design to make the structure appear less visually rigid. We determined that a superimposed packing of elements that were rounded but not circular would best achieve our visual aesthetic goal. Our written algorithm created this superimposed packing on the rectangular tiling, which we applied to the overall pavilion form. Transforming the pavilion into a tessellated structure reduced the total number of producible elements from over two thousand to fifteen, with each finalized tile featuring a packing of fifteen three-dimensional components. The finalized structural tile can be seen in Figure 11.

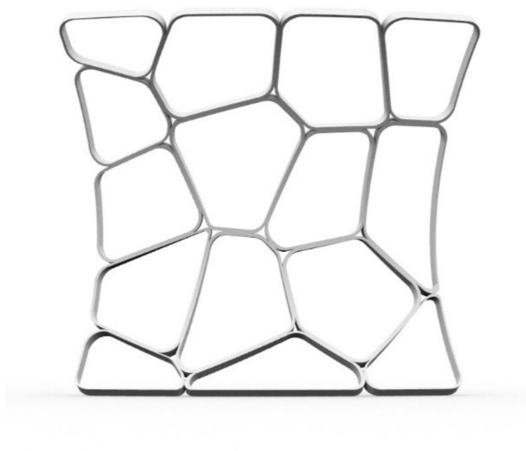


Figure 11: Finalized Structural Tile of Pavilion

Material Selection

With the design of the pavilion complete, we then selected our materials. As per the requirements of the Non Architecture Waste Pavilion Competition, our pavilion had to be made completely of recycled materials. We selected materials based on their relevance to WPI and their structural capabilities.

Polylactic Acid (PLA)

We decided to use Polylactic Acid, or PLA, as our primary material, as it is one of the most wasted on WPI's campus due to the school's considerable number of 3D printers. PLA is the most common 3D printing material available today. In fact, nearly all the 3D printers on WPI's campus use PLA. Compared to other 3D printing materials, PLA prints at relatively low temperatures and experiences much slower degradation at ambient temperature (25° C) (Deroiné, et al., 2014). Although PLA is composed of fermented corn starch and sugar cane, it does not biodegrade well, meaning it typically goes directly to a landfill after use. PLA has a recorded tensile strength of 47.8 MPa, a density of between 1.11 and 1.24 g/cm^3, a Young's Modulus of 4000 GPa, and a flexural strength of 80 MPa, making PLA a brittle material (Deroiné, et al., 2014). The global increase in plastic waste and PLA's prevalence on campus made it a prime material for our pavilion.

Figure 12: Polylactic Acid (PLA)

Jute Bast Fibers

Because PLA is a brittle material, it is not intended for use in largescale structures. We decided to create a composite material with PLA to increase its structural capabilities. Current research including a 2022 study conducted through the University of Applied Sciences in Bremen, Germany suggests that the integration of bast fibers with PLA improves the material's overall tensile strength (Graupner, Poonsawat, Narkpiban, & Müssig, 2022). Bast fibers are obtained from the outer layers of the stems of certain plants, including hemp, jute, and bamboo (Jones, Ormondroyd, Curling, & Popescu, 2017). We selected jute for our composite material due to its abundance, accessibility, and overall tensile strength. Jute Fibers have a tensile strength between 200 and 440 MPa, a density of 1.3 g/cm^3, and a Young's Modulus between 26 and 32 GPa (Salman, 2020).

Figure 13: Jute Fibers

Composite Material

Jute-PLA composites have been found to exceed the tensile capabilities of PLA alone. A 2021 study from the Vellore Institute of Technology found that 40% by volume jute composites had an average tensile strength of 58.82 MPa, exceeding the tensile strength of PLA alone (Shrivastava & Dondapati, 2021). Another study conducted by the University of Alberta in 2016 concluded that jute fibers were most effectively combined with PLA at a 20% by volume ratio (George, Chae, & Bressler, 2016). Additionally, the study found that jute was one of the most effective bast fibers for PLA composites due to its high cellulosic content, which increases adhesion with PLA (George, Chae, & Bressler, 2016). Because the study of bast fiber and PLA composites is fairly new, there is still great variation in data and deliberation on the material's true tensile capabilities.

Experimental Design

To achieve our second goal of understanding the feasibility of recycled PLA as a structural material, we conducted our own experimental tests on the tensile strength of PLA/jute composites. We created three variations of PLA for testing: a control made of 100% PLA, a 5% jute by mass PLA composite, and a 10% jute by mass PLA composite. Each of these variations underwent extensive tensile testing in a controlled environment, a high thermal environment, and a high moisture environment. We decided upon tensile testing because PLA typically is much weaker in tension than it is in compression. Additionally, recent studies on similar experiments were conducted in tension as well, providing us reference points for data comparison. We completely designed the experimental process followed to find our own tensile testing results.

Table 1: The Nine Variations of Tensile Tests Conducted, Each with Three Samples

Controlled Environment	High Thermal Environment	High Moisture Environment
0% Jute, 100% PLA	0% Jute, 100% PLA	0% Jute, 100% PLA
5% Jute, 95% PLA	5% Jute, 95% PLA	5% Jute, 95% PLA
10% Jute, 90% PLA	10% Jute, 90% PLA	10% Jute, 90% PLA

Composite Sample Preparation

To conduct our tensile testing, we first had to create our sample materials. We collected 1.5 kilograms of PLA waste from the Makerspace in WPI's Innovation Studio, which houses the majority of WPI's 3D printers. We based our sample preparation methods heavily on a 2020 WPI Major Qualifying Project, *Reuse Plastic for 3D Printing* (Feng, Kennedy, Miyajima, Ng, & Seo, 2020). The report detailed the process of recycling PLA from printed form back to filament, where the PLA must be shredded, washed, and dehydrated before being reformed and spun to become filament. We adapted this procedure to instead recycle PLA from printed form to molded form, allowing us to customize the forms we produced for tensile testing and prototyping components of our pavilion.

For the first step of our recycling process, shredding, we utilized the shredder designed and built as part of the 2020 *Reuse Plastic for 3D Printing* MQP. The shredder's base is a standard wood planer fastened to a cart. Atop the planer is a box with an eight-inch square hopper, both made from plywood and plexiglass, which encloses two claws that run back and forth over the planer. The claws are designed with interlocking teeth to push PLA down to the planer blade. To shred our PLA, we placed the waste prints in the hopper with the claws in an open position. Once the box and hopper were full of the waste PLA, we turned on the planer and unlocked the claws, moving them back and forth to force PLA through the planer. The shredded PLA then dropped into a five-gallon bucket below the shredding machine. This process was repeated until all PLA waste was shredded into chips from its original forms.

Figure 14: PLA Shredder

Varying chip sizes emerged from the PLA's first pass through the shredder. Based on Reuse Plastic for 3D Printing, the chips had to reach a diameter of 7 mm or smaller to be recycled effectively. We reduced this standard to 4 mm in diameter because we knew the chips would be molded instead of reheated, requiring them to be much finer. Most chips that emerged from the first shred were between 20 and 40 mm in diameter. After using a sieve to filter out the properly sized chips, we placed the large chips back in the shredder and repeated the process until all chips reached our desired size of less than 4 mm in diameter. Most chips took three passes through the shredder to reach the desired size, while some still needed additional shredding in a food processor to reach the 4 mm diameter standard. At the end of the shredding process, we collected approximately 1.2 kilograms of shredded PLA waste.

Figure 15: Shredded PLA at Varying Sizes

With the shredding complete, we then washed the PLA to ensure no dirt and debris were included in our samples. All the shredded PLA was rinsed with cold water multiple times in a clean 5-gallon bucket. The PLA was then laid out on aluminum sheets and left to dry for several hours. Once air dried, the sheets of shredded PLA were placed into a large oven at 40° C for six hours to be dehydrated. The dehydration of PLA is important because trapped moisture has the potential to break the interior polymer chains and weaken the material (Feng, Kennedy, Miyajima, Ng, & Seo, 2020).

In *Reuse Plastic for 3D Printing*, the shredded, washed, and dehydrated PLA then was reformed into filament. For the purpose of our project, we adapted the remaining procedure for molded composites and instead prepared our jute. We bound our jute fiber strands by hand and finely cut them down to 4 mm in length. We heated the jute for a few minutes in the oven to ensure it was dry, then weighed it to create our three sample materials.

Our three samples, 1) the control made of 100% PLA, 2) the 5% jute by mass PLA composite, and 3) the 10% jute by mass PLA composite, were made in batches. We split our prepared PLA into approximate thirds and weighed each sample. Sample 1, the control, was untouched, while samples 2 and 3 had to be effectively mixed. Once the 5% and 10% PLA by mass were added to samples 2 and 3 respectively, each sample was placed in the cleaned food processor and spun for two minutes. This process ensured that the jute was distributed evenly through the PLA samples to create accurate composites.

Dog Bone Development and Molding

The three samples then had to be molded into dog bones for tensile testing. We created our molds in accordance with ASTM standards for polymers (American Society for Testing and Materials, 2012), as detailed in Figure 16.

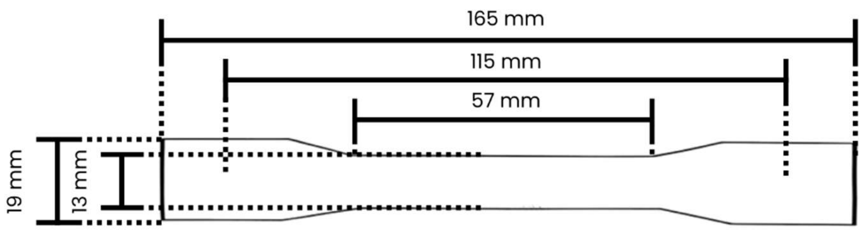


Figure 16: Dog Bone Dimensions for Testing by ASTM Standards

To create our mold, we first 3D printed an inverse of our desired dog bone mold with a large rim around the edges, shown in Figure 17.

Figure 17: Inverse 3D Printed Mold

We used Mold Star 30, a two-part hardening silicone that is heat resistant up to 232° C. After creating the silicone from even parts of its components, we poured it over the inverse dog bone mold. The mold was then placed on a vibrating table for forty-five minutes, the time taken to reach the silicone's setpoint, and left to fully cure for an additional five hours and fifteen minutes to ensure the silicone was fully hardened. We made two molds to increase the efficiency of our dog bone production.

Figure 18: Silicone Mold Formation on Vibrating Table

With our molds created, we began to form our dog bones from our sample materials. For each sample, we filled the mold tightly and ensured the PLA or PLA composite was packed down to avoid air bubbles. The filled molds were then placed on a tray in a sealed oven under a fume hood at 200° C for one hour.

Figure 19: Composite Material Melted into Mold

After leaving the mold to cool for an additional four hours, we removed the sample dog bones from the mold. Due to the settling of the PLA chips during the melting process, the tops of the dog bones were not level, but rather bumpy and non-uniform. To correct this, we used a drill press to file down the top faces of the dog bones and ensure they met the standardized dog bone specifications. We followed a similar process for molding and creating our pavilion component, as seen in Figure 20, but instead used a belt sander to flatten the uneven surfaces. Using this procedure, we created twenty-seven dog bones, or nine of each PLA sample type, and one complete pavilion component.

Figure 20: Molding Process of Pavilion Component

Composite Material Tensile Testing

We conducted our tensile tests under three conditions: 1) a controlled environment, 2) a warm thermal environment of 32 ° C, and 3) a high moisture environment. For each of the three conditions, we tested three control dog bones, three 5% jute by mass dog bones, and three 10% jute by mass dog bones. The controlled environment occurred at ambient temperature and standard interior humidity, the warm environment tensile tests took place in a thermal chamber set to 32 ° C, and the high moisture tensile tests were conducted on dog bones that had been submerged in water for 24 hours.

Figure 21: Tensile Test of Dog Bone

Structural Analysis of Pavilion

After determining the tensile strength of the PLA composites, we utilized the data to simulate our pavilion in ANSYS Fluent and test its structural capabilities. ANSYS Fluent is an engineering software used to simulate fluid flow, heat transfer, and deformation. We simulated three cases: 1) a control, 2) our pavilion exposed to high thermal conditions of 32° C, and 3) our pavilion exposed to extreme moisture conditions. In each case, we then simulated the pavilion's deformation under its own weight, as well as the pavilion's deformation under a wind load of 11.18 m/s (25.02 mph), the highest recorded wind velocity in the state of Massachusetts. A pressure of 25 psf was assumed for the testing, which is standard for the design of low-rise buildings.

Table 2: The Six Variations of ANSYS Fluent Simulations Conducted Based on the Three Environmental Conditions

0% Jute, 100% PLA	**5% Jute, 95% PLA**	**10% Jute, 90% PLA**
Self-Weight	Self-Weight	Self-Weight
Wind Load	Wind Load	Wind Load

RESULTS & FINDINGS

Our project focused on not only the complete development of an architectural form, but on better understanding the structural capabilities of PLA composites. The following section describes the finalized architectural form and explains the additional tasks taken on to introduce our project and its message to WPI's campus. This section also analyzes the results of our tensile testing and details the structural analysis of the finalized pavilion form.

Architectural Design

Our pavilion form underwent multiple iterations before reaching its finalized state. As detailed in the Methodology, our pavilion featured six unique paths each meant to convey a specific emotion. By using Rhino and Grasshopper, we developed a pattern for the structural components that was tessellated yet had the appearance of being random. The inner details of the structural components followed a similar tessellation pattern to make the construction of our pavilion more feasible in real-world applications. We focused on developing the structural components of our pavilion for our project, but we included further developments to the pavilion, such as the fins, in the final renders and competition submission. The finalized pavilion form developed in Rhino can be seen in Figure 22.

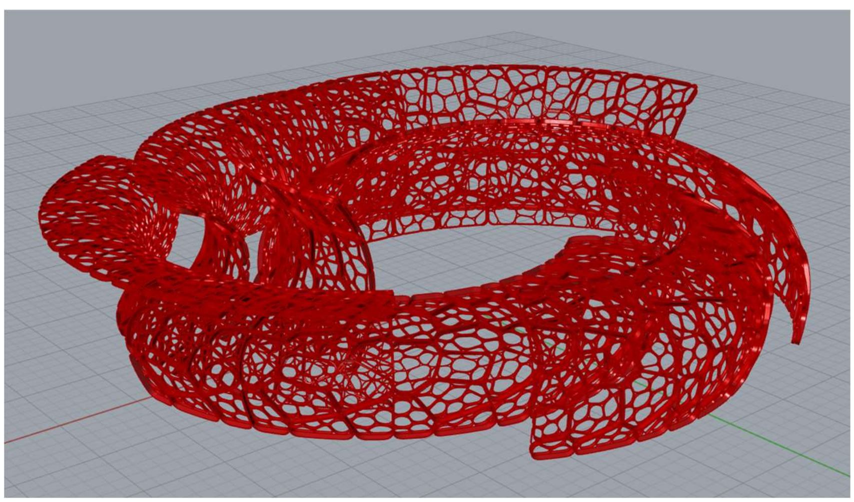


Figure 22: Finalized Pavilion form in Rhino

Non Architecture Competition Results

We entered our finalized pavilion in the 2023 Non Architecture International Waste Pavilion Competition. We were allowed a single render of the pavilion, one paragraph describing our purpose, and five words to summarize the intentions behind the design. Our competition render can be seen in Figure 23 and our competition brief and keywords can be found in Figure 24. Our pavilion was a finalist in the competition, earning a publication in the Non Architecture Competitions Journal.

Figure 23: Non Architecture Competition Render

"When looking at a person, it is challenging to understand what they are going through. Mental health awareness is a rising issue, especially on college campuses. Sometimes, it can feel like everyone is on their own path, lost in an entanglement of their emotions. Motus demonstrates that although everyone's mental health journey may appear different, it is the journey itself that brings people together to combat the mental health crisis. There are six points of entry to the pavilion, each representing an emotion one may feel on their mental health journey. The six paths, distracted, anxious, isolated, overwhelmed, resentful, and secure, although innately different, unite at the central gathering point. Our pavilion dually sheds light on the college mental health crisis and the wastefulness of innovation. 3D printed plastic is one of the most discarded products at our engineering university, hence Motus is constructed completely of recycled plastic. A combination of largescale molding and 3D printing is utilized to achieve the complex form. Our hope is to bring awareness to our college campus on the interconnected nature of mental health through the relevant usage of 3D printed waste and our study of architectural engineering."

#community engagement #recycling #mental health #3D printing #acceptance

Figure 24: Non Architecture Competition Brief

Connections with the Center for Well-Being

During the laboratory-based phase of our project, we contacted WPI's Center for Well-Being (CWB) to inform them of our project and its purpose in representing WPI students' mental health. Additionally, we wanted to discuss the potential to display a scaled component of our design somewhere within the CWB. During initial conversations with Paula Fitzpatrick, Director of the Center for Well-Being, we learned that the CWB is hoping to expand their grounds to the outdoors, creating not only a new entryway to Daniels Residence Hall, but a space for outdoor mindfulness activities to be held. The exterior expansion is only in the conceptual stage as funding for the project is short, but a local landscape architecture firm came for an initial visit to provide estimates. With our background in architectural engineering and our prior project research, we offered Paula Fitzpatrick to produce some initial site plans and renders for the project. This not only allowed the CWB to save money on project development costs but provided us with a direct application for our research and ultimate project goal.

Center for Well-Being Developments

After initial discussions, we began to meet regularly with Paula Fitzpatrick to learn more about the CWB's vision for the outdoor space. In order to make the project more financially feasible, we organized the design into phases, which would allow for the CWB to proceed with renovating as they have the budget to do so. An initial concept for the lawn that we felt aligned with our pavilion was 'labyrinth', the idea that the space would feature multiple paths and spaces for students to decompress at their own pace. The discussed first phase of the project included regrading the ground, removing and replacing the existing walkways, and relandscaping. Completion of the first phase would make the lawn usable to students until further developments occur. Because our project focuses on sustainability in construction, we also tried to preserve as much of the preexisting landscaping in the first phase of the renovation. As design developments continued, the potential to renovate the CWB lawn increased. The proposed renovations included a new entryway to Daniels Residence Hall, and as a result, WPI's Housing and Residential Experience Center (HRE Center) joined the design process. The designs would create a common outdoor space for future WPI residential developments to utilize, making the proposal equally appealing to the HRE Center. Current versions of the CWB Lawn design can be seen in Figure 25 and Figure 26. As of Spring 2024, these lawn designs are in the finalization stages, and construction is set to begin in Summer 2024. The new lawn space will include a redesigned and scaled version of the waste pavilion for future WPI students to enjoy.

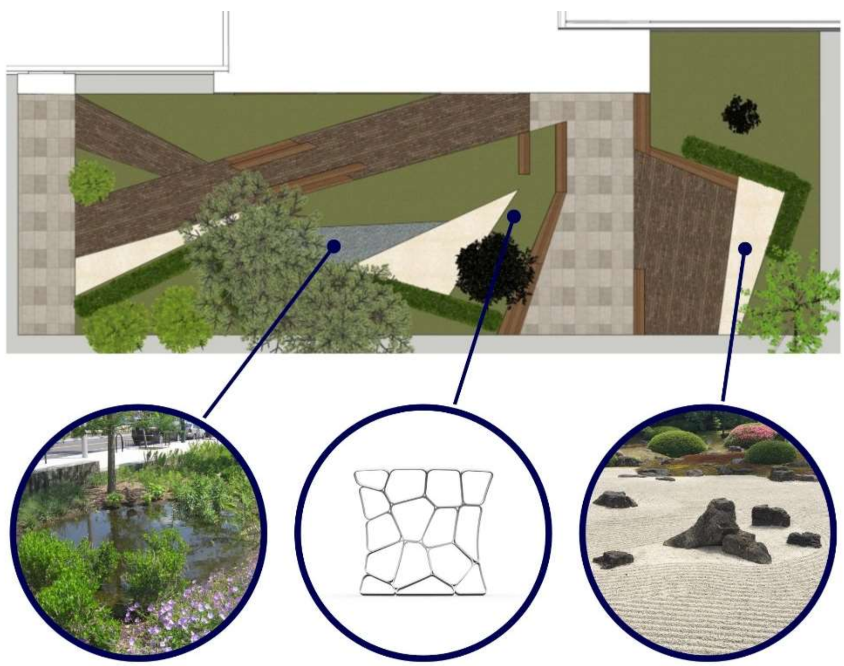


Figure 25: Site Plan for CWB Lawn Development

Figure 26: *Render of CWB Lawn Development*

Mechanical Strength of Composite Material

Our tensile tests were conducted over the span of two weeks in a controlled laboratory environment. Conditions of the space and equipment were kept consistent to reduce potential error in our results. A total of twenty-seven tests were conducted: nine under each condition, three of each being for different percentages of jute by mass. The averages of the three tests for each composition were taken for comparison. All dog bones were 58 mm in length with total areas ranging from 44.2 mm^2 to 70.2 mm^2. The complete data from these tensile tests, including the maximum load, tensile strength, and tensile strain at maximum load for each trial, can be found in the Appendix.

For condition 1) the controlled environment, the 0% jute by mass dog bones had an average tensile strength of 14.24 MPa, the 5% jute by mass dog bones had an average tensile strength of 9.30 MPa, and the 10% jute by mass dog bones had an average tensile strength of 11.92 MPa.

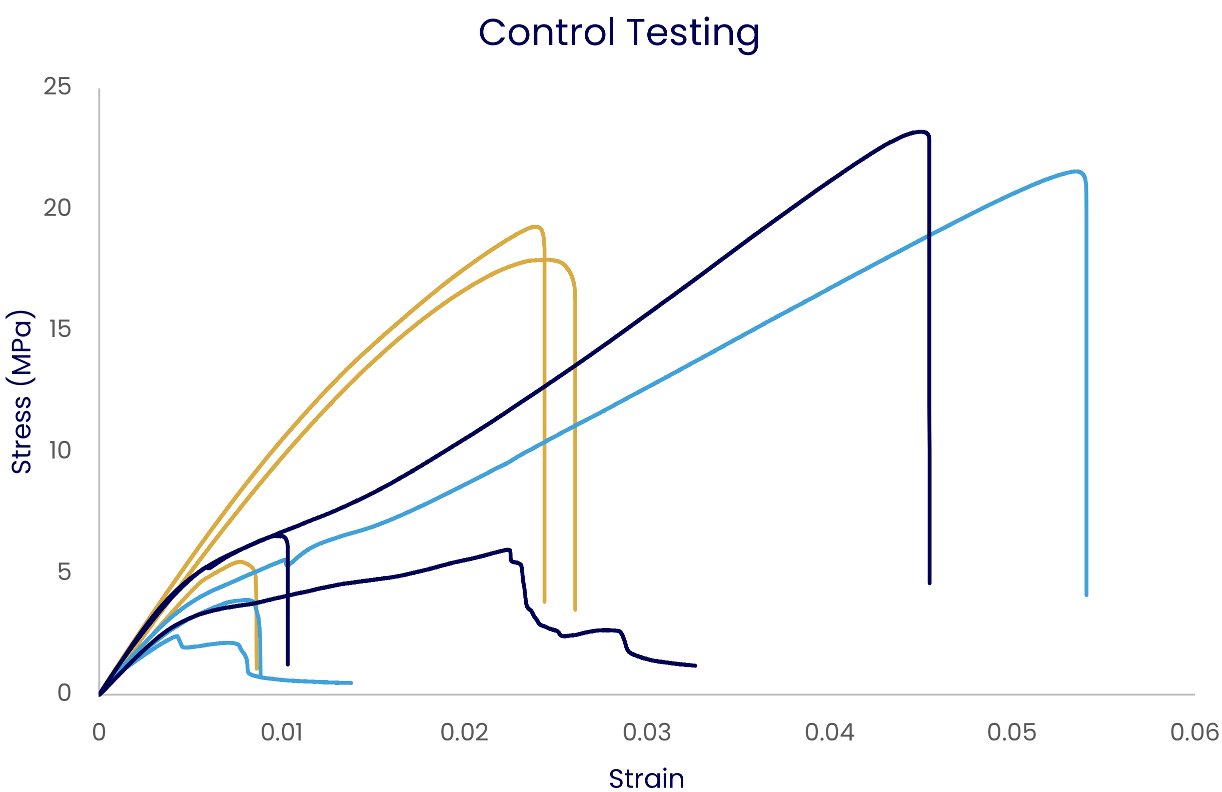


Figure 27: Stress-Strain Curves of Control Tensile Tests

The 0% jute by mass dog bones were the highest performing of the controlled environment, followed by the 10% jute by mass dog bones. Test 3 of the 5% jute by mass dog bones recorded a tensile strength of 21.58 MPa, and Test 2 of the 10% jute by mass dog bones had a tensile strength of 23.21 MPa, the two highest under those conditions. The 0% jute by mass dog bones were the most consistent in tensile strength compared to the two composites.

For condition 2) the warm thermal environment of 32 ° C, the 0% jute by mass dog bones had an average tensile strength of 3.02 MPa, the 5% jute by mass dog bones had an average tensile strength of 15.44 MPa, and the 10% jute by mass dog bones had an average tensile strength of 14.79 MPa.

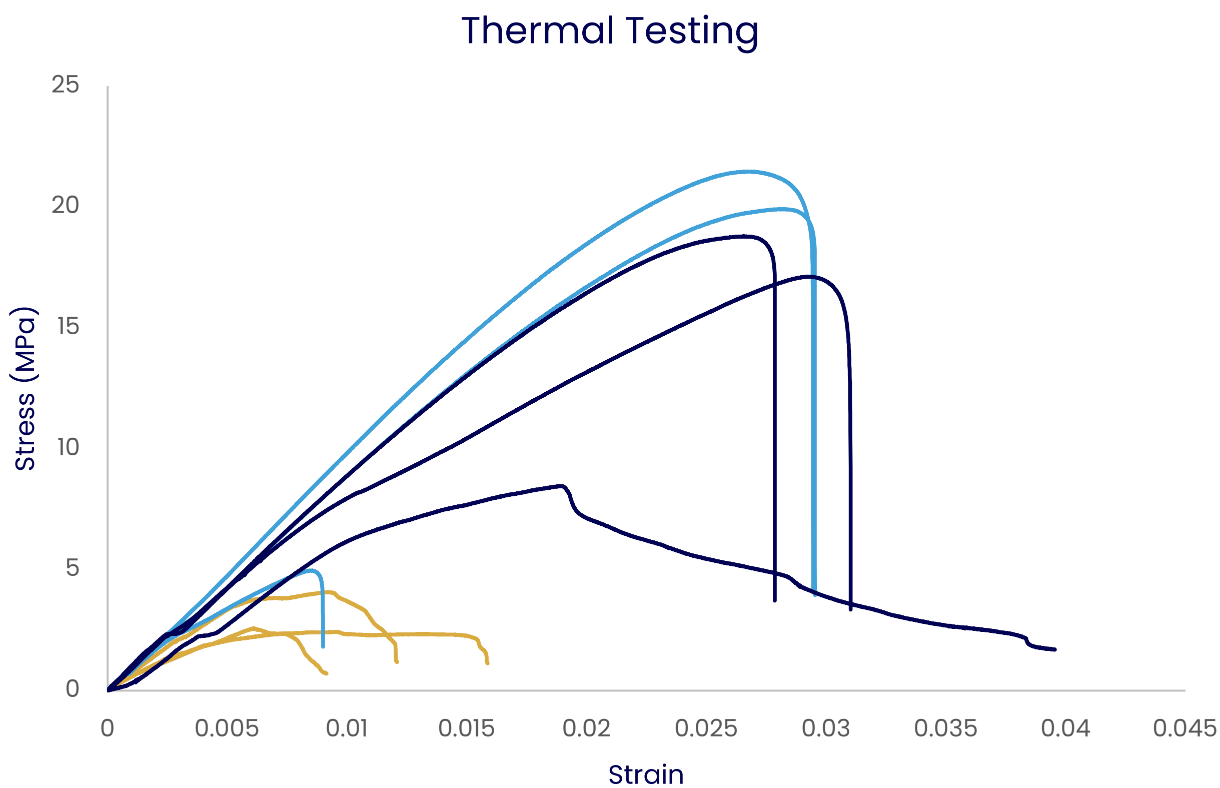


Figure 28: Stress-Strain Curves of High Thermal Tensile Tests

The 5% by mass dog bones had the best tensile performance of the thermal environment, followed closely by the 10% jute by mass dog bones. The 0% jute by mass dog bones performed significantly worse in thermal conditions than in controlled conditions, with average tensile strengths of 14.24 MPa and 3.02 MPa, respectively. Since PLA is a brittle material that is molded by heat, it is understandable that reexposure to heat would negatively impact the material's tensile capabilities. Test 2 of the 5% jute by mass dog bones recorded a tensile strength of 21.47 MPa, which is comparable to its maximum performing test in the controlled environment.

Finally, for condition 3) the high moisture environment, the 0% jute by mass dog bones had an average tensile strength of 9.03 MPa, the 5% jute by mass dog bones had an average tensile strength of 15.96 MPa, and the 10% jute by mass dog bones had an average tensile strength of 12.37 MPa.

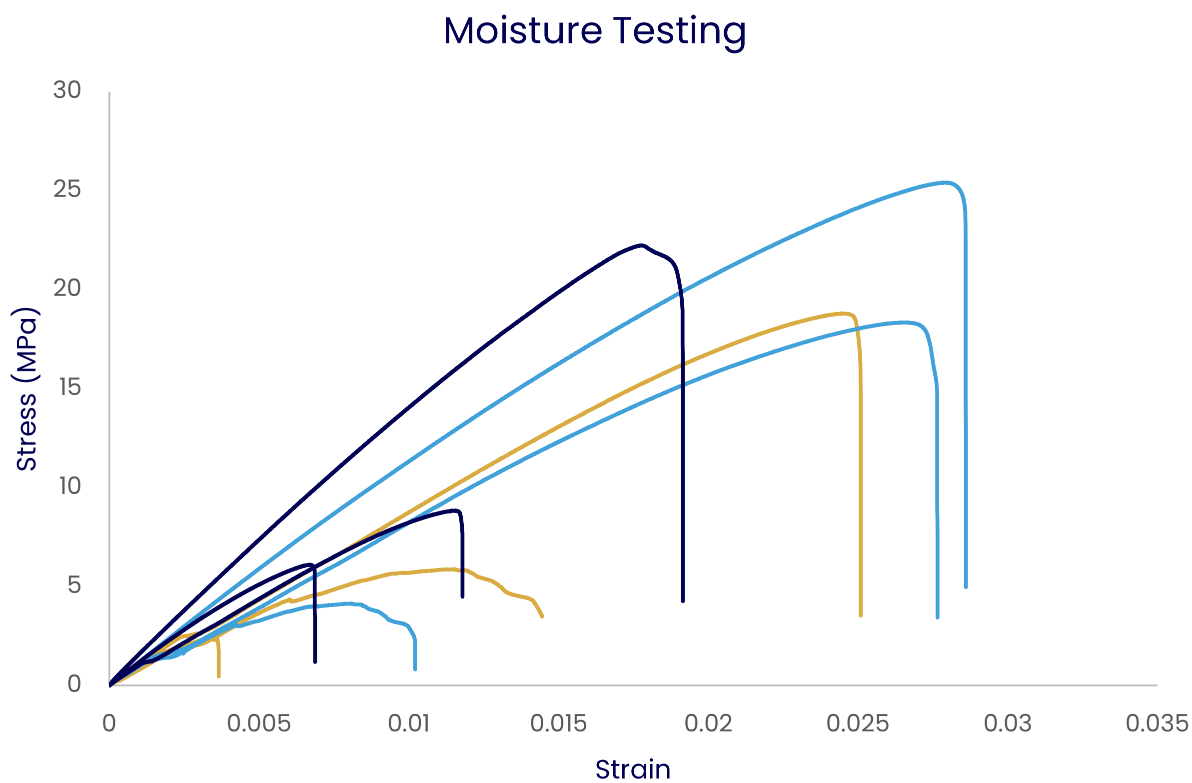


Figure 29: Stress-Strain Curves of High Moisture Tensile Tests

Similar to the thermal environment, the 5% by mass dog bones had the best tensile performance, with the 10% jute by mass dog bones being the second best performing. Test 3 of the 5% jute by mass dog bones had a tensile strength of 25.41 MPa, the highest recorded tensile strength of all dog bone trials under every tested condition. The 0% jute by mass dog bones performed better in moisture conditions than thermal conditions, with average tensile strengths of 3.02 MPa and 9.03 MPa, respectively. However, both tests were significantly below the tensile strength of the 0% jute by mass dog bones tested in the controlled environment, which had an average tensile strength of 14.24 MPa. Of the 5% jute by mass tests across all three environments, the thermal and moisture tests with average tensile strengths of 15.44 MPa and 15.96 MPa were equally high performing in comparison to the other composite levels. The 10% jute by mass tests had the highest average tensile strength of 14.79 MPa under thermal conditions, performing better than the control, which had an average tensile strength of 11.92 MPa. Thus, in the samples with 0% jute by mass, the tensile capabilities were weakened under thermal and moisture conditions, while in the jute composite samples, the tensile capabilities were improved.

Table 3: Summarized Average Tensile Strength Results of Composite Material

Testing Environment	Composite Type	Average Tensile Strength (MPa)
Control	0% Jute	14.24
	5% Jute	9.30
	10% Jute	11.92
Thermal	0% Jute	3.02
	5% Jute	15.44
	10% Jute	14.79
Moisture	0% Jute	9.03
	5% Jute	15.96
	10% Jute	12.37

When considering the best performing of the three composites across the three testing environments, the 0% jute by mass samples do not compete with the two jute composites. Although the 0% jute by mass dog bones were the best performing in the controlled environment, they fell short when compared to the tensile strength of both the 5% and 10% composites in thermal and moisture conditions. For PLA to be used structurally in real-world applications, it needs to be able to withstand uncontrolled conditions.

The 5% and 10% jute by mass samples performed similarly in all three tests, with the 10% jute by mass samples performing higher in the controlled test and the 5% jute by mass samples performing higher in thermal and moisture conditions. By the same logic that determines the 0% jute by mass samples to be low performing, the 5% just by mass is the best performing of the two composites, as it yielded the highest average tensile strength for challenging conditions of both temperature and moisture.

Findings from Structural Analysis of Pavilion

Under the assumption that our pavilion design would be constructed of 5% jute by mass composite, we utilized ANSYS to simulate our structure under various conditions determined through the dog bone trials. We tested the deformation of the structure under its own weight, the deformation of the structure due to wind loads of 11.18 m/s (25.02 mph), and the yielding of the structure due to wind loads. Each of these three were conducted on conditions from each of the three cases of 5% jute by mass composite from the averages of the tensile testing: 1) the control, where the average tensile strength was found to be 9.30 MPa, 2) the pavilion exposed to high thermal conditions of 32° C, where the average tensile strength was found to be 15.44 MPa, and 3) the pavilion exposed to extreme moisture conditions, where the average tensile strength was found to be 15.96 MPa. We determined the density of the 5% jute by mass composite to be 1.186 g/cm^3. Additionally, we calculated the modulus of elasticity for each of the three cases; the 5% jute by mass dog bones had an average modulus of elasticity of 354.87 in the controlled environment, 735.06 in the high thermal environment, and 624.11 in the high moisture environment. Based on this data, ANSYS produced renders that highlighted the areas of greatest deflection and yielding when using the 5% jute by mass composite.

In Figure 30 and Figure 31, areas of high deformation or yielding can be seen in red, while areas of lesser deformation can be seen in colors along the spectrum up to zero deformation, represented by dark blue.

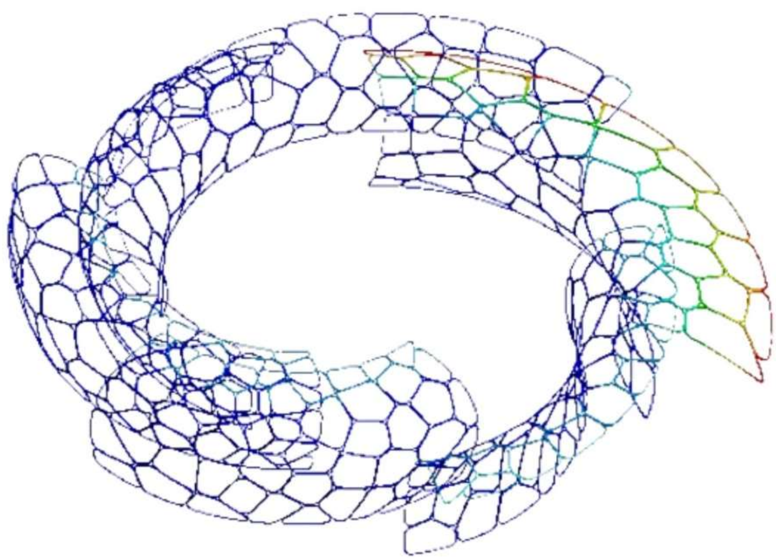


Figure 30: ANSYS Fluent Simulation Results for Pavilion Under Self-Weight

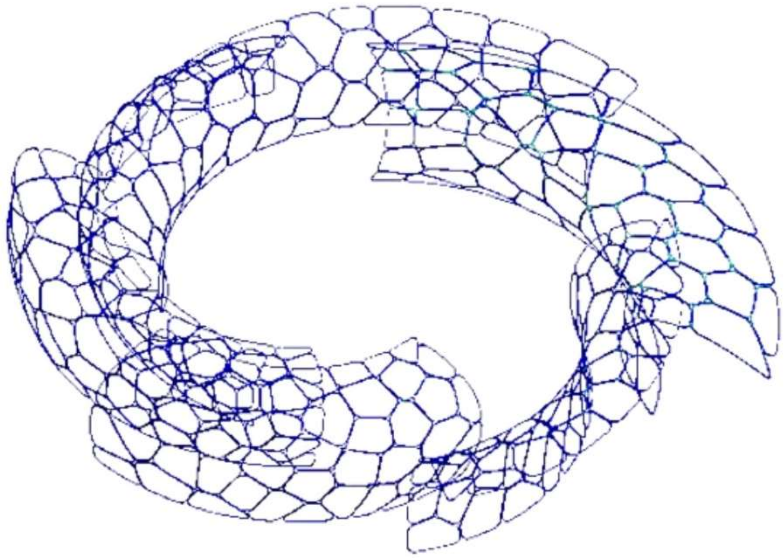


Figure 31: ANSYS Fluent Simulation Results for Pavilion Under Wind Load

CONCLUSION

The tensile test results supported the feasibility of jute-PLA composites as a structural material and highlighted the potential for PLA waste to be repurposed instead of contributed to general waste production. As a result, we gained a greater understanding of the feasibility of using recycled PLA as a structural material. Additionally, we were successful in designing a pavilion that addressed both WPI's mental health crisis and the generation of plastic waste on campus. After earning a finalist designation in the 2023 Non Architecture Waste Pavilion Competition, we were able to develop the pavilion in ways that will benefit the WPI community and Center for Well-Being for years to come. We look forward to seeing our pavilion and subsequent CWB lawn designs take form and serve their purpose in promoting well-being on campus.

Purpose of Work

The work in this project highlights the potential for heightened architectural development through the integration of mathematics. The architectural design of the project was greatly enhanced by the application of tilings, both aesthetically in the form of visual appeal and functionally in the form of component modularization. The combination of architectural and mathematical perspectives allowed for the pavilion purpose to extend beyond the competition and have potential long-standing impacts on WPI campus through the Center for Well-Being.

Limitations

Various limitations were encountered throughout the design process that could be improved upon for future extensions of the project. In the design phase of the project, we were unable to fully develop the modular skin façade for the pavilion walls. Material selection, attachment style, and replaceability of the fins would need to be considered before the pavilion components could be created to their fully intended design. Additionally, the attachment of individual pavilion components was not investigated in this project, and determining methods for attaching structural components would require further testing with the PLA-just composite.

During the laboratory phase of the project, certain equipment limitations affected the overall accuracy of our testing. Most notably, we were unable to remove all air bubbles from our dog bone samples, creating potentially weak points in the samples during the tensile tests. Additionally, an industrial shredder would be required for further development of the composite material. Although the student-made shredder was able to achieve its purpose, the shredding process was extensive and could have been shortened significantly by a more effective piece of machinery. By the end of the shredding process, the student-made shredder fell apart and was disassembled into parts with hopes of being developed in the future for other recycling related projects.

Future Work

As developments for the Center for Well-Being lawn space are finalized, we hope to continue contributing to the design and development processes until the project is completed. Additional developments may need to be considered as the lawn space grows and serves new purposes over its years with the CWB.

Further experiments are necessary to determine the true structural capabilities of jute-PLA composites as a structural material. Compression, shear, and low thermal, and low moisture tests would be beneficial for a more complete understanding of PLA-jute composite capabilities in an outdoor, variably conditioned environment. Additional testing is required to understand the feasibility of composites with other bast fibers or plastic types.

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APPENDIX

Below is the complete data from all twenty-seven tensile tests conducted. This data includes the maximum load, tensile strength, and tensile strain at maximum load for each trial.

Table 4: Complete Tensile Testing Data

Testing Environment	Composite Type	Maximum Load (N)	Tensile Strength (MPa)	Average Tensile Strength (MPa)	Area (mm^2)	Length (mm)	Tensile Strain (Extension) Gauge Length (mm)	Tensile Strain (Extension) at Maximum Load (mm/mm)	E (stress/strain)	Average E
Control	0% Jute	242.414	5.48		44.2	58	58	0.01	548	
		878.062	19.3		45.5	58	58	0.02	965	
		816.047	17.94	14.24	45.5	58	58	0.02	897	803.333
	5% Jute	203.254	3.91		52	58	58	0.01	391	
		170.064	2.42		70.2	58	58	0.01	242	
		1178.07	21.58	9.30	54.6	58	58	0.05	431.6	354.867
	10% Jute	295.391	5.98		49.4	58	58	0.02	299	
		1146.81	23.21		49.4	58	58	0.05	464.2	
		324.117	6.56	11.92	49.4	58	58	0.01	656	473.067
Thermal	0% Jute	107.391	2.43		44.2	58	58	0.01	243	
		113.6	2.57		44.2	58	58	0.01	257	
		180.08	4.07	3.02	44.2	58	58	0.01	407	302.333
	5% Jute	218.621	4.95		44.2	58	58	0.01	495	
		1116.3	21.47		52	58	58	0.03	715.67	
		1087.21	19.89	15.44	44.2	58	58	0.02	994.5	735.056
	10% Jute	976.921	18.79		52	58	58	0.03	626.33	
		450.735	8.46		53.3	58	58	0.02	423	
		912.247	17.12	14.79	53.3	58	58	0.03	570.67	540
Moisture	0% Jute	1001.69	18.79		53.3	58	58	0.02	939.5	
		258.348	5.87		52	58	58	0.02	293.5	
		112.768	2.42	9.03	54.6	58	58	0.02	121	451.333
	5% Jute	220.661	4.14		53.3	58	58	0.01	414	
		1120.75	18.34		61.1	58	58	0.03	611.33	
		1255.21	25.41	15.96	49.4	58	58	0.03	847	624.111
	10% Jute	1098.39	22.23		49.4	58	58	0.02	1111.5	
		298.388	6.12		61.1	58	58	0.02	306	
		453.835	8.75	12.37	54.6	58	58	0.03	291.67	569.722