#### Abstract

Conventional ultrasound imaging systems use array transducers for focusing and beam steering, to improve lateral resolution and permit real-time imaging. This thesis research investigates a different use of array transducers, where the acoustic field and the receiver characteristics are designed such that the energy of the output signal from targets of a specified geometry is maximized. The output signal is the sum of the received signals obtained using all the possible combinations of transducer array elements as transmitter and receiver. This work is based on annular array transducers, but is applicable for any array configuration.

The first step is the development of software for the efficient modeling of the wave interaction between transmitted field and target, and between the transducer and receiver field. Using this software, we have calculated the received signal for each combination of an array element as transmitter and the same or another array element as receiver, leading to an  $N \times N$  received signal matrix for an N element array transducer. A waveform optimization algorithm is then implemented for the purpose of determining the set of delays for the individual array elements, which maximizes the energy of the sum of the received signals. In one implementation of this algorithm, the received signal with the maximum energy is considered as a reference signal, and specific delays are applied to the other signals so that any two signals produce a maximum correlation. This leads to an  $N \times N$  delay matrix, which, however, is not readily implemented in a practical real-time system, which uses all the elements in an array transducer simultaneously to customize

acoustic fields. Hence, the values in this delay matrix are fed into a linear programming optimizer tool to obtain a set of delay values, which makes its implementation practical.

The optimized set of delays thus obtained is used to maximize the energy of the received signal for a given transducer and target geometry and hence to enhance the reflectivity of that target. It is also important to check the robustness of the optimized set of delays obtained above, for a given target geometry. Robustness refers to the sensitivity of the optimization to variation in target geometry. This aspect is also evaluated as a part of this thesis work.

### Acknowledgement

This thesis most definitely wouldn't have been completed without the encouragement, motivation and best wishes that I have received in the past few years from several people. I would like to take this opportunity to express my gratitude to all these people. Above all, I would like to say a big big thank you to my advisor Prof. Peder.C.Pedersen. Working as a part of the ultrasound lab has given me the opportunity to learn about some new trends in the Medical imaging industry, thanks to Prof. Pedersen. His primary objective to produce novel and good quality work taught me a lot of things in addition to the research oriented matter, which will definitely help me in the long run. I am especially grateful for the cooperation he has offered in the last two years and the time he has spent in refining my thesis write-up. I would also like to express my gratitude to Prof. Reinhold Ludwig and Prof. Sergey Makarov for readily accepting to be on my thesis committee and reviewing my work.

I would like to thank my lab mates: Dalys Sebastian, Deepti Sukhwani, Ruben Lara Montalvo and Carsten Poulsen for all their help, great memories, and a wonderful and fun work atmosphere. I have to sincerely acknowledge my dear friends: Mili, Siddhesh, Kamal-Renu and Rahul-Shivani; my roomies: Vishal, Dharmesh, Vinay and Ritesh for all the encouragement that I have received through the ups and downs I encountered while completing this thesis. Thanks to all my other friends and colleagues at EMC for the positive advice and support.

Last but not at all the least, this really wouldn't have been possible without the blessings and best wishes from my parents, my dear brother Akshay and my family back home. They are my source of motivation and I owe this experience to them.

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