IQP JAG 9003: MA Teacher Licensing

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Chapter 1

Tahanto Regional High School is a small, regional high school located in Boylston, Massachusetts. The towns that make up the region are Boylston and Berlin. However, those are not the only towns that are represented by the student population due to school choice. Other towns include: Clinton, Millbury, Leominster, and the city of Worcester. Conversely, students from Boylston and Berlin leave the school district and pursue other options at other surrounding high schools, such as Hudson High School, Assabet Regional Vocational Technical High School, Parker Charter School, St. John's Shrewsbury and St. Peter-Marian. There are 488 students[1] (48% male, 52% female) currently enrolled in Tahanto with 35 full-time teachers present.[2] The 13:1 student-teacher ratio is the same as the state average student-teacher ratio.[3] The student population is not very diverse at all. The breakdown of the students' ethnic background are: 95% White (as opposed to 72% state average), 2% Hispanic (13%), Asian/Pacific Islander 2% (5%), American Indian <1% (<1%) and Black, Non-Hispanic <1% (8%).[4]

These figures are a direct correlation to the demographics of the towns themselves. The town of Boylston is a rural, typical New England town. It is 19.67 square miles in area and has a population of 3,517 with 96.7% being White, 1.4% Asian, 0.7% African-American and 0.8% 2+ races.[5] As of the 2000 census, the town of Boylston's per capita in come was $32,274, compared to the national average of $21,587.[6] The town of Berlin is very similar in figures (in terms of race percentages). Berlin, like Boylston, is a rural, typical New England town. It is 13.09 square miles in area and has a population of 2,293 with 97.6% White, 1.0% Asian, 0.2% African-American and 0.8% 2+ races.[7] Berlin, however, has not developed economically as much as Boylston has recently. Berlin's per capita income, according to the 2000 census, was $28,915, as opposed to the national average of $21,587.[8]

The state of Massachusetts tests all students throughout their educational career with a standardized test known as the MCAS. Once a student reaches the 10th grade, their MCAS

[1] www.greatschools.net/modperl/browse_school/ma/196 ; accessed Dec. 12, 2007
[2] www.publicschoolreview.com/school_ov/school_id/37227 ; accessed Dec. 12, 2007
[3] www.greatschools.net/modperl/browse_school/ma/196 ; accessed Dec. 12, 2007
[4] www.greatschools.net/modperl/browse_school/ma/196 ; accessed Dec. 12, 2007
[5] www.mass.gov/dhed/iprofile/039.pdf "Demographics" ; accessed Dec. 1, 2007
[6] www.epodunk.com/cgi-bin/genInfo.php?locIndex=2884#econ "Economy" ; accessed Dec. 1, 2007
[7] www.mass.gov/dhed/iprofile/028.pdf "Demographics" ; accessed Dec. 1, 2007
[8] www.epodunk.com/cgi-bin/genInfo.php?locIndex=2874 "Economy" ; accessed Dec. 1, 2007

scores determine whether or not they can graduate high school. However, the MCAS also tests the school system to see how well the teachers perform. To do this, the state collects the MCAS score data on every school so they can plot an Adequate Yearly Progress (AYP) chart to see how each school performs according to the state's regulations. Once the state has the data, and plots it, they analyze how the school has performed, and it gives the school a rating based on performance and rates how they can improve in both English/Language Arts and Mathematics. Tahanto's ratings are shown in the table below:

Subject	Performance Rating	Improvement Rating
English/Language Arts	Very High	On Target
Mathematics	High	On Target

Data obtained on Dec. 1, 2007 from: http://profiles.doe.mass.edu/ayp/ayp_report/school.aspx?fycode=2007&orgcode=06200505

There are certain guidelines that the state places on the AYP, which includes: participation, performance, improvement and graduation rates. The table of how Tahanto performed according to the state's regulations is shown in the table below:

	Participation (95%)	Performance (Eng/LA=85.4%) (Math=76.5%)	Improvement	Graduation Rate (92% att. G1-8) (55% grad. rate)
Subject; Met Target	Eng/LA; YES Math; YES	Eng/LA; YES Math; YES	Eng/LA; YES Math; YES	Eng/LA; YES Math; YES
Subject; Actual	Eng/LA; 99% Math; 100%	Eng/LA; 96.4% -Spec. Ed: 85.3% Math; 81.8% -Spec. Ed: 59.2%	Eng/LA; +0.7% Math; +4.6%	Eng/LA; 95% Math; 96%

Data obtained on Dec. 1, 2007 from:
http://profiles.doe.mass.edu/ayp/ayp_report/school.aspx?fycode=2007&orgcode=06200505
*State regulations in parenthesis; The improvement percentages are based on the previous year's data.

In 2006, Tahanto performed very well on the MCAS. The results are based on a Composite Performance Index (CPI), which is a score out of 100 that the school achieves based on students' performances. In English, Tahanto scored a 98.3/100 on the CPI, which placed them 9th in the state out of 284 school districts in the state.[9] In Math, Tahanto scored 93.8/100 on the CPI,

[9] http://profiles.doe.mass.edu/mcas.aspx ; accessed Dec. 1, 2007

which placed them 68th in the state.[10] MCAS scores are based on four different categories: advanced, proficient, needs improvement and failed. The table below shows the breakdown of the scores Tahanto students had on the 2006 MCAS.

English/Language Arts:

Score	Number of Students	Percentage of Class
Advanced	24	33%
Proficient	41	57%
Needs Improvement	5	7%
Failed	2	3%

Data obtained on Dec. 1, 2007 from: http://profiles.doe.mass.edu/mcas.aspx

Mathematics:

Score	Number of Students	Percentage of Class
Advanced	29	40%
Proficient	31	42%
Needs Improvement	8	11%
Failed	5	7%

Data obtained on Dec. 1, 2007 from: http://profiles.doe.mass.edu/mcas.aspx

Tahanto has many advantages as opposed to some of the other surrounding schools. These advantages include a favorable student-teacher ratio (13:1) and a very small student body so the teachers can get to know the students on a more personal level. These advantages seem to have a direct effect on how well the students of Tahanto perform on the MCAS. Tahanto's performance in English/Language Arts was "very high" and they were "on target" for improvement rating. Also, 90% of the 10th grade students scored advanced or proficient. The school's performance in Math was "high" and, like English/Language Arts, they were "on target" for their improvement rating. Also, 82% of the 10th grade students scored advanced or proficient.

[10] http://profiles.doe.mass.edu/mcas.aspx ; accessed Dec. 1, 2007

Chapter 2

The following is a list of courses that were taught during the student teaching process: Algebra I Level 2, Precalculus Level 0 (Pre-AP), AP Calculus AB and AP Calculus BC. Tahanto Regional High School is a Middle/High School with grades 7-12. Students are tested in the sixth grade at the feeder schools, Berlin and Boylston Elementary Schools, and school-choice students are tested during the summer and then placed in a mathematics class based on their performance on the entrance exam. Incoming students are placed in 7th grade math, 7th grade Pre-Algebra, or 8th grade honors (Algebra I Level I) depending on their performance. The track that students take depends on where they have been placed.

	Level 2	Level 1	Level 0
7th grade	7th grade math	Pre-Algebra	Algebra I
8th grade	Pre-Algebra	Algebra I	Geometry I
9th grade	Algebra	Geometry I	Algebra II
10th grade	Geometry II	Algebra II	Precalculus
11th grade	Algebra II-II	Precalculus	AP Calculus AB
12th grade	(optional) Functions Statistics and Trigonometry Or Personal Finance	AP Calculus AB	AP Calculus BC

If students are shown to be successful, they may cross over into a higher level of math. Therefore, the courses that I taught had students of different grade levels to include sophomores taking Precalculus and one junior taking AP Calculus BC.

The Algebra I Level 2 class that I taught had freshman and 11 out of 12 were on an IEP.[11] The level 2 curriculum is based on the Discovering Series.[11] The Discovering Algebra series works with data from the physical and social sciences, emphasizes techniques for data analysis, and uses technology tools such as graphing calculators to help visualize and explore algebraic concepts.[11] The material is designed to have the students "discover" the algebra while performing different explorations throughout the text. As the students do the investigations, they

[11] Meeting with Francene Gleason. 19 December 2007.

are required to think about the examples and exercises and need to discuss with each other the mathematics before they are given instruction. Steps are given to guide the students through an exploration. This method of learning provides the students with a hands-on approach and reinforces concepts that appeal to visual learners. The general philosophy in this course is that all students are capable of learning mathematics and that all students will be successful because of the process of discovering the mathematics. This series is used for Geometry Level II and for Algebra II Level II.[11] Students are required to take three years of mathematics and thus this series ends at Algebra II. If students wish to continue their mathematics education, they may take Functions, Statistics, and Trigonometry (a more rigorous course), or they may enroll in Personal Finance.

In order to be successful in the Algebra II Level II course, the students must have mastered 7th grade math and pre-algebra. Students engage in problem solving, communicating, reasoning, and connecting to understand and apply concepts of variable expression and equation. They represent situations and number patterns with tables, graphs, and verbal rules and equations, and explore the interrelationships of these representations. Students analyze tables and graphs to identify properties and relationships and demonstrate an ability to solve linear equations using concrete, informal and formal methods. Students describe strategies used to explore inequalities and nonlinear equations. Students apply algebraic methods to solve a variety of real-world problems. Students explore and describe a variety of ways to solve equations including hands-on activities, trial and error, and numerical analysis. Students know and apply algebraic procedures for solving equations and inequalities.

The strands that run through the Algebra I-II course are number sense and operations, patterns, relations, and algebra, data analysis, statistics, and probability.[1] In the number sense and operations strand, the students are encouraged to improve their understanding of numbers.[11] They are introduced to a variety of ways to represent numbers and learn to better understand number systems on higher level than in middle school. They learn how operations are related to one another by learning the properties of numbers. Connections are made to real-life situations and students develop an ability to make estimates based on their understanding of number sense. In the patterns, relations, algebra strand, students develop a deeper understanding of relations and functions and patterns.[11] They learn how to define functions algebraically, numerically, and graphically. They learn how to interpret data and create functions based on the data. They explore a variety of patterns including linear, quadratic, and exponential. In the data analysis and probability strands, the students develop their understanding of how to represent data in tables, stat plots, box plots, histograms, and circle graphs.[11] They use statistical analysis to analyze the data in the various representations.

The Precalculus course at Tahanto is offered at only one level. It is an accelerated math course designed to prepare the students for an AP Calculus class. The classes that I taught were a mix of sophomores, juniors, and seniors. The curriculum is based on the textbook, *Precalculus: Graphing and Data Analysis*, by Michael Sullivan and Michael Sullivan III and is published by Prentice Hall. The course is designed for students where it might be their last math class they ever take and for others who are preparing for an AP Calculus course or preparing for college. Because of the rigor of the course, it is considered a Level 0 course. This series is designed to be mathematically comprehensive and provides excellent preparation for courses taken in the future. The material is presented to reach students of varying learning styles and abilities. Graphing technology is integrated throughout the course. The graphing calculator is used as a tool for discovering mathematics and for problem solving. Students study a full catalog of functions at the beginning of the course. Students explore the twelve basic functions and their algebraic properties and are encouraged to connect the math algebraically, graphically, and numerically in preparation for AP Calculus. The students are given sufficient opportunity to modeling using real-world applications. They learn domain, range, symmetry, continuity, end behavior, asymptotes, extrema, and periodicity. Once students have a comfortable understanding of functions, the rest of the course consists of studying the various types of functions in depth and modeling the behavior of functions as well as introducing topics of calculus. In order for students to be successful in Precalculus they must have obtained a grade of no lower than an 88 in Algebra II – Level I or an A in Functions, Statistics, and Trigonometry. Students are only allowed to take the course with permission from the instructor.

The strands that run through the Precalculus course include number sense and operations strand, the patterns, relations, and algebra strand, the geometry strand, and the data analysis, statistics, and probability strand.[11] Complex numbers are developed more at this level and polar coordinates are introduced. Mathematical induction is used to prove theorems when taught sequences and series. Students delve deeper into the study of functions. Polynomials are examined more closely. Trigonometric functions are introduced including proving and using identities and relating trig functions to geometry. Students learn to graph trig functions and apply transformations to the graphs. Ultimately the student is given a solid base for taking an advanced placement calculus course. A sample of the Pre-Calculus curriculum framework and the learning standards for Pre-Calculus are located in the appendix under Figures 13 and 14 respectively.

The AP Calculus AB class consisted of 21 students. AP Calculus is taught using *Calculus: Graphical, Numerical, Algebraic* by Demana, Finney, Waits, Kennedy. The curriculum is designed around the new advanced placement calculus course description.[11] Calculus is explored using the rule of four: algebraic, numerical, graphical, and verbal.[11] Students are expected to use the rule of four to investigate and solve problems. The use of technology is integrated throughout the book. The goals of the advanced placement calculus curriculum as described in the course description at AP Central by CollegeBoard are reflected throughout the text. Students are rigorously taught with enough time allowed during the month of April to practice for the AP exam in May. In order to be successful in AP Calculus AB, the student must have obtained an average of no lower than a B+ in Precalculus and must be recommended to the course by the instructor of Precalculus. The AP Calculus syllabus is located in the appendix under Figure 12.

The AP Calculus BC course consists of 6 students. This course is taught using *Calculus of a Single Variable* by Larson, Hostetler, and Edwards. This program offers comprehensive coverage of the material required by students in BC Calculus as noted in the BC Calculus description put out by AP Central. The program offers a curriculum that is pedagogically sound and comprehensible. Students in BC Calculus are the top students in the school. To reach this class, students must have entered the 7th grade enrolled in Algebra I Level I.

Chapter 3

Throughout the course of my student teaching, it was required that I develop lesson plans based on the curriculum. Listed are some of the different lesson plans that I developed along with an explanation of the rationale behind each lesson.

Real-Life Application Quadratics

This lesson plan was designed to allow the students to discover and explore the quadratic relationship between height and time within the context of a real-life application. Students were introduced to quadratic functions via an example of a real-life situation. After watching a video on the overhead computer, students were given questions to answer based on the application. After the lesson was completed, students were given other examples to do for homework. All of the homework questions were on applications of quadratics.

Real Life Application-Quadratic Functions

Objective: To discover and explore the quadratic relationship between height and time within the context of a real-life situation: a pilot being ejected from an aircraft via the ejection seat.

Using the idea of a pilot being ejected from an ejection seat, students will explore and manipulate a quadratic equation that models the change in height over time of a pilot being ejected from his or her jet and then falling to the ground with the aid of a parachute.

Background: In collaboration with the Naval Air Warfare Center Weapons Division at China Lake, CA and the Canadian National Defense Headquarters, the NASA Dryden Flight Research Center has been conducting flight research obtained through a series of ejection tests. Results from research carried out since 1995 show that NASA Dryden successfully replaced the current parachute canopy used in F/A-18 aircraft in the United States and Canåda with a modified C-9 parachute canopy. The new escape system has proven so far to be safer and provide a slower descent and softer landing with reduced pressure on the parachute when it opens.

Model: $-16t^2 + 608t + 4{,}482$

1. **What is the maximum height (relative to the ground) reached by the pilot after being ejected?**

2. **How many feet above the jet was the pilot ejected?**

3. **When did the pilot reach the ground with the aid of his or her parachute?**

Biorhythms-Precalculus

In preparing this lesson plan, I focused on an application of the trigonometric functions. Students were required to research what biorhythms actually were prior to this assignment. This lesson allowed the students to see a real-life application of sinusoidal functions. Students needed to have prior knowledge on how to graph sine and cosine graphs.

Biorhythms

Biorhythm theory states that a person's biological functioning is controlled by three phenomena that vary sinusoidally with time. It uses the graphs of three simple sine functions to make predictions about an individual's physical, emotional, and intellectual potential for a particular day. The graphs are given by y=Asin(Bx) where x=0 corresponds to a person's day of birth and where A=1 is used to denote 100% potential.

The theory states that the physical cycle affects aspects of the body. It encompasses your energy levels, your resistance, your overall physical strength, your eye-hand coordination, your endurance, and your resistance to disease. During the positive half of the cycle, you will feel your best. During the down half of the cycle, you are likely to have less energy and less vitality.

The emotional cycle influences our emotional states, affecting love and hate, optimism and pessimism, passion and coldness, and depression and elation. At the peak, you are feeling most creative, loving, and warm and are probably more open in your relationships. Lower levels have an inclination to leave you withdrawn, less cooperative, irritated, and negative.

The intellectual cycle influences our memory, alertness, speed of learning, reasoning ability, and accuracy of computation. At the peak, you will be most intellectually responsive and open to accepting and understanding new ideas, theories, and approaches. At lower levels, you will have difficulty grasping new ideas and concepts. •

Graphing the Sinusoidal Function y=Asin(Bx)

- If the physical cycle has a period of 23 days, then $\frac{2\pi}{B} = 23$. Solving, we find that B=_______. The physical biorhythm can be described by the equation $y = \sin((\frac{2\pi}{23})x)$. Enter this in your calculator.
- If the emotional cycle has a period of 28 days, them B=________, and the emotional biorhythm can be described by y=___________. Enter this in your calculator.
- If the intellectual cycle has a period of 33 days, then B=_______, and the intellectual biorhythm can be described by y=___________. Enter this in your calculator.

How to Find Your Window
The x-min is the number of days old you are as of today, x-max can be 30 more days than that. To find the number of days old you are, use the days between dates feature on the calculator.

Example: from November 13, 1973 through January 15, 2006
dbd(11.1373,01.1506)

x-min=_______ x-max= x-min + 30= _________

Questions

1. Label your graph completely.

2. Safety experts have often warned drivers that emotional and sensitivity factors contribute to many accidents. Data collected from several sources indicate that an unusually high percentage of careless or self-caused accidents occur on critical days of the 28-day cycle. What are your critical days of the emotional cycle for the current month?

3. Some doctors claim that the fluctuations of the intellectual cycle are closely related to the secretion of the thyroid gland. According to some studies, the first 16.5 days of this cycle are when students and others are more able to absorb new concepts and make more progress. Using the current month, describe where you are in your intellectual cycle. Be sure to include your critical days in your description.

4. According to some hospital research, a patient does better if he or she chooses to have an optional procedure done on a day when the physical cycle is high and avoids critical days. Assuming the research is correct and that you need some elective surgery, show what days of your physical cycle would be high and what critical days should be avoided during the current month.

5. Maxine said that the emotional cycle always crosses the median point on the same day of the week on alternate weeks. Is she correct?

6. Tony said that if you examine the emotional cycle on alternate weeks, it produces a pattern wherein the day of the week is always a high or low. Is he correct?

7. All three life cycles start at zero at birth because it is the beginning of independent life for the organism. How long does it take for all three cycles to get back together again at zero?

8. Mark McGuire hit his 70th home run September 27, 1998. Mark was born October 1, 1963. As his biorhythm advisor, what would you tell him after observing his graphs?

9. Greg Maddux had his 300th victory on August 7, 2004. He failed to get his 300th on Aug 1. Greg was born April 14, 1966. As his biorhythm advisor, what would you tell him after observing his graphs?

Trig Project-Precalculus

This lesson plan was designed to show the students how the sine and cosine graphs are transformed based on the changes in amplitude, period, phase shift and vertical shift. Students needed to know how to draw the sine and cosine graphs because they were required to draw the graphs on a coordinate plane, then draw several graphs which had different amplitudes and analyze the changes that took place. The students also had to do that for period, phase shift and vertical shift changes.

Title: Exploring Variations in Trig Functions

Overview: This lesson will demonstrate the various translations of $y = A \sin B(x + D) + C$ based upon changes in A, B, C, and D.

Grade Level: Grades 10, 11, and 12; Precalculus

Prerequisite Knowledge:
Students must have basic knowledge of the parent graphs of the trigonometric functions of sine and cosine.

Objectives:

- The student will be able to predict the resulting graph from any single change of parameter in a trigonometric function.
- Students in each group must confer and agree on the prediction of the above changes.
- Each student will be required to perform a specific duty in compiling the discoveries of the group.

Evaluation:
Student achievement will be based on the student summary sheets and graphs of the functions. Students will be graded on neatness, accuracy, and overall understanding.

Extension/Follow-Up:

- In future lessons explore periodic real-world phenomena using the sine and cosine functions – sound waves, radio frequency, tides and hours of daylight.
- Have a student bring in one of their musical instruments. A CBL (TI Calculator Based Laboratory) would then be hooked up to a microphone and different pitches would be tried. For each pitch, one group of students will come up and carefully figure out the period of the curve. The frequency may now be found by using the equation $f=2\pi/p$ where f is the frequency, and p is the period of the graph.

Curve Sketching-Calc AB and BC

This lesson plan was developed to help students gain the understanding of what the graph of a function looks like using the function's first and second derivatives. Students needed to know how to obtain a function's derivative and how to apply it to graph analysis. Also, students had to be able to connect the graph of the derivative to the graph of the function without having a function. This allowed the students to gain a deeper understanding of how the graphs of the first and second derivative relate to one another.

The Shape of a Graph, Part I

In the previous section we saw how to use the derivative to determine the absolute minimum and maximum values of a function. However, there is a lot more information about a graph that can be determined from the first derivative of a function. We will start looking at that information in this section. The main idea we'll be looking at in this section we will be identifying all the relative extrema of a function.

Let's start this section off by revisiting a familiar topic from the previous chapter. Let's suppose that we have a function, $f(x)$. We know from our work in the previous chapter that the first derivative, $f'(x)$, is the rate of change of the function. We used this idea to identify where a function was increasing, decreasing or not changing.

Before reviewing this idea let's first write down the mathematical definition of increasing and decreasing. We all know what the graph of an increasing/decreasing function looks like but sometimes it is nice to have a mathematical definition as well. Here it is.

Definition

1. Given any x_1 and x_2 from an interval *I* with $x_1 < x_2$ if $f(x_1) < f(x_2)$ then $f(x)$ is ***increasing*** on *I*.
2. Given any x_1 and x_2 from an interval *I* with $x_1 < x_2$ if $f(x_1) > f(x_2)$ then $f(x)$ is ***decreasing*** on *I*.

This definition will actually be used in the proof of the next fact in this section.

Now, recall that in the previous chapter we constantly used the idea that if the derivative of a function was positive at a point then the function was increasing at that point and if the derivative was negative at a point then the function was decreasing at that point. We also used the fact that if the derivative of a function was zero at a point then the function was not changing at that point. We used these ideas to identify the intervals in which a function is increasing and decreasing.

The following fact summarizes up what we were doing in the previous chapter.

Fact

1. If $f'(x) > 0$ for every *x* on some interval *I*, then $f(x)$ is increasing on the interval.
2. If $f'(x) < 0$ for every *x* on some interval *I*, then $f(x)$ is decreasing on the interval.
3. If $f'(x) = 0$ for every *x* on some interval *I*, then $f(x)$ is constant on the interval.

The proof of this fact is in the Proofs From Derivative Applications section of the Extras chapter.

Let's take a look at an example. This example has two purposes. First, it will remind us of the increasing/decreasing type of problems that we were doing in the previous chapter. Secondly, and maybe more importantly, it will now incorporate critical points into the solution. We didn't know about critical points in the previous chapter, but if you go back and look at those examples, the first step in almost every increasing/decreasing problem is to find the critical points of the function.

Example 1 Determine all intervals where the following function is increasing or decreasing.

$$f(x) = -x^5 + \frac{5}{2}x^4 + \frac{40}{3}x^3 + 5$$

Solution

To determine if the function is increasing or decreasing we will need the derivative.

$$\begin{aligned} f'(x) &= -5x^4 + 10x^3 + 40x^2 \\ &= -5x^2\left(x^2 - 2x - 8\right) \\ &= -5x^2(x-4)(x+2) \end{aligned}$$

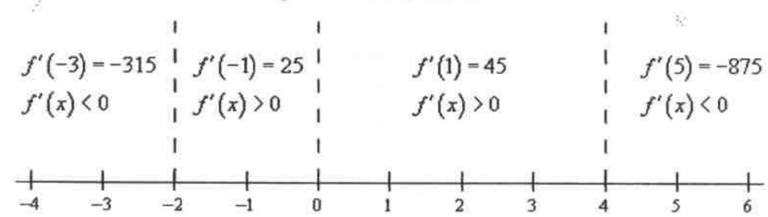
Note that when we factored the derivative we first factored a "-1" out to make the rest of the factoring a little easier.

From the factored form of the derivative we see that we have three critical points : $x = -2$, $x = 0$, and $x = 4$. We'll need these in a bit.

We now need to determine where the derivative is positive and where it's negative. We've done this several times now in both the Review chapter and the previous chapter. Since the derivative is a polynomial it is continuous and so we know that the only way for it to change signs is to first go through zero.

In other words, the only place that the derivative *may* change signs is at the critical points of the function. We've now got another use for critical points. So, we'll build a number line, graph the critical points and pick test points from each region to see if the derivative is positive or negative in each region.

Here is the number line and the test points for the derivative.



Make sure that you test your points in the derivative. One of the more common mistakes here is to test the points in the function instead! Recall that we know that the derivative will be the same sign in each region. The only place that the derivative can change signs is at the critical points and we've marked the only critical points on the number line.

So, it looks we've got the following intervals of increase and decrease.

$$\text{Increase}: \quad -2 < x < 0 \text{ and } 0 < x < 4$$
$$\text{Decrease}: \quad -\infty < x < -2 \text{ and } 4 < x < \infty$$

Note that often the fact that only a single point separates the two intervals of increase will be ignored and the interval will be written $-2 < x < 4$.

In this example we used the fact that the only place that a derivative can change sign is at the critical points. Also, the critical points for this function were those for which the derivative was zero. However, the same thing can be said for critical points where the derivative doesn't exist. This is nice to know. A function can change signs where it is zero or doesn't exist. In the previous chapter all our examples of this type had only critical points where the derivative was zero. Now, that we know more about critical points we'll also see an example or two later on with critical points where the derivative doesn't exist.

How that we have the previous "reminder" example out of the way let's move into some new material. Once we have the intervals of increasing and decreasing for a function we can use this information to get a sketch of the graph. Note that the sketch, at this point, may not be super accurate when it comes to the curvature of the graph, but it will at least have the basic shape correct. To get the curvature of the graph correct we'll need the information from the next section.

Let's attempt to get a sketch of the graph of the function we used in the previous example.

Example 2 Sketch the graph of the following function.

$$f(x) = -x^5 + \frac{5}{2}x^4 + \frac{40}{3}x^3 + 5$$

Solution

There really isn't a whole lot to this example. Whenever we sketch a graph it's nice to have a few points on the graph to give us a starting place. So we'll start by the function at the critical points. These will give us some starting points when we go to sketch the graph. These points are,

$$f(-2) = -\frac{89}{3} = -29.67 \qquad f(0) = 5 \qquad f(4) = \frac{1423}{3} = 474.33$$

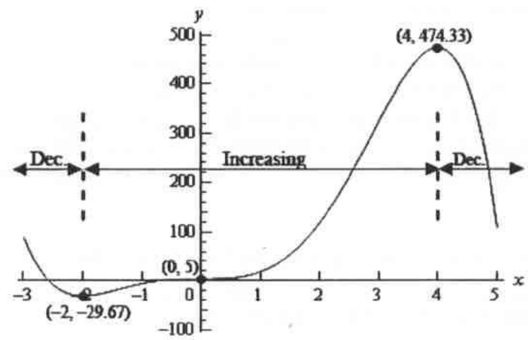
Once these points are graphed we go to the increasing and decreasing information and start sketching. For reference purposes here is the increasing/decreasing information.

$$\text{Increase : } -2 < x < 0 \text{ and } 0 < x < 4$$
$$\text{Decrease : } -\infty < x < -2 \text{ and } 4 < x < \infty$$

Note that we are only after a sketch of the graph. As noted before we started this example we won't be able to accurately predict the curvature of the graph at this point. However, even without this information we will still be able to get a basic idea of what the graph should look like.

To get this sketch we start at the very left of the graph and knowing that the graph must be decreasing and will continue to decrease until we get to $x = -2$. At this point the function will continue to increase until it gets to $x = 4$. However, note that during the increasing phase it does need to go through the point at $x = 0$ and at this point we also know that the derivative is zero here and so the graph goes through $x = 0$ horizontally. Finally, once we hit $x = 4$ the graph starts, and continues, to decrease. Also, note that just like at $x = 0$ the graph will need to be horizontal when it goes through the other two critical points as well.

Here is the graph of the function. We, of course, used a graphical program to generate this graph, however, outside of some potential curvature issues if you followed the increasing/decreasing information and had all the critical points plotted first you should have something similar to this.



Let's use the sketch from this example to give us a very nice test for classifying critical points as relative maximums, relative minimums or neither minimums or maximums.

Recall Fermat's Theorem from the Minimum and Maximum Values section. This theorem told us that all relative extrema (provided the derivative exists at that point of course) of a function will be critical points. The graph in the previous example has two relative extrema and both occur at critical points as the Fermat's Theorem predicted. Note as well that we've got a critical point that isn't a relative extrema ($x = 0$). This is okay since Fermat's theorem doesn't say that all critical points will be relative extrema. It only states that relative extrema will be critical points.

In the sketch of the graph from the previous example we can see that to the left of $x = -2$ the graph is decreasing and to the right of $x = -2$ the graph is increasing and $x = -2$ is a relative minimum. In other words, the graph is behaving around the minimum exactly as it would have to be in order for $x = -2$ to be a minimum. The same thing can be said for the relative maximum at $x = 4$. The graph is increasing of the left and decreasing on the right exactly as it must be in order for $x = 4$ to be a maximum. Finally, the graph is increasing on both sides of $x = 0$ and so this critical point can't be a minimum or a maximum.

These ideas can be generalized to arrive at a nice way to test if a critical point is a relative minimum, relative maximum or neither. If $x = c$ is a critical point and the function is decreasing to the left of $x = c$ and is increasing to the right then $x = c$ must be a relative minimum of the function. Likewise, if the function is increasing to the left of $x = c$ and decreasing to the right then $x = c$ must be a relative maximum of the function. Finally, if the function is increasing on both sides of $x = c$ or decreasing on both sides of $x = c$ then $x = c$ can be neither a relative minimum nor a relative maximum.

These idea can be summarized up in the following test.

First Derivative Test

Suppose that $x = c$ is a critical point of $f(x)$ then,

1. If $f'(x) > 0$ to the left of $x = c$ and $f'(x) < 0$ to the right of $x = c$ then $x = c$ is a relative maximum.
2. If $f'(x) < 0$ to the left of $x = c$ and $f'(x) > 0$ to the right of $x = c$ then $x = c$ is a relative minimum.
3. If $f'(x)$ is the same sign on both sides of $x = c$ then $x = c$ is neither a relative maximum nor a relative minimum.

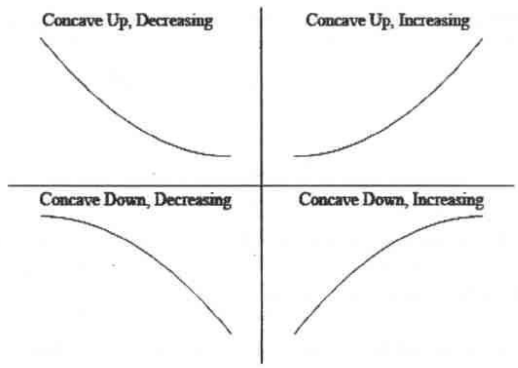
It is important to note here that the first derivative test will only classify critical points as relative extrema and not as absolute extrema. As we recall from the Finding Absolute Extrema section absolute extrema are largest and smallest function value and may not even exist or be critical points if they do exist.

The first derivative test is exactly that, a test using the first derivative. It doesn't ever use the value of the function and so no conclusions can be drawn from the test about the relative "size" of the function at the critical points (which would be needed to identify absolute extrema) and can't even begin to address the fact that absolute extrema may not occur at critical points.

The Shape of a Graph, Part II

In the previous section we saw how we could use the first derivative of a function to get some information about the graph of a function. In this section we are going to look at the information that the second derivative of a function can give us a about the graph of a function.

Before we do this we will need a couple of definitions out of the way. The main concept that we'll be discussing in this section is concavity. Concavity is easiest to see with a graph (we'll give the mathematical definition in a bit).



So a function is **concave up** if it "opens" up and the function is **concave down** if it "opens" down. Notice as well that concavity has nothing to do with increasing or decreasing. A function can be concave up and either increasing or decreasing. Similarly, a function can be concave down and either increasing or decreasing.

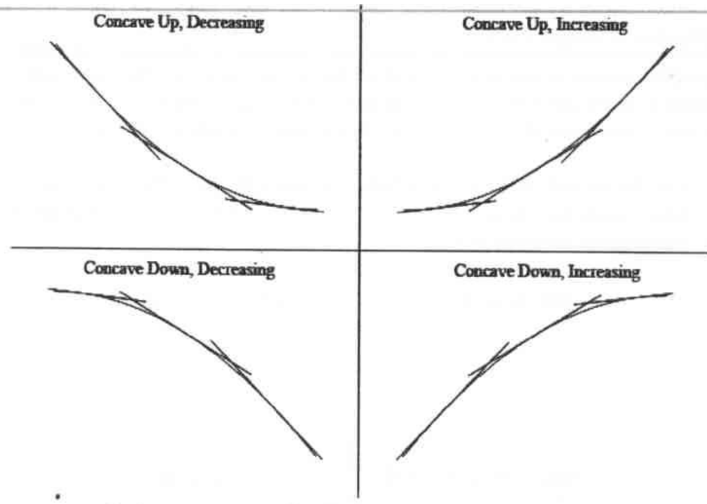
It's probably not the best way to define concavity by saying which way it "opens" since this is a somewhat nebulous definition. Here is the mathematical definition of concavity.

Definition 1

Given the function $f(x)$ then

1. $f(x)$ is **concave up** on an interval I if all of the tangents to the curve on I are below the graph of $f(x)$.
2. $f(x)$ is **concave down** on an interval I if all of the tangents to the curve on I are above the graph of $f(x)$.

To show that the graphs above do in fact have concavity claimed above here is the graph again (blown up a little to make things clearer).



So, as you can see, in the two upper graphs all of the tangent lines sketched in are all below the graph of the function and these are concave up. In the lower two graphs all the tangent lines are above the graph of the function and these are concave down.

Again, notice that concavity and the increasing/decreasing aspect of the function is completely separate and do not have anything to do with the other. This is important to note because students often mix these two up and use information about one to get information about the other.

There's one more definition that we need to get out of the way.

Definition 2

A point $x = c$ is called an **inflection point** if the function is continuous at the point and the concavity of the graph changes at that point.

Now that we have all the concavity definitions out of the way we need to bring the second derivative into the mix. We did after all start off this section saying we were going to be using the second derivative to get information about the graph. The following fact relates the second derivative of a function to its concavity. The proof of this fact is in the Proofs From Derivative Applications section of the Extras chapter.

Fact

Given the function $f(x)$ then,

1. If $f''(x) > 0$ for all x in some interval I then $f(x)$ is concave up on I.
2. If $f''(x) < 0$ for all x in some interval I then $f(x)$ is concave down on I.

Notice that this fact tells us that a list of possible inflection points will be those points where the second derivative is zero or doesn't exist. Be careful however to not make the assumption that just because the second derivative is zero or doesn't exist that the point will be an inflection point. We will only know that it is an inflection point once we determine the concavity on both sides of it. It will only be an inflection point if the concavity is different on both sides of the point.

Now that we know about concavity we can use this information as well as the increasing/decreasing information from the previous section to get a pretty good idea of what a graph should look like. Let's take a look at an example of that.

Example 1 For the following function identify the intervals where the function is increasing and decreasing and the intervals where the function is concave up and concave down. Use this information to sketch the graph.

$$h(x) = 3x^5 - 5x^3 + 3$$

Solution

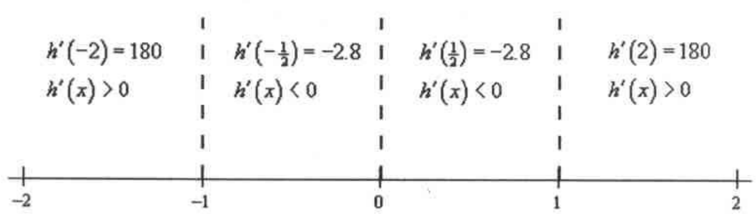
Okay, we are going to need the first two derivatives so let's get those first.

$$h'(x) = 15x^4 - 15x^2 = 15x^2(x-1)(x+1)$$

$$h''(x) = 60x^3 - 30x = 30x(2x^2 - 1)$$

Let's start with the increasing/decreasing information since we should be fairly comfortable with that after the last section.

There are three critical points for this function : $x=-1$, $x=0$, and $x=1$. Below is the number line for the increasing/decreasing information.



So, it looks like we've got the following intervals of increasing and decreasing.

Increasing : $-\infty < x < -1$ and $1 < x < \infty$

Decreasing : $-1 < x < 1$

Note that from the first derivative test we can also say that $x=-1$ is a relative maximum and that $x=1$ is a relative minimum. Also $x=0$ is neither a relative minimum or maximum.

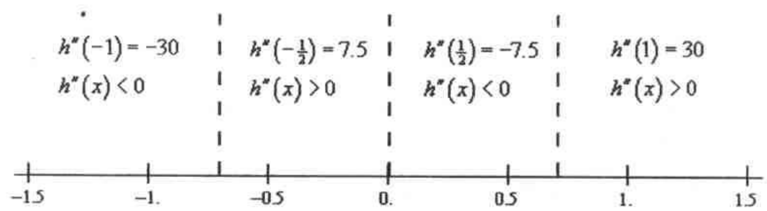
Now let's get the intervals where the function is concave up and concave down. If you think about it this process is almost identical to the process we use to identify the intervals of increasing and decreasing. This only difference is that we will be using the second derivative instead of the first derivative.

The first thing that we need to do is identify the possible inflection points. These will be where the second derivative is zero or doesn't exist. The second derivative in this case is a polynomial and so will exist everywhere. It will be zero at the following points.

$$x = 0,\ x = \pm\frac{1}{\sqrt{2}} = \pm 0.7071$$

As with the increasing and decreasing part we can draw a number line up and use these points to divide the number line into regions. In these regions we know that the second derivative will always have the same value since these three points are the only places where the function *may* change sign. Therefore, all that we need to do is pick a point from each region and plug it into the second derivative. The second derivative will then have that sign in the whole region from which the point came from

Here is the number line for this second derivative.



So, it looks like we've got the following intervals of concavity.

$$\text{Concave Up}: \ -\frac{1}{\sqrt{2}} < x < 0 \text{ and } \frac{1}{\sqrt{2}} < x < \infty$$

$$\text{Concave Down}: \ -\infty < x < -\frac{1}{\sqrt{2}} \text{ and } 0 < x < \frac{1}{\sqrt{2}}$$

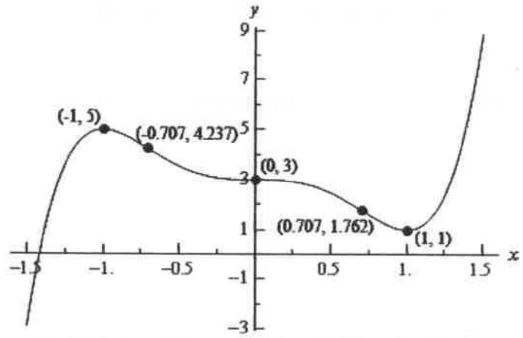
This also means that

$$x = 0,\ x = \pm\frac{1}{\sqrt{2}} = \pm 0.7071$$

are all inflection points.

All this information can be a little overwhelming when going to sketch the graph. The first thing that we should do is get some starting points. The critical points and inflection points are good starting points. So, first graph these points. Now, start to the left and start graphing the increasing/decreasing information as we did in the previous section when all we had was the increasing/decreasing information. As we graph this we will make sure that the concavity information matches up with what we're graphing.

Using all this information to sketch the graph gives the following graph.



We can use the previous example to get illustrate another way to classify some of the critical points of a function as relative maximums or relative minimums.

Notice that $x=-1$ is a relative maximum and that the function is concave down at this point. This means that $f''(-1)$ must be negative. Likewise, $x=1$ is a relative minimum and the function is concave up at this point. This means that $f''(1)$ must be positive.

As we'll see in a bit we will need to be very careful with $x=0$. In this case the second derivative is zero, but that will not actually mean that $x=0$ is not a relative minimum or maximum. We'll see some examples of this in a bit, but we need to get some other information taken care of first.

It is also important to note here that all of the critical points in this example were critical points in which the first derivative were zero and this is required for this to work. We will not be able to use this test on critical points where the derivative doesn't exist.

Here is the test that can be used to classify some of the critical points of a function. The proof of this test is in the Proofs From Derivative Applications section of the Extras chapter.

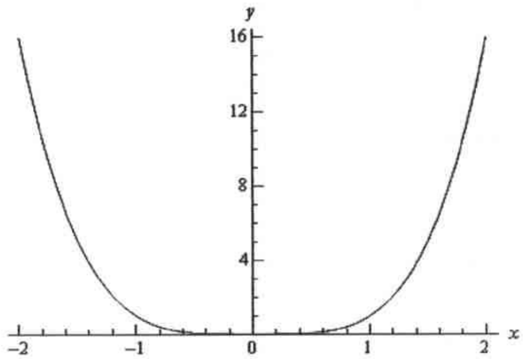
Second Derivative Test

Suppose that $x=c$ is a critical point of $f'(c)$ such that $f'(c)=0$ and that $f''(x)$ is continuous in a region around $x=c$. Then,

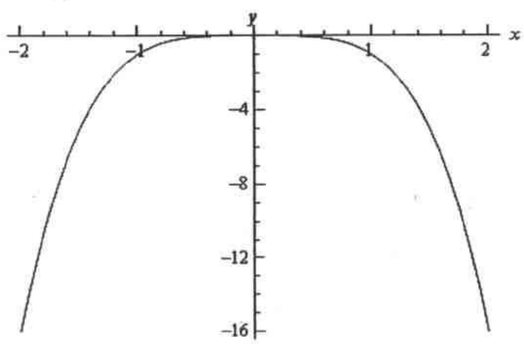
1. If $f''(c)<0$ then $x=c$ is a relative maximum.
2. If $f''(c)>0$ then $x=c$ is a relative minimum.
3. If $f''(c)=0$ then $x=c$ can be a relative maximum, relative minimum or neither.

The third part of the second derivative test is important to notice. If the second derivative is zero then the critical point can be anything. Below are the graphs of three functions all of which have a critical point at $x = 0$, the second derivative of all of the functions is zero at $x = 0$ and yet all three possibilities are exhibited.

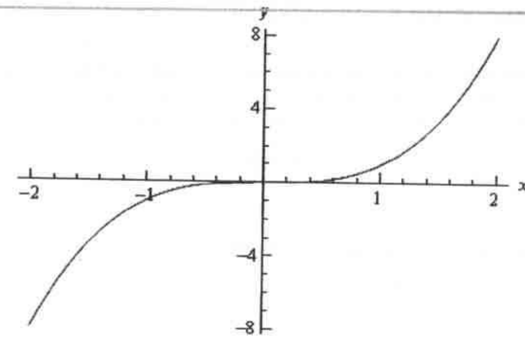
The first is the graph of $f(x) = x^4$. This graph has a relative minimum at $x = 0$.



Next is the graph of $f(x) = -x^4$ which has a relative maximum at $x = 0$.



Finally, there is the graph of $f(x) = x^3$ and this graph had neither a relative minimum or a relative maximum at $x = 0$.



So, we can see that we have to be careful if we fall into the third case. For those times when we do fall into this case we will have to resort to other methods of classifying the critical point. This is usually done with the first derivative test.

Let's go back and relook at the critical points from the first example and use the Second Derivative Test on them, if possible.

Example 2 Use the second derivative test to classify the critical points of the function,

$$h(x) = 3x^5 - 5x^3 + 3$$

Solution

Note that all we're doing here is verifying the results from the first example. The second derivative is,

$$h''(x) = 60x^3 - 30x$$

The three critical points ($x = -1$, $x = 0$, and $x = 1$) of this function are all critical points where the first derivative is zero so we know that we at least have a chance that the Second Derivative Test will work. The value of the second derivative for each of these are,

$$h''(-1) = -30 \qquad h''(0) = 0 \qquad h''(1) = 30$$

The second derivative at $x = -1$ is negative so by the Second Derivative Test this critical point this is a relative maximum as we saw in the first example. The second derivative at $x = 1$ is positive and so we have a relative minimum here by the Second Derivative Test as we also saw in the first example.

In the case of $x = 0$ the second derivative is zero and so we can't use the Second Derivative Test to classify this critical point. Note however, that we do know from the First Derivative Test we used in the first example that *in this case* the critical point is not a relative extrema.

Taking It To The Limit Lab

This lab was designed for the students to study the behavior of a function f(x) near a specified point. While this is sometimes a straightforward process, it can also be quite subtle. By gaining an intuitive feel for the notion of limits, it helps to lay a solid foundation for success in Calculus. Students were required to use their graphing calculators to obtain tables of values for specific functions and using the table, they had to draw conclusions about the behavior of the function near the specified point.

LAB 2:
Taking it to the Limit: Introduction to the Limits of Functions

INTRODUCTION:

In this lab we shall study the behavior of a function f(x) near a specified point. While this is sometimes a straightforward process, it can also be quite subtle; in many instances in calculus the process for finding a limit must be applied carefully. By gaining an intuitive feel for the notion of limits, you will be laying a solid foundation for success in calculus.

OBJECTIVES:

- To develop an intuitive understanding of the nature of limits.
- To lay the foundation for the frequent use of limits in calculus.
- To experience the power and the peril of investigating limits by successively closer evaluation.

MATERIALS:

All the investigations and questions appear on these sheets.

DIRECTIONS:

The directions are given for each question. Answer each on the appropriate sheet.

Question 1: Consider the function f(x) defined by $f(x) = \frac{x^4 - 1}{x - 1}$.

a. By successive evaluation of f (x) at x = 1.8, 1.9, 1.99, 1.999, and 1.9999, what do you think happens to the values of f(x) as x approaches 2 from the left?

b. Do a similar investigation on f(x) for values of x slightly greater than 2: x = 2.2, 2.1, 2.01, 2.001, 2.0001, and 2.00001. Comment on the results, as x approaches 2 from the right.

c. What you found in parts a and b should be $\lim_{x \to 2} (f(x)) = 15$

In this particular case, you could have "cheated" by immediately evaluating f(x) at x = 2. Use your graphing calculator to plot a graph of the function between 1.8 and 2.2 to show what happens in this situation. Show the graph below…

Question 2

Use the same function f(x) as in question 1, but this time consider what happens as x approaches 1.

a. Study this situation experimentally as you did in parts a and b in #1. To investigate the limit as x approaches 1 from the left use x = .8, .9, .99, .999 and .9999. Write your conclusion about what happens to the f(x) values when x approaches 1 from the left.

b. To investigate the limit as x approaches 1 from the right use x = 1.2, 1.1, 1.01, 1.001, and 1.0001. Write your conclusion about what happens to the f(x) values when x approaches 1 from the right.

c. What happens when you try to "cheat" and evaluate f(x) directly at x = 1.

Question 3

By calculator experimentation, try to determine the values of the following limits. Use function evaluation as you approach the "target" value from the left and from the right. Show your T-CHARTS.

a. $\lim_{x \to 0} \frac{\sin(10x)}{x}$

b. $\lim_{x \to 1} g(x)$, *where* $g(x) = \begin{cases} \frac{x^4 - 1}{x - 1}, & \text{for } x < 1 \\ 17, & \text{for } x = 1 \\ 14 - \frac{10}{x}, & \text{for } x > 1 \end{cases}$

c. $\lim_{x \to 0} (1 + x)^{1/x}$

(HINT: This limit is a famous mathematical constant.)

Question 4

It is important to be aware that limits can sometimes fail to exist. Investigate the following limits and explain why you know (think) that each does not exist. You may either use T-CHARTS or graphs to explore. Show your work and explain your conclusion for each:

a. $\lim_{x \to 2} \frac{x}{x-2}$

b. $\lim_{x \to 0} \frac{\sin(10x)}{x^2}$

c. $\lim_{x \to 0} \sin(\frac{1}{x})$

d. $\lim_{x \to 0} \frac{abs(x)}{x}$

e. $\lim_{x \to 4} f(x) = \begin{cases} (x+2)^3, & for\ x < 4 \\ e^x, & for\ x > 4 \end{cases}$

Question 5

Find the limit, if it exists,

as x approaches ∞. *It is necessary to check as x approaches both* $+\infty$ *and* $-\infty$ (HINTS: The limit exists for two of these functions. Use your calculator to evaluate for appropriate values of x, and use some prior mathematics knowledge.)

a. $f(x) = \dfrac{3x^4 - x^2 + 10}{2x^4 + \dfrac{5}{x}}$

b. $f(x) = \dfrac{\sin(x)}{1 + x^2}$

c. $f(x) = \dfrac{4 - 3/x}{\sin(x)}$

input $x = a$	output $f(a)$	left-hand limit $\lim_{x \to a^-} f(x)$	right-hand limit $\lim_{x \to a^+} f(x)$	limit $\lim_{x \to a} f(x)$	Is f continuous at $x = a$? (yes or no)

Integration Lesson Plan

This lesson plan outline included all materials used for teaching Integration to AP Calculus AB. I wrote this lesson plan in order to make sure I was following the standards for the AP Calculus Course Description. I used a number of materials from outside sources to include past AP problems and explorations from Larson's textbook and a few that I created on my own.

Lesson Plan Outline for Integration

Lesson 1: Antiderivatives and Indefinite Integration (2-3 Days)

Objectives:

- Write the general solution of a differential equation
- Use indefinite integral notation for antiderivatives
- Use basic integration rules to find antiderivatives
- Find a particular solution of a differential equation

Notes (teacher generated):

Introduction to Integration

- Definition of Antiderivative
- Differential Equations
- Basic Integration Rules
    - Applying the basic integration rule
        - Sum/difference rule
        - Constant Rule
        - Constant Multiple Rule
    - Rewriting before integrating $\int \frac{x+1}{\sqrt{x}}$ dx
    - Integrating polynomial functions $\frac{x^{n+1}}{n+1}$ +C
- Initial Conditions and Particular Solutions
    - Finding a particular solution

More Integration: Trig Indefinite Integrals

- Trig Integrals
- Physics application (position, velocity, acceleration)

Activities:

- Exploration: Finding Antiderivatives Worksheet
- Writing About Concepts Exercises 65 and 66 (Larson)
- Solving a Vertical Motion Problem

Homework: p255 #1-34; p257#67-74, 77-79; p255 33-42

AP Problems: 1993 #2, 1995 #2, 1997 #1, 1999 #1

Lesson 2: Area and the Definite Integral (1-2 days)

Objectives:

- Use sigma notation to write and evaluate a sum
- Understand the concept of area
- Approximate the area of a plane region
- Find the area of a plane region using limits

Notes (teacher generated):

Area - Definite Integrals

- Sigma notation
- Estimating definite integrals using the sums of rectangles

Homework: p268 #31-34 Find the limit of s(n) as n→ ∞; p269 #75 Worksheet (Larson)

Extra Credit: View article @ www.matharticles.com "Looking at $\sum_{k=1}^{n} k$ and $\sum_{k=1}^{n} k^2$ Geometrically" Read and Write Summary

Lesson 3: Riemann Sums and Definite Integrals (2-3 days)

Objectives:

- Understand the definition of a Riemann sum
- Evaluate a definite integral using limits
- Evaluate a definite integral using properties of definite integrals

Notes (teacher generated):

Definition of Definite Integral

- Definition of Riemann sum
  - LRAM, MRAM, RRAM
- Definition of a definite integral
- Evaluating a definite integral as a limit
- The definite integral as the area of a region
- Properties of definite integrals

Definite Integrals

- Setting up integrals to represent area
- Rules of definite integrals
  - Constant multiple rule
  - Sum and difference rule

Activities: Think About It p280 #47 + 48 (Larson)

Worksheets: Definite and Indefinite Integrals Multiple Choice

Homework: p278 13-22; p279 23-32; p279 33-46

AP Problems: 2003 #3 , 1998 #3, 1999 #3

Extra Credit: View article @ www.matharticles.com "The Evolution of Integration" Read and Summarize

Lesson 4: The Fundamental Theorem of Calculus (2-3 days)
Objectives:
- Evaluate a definite integral using the Fundamental Theorem of Calculus
- Understand and use the Mean Value Theorem for Integrals
- Find the average value of a function over a closed interval
- Understand and use the Second Fundamental Theorem of Calculus

Notes (teacher generated)

Fundamental Theorem of Calculus
- Definition of Fundamental Theorem of Calculus
- Guidelines for using FTC
- Evaluating a definite integral

Using FTC to find area
- Mean value theorem for integrals
- Average value of a function

Second Fundamental Theorem of Calculus
- Interpretation of an integral as an accumulation function

Activities:
Writing About Concepts p292 #53-60
Exploration Graphing Calculator #1, #2
Detective Hat Function Worksheet
Worksheet: Integrals and the FTC Multiple Choice
Worksheet: FTC II
The Accumulation Function Activity by Rahn

Homework: p 291 # 2-22 even; p291 27-32; p291 43-50

AP Problems:
1994 #6, 1995 #6, 1995 BC #6, 1997 #5, 1999 #5
Multiple choice AP problems packets on FTC only

Extra Credit: Project: Demonstrating the Fundamental Theorem p294 (Larson)

Lesson 5: Integration by Substitution (2-3 days)
Objectives:

- Use pattern recognition to find an indefinite integral
- Use a change of variables to find an indefinite integral
- Use the General Power Rule for Integration to find an indefinite integral
- Use a change of variables to evaluate a definite integral
- Evaluate a definite integral involving an even or odd function

Notes (teacher generated)

Integration by substitution

- U-substitution method for integration
- Multiplying and dividing by a constant
- Change of variables

Using u-substitution with definite integrals

- U-substitution method for definite integrals
- Integration of even or odd functions

Activities:

Worksheet Integration II
Exploration p300 (Larson)

Homework: p304 #1-38; p305 43-56; p305 #63-70

AP Problems:

Lesson 6: Numerical Integration:

Objectives:

- Approximate a definite integral using the Trapezoidal Rule

Notes (teacher generated)

Trapezoidal Rule

Activities:

Worksheet Fundamental Theorem Trapezoidal Rule
P315 #48 (Larson)

Homework:

P315 #51 + #52
P314 #2-10 even

AP Problems:

Review for exam:
Exploring Numerical Integration on the Graphing Calculator by Rahn
Worksheet: Fundamental Theorem Review Multiple Choice
Worksheet: Definite and Indefinite Integrals Review

Exponential Functions

This lesson plan was designed for a low-level algebra I class. The purpose of this lesson was to help students learn about an exponential relationship and how it relates to real-world situations. This lesson introduces the exponential function through graphing, investigating patterns of graphs and making connections to the real world. I developed activities that were easy for low level students to grasp the essential understanding of exponential functions.

Lesson Plan: Exponential Functions   Algebra I-Level II/III

Essential Understanding: The purpose of this lesson is to help students learn about an exponential relationship and how it relates to real-world situations. This lesson introduces the exponential function through graphing, investigating patterns of graphs and making connections to the real world.

Content Standards (Massachusetts Curriculum Frameworks):

- Students will describe, complete, extend, generalize, and create a wide variety of patterns in the area of exponential relationships. (DOE A1.P.1 and 7)
    - Translate patterns into algebraic functions (data from tables).
    - Graph exponential functions with the aid of a graphing utility.
- Students will connect mathematics to other learning by understanding the interrelationships of mathematical ideas and the role that mathematics and mathematical modeling play in other disciplines and in life. (DOE A1.D.1 and 2, AI .P.11)
    - Solve problems by looking for and using a pattern.
    - Develop and evaluate inferences and predictions that are based on data.
    - Solve problems involving growth and decay.
    - Investigate real-world situations.
    - Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.
    - Use mathematical models to represent and understand quantitative relationships.

Objectives:
Students will know and be able to:

- Develop a mathematical model using real life data researched on the Internet or in the library.
- Calculate lines of best fit (fit an exponential growth/decay model) to the data using technology.
- Develop a mathematical model, explain and defend their model, and justify their reasoning.
- Discuss or debate affects of different types of exponential growth (i.e. population, aids, and bacteria).
- Develop their own set of questions related to their data sets.

Prerequisites: Students should know

- exponential notation
- how to create a table of values
- how to graph data on an xy-coordinate system
- how to evaluate an equation

Classroom Activities:

1. "Skittles and Skittles and More Skittles, OH MY!!!" Activity
2. Modeling Exponential Growth and Decay using Skittles and the TI-83 Graphing Calculator

Extensions:

1. Have students investigate a number of situations involving the equation $y=2^x$. Have them look at how much money would be earned by starting out with a penny on the first day and doubling the amount on each successive day.
2. Have students discuss what happens if they start with two bacteria and the number of bacteria doubles every hour.
3. Consider the number of sections created if you repeatedly fold a paper in half.
4. To establish relevance, have students consider where they have encountered information about and had experience with these key concepts before. Tell them to consider real world and career applications.

Function Derivative Match Activity

This activity was designed for AP Calculus AB as a refresher to remember their Precalculus topics. The students were given function graphs, domains, ranges, equations, and a summary sheet. They needed to match the graphs with the graphs of their equations and domains and ranges. They were to assess themselves based on a rubric. This was done without the aid of a graphing calculator. Students had to rely on their past knowledge in order to complete the activity successfully.

Function Match Lab Activity

Directions:

Choose a partner. Once all students are paired up, **CAREFULLY** cut the lab sheets that contain graphs, domains, ranges and equations along the dotted lines. Be sure that the function number is not cut off of the graph.

Working with your partner, match each graph with its appropriate domain, range, and equation. (Each graph does have a matching equation, domain and range.)

When your group is finished, summarize your results on the Summary sheet. Then report to the teacher to check your answers.

NOTE: THIS LAB IS TO BE DONE WITHOUT THE USE OF A GRAPHING CALCULATOR!!!!!

Materials:

- 1 Function Matching lab packet
- 12 function graphs
- 12 domains
- 12 ranges
- 12 equations
- 1 Summary sheet
- 1 pair of scissors

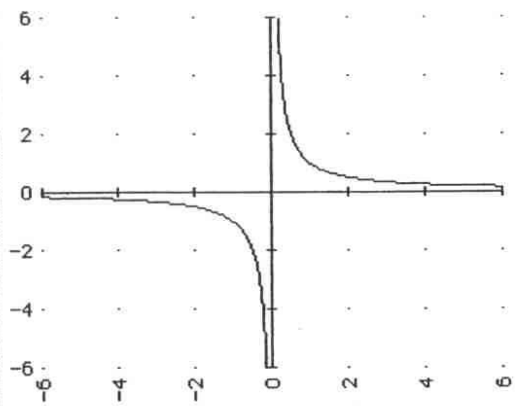
NAMES ______________________

Function Match
Summary

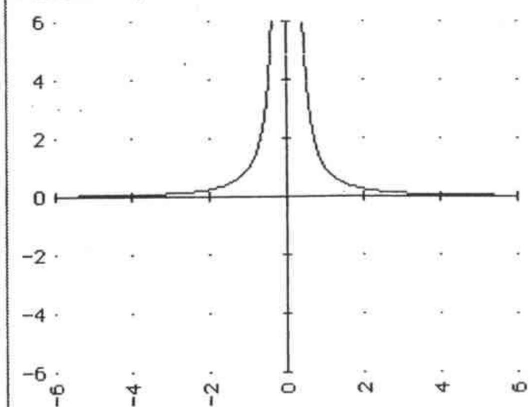
FUNCTION #1 Equation: ______________ Domain: ________ Range: ________	**FUNCTION #2** Equation: ______________ Domain: ________ Range: ________
FUNCTION #3 Equation: ______________ Domain: ________ Range: ________	**FUNCTION #4** Equation: ______________ Domain: ________ Range: ________
FUNCTION #5 Equation: ______________ Domain: ________ Range: ________	**FUNCTION #6** Equation: ______________ Domain: ________ Range: ________
FUNCTION #7 Equation: ______________ Domain: ________ Range: ________	**FUNCTION #8** Equation: ______________ Domain: ________ Range: ________
FUNCTION #9 Equation: ______________ Domain: ________ Range: ________	**FUNCTION #10** Equation: ______________ Domain: ________ Range: ________
FUNCTION #11 Equation: ______________ Domain: ________ Range: ________	**FUNCTION #12** Equation: ______________ Domain: ________ Range: ________

Range $y \in \Re$	Range $y \neq 0$
Range $y \neq 0$	Range $y > 0$
Range $y > 0$	Range $y > 0$
Range $y > 0$	Range $y \geq 0$
Range $y \geq 0$	Range $y \neq -2$
Range $y = 1$	Range $y \geq 1$

Function #1



Function #2



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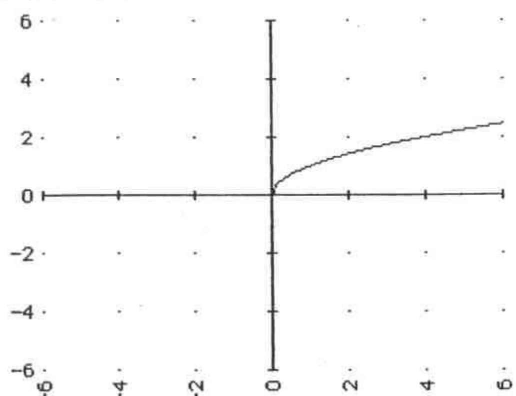

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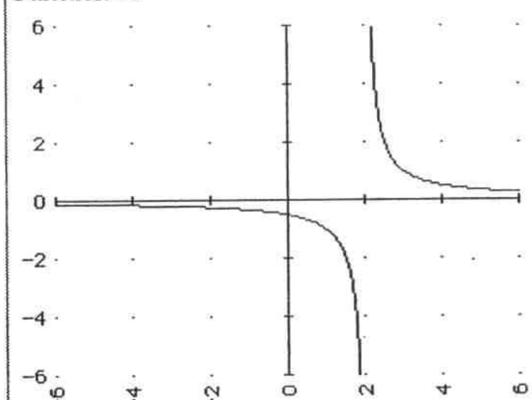

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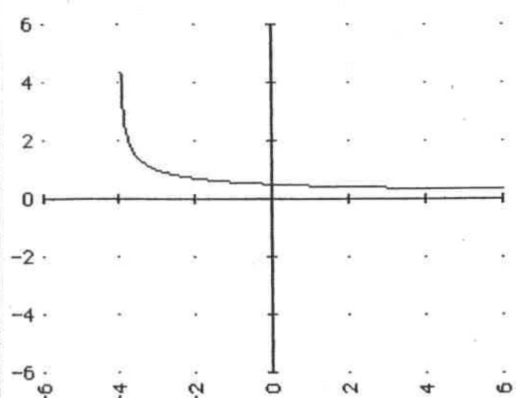
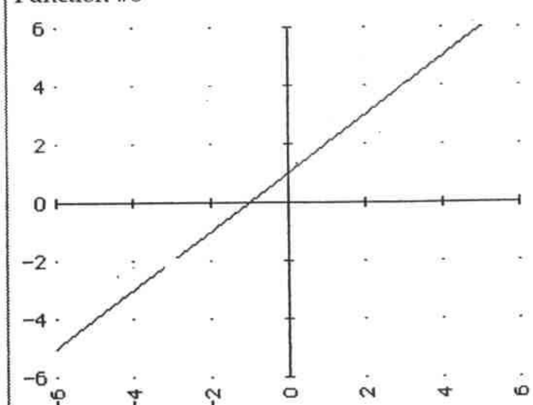
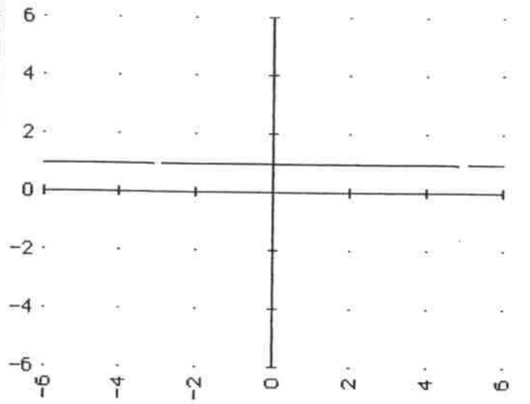


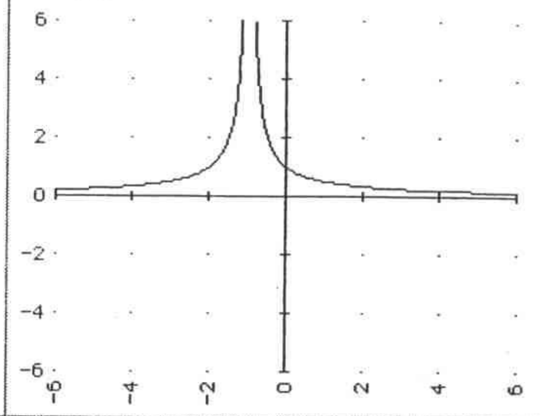
Function #6



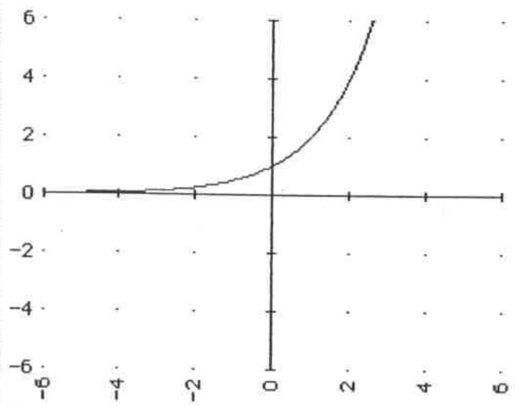
Function #7



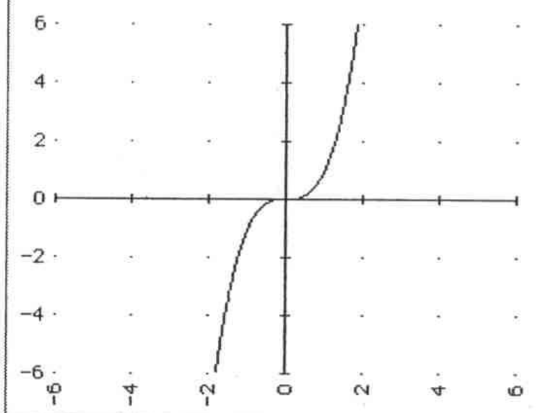
Function #8



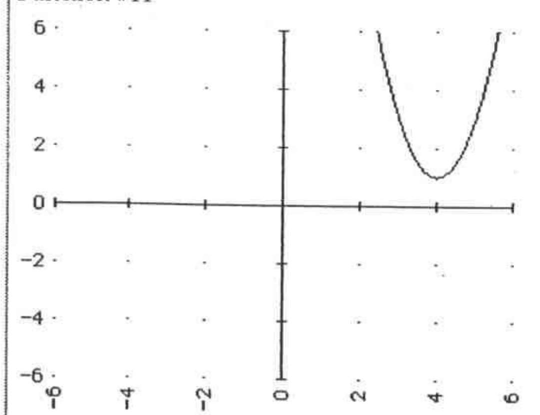
Function #9



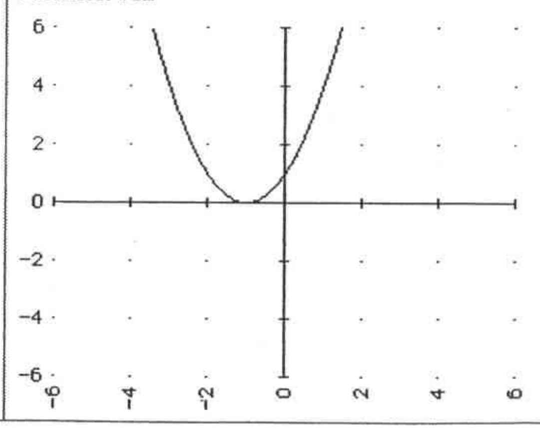
Function #10



Function #11



Function #12



Domain $x \in \Re$	Domain $x \in \Re$
Domain $x \in \Re$	Domain $x \neq -3$
Domain $x \neq 0$	Domain $x \neq 0$
Domain $x \neq 3$	Domain $x \neq -3, 5$
Domain $x \neq -1$	Domain $x \geq 0$
Domain $x > -4$	Domain $x \neq 2$

Equation $y = \frac{1}{x}$	Equation $y = \frac{x^3 - x^2 - 5x - 3}{x - 3}$
Equation $y = 2^x$	Equation $y = \frac{1}{x^2}$
Equation $y = \frac{(x-5)(x+3)}{x^2 - 2x - 15}$	Equation $y = \sqrt{x}$
Equation $y = x^3$	Equation $y = 2x^2 - 16x + 33$
Equation $y = \frac{1}{x - 2}$	Equation $y = \frac{1}{\sqrt{x + 4}}$
Equation $y = \frac{x^2 + 4x + 3}{x + 3}$	Equation $y = \frac{1}{\sqrt{x^2 + 2x + 1}}$

Chapter 4

There are five courses which I am teaching: (2) Pre-Calculus courses, AP Calculus AB and BC, and a low-level algebra course. The class-sizes range from 6 to 23 students. These sizes make it much easier for the instructor to be able to recognize who is present and who is paying attention during each class. Unfortunately, no matter how many students are present and are paying attention, there are different learning strengths in each of the classes. However, no matter what their personal learning strengths are, having the students go up to the board to work on example problems is critical. That is extremely important because it shows the instructor how well the students have obtained the knowledge just presented, it gets all of the students involved and it reaches those students who might not "get it" in lecture or note-taking form. Although not all of the students can go up to the board at once, the students that didn't go up would work on problems on their own at their desk. In the case of the AP courses, having the students work in groups (at the board) is better because of the complexities of the various problems. It also gets the students collaborating and talking the problem out and going through the process step-by-step. If a student does happen to miss a class or two, there are a few things an instructor can do to utilize class time effectively. In the classes other than the AP courses, the instructor can briefly (<5min) go over what was discussed the day before and lead into the next topic. If the absent student has any questions about the previous topic, the instructor can tell them to see them after class or during the time when the students are working at the board, at which time the instructor can give them an overview of what was previously discussed, as well as have them do extra work (i.e. read and do several problems) from the last section to make sure they understand it correctly. However, if a majority of the students didn't have a clear understanding of the material, then the instructor would have to spend an extra day on the given topic. In the AP courses, it would be much harder because the schedule is strictly guided. If a student misses a section it would be up to them to get the notes and learn the material from the instructor because all of the lessons must be taught and each class is valuable. Unlike the Worcester schools, there isn't a high mobility rate at Tahanto Regional High School. This year, in the courses I teach, there are three exchange students, one of which is a foreign exchange student. The two non-foreign exchange students came at the beginning of the year and were able to settle in from the start. The foreign exchange student came to Tahanto in the middle of the second term. When the student came for the first day, they sat in on the new lecture and started learning the material being presented that day. During the class, the instructor would ask various questions to get a sense of what the student had previously learned in Japan.

In one section of Pre-Calculus, there are 23 students and one student is the foreign exchange student from Japan. This collective group of students is an ideal class because all of them are high attentive learners who enjoy learning new lessons, there is not an attendance problem and, for the most part, do not cause any problems that hinders others from learning the new materials presented during class time. It is very fortunate that those students attend class on a regular-basis and are very attentive because the instructor can go about their daily lesson plans without being disrupted. There are a few different learning styles, most notably note-taking, that are present in that class. Overall, that class is a good note-taking class with part of the class time used for a lecture. They are a good note-taking class because they pay attention and are attentive. The other part of the class that is very useful, for the teacher and the students, is for them to go the board and attempt problems from the section just taught.

In the other section of Pre-Calculus, there are also 23 students with both non-foreign exchange students. In this class, there is one student with an individualized education plan (IEP). Although this student has an IEP, this student works very diligently and understands the material extremely well. This student is one of the better students in the class and earns high marks during each term. Similarly to the other Pre-Calculus class, there are different learning styles present, but they learn better with an instructor led discussion on the topic. This is difficult because of the conflict of learning styles between the two classes. Since there is this difference, during some topics the instructor will have notes already prepared (for both sections) and hand it out to each student and then go over them with the students. During the other topics, the instructor will have the students (in both sections) take notes on their own. However, during the latter part of class (in both sections), the students will come up to the board and work on problems to get a hands-on feel for each of the different problems.

The AP Calculus-AB course is a challenging course to teach because some of the students in the class should not be in it at all. Tahanto's small population wouldn't allow for a regular Calculus course which was originally offered at the end of last year. So, the students who were signed up for regular Calculus were then added to the AP course. This is difficult for the teacher because these select few students know they shouldn't be in the class and cause disruptions during class time when others try to learn. There is mostly behavioral and attendance problems in that class, which makes it hard for an instructor to teach the lesson plan completely. For the students who actually want to learn, there are a couple learning styles that are present. The main learning style present is an instructor based discussion. The students like talking about what is being presented and have the instructor walk through a problem or two to get a better understanding of how to solve a problem.

The AP Calculus-BC course is a relatively easy course to teach because there are 6 students and each student is eager to learn. This class is made up of high intellectual students with no issues (both in and out of school) which makes it much easier for the instructor to teach the lessons. Overall, this class has essentially one learning style, instructor based discussion. These students like to think about what is being presented as the instructor talks about it. If they have any questions about any topics, it is much easier for the instructor to go through it step by step because they will all pay attention.

The low-level Algebra course is the most demanding course to teach for several reasons. One reason is because the class is made up of 14 students, each having a different IEP. Another reason is they have trouble doing basic mathematical functions (i.e. long division). This is extremely difficult for the instructor because they have to prepare a plan that will meet the needs of all the students. Although, they have difficulties understanding the lesson concepts, they are eager to learn and work hard at trying to understand the material. The students do not cause problems during class and attend class on a regular basis. These students learn best when it's hands-on learning. The optimal lesson would be to give them 5-10 minutes of notes then have a mini-project that incorporates the lesson to show them the real-life applications of the lesson. For example, when they learned about slope I took them around the school and had them act like building inspectors and measure the rise and run of the stairs around the school to see if the slope of the stairs were up to state codes.

Chapter 5

Involving students in the process of assessment helps them to assess their own work. It helps students to deepen their understanding of the learning process. Students are able to learn more effectively when they are provided with opportunities to play an active role.

In the Route 66 Algebra I lesson plan, I chose to use a rubric to assess that final project. It was a summative assessment designed to look at the whole lesson and what the students were able to get out of the lesson. I gave the rubric out to the students in the beginning so that they could see what the expectations were and what they would be assessed on. I provided the students with recommendations, commendations, and feedback to help them determine what steps they needed to take to further their learning. Sharing the objectives with them in the beginning allowed them to better understand the learning goals for which they were striving. I tried to create an environment that promoted risk-taking since students must feel supported and confident in their environment in order to take risks. By having the students write journal entries helped to connect the lesson to the real world. Students are more likely to retain material if they can connect it to the real world. Also, students are more motivated by assignments that they think are real. Not only were students assessed by the teacher, they assessed themselves as well and they assessed the projects of their peers while they were being presented. Included in the lesson plan were some performance assessments. I made lists of tasks that the students were required to complete at different phases of the project.

Route 66 Project Rubric **Teacher Evaluation**

Name: Assessment
Date: _______________

	Advanced 16-20 points	Proficient 11-15 points	Needs Improvement 6-10 points	Warning 0-5 points
Organization	Extremely well organized; logical format that was easy to follow; the organization enhanced the effectiveness of the project	Presented in a thoughtful manner; there were signs of organization and most transitions were easy to follow, but at times ideas were unclear	Somewhat organized; ideas were not presented coherently and transitions were not always smooth,	Choppy and confusing; format was difficult to follow; transitions of ideas were abrupt
Content Accuracy	Completely accurate; all facts were precise and explicit	Mostly accurate; a few inconsistencies or errors in information	Somewhat accurate; more than a few inconsistencies or errors in information	Completely inaccurate; the facts in this project were misleading to the audience
Research	Went above and beyond to research information; solicited material in addition to what was provided; brought in personal ideas and information to enhance project	Did a very good job of researching; utilized materials provided to their full potential; at times took the initiative to find information outside of school	Used the material provided in an acceptable manner, but did not consult any additional resources	Did not utilize resources effectively; did little or no fact gathering on the topic
Creativity	Was extremely clever and presented with originality; a unique approach that truly enhanced the project	Was clever at times; thoughtfully and uniquely presented	Added a few original touches to enhance the project but did not incorporate it throughout	Little creative energy used during this project
Presentation Mechanics	Was engaging, provocative, and captured the interest of the audience and maintained this throughout the entire presentation; great variety of visual aids and multimedia; visual aids were colorful and clear	Was well done and interesting to the audience; was presented in a unique manner and was very well organized; some use of visual aids	Was at times interesting and was presented clearly and precisely; was clever at times and was organized in a logical manner; limited variety of visual aids and visual aids were not colorful or clear	Was not organized effectively; was not easy to follow and did not keep the audience interested; no use of visual aids

The function derivative match activity was also assessed using a rubric. It allowed the students to see what the expectations were before the activity was completed. They needed to rely on past knowledge to be successful in the activity and the rubric scored them according to their essential understanding of functions and their derivatives.

Function Derivative Match Activity

	Advanced 21-25pts	Proficient 16-20pts	Acceptable 10-15pts	Needs Improvement 0-9pts
Understanding of the relationship between the derivative of a function and the key features of its graph	Demonstrates a thorough understanding of the relationship between the derivative of a function and the key features of its graph	Demonstrates an understanding of the relationship between the derivative of a function and the key features of its graph	Demonstrates a partial understanding of the relationship between the derivative of a function and the key features of its graph	Demonstrates some basic understanding of the relationship between the derivative of a function and the key features of its graph
Ability to independently interpret the rates of change of a function using the different descriptions	Very capably and independently determines and interprets rates of change of functions drawn from a variety of descriptions	Independently determines and interprets rates of change of functions drawn from a variety of descriptions	With some assistance, determines and interprets rates of change of functions drawn from a variety of descriptions	Requires a great deal of assistance to determine and interpret rates of change of functions drawn from a variety of descriptions
Understanding of how to analyze functions using differential calculus	Independently and with a thorough understanding analyzes functions using differential calculus	Independently analyzes functions using differential calculus	Beginning to analyze functions using differential calculus, regular review is needed	Exhibits limited understanding when analyzing functions using differential calculus. Additional review and support is needed
Ability to correctly match each of the descriptions with each function	Able to thoroughly complete activity with no support and no errors (all correct)	Able to thoroughly complete activity with no support but has minimal errors (17-19 correct)	Completes activity with minimal support and with minimal errors (12-16) correct)	Unable to complete activity without support and many errors (0-11 correct)

The AP Calculus Graph Theory assessment was a formative assessment. I chose problems that were related to those on the AP exam. Since the course is designed with the understanding that students are going to take the AP exam in May, I wanted to make sure they could use the tools that were taught to complete problems similar to those on the AP exam.

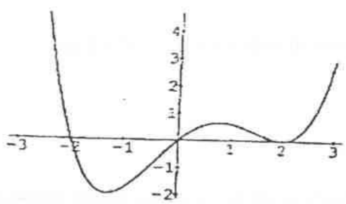
A.P. CALCULUS
GRAPH THEORY

NAME Assessment # 2
DATE________________

ALL ANSWERS MUST BE JUSTIFIED

1. Find where the function $f(x) = (1000 - x)^2 + x^2$ is increasing and where it is decreasing.

2. The graph below is **the derivative of a function** f (x)



a. Find where f (x) is increasing and where it is decreasing.

b. Find all relative maximum(s) and relative minimum(s) of f (x).

c. If f (-3) = -2, sketch a possible graph of f (x).

Assessment #2 (cont.)

3. A function f (x) is continuous on the interval $-3 \leq x \leq 3$ and its first and second derivatives have the values given in the table:

x	(-3 , 1)	-1	(-1 , 0)	0	(0 , 1)	1	(1 , 3)
$f'(x)$	Positive	0	Negative	Neg.	Negative	0	Negative
$f''(x)$	Negative	Neg.	Negative	0	Positive	0	Negative

a. What are the x-coordinates of all relative maxima and minima of f (x) on $-3 \leq x \leq 3$?

b. What are the x-coordinates of all points of inflection of f (x) on $-3 \leq x \leq 3$?

c. Sketch a possible graph for f (x) which satisfies all of the given properties.

The Calculus Derivatives Quiz was designed to make sure the students were able to use what they were taught to solve a variety of problems. The quiz had both a calculator and non-calculator section. Students needed to be able to find derivatives without the use of a calculator.

On the part of the exam where the calculator was able to be used, students were required to show all of the steps needed to solve the problem for full credit. The calculator was to be used a tool to find intercepts or critical points on a graph but the problems needed to be solved using calculus.

CALCULUS – DERIVATIVES

NAME Assessment #1

DATE________________

PART 1 NO CALCULATOR MAY BE USED   Select the correct answer for each:

1. For $y=\frac{2x+3}{3x-2}$, dy / dx = a. $\frac{-13}{(3x-2)^2}$ b. $\frac{-13}{3x-2}$ c. $\frac{13}{(3x-2)^2}$

d. $\frac{13}{3x-2}$ e. $\frac{2}{3}$

2. Find the derivative of the function $f(x)=(10-5x)^4$

a. $-20(10-5x)$ *b.* $-20(10-5x)^3$ *c.* $20(10+5x)$ *d.* $40(10-5x)^3$ *e.* $-40(10-5x)^3$

3. If

$f(x)=\sqrt{(x^2+2)^3}$, *then* $f'(x)=$ *a.* $\frac{3\sqrt{x^2+2}}{2}$ *b.* $3x\sqrt{x^2+2}$

c. $\sqrt{6x(x^2+2)^2}$ *d.* $\frac{3x}{\sqrt{x^2+2}}$ *e.* $\frac{4x}{3\sqrt[3]{x^2+2}}$

4. If

$f(x)=\frac{1}{2\sin(2x)}$, *then* $f'(x)=$ *a.* $-\csc(2x)ctn(2x)$ *b.* $-\csc^2(2x)$

c. $-2\csc^2(2x)$ *d.* $-2\csc(2x)ctn(2x)$ *e.* $\frac{-2\cos(2x)}{(2\sin(2x))^2}$

PAGE 2 DERIVATIVES AND GRAPH THEORY Name Assessment #1 (cont.)
CALCULATORS MAY BE USED AS NEEDED FOR THIS PAGE

5. Given $y = x^2 - \frac{x^3}{6}$

   a. State where y is increasing. Explain.

   b. State where y is decreasing. Explain.

6. $y = x^3 + 4x^2 - 3x + 2$ **on [-5 , 1]**

   a. State the coordinates of each stationary point, and classify each as a **relative max or min or absolute max or min.**

   b. State the coordinates of each endpoint on the interval given, and classify each as a **relative max or min or absolute max or min.**

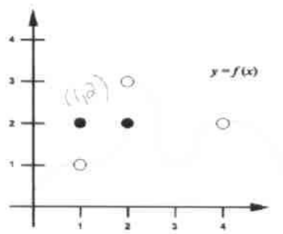
7. Given the curve $y = x^3 - 6x^2$.

   a. State where y is concave up and where it is concave down.

   b. Write the equation for the tangent line to the curve y at its point of inflection.

The AP Calculus BC test on limits was an assessment that was given at the end of the review of limits from a prior AP AB Calculus course. It was a summative assessment that was designed to assess the knowledge after teaching was completed.

AP Calculus BC Test Limits Name:__________________



Find the following using f(x) above:

1. $\lim_{x \to 1} f(x)$
2. $\lim_{x \to 4} f(x)$
3. $\lim_{x \to 2} f(x)$
4. f(2)

Find the following limits

5. $\lim_{x \to -3} \frac{\sqrt{1-x}-2}{x+3}$

6. $\lim_{x \to 2} \frac{x-2}{x^2-4}$

7. $\lim_{x \to \frac{\pi}{4}} \tan x$

8. $\lim_{x \to 3} 15$

9. $\lim_{x \to 2} \frac{x-2}{x^2-2x}$

10. $\lim_{x\to 6} \sqrt{25 - x^2}$

11. $\lim_{n\to \infty} \left(1 + \frac{1}{n}\right)^n$

12. $\lim_{x\to 3^+} \frac{x-3}{|x-3|}$

Given the $\lim_{x\to 3} f(x) = 5$ and $\lim_{x\to 3} g(x) = 9$, find the indicated limits in the problems below.

13. $\lim_{x\to 3} \sqrt{g(x)}$

14. $\lim_{x\to 3} \frac{\sqrt{g(x)} + 2f(x)}{g(x) - f(x)}$

Evaluate the indicated limit. Use -∞ or ∞ where appropriate.

15. $\lim_{x\to \infty} \frac{2-3x^3}{7+4x^3}$

16. $\lim_{x\to \infty} \frac{4x^3+2x-9}{8-7x^2}$

17. $\lim_{x\to 0} \frac{\sin 4x}{7x}$

18. $\lim_{x\to 0} \frac{\tan 4x}{\sin 8x}$

19. $\lim_{x\to 0} \frac{1-\cos x}{x}$

Sketch a graph of the following piecewise defined function and discuss its continuity using limits.

20. $f(x) = \begin{cases} 4 - x^2, & x < 1 \\ 2x + 3, & x \geq 1 \end{cases}$

21. Discuss continuity at a point using limits. (hint: 3 cases)

22. Draw an example of a function with a removable discontinuity at $x = 2$.

23. Draw an example of a function with a nonremovable discontinuity at x = 4.

24. Name three instances where a limit might fail to exist.

a.

b.

c.

25. Draw a picture and explain the Intermediate Value Theorem.

In most of the classes that I taught, I used “Do Now’s” to begin the class. These “do now’s” could be problems that required students to remember topics that were previously taught or they could be problems that were taught in a previous math class. This allowed me to walk around the room, take attendance, pass papers back, or check for understanding by glancing at the work being done by the students. This type of assessment was great for an AP Calculus class because students had to remember all of the topics that were previously taught and if they were incorrect in any of their work, they knew what they needed to do to study for an upcoming exam or even for the AP exam.

DO NOW

1. Find $\frac{dy}{dx}$ if $x + 2xy - y^2 = 2$.

2. Write an equation of the line tangent to the graph of $f(x) = x(1\text{-}2x)^3$ at the point (1, -1)

3. $\int \frac{\cos\theta}{\sqrt{(1+\sin\theta)}}\, d\theta$

Do Now Calculus

1. $\int x\sqrt{x^2+2}\,dx$

2. $\int (1+2x)^4(2)dx$

3. $\int x^3(x^4-1)^5dx$

4. $\int \frac{x}{(1-x^2)^3}\,dx$

5. $\int x^3\sqrt{x^4+5}\;dx$

6. $\int \pi\sin(\pi x)dx$

7. $\int 4x^3\sin(x^4)dx$

8. $\int \sin 2x \cos 2x$ dx

9. $\int \sec^2 4x\, dx$

10. $\int \sec 3x \tan 3x\, dx$

The AP Calculus AB Quiz on integration was an assessment used to assess students understanding of integration and u-substitution. I designed the quiz by choosing different problems that required students to understand all of the methods of integration rather than one method.

AP Calculus AB
Quiz Integration

Name:___________________

1. $\int \sin 5x \, dx$

2. $\int x^{\frac{3}{2}} + \sqrt{x} \, \mathrm{dx}$

3. $\int x \cos x^2 \, \mathrm{dx}$

4. $\int \frac{x^3 + x + 4}{x^2} \, \mathrm{dx}$

5. $\int \frac{(1+\ln x)^2}{x} \, \mathrm{dx}$

6. $\int x\sqrt{2-x} \; dx$

7. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, \mathrm{dx}$

8. $\int (x^3 - 3)^4 \, x^2 dx$

9. $\int \frac{4}{x^2} + \frac{3}{x^3} \, \mathrm{dx}$

10. $\int \frac{1}{x} \sin(\ln x) dx$

The AP Calculus AB exam on precalculus topics was a summative assessment given to the students to assess their knowledge of precalculus topics that students needed to master in order to take an AP class. The students were given this assessment prior to learning any AP topics. Students needed to obtain mastery of these topics in order to be in the class.

AP Calculus AB Exam: Name:

Precalculus Topics

1. Write an equation for the specified line:
   a. Through (1, -6) with slope 3.

   b. Through (-1, 2) and (-3, 6).

   c. The vertical line through (0, -3).

   d. The horizontal line through (0,2).

   e. Through (3,1) and parallel to $2x - y = -2$.

   f. Through (-2, -3) and perpendicular to $4x + 3y = 12$.

2. Name the test for symmetry: (how do you determine it algebraically)
   a. About the y-axis.
   b. About the x-axis.
   c. About the origin.

3. Find the domain of the following functions:
   a. $Y = x^2 + 1$.
   b. $Y = 1 - \cos x$.
   c. $Y = \sqrt{4 - x}$
   d. $Y = \frac{7}{x}$
   e. $Y = \frac{4}{\sqrt{x+2}}$
4. Find the x and y-intercepts of the following:
   a. $Y = x^3 - 4x$.

   b. $Y = 2x^2 + 8x + 7$

   c. $4x - 3y = 12$

5. Find the points of intersection of $x^2 - y = 3$ and $x - y = 1$.

6. Find f(2a) if f(x) is $2x^3 + 3$.

7. Find f(x-1) if f(x) is $3x^2 + 2x - 5$.

8. Describe the shift of y = f(x) in each of the following instances:
   a. Y = 3f(x-2) + 1.

   b. Y = f(-x)

   c. Y = -f(x)

9. Define the 6 trig functions in terms of the sides of a right triangle.

10. Recreate the "Cheat chart" and label using degrees and radians.

11. Evaluate:

a. ln e
b. ln 1
c. ln 0
d. ln e^x
e. $e^{\ln x}$

12. What determines how many turning points a polynomial will have?

13. Find the exact value of the following:

a. $\sin\frac{\pi}{4}$
b. $\cos\frac{-7\pi}{6}$
c. $\tan\frac{\pi}{2}$
d. $\sin\frac{11\pi}{6}$

14. Sketch the following functions:

a. Identity function

b. Cube function

c. Square function

d. Absolute value function

e. Exponential function

f. Logarithmic function

g. Rational function

15. Given y = - 2 - $3\sin 5(x - \frac{\pi}{4})$ find the following:

a. The period:

b. The phase shift:

c. The vertical shift:

d. The amplitude

Conclusion

When I was looking at colleges and sat down with my parents to discuss what I wanted to do after I graduated college, I told them I wanted to be a high school math teacher and hopefully coach baseball and/or football. This experience has helped me support my case for my aspirations of becoming a math teacher because I found my true enjoyment. I benefited from having a wonderful mentor and a good group of kids to work with and who were eager to learn. I took pleasure in every step of the way, from my practicum to taking the grueling MTEL up at Fitchburg High School, which took a total of 10 hours. However, those 10 hours were rewarding because it was a couple days after I found out I passed them that I encountered one of the best moments while doing my practicum. The Algebra I-Level II class brought in donut holes from Dunkin' Donuts and a card with all of their signatures saying how much they appreciated the time I spent with them. It was then, when I realized that kids really do care about learning and can be motivated to learn new material. I truly believe this project has helped me mature as a grown adult because I had to take full responsibility of the students and it was up to me to keep them under control and make sure the lesson plans were taught in the class period.

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Appendix: Miscellaneous Materials Used

Algebra I-II/III Name:
Homework
Exponential Functions

"I'm rich!", OR "Am I?"

Today is your lucky day. You just received notice that a rich relative wants to give you money. You have a few options to consider before accepting the money.

Option #1: They would give you $1000 per year until you reach the age of 30.

OR

Option #2: They would give you one dollar this year, 2 dollars next year, 4 dollars the next year and would continue in this pattern of doubling the amount each year until you are 30.

1. Create two tables of values, one for each option.
2. Graph your results on graph paper. Put the money on the vertical axis and the years on the horizontal axis. You will graph two lines. Use colored pencils to distinguish them from each other. Provide a key.

Which option would you choose? Why? Provide a mathematical reason for your answer.

If you only received the money for 10 years, which would be the better option?

Figure 1: Exponential Worksheet

Algebra I-II/III

Homework

Exponential Functions

Name:

SICK AND TIRED OF MATH???????

(NEVER!!!!)

You have just developed a bacterial infection called "Mathisinfectious". Unfortunately this bacterium does not exempt you from homework. Since this is the beginning of your infection, you only have two bacteria in your system at this moment. The bacteria double every hour. The antibiotic to cure this infection will not be effective until there are more than 15,000 bacteria in your system. If it is 9:00 a.m. right now, when will you be able to start taking your antibiotic?

Create a table of values.

Hour	# Bacteria
0	2

Figure 2: Exponential Worksheet

"Skittles and Skittles and More Skittles, OH MY!!!" Activity

To help make predictions in real-world situations, researchers often use experiments known as simulations. The results of the simulations are gathered and analyzed. This data is then compared with known information about the actual population. If the results seem questionable, the simulation may be revised. This modeling process can be summarized by the following four steps:

- Creating a model
- Translating the model into mathematics using the mathematics
- Relating the results to the real-world situation
- Revising the model

In the following explorations, you investigate this modeling process using a population of Skittles.

MATERIALS:
A large flat container with a lid
A package of Skittles
A graphing calculator
Graph paper
Worksheets (teacher prepared)
Group Exploration Job Sheet

Before beginning the simulation, read Steps 1-7 below and predict how you think the number of Skittles will change.

1. Place 2 Skittles in your container. This is the initial population.
2. After closing the lid, shake the container.
3. Open the lid and count the number of Skittles with the marked side up.
4. Skittles reproduce asexually (by themselves). Reproduction is triggered when the marked side of a Skittle is exposed to light. Add one Skittle to the container for each mark counted.
5. Record the number of Skittles now in the container. This is the end of one "Shake".
    a. The end of each "Shake" represents the end of one time period.
    b. The number of Skittles present at the end of a "Shake" is the total population at that time.

(a)

c. Remember that at "Shake" 0, the number of Skittles was 2.

6. A method of recording and organizing your data has been provided for you.
7. Repeat Parts 2-5 for 15 "Shakes".

(b)

Figure 3 (a) and (b): Exponential Activity

GROUP EXPLORATION JOB SHEET

Please write the name of your group members in the space provided next to his/her job.

Recorder:_____________________________ records the information neatly and accurately.

Counter: _____________________________counts the data and relays the number to the recorder.

Shaker: _____________________________shakes the data and adds the appropriate number of "Skittles" after each shake.

DO NOT EAT THE DATA!!!!!!!!!!!! (GERMS)
You will be provided with your own data when the activity is completed.

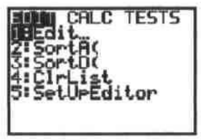
Figure 4: Exponential Worksheet Responsibility Chart

MODELING EXPONENTIAL GROWTH AND DECAY USING THE TI-83
GRAPHING CALCULATOR

After collecting the data from the "Skittles" activity, you are ready to enter the data into your graphing calculator.

TO ENTER DATA INTO A LIST:

When you press the STAT key you will see the following window.



Press ENTER to edit a list.
Simply enter the data for x in L1.
Press the right arrow key to get to L2 and enter the data for y into L2.

TO CLEAR LISTS:

Press STAT , ENTER for edit, and use the up arrow key to place the cursor on the list name, press CLEAR, then ENTER.

WARNING!!!!!
Pressing the DEL key instead of CLEAR will delete the list from the calculator.
You can get it back with the INS key.

PREPARING TO GRAPH:

Press STATPLOT (2^{nd} Y=)
Press ENTER for PLOT 1.
Move the cursor to ON, and press ENTER
Make sure the list names you want appear opposite Xlist and Ylist

Figure 5: Exponential Worksheet Help

Summative Assessment
Exponential Functions
Due date:___________

Name:
Algebra I-II/III

RAFT

ROLE: You are a biologist studying the growth of a new disease that is feared to be the new pandemic.

- Research current data on Avian flu, SARS in China, West Nile Virus, Lyme Disease, AIDS or any other virus or bacteria.

AUDIENCE: You are presenting information regarding your research to a team of doctors that are looking for a cure.

FORM: Produce a number of graphs detailing the growth rate of the disease and the affect on society if allowed to continue without treatment.

- Create a scatter plot of your data.
- Calculate the exponential model with the aid of technology.
- Explain the results of your findings and defend the model you came up with.
- Justify your reasoning.
- What are the implications of your results?

TIME: PRESENT

Use present day data researched on the Internet or in the library or in medical journals.

Figure 6: Exponential Activity/Assessment

Graphing Calculator Exploration: Derivatives of composite functions: Examples with powers

Purpose: To illustrate the geometric interpretation of the chain rule as a relation between slopes of tangent lines and the chain rule for a doubly composite function w(z(y(x))). This worksheet uses the chain rule and the formula for differentiating Ax^n for constants A and n.

Problem #1:

A. Graph $y = \sqrt{x}$ with its tangent line at $x = 4$. Use a differentiation formula to find dy/dx at $x = 4$ and put this value under the drawing.

B. Graph $z = \frac{1}{4} y^3$ with its tangent line at $y = 2$. Use a differentiating formula to find dz/dy at $y = 2$ and put this value under the drawing.

C. Substitute the formula for y in terms of x from part A into the formula for z in terms of y from part B to express z as a function of x. Generate the graph of z as a function of x for $-1 < x < 8$, and $-1 < z < 5$.

(a)

D. Use differentiation formulas and the equation from part C to find the derivative dz/dx at x = 4 and put the value under the graph. Estimate the slope of the tangent line at x = 4 using your graph.

E. The chain rule for differentiating z(y(x)) with respect to x reads

$$dz/dx = dz/dy * dy/dx$$

if the values of the variables are omitted. Use this formula to explain how the slopes of the tangent lines in the graphs from Parts A,B,and C are related.

Problem #2:

A. Graph w = 4/z and its tangent line at z = 2. Use a differentiation formula to find the derivative dw/dz at z = 2 and put its value under the drawing. Estimate the slope of the tangent line at z = 2 using your graph.

B. Substitute the formula for z in terms of x from Problem 1C into the formula for w in terms of z from part A of this problem to express w as a function of x. Check your formula by generating the graph for $-1 < x < 8$ and $-1 < w < 6$. Draw the graph.

C. Use differentiation formulas and the equation from part B to find the derivative dw/dx at x = 4 and put its value under the drawing.

(b)

Problem #3: In Problem 1 we used the rule dz/dx = dz/dy * dy/dx. Give the analogous formula for expressing dw/dx in terms of dw/dz, dz/dy, and dy/dz. This type of formula is what gives the "chain" rule its name.

Problem #4: Use the chain rule from Problem 3 to explain how the slopes of the tangent lines in Problem 1A, 1B, 2A, and 2B are related.

(c)

Figure 7 (a), (b) and (c): Derivatives of Composite Functions Worksheets

Exploration Graphing Calculator Fundamental Theorem of Calculus

#1 Graphing NINT f p298 Finney

Use NINT to graph the function

$F(x) = \int_3^x \tan t \, dt + 5$

1. Graph the function y = F(x) in the window $[-10,10]$ by $[-10,10]$. You will probably wait a long time and see no graph. Break out of the graphing program if necessary.
2. Recall that the graph of the function y = tan x has vertical asymptotes. Where do they occur on the interval $[-10,10]$?
3. When attempting to graph the function $F(x) = \int_3^x \tan t \, dt + 5$ on the interval $[-10,10]$, your graphing calculator begins by trying to find F(-10). Explain why this might cause a problem for your calculator.
4. Set your viewing window so that your calculator graphs only over the domain of the continuous branch of the tangent function that contains the point (3, tan 3).
5. What is the domain in step 4? Is it an open interval or a closed interval?
6. What is the domain of F(x)? Is it an open interval or a closed interval?
7. Your calculator graphs over the closed interval $[x_{min}, x_{max}]$. Find a viewing window that will give you a good look at the graph of F and produce the graph on your calculator.
8. Describe the graph of F.

Figure 8: Fundamental Theorem of Calculus Exploration

Exploration: Finding Antiderivatives Name:

For each derivative, describe the original function F

1. $F'(x) = 2x$
2. $F'(x) = x$
3. $F'(x) = x^2$
4. $F'(x) = \frac{1}{x^2}$
5. $F'(x) = \frac{1}{x^3}$
6. $F'(x) = \cos x$

What strategy did you use to find F?

Figure 9: Integral Worksheet/Assessment

Volumes of Solids of Revolution

Cross Sections

What is a cross section?

A cross section of a three dimensional solid would be the two dimensional shape you get when you slice the solid. For example, think of a loaf of bread. When we slice the bread, we get a cross section of the bread. A cross section would be like cutting an object and looking at the face of what you just cut. Similar to the disk method, you will be given a region defined by a number of functions. You will graph the region and lay that region flat so that any solid you build on that region will have the same cross section no matter where you slice it.

We are going to view some of these solids in class at:

http://astro.temple.edu/~dhill001/sectionmethod/sectiongallery.html

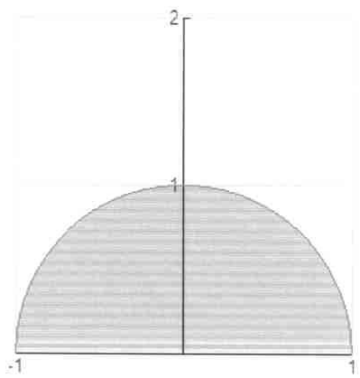
The volume would be the area of a cross section times the height.

For example: Let's consider the volume of the solid whose base is the function $y = \sqrt{1-x^2}$ and whose cross sections perpendiculars to the x-axis are squares.

The volume can be found by summing all of the areas of each square in the region it is bounded by.

(a)

This region would like:



If the base of the solid is this semicircle and the cross sections are squares, imagine that an infinite number of squares are rising up off of this page and the base of the squares lie perpendicular to the x-axis.

The area of this particular cross section would be $A = s^2$, and s would equal $\sqrt{1-x^2}$. The heights of the squares are going to vary according to where the square is located in our bounds.

$V = \int_{-1}^{1} s^2\, dx$

$V = \int_{-1}^{1} (\sqrt{1-x^2})^2\, dx$

$V = \frac{4}{3}$

Now consider the same base but change the cross sections to semicircles.

The area of a semicircle is $\frac{1}{2}\pi r^2$.

$V = \int_{-1}^{1} \frac{1}{2}\pi r^2\, dx$

(b)

The radius r would equal $\frac{\sqrt{1-x^2}}{2}$

Therefore, $V = \frac{1}{2}\pi \int_{-1}^{1} (\frac{\sqrt{1-x^2}}{2})^2 \ dx$

$V = \frac{\pi}{6}$

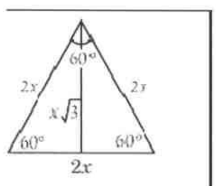
Now consider equilateral triangles with the base the same region.

The area of the cross section would equal the area of a triangle.

The area of a triangle is $A = \frac{1}{2}BH$.

The base of the triangle would be $y = \sqrt{1-x^2}$.

The height is more difficult to find. Remember from Geometry:



Area of an equilateral triangle is $\frac{\sqrt{3}}{4} base^2$

Therefore our equilateral triangle area would be $\frac{\sqrt{3}}{4}\left(\sqrt{1-x^2}\right)^2$.

Which would yield a volume of $V = \int_{-1}^{1} \frac{\sqrt{3}}{4}\,(1-x^2)\,dx$.

$V = \frac{\sqrt{3}}{3}$

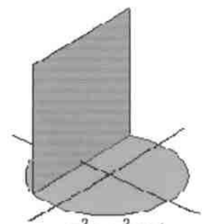
(c)

Consider another example from:

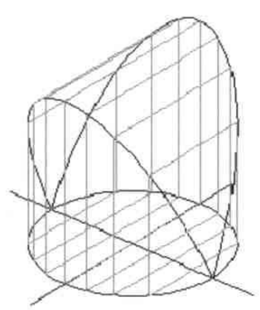
http://www.ies.co.jp/math/java/samples/renshi.html

There is a solid whcese bottom face is the circle $x^2+y^2 \leqq 1$
And every cross-section of the solid perpendicular to x-axis is a square.

Find the volume of the solid

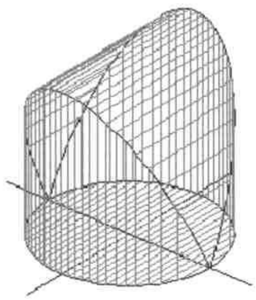


If we were to make the solid it would look like this:



As we keep accumulating squares, it would look like this:

(d)



The area of the cross sections are squares and the area of a square is $A = s^2$.

Solve the equation of the circle for y and get $y = \pm\sqrt{1-x^2}$.

Our base would be twice $\sqrt{1-x^2}$. because of the symmetry of the graph.

The volume would be $V = \int_{-1}^{1}(2\sqrt{1-x^2})^2dx$

$V = \frac{16}{3}$ cubed units

Utilize the internet to help you visualize these solids. We had to draw them by hand many years ago while walking uphill in three feet of snow backwards and barefoot (ha-ha).

Homework: p 465-466 # 61, 62, 63

(e)

Figure 10 (a), (b) and (c): Cross-section lesson

Applications of Quadratic Functions

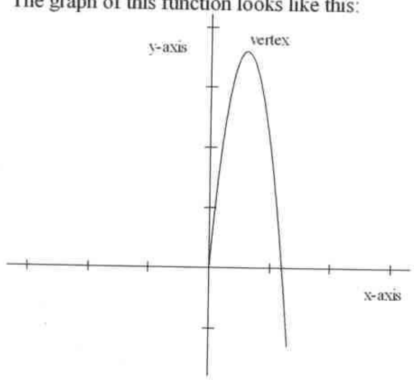
Quadratic functions have many applications. Let us look at a few examples:

Example 1:

An agency sells tickets for an opera. Based on previous experience, this agency has determined that the profit they can make on selling x tickets is given by the function $P(x) = 12x - 0.1x^2$. What is the maximum profit they can expect to make and how many tickets do they have to sell in order to earn this maximum profit?

Solution:

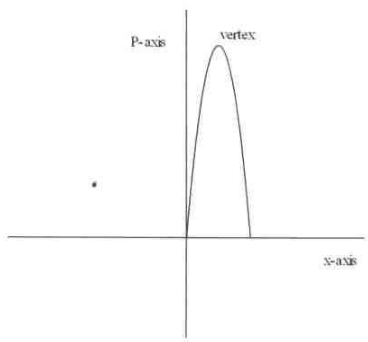
First we take a look at the function $f(x) = -0.1x^2 + 12x$ and we know that we can write this as $y = -0.1x^2 + 12x$. Since the first coefficient of this function is negative we know that the graph of this function if a parabola opening downwards, in other words, a parabola with a maximum y-value.
The graph of this function looks like this:



Clearly, if we want to find the maximum y-value of this parabola we have to find the coordinates of the vertex. We can do this by using the vertex theorem which states that the vertex of function $f(x) = ax^2 + bx + c$ is the point $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

In this case we have a = -0.1, b = 12 and c = 0. So $-\frac{b}{2a} = -\frac{12}{2\cdot(-0.1)} = 60$. We have found that the x-coordinate of the vertex is 60. The y-coordinate of the vertex is then $f(60) = -0.1\cdot 60^2 + 12\cdot 60 = -360 + 720 = 360$. We now see that the maximum y-value is y = 360 for x = 60.

Let us now return to the original function $P(x) = 12x - 0.1x^2$. Shifting the two terms around we get $P(x) = -0.1x^2 + 12x$ and we realize that we have the same exact function as before except that the vertical axis is now called *P(x)*, the profit axis. This looks like this:



Using the same reasoning as above we find that the maximum profit is P = \$360 for x = 60, the number of tickets sold.

2

(b)

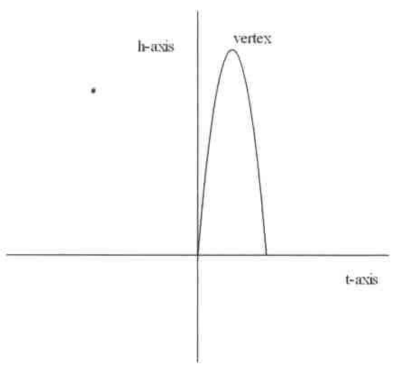
Example 2:

An object fired vertically upward with an initial speed v_0 is after t seconds at a height of $h(t) = v_0 \cdot t - 4.9 \cdot t^2$, in meters. Given that this object has an initial speed of 68.6 m/s, what is the maximum height it will reach?

Solution:

Filling in the initial speed we get $h(t) = 68.6t - 4.9t^2$ which can be rewritten as $h(t) = -4.9t^2 + 68.6t$. This again is the equation of a parabola but now in a coordinate system in which the horizontal axis is t, the time axis, and the vertical axis is the h, the height axis.

The graph is:



If we want to find the coordinates of the vertex we have to use the vertex formula again. In this case, with a = -4.9, b = 68.6 and c = 0 we get $-\frac{b}{2a} = -\frac{68.6}{2 \cdot (-4.9)} = 7$. So we found that the maximum height is attained at t = 7 sec. To find the maximum height we have to find the y-coordinate of the vertex: the maximum height is $h(7) = 68.6 \cdot 7 - 4.9 \cdot 7^2 = 240.1\ meters$

We often have to do a bit more work to solve a problem. This is the case when we do not know the equation of the function to be maximized or minimized, and we have to find it ourselves. Let us look at a few examples of this type of problem:

(c)

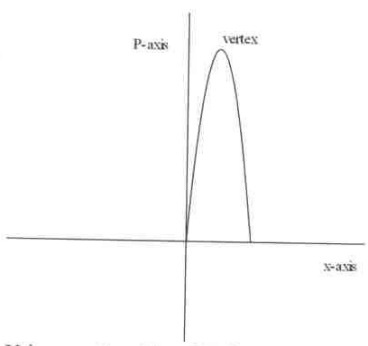
Example 3:

Assume that a company knows that the cost to produce x items is given by the cost function $C(x) = 5x^2 + 800x$ dollars. It also knows that the revenue from x items is given by the revenue function $R(x) = 1000x + 200$. Find the maximum profit they can expect and how many of these items they have to produce and sell to make this maximum profit.

Solution:

Let us explore the situation a bit before coming to the real solution of the problem.
The cost to produce 10 items is $C(10) = 5 \cdot 10^2 + 800 \cdot 10 = \$8{,}500$.
The revenue from selling 10 items is $R(10) = 1000 \cdot 10 + 200 = \$10{,}200$. Since profit is defined as revenue minus cost as in the formula $P = R - C$, then producing and selling 10 items will mean a profit of $1,700.
Now, let us look at the situation of producing and selling 15 items. The cost to produce 15 items is $C(15) = \$13{,}125$ and the revenue is $R(15) = \$15{,}200$. The profit in this case will be $2,075.
Last, let us see what happens when 50 items are produced and sold. In this case, after calculating C(50) and R(50) we find that a loss (when a profit is negative we call it a loss) of $2,300 will be made.
You see now that it is a reasonable question to ask how many items must be produced and sold to maximize the profit. We will come to that now.

As mentioned above, profit is defined as revenue minus cost, the formula $P = R - C$. Inserting the cost and revenue functions given in the problem into this formula will give us:
$\mathrm{P(x)} = (1000x + 200) - (5x^2 + 800x) = -5x^2 + 200x + 200$. This is an equation of a parabola opening downward, a parabola with a maximum. We have already seen the graph in example 1:



Using a = -5 and b = 200 the vertex formula gives us $(20, f(20)) = (20, 2200)$ as a vertex. So in this case the maximum profit will be $2,200 for x = 20 items produced and sold.

(d)

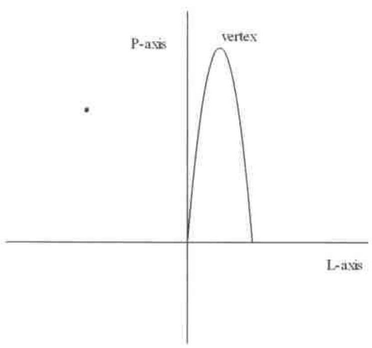
Example 4:

The sum of two integers is 30. What is a maximum product of these two numbers?

Solution:

Using L and S as the larger and smaller numbers respectively, we get $L+S=30$ and $P=L\cdot S$.

The maximum value of the product can be found after writing the product as either a function of L or of S. Writing P as a function of L we get
$P(L)=L(30-L)=-L^2+30L$. The graph of this function in an L, P coordinate system, where L stands for the larger number and P for the product is:



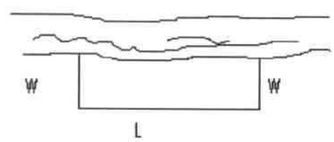
Using the vertex formula, with a = -1 and b = 30 we get the coordinates of the vertex as (15, 225). So the maximum value of the product is 225 for L = 15, and this makes S = 15 as well.

Example 5:

A farmer wants to build a rectangular fence near a river, and will use 120 ft of fencing. What are the dimensions of the largest region that can be enclosed if the side next to the river is not fenced?

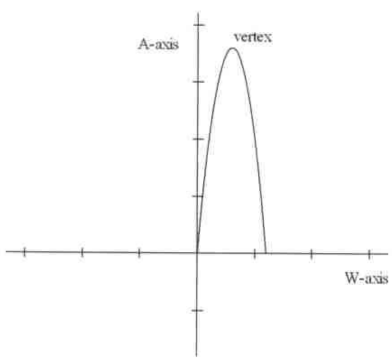
Solution:

Draw a sketch:



We see that the farmer has to use the 120 ft on three pieces of wire. Say that the longest piece is called L and the shorter two pieces are W each. You then get 120 = L + 2W. Solve for L (L = 120 - 2W) and substitute this in the equation for area.

As for the equation for area, we use A = W(L) and combine this with the previous equation to get $A = W(120 - 2W) = -2W^2 + 120W$. This is yet another equation of a parabola, now in a W, A coordinate system in which W stands for the width and A for the area. The graph looks like:



We find the vertex in the usual way to get (W, A) = (30, 1800). So we have found a maximum area of 1800 ft^2 for a width of 30 ft. Using L = 120 - 2W, we end up with a length of 60 ft.

(f)

Figure 11 (a) – (f): Quadratic formula lesson

AP Calculus AB
Syllabus

School: Tahanto Regional High School
Course: AP Calculus AB
Text: ***Calculus: Graphical, Numerical, Algebraic*** by Finney, Ross L., Franklin Demana, Bert Waits, and Daniel Kennedy, 2003 edition
Teacher: Ms. Francene Gleason

Learning Calculus is a process; it does not come all at once. Be patient, persevere, ask questions, discuss ideas and work with classmates, and seek extra help when you need it right away. The rewards of learning calculus will be very satisfying.

Philosophy: The main objective of an Advanced Placement course in calculus is to provide students with a full academic year of work in calculus equivalent to courses offered in colleges and universities. It is expected that all of my students who take an AP course in calculus will take the AP exam in May to receive placement or credit at a college or university. Students taking this course should have an excellent mathematical background, be highly motivated to learn the material and possess an interest in mathematics. This course is designed to provide students' with the opportunity to explore higher-level mathematics by developing the concepts of calculus as outlined in the AP Calculus AP Course Description. In addition, broad concepts and widely applicable methods are emphasized. The course utilizes a multirepresentational approach to calculus, with concepts, results, and problems being expressed geometrically, numerically, analytically, and verbally. Technology is used regularly to reinforce and verify analytical results, implement experimentation, and to assist in interpreting results. This will enable students to build a richer, more meaningful understanding of calculus. (The College Board – AP Calculus Course Description)

Course Content:

Chapter One: Prerequisites for Calculus **Days 4-6**

It is expected that students in my AP Calculus class complete Chapter One over the summer and complete an additional packet of teacher prepared worksheets that will be collected on the first day of class. After a quick review of the material, students are given an exam the first week of class. Mastery of a minimum of 85% is required to remain in the course.

- Lines
- Functions and Graphs
- Exponential Functions
- Parametric Equations
- Functions and Logarithms
- Trigonometric functions

(a)

Objectives:

- Students will have a complete understanding of linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric and piecewise defined functions.
- Students will be familiar with the properties of functions, the algebra of functions and the graphs of functions.
- Students will understand the language of functions (domain, range, odd and even, periodic, symmetry, zeros, intercepts, etc.)
- Students will know the values of the trigonometric functions.
- Students will know how to use technology to aid in the understanding of functions.

Activities or Projects:

- ✓ ***Calculus Explorations*** by Paul A. Foerster #37, 40 and 2.
- ✓ Function Match Activity (domain, range, equation, and graph match)
- ✓ Sort of (FUN)ction Activity (identify similarities and differences of functions represented graphically, numerically, algebraically, and with tables)

Chapter 2: Limits and Continuity **Days 9-11**

- Rates of Change and Limits
- Limits involving infinity
- Continuity
- Rates of Change and Tangent Lines

Objectives:

- Students will gain an intuitive understanding of the limiting process.
- Students will calculate limits using algebra.
- Students will estimate limits from graphs or tables of data.
- Students will understand asymptotes in terms of graphical behavior.
- Students will describe asymptotic behavior in terms of limits involving infinity.
- Students will compare relative magnitudes of functions and their rates of change.
- Students will gain an intuitive understanding of continuity.
- Students will understand continuity in terms of limits.
- Students will gain an understanding of graphs of continuous functions.

Activities or Projects:

- ✓ ***Calculus Explorations*** by Paul A. Foerster #5, 6, 8, 9, 10
- ✓ Taking It To The Limit Lab (limits graphically, numerically, tables)
- ✓ Sample AP Questions

Chapter 3: Derivatives **Days 25-30**

- Derivative of a Function
- Differentiability
- Rules for Differentiation
- Velocity and Other Rates of Change
- Derivatives of Trigonometric Functions
- Chain rule

(b)

- Implicit Differentiation
- Derivatives of Inverse Trigonometric Functions
- Derivatives of Exponential and Logarithmic Functions

Objectives:

- Students will understand derivatives presented graphically, numerically, and analytically.
- Students will interpret derivatives as instantaneous rates of change.
- Students will define derivatives as the limit of the difference quotient.
- Students will understand the relationship between differentiability and continuity.
- Students will understand the meaning of the slope of a curve at a point including points at which there are vertical tangents and points at which there are no tangents.
- Students will realize local linear approximation and tangent line to a curve at a point.
- Students will comprehend the instantaneous rate of change as the limit of average rate of change.
- Students will approximate rate of change from graphs and tables of values.
- Students will connect the characteristics of graphs of f and f'.
- Students will have knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- Students will be able to use the rules of differentiation to calculate derivatives of sums, products, and quotients including second and higher order derivatives.
- Students will be able to use derivatives to analyze straight-line motion and solve other problems involving rates of change.
- Students will be able to find derivatives using implicit differentiation and the chain rule.

Activities or Projects:

- ✓ Sample AP Questions
- ✓ ***Calculus Explorations*** by Paul A. Foerster #12, 23,14, 16, 17, 18, 20, 21, 15, 19, 65, 66, 25, 13,
- ✓ Weakest Link Activity (decomposition of composite functions)
- ✓ Delicious Derivatives Activity (for chain rule)

Chapter 4: Applications of Derivatives **Days 23-27**

- Extreme Values of Functions
- Mean Value Theorem
- Connecting f' and f'' with the Graph of f
- Modeling and Optimization
- Linearization and Newton's Method
- Related Rates

Objectives:

- Students will understand the relationship between the increasing and decreasing behavior of f and the sign of f'.
- Students will be able to correspond the characteristics of the graphs of f, f', and f''.
- Students will be able to apply the Mean Value Theorem and understand its geometric consequence.

(c)

- Students will connect the characteristics of graphs of f and f'.
- Students will translate verbal descriptions into equations involving derivatives and vice versa.
- Students will be able to determine the local or global extreme values of a function.
- Students will analyze curves, including the notion of monotonicity and concavity.
- Students will be able to solve application problems involving finding minimum and maximum values of a function.
- Students will be able to solve related rates problems.
- Students will be able to find linearization and use Newton's method to approximate the zeros of a function.
- Students will be able to estimate the change in a function using differentials.

Activities or Projects:

- ✓ ***Calculus Explorations*** by Paul A. Foerster #29, 30, 48, 49, 67, 68
- ✓ Which is Which Activity (connecting the graphs of f, f', and f''
- ✓ Sample AP Questions
- ✓ Function Derivative Match Activity (Match functions, verbal descriptions of functions, derivatives, and verbal descriptions of derivatives)

Chapter 5: The Definite Integral **Days 20-26**

- Estimating with Finite Sums
- Definite Integrals
- Definite Integrals and Antiderivatives
- Fundamental Theorem of Calculus
- Trapezoidal Rule

Objectives:

- Students will be able to approximate the area under the graph of a nonnegative continuous function by using rectangle approximation methods.
- Students will be able to interpret the area under a graph as a net accumulation of a rate of change.
- Students will be able to express the area under a curve as a definite integral and as a limit of Riemann sums.
- Students will be able to compute the area under a curve using a numerical integration procedure.
- Students will be able to apply the rules for definite integrals and find the average value of a function over a closed interval.
- Students will be able to apply the Fundamental Theorem of Calculus.
- Students will understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- Students will be able to approximate the definite integral by using the Trapezoidal Rule.
- Students will be able to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

Activities or Projects:

- ✓ ***Calculus Explorations*** by Paul A. Foerster #30,32-35
- ✓ Worksheets from AP Calculus AB/BC College Board conference

(d)

✓ Calculator explorations

Chapter 6: Differential Equations and Mathematical Modeling **Days 20-22**

- Slope Fields and Euler's Method
- Antidifferentiation by Substitution
- Exponential Growth and Decay

Objectives:

- Students will be able to construct antiderivatives using the Fundamental Theorem of Calculus.
- Students will be able to find antiderivatives of polynomials, exponentials, and selected trigonometric functions as well as linear combinations of these functions.
- Students will be able to solve initial value problems.
- Students will be able to construct slope fields using technology and interpret slope fields as visualizations of differential equations.
- Students will understand geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.
- Students will be able to compute indefinite and definite integrals by the method of substitution.
- Students will be able to solve problems involving exponential growth and decay in a variety of applications.
- Students will be able to use Euler's method and the improved Euler's method to find approximate solutions to differential equations with initial values.

Activities or Projects:

✓ ***Calculus Explorations*** by Paul A. Foerster #44, 46
✓ Sample AP Questions
✓ Calculator explorations

Chapter 7: Applications of Definite Integrals **Days 15-20**

- Integral as Net Change
- Areas in the plane
- Volume
- Applications from Science and Statistics

Objectives:

- Students will be able solve problems in which a rate is integrated to find the net change over time in a variety of applications.
- Students will be able to use integration to calculate areas of regions in a plane.
- Students will be able to use integration (by disks, washers, and cross sections) to calculate the volumes of solids.
- Students will be able to find specific antiderivatives using initial conditions, including applications to motion along a line.
- Students will be able to adapt their knowledge of integral calculus to model problems involving rates of change in a variety of applications, possibly in unfamiliar contexts.

(e)

Activities or Projects:

- ✓ ***Calculus Explorations*** by Paul A. Foerster #50-52
- ✓ Sample AP Questions

Technology:

Classroom demonstration is done with a TI-84 Plus Graphing Calculator. I use an overhead computer projection screen with TI –SmartView Software or I use a view screen panel. Students in the course are required to own a graphing calculator. Most students have a TI-83, TI-83 Plus, TI-84 or TI -84 Plus. Students who cannot afford a calculator may rent one from the school for a minimal fee or one will be provided for in-class use. Each student must be able to graph a function, find intersections and roots of functions, calculate numerical derivatives and approximate definite integrals using the graphing calculator. Students must be able to perform the calculus for non-calculator exams. Students are given programs for slope fields, Riemann Sums, area, and Trapezoidal rule. I use the Internet extensively in my course to help the students to visualize and to understand complex concepts. Students are regularly given web addresses for further enhancement of a topic. I also use Geometer's Sketchpad Version 4, and a CD-ROM Journey Through Calculus by Bill Ralph to help students fully explore and appreciate the dynamics of calculus.

Teaching Strategies:

I believe that every student can learn calculus if given the opportunity. I regularly use a Table PC with Math Journal software to generate notes for the students. I try to minimize lecture to the teaching of new topics so that students have the opportunity for classroom activities and projects that will enhance their knowledge of the curriculum. Most of the class time is used for students to practice and ask questions. Students are required to speak with correct mathematical vocabulary and are required to verbalize strategies used to solve problems. I emphasize that they can reach a solution but they must justify how they arrived at the solution. I place a lot of emphasis on how concepts are connected graphically, numerically, analytically, and verbally. Students must practice calculus in order to learn calculus. Thus, homework is given almost every night. I frequently give take-home exams so as not to use class time and encourage the students to form study groups to complete them. Students must pass in their own worked solutions for credit. Beginning in January, I hold AP Calc Sunday night parties at students' homes. I travel to a pre-designated house and we practice ONLY AP Multiple Choice or Open-Response questions for 2 hours. By the end of April we usually cover all released multiple-choice problems from 1969 - 2003 and many open-response questions from selected years. Students are not required to attend these sessions but are encouraged to attend and usually 90% or better will attend.

Assessment and Evaluation:

I use a variety of tools to assess the students in my class. Students are given mini quizzes daily that may consist of one to three multiple choice questions from old AP exams dating back to 1969 or a portion of a single open-response question from past exams. These are timed and scored using the scoring guidelines put forth by the College Board. These quizzes account for 25% of each student's quarter grade. In addition to the quizzes, I give in-class tests and take-home tests. These tests usually include at least one problem from a past AP exam. Tests account for 50% of each student's quarter grade. The remaining 25% of a student's grade is based on projects, worksheets, classroom activities, and homework. Students are given a midterm exam if they do not have a 90% or above average in the class. This is school policy. The same applies to the final at the end of the year.

(f)

Activities and Projects:
Projects and activities are given from a variety of sources to include but not limited to:

- ✓ *Advanced Placement Correlations and Preparation*, Calculus, 2003, by Finney, Demana, Waits, and Kennedy that accompanies the textbook
- ✓ *Calculus Explorations* by Paul A Foerster
- ✓ Web-based activities and labs from James Rahn's website http://www.jamesrahn.com/
- ✓ AP Calculus Multiple Choice Questions Collection 1969-1998, College Board
- ✓ AP Calculus Open Response Questions Collection, available online at AP Central
- ✓ *Fast Track to a 5, Preparing for the AP Calculus AB and Calculus BC Examinations,* Sharon Cade, Rhea Caldwell, and Jeff Lucia
- ✓ *Calculus of a Single Variable*, Larson, Hostetler, and Edwards, 8th edition, 2006
- ✓ *5 Steps to a 5, AP Calculus AB/BC,* McGraw Hill Second edition
- ✓ *Geometer's Sketchpad* software, version 4

After the Exam in May:
After the exam, students are given a number of projects to complete. One of the projects involves answering the questions, "What is a derivative?" and "What is an integral?" They are asked to create a presentation that would explain these topics to students with knowledge of mathematics through Algebra I and then they present their projects to my Algebra I class. Students use a variety of media to create these projects including creating lyrics and writing music, creating poetry, making a story book, creating a video or power point presentation, or creating a video game.

Another project involves reading a book from a selected list of books and writing a report detailing the mathematics in the readings. Some of the books chosen are:

- *Zero, the Biography of a Dangerous Idea,* Charles Seife, Penguin Books, 2000.
- *The Millennium Problems*, Keith Devlin, Basic Books, 2002.
- *Euclid in the Rainforest: Discovering Universal Truths in Logic and Math,* Joseph Mazur, Pi Press, 2005.

(g)

This verifies that I have read and understand the information as it was explained in the course overview and discussed in class.

Student Name:____

Student Signature:______________________________________

This verifies that I have read and discussed the information as it was explained in the course overview with my son or daughter.

Parent Name:____

Parent Signature:____

(h)

Figure 12 (a) – (h): Actual AP Calculus AB Syllabus used by mentor teacher

CURRICULUM FRAMEWORK: **Course: Precalculus**

DOE Standard Number and State Standard	Learning Outcomes	Assessment	Resources	Suggested Activities
12.P.8 Solve a variety of equations and inequalities using algebraic, graphical, and numerical methods, including the quadratic formula; use technology where appropriate. Include polynomial, exponential, logarithmic, and trigonometric functions; expressions involving absolute values; trigonometric relations; and simple rational expressions.	Use the distance and midpoint formulas. Use algebra to solve geometry problems.	Worksheets Applications of the Distance and Midpoint Formulas Worksheet: Verifying Right Triangles Writing to Learn: Prove that the midpoint of the hypotenuse of any right triangle is equidistant from the three vertices.	**Chapter 1: Graphs** *Precalculus Graphing and Data Analysis by Sullivan and Sullivan, 2001* Board Problems 1, 5, 15, 21, 39, 49 1.1 Rectangular Coordinates; Graphing Utilities Power Point Presentation 1.1	Calculate the distance from home plate to second base on a baseball diamond. Have students discuss the difference between the coordinates of points in the plane and those on their graphing utility. 1.1 Exercises in textbook. Finding the coordinates of a point shown on a graphing utility screen. Using Algebra to solve geometry problems.
12.P.8	Graph equations by hand and with a graphing utility. Find intercepts. Test an equation for symmetry with respect to the x-axis, y-axis, and origin.	Quiz: Symmetry Algebraically and Graphically	**Chapter 1: Graphs** *Precalculus Graphing and Data Analysis by Sullivan and Sullivan, 2001* 1.2 Graphs of Equations Power Point Presentation 1.2 Internet Activity	Have students experiment with the window of their graphing utility with various functions. 1.2 Exercises in textbook. Graphing Calculator Exploration: Graphing an equation using a graphing utility and find intercepts Solving equation for y= Finding intercepts from a graph and an equation.

Berlin~Boylston Regional School District Curriculum Map ***11/03***

Figure 13: Sample curriculum framework for Pre-Calculus

Learning Standards for Precalculus

NOTE: The parentheses at the end of a learning standard contain the code number for the corresponding standard in the two-year grade spans.

NUMBER SENSE AND OPERATIONS

Understand numbers, ways of representing numbers, relationships among numbers, and number systems

Understand meanings of operations and how they relate to one another

Compute fluently and make reasonable estimates

Students engage in problem solving, communicating, reasoning, connecting, and representing as they:

PC.N.1 Plot complex numbers using both rectangular and polar coordinates systems. Represent complex numbers using polar coordinates, i.e., $a + bi = r(\cos\theta + i\sin\theta)$. Apply DeMoivre's theorem to multiply, take roots, and raise complex numbers to a power.

PATTERNS, RELATIONS, AND ALGEBRA

Understand patterns, relations, and functions

Represent and analyze mathematical situations and structures using algebraic symbols

Use mathematical models to represent and understand quantitative relationships

Analyze change in various contexts

Students engage in problem solving, communicating, reasoning, connecting, and representing as they:

PC.P.1 Use mathematical induction to prove theorems and verify summation formulas, e.g., verify $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

PC.P.2 Relate the number of roots of a polynomial to its degree. Solve quadratic equations with complex coefficients.

(a)

Patterns, Relations, and Algebra (continued)

PC.P.3 Demonstrate an understanding of the trigonometric functions (sine, cosine, tangent, cosecant, secant, and cotangent). Relate the functions to their geometric definitions.

PC.P.4 Explain the identity $\sin^2\theta + \cos^2\theta = 1$. Relate the identity to the Pythagorean theorem.

PC.P.5 Demonstrate an understanding of the formulas for the sine and cosine of the sum or the difference of two angles. Relate the formulas to DeMoivre's theorem and use them to prove other trigonometric identities. Apply to the solution of problems.

PC.P.6 Understand, predict, and interpret the effects of the parameters a, ω, b, and c on the graph of $y = a\sin(\omega(x - b)) + c$; similarly for the cosine and tangent. Use to model periodic processes. (12.P.13)

PC.P.7 Translate between geometric, algebraic, and parametric representations of curves. Apply to the solution of problems.

PC.P.8 Identify and discuss features of conic sections: axes, foci, asymptotes, and tangents. Convert between different algebraic representations of conic sections.

PC.P.9 Relate the slope of a tangent line at a specific point on a curve to the instantaneous rate of change. Explain the significance of a horizontal tangent line. Apply these concepts to the solution of problems.

Geometry

Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

Specify locations and describe spatial relationships using coordinate geometry and other representational systems

Apply transformations and use symmetry to analyze mathematical situations

Use visualization, spatial reasoning, and geometric modeling to solve problems

Students engage in problem solving, communicating, reasoning, connecting, and representing as they:

PC.G.1 Demonstrate an understanding of the laws of sines and cosines. Use the laws to solve for the unknown sides or angles in triangles. Determine the area of a triangle given the length of two adjacent sides and the measure of the included angle. (12.G.2)

PC.G.2 Use the notion of vectors to solve problems. Describe addition of vectors, multiplication of a vector by a scalar, and the dot product of two vectors, both symbolically and geometrically. Use vector methods to obtain geometric results. (12.G.3)

PC.G.3 Apply properties of angles, parallel lines, arcs, radii, chords, tangents, and secants to solve problems. (12.G.5)

(b)

MEASUREMENT

Understand measurable attributes of objects and the units, systems, and processes of measurement

Apply appropriate techniques, tools, and formulas to determine measurements

Students engage in problem solving, communicating, reasoning, connecting, and representing as they:

PC.M.1 Describe the relationship between degree and radian measures, and use radian measure in the solution of problems, in particular problems involving angular velocity and acceleration. (12.M.1)

PC.M.2 Use dimensional analysis for unit conversion and to confirm that expressions and equations make sense. (12.M.2)

DATA ANALYSIS, STATISTICS, AND PROBABILITY

Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them

Select and use appropriate statistical methods to analyze data

Develop and evaluate inferences and predictions that are based on data

Understand and apply basic concepts of probability

Students engage in problem solving, communicating, reasoning, connecting, and representing as they:

PC.D.1 Design surveys and apply random sampling techniques to avoid bias in the data collection. (12.D.1)

PC.D.2 Apply regression results and curve fitting to make predictions from data. (12.D.3)

PC.D.3 Apply uniform, normal, and binomial distributions to the solutions of problems. (12.D.4)

PC.D.4 Describe a set of frequency distribution data by spread (variance and standard deviation), skewness, symmetry, number of modes, or other characteristics. Use these concepts in everyday applications. (12.D.5)

PC.D.5 Compare the results of simulations (e.g., random number tables, random functions, and area models) with predicted probabilities. (12.D.7)

(c)

Figure 14 (a) – (c): Learning standards for Pre-Calculus used at Tahanto Regional High School