



# Design of an Inversion Mechanism

A Major Qualifying Project proposal to be submitted to the faculty  
of Worcester Polytechnic Institute in partial fulfillment of the  
requirements for the Degree of Bachelor of Science

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## **Abstract**

The goal of this project was to create a mechanism that picks up a part, inverts it 180 degrees, and places it in a new location in its new orientation. This task was completed through the use of the design process. Ideas were brainstormed, drawn up, and evaluated. One design that was deemed a viable option was then modeled using Pro/ENGINEER. After modeling, the design was analyzed for various attributes such as stress, deflection, and fatigue failure. The result of this work is the creation of an inverting mechanism that uses a system of bevel gears with grippers attached to hold, rotate, and move the part. With the part in the grippers, as the rotating gear moves along the stationary gear, the part is flipped over 180 degrees. The part is brought to the grippers and removed from the grippers by the use of tooling that is stationary above the pick-up and drop-off locations. This mechanism provides a new way to access both sides of the part being moved as well as new tooling that could be modified and applied in several other applications.

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## **Introduction**

The sponsor is in need of a mechanism that picks up a part from one position, inverts it 180 degrees, and then releases it in a new location. The company will be using this mechanism on a new machine. This means that although the design envelope and speed of the mechanism are specified, the problem is very open-ended as to what type of mechanism could be used (i.e. linkage, gear train, etc.). Through the design process, many ideas have been brainstormed, preliminary designs were drawn up and analyzed, and one final design was picked, modeled, analyzed, and is fully described in this report.

## **Problem Statement**

Design a mechanism that will grip a part, invert it 180 degrees and then release it in a new location away from the original position.

## **Task Specifications**

1. Must be able to transfer 180 parts per minute
2. Must flip part 180 degrees between pick-up and drop-off locations
3. Must be self-contained, i.e. must include all parts, motors, etc. within it
4. Must fit mechanism within a design envelope of 50cm x 50cm x 70cm (W x D x H)
5. Must place the part within 0.5 mm of target location
6. Must not cause any visual or structural damage to the part
7. Must not touch any of the specified sections of the part
8. Must be in constant contact with the part from pick-up to drop-off point
9. Must be compatible with existing assembly equipment

10. Must be designed so that all parts have infinite life (1 million cycles)
11. Must be repairable and must be able to be assembled by a trained mechanic
12. Must not contain any attachments between moving parts except fasteners (i.e. no welds between moving parts)

## **Background**

### **Grippers**

When designing a gripper for a particular use, there are many factors to consider. Some grippers may grip objects better than others. The gripping abilities of the mechanism are based on various properties.

Grippers can be pneumatic, mechanical or vacuum actuated.

The force with which the gripper holds onto the object, the material of the object, and the material of the gripper all affect how well the gripper holds the part in place. The gripper should exert enough force on the object so that it does not shift or fall during movement, but not so much force that it causes any cosmetic or structural damage to the part. The interaction between the material of the part and the material of the gripper is important as well. The material of the gripper will differ based on the part that it is holding. If there is not enough friction between the two materials, the gripper may drop the part or may need to exert more force on the part in order to hold it tightly.

One other issue to consider when designing a gripper for a specific use is whether or not there are certain areas of the part that cannot be touched. If there is a significant amount of space that cannot come into direct contact with the gripper, a vacuum or suction gripper might be considered instead of a gripper that resembles fingers with rubber ends. With this type of gripper, there is less surface area on the part that comes into contact with the gripper. At the same time, there needs to be enough surface area in contact with the gripper to generate sufficient force to hold the part.

## **Gear Backlash**

Backlash is the result of clearance between the teeth in gears. Space between teeth causes relative motion between the gears. Anti-backlash gears are split into two gears, each half the thickness of the original. One is fixed to the shaft and the other is allowed to rotate around the stationary shaft, but is preloaded with springs to the fixed gear. Springs pull the two gears apart radially. With this configuration, the free gear is always pushed the opposite direction of the stationary gear so that at all times, the pinion is in contact with both gears. The fixed gear is on one side of the tooth and the free gear is on the other.

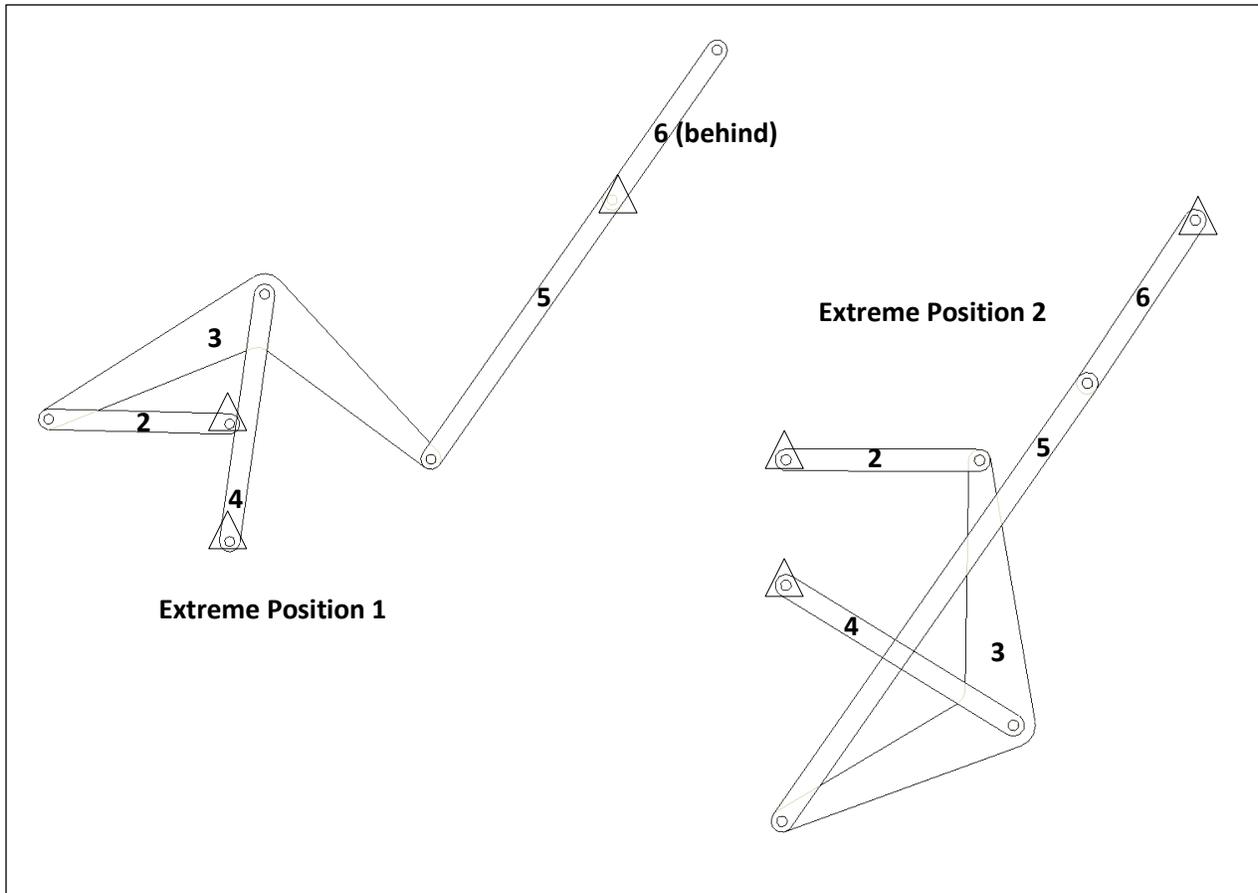
## **Preliminary Designs**

### **Linkages**

In the beginning of this project, several different types of linkages were considered for this problem including the Stephenson III Six-bar Linkage and a Modified Chebyshev Linkage.

#### *Stephenson III Six-bar Linkage*

The Stephenson III Six-bar Linkage was one option as a solution to the problem. This linkage is modeled in Pro/Engineer and is shown in Figure 1.



**Figure 1: Stephenson III Six-bar Linkage**

The linkage has links 2, 4 and 6 pivoted to ground. Link 6 is the input link and is driven at a constant speed through 360 degrees. During this motion, link 2 moves through 180 degrees. The 180 degrees gives the desired inversion of the part. After link 2 moves 180 degrees, it travels back along the same path to its start position while the crank is finishing its 360 degree rotation.

### *Chebyshev Four-bar Linkage*

A second linkage researched is the Chebyshev four-bar linkage. This linkage was originally intended to create approximately straight line motion at the coupler point 'P'. Because it is a Grashof double rocker, the coupler is flipped nearly 180° as it moves in that straight line. The original link lengths and configuration of the Chebyshev are shown in Figure 2.

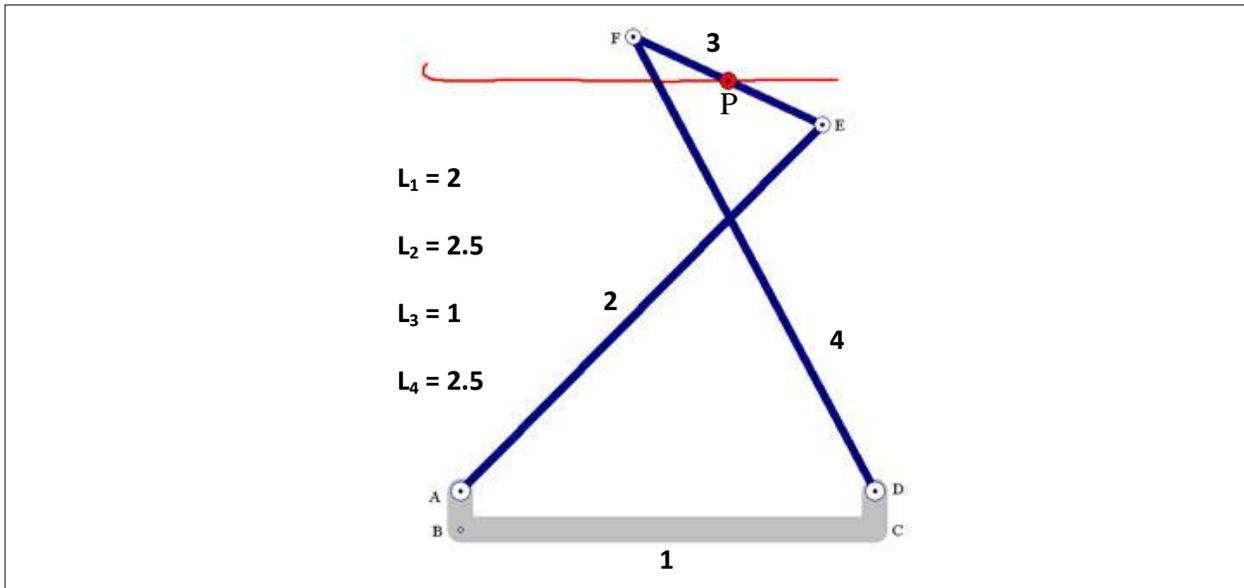


Figure 2: Chebyshev Linkage

A driver dyad can be added to create a six-bar linkage with 360° input. Because the inversion mechanism does not need to move in a straight line along its path as it turns over, the link lengths can be altered to optimize the inversion of the coupler link. These changes were applied to the link lengths; the resulting linkage is shown in Figure 3.

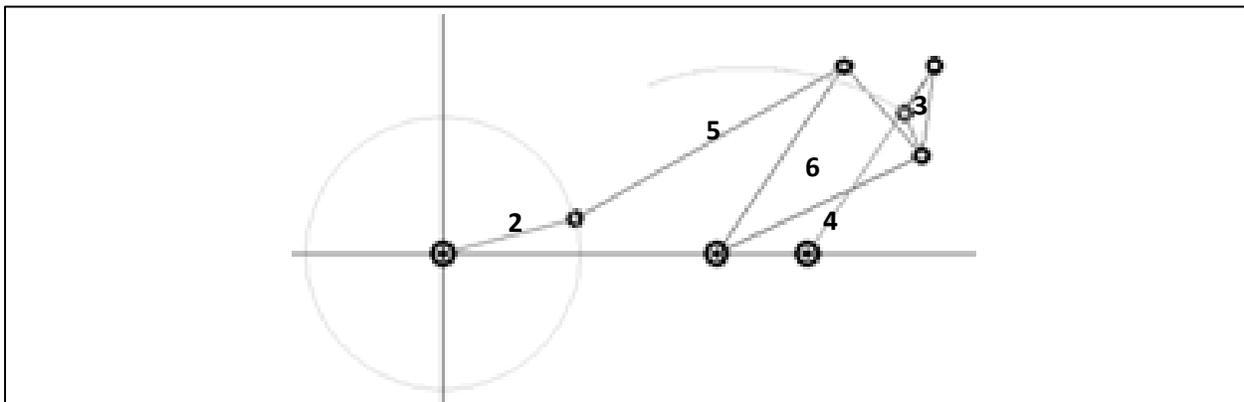
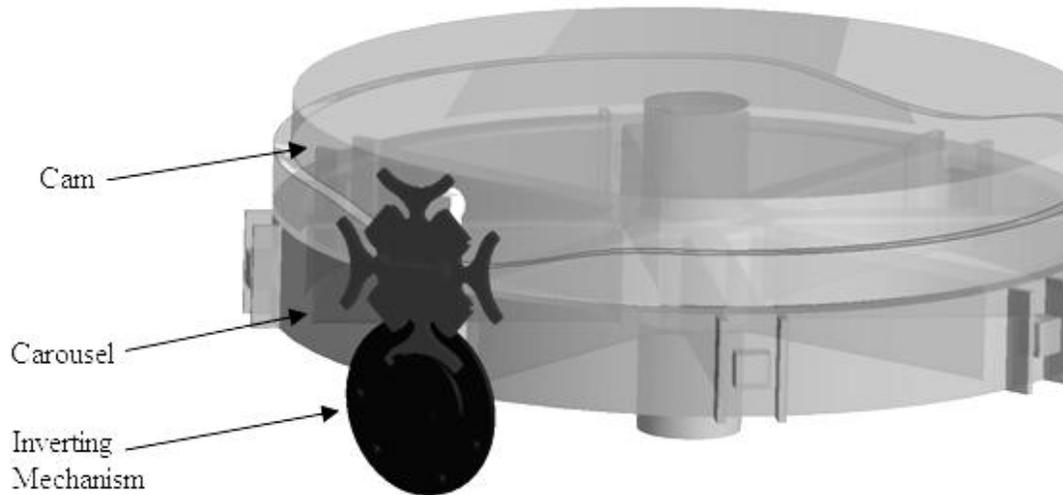


Figure 3: Modified Chebyshev Linkage

The link 3 represents the coupler which is inverted as link 2 rotates 360°.

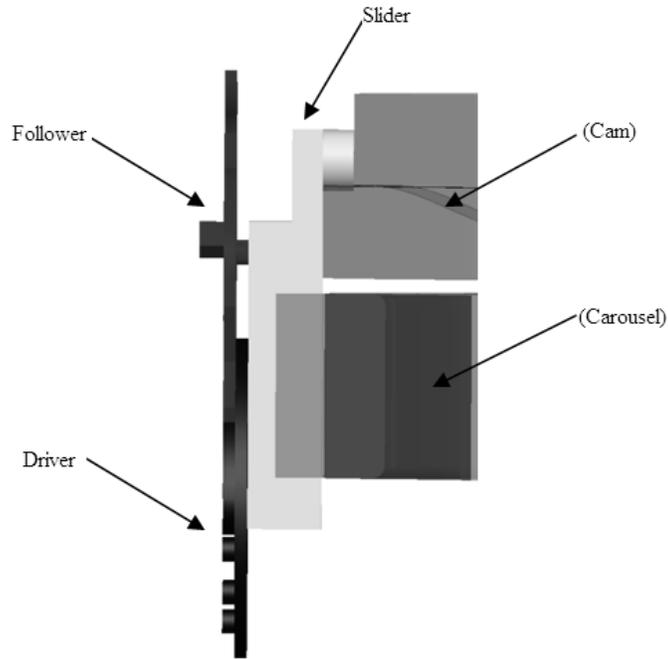
## Carousel

A carousel design was also investigated as a possible solution to this problem. The basic layout of this design is shown in Figure 4. It consists of three basic components, the carousel, cam and the inverting mechanism assembly.



**Figure 4: Full Model of Two-Part Mechanism**

The carousel rotates 360 degrees. There are 8 spokes on the carousel, each of which has a separate inverting mechanism attached to it. The inverting mechanism consists of a slider, and an inverting driver and follower as shown in Figure 5.



**Figure 5: Preliminary Inverting Mechanism Details**

The slider attaches directly to the carousel and is free to slide in the vertical direction. The inverter is attached to the slider so that as the slider moves up and down, so does the inverter. This motion allows the gripper, which is attached to the inverter follower, to raise and lower the part as it approaches and moves away from the pick-up and drop-off locations. The roller attached to the top of the slider allows it to interact with the cam. This roller slides on the cam, shown above the carousel in Figure 5. As the cam turns and the roller runs up and down on the cam surface, the slider moves up and down to bring the part to and from the pick-up and drop-off nests.

## **Final Design**

### **Description**

This design uses a system of bevel gears to carry the part from pick-up location to drop-off location while turning it over. The team designed several assemblies that work together to control the part at all points throughout its movement. This includes the frame assembly, gear assembly, gripper assembly, activator assembly and solenoid and rail assembly. Each of these assemblies incorporates numerous parts, both manufactured and purchased, that are labeled in the following section.

### **Annotated Pictures and Parts List**

The following is a series of annotated pictures and several tables that detail the assemblies and parts involved in this device. The frame assembly consists of the table, stanchions and cross-members that support the mechanism. The gear assembly consists of the gears, shafts and bearings and is driven by a servo to rotate the attached assemblies 360 degrees. The gripper assembly is attached to each of the small planet gears and holds the part as it turns over and rotates about the center of the sun gear. The activator assembly incorporates a leveling plate, vacuum and cam that level the gripper assembly, attach to the part and then open the gripper arms to release the part. The solenoid and rail assembly consists of a rail and guide block driven by a solenoid to control the vertical motion of the activator and its components.

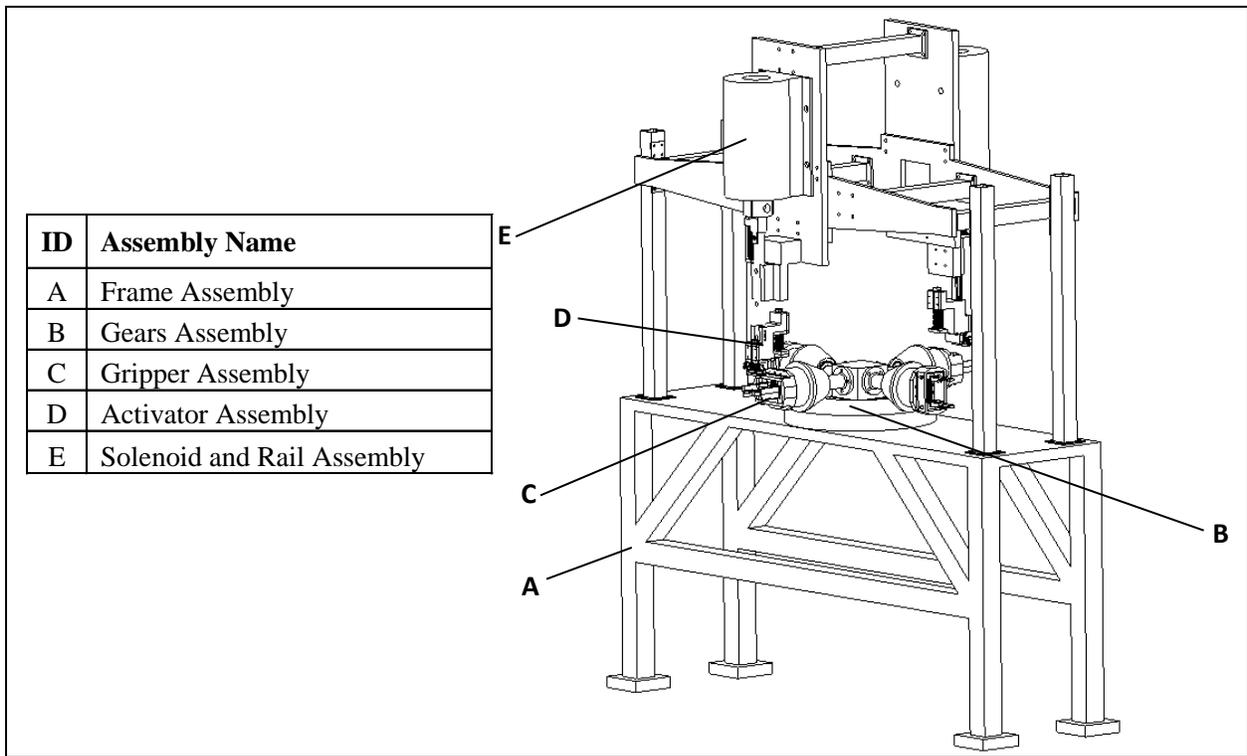


Figure 6: View of Entire Model

ID	Part Name	Part Type*	ID	Part Name	Part Type*
<b>A</b>	<b>Frame Assembly</b>		<b>D</b>	<b>Activator Assembly</b>	
AA	Table	M	DA	Activator	M
AB	Stanchions	M	DB	Vacuum Slider	M
AC	Table Bridge Plates	M	DC	Vacuum Slider Pin	M
AD	Center Cross-Members	M	DD	Vacuum Slider Fastening Plates	M
AE	End Cross-Members	M	DE	Rubber Seals	M
AF	Frame Bolts	P	DF	Leveling Slider	M
<b>B</b>	<b>Gears Assembly</b>		DG	Leveling Slider Pin	M
BA	Gear Base	M	DH	Leveling Slider Fastening Plates	M
BB	Gear Base Flange	M	DI	Vacuum slider spring	P
BC	Stepped Gear Shaft	M	DJ	Fastening Plate Bolts	P
BD	Large Sun Gear	P	DK	Hose Fittings	P
BE	Small Planet Gear	P	DL	Leveling Slider spring	P
BF	Small Gear Nut	P	DM	Activator Set Screws	P
BG	Stepped gear shaft bolts	P	DN	Activator-to-Rail Bolt	P
BH	Gear base flange bolts	P	<b>E</b>	<b>Solenoid and Rail Assembly</b>	
BI	Large Gear Ball Bearing	P	EA	Solenoid Plate	M
BJ	Large Gear Thrust Bearing	P	EB	Yoke	M
BK	Small Gear Ball Bearing (Small ID)	P	EC	Shelf	M
BL	Small Gear Ball Bearing (Large ID)	P	ED	Yoke pin	M
<b>C</b>	<b>Gripper Assembly</b>		EE	THK Rail	P
CA	Gripper Base	M	EF	THK Guide Block	P
CB	Upper Gripper Arm Pair	M	EG	Yoke spring	P
CC	Lower Gripper Arm Pair	M	EH	Solenoid	P
CD	Gripper Arm Pin	M	EI	Solenoid screws	P
CE	Gripper Spring Pin	M	EJ	Solenoid plate screws	P
CF	Gripper pin snap rings	P	EK	Shelf screws	P
CG	Gripper spring	P	EL	Guide block screws	P
CH	Gripper base to small gear bolts	P	EM	Yoke-to-rail bolt	P

\* M - Machined, P - Purchased

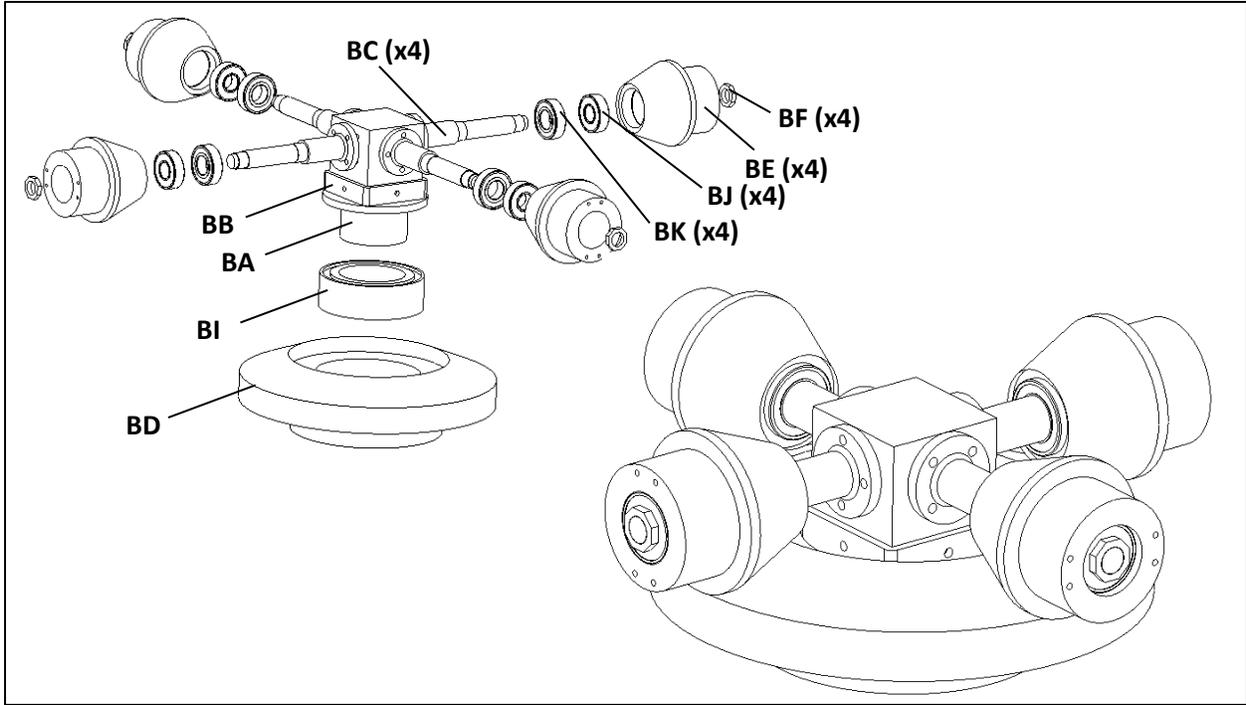


Figure 7: Gears Assembly [B]

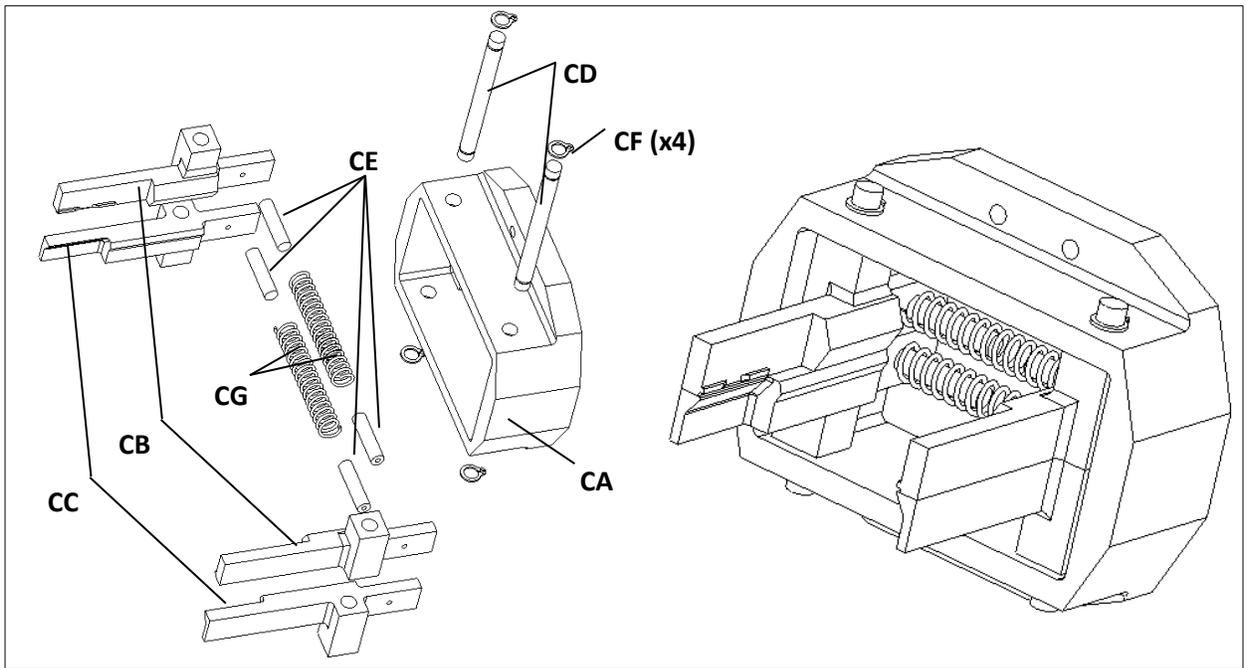


Figure 8: Gripper Assembly [C]

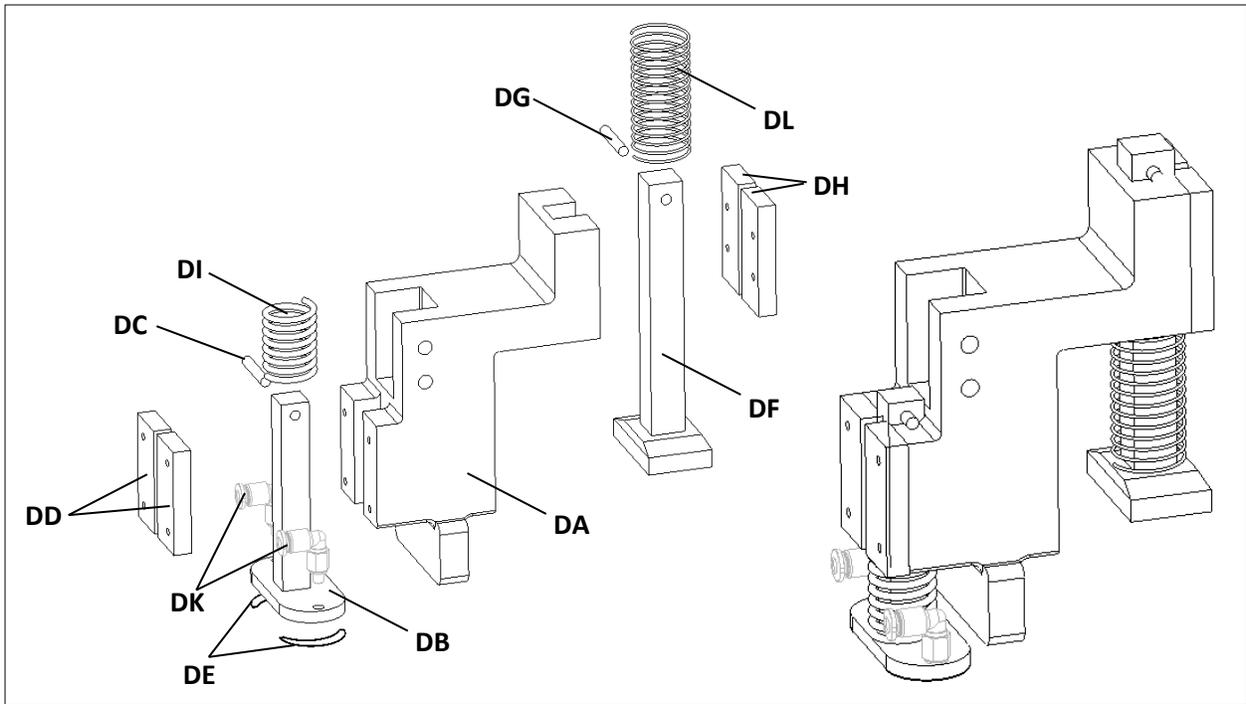


Figure 9: Activator Assembly [D]

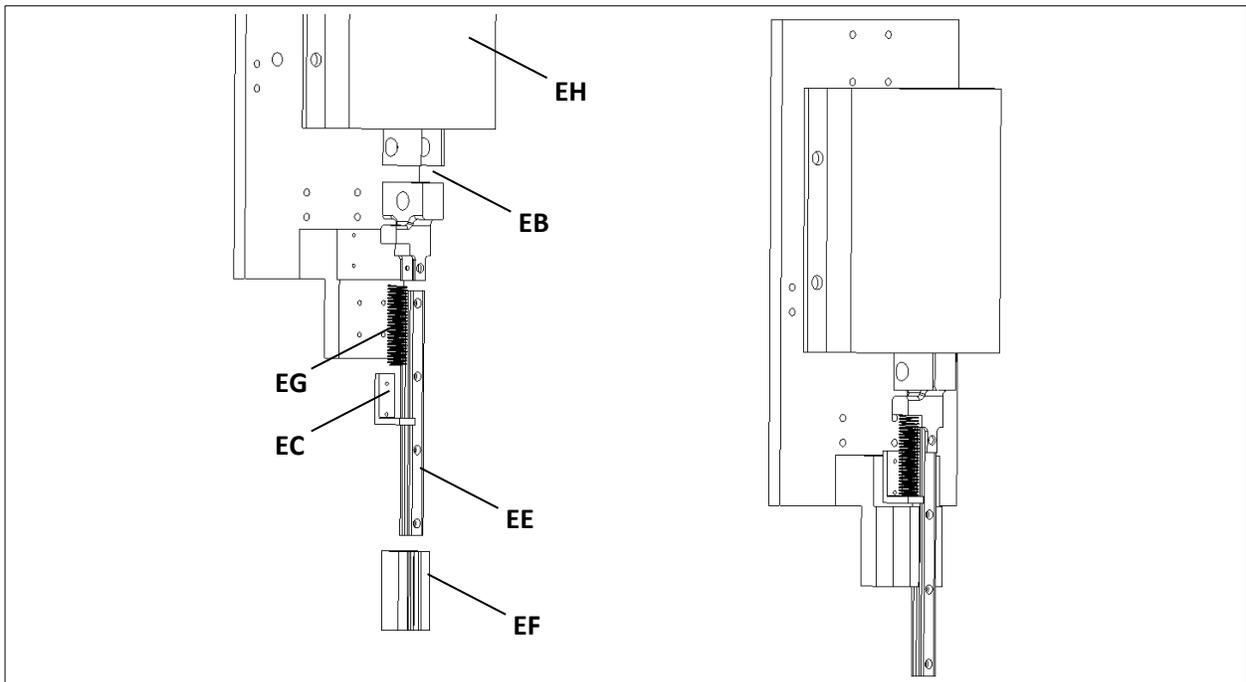


Figure 10: Rail and Solenoid Assembly [E]

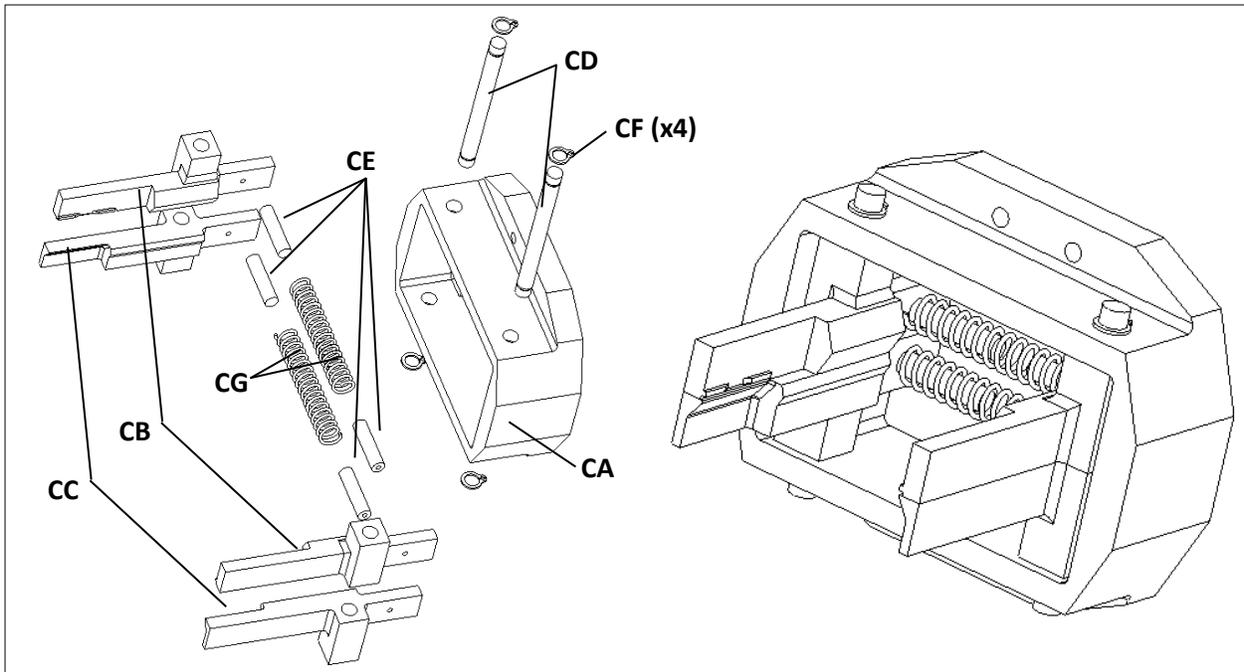
## Gripper Assembly

The requirements of the gripper assembly [C] are to:

1. Hold the part without passively opening
2. Allow the part to enter from one side and leave from the other.

These are necessary functions of the gripper assembly, but along with these came a number of constraints. When the gripper arms are holding the part, there are specified areas of the part which cannot come into contact with the arms. This area includes most of the top surface of the part. They also need to hold it in such a way that there is no damage to the fragile part. Because the grippers will be turning over and moving around the carousel rapidly, low mass and moment of inertia are desired traits. Therefore, the grippers were designed to be as compact as possible.

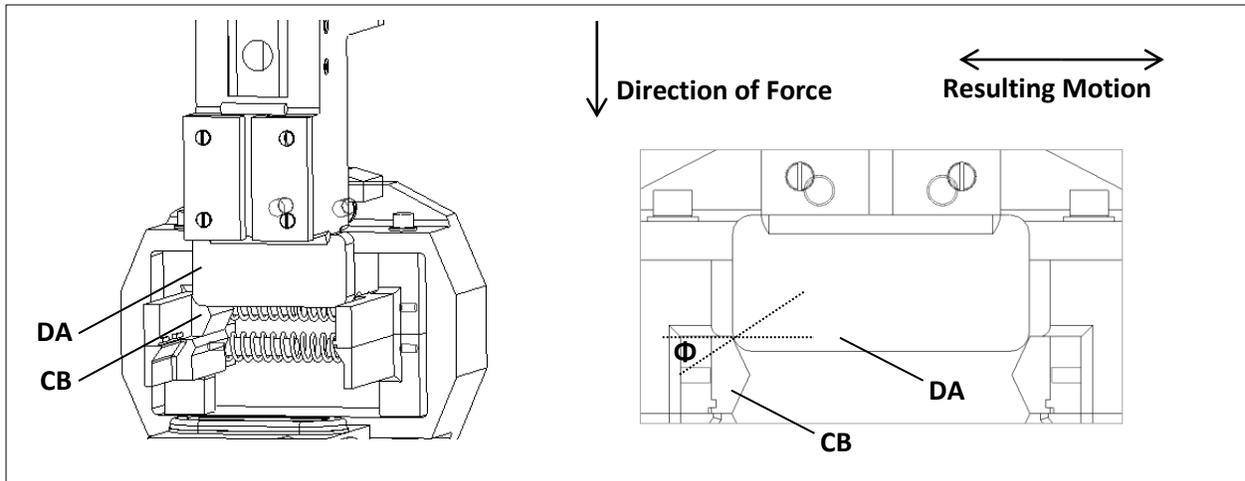
The general layout of the gripper assembly [C] is shown in Figure 11. The gripper assembly contains a base [CA] with four arms [CB & CC] and is symmetrical. The team modeled the arms on the right side then simply made mirror copies for the left side of the gripper assembly. There are distinct differences between the upper [CB] and lower [CC] arms (Note that “lower” here refers to the arms which are at the bottom when the part is being picked up. Later, when the part is dropped off these arms are actually on top and the “upper” arms are on the bottom).



**Figure 11: Gripper Assembly General Layout**

The bottom arms [CC] support the part as it is placed in the gripper at the pick-up location. They need to support the weight of the part as well as the vertical force applied by the activator [DA]. The top arms [CB] contour at their ends to the shape of the part to prevent it from rotating out of place. These arms also include a contoured lip that mates with the lip on the edge of the part to prevent it from falling out of this side of the gripper when it is turned over. With this configuration, the part will always be held in place from translation or rotation in any direction relative to the gripper.

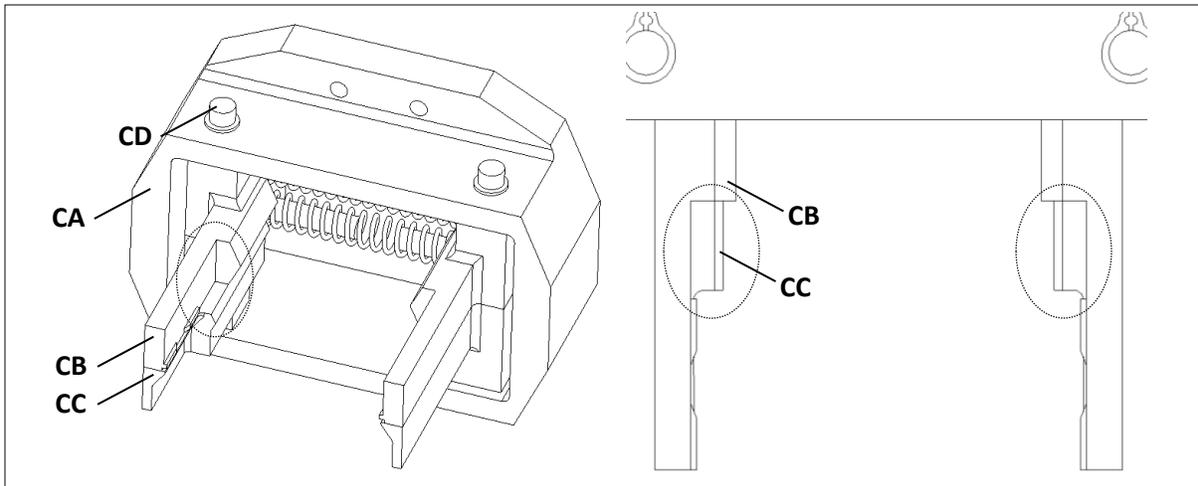
Next, the team created a way to open and close the gripper arms [CB & CC]. Because one of the functions of the grippers is to hold the part stationary while the carousel is in motion, a spring was added to prevent the grippers from releasing the part prematurely. The arms will be forced open using a cam motion driven from a vertical activator [DA]. The angled surfaces of the gripper arms have a specific shape according to the pressure angle of the interaction with the activator. This interaction is illustrated in Figure 12.



**Figure 12: Cam Interaction and Pressure Angle**

The pressure angle ( $\Phi$ ) is the angle between the direction of the resulting motion and the normal of the tangent between the two surfaces in contact. The acceptable maximum pressure angle is 30 degrees. The team designed this angle to be 25 degrees. The activator [DA] interacts with the gripper arms [CB & CC] to push them outward as shown in Figure 12. The opening between the gripper arms works well because it also allows the activator assembly [D] to grab the part and carry it downward through the gripper assembly into the nest. The design of this pusher is discussed in the Activator Assembly section.

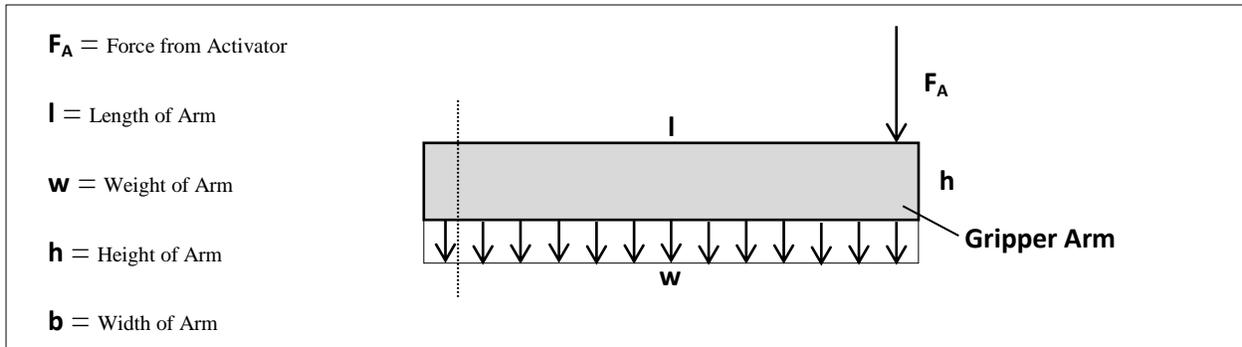
The gripper assembly [C] must work at both the pick-up and drop-off nests because it will be carried around to each, but the activator assemblies [D] at the nest locations can be different because they are stationary. Knowing that, the gripper assembly was designed to interact with two different activators in deliberate ways. At the pickup nest, only the bottom arms [CC] will be opened. Because the top arms [CB] remain stationary, the part is locked in and cannot continue out of the top of the gripper. This isolation of the arms is achieved by shortening the length of the cam section of the upper arm as shown in Figure 13.



**Figure 13: Top View of Gripper Arms - with noted cam slots**

Because the protruding cam surface at a specific section of the arm was omitted, the activator will only interact with the bottom arm [CC] when it passes through that section. At the drop-off nest, both sets of arms are opened to release the part. It is not actually necessary to open the “lower” arms [CC] at this point, but there is little adverse effect on the operation by doing so. A slot (shown in Figure 13 in dotted oval) would need to be cut into the arm to prevent its opening at the drop-off. This would increase cost in manufacturing and create extraneous stress concentrations on the arm.

Each set of arms in the gripper assembly has a single pin [CD] connecting it to the base [CA]. At this point in the design the accuracy of the part placement was investigated. In order to avoid interference between the gears and the nest, the part is held at the end of the arms. Because the location of the part is far from where the arms are pinned, there is concern as to the error in the placement of the part due to cantilever beam deflection of the arms and clearance in the pins. This error was calculated; the full calculation is included in Appendix A: Calculations. Figure 14 shows a diagram of the arms [CB & CC] considering bending.



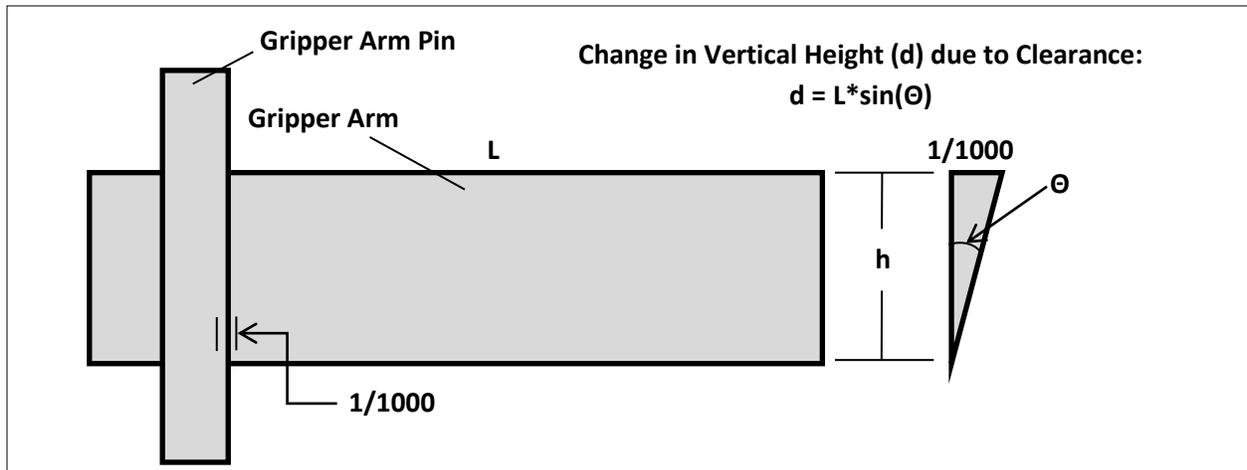
**Figure 14: Bending Forces on Gripper Arm**

The arms were originally designed with a very small height,  $h$ , to keep their mass low. This small height made them extremely susceptible to bending. The team will only allow 2/1000” of error in the part placement .Even though they are made of steel it was found that the end of the beam would have an unacceptable deflection with the original beam cross section. The moment of inertia of a beam has large impact on its cantilever deflection. The equation governing moment of inertia in this case is:

$$I = \frac{b * h^3}{12}$$

The height,  $h$ , has a cubic value in this equation, so we increased  $h$  to dramatically increase the moment of inertia which decreases the bending to within the acceptable range.

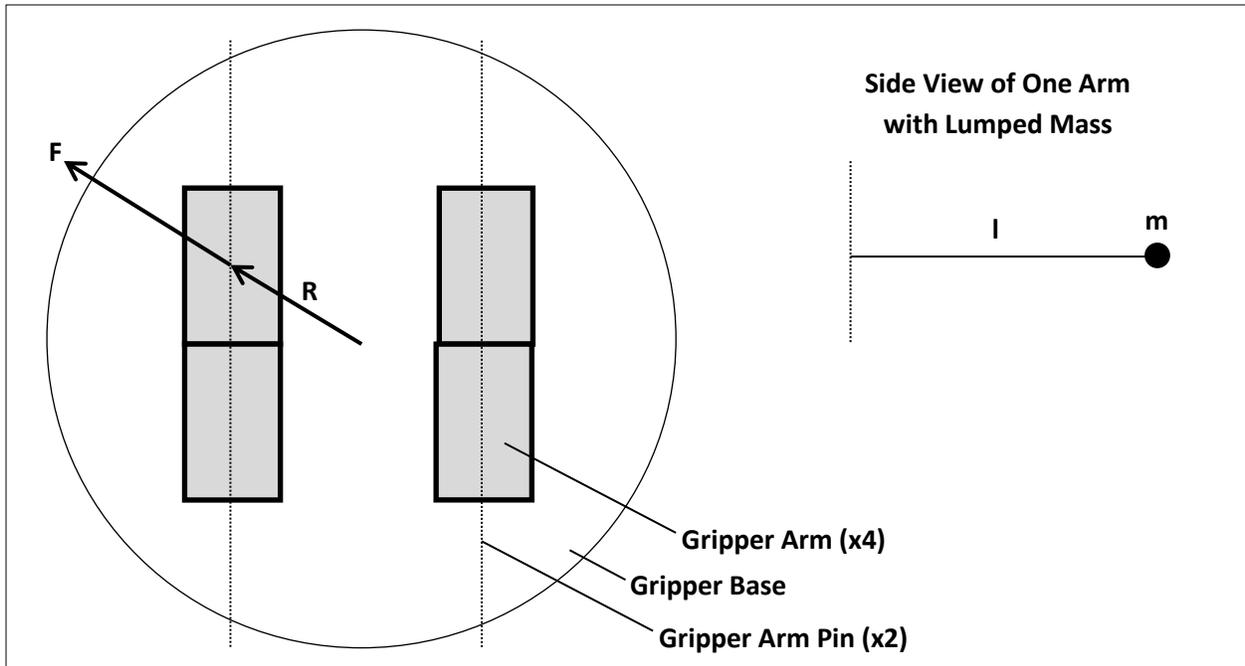
The team also calculated the error in the height of the part due to the clearance in the pins holding the arms. Figure 15shows how the clearance in the pins affects the beam.



**Figure 15: Clearance in the Gripper Pins**

The team assumed that there would be 0.001 in clearance between the pin and the hole. Neglecting bending, this resulted in unacceptable displacement at the end of the beam. To decrease this error, the height of the pins was doubled to decrease the pin clearance error to an acceptable value. The combined error due to pin clearance and bending is now within the acceptable range of less than 0.002 in.

The final aspect in the initial design of the gripper assembly is the inclusion of springs. The purpose of including springs is to hold the grippers closed around the part. The forces acting against them will be the centrifugal force from rotation of the small planet gear [BE], and the opening force of the activator [DA]. A diagram of these forces is shown in Figure 16.



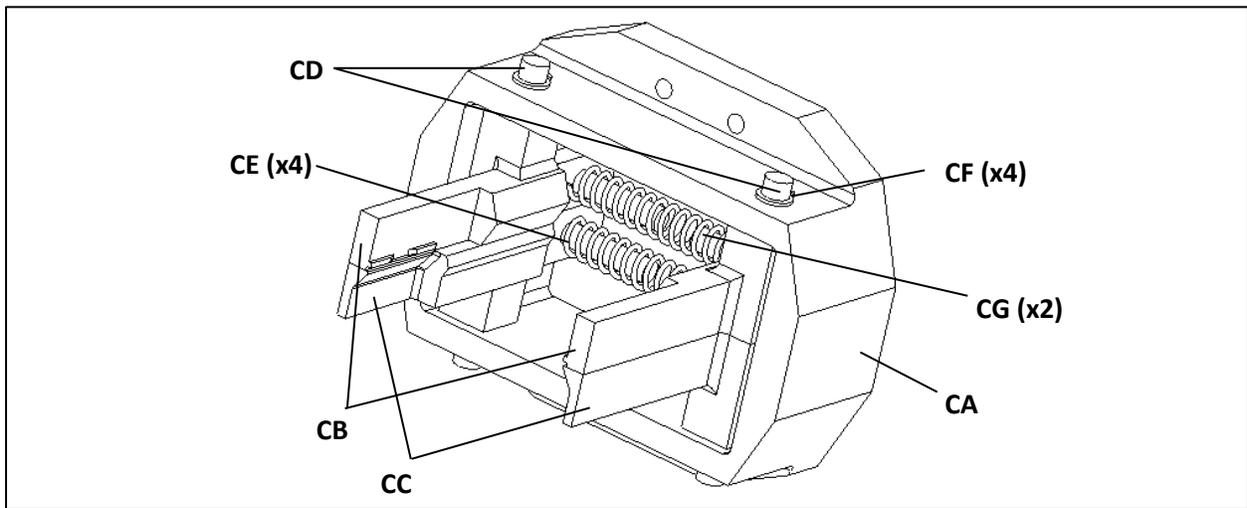
**Figure 16: Free Body Diagram of Rotation of Grippers**

Ideally, these springs should be just strong enough to overcome the force from rotation and apply a holding force on the part. If they are too strong, the activator will not be able to force the gripper arms open. A full calculation showing the evaluation of the centrifugal force is included in Appendix A: Calculations. This calculation is carried out for one individual arm. The arm is modeled as a beam with a pin at the end; this was then conservatively assumed to be a lumped mass at the end of the beam. Considering the production speed and the gear ratio, the radial speed of the gripper was calculated and used to find the centrifugal force. The centrifugal force is:

$$f = m * \frac{v^2}{r}$$

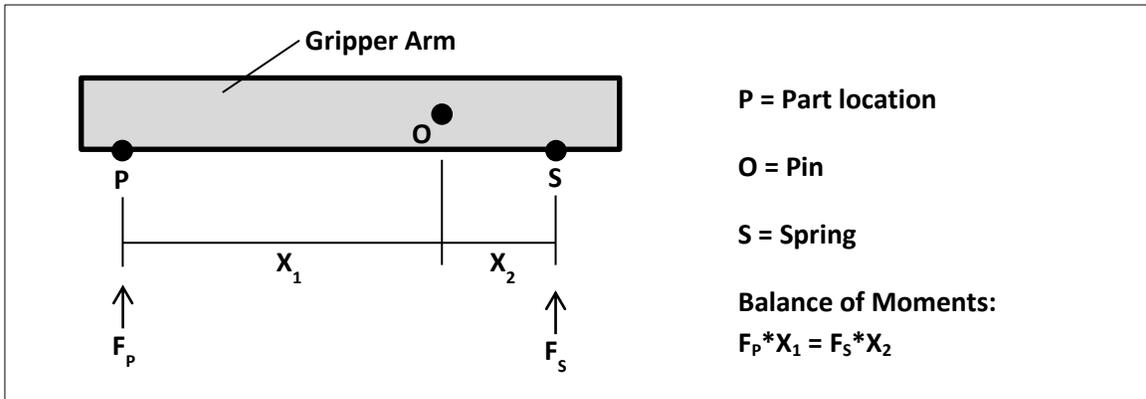
Above,  $m$  is the mass of the lumped model,  $v$  is the tangential speed, and  $r$  is the radius from the center of rotation. This small value is insignificant in comparison to the force needed to be applied to hold the part. It has been considered negligible.

Compression springs are used to hold the gripper arms shut. Overall, compression springs are reliable, easy to install, less expensive and more readily available than other types of springs. The team aimed to use compression springs wherever possible throughout this design. While compression springs work to push things apart; the gripper arms [CB & CC] need to be forced together. In order to incorporate the compression springs into the gripper assembly, the team extended the gripper arms in the negative direction past the point where they were pinned [CD]. A rod [CE] is run through the center of each spring from the arms to constrain the springs [CG] from falling out of the gripper assembly. The new gripper assembly is shown in Figure 17.



**Figure 17: Full Gripper Assembly**

The springs are now placed in the back end of the gripper assembly where they will not interfere with the activator. The force exerted on the part from the springs can be calculated using the distances from the pins Figure 18.



**Figure 18: Spring Forces on Gripper Arm**

Further modifications were later made to the gripper assembly to account for interaction with the activator assembly and for manufacturability. These modifications are discussed in those sections of the report.

## Solenoid and Rail Assembly

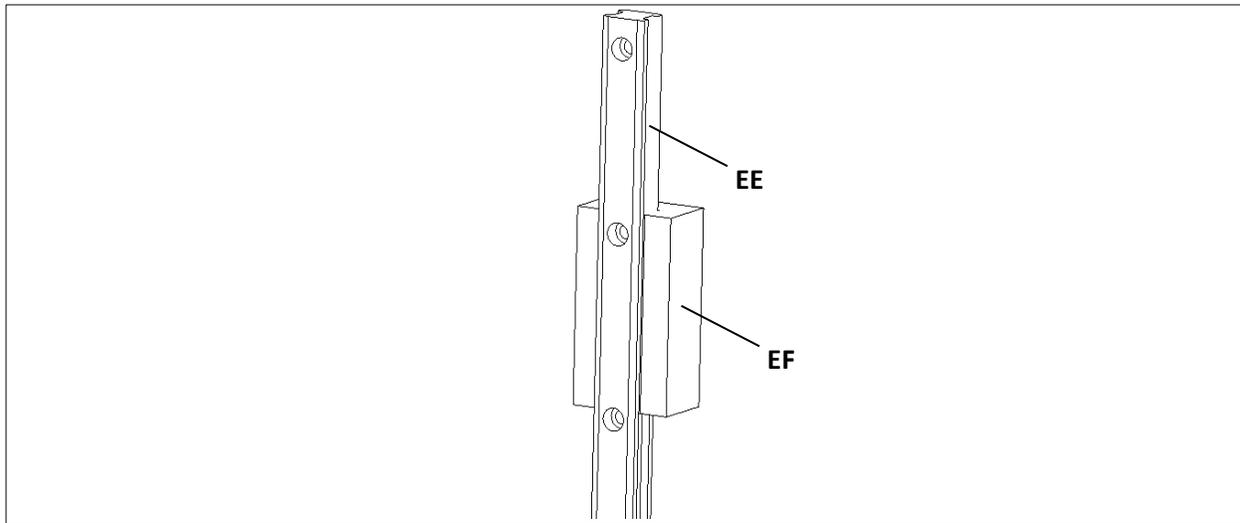
### Solenoid

For this design, the team decided that a solenoid is the best solution to the problem of how to power vertical motion of the activator assembly. A solenoid was chosen that has enough force to push the activator assembly down while combatting the strength of the spring used to return the activator assembly to its start position. The solenoid also needed to have a response time that was fast enough for the production rate. Since solenoids are powered by electricity, the response time was not an issue. The chosen solenoid has a maximum response time of 60ms, which is well within the allowable range. Another important consideration was the maximum stroke of the solenoid. A solenoid with the correct stroke was needed in order to precisely place all of the parts of the activator assembly during operation. A standard solenoid was chosen from a catalog and the specs from that catalog can be found in Appendix B: Standard Parts.

### Rail and Guide Block

In this mechanism, there is need for a linear motion system which consists of a rail [EE] and a guide block [EF], shown in Figure 19. This is necessary because the motion of the solenoid [EH] needs to be

directed vertically, and only vertically, so that the activator assembly moves the part precisely while remaining level. The linear motion system chosen for this mechanism is one from THK Rail. A prefabricated system was chosen because all sizing and bearing ratios are predetermined. This also prevents the need for on-site manufacturing and eases replacement through the use of standard parts. The system chosen was mocked-up in Pro/Engineer and is shown in Figure 19.

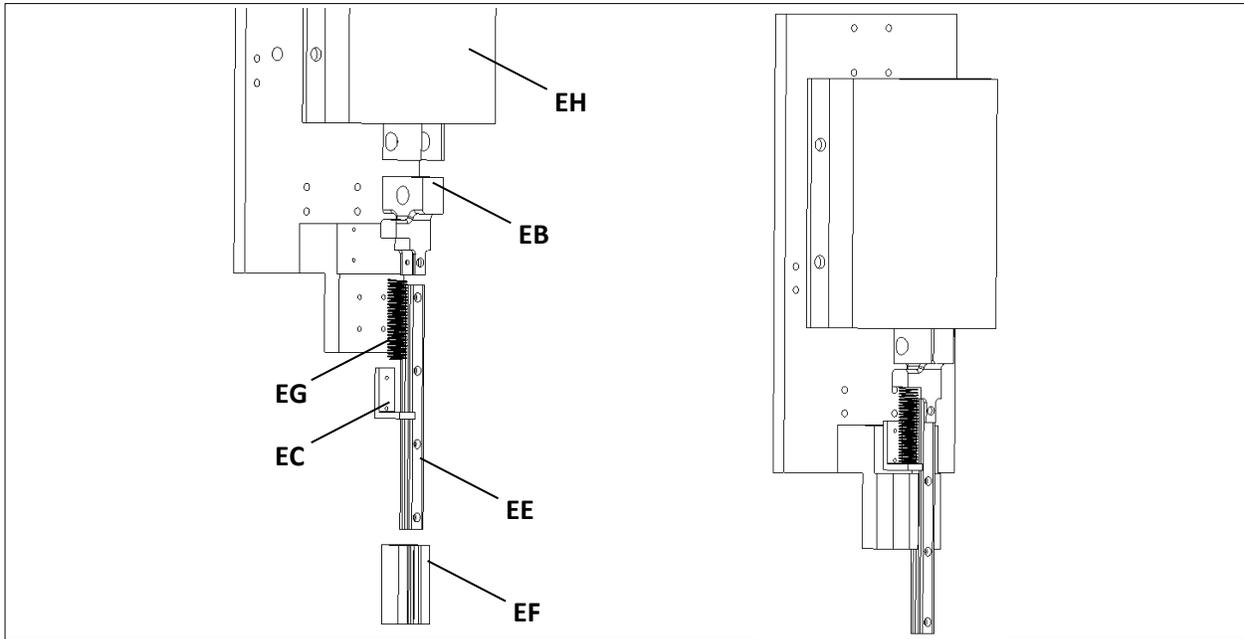


**Figure 19: Mock-up of Linear Motion System**

This mechanism uses the guide block as the stationary part in the linear motion system. The rail is then left with one degree of freedom. Because the system is set up in this fashion, the weight of the rail becomes much more important than the weight of the guide block, since the guide block is grounded. Because the weight of the rail is being supported from above, the minimum rail weight possible is ideal.

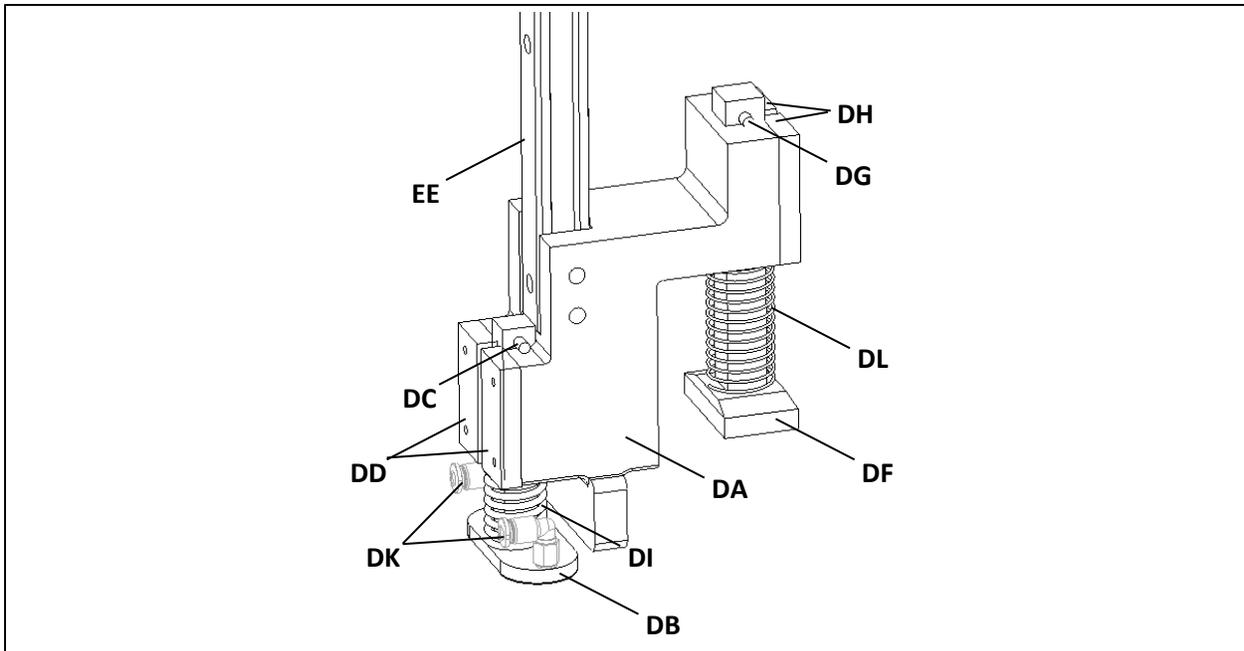
When sizing for the appropriate linear motion system, the length of the guide block and the weight of the rail were considered. The length of the guide block is important because it determines the contact length between the rail and the guide block. The contact length can be maximized by either using two guide blocks on one rail, or by using one guide block at a longer length. The latter was chosen for this design. With a long guide block, the rail will be more stable and have less freedom to move in anything but a vertical direction. Also, deflection and vibration of the rail will be minimized, as the rail will have more

stability. The assembly of this system with the remainder of the Solenoid and Rail Assembly is shown in Figure 20.



**Figure 20: Solenoid and Rail Assembly**

The solenoid and rail assembly attaches to the rest of the mechanism in various locations. The shelf [EC] and the guide block [EF] attach to the solenoid base [EA]. The rail [EE] runs through the guide block and connects at the top to the yoke [EB] with a pin. The bottom of the rail connects to the activator [DA] with one screw through the rail and with two set screws through the activator to keep the activator tightly fit against the rail. This connection can be seen in Figure 21.



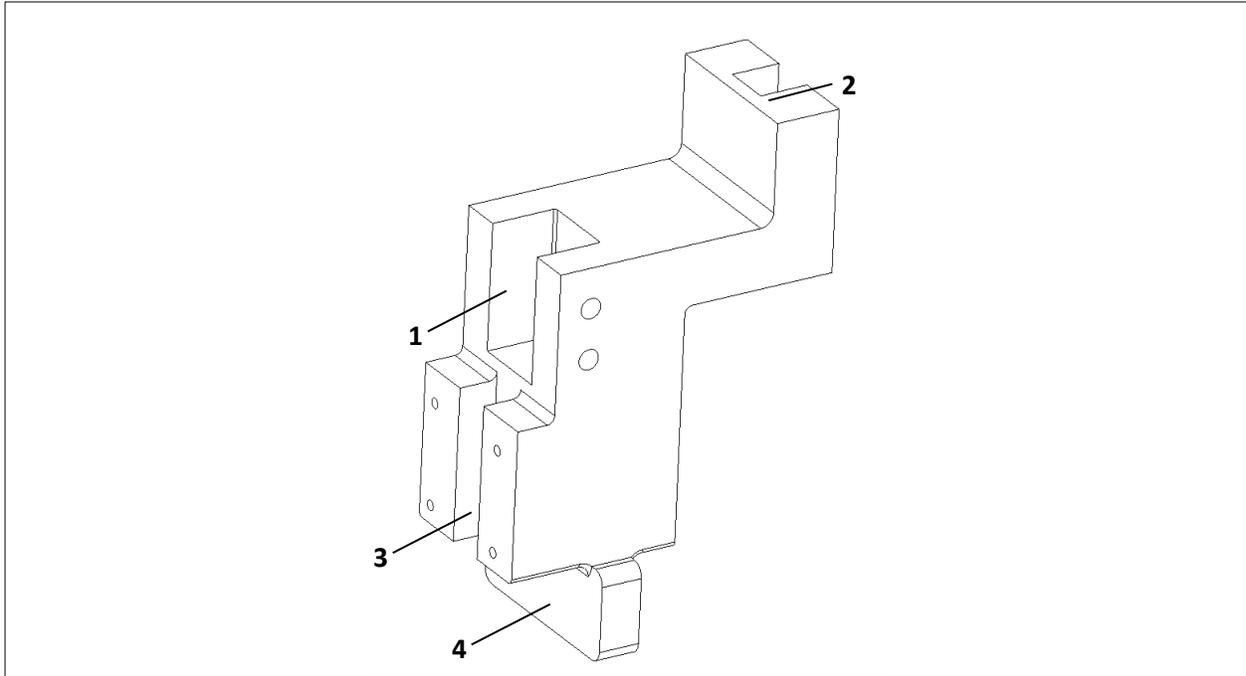
**Figure 21: Rail-to-Activator Connection**

The rail has bolt holes predrilled at 60 millimeters apart (on center). Any section and any length of the rail can be purchased. This means that a rail can be purchased that has a bolt hole a specific distance from each/either end as necessary. The position of the lower bolt hole, the position of the upper bolt hole, and the length of the bar (which determines how all other parts in the assembly line up and connect) were considered in the selection of this system.

The rail moves downward as the solenoid is activated and outputs a stroke of 50 mm. When the activator reaches the bottom of the stroke, it needs to be lifted back up through the gripper assembly [C]. The upward vertical motion is achieved by the use of a compression spring between the shelf [EC] and the yoke [EB]. Once the solenoid has finished its full stroke and is turned off, the spring will return the entire system to the original position.

## **Activator Assembly**

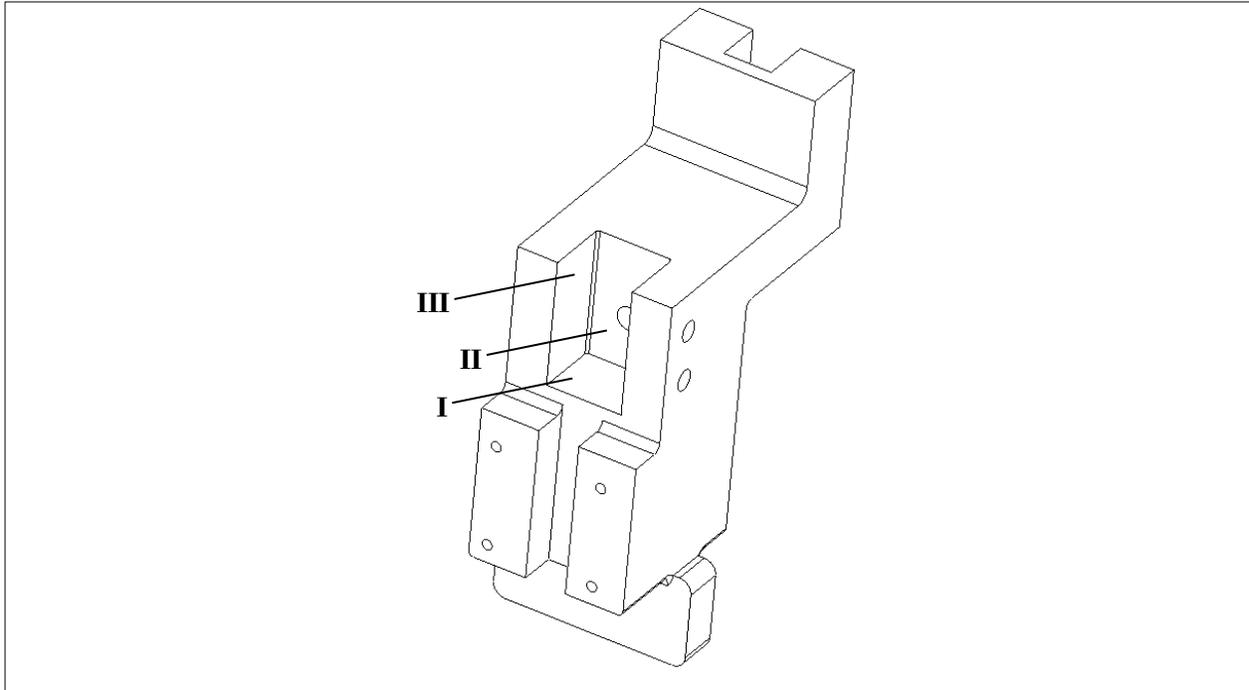
The activator assembly [D] is attached to the vertical rail [EE] and interacts with the gripper assembly [C] as it carries the part to and from the gripper arms. Figure 22 labels the components of the activator.



**Figure 22: Activator Components**

Each component is labeled in the order that it acts in the device. Number one is the area which attaches to the rail, two is a slot for a leveling slider [DF], three is a slot for the vacuum slider [DB], and four is the cam interacting with the gripper arms [CB & CC].

The primary function of the activator assembly is to open the gripper arms to grab and deliver the part. This can be broken down into many sub-functions. One crucial aspect of this design was the consideration of tolerances of each feature of the part. Each feature is built from specified datum planes at the base of the activator. These planes are highlighted in Figure 23. They are the three that align with the rail: at the bottom of the rail [I], on the back side of the rail [II], and on the back edge of the rail [III].



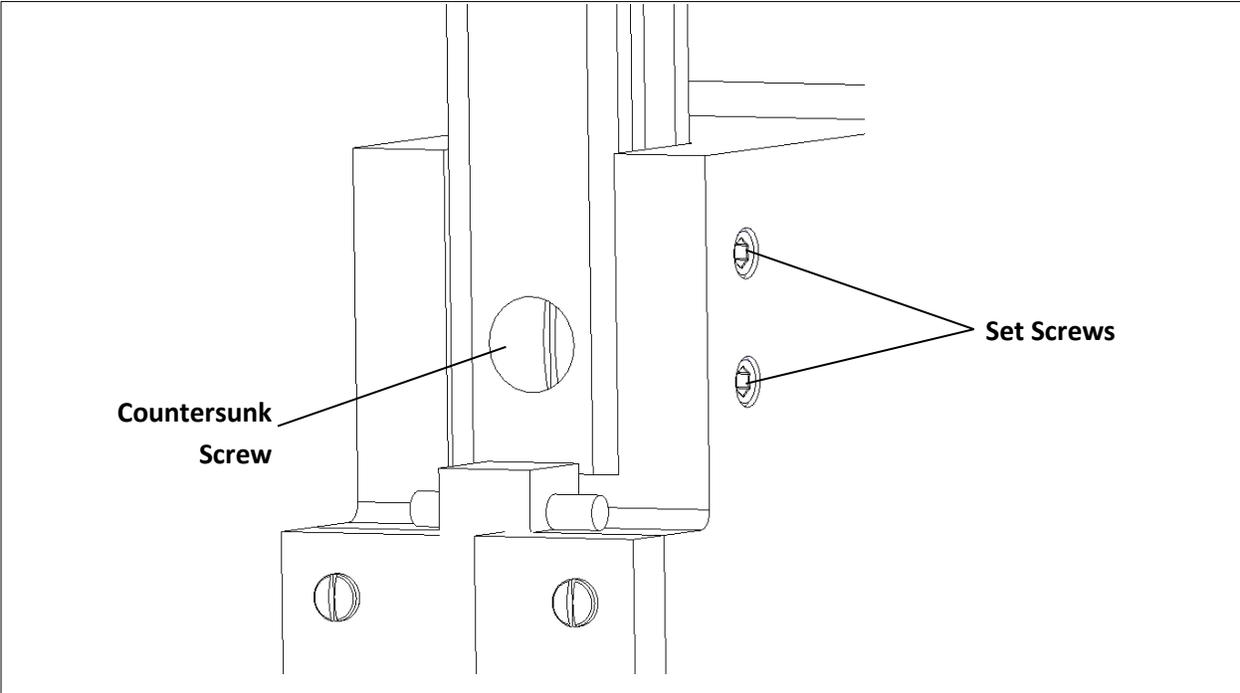
**Figure 23: Activator Reference Planes**

The vertical spacing of the components of the activator is crucial to the functionality of each component. In order to function properly the activator assembly must first level the gripper assembly, then connect to the part, then open the arms to release the part. With all of these steps completed the activator can then continue downward to complete the stroke of the solenoid and place the part in its target location. Note that at part pick-up the activator will move downward through the empty gripper, attach to the part at the bottom, then bring it up into the gripper leaving the part latched in the gripper as the activator continues up and out of the way. At drop-off the part will be moved from the gripper to the target location. For a better view of this motion, please see the videos attached to this report.

### **Rail Attachment**

It is crucial that the activator also have perfectly vertical motion. All moving components will be assembled precisely and without welding so that they can be replaced if need be. To ensure it is located accurately, the activator must be lined up with the front and side planes of the rail. This component is detailed in Figure 24. One screw is placed on the activator to align with the pre-tapped holes in the rail.

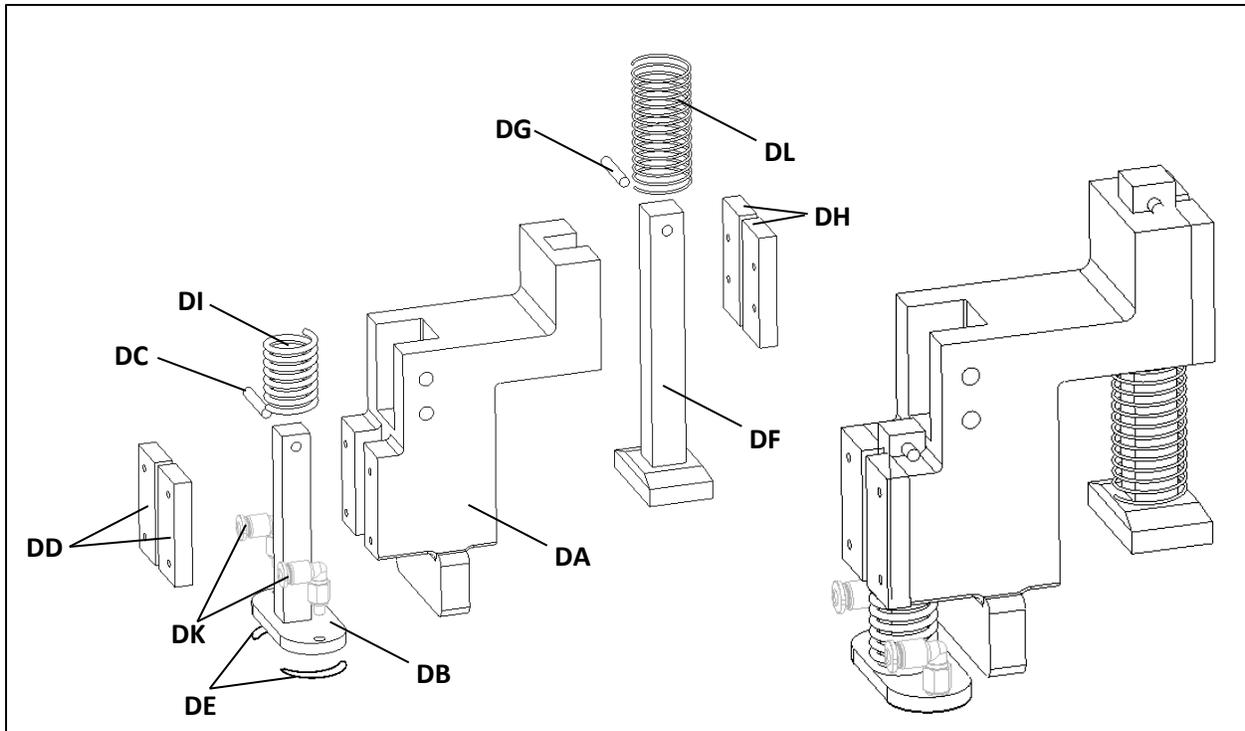
This countersunk screw, in conjunction with the set screws, ensures that the activator has zero degrees of freedom with relation to the rail.



**Figure 24: Rail Attachment**

**Leveling Slider**

The gripper assembly must be leveled to assure proper attachment to the part. In order to do this, the leveling slider sub-assembly is used. A flat plate (leveling slider [DF]) is attached to the back side of the activator assembly [D] to mate with a parallel surface on the gripper assembly [C]. This sub-assembly is shown in Figure 25.



**Figure 25: Exploded Assembly of Activator and Sliders**

The leveling slider sub-assembly also includes a pin [DG], two fastening plates [DH], a compression spring [DL] and four screws [DJ, not shown]. The spring is placed around the leveling slider, and then the leveling slider is placed into the groove on the activator and held in place with the pin, fastening plates and screws.

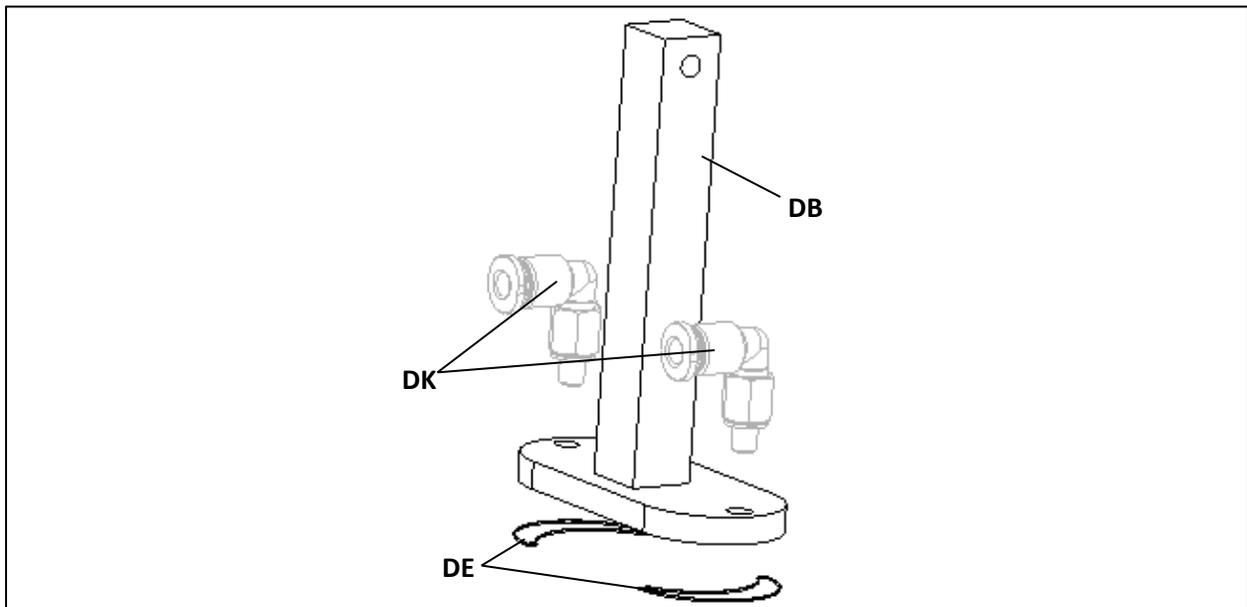
This leveling slider is the first part of the activator assembly to interact with the gripper assembly. The leveling slider is suspended behind the rest of the activator components, so it must be stiffened to ensure that the activator part will not break. An analysis of the activator was conducted to ensure this extension is strong enough. It is included in the stress analysis section of this report. The bottom part of the leveling slider [DF] comes into contact with the gripper assembly. It levels the assembly and then the spring compresses as the activator continues downward.

## Vacuum Slider

The vacuum slider sub-assembly is responsible for attaching to the part. This sub-assembly is shown in Figure 25 as well.

The vacuum slider sub-assembly is made up of a slider [DB], fastening plates [DD], a pin [DC], a spring [DI] and four screws [DJ]. These parts are assembled just the same as the leveling slider. Unique to this side of the activator are the hose fittings [DK] and rubber seals [DE].

At the lower end of the vacuum slider, an oval shape is formed. This shape is made to fit exactly over the part. A rubber seal is attached on the underside of each end of the oval. This seal is shaped to contour to the part.



**Figure 26: Detailed View of Vacuum Assembly with Rubber Seals and Hose Fittings**

Within the contour of the rubber seal, there is a tapped hole through the part. From above, one hose fitting screws into each of these tapped holes. This allows a vacuum hose to be attached to the part on each fitting. Once the vacuum is turned on, the rubber contour will seal to the part and allow the mechanism to carry the part to and from the gripper arms.

This assembly comes into contact with the gripper assembly next (after the leveling slider). The rubber seals on the vacuum slider touch the part and suction is applied through the vacuum hoses. As the part is attached, the spring begins to compress and the remainder of the activator continues downward.

It is crucial that this part of the activator which holds the vacuum be at the correct vertical height to assure that it meshes with the part correctly. Therefore, the distance between the vacuum attachment and the activator reference planes from Figure 23 will have a very tight tolerance.

The bearing ratios for both the vacuum slider and the leveler were calculated in the same way. The formula and calculations for finding the equivalent diameter are shown in Figure 27.

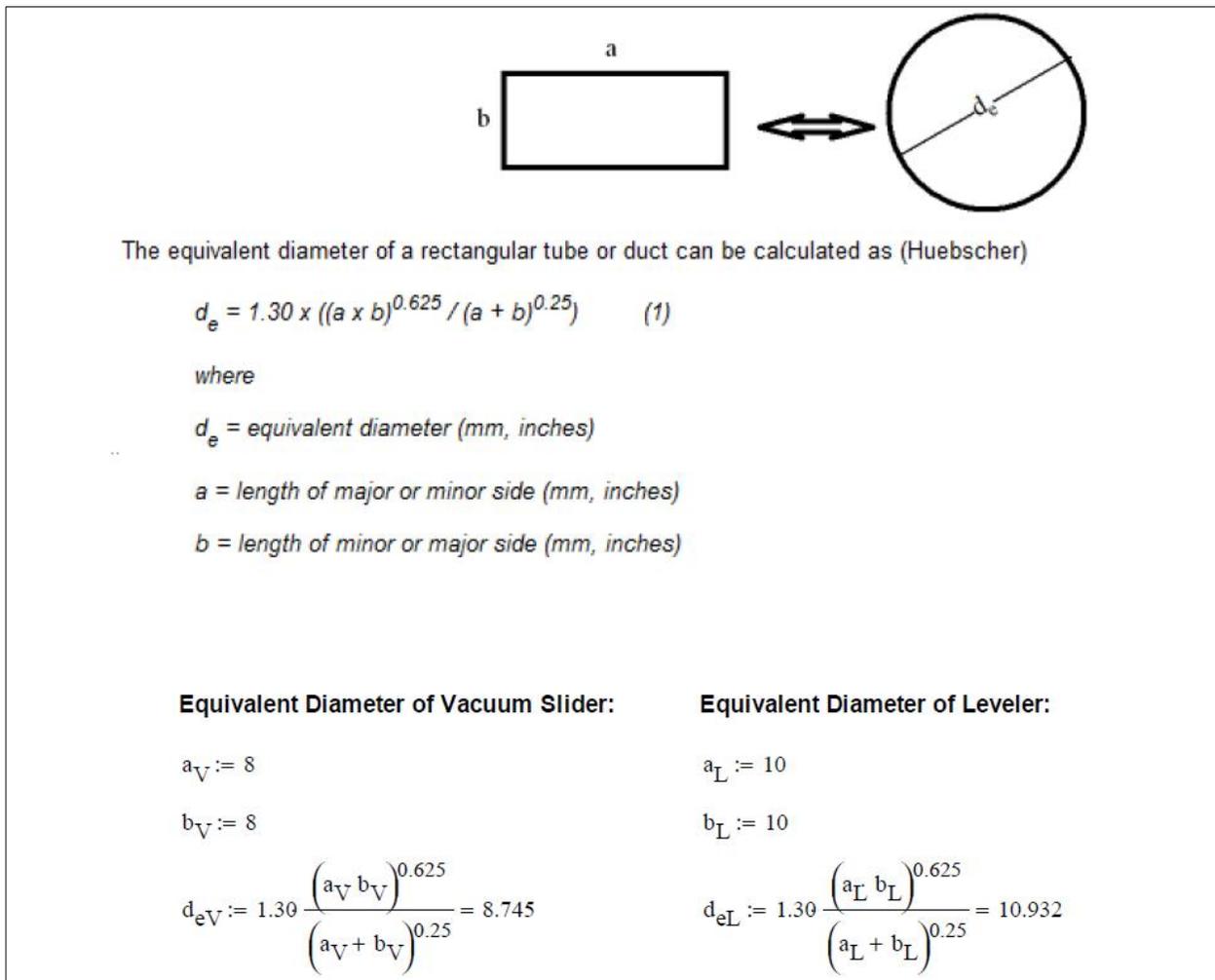


Figure 27: Bearing Ratio Calculations

The contact length is determined by multiplying the equivalent diameter by two. With this, the bearing ratio will be optimized. The contact length for the vacuum slider is 30 mm.

### **Gripper Interaction Cam**

The interaction cam opens the gripper arms to release the part. It is the last component of the activator to reach the gripper assembly. The vertical height of this cam will be different at the drop-off location than it is at the pick-up location. This difference of location allows the activator to interact with the gripper assembly differently at each location by selectively opening each gripper arm.

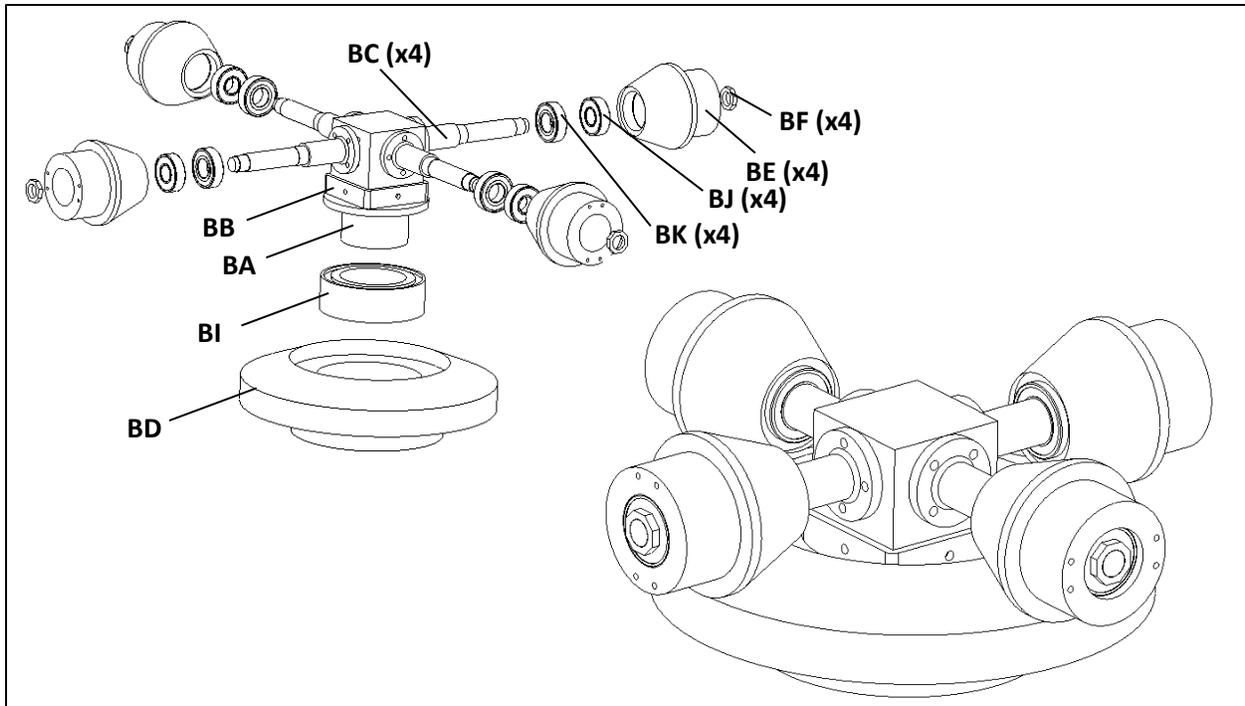
### **Gear Assembly**

When designing the gear assembly there were several factors that the team had to take into account:

- Type of gears
- Size of the large bevel gear
- Gear ratio between the large bevel gear and the small bevel gears
- Material of the two gears
- How the gears would be interfaced together

### **Gears**

The material needed and the way in which the gears needed to be interfaced were driven by the problem. The material for the gears has to be steel. The teeth have to be hardened in order to ensure that the gears will have enough life to deal with the production requirements of the problem. The gear assembly is shown in Figure 28.



**Figure 28: Gears Assembly**

The gears must be interfaced such that the arms [BC] for the small bevel gears [BE] must be perpendicular to the rotation shaft that will be attached to the servo motor to ensure that the gripper is flat at both the pick-up and drop-off points. When the team started researching what type of gears would best fit the application, it was obvious that bevel gears would be needed in order to achieve the perpendicular shaft requirement. The team then went further and chose spiral bevel gears for this problem because they are quieter and experience less contact force than traditional bevel gears. The distance between the pick-up nest and the drop-off nest had to be a particular distance so the selected gear needed to fit within that distance. The team chose a gear that has a diameter as close to the nest distance as possible in order to minimize any cantilevering of the gripper assembly. When choosing a gear ratio it was crucial to ensure that the ratio would allow the gripper assembly to flip 180 degrees within 180 degrees of motion of the servo motor, thus inverting the part as it reaches the opposite side of the base gear. The team chose a gear ratio of 1:3 in order to keep the small bevel gears as small as possible while still having enough surface area to attach the gripper assembly. At a 1:1 ratio, the carrier gears were too large to assemble the device.

The number of small bevel gears was chosen based on their size and what would fit on the large bevel gear.

### **Arms**

The arms [BC] of the gear assembly were designed with many design considerations taken into account. The most important of these design considerations was the diameters of the steps in the shaft. The arms had to be strong enough to support the weight of the small bevel gears [BE] without deflecting more than 0.001 inches. After the minimum diameter was found using static beam analysis, the steps of the arms were designed so that the arm-to-carrier-gear bearings could be adequately secured. In order to achieve this assembly and secure the bearings properly, it was decided that two bearings with the same outer diameter but different inner diameters were needed. The decision to use two bearings with different inner diameters dictated the geometry of the arms. The geometry dictated by the bearings is such that the shaft steps up larger and larger as one looks closer to the central connection of the arms. This allows for the inner-most bearing to fit over the outer steps of the arms and then press fit onto the larger steps of the shaft. The smaller inner diameter bearing can then be pressed onto the smaller steps of the arms.

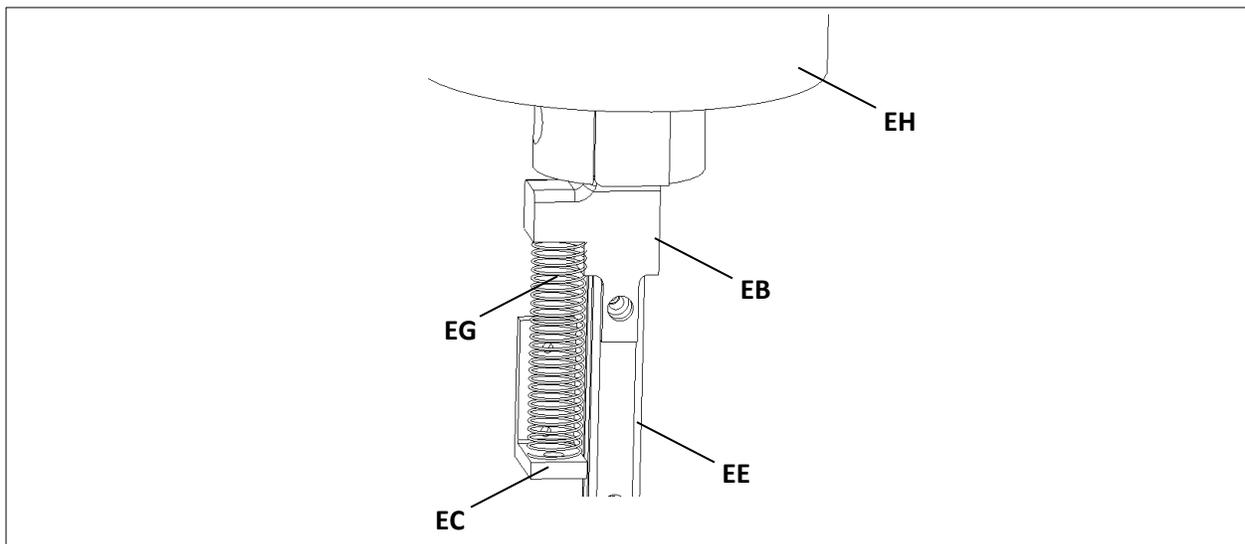
### **Bearings**

In order to reduce friction between the shaft and the small bevel gears [BE] and to increase life of those parts, bearings were needed. Radial ball bearings were chosen for the small bevel gears since the loads on the bearings are not axial. A bearing was needed below the arms assembly inside of the large bevel gear [BD] as well. In this instance, both axial and radial loads needed to be accounted for. The team decided to use a dual direction bearing to solve this problem. The bearing utilizes two rows of balls offset to handle both radial and axial loads, a necessity for this application. Standard bearings were chosen from catalogs in order to keep the cost of the bearings down and to make them easier to obtain in quantity.

## Manufacturing

Since many of parts of this mechanism are customized for its operation, it was necessary to ensure that machining these parts would be as simple as possible in order to reduce costs. Some of the most complex parts include the yoke [EB], the activator [DA] and the gripper arms [CB and CC], each shown in the figures to follow. Additionally, many parts have crucial tolerances in order for the alignment of all of the parts to work out correctly.

The manufacturing of the yoke [EB] had to be considered. The placement of the yoke [EB] is shown in Figure 29.

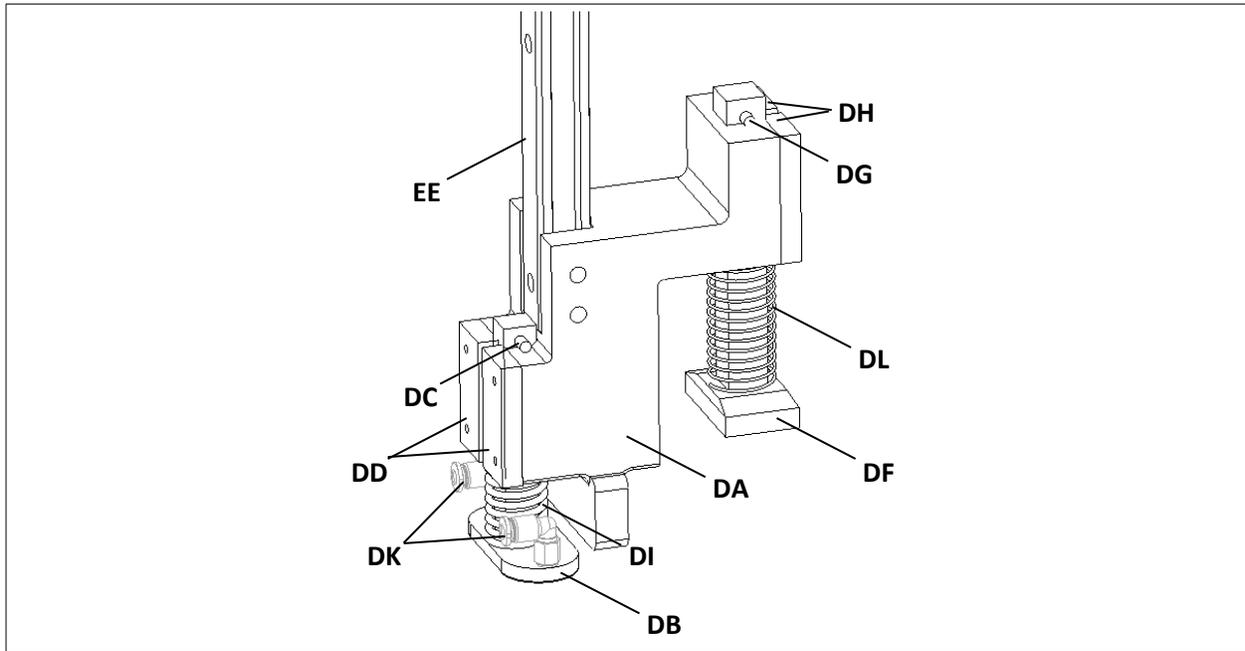


**Figure 29: Yoke [EB] Placement**

The yoke [EB] attaches to three other parts and experiences many different forces. The yoke [EB] connects to the rail [EE], the yoke [EB] spring [EG] and the solenoid [EH]. All of these parts are prefabricated, and therefore have pre-designated dimensions. The most important dimension on this part is the dimension from the hole where the yoke [EB] connects to the solenoid to the hole where the yoke [EB] connects to the rail. This dimension is important to ensure proper vertical alignment of the activator, leveling slider and vacuum slider below. If any of these parts is not in the proper place, the part may not

be picked up correctly or could potentially fall through the grippers. This could happen if the activator arrived at the gripper arms too soon and pushed them open.

The next part that was considered for its complexity was the activator [DA], shown in Figure 30.

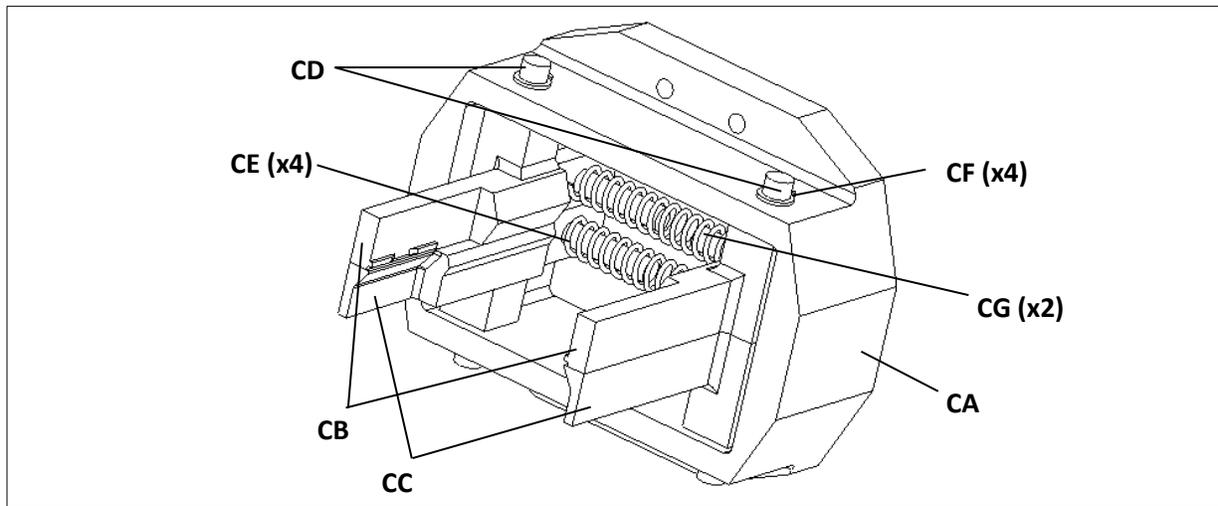


**Figure 30: Activator Assembly**

The activator is the connection between the vertical motion of the rail [EE] and the picking up of the part by the vacuum slider [DB]. The hole in the activator that connects the rail to the activator, as well as its surrounding walls, has crucial placement. This, again, will determine the accuracy of the vertical position of the vacuum slider, the leveling slider, and the activator cam surface. Additionally, the walls of the activator that surround the rail should have a tight tolerance. The rail will be held tightly against one wall with set screws inserted through the opposing wall. This fit will determine the horizontal position that centers the vacuum slider over the part. If the vacuum slider is not perfectly centered over the part, it could hit the gripper arms and cause the part to fall before the vacuum has a chance to apply a suction force. Other parts of the activator that have crucial dimensions are the slots into which the vacuum slider and the leveling slider [DF] are inserted. These parts have to be perfectly aligned so that the bottom

feature on each slider is aligned in both horizontal directions. Several final features that were added to the activator were filets in any corners that do not need to be precise angles. This reduces stress concentrations and also reduces the need for such precision in these areas of the part.

The final parts that were considered for their complexity are the gripper arms [CB and CC], shown in Figure 31.



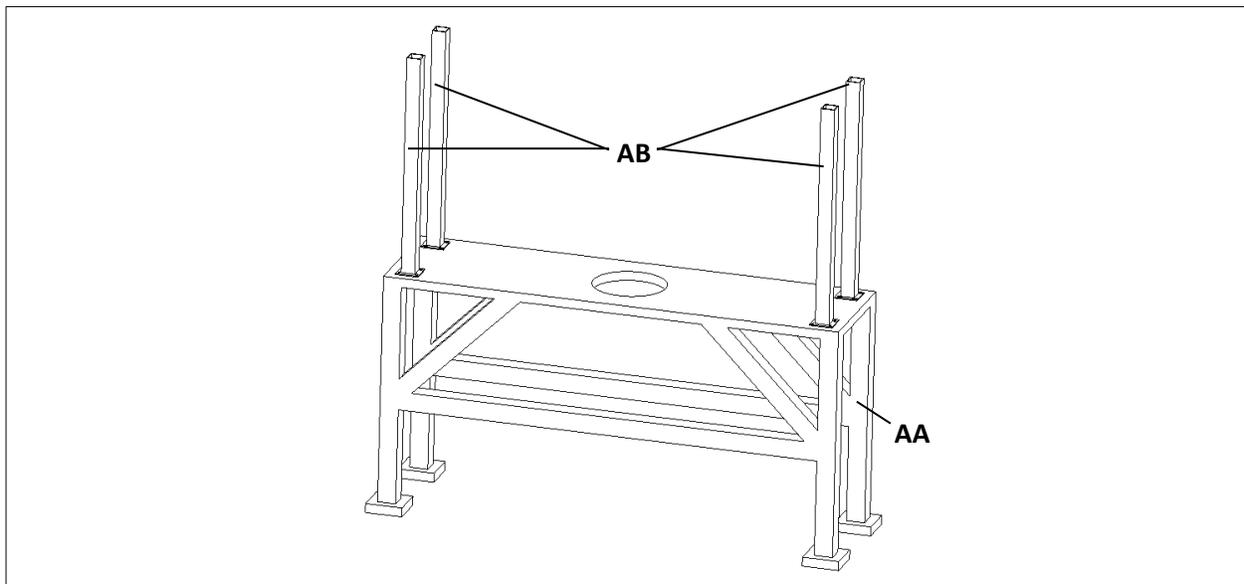
**Figure 31: Gripper Assembly**

The gripper arms [CB and CC] are essentially a very complicated beam, held in place by a pin [CD]. The location of each gripper arm is crucial. This means that the hole that the large pin goes through that holds the gripper arms needs to be placed correctly. Additionally, each feature along each gripper arm performs a particular task and each of these tasks needs to be executed precisely. The feature on the top grippers [CB] at the outermost end of the beam (the end away from the gears) is where the part is held. This is possibly the most crucial part of the entire mechanism. One main specification for this entire design is that the part is always held rigid in all directions of translation and rotation. This is done to ensure that the part is never dropped and does not incur any damage. This particular feature on the end of the gripper arms holds the part during its inversion. If the contour of the gripper arms is not perfect, the part could slip away from the grippers.

Three main components of this design have been discussed for their complexity. They are certainly not the only parts in this device that have crucial tolerances, but are simply considered some of the most complex parts to be manufactured.

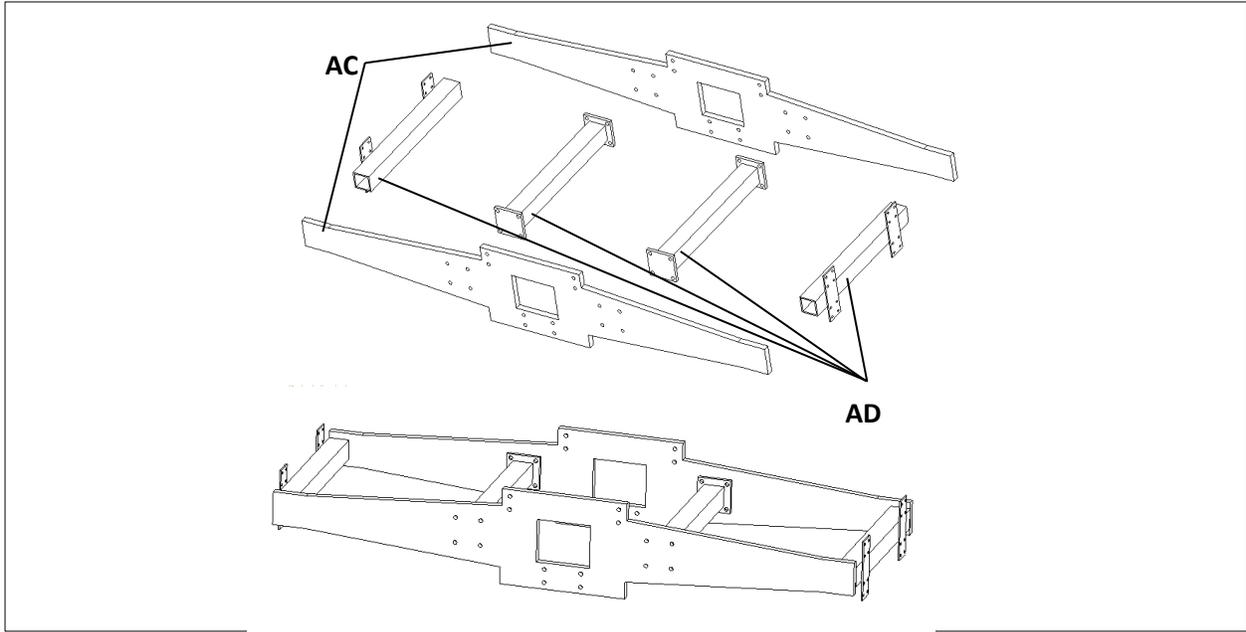
## Assembly

First, the table base [AA] and the stanchions [AB] are to be assembled. The stanchions are attached to the oval slots in the table. Later, the distance between stanchions can be modified to correctly align other parts of the assembly simply by loosening the stanchion bolts and sliding the stanchions in the slots until the stanchions are in their correct position. This assembly is shown in Figure 32.



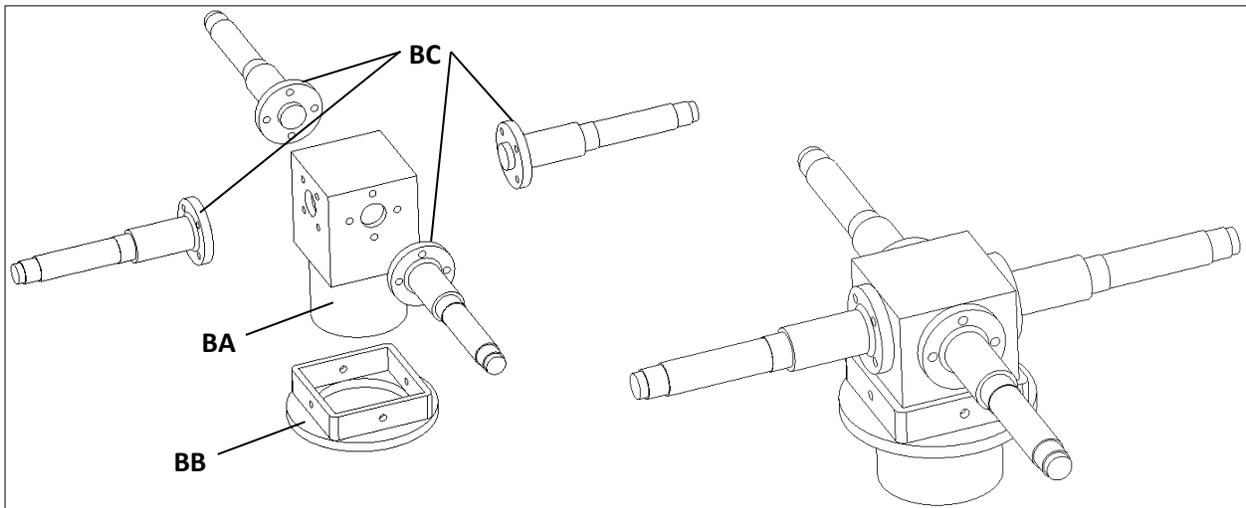
**Figure 32: Table Base and Stanchion Assembly**

Next, the table bridge is assembled. This is done by bolting the two table bridge plates [AC] opposing each other with cross-members [AD and AE] in between. This part will later attach to the table and stanchions. This is shown in Figure 33.



**Figure 33: Table Bridge Assembly**

. The gear base flange [BB] is attached to the gear base [BA]. Then, the stepped gear shafts [BC] are attached to the gear base. There are four of the same arms, but they are attached at different angles. The bolt circle for one arm is 45 degrees different from the next so that the screws do not interfere with each other in the middle of the gear base. Figure 34 shows the gear base assembly.



**Figure 34: Gear Base Assembly**

Then, the gears are attached to the gear base. First, the bearings are added. A thrust bearing [BI] is press-fit into the large gear [BD]. Next, the shaft is press-fit onto the bearing. Then, the inside bearings [BK] for the small gears are press-fit onto each of the shafts. Then, the outer bearings [BJ] for the small gears are press-fit and a nut [BF] is screwed on to the end of each shaft. Finally, the small gears [BE] are press-fit over the bearings and aligned properly with the large, stationary gear below. The small gear should be aligned so that the pre-drilled holes for the attachment of the gripper base are at the top and bottom of the gear near the tooling stations. The holes should be on the left and right sides of the gear for the two small gears that are between stations. The entire gear assembly is shown in Figure 35.

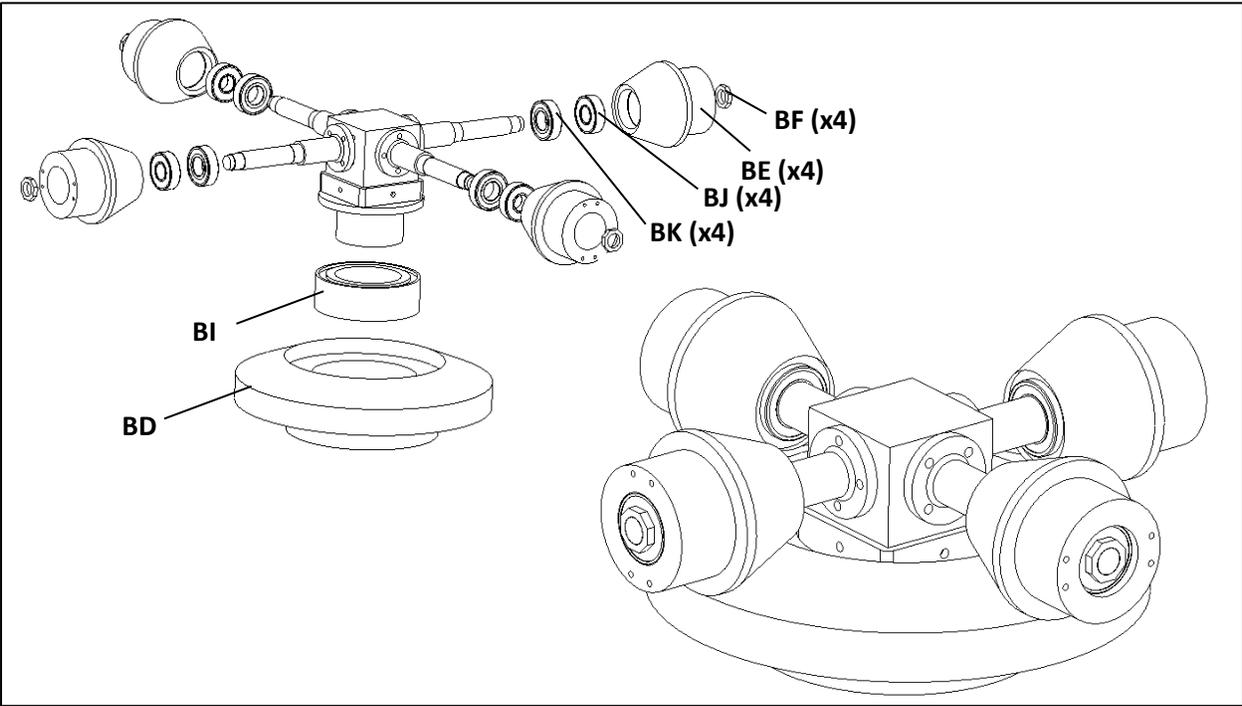
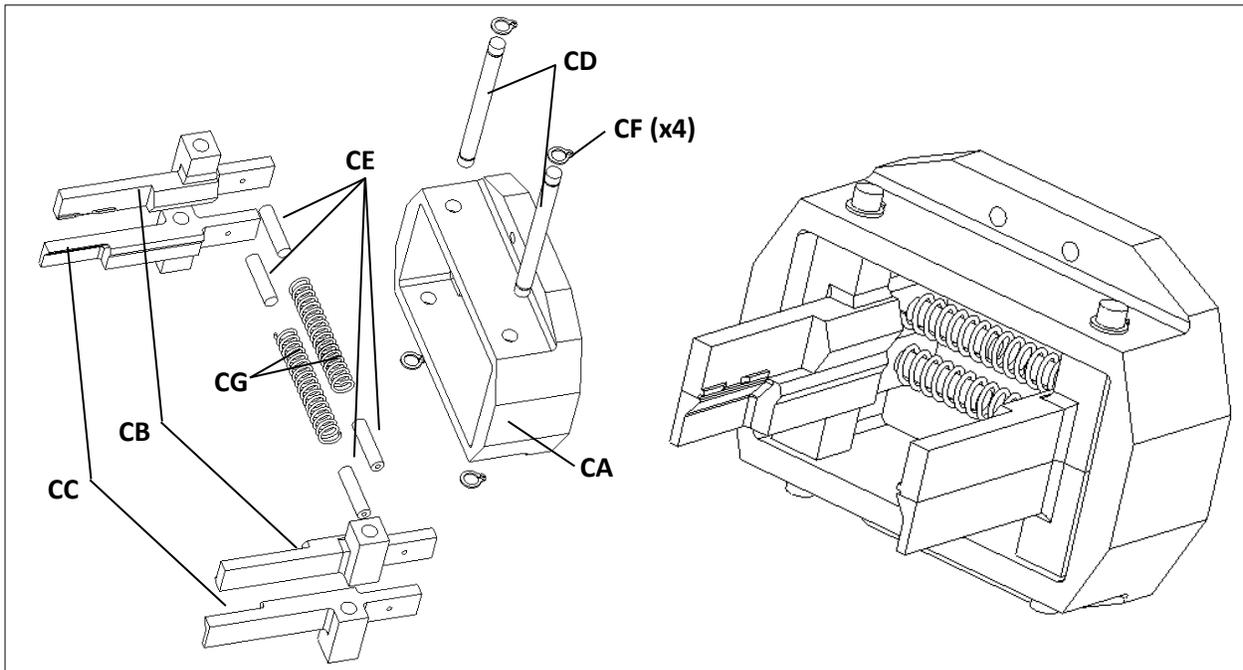


Figure 35: Gears, Bearings and Gear Base Assembly

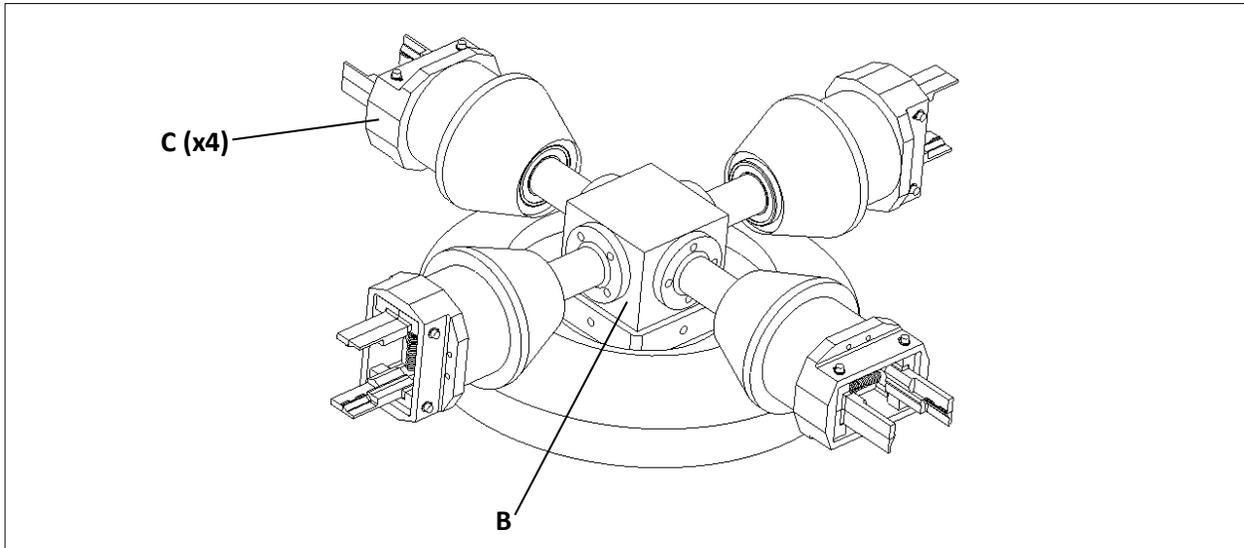
The grippers are then sub-assembled. The gripper base [CA] is the base of this sub-assembly. First, threaded pins [CE] are screwed into the tapped holes at the back end of each of the gripper arms [CB and CC], facing towards the center of the arms. All four threaded pins are the same. The bottom arms are aligned and spring [CG] is inserted on the threaded pins between the arms. The same is done for the top arms, with a second spring. The gripper arms are put into place in their proper location and orientation and held in place by the vertical pins [CD]. In order to get the gripper arms into place in the gripper base, the springs will have to be compressed. This compression, on the opposite side of the pin from where the part is being held, is what holds the part in place. The pins are then held in place by two snap rings [CF] each, one at the top and one at the bottom. The sub-assembly of the grippers is shown in Figure 36.



**Figure 36: Sub-assembly of Grippers**

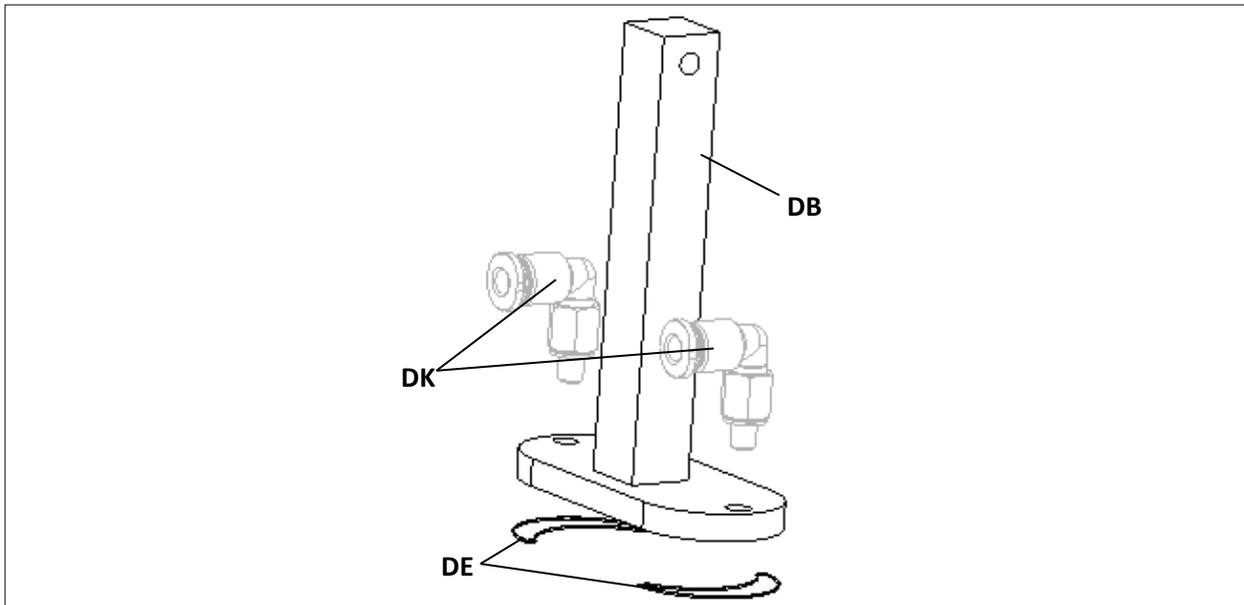
The gripper sub-assembly [C] is then attached to the gears sub-assembly [B]. Four gripper sub-assemblies are attached, one assembly to each of the four small gears. Each gripper sub-assembly is attached with four screws, two near the top of the gripper base and two near the bottom. As mentioned previously, the small gears are each rotated 90 degrees from one another. This is so that as these gripper sub-assemblies

are attached, they are at the proper orientation around the gear assembly so that as the gear base rotates and the small gears rotate, the part consequently is inverted. This assembly is shown in Figure 37.



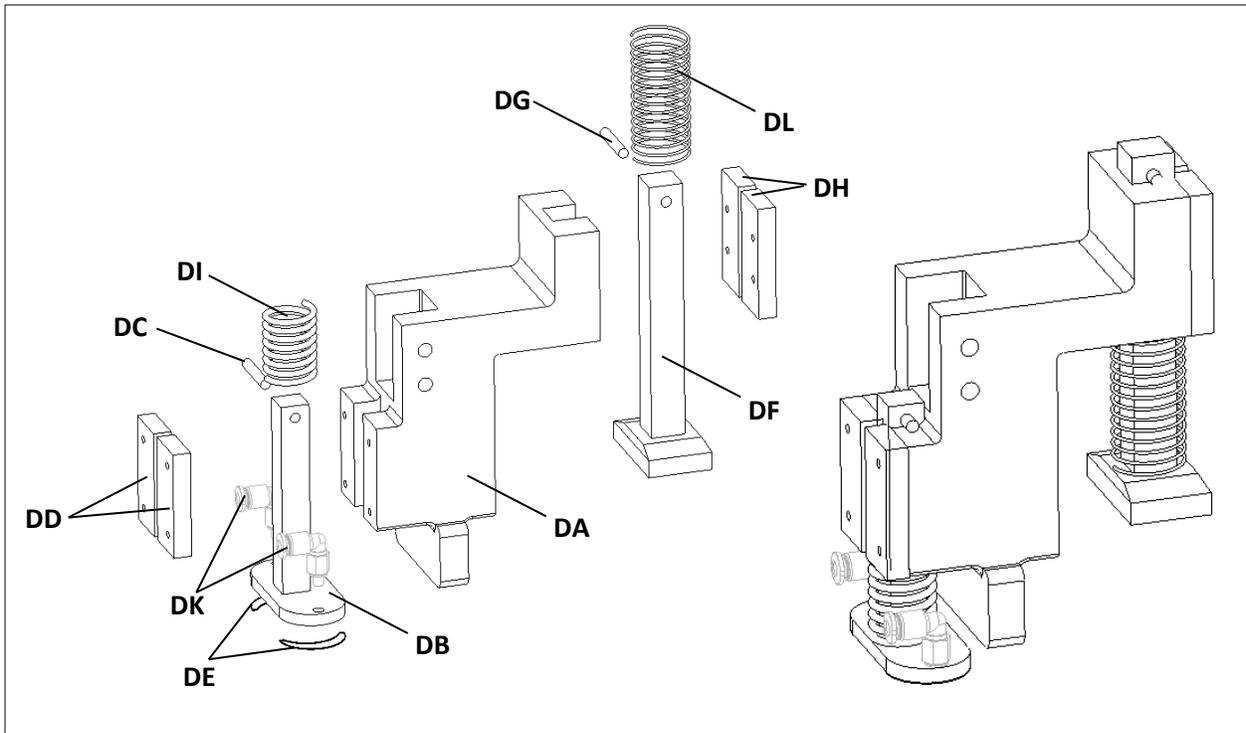
**Figure 37: Gears and Grippers Assembly**

Next, the vacuum and its fittings are sub-assembled. Two prefabricated hose fittings [DK] are screwed into the tapped holes on the vacuum slider [DB]. They both should end up facing the same direction (this direction is shown in the sub-assembly drawing). With the hose fittings in this orientation, the hose that is connected to the device should not get in the way of any moving parts. Also, the molded rubber suction pieces [DE] need to be attached to the under-side of the vacuum slider. With these parts all properly assembled, the vacuum slider will be able to apply suction to the part and the suction will hold the part against the rubber fittings, so as not to cause any cosmetic damage to the part. The assembly of the vacuum slider and its fittings is shown in Figure 38.



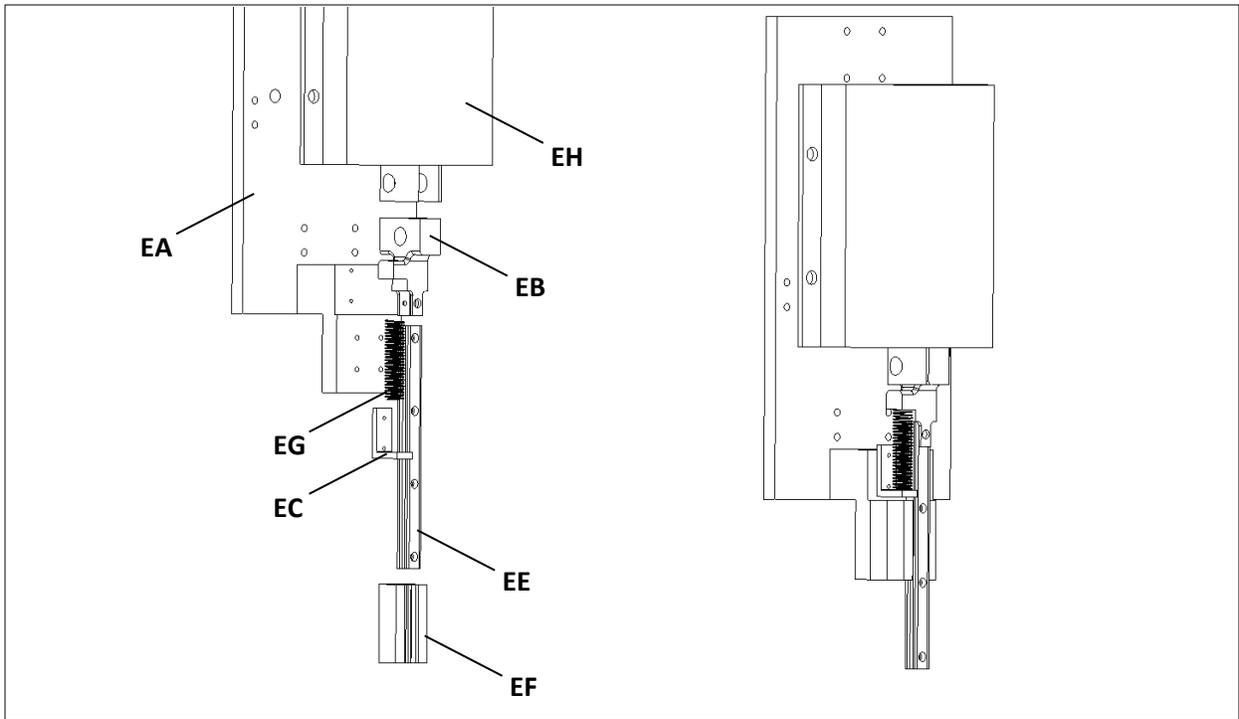
**Figure 38: Sub-assembly of Vacuum**

Next, the activator sub-assembly should be assembled. There are two activator assemblies, one for the pick-up station and one for the drop-off station. They are assembled the same, but the activator is slightly modified for the specific task at the different stations. The vacuum slider [DB] and leveling slider [DF] are both added to the activator sub-assembly in the same fashion. The spring [DI and DL] is slid onto the slider and the slider is placed in its slot in the activator [DA]. The spring is held in compression and the pin [DC and DG] is press-fit at the top of the slider. This pin prevents the slider from moving in a downward vertical direction further than is intended. Then, the two fastening plates [DD and DH] are added on the face of each side of the activator to hold in each slider. The fastening plates are different (both length and width) for each of the two sliders, but the two screws that hold each plate in are the same for all four plates. This completes the assembly of the activator sub-assembly. The activator sub-assembly for the pick-up station is shown in Figure 39.



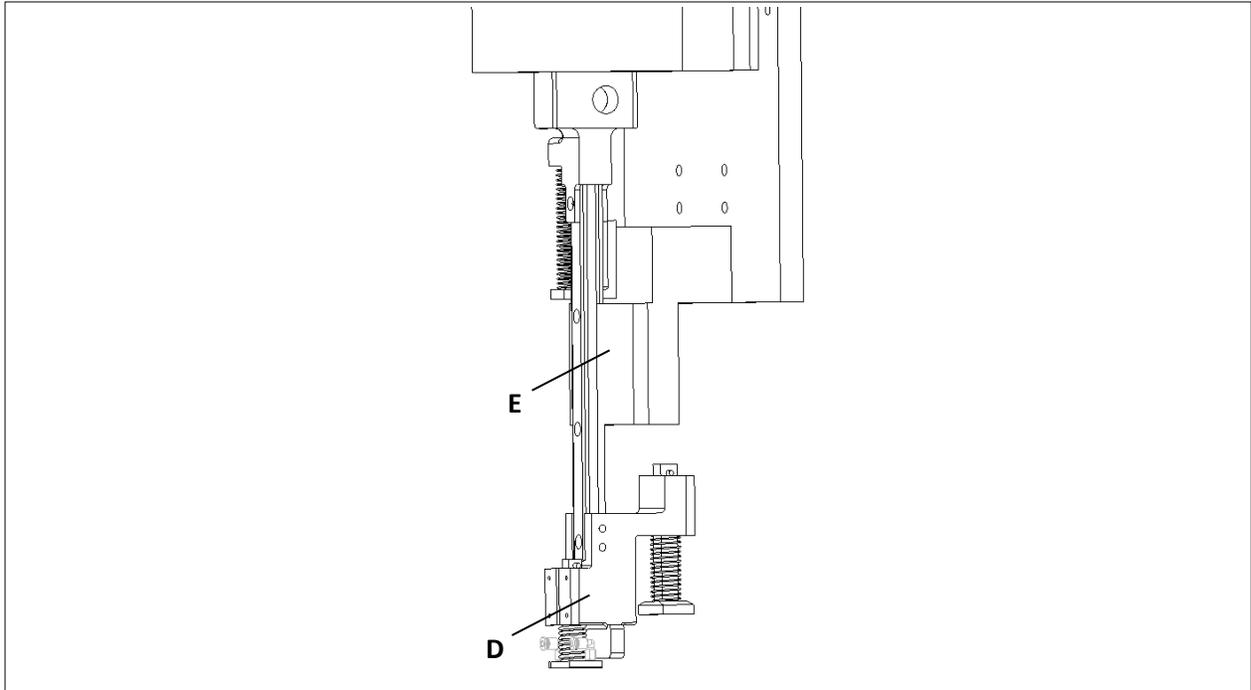
**Figure 39: Assembly of Activator with Sliders**

The rail [EE], the guide block [EF] and the solenoid [EH] are then assembled in a sub-assembly with other various components. The solenoid plate [EA] is the base fixture for this sub-assembly. First, the purchased solenoid is attached to the solenoid plate with four screws. Then, the purchased rail and guide block are assembled. The rail is slid into the guide block and then from the back side of the solenoid plate, the guide block is screwed into place. Next the yoke [EB] is put into place. It is bolted to the top end of the rail in the pre-drilled bolt hole. It is pinned to the solenoid plunger with a press-fit pin. Then the shelf [EC] is bolted to the solenoid plate. It is placed to the side of the rail and holds the spring [EG] that is placed between the shelf and the yoke [EB] and allows the rail to return vertically after the solenoid has finished its downward stroke. The sub-assembly of this section of the mechanism is shown in Figure 40.



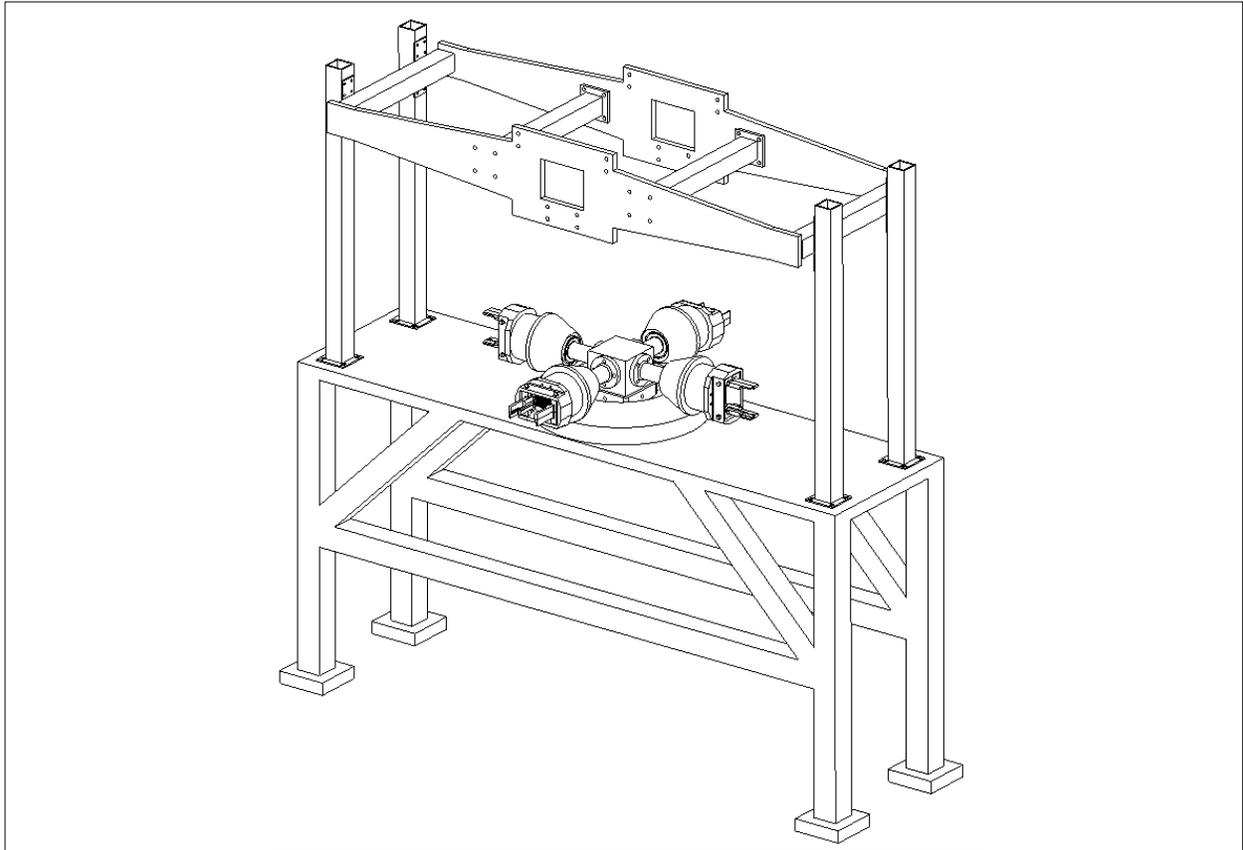
**Figure 40: Assembly of Solenoid and Rail**

The solenoid and rail sub-assembly [E] is then added to the activator sub-assembly [D]. This is done by sliding the rail into the top, center slot on the activator and screwing them together through the hole in the rail and the tapped hole in the activator. Then, set screws are inserted into the side of the activator. These screws can later be adjusted to ensure that the activator is properly centered below the solenoid and above the grippers. Figure 41 shows how these two sub-assemblies are combined.



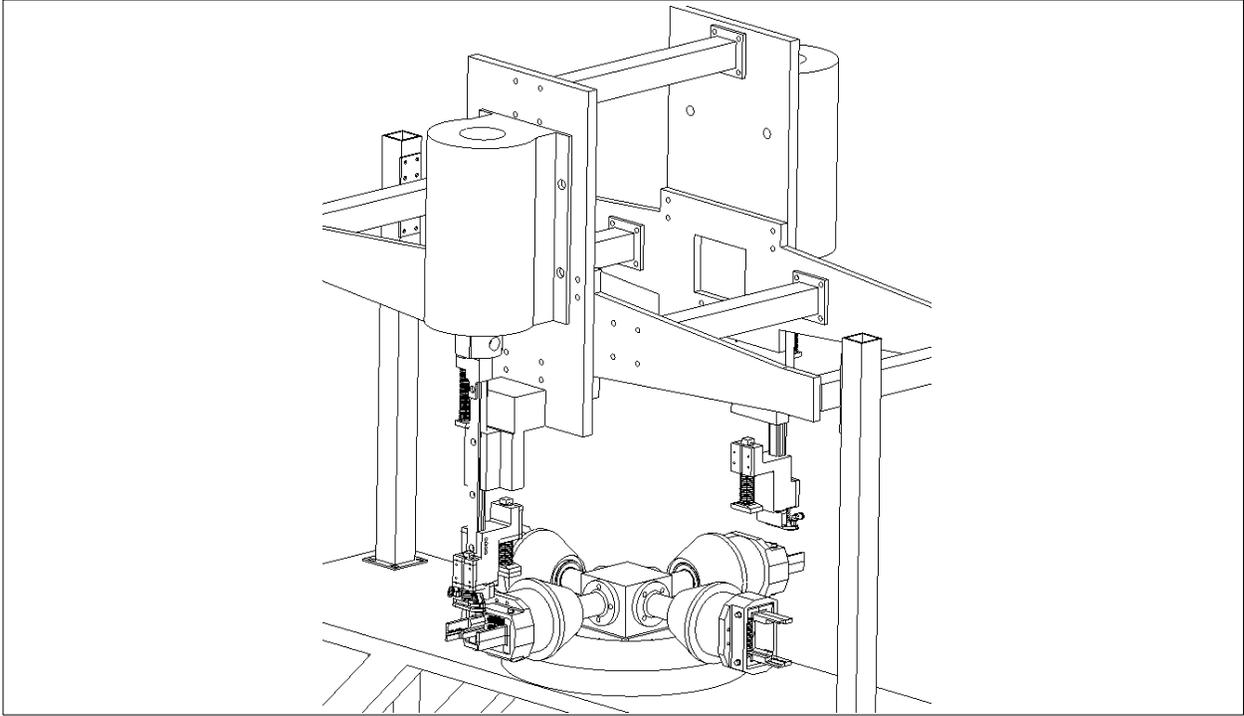
**Figure 41: Assembly of Activator to Rail**

Then, the assembly of all of the aforementioned sub-assemblies is begun. The gear and gripper sub-assembly is screwed to the table. The large gear fits into a hole in the table and is screwed from the underside of the top table surface. This holds the large gear stationary but allows the gear base shaft that comes down through the center of the thrust bearing to be accessed by the servo motor. This shaft will rotate, which will in turn rotate the small gears, the grippers, and therefore the part. Next, the table bridge is added to the full assembly of the mechanism. It is bolted to the stanchions at its four corners. The height of the table bridge is crucial to the overall vertical placement of the tooling. The assembly of the table assembly, the table bridge assembly and the gears and grippers assembly is shown in Figure 42.



**Figure 42: Assembly of Table, Gears and Grippers**

The final part of this assembly is the tooling (the solenoid, rail and activator) sub-assembly. Two of these sub-assemblies will be in the mechanism as a whole as mentioned previously, one with the pick-up activator and one with the drop-off activator. One tooling sub-assembly is bolted to the table bridge above one of the sets of gripper arms. Then, a cross-member is bolted to the solenoid plate near the top. This will provide support for each of the solenoid plates and thus the tooling stations as the mechanism moves up and down and creates a torque on the table bridge. After attaching the cross-member to one side, the second tooling station is added and is bolted to the solenoid plate and the cross-member. This final assembly step is shown in Figure 43.



**Figure 43: Assembly of All Remaining Parts**

With the completion of these steps, the inversion mechanism is assembled. Post-assembly, several measurements should be taken to ensure that all parts are properly aligned. If any part is not properly aligned, some parts may need to be re-cut and some may need washers added. Once full alignment has been completed, the mechanism should pick up a part, flip it over, and drop it off without any collisions or dropping of the part. The final assembly is shown in Figure 44 as it would look once all parts and assemblies have been attached and aligned.

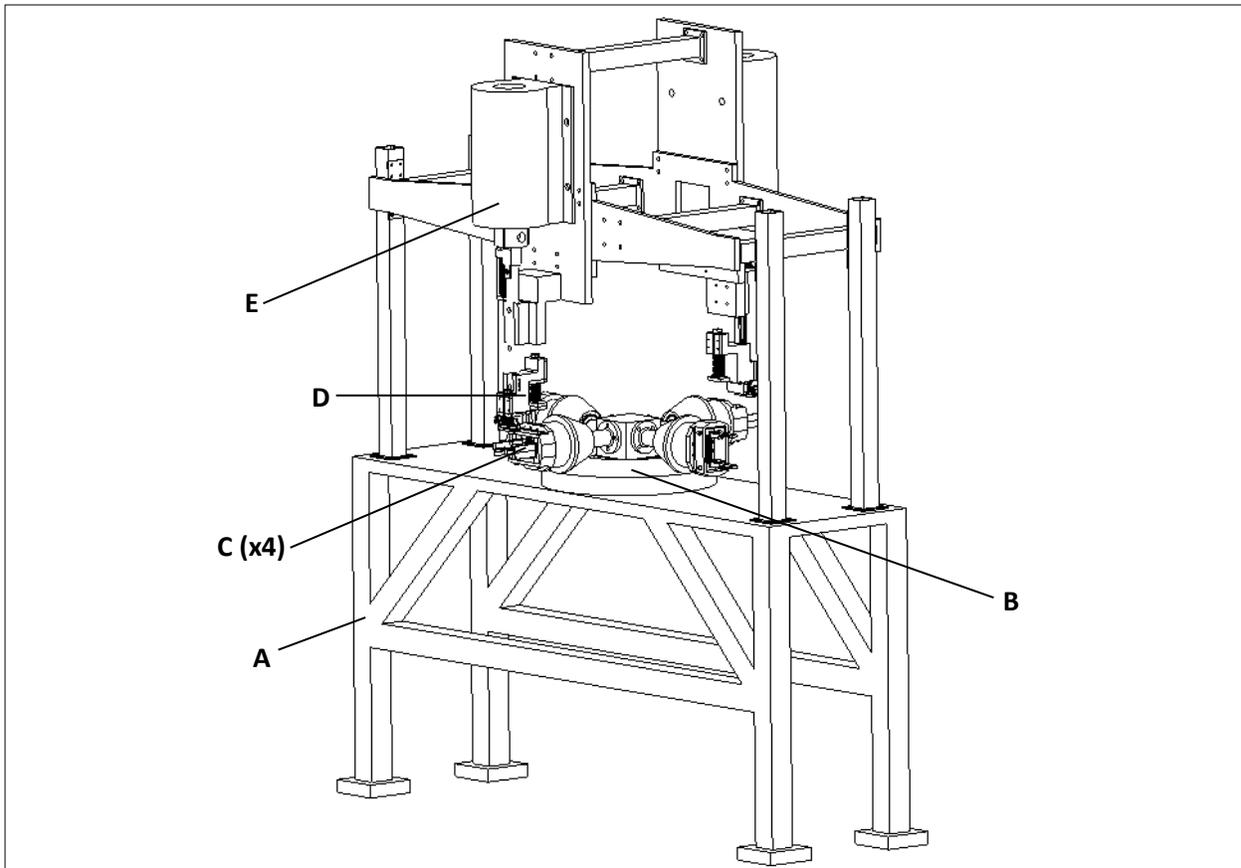


Figure 44: Final Assembly

## Results and Analysis

For this project it is crucial that all parts that will be locating the part within the overall machine are located extremely precisely. In addition, the lifetime cycles of the parts are crucial as the production rate is very high. To ensure that the precision and lifetime satisfied the necessary levels, particular analyses were performed on crucial parts. For parts that are critical to location of the part bending analysis was necessary to ensure that the loads applied to those parts would not deflect them more than what is allowed by the precision of placement. In addition to bending, some parts needed to be analyzed for displacement due to clearance issues. The tolerance achievable during manufacturing will impact precision placement after assembly and it was necessary to check that the displacement caused from clearance wouldn't displace the part outside of allowable ranges. For parts that were constantly being put under variable

stresses it was necessary to perform fatigue calculations in order to ensure that the parts would hold up for an extended period of time under the high production rates needed in this application. All fasteners and pins needed to be analyzed for shearing and tear out to ensure that they would not fail during operation. All detailed analyses are included in Appendix A: Calculations. This section outlines the analyses that were carried out and describes the general methods.

## **Bolts, Screws and Pin**

Table 1 summarizes the analysis required for all of the bolts, screws and pins in the entire design. The categories of analysis are broken down as: overall stress analysis, clearance check, shearing, tearout, and none necessary. Each part is listed with the appropriate analyses checked off. Overall stress analysis applies to pins which will be thoroughly checked for fracture due to stresses in all directions. Clearance check is a calculation of the error in the placement of a component due to tolerances in the fit of a pin, screw, or bolt. Shearing applies to parts which are only a concern for shearing (shearing is a part of the full stress calculation, but these parts need only the shearing analyzed). Tearout analysis is for pins which may be in danger of ripping out of the material around them. Some bolts, screws and pins have no necessary analysis because they are bulky enough that there is no fear that they will break. They are located in areas of the machine which have enough open space that we were able to make fasteners large enough that they will not break.

**Table 1: Summary of Analysis of Bolts, Screws and Pins**

ID	Part Name	Analysis Required				
		Overall Stress Analysis	Clearance Check	Shearing	Tear out	None necessary
<b>A</b>	<b>Frame Assembly</b>					
AE	Frame Bolts					X
<b>B</b>	<b>Gears Assembly</b>					
BG	Stepped gear shaft bolts	X				
BH	Gear base flange bolts					
<b>C</b>	<b>Gripper Assembly</b>					
CD	Gripper Arm Pin		X			
CE	Gripper Spring Pin					X
CH	Gripper base to small gear bolts			X		
<b>D</b>	<b>Activator Assembly</b>					
DC	Vacuum Slider Pin			X		
DG	Leveling Slider Pin			X	X	
DJ	Fastening Plate Bolts					X
DM	Activator Set Screws					X
DN	Activator-to-Rail Bolt					
<b>E</b>	<b>Solenoid and Rail Assembly</b>					
ED	Yoke pin			X		
EI	Solenoid screws					X
EJ	Solenoid plate screws					X
EK	Shelf screws					X
EL	Guide block screws					X
EM	Yoke-to-rail bolt					

## Other Parts

Table 2: Summary of Analysis of Other Parts

ID	Part Name	Analysis Required					
		Stress Concen.	Bending	Clearance/ Backlash	Buckling	Dynamic	None Necessary
<b>A</b>	<b>Frame Assembly</b>						X
AA	Table						X
AB	(Vertical Posts)						X
AC	("Bridge")						X
AD	(Perpendicular Supports)						X
<b>B</b>	<b>Gears Assembly</b>						X
BA	Gear Base						X
BB	Gear Base Flange						
BC	Stepped Gear Shaft	X	X			X	
BD	Large Bevel Gear			X			
BE	Small Bevel Gear			X			
BF	Small Gear Nut						X
<b>C</b>	<b>Gripper Assembly</b>						X
CA	Gripper Base						X
CB	Upper Gripper Arm Pair	X	X			X	
CC	Lower Gripper Arm Pair	X	X			X	
CF	Gripper pin snap rings						X
<b>D</b>	<b>Activator Assembly</b>					X	
DA	Activator		X				
DB	Vacuum Slider						
DD	Vacuum Slider Fastening Plates						X
DE	Rubber Seals						X
DF	Leveling Slider						
DH	Leveling Slider Fastening Plates						X
DK	Hose Fittings						X
<b>E</b>	<b>Solenoid and Rail Assembly</b>						
EA	Solenoid Plate						X
EB	Yoke						

EC	Shelf						
EE	THK Rail						X
EF	THK Guide Block						X
EH	Solenoid						

## Springs

Multiple springs are needed in the design produced by the team. Each of these springs had to be analyzed to ensure that each was able to compress the required amount while providing the correct force at that compressed length. Once the size requirements of each spring were found a search was begun to find springs that would fit the design. It was found that custom springs would be needed for this design due to the compression, force, and size requirements. Each of these springs must also have a dynamic fatigue safety factor that is great than 1.5 in order to ensure that the springs don't fail before reaching infinite life. The following table shows the required specifications of each spring.

**Table 3: Table of Spring Specifications**

Spring Specifications			
Spring Application	Original Compressed Length [mm]	Final Compressed Length [mm]	Final Compressed Force [N]
Solenoid	60.6	10.6	$40 < F < 50$
Leveler	34.77	9.567	$20 < F < 30$
Vacuum	19.425	9.425	$10 < F < 20$

## Timing

The timing of the interactions between the components of this mechanism are crucial to the control of the part. Specifically, the servo motor on the gear assembly must mesh with the solenoid timing correctly, and the activator assembly must be located precisely with the gripper assembly. The given speed of operation for this machine is 180 parts per minute which equates to 3 parts per second. This means that

every 0.33 seconds one part will be picked up and one will be released, but this is not necessarily the same part.

In reality, this mechanism will use half of the 0.33 second index to carry the part around the base, and half to move it in and out of the gripper assemblies. This results in an angular speed of 540 degrees per second around the base (because one index is 90 degrees per 0.17 seconds). The solenoid is able to complete its stroke and return to its upright position in the remaining 0.17 seconds. This results in a 25% duty cycle for the solenoid as it is only activated when it is in the down position.

One of the finest details in designing the motion of this mechanism was the height controls in the activator assembly. Each part must hit the gripper or part holder at the correct time and in the correct place in order to function properly. These interactions are different at the pick-up and drop-off locations.

**Pick-up:** At the pick-up location the start point has an empty gripper with the part located in the part holder below. As the solenoid comes down, the leveling plate is the first point of contact. The leveling plate aligns with the top flat face of the gripper to force it to be exactly level. That being done, the next interaction is the activator cam surface hitting the bottom grippers [CC]. At the pickup location the activator passes through the top grippers [CB] without interacting with them at all. The activator opens the bottom arms, which allows the vacuum part to pass through without any contact. At the bottom of the stroke the vacuum contacts the part. Next, the solenoid deactivates and the entire assembly is moved back up by the solenoid spring. As the vacuum passes through the top gripper arms [CB] the part is caught and held in place between the arms. The activator assembly continues upward as the bottom gripper arms [CC] close simultaneously, thus holding the part securely in place.

**Drop-off:** At the drop-off location the start point has the part in the gripper assembly. As the solenoid comes down, the first interaction is the leveling plate with the gear. Next, the vacuum contacts the part and attaches to it. As the solenoid continues down from that point the activator begins to open both pairs of gripper arms as the vacuum slider compresses (the part has not yet moved). When the

gripper arms are fully opened, the part is able to move downward and the vacuum slider snaps down toward the part holder. At the end of the solenoid stroke the part is placed in the part holder. The entire assembly then returns back up through the gripper and out of the way. This configuration allows the vacuum to securely contact the part without having the gripper arms slide out from under it before it is being held.

## **Conclusions**

The goal of this project was to create a mechanism that picks up a part, inverts it 180 degrees, and places it in a new location in its new orientation. This task was completed through the use of the design process. Ideas were brainstormed, drawn up, and evaluated. One design that was deemed a viable option was then modeled using Pro/ENGINEER. After modeling, the design was analyzed for various attributes such as stress, deflection, failure and fatigue. The result of this work is the creation of an inverting mechanism. The mechanism uses a system of bevel gears with grippers attached to hold, rotate, and move location of the part. With the part in the grippers, as the rotating gear moves along the stationary gear, the part is flipped over. The part is brought to the grippers and removed from the grippers by the use of tooling that is stationary above the pick-up and drop-off locations. The team found that even under applied loads, the part still remains at a precise location for the assembly process. Through the analysis of the parts involved in this mechanism, it has been determined that all parts will stand up to the loads that they are being subjected to and will have infinite life, as required. During the assembly and animation stages of the solid model in Pro/ENGINEER, many parts required adjustments and modifications in order to make every part work correctly within the device. Also, with this it became clear that the device would transfer and turn over parts in the allotted time. This mechanism provides a new way to access both sides of the part being moved as well as new tooling that could be modified and applied in several other applications.

## **Recommendations**

The nine-month span of this project from the explanation of the problem to the animation of the final model has produced great results. The mechanism that has been developed has potential for use in the application that it was designed for, but could still use several final touches. A cost analysis of all parts involved in the mechanism needs to be done. This involves the cost of pre-fabricated parts as well as the cost to manufacture the custom parts and the cost to assemble the mechanism as a whole. Additionally, prototyping and testing are necessary before the mechanism is put to full use. A mock-up of the mechanism should be made and put through real-time motions and forces in order to determine its ability to withstand normal, everyday operation.

## Bibliography

Gillis, C. (2010, September). Engineer. (C. Brown, M. Dooman, & S. Lax, Interviewers)

Maynard, C. (2010, September). Engineer. (C. Brown, M. Dooman, & S. Lax, Interviewers)

Nicholas P. Chironis, N. S. (2007). *Mechanisms and Mechanical Devices Sourcebook*. New York: McGraw-Hill.

Norton, R. L. (2006). *Machine Design* (3rd ed.). Upper Saddle River, NJ: Pearson Prentice Hall.

Norton, R. L. (2008). *Design of Machinery* (4th ed.). New York, NY: McGraw-Hill.

X-Y-Z: *Indexing Assembly Systems*. (n.d.). Retrieved October 5, 2010, from Assembly Magazine:

<http://www.assemblymag.com/CDA/Articles/Howto/43cbd62c8194a010VgnVCM100000f932a8>

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## Appendix A: Calculations

### Gear Arm Bolt Analysis

$$\text{Force}_{\text{Solenoid}} := 150\text{N}$$

$$\text{Volume}_{\text{Smallgearandgrripper}} := 73.979\text{mm}^3 + 254906\text{mm}^3 = 2.55 \times 10^5 \cdot \text{mm}^3$$

$$\rho := 7800 \frac{\text{kg}}{\text{m}^3}$$

$$g := 9.81 \frac{\text{m}}{\text{s}^2}$$

$$F := \text{Volume}_{\text{Smallgearandgrripper}} \cdot \rho \cdot g + \text{Force}_{\text{Solenoid}} = 169.511 \cdot \text{N}$$

$$c := \frac{5.5\text{mm}}{2} = 2.75 \cdot \text{mm}$$

$$D := 5.5\text{mm}$$

$$l := 138.5\text{mm}$$

$$M := F \cdot l = 23.477 \cdot \text{N} \cdot \text{m}$$

$$\sigma_{xx} := \frac{M \cdot c}{\left( \frac{\pi \cdot D^4}{64} \right)} = 1.437 \cdot \text{GPa}$$

Units are all in Mpa

$$\sigma_{xx} := 1.437 \cdot 10^3 \quad \sigma_{yy} := 0 \quad \sigma_{zz} := 0$$

$$\tau_{xy} := 0 \quad \tau_{xz} := 0 \quad \tau_{yz} := 0$$

$$\tau_{yx} := \tau_{xy} \quad \tau_{zx} := \tau_{xz} \quad \tau_{zy} := \tau_{yz}$$

$$0 = \sigma^3 - C_2 \cdot \sigma^2 - C_1 \cdot \sigma - C_0$$

$$C_2 := \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$$

$$C_1 := \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2 - \sigma_{xx} \cdot \sigma_{yy} - \sigma_{xx} \cdot \sigma_{zz} - \sigma_{yy} \cdot \sigma_{zz}$$

$$C_0 := \sigma_{xx} \cdot \sigma_{yy} \cdot \sigma_{zz} + 2 \cdot \tau_{xy} \cdot \tau_{xz} \cdot \tau_{yz} - \sigma_{xx} \cdot \tau_{yz}^2 - \sigma_{yy} \cdot \tau_{xz}^2 - \sigma_{zz} \cdot \tau_{xy}^2$$

$$C_2 = 1.437 \times 10^3 \quad C_1 = 0 \quad C_0 = 0$$

$$\text{Coef} := \begin{pmatrix} -C_0 \\ -C_1 \\ -C_2 \\ 1 \end{pmatrix} \quad r := \text{polyroots}(\text{Coef}) \quad r = \begin{pmatrix} 0 \\ 0 \\ 1.437 \times 10^3 \end{pmatrix}$$

$$\sigma_1 := r_2 \quad \sigma_1 = 1.437 \times 10^3$$

$$\sigma_2 := r_1 \quad \sigma_2 = 0$$

$$\sigma_3 := r_0 \quad \sigma_3 = 0$$

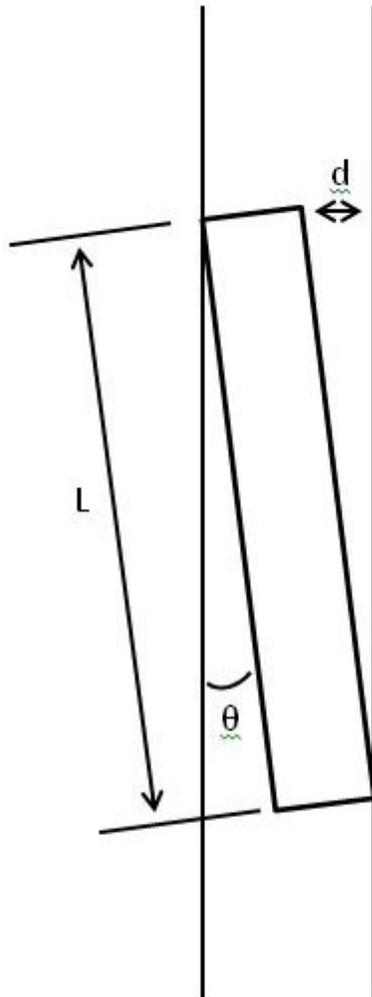
$$\tau_{13} := \left| \frac{\sigma_1 - \sigma_3}{2} \right| = 718.5$$

$$\tau_{12} := \left| \frac{\sigma_1 - \sigma_2}{2} \right| = 718.5$$

$$\tau_{23} := \left| \frac{\sigma_2 - \sigma_3}{2} \right| = 0$$

## Gripper Pin Clearance

Drawing:



$d$  = clearance between pin and hole (capability of machining)

$L$  = total length of pin

$\theta$  = angle gripper arms rotate down or up due to clearance

### Solving for displacement:

$$d := 0.0254\text{mm}$$

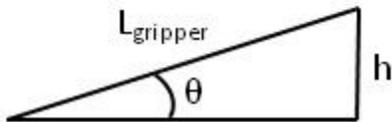
$$L_{\text{pin}} := 50\text{mm}$$

$$L_{\text{gripper}} := 46.5\text{mm}$$

$$\sin(\theta) = \frac{d}{L_{\text{pin}}}$$

$$\theta := \text{asin}\left(\frac{d}{L_{\text{pin}}}\right) = 5.08 \times 10^{-4}$$

displacement of end of gripper arm:



$h$  = displacement at end of gripper arm

$$\sin(\theta) = \frac{h}{L_{\text{gripper}}}$$

$$h := L_{\text{gripper}} \cdot \sin(\theta) = 0.024\text{mm}$$

$$h = 9.3 \times 10^{-4}\text{in}$$

## Gripper Base Bolts

$$V_{\text{gripassem}} := 73594.5 \text{mm}^3 = 7.359 \times 10^{-5} \cdot \text{m}^3$$

$$\rho_{\text{st}} := 7800 \frac{\text{kg}}{\text{m}^3}$$

$$g_{\text{st}} := 9.81 \frac{\text{m}}{\text{s}^2}$$

$$F_{\text{st}} := (V_{\text{gripassem}}) \cdot \rho \cdot g = 5.631 \cdot \text{N}$$

$$r_{\text{bolt}} := \frac{2.75}{2} \text{mm}$$

$$A_{\text{onebolt}} := \pi \cdot r_{\text{bolt}}^2 = 5.94 \times 10^{-6} \text{m}^2 \quad A_{\text{fourbolts}} := A_{\text{onebolt}} \cdot 4 = 2.376 \times 10^{-5} \text{m}^2$$

$$\tau_{\text{st}} := \frac{F}{A_{\text{fourbolts}}} = 0.237 \cdot \text{MPa}$$

$$S_{\text{st}} := 434 \text{MPa}$$

$$S_{\text{st,Min}} := 0.577 \cdot S_{\text{y}} = 250.418 \cdot \text{MPa}$$

## Vacuum Slider Pin

$$V_{\text{vacuum}} := 5804.5 \text{ mm}^3 = 5.805 \times 10^{-6} \cdot \text{m}^3$$

$$V_{\text{vspring}} := 623.937 \text{ mm}^3 = 6.239 \times 10^{-7} \cdot \text{m}^3$$

$$\rho_{\text{m}} := 7800 \frac{\text{kg}}{\text{m}^3}$$

$$g_{\text{m}} := 9.81 \frac{\text{m}}{\text{s}^2}$$

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$$F_{\text{m}} := (V_{\text{vacuum}} + V_{\text{vspring}}) \cdot \rho \cdot g = 0.492 \cdot \text{N}$$

$$r_{\text{pin}} := \frac{2.5}{2} \text{ mm}$$

$$A_{\text{m}} := \pi \cdot r_{\text{pin}}^2 = 4.909 \times 10^{-6} \text{ m}^2$$

$$\tau_{\text{m}} := \frac{F}{A} = 0.1 \cdot \text{MPa}$$

$$S_{\text{m}} := 434 \text{ MPa}$$

$$S_{\text{y}} := 0.577 \cdot S_y = 250.418 \cdot \text{MPa}$$

## Leveling Slider Pin

$$V_{\text{leveler}} := 9440.76 \text{mm}^3 = 9.441 \times 10^{-6} \cdot \text{m}^3$$

$$V_{\text{Ispring}} := 167.319 \text{mm}^3 = 1.673 \times 10^{-7} \cdot \text{m}^3$$

$$\rho := 7800 \frac{\text{kg}}{\text{m}^3}$$

$$g := 9.81 \frac{\text{m}}{\text{s}^2}$$

$$F_{\text{w}} := (V_{\text{leveler}} + V_{\text{Ispring}}) \cdot \rho \cdot g = 0.735 \cdot \text{N}$$

$$r_{\text{pin}} := \frac{2.75}{2} \text{mm}$$

$$A_{\text{w}} := \pi \cdot r_{\text{pin}}^2 = 5.94 \times 10^{-6} \text{m}^2$$

$$\tau := \frac{F}{A} = 0.124 \cdot \text{MPa}$$

$$S_y := 434 \text{MPa}$$

$$S_{ys} := 0.577 \cdot S_y = 250.418 \cdot \text{MPa}$$

Radius of Pin	$r_{\text{pin}} := \frac{2.5}{2} \text{mm}$
Density of Steel	$\rho := 7800 \frac{\text{kg}}{\text{m}^3}$
Volume of Leveling Slider	$V_{\text{leveler}} := 9440.76 \text{mm}^3 = 9.441 \times 10^{-6} \cdot \text{m}^3$
Volume of Leveling Slider Spring	$V_{\text{spring}} := 167.319 \text{mm}^3 = 1.673 \times 10^{-7} \cdot \text{m}^3$

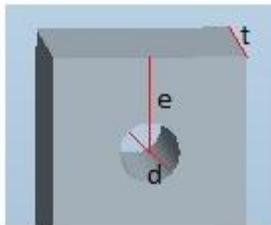
### Shear Stress Analysis:

Force due to Weight of Parts	$F_{\text{ww}} := (V_{\text{leveler}} + V_{\text{spring}}) \cdot \rho \cdot g = 0.735 \cdot \text{N}$
Cross-sectional Area of Pin	$A_{\text{ww}} := \pi \cdot r_{\text{pin}}^2 = 4.909 \times 10^{-6} \text{m}^2$
Shear Stress on Pin	$\tau := \frac{F}{A} = 0.15 \cdot \text{MPa}$
Yield Strength of Steel	$S_y := 434 \text{MPa}$
Yield Stress of Steel	$S_{ys} := 0.577 \cdot S_y = 250.418 \cdot \text{MPa}$

The shear stress on the pin is less than yield stress of steel, therefore the leveling slider pin will not shear due to the weight of the parts that it is supporting.

### Tearout Analysis:

$e := 4.75 \text{mm}$
$d := 3 \text{mm}$
$t := 9.53675 \text{mm}$



Affective Area	$A_{\text{eff}} := 2 \cdot e \cdot t = 90.599 \cdot \text{mm}^2$	+
Force due to Leveling Slider Spring	$P := 46.45 \text{N}$	
Shear Stress of Tearout	$\tau_{\text{tearout}} := \frac{P}{A_{\text{eff}}} = 0.513 \cdot \text{MPa}$	

The shear stress of the tearout is less than the yield stress of steel, therefore the leveling slider pin will not tearout of the leveling slider due to the force of the leveling slider spring.

## Yoke-to-Rail Bolt

$$V_{\text{rail}} := 31273.6 \text{mm}^3 = 3.127 \times 10^{-5} \cdot \text{m}^3$$

$$V_{\text{activassem}} := 89487.6 \text{mm}^3 = 8.949 \times 10^{-5} \cdot \text{m}^3$$

$$\rho_{\text{steel}} := 7800 \frac{\text{kg}}{\text{m}^3}$$

$$g_{\text{steel}} := 9.81 \frac{\text{m}}{\text{s}^2}$$

$$F_{\text{steel}} := (V_{\text{rail}} + V_{\text{activassem}}) \cdot \rho \cdot g = 9.24 \cdot \text{N}$$

$$r_{\text{bolt}} := \frac{4.5}{2} \text{mm}$$

$$A_{\text{bolt}} := \pi \cdot r_{\text{bolt}}^2 = 1.59 \times 10^{-5} \text{m}^2$$

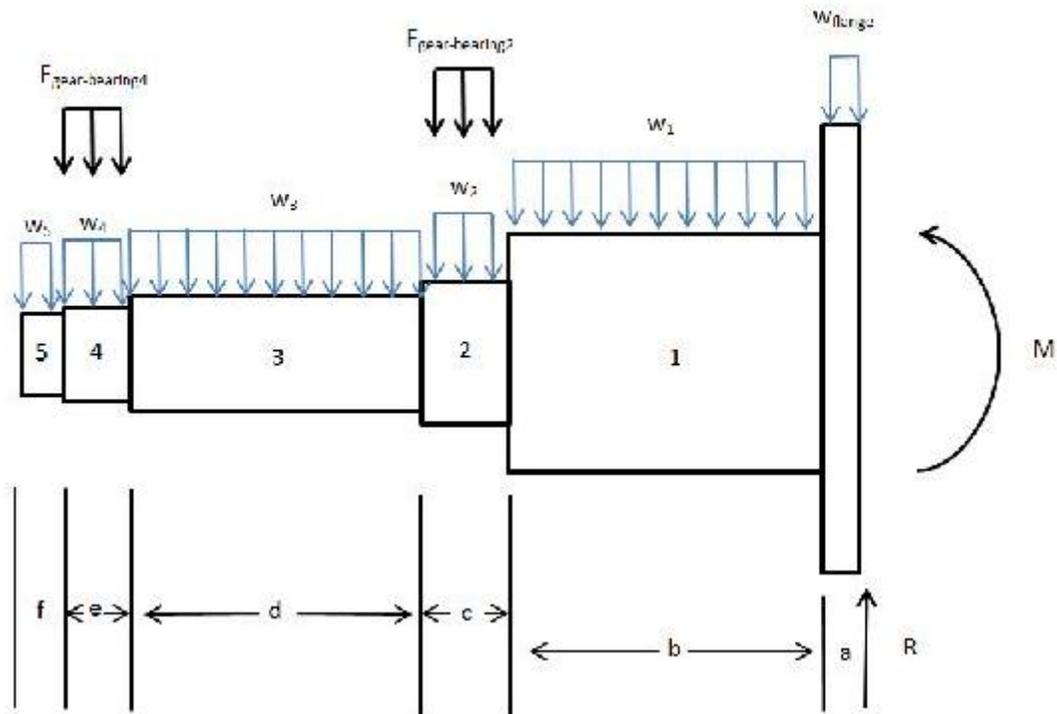
$$\tau_{\text{steel}} := \frac{F}{A} = 0.581 \cdot \text{MPa}$$

$$S_{\text{steel}} := 434 \text{MPa}$$

$$S_{\text{MEQV}} := 0.577 \cdot S_y = 250.418 \cdot \text{MPa}$$

## Gear Arms

Drawing :



$M$  = moment reaction at center

$R$  = force reaction at center

$w_{flange}$  = weight of the flange

$w_1$  = weight of the first section of the shaft

$w_2$  = weight of the second section of the shaft + the weight of the bearing

$F_{gear-bearing2}$  = force from the smaller bevel gear acting at bearing on section 2

$w_3$  = weight of the third section of the shaft

$w_4$  = weight of the fourth section of the shaft + weight of the bearing

$F_{gear-bearing4}$  = force from the smaller bevel gear acting at bearing on section 4

$w_5$  = weight of the fifth section of the shaft + weight of the end nut

$L$  = total length of shaft ( $a+b+c+d+e+f$ )

## Initial Values:



$$\rho_{\text{steel}} := 7850 \frac{\text{kg}}{\text{m}^3} \quad \text{density of steel}$$

$$E := 210\text{GPa} \quad \text{modulus of elasticity of steel}$$

$$G := 75\text{GPa} \quad \text{modulus of rigidity of steel}$$

$$a := 7\text{mm}$$

$$b := 47.5\text{mm}$$

$$c := 12\text{mm}$$

$$d := 62\text{mm}$$

$$e := 12\text{mm}$$

$$f := 5\text{mm}$$

$$L := a + b + c + d + e + f = 145.5\text{mm}$$

$$d_{\text{flange}} := 50\text{mm} \quad \text{diameter of the flange}$$

$$d_1 := 25\text{mm} \quad \text{diameter of the first section}$$

$$d_2 := 17\text{mm} \quad \text{diameter of the second section}$$

$$d_3 := 16.5\text{mm} \quad \text{diameter of the third section}$$

$$d_4 := 16\text{mm} \quad \text{diameter of the fourth section}$$

$$d_5 := 15\text{mm} \quad \text{diameter of the fifth section}$$

$$m_{\text{carrier}} := 1.8\text{kg} \quad \text{mass of the carrier bevel gear}$$

### cross sectional areas:

$$A_{\text{flange}} := \pi \cdot \left( \frac{d_{\text{flange}}}{2} \right)^2 = 1.963 \times 10^3 \cdot \text{mm}^2$$

$$A_1 := \pi \cdot \left( \frac{d_1}{2} \right)^2 = 490.874 \cdot \text{mm}^2$$

$$A_2 := \pi \cdot \left( \frac{d_2}{2} \right)^2 = 226.98 \cdot \text{mm}^2$$

$$A_3 := \pi \cdot \left( \frac{d_3}{2} \right)^2 = 213.825 \cdot \text{mm}^2$$

$$A_4 := \pi \cdot \left(\frac{d_4}{2}\right)^2 = 201.062 \cdot \text{mm}^2$$

$$A_5 := \pi \cdot \left(\frac{d_5}{2}\right)^2 = 176.715 \cdot \text{mm}^2$$

**weight per unit length of each section:**

$$w_{\text{flange}} := \rho_{\text{steel}} \cdot A_{\text{flange}} \cdot g = 151.154 \cdot \frac{\text{N}}{\text{m}}$$

$$w_1 := \rho_{\text{steel}} \cdot A_1 \cdot g = 37.789 \cdot \frac{\text{N}}{\text{m}}$$

$$w_2 := \rho_{\text{steel}} \cdot A_2 \cdot g = 17.473 \cdot \frac{\text{N}}{\text{m}}$$

$$w_3 := \rho_{\text{steel}} \cdot A_3 \cdot g = 16.461 \cdot \frac{\text{N}}{\text{m}}$$

$$w_4 := \rho_{\text{steel}} \cdot A_4 \cdot g = 15.478 \cdot \frac{\text{N}}{\text{m}}$$

$$w_5 := \rho_{\text{steel}} \cdot A_5 \cdot g = 13.604 \cdot \frac{\text{N}}{\text{m}}$$

**weight per unit length of bevel gear:**

$$w_{\text{carrier}} := \frac{m_{\text{carrier}} \cdot g}{c + e} = 735.499 \cdot \frac{\text{N}}{\text{m}}$$

**Moments of inertia:**

$$I_{zz}(x) := \begin{cases} \left(\frac{\pi}{64} \cdot d_{\text{flange}}^4\right) & \text{if } 0 \leq x \leq a \\ \left(\frac{\pi}{64} \cdot d_1^4\right) & \text{if } a < x \leq (a + b) \\ \left(\frac{\pi}{64} \cdot d_2^4\right) & \text{if } (a + b) < x < (a + b + c) \\ \left(\frac{\pi}{64} \cdot d_3^4\right) & \text{if } a + b + c < x \leq (a + b + c + d) \\ \left(\frac{\pi}{64} \cdot d_4^4\right) & \text{if } (a + b + c + d) < x \leq (a + b + c + d + e) \\ \left(\frac{\pi}{64} \cdot d_5^4\right) & \text{if } (a + b + c + d + e) < x \leq L \end{cases}$$

### Solving for reaction forces and moments:

$$\underline{R} := 0 \quad M_1 := 0$$

Given

$$R - w_{\text{flange}} \cdot a - w_1 \cdot b - w_2 \cdot c - w_3 \cdot d - w_4 \cdot e - w_5 \cdot f - w_{\text{carrier}} \cdot c - w_{\text{carrier}} \cdot e = 0$$

$$\underline{R} := \text{Find}(R) = 21.989 \text{ N}$$

Given

$$\begin{aligned} M_1 + w_{\text{flange}} \cdot a \cdot \frac{a}{2} + w_1 \cdot b \cdot \left( a + \frac{b}{2} \right) + w_2 \cdot c \cdot \left( a + b + \frac{c}{2} \right) + w_{\text{carrier}} \cdot c \cdot \left( a + b + \frac{c}{2} \right) \dots &= 0 \\ + w_3 \cdot d \cdot \left( a + b + c + \frac{d}{2} \right) + w_4 \cdot e \cdot \left( a + b + c + d + \frac{e}{2} \right) + w_{\text{carrier}} \cdot e \cdot \left( a + b + c + d + \frac{e}{2} \right) \dots & \\ + w_5 \cdot f \cdot \left( a + b + c + d + e + \frac{f}{2} \right) & \end{aligned}$$

$$\underline{M_1} := \text{Find}(M_1) = -1.927 \text{ N}\cdot\text{m}$$



### Singularity functions:



$$\underline{S}(x, z) := \text{if}(x \geq z, 1, 0) \quad C_1 := 0 \quad C_2 := 0 \quad C_3 := 0 \quad C_4 := 0$$

$$x := 0, 0.005 \cdot L \dots L$$

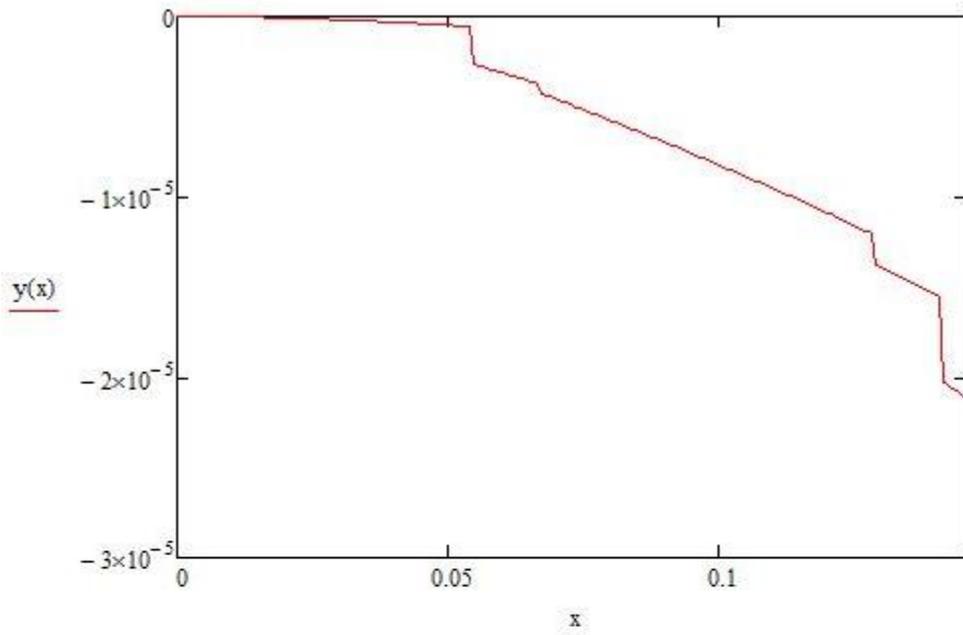
$$\begin{aligned} q(x) := & -w_{\text{flange}} \cdot S(x, 0) + w_{\text{flange}} \cdot S(x, a) \dots \\ & + -w_1 \cdot S(x, a) + w_1 \cdot S(x, a + b) - w_2 \cdot S(x, a + b) \dots \\ & + w_2 \cdot S(x, a + b + c) - w_{\text{carrier}} \cdot S(x, a + b) \dots \\ & + w_{\text{carrier}} \cdot S(x, a + b + c) - w_3 \cdot S(x, a + b + c) \dots \\ & + w_3 \cdot S(x, a + b + c + d) - w_4 \cdot S(x, a + b + c + d) \dots \\ & + w_4 \cdot S(x, a + b + c + d + e) \dots \\ & + -w_{\text{carrier}} \cdot S(x, a + b + c + d) \dots \\ & + w_{\text{carrier}} \cdot S(x, a + b + c + d + e) \dots \\ & + -w_5 \cdot S(x, a + b + c + d + e) \end{aligned}$$

$$\begin{aligned}
\underline{\underline{V(x)}} := & R \cdot S(x, 0) - w_{\text{flange}} \cdot S(x, 0) \cdot (x - 0)^1 + w_{\text{flange}} \cdot S(x, a) \cdot (x - a)^1 \dots \\
& + -w_1 \cdot S(x, a) \cdot (x - a)^1 + w_1 \cdot S[x, (a + b)] \cdot [x - (a + b)]^1 - w_2 \cdot S[x, (a + b)] \cdot [x - (a + b)]^1 \dots \\
& + w_2 \cdot S[x, (a + b + c)] \cdot [x - (a + b + c)]^1 - w_{\text{carrier}} \cdot S[x, (a + b)] \cdot [x - (a + b)]^1 \dots \\
& + w_{\text{carrier}} \cdot S[x, (a + b + c)] \cdot [x - (a + b + c)]^1 - w_3 \cdot S[x, (a + b + c)] \cdot [x - (a + b + c)]^1 \dots \\
& + w_3 \cdot S[x, (a + b + c + d)] \cdot [x - (a + b + c + d)]^1 - w_4 \cdot S[x, (a + b + c + d)] \cdot [x - (a + b + c + d)]^1 \dots \\
& + w_4 \cdot S[x, (a + b + c + d + e)] \cdot [x - (a + b + c + d + e)]^1 \dots \\
& + -w_{\text{carrier}} \cdot S[x, (a + b + c + d)] \cdot [x - (a + b + c + d)]^1 \dots \\
& + w_{\text{carrier}} \cdot S[x, (a + b + c + d + e)] \cdot [x - (a + b + c + d + e)]^1 \dots \\
& + -w_5 \cdot S[x, (a + b + c + d + e)] \cdot [x - (a + b + c + d + e)]^1 + C_1
\end{aligned}$$

$$\begin{aligned}
M(x) := & M_1 \cdot S(x, 0) + R \cdot S(x, 0) \cdot (x - 0)^1 - \frac{w_{\text{flange}}}{2} \cdot S(x, 0) \cdot (x - 0)^2 + \frac{w_{\text{flange}}}{2} \cdot S(x, a) \cdot (x - a)^2 \dots \\
& + \frac{-w_1}{2} \cdot S(x, a) \cdot (x - a)^2 + \frac{w_1}{2} \cdot S(x, a + b) \cdot [x - (a + b)]^2 - \frac{w_2}{2} \cdot S(x, a + b) \cdot [x - (a + b)]^2 \dots \\
& + \frac{w_2}{2} \cdot S(x, a + b + c) \cdot [x - (a + b + c)]^2 - \frac{w_{\text{carrier}}}{2} \cdot S(x, a + b) \cdot [x - (a + b)]^2 \dots \\
& + \frac{w_{\text{carrier}}}{2} \cdot S(x, a + b + c) \cdot [x - (a + b + c)]^2 - \frac{w_3}{2} \cdot S(x, a + b + c) \cdot [x - (a + b + c)]^2 \dots \\
& + \frac{w_3}{2} \cdot S(x, a + b + c + d) \cdot [x - (a + b + c + d)]^2 - \frac{w_4}{2} \cdot S(x, a + b + c + d) \cdot [x - (a + b + c + d)]^2 \dots \\
& + \frac{w_4}{2} \cdot S(x, a + b + c + d + e) \cdot [x - (a + b + c + d + e)]^2 \dots \\
& + \frac{-w_{\text{carrier}}}{2} \cdot S(x, a + b + c + d) \cdot [x - (a + b + c + d)]^2 \dots \\
& + \frac{w_{\text{carrier}}}{2} \cdot S(x, a + b + c + d + e) \cdot [x - (a + b + c + d + e)]^2 \dots \\
& + \frac{-w_5}{2} \cdot S(x, a + b + c + d + e) \cdot [x - (a + b + c + d + e)]^2 + C_1 \cdot x + C_2
\end{aligned}$$

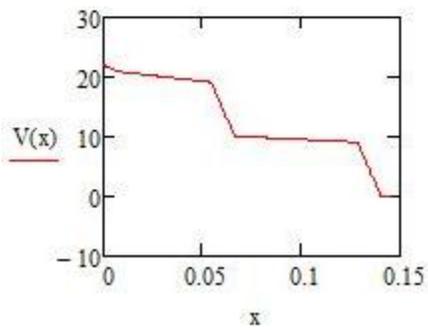
$$\theta(x) := \frac{1}{E \cdot I_{zz}(x)} \left[ \begin{aligned} & M_1 \cdot S(x, 0) \cdot (x - 0)^1 + \frac{R}{2} \cdot S(x, 0) \cdot (x - 0)^2 - \frac{w_{\text{flange}}}{6} \cdot S(x, 0) \cdot (x - 0)^3 + \frac{w_{\text{flange}}}{6} \cdot S(x, a) \cdot (x - a)^3 \dots \\ & + \frac{-w_1}{6} \cdot S(x, a) \cdot (x - a)^3 + \frac{w_1}{6} \cdot S(x, a + b) \cdot [x - (a + b)]^3 - \frac{w_2}{6} \cdot S(x, a + b) \cdot [x - (a + b)]^3 \dots \\ & + \frac{w_2}{6} \cdot S(x, a + b + c) \cdot [x - (a + b + c)]^3 - \frac{w_{\text{carrier}}}{6} \cdot S(x, a + b) \cdot [x - (a + b)]^3 \dots \\ & + \frac{w_{\text{carrier}}}{6} \cdot S(x, a + b + c) \cdot [x - (a + b + c)]^3 - \frac{w_3}{6} \cdot S(x, a + b + c) \cdot [x - (a + b + c)]^3 \dots \\ & + \frac{w_3}{6} \cdot S(x, a + b + c + d) \cdot [x - (a + b + c + d)]^3 - \frac{w_4}{6} \cdot S(x, a + b + c + d) \cdot [x - (a + b + c + d)]^3 \dots \\ & + \frac{w_4}{6} \cdot S(x, a + b + c + d + e) \cdot [x - (a + b + c + d + e)]^3 \dots \\ & + \frac{-w_{\text{carrier}}}{6} \cdot S(x, a + b + c + d) \cdot [x - (a + b + c + d)]^3 \dots \\ & + \frac{w_{\text{carrier}}}{6} \cdot S(x, a + b + c + d + e) \cdot [x - (a + b + c + d + e)]^3 \dots \\ & + \frac{-w_5}{6} \cdot S(x, a + b + c + d + e) \cdot [x - (a + b + c + d + e)]^3 + \frac{C_1}{2} \cdot x^2 + C_2 \cdot x + C_3 \end{aligned} \right]$$

$$y(x) := \frac{1}{E \cdot I_{zz}(x)} \left[ \begin{aligned} & \frac{M_1}{2} \cdot S(x, 0) \cdot (x - 0)^2 + \frac{R}{6} \cdot S(x, 0) \cdot (x - 0)^3 - \frac{w_{\text{flange}}}{24} \cdot S(x, 0) \cdot (x - 0)^4 + \frac{w_{\text{flange}}}{24} \cdot S(x, a) \cdot (x - a)^4 \dots \\ & + \frac{-w_1}{24} \cdot S(x, a) \cdot (x - a)^4 + \frac{w_1}{24} \cdot S(x, a + b) \cdot [x - (a + b)]^4 - \frac{w_2}{24} \cdot S(x, a + b) \cdot [x - (a + b)]^4 \dots \\ & + \frac{w_2}{24} \cdot S(x, a + b + c) \cdot [x - (a + b + c)]^4 - \frac{w_{\text{carrier}}}{24} \cdot S(x, a + b) \cdot [x - (a + b)]^4 \dots \\ & + \frac{w_{\text{carrier}}}{24} \cdot S(x, a + b + c) \cdot [x - (a + b + c)]^4 - \frac{w_3}{24} \cdot S(x, a + b + c) \cdot [x - (a + b + c)]^4 \dots \\ & + \frac{w_3}{24} \cdot S(x, a + b + c + d) \cdot [x - (a + b + c + d)]^4 - \frac{w_4}{24} \cdot S(x, a + b + c + d) \cdot [x - (a + b + c + d)]^4 \dots \\ & + \frac{w_4}{24} \cdot S(x, a + b + c + d + e) \cdot [x - (a + b + c + d + e)]^4 \dots \\ & + \frac{-w_{\text{carrier}}}{24} \cdot S(x, a + b + c + d) \cdot [x - (a + b + c + d)]^4 \dots \\ & + \frac{w_{\text{carrier}}}{24} \cdot S(x, a + b + c + d + e) \cdot [x - (a + b + c + d + e)]^4 \dots \\ & + \frac{-w_5}{24} \cdot S(x, a + b + c + d + e) \cdot [x - (a + b + c + d + e)]^4 + \frac{C_1}{6} \cdot x^3 + \frac{C_2}{2} \cdot x^2 + C_3 \cdot x + C_4 \end{aligned} \right]$$

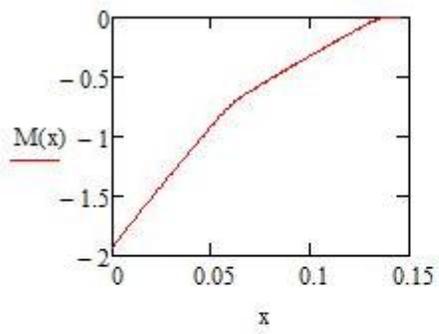


Max deflection:  $y(L) = -0.021$ -mm

### Stress Concentrations and Critical Sections:



$V_{\max} := V(0) = 21.989$  N



$$M_{\max} := |M(0)| = 1.927 \cdot \text{N} \cdot \text{m}$$

$$\sigma_x := M_{\max} \cdot \frac{0.5 \cdot d_{\text{flange}}}{I_{zz}(0)} = 0.157 \cdot \text{MPa}$$

$$r_{\text{curve}} := 0.7 \text{mm}$$

Determination of stress concentration factors:

2]

From table E-2

$$X := \begin{pmatrix} 6^3 & 6^2 & 6 & 1 \\ 3^3 & 3^2 & 3 & 1 \\ 2^3 & 2^2 & 2 & 1 \\ 1.5^3 & 1.5^2 & 1.5 & 1 \\ 1.2^3 & 1.2^2 & 1.2 & 1 \\ 1.1^3 & 1.1^2 & 1.1 & 1 \\ 1.07^3 & 1.07^2 & 1.07 & 1 \\ 1.05^3 & 1.05^2 & 1.05 & 1 \\ 1.03^3 & 1.03^2 & 1.03 & 1 \\ 1.02^3 & 1.02^2 & 1.02 & 1 \\ 1.01^3 & 1.01^2 & 1.01 & 1 \end{pmatrix} \quad Y := \begin{pmatrix} 0.87868 \\ 0.89334 \\ 0.90879 \\ 0.93836 \\ 0.97098 \\ 0.95120 \\ 0.97527 \\ 0.98137 \\ 0.98061 \\ 0.96048 \\ 0.91938 \end{pmatrix}$$

$$r_1 := 2$$

	0	1	2	3	4	5	6	7	8
$X^T =$	216	27	8	3.375	1.728	1.331	1.225	1.158	1.093
1	36	9	4	2.25	1.44	1.21	1.145	1.103	1.061
2	6	3	2	1.5	1.2	1.1	1.07	1.05	1.03
3	1	1	1	1	1	1	1	1	...

$$U := (X^T \cdot X)^{-1} \cdot (X^T \cdot Y)$$

$$U = \begin{pmatrix} -8.795 \times 10^{-4} \\ 0.016 \\ -0.088 \\ 1.039 \end{pmatrix}$$

$$U_0 = -8.795 \times 10^{-4}$$

$$U_1 = 0.016$$

---

$$U_2 = -0.088$$

$$U_3 = 1.039$$

$$\hat{A}(r) := U_0 \cdot r^3 + U_1 \cdot r^2 + U_2 \cdot r + U_3$$

$$A(2) = 0.918$$

$$Y_b := \begin{pmatrix} -0.33243 \\ -0.30860 \\ -0.28598 \\ -0.25759 \\ -0.21796 \\ -0.23757 \\ -0.20958 \\ -0.19653 \\ -0.18381 \\ -0.17711 \\ -0.17032 \end{pmatrix}$$

$$U_b = (X^T \cdot X)^{-1} \cdot (X^T \cdot Y_b)$$

$$U_b = \begin{pmatrix} -9.525 \times 10^{-3} \\ 0.105 \\ -0.357 \\ 0.074 \end{pmatrix}$$

$$U_{b_0} = -9.525 \times 10^{-3}$$

$$U_{b_1} = 0.105$$

$$U_{b_2} = -0.357$$

$$U_{b_3} = 0.074$$

$$B(r) := U_{b_0} \cdot r^3 + U_{b_1} \cdot r^2 + U_{b_2} \cdot r + U_{b_3}$$

$$B(2) = -0.295$$

$$K_{tbend}(r) := A(r) \cdot \left( \frac{r_{curve}}{50mm} \right)^{B(r)}$$

$$K_{tbend}(2) = 3.229$$



#### Determination of lifetime cycles and safety factor:



$$\sigma_{xconc} := \sigma_x \cdot K_{tbend}(2) = 0.507 \cdot \text{MPa}$$

$$\tau_{shearmax} := \frac{4}{3} \cdot \frac{V(0)}{A_{flange}} = 14.932 \cdot \text{kPa}$$

$$\sigma_{1max} := \frac{\sigma_{xconc}}{2} + \tau_{shearmax} = 268.428 \cdot \text{kPa}$$

$$\sigma_{3max} := \frac{\sigma_{xconc}}{2} - \tau_{shearmax} = 238.564 \cdot \text{kPa}$$

$$\tau_{13max} := \frac{\sigma_{1max} - \sigma_{3max}}{2} = 14.932 \cdot \text{kPa}$$

#### Von Mises Effective Stresses:

$$\sigma'_{max} := \sqrt{\sigma_{1max}^2 - \sigma_{1max} \cdot \sigma_{3max} + \sigma_{3max}^2} = 254.812 \cdot \text{kPa}$$

$$\sigma'_{min} := 0 \text{ Pa}$$

$$\sigma'_{alt} := \frac{\sigma'_{max}}{2} = 127.406 \cdot \text{kPa}$$

$$\sigma'_{mean} := \frac{\sigma'_{max}}{2} = 127.406 \cdot \text{kPa}$$

$S_{ut} := 518.8\text{MPa}$       AISI 1040 Steel

$S_y := 353.4\text{MPa}$

$T_{oper} := 200$

$R_{\text{sur}} := 0.99999$

load := "bending"

surface := "machined"

$S_{eun} := \begin{cases} \text{return } (0.5 \cdot S_{ut}) & \text{if } S_{ut} \leq 1400\text{MPa} \\ (700\text{MPa}) & \text{otherwise} \end{cases}$

$C_{load} := \begin{cases} \text{return } 1 & \text{if load = "bending"} \\ \text{return } 1 & \text{if load = "torsion"} \\ \text{return } 0.7 & \text{if load = "axial"} \end{cases}$

$C_{size} := \begin{cases} \text{return } \left[ 1.189 \cdot \left( \frac{d_{flange}}{m} \right)^{-0.097} \right] & \text{if } 0.008\text{m} < d_{flange} \leq 0.250\text{m} \\ \text{return } 0.6 & \text{if } d_{flange} > 0.250\text{m} \end{cases}$

$A_{\text{sur}} := \begin{cases} \text{return } 1.34 & \text{if surface = "ground"} \\ \text{return } 2.70 & \text{if surface = "machined"} \\ \text{return } 2.7 & \text{if surface = "cold_rolled"} \\ \text{return } 14.4 & \text{if surface = "hot_rolled"} \\ \text{return } 39.9 & \text{if surface = "forged"} \end{cases}$

$b_{\text{sur}} := \begin{cases} \text{return } (-0.085) & \text{if surface = "ground"} \\ \text{return } (-0.265) & \text{if surface = "machined"} \\ \text{return } (-0.265) & \text{if surface = "cold_rolled"} \\ \text{return } (-0.718) & \text{if surface = "hot_rolled"} \\ \text{return } (-0.995) & \text{if surface = "forged"} \end{cases}$

A = 2.7

b = -0.265

---

$$C_{\text{surface}} := A \cdot \left( \frac{S_{\text{ut}}}{\text{ksi}} \right)^b$$

$$C_{\text{temp}} := \begin{cases} \text{return } 1 & \text{if } T_{\text{oper}} \leq 450 \\ [1 - 0.0032 \cdot (T_{\text{oper}} - 840)] & \text{otherwise} \end{cases}$$

$$C_{\text{reliability}} := \begin{cases} \text{return } 1.000 & \text{if } R = 0.50 \\ \text{return } 0.897 & \text{if } R = 0.90 \\ \text{return } 0.814 & \text{if } R = 0.99 \\ \text{return } 0.753 & \text{if } R = 0.999 \\ \text{return } 0.702 & \text{if } R = 0.9999 \\ \text{return } 0.659 & \text{if } R = 0.99999 \end{cases}$$

$$C_{\text{total}} := C_{\text{load}} \cdot C_{\text{size}} \cdot C_{\text{surface}} \cdot C_{\text{temp}} \cdot C_{\text{reliability}} = 0.9$$

$$S_e := C_{\text{total}} \cdot S_{\text{eun}} = 233.525 \cdot \text{MPa}$$

$$S_m := \begin{cases} \text{return } (0.75 \cdot S_{\text{ut}}) & \text{if load = "axial"} \\ (0.9 \cdot S_{\text{ut}}) & \text{otherwise} \end{cases}$$

$$S_m = 466.92 \cdot \text{MPa}$$

$$z := -3$$

$$b_{\text{ww}} := \frac{1}{z} \cdot \log \left( \frac{S_m}{S_e} \right) = -0.1$$

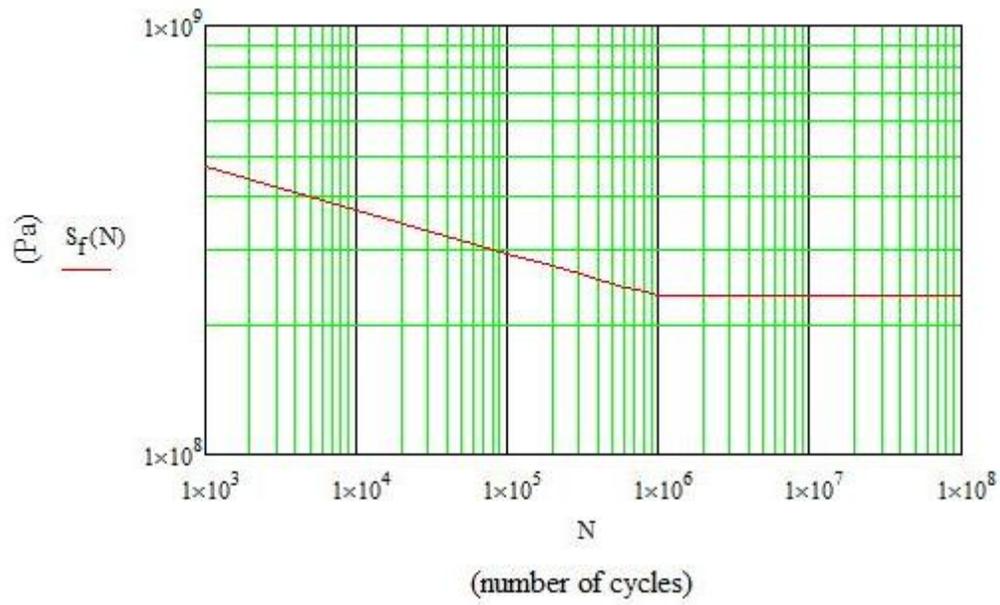
$$a_{\text{ww}} := \frac{S_m}{1000^{b_{\text{ww}}}} = 933.58 \cdot \text{MPa}$$

$$S_f = a \cdot N^b$$

Graphing the S-N diagram:

$$N_{\text{ww}} := 10^3, 10^5 \dots 10^8$$

$$S_f(N) := \begin{cases} \text{return } a \cdot N^b & \text{if } N < 10^6 \\ S_e & \text{otherwise} \end{cases}$$



$$N = 10^6$$

Given

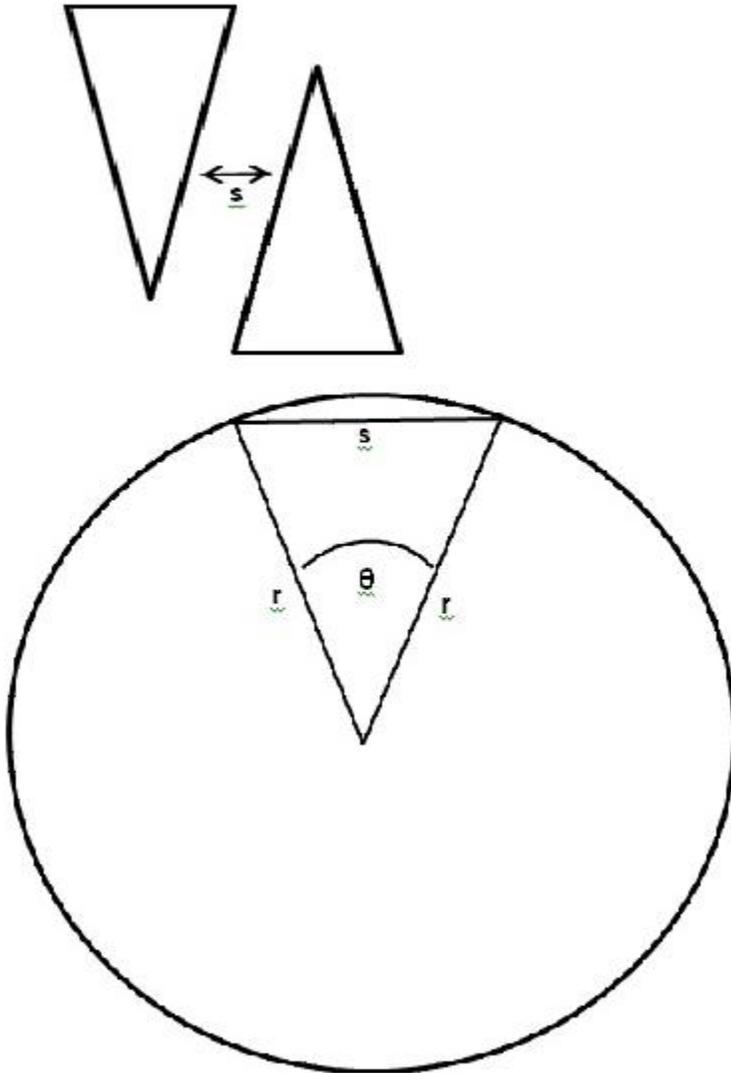
$$\sigma'_{\text{mean}} = a \cdot N^b$$

$$\text{Find}(N) = 3.411 \times 10^{38}$$

$$\text{Safety}_{\text{factor}} := \frac{S_e \cdot S_{\text{ut}}}{\sigma'_{\text{alt}} \cdot S_{\text{ut}} + \sigma'_{\text{mean}} \cdot S_e} = 1.264 \times 10^3$$

## Gear Backlash

Drawing:



$s$  = distance between teeth of small carrier gear and large stationary gear due to backlash

$r$  = radius of large stationary gear

$\theta$  = angle carrier gear moves through from 0 tolerance between teeth to maximum backlash

## Solving for part displacement due to backlash:

$$r_{\text{stationary}} := 270\text{mm}$$

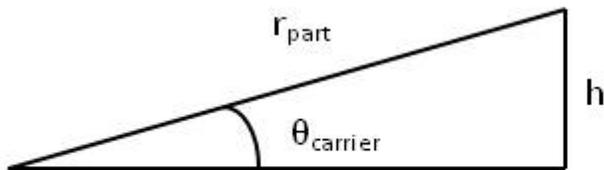
$$r_{\text{carrier}} := 90\text{mm}$$

$$s_{\text{max}} := 0.36\text{mm}$$

$$\theta_{\text{arm}} := \frac{s_{\text{max}}}{r_{\text{stationary}}} = 1.333 \times 10^{-3} \quad \text{maximum angle of movement in radians}$$

for each angle of movement in the arms that hold the carrier gears, the carrier gears go through 3 degrees of rotation.

$$\theta_{\text{carrier}} := \theta_{\text{arm}} \cdot 3 = 4 \times 10^{-3}$$



$h$  = maximum displacement of part

$r_{\text{part}}$  = distance from center of part to end of part (lengthwise)

$$r_{\text{part}} := 22.225\text{mm}$$

$$\sin \theta_{\text{carrier}} = \frac{h}{r_{\text{part}}}$$

$$h := r_{\text{part}} \cdot \sin(\theta_{\text{carrier}}) = 0.089\text{mm}$$

$$h = 3.5 \times 10^{-3}\text{-in}$$

This is the distance that the leveling plate is making up for. By using the leveling plate, this distance is removed and the part is at the correct orientation

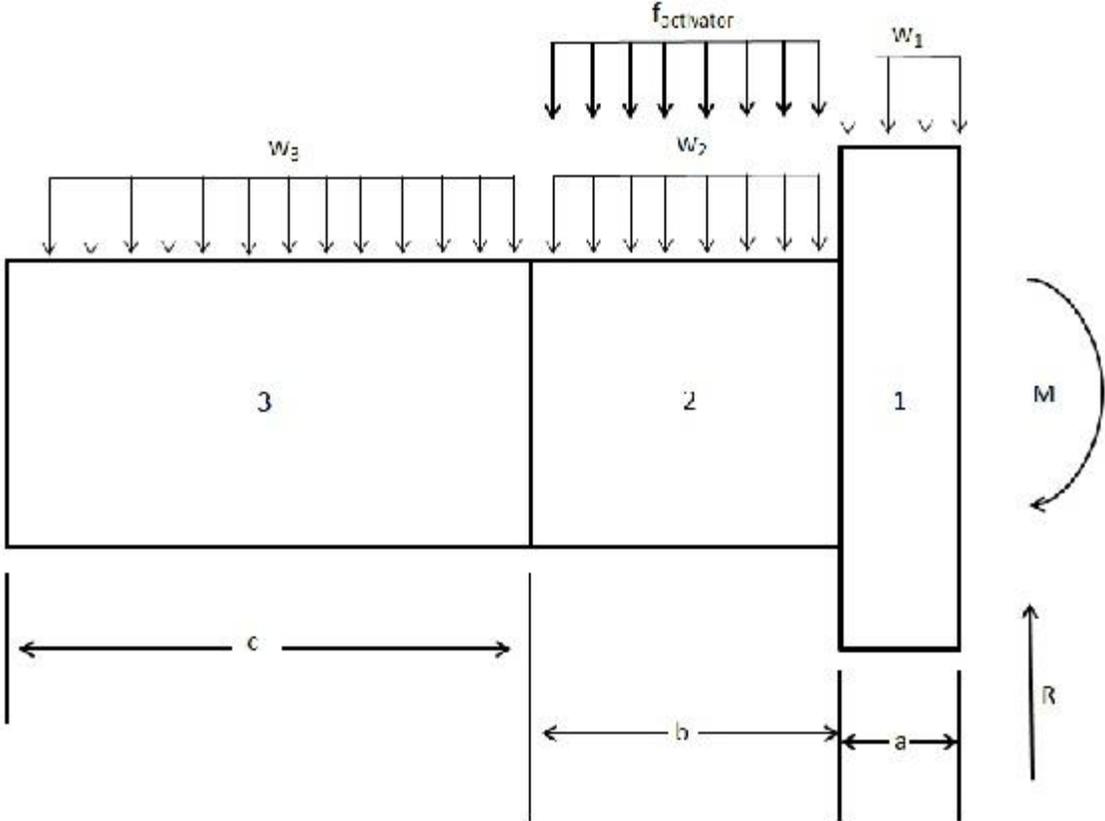
# Gripper Arms Analysis

## Top Arm:

### Drawings :



#### Front View:



$M$  = moment reaction at pin

$R$  = force reaction at pin

$w_1$  = weight of section 1

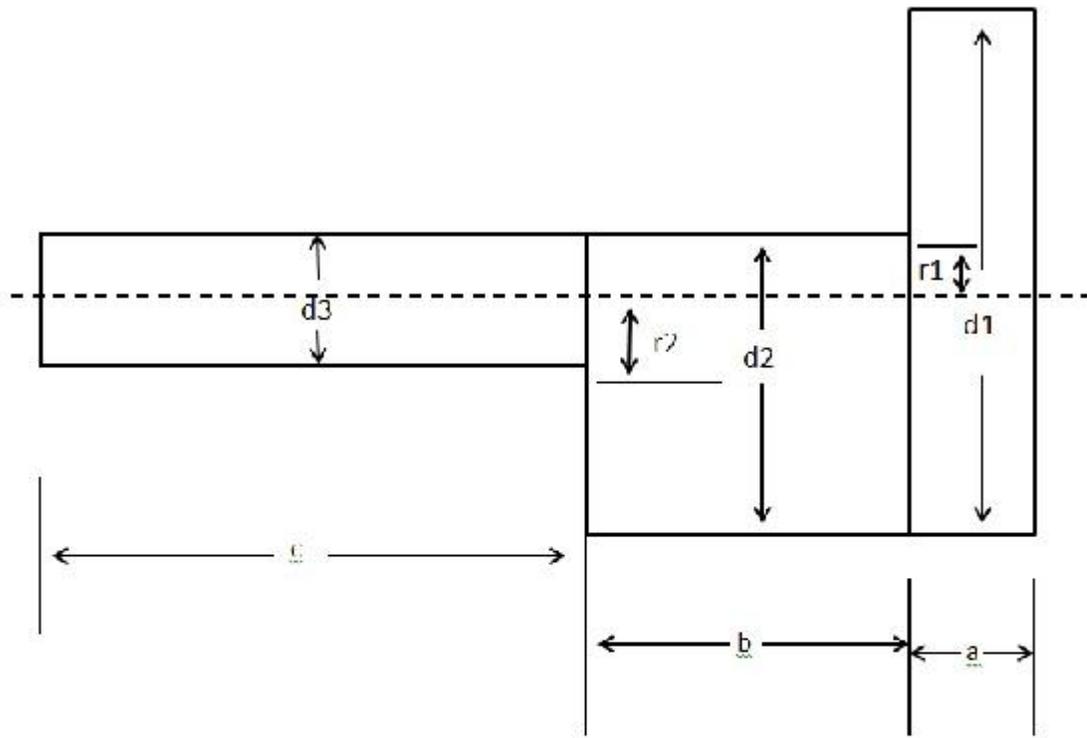
$w_2$  = weight of section 2

activator = force due to the activator opening the arms

$w_3$  = weight of section 3

$L$  = total length ( $a+b+c$ )

Top View:



$d_1$  = thickness of section 1

$r_1$  = distance from center of mass of section 1 to the z-axis (dotted line)

$d_2$  = thickness of section 2

$r_2$  = distance from center of mass of section 2 to the z-axis (dotted line)

$d_3$  = thickness of section 3

## Initial Values:

---



$$\rho_{\text{steel}} := 7850 \frac{\text{kg}}{\text{m}^3} \quad \text{density of steel}$$

$$E := 210\text{GPa} \quad \text{modulus of elasticity of steel}$$

$$\overset{\sim}{G} := 75\text{GPa} \quad \text{modulus of rigidity of steel}$$

$$a := 5\text{mm}$$

$$b := 11.5\text{mm}$$

$$\overset{\sim}{c} := 30\text{mm}$$

$$\overset{\sim}{L} := a + b + c = 46.5\text{-mm}$$

$$h_1 := 20\text{mm} \quad \text{height of section 1}$$

$$h_2 := 10\text{mm} \quad \text{height of section 2}$$

$$h_3 := h_2 = 10\text{-mm} \quad \text{height of section 3}$$

$$d_1 := 15\text{mm}$$

$$d_2 := 9\text{mm}$$

$$d_3 := 4\text{mm}$$

$$r_1 := 0.5\text{mm}$$

$$r_2 := 2.5\text{mm}$$

$$f_{\text{activator}} := 150\text{N}$$

### Cross sectional areas:

$$A_1 := h_1 \cdot d_1 = 300\text{-mm}^2$$

$$A_2 := h_2 \cdot d_2 = 90\text{-mm}^2$$

$$A_3 := h_3 \cdot d_3 = 40\text{-mm}^2$$

### mass of each section:

$$m_1 := A_1 \cdot a \cdot \rho_{\text{steel}} = 0.012\text{kg}$$

$$m_2 := A_2 \cdot b \cdot \rho_{\text{steel}} = 8.125 \times 10^{-3}\text{kg}$$

$$m_3 := A_3 \cdot c \cdot \rho_{\text{steel}} = 9.42 \times 10^{-3}\text{kg}$$

**weight per unit length of each section:**

$$w_1 := \rho_{\text{steel}} \cdot A_1 \cdot g = 23.095 \cdot \frac{\text{N}}{\text{m}}$$

$$w_2 := \rho_{\text{steel}} \cdot A_2 \cdot g = 6.928 \cdot \frac{\text{N}}{\text{m}}$$

$$w_3 := \rho_{\text{steel}} \cdot A_3 \cdot g = 3.079 \cdot \frac{\text{N}}{\text{m}}$$

**force per unit length due to activator:**

$$F_{\text{activator}} := \frac{f_{\text{activator}}}{b} = 1.304 \times 10^4 \cdot \frac{\text{N}}{\text{m}}$$

**Moments of Inertia:**

$$I_{zz}(x) := \begin{cases} \frac{d_1 \cdot h_1^3}{12} + A_1 \cdot r_1^2 & \text{if } 0 \leq x \leq a \\ \frac{d_2 \cdot h_2^3}{12} + A_2 \cdot r_2^2 & \text{if } a < x \leq a + b \\ \frac{d_3 \cdot h_3^3}{12} & \text{if } a + b < x \leq L \end{cases}$$

**Solving for reaction force and moment:**

$$\underline{\underline{R}} := 0 \quad \underline{\underline{M}}_1 := 0$$

Given

$$R - w_1 \cdot a - w_2 \cdot b - F_{\text{activator}} \cdot b - w_3 \cdot c = 0$$

$$\underline{\underline{R}} := \text{Find}(R) = 150.288 \text{ N}$$

Given

$$M_1 + w_1 \cdot a \cdot \left(\frac{a}{2}\right) + w_2 \cdot b \cdot \left(a + \frac{b}{2}\right) + F_{\text{activator}} \cdot b \cdot \left(a + \frac{b}{2}\right) + w_3 \cdot c \cdot \left(a + b + \frac{c}{2}\right) = 0$$

$$\underline{\underline{M}}_1 := \text{Find}(M_1) = -1.617 \cdot \text{N} \cdot \text{m}$$

## Singularity Functions:



$$S(x, z) := \text{if}(x \geq z, 1, 0) \quad C_1 := 0 \quad C_2 := 0 \quad C_3 := 0 \quad C_4 := 0$$

$$x := 0, 0.005 \cdot L \dots L$$

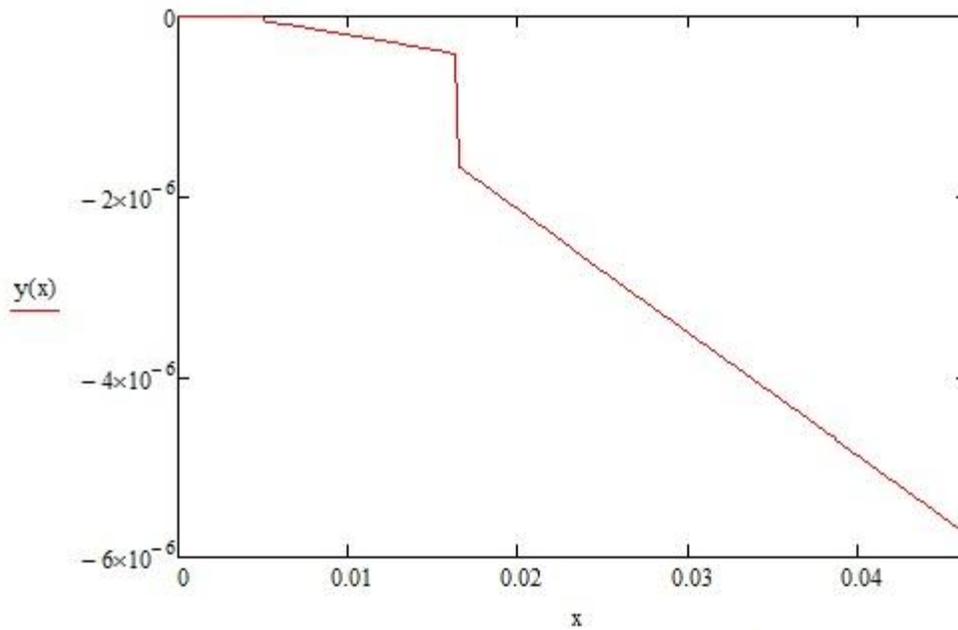
$$\begin{aligned} q(x) := & -w_1 \cdot S(x, 0) + w_1 \cdot S(x, 0) \dots \\ & + -w_2 \cdot S(x, a) + w_2 \cdot S[x, (a + b)] - F_{\text{activator}} \cdot S(x, a) \dots \\ & + F_{\text{activator}} \cdot S[x, (a + b)] \dots \\ & + -w_3 \cdot S[x, (a + b)] \end{aligned}$$

$$\begin{aligned} V(x) := & R \cdot S(x, 0) - w_1 \cdot S(x, 0) \cdot (x - 0)^1 + w_1 \cdot S(x, 0) \cdot (x - 0)^1 \dots \\ & + -w_2 \cdot S(x, a) \cdot (x - a)^1 + w_2 \cdot S[x, (a + b)] \cdot [x - (a + b)]^1 - F_{\text{activator}} \cdot S(x, a) \cdot (x - a)^1 \dots \\ & + F_{\text{activator}} \cdot S[x, (a + b)] \cdot [x - (a + b)]^1 \dots \\ & + -w_3 \cdot S[x, (a + b)] \cdot [x - (a + b)]^1 \end{aligned}$$

$$\begin{aligned} M(x) := & M_1 \cdot S(x, 0) + R \cdot S(x, 0) \cdot (x - 0)^1 - \frac{w_1}{2} \cdot S(x, 0) \cdot (x - 0)^2 + \frac{w_1}{2} \cdot S(x, 0) \cdot (x - 0)^2 \dots \\ & + \frac{-w_2}{2} \cdot S(x, a) \cdot (x - a)^2 + \frac{w_2}{2} \cdot S[x, (a + b)] \cdot [x - (a + b)]^2 - \frac{F_{\text{activator}}}{2} \cdot S(x, a) \cdot (x - a)^2 + \frac{F_{\text{activator}}}{2} \cdot S[x, (a + b)] \cdot [x - (a + b)]^2 \dots \\ & + \frac{-w_3}{2} \cdot S[x, (a + b)] \cdot [x - (a + b)]^2 \end{aligned}$$

$$\theta(x) := \frac{1}{E \cdot I_{zz}(x)} \left[ \begin{aligned} & M_1 \cdot S(x, 0) \cdot (x - 0)^1 + \frac{R}{2} \cdot S(x, 0) \cdot (x - 0)^2 - \frac{w_1}{6} \cdot S(x, 0) \cdot (x - 0)^3 + \frac{w_1}{6} \cdot S(x, 0) \cdot (x - 0)^3 \dots \\ & + \frac{-w_2}{6} \cdot S(x, a) \cdot (x - a)^3 + \frac{w_2}{6} \cdot S[x, (a + b)] \cdot [x - (a + b)]^3 - \frac{F_{\text{activator}}}{6} \cdot S(x, a) \cdot (x - a)^3 \dots \\ & + \frac{F_{\text{activator}}}{6} \cdot S[x, (a + b)] \cdot [x - (a + b)]^3 \dots \\ & + \frac{-w_3}{6} \cdot S[x, (a + b)] \cdot [x - (a + b)]^3 \end{aligned} \right]$$

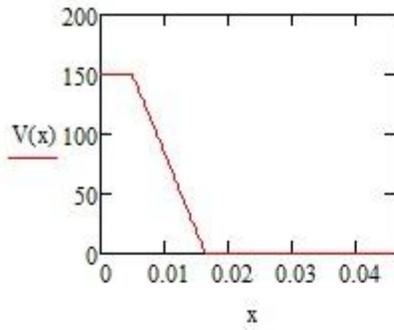
$$y(x) := \frac{1}{E \cdot I_{zz}(x)} \left[ \begin{aligned} & \frac{M_1}{2} \cdot S(x, 0) \cdot (x - 0)^2 + \frac{R}{6} \cdot S(x, 0) \cdot (x - 0)^3 - \frac{w_1}{24} \cdot S(x, 0) \cdot (x - 0)^4 + \frac{w_1}{24} \cdot S(x, a) \cdot (x - a)^4 \dots \\ & + \frac{-w_2}{24} \cdot S(x, a) \cdot (x - a)^4 + \frac{w_2}{24} \cdot S[x, (a + b)] \cdot [x - (a + b)]^4 - \frac{F_{activator}}{24} \cdot S(x, a) \cdot (x - a)^4 \dots \\ & + \frac{F_{activator}}{24} \cdot S[x, (a + b)] \cdot [x - (a + b)]^4 \dots \\ & + \frac{-w_3}{24} \cdot S[x, (a + b)] \cdot [x - (a + b)]^4 \end{aligned} \right]$$



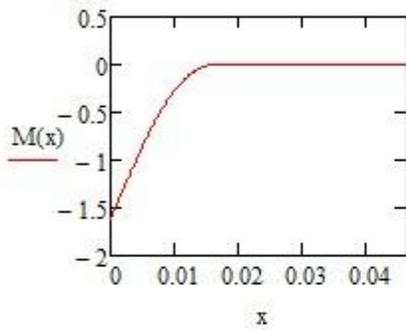
$$y(L) = -5.764 \times 10^{-3} \cdot \text{mm}$$

**Max deflection:**

$$y(L) = -2.269 \times 10^{-4} \cdot \text{in}$$



$$V_{\max} := V(0) = 150.288 \text{ N}$$



$$M_{\max} := |M(0)| = 1.617 \cdot \text{N} \cdot \text{m}$$

#### Determination of lifetime cycles and safety factor:



$$\sigma_x := M_{\max} \cdot \frac{0.5 \cdot d_1}{I_{zz}(0)} = 1.203 \cdot \text{MPa}$$

$$\tau_{\text{shearmax}} := \frac{4}{3} \cdot \frac{V(0)}{A_1} = 0.668 \cdot \text{MPa}$$

$$\sigma_{1\max} := \frac{\sigma_x}{2} + \tau_{\text{shearmax}} = 1.27 \times 10^3 \cdot \text{kPa}$$

$$\sigma_{3\max} := \frac{\sigma_x}{2} - \tau_{\text{shearmax}} = -66.249 \cdot \text{kPa}$$

$$\tau_{13\max} := \frac{\sigma_{1\max} - \sigma_{3\max}}{2} = 667.945 \cdot \text{kPa}$$

### Von Mises Effective Stresses:

$$\sigma'_{\max} := \sqrt{\sigma_{1\max}^2 - \sigma_{1\max} \cdot \sigma_{3\max} + \sigma_{3\max}^2} = 1.304 \times 10^3 \cdot \text{kPa}$$

$$\sigma'_{\min} := 0 \text{ Pa}$$

$$\sigma'_{\text{alt}} := \frac{\sigma'_{\max}}{2} = 652.014 \cdot \text{kPa}$$

$$\sigma'_{\text{mean}} := \frac{\sigma'_{\max}}{2} = 652.014 \cdot \text{kPa}$$

$$S_{\text{ut}} := 518.8 \text{ MPa} \quad \text{AISI 1040 Steel}$$

$$S_y := 353.4 \text{ MPa}$$

$$T_{\text{oper}} := 200$$

$$R_{\text{sw}} := 0.99999$$

$$\text{load} := \text{"bending"}$$

$$\text{surface} := \text{"machined"}$$

$$S_{\text{eun}} := \begin{cases} \text{return } (0.5 \cdot S_{\text{ut}}) & \text{if } S_{\text{ut}} \leq 1400 \text{ MPa} \\ (700 \text{ MPa}) & \text{otherwise} \end{cases}$$

$$C_{\text{load}} := \begin{cases} \text{return } 1 & \text{if load = "bending"} \\ \text{return } 1 & \text{if load = "torsion"} \\ \text{return } 0.7 & \text{if load = "axial"} \end{cases}$$

$$C_{\text{size}} := \begin{cases} \text{return } \left[ 1.189 \cdot \left( \frac{d_1}{\text{m}} \right)^{-0.097} \right] & \text{if } 0.008 \text{ m} < d_1 \leq 0.250 \text{ m} \\ \text{return } 0.6 & \text{if } d_1 > 0.250 \text{ m} \end{cases}$$

$$A_{\text{sw}} := \begin{cases} \text{return } 1.34 & \text{if surface = "ground"} \\ \text{return } 2.70 & \text{if surface = "machined"} \\ \text{return } 2.7 & \text{if surface = "cold_rolled"} \\ \text{return } 14.4 & \text{if surface = "hot_rolled"} \\ \text{return } 39.9 & \text{if surface = "forged"} \end{cases}$$

$$b_{\text{surf}} := \begin{cases} \text{return } (-0.085) & \text{if surface} = \text{"ground"} \\ \text{return } (-0.265) & \text{if surface} = \text{"machined"} \\ \text{return } (-0.265) & \text{if surface} = \text{"cold_rolled"} \\ \text{return } (-0.718) & \text{if surface} = \text{"hot_rolled"} \\ \text{return } (-0.995) & \text{if surface} = \text{"forged"} \end{cases}$$

$$A = 2.7$$

$$b = -0.265$$

$$C_{\text{surface}} := A \cdot \left( \frac{S_{\text{ut}}}{\text{ksi}} \right)^b$$

$$C_{\text{temp}} := \begin{cases} \text{return } 1 & \text{if } T_{\text{oper}} \leq 450 \\ [1 - 0.0032 \cdot (T_{\text{oper}} - 840)] & \text{otherwise} \end{cases}$$

$$C_{\text{reliability}} := \begin{cases} \text{return } 1.000 & \text{if } R = 0.50 \\ \text{return } 0.897 & \text{if } R = 0.90 \\ \text{return } 0.814 & \text{if } R = 0.99 \\ \text{return } 0.753 & \text{if } R = 0.999 \\ \text{return } 0.702 & \text{if } R = 0.9999 \\ \text{return } 0.659 & \text{if } R = 0.99999 \end{cases}$$

$$C_{\text{total}} := C_{\text{load}} \cdot C_{\text{size}} \cdot C_{\text{surface}} \cdot C_{\text{temp}} \cdot C_{\text{reliability}} = 1.012$$

$$S_e := C_{\text{total}} \cdot S_{\text{eun}} = 262.454 \cdot \text{MPa}$$

$$S_m := \begin{cases} \text{return } (0.75 \cdot S_{\text{ut}}) & \text{if load} = \text{"axial"} \\ (0.9 \cdot S_{\text{ut}}) & \text{otherwise} \end{cases}$$

$$S_m = 466.92 \cdot \text{MPa}$$

$$z := -3$$

$$b_{\text{ax}} := \frac{1}{z} \cdot \log \left( \frac{S_m}{S_e} \right) = -0.083$$

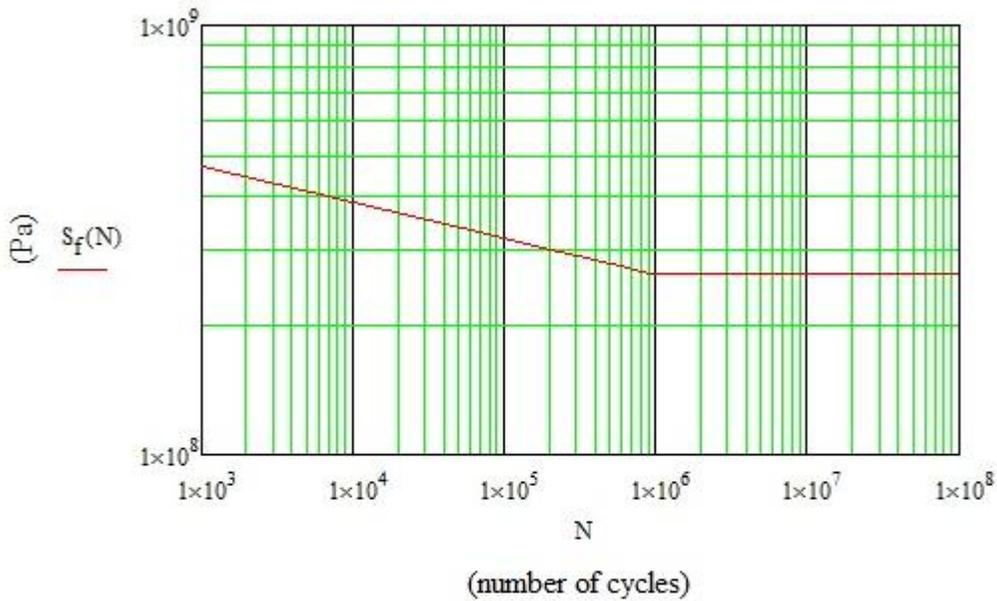
$$a_{\text{ax}} := \frac{S_m}{1000^{b_{\text{ax}}}} = 830.677 \cdot \text{MPa}$$

$$S_f = a \cdot N^b$$

Graphing the S-N diagram:

$$N := 10^3, 10^5 \dots 10^8$$

$$S_f(N) := \begin{cases} \text{return } a \cdot N^b & \text{if } N < 10^6 \\ S_e & \text{otherwise} \end{cases}$$



$$N := 10^6$$

Given

$$\sigma'_{\text{mean}} = a \cdot N^b$$

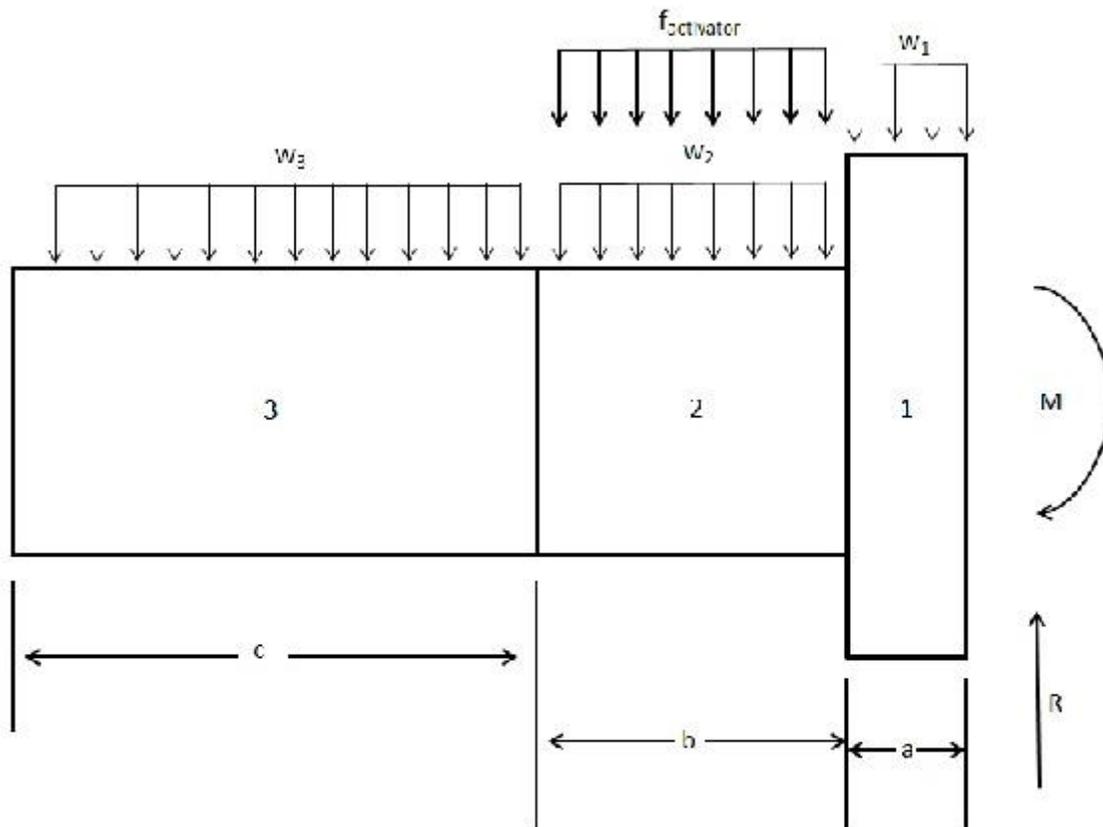
$$\text{Find}(N) = 1.713 \times 10^{37}$$

$$\text{Safety factor} := \frac{S_e \cdot S_{\text{ut}}}{\sigma'_{\text{alt}} \cdot S_{\text{ut}} + \sigma'_{\text{mean}} \cdot S_e} = 267.303$$

## Bottom Arm:

### Drawings :

Front View:



$M$  = moment reaction at pin

$R$  = force reaction at pin

$w_1$  = weight of section 1

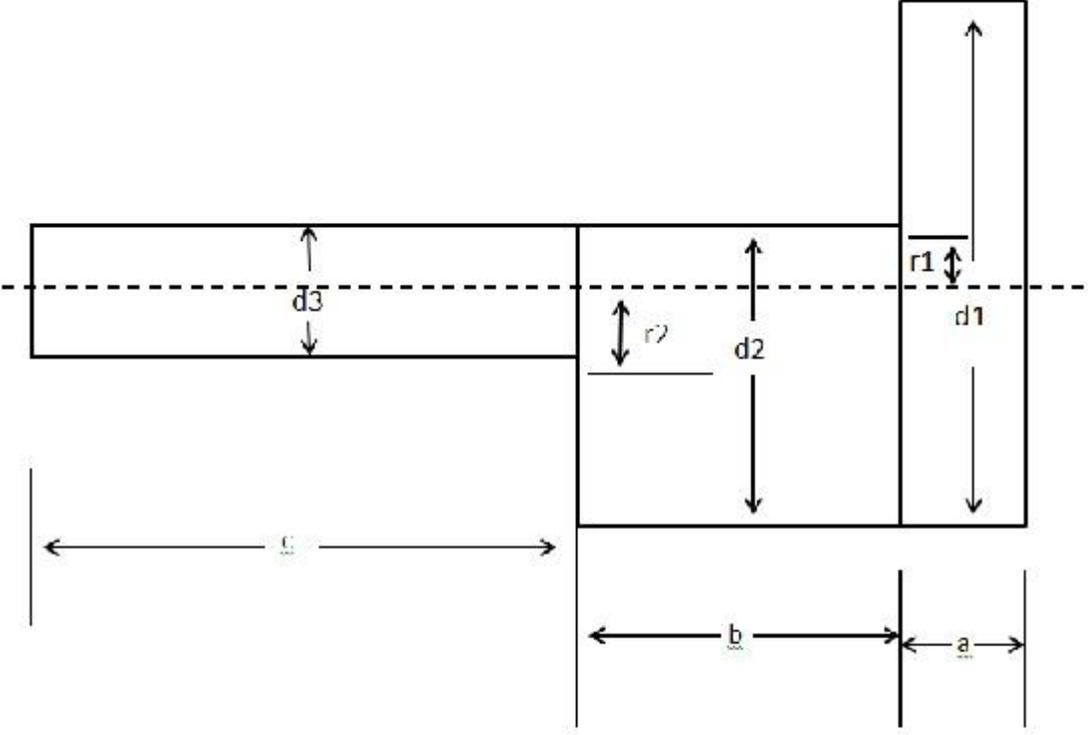
$w_2$  = weight of section 2

$f_{\text{activator}}$  = force due to the activator opening the arms

$w_3$  = weight of section 3

$L$  = total length ( $a+b+c$ )

Top View:



$d_1$  = thickness of section 1

$r_1$  = distance from center of mass of section 1 to the z-axis (dotted line)

$d_2$  = thickness of section 2

$r_2$  = distance from center of mass of section 2 to the z-axis (dotted line)

$d_3$  = thickness of section 3

## Initial Values:



$$\rho_{\text{steel}} := 7850 \frac{\text{kg}}{\text{m}^3} \quad \text{density of steel}$$

$$E := 210\text{GPa} \quad \text{modulus of elasticity of steel}$$

$$G := 75\text{GPa} \quad \text{modulus of rigidity of steel}$$

$$a := 5\text{mm}$$

$$b := 21.5\text{mm}$$

$$c := 20\text{mm}$$

$$L := a + b + c = 46.5\text{mm}$$

$$h_1 := 20\text{mm} \quad \text{height of section 1}$$

$$h_2 := 10\text{mm} \quad \text{height of section 2}$$

$$h_3 := h_2 = 10\text{mm} \quad \text{height of section 3}$$

$$d_1 := 15\text{mm}$$

$$d_2 := 9\text{mm}$$

$$d_3 := 4\text{mm}$$

$$r_1 := 0.5\text{mm}$$

$$r_2 := 2.5\text{mm}$$

$$f_{\text{activator}} := 150\text{N}$$

### Cross sectional areas:

$$A_1 := h_1 \cdot d_1 = 300\text{mm}^2$$

$$A_2 := h_2 \cdot d_2 = 90\text{mm}^2$$

$$A_3 := h_3 \cdot d_3 = 40\text{mm}^2$$

### mass of each section:

$$m_1 := A_1 \cdot a \cdot \rho_{\text{steel}} = 0.012\text{kg}$$

$$m_2 := A_2 \cdot b \cdot \rho_{\text{steel}} = 0.015\text{kg}$$

$$m_3 := A_3 \cdot c \cdot \rho_{\text{steel}} = 6.28 \times 10^{-3}\text{kg}$$

weight per unit length of each section:

$$w_1 := \rho_{\text{steel}} \cdot A_1 \cdot g = 23.095 \cdot \frac{\text{N}}{\text{m}}$$

$$w_2 := \rho_{\text{steel}} \cdot A_2 \cdot g = 6.928 \cdot \frac{\text{N}}{\text{m}}$$

$$w_3 := \rho_{\text{steel}} \cdot A_3 \cdot g = 3.079 \cdot \frac{\text{N}}{\text{m}}$$

force per unit length due to activiator:

$$F_{\text{activator}} := \frac{f_{\text{activator}}}{b} = 6.977 \times 10^3 \cdot \frac{\text{N}}{\text{m}}$$

Moments of Inertia:

$$I_{zz}(x) := \begin{cases} \frac{d_1 \cdot h_1^3}{12} + A_1 \cdot r_1^2 & \text{if } 0 \leq x \leq a \\ \frac{d_2 \cdot h_2^3}{12} + A_2 \cdot r_2^2 & \text{if } a < x \leq a + b \\ \frac{d_3 \cdot h_3^3}{12} & \text{if } a + b < x \leq L \end{cases}$$

Solving for reaction force and moment:

$$\underline{\underline{R}} := 0 \quad \underline{\underline{M}}_1 := 0$$

Given

$$R - w_1 \cdot a - w_2 \cdot b - F_{\text{activator}} \cdot b - w_3 \cdot c = 0$$

$$\underline{\underline{R}} := \text{Find}(R) = 150.326 \text{ N}$$

Given

$$M_1 + w_1 \cdot a \cdot \left(\frac{a}{2}\right) + w_2 \cdot b \cdot \left(a + \frac{b}{2}\right) + F_{\text{activator}} \cdot b \cdot \left(a + \frac{b}{2}\right) + w_3 \cdot c \cdot \left(a + b + \frac{c}{2}\right) = 0$$

$$\underline{\underline{M}}_1 := \text{Find}(M_1) = -2.367 \cdot \text{N} \cdot \text{m}$$

## Singularity Functions:



$$S(x, z) := \text{if}(x \geq z, 1, 0) \quad C_1 := 0 \quad C_2 := 0 \quad C_3 := 0 \quad C_4 := 0$$

$$x := 0, 0.005 \cdot L \dots L$$

+

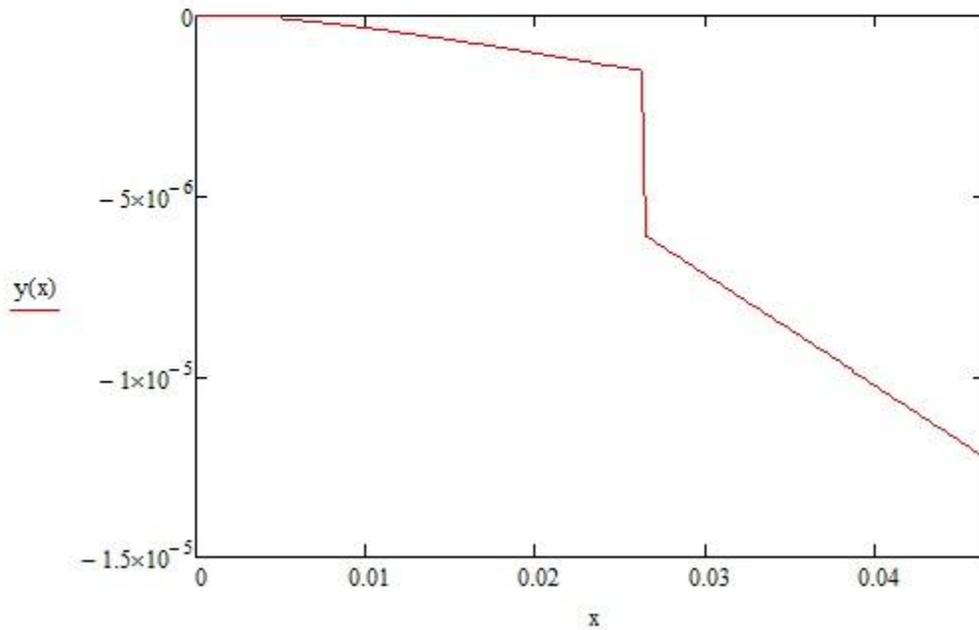
$$\begin{aligned} q(x) := & -w_1 \cdot S(x, 0) + w_1 \cdot S(x, 0) \dots \\ & + -w_2 \cdot S(x, a) + w_2 \cdot S[x, (a + b)] - F_{\text{activator}} \cdot S(x, a) \dots \\ & + F_{\text{activator}} \cdot S[x, (a + b)] \dots \\ & + -w_3 \cdot S[x, (a + b)] \end{aligned}$$

$$\begin{aligned} V(x) := & R \cdot S(x, 0) - w_1 \cdot S(x, 0) \cdot (x - 0)^1 + w_1 \cdot S(x, 0) \cdot (x - 0)^1 \dots \\ & + -w_2 \cdot S(x, a) \cdot (x - a)^1 + w_2 \cdot S[x, (a + b)] \cdot [x - (a + b)]^1 - F_{\text{activator}} \cdot S(x, a) \cdot (x - a)^1 \dots \\ & + F_{\text{activator}} \cdot S[x, (a + b)] \cdot [x - (a + b)]^1 \dots \\ & + -w_3 \cdot S[x, (a + b)] \cdot [x - (a + b)]^1 \end{aligned}$$

$$\begin{aligned} M(x) := & M_1 \cdot S(x, 0) + R \cdot S(x, 0) \cdot (x - 0)^1 - \frac{w_1}{2} \cdot S(x, 0) \cdot (x - 0)^2 + \frac{w_1}{2} \cdot S(x, 0) \cdot (x - 0)^2 \dots \\ & + \frac{-w_2}{2} \cdot S(x, a) \cdot (x - a)^2 + \frac{w_2}{2} \cdot S[x, (a + b)] \cdot [x - (a + b)]^2 - \frac{F_{\text{activator}}}{2} \cdot S(x, a) \cdot (x - a)^2 + \frac{F_{\text{activator}}}{2} \cdot S[x, (a + b)] \cdot [x - (a + b)]^2 \dots \\ & + \frac{-w_3}{2} \cdot S[x, (a + b)] \cdot [x - (a + b)]^2 \end{aligned}$$

$$\theta(x) := \frac{1}{E \cdot I_{zz}(x)} \left[ \begin{aligned} & M_1 \cdot S(x, 0) \cdot (x - 0)^1 + \frac{R}{2} \cdot S(x, 0) \cdot (x - 0)^2 - \frac{w_1}{6} \cdot S(x, 0) \cdot (x - 0)^3 + \frac{w_1}{6} \cdot S(x, 0) \cdot (x - 0)^3 \dots \\ & + \frac{-w_2}{6} \cdot S(x, a) \cdot (x - a)^3 + \frac{w_2}{6} \cdot S[x, (a + b)] \cdot [x - (a + b)]^3 - \frac{F_{\text{activator}}}{6} \cdot S(x, a) \cdot (x - a)^3 \dots \\ & + \frac{F_{\text{activator}}}{6} \cdot S[x, (a + b)] \cdot [x - (a + b)]^3 \dots \\ & + \frac{-w_3}{6} \cdot S[x, (a + b)] \cdot [x - (a + b)]^3 \end{aligned} \right]$$

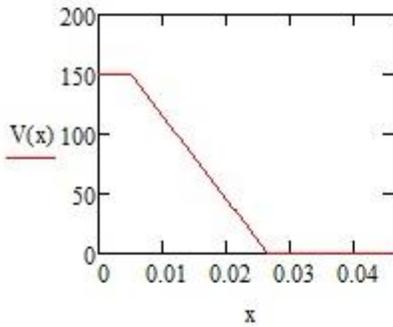
$$y(x) := \frac{1}{E \cdot I_{zz}(x)} \left[ \begin{aligned} & \frac{M_1}{2} \cdot S(x, 0) \cdot (x - 0)^2 + \frac{R}{6} \cdot S(x, 0) \cdot (x - 0)^3 - \frac{w_1}{24} \cdot S(x, 0) \cdot (x - 0)^4 + \frac{w_1}{24} \cdot S(x, a) \cdot (x - a)^4 \dots \\ & + \frac{-w_2}{24} \cdot S(x, a) \cdot (x - a)^4 + \frac{w_2}{24} \cdot S[x, (a + b)] \cdot [x - (a + b)]^4 - \frac{F_{activator}}{24} \cdot S(x, a) \cdot (x - a)^4 \dots \\ & + \frac{F_{activator}}{24} \cdot S[x, (a + b)] \cdot [x - (a + b)]^4 \dots \\ & + \frac{-w_3}{24} \cdot S[x, (a + b)] \cdot [x - (a + b)]^4 \end{aligned} \right]$$



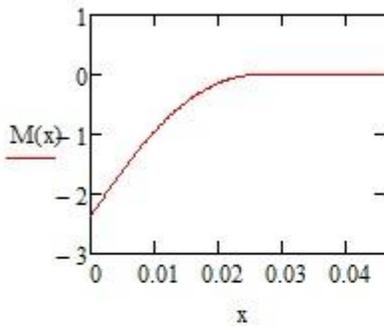
**Max deflection:**

$$y(L) = -0.012 \cdot \text{mm}$$

$$y(L) = -4.829 \times 10^{-4} \cdot \text{in}$$



$$V_{\max} := V(0) = 150.326 \text{ N}$$



$$M_{\max} := |M(0)| = 2.367 \text{ N}\cdot\text{m}$$

#### Determination of lifetime cycles and safety factor:



$$\sigma_x := M_{\max} \cdot \frac{0.5 \cdot d_1}{I_{zz}(0)} = 1.762 \text{ MPa}$$

$$\tau_{\text{shearmax}} := \frac{4}{3} \cdot \frac{V(0)}{A_1} = 0.668 \text{ MPa}$$

$$\sigma_{1\max} := \frac{\sigma_x}{2} + \tau_{\text{shearmax}} = 1.549 \times 10^3 \text{ kPa}$$

$$\sigma_{3\max} := \frac{\sigma_x}{2} - \tau_{\text{shearmax}} = 213.044 \text{ kPa}$$

$$\tau_{13\max} := \frac{\sigma_{1\max} - \sigma_{3\max}}{2} = 668.116 \text{ kPa}$$

#### Von Mises Effective Stresses:

$$\sigma'_{\max} := \sqrt{\sigma_{1\max}^2 - \sigma_{1\max} \cdot \sigma_{3\max} + \sigma_{3\max}^2} = 1.455 \times 10^3 \text{ kPa}$$

$$\sigma'_{\min} := 0 \text{ Pa}$$

$$\sigma'_{\text{alt}} := \frac{\sigma'_{\max}}{2} = 727.251 \cdot \text{kPa}$$

$$\sigma'_{\text{mean}} := \frac{\sigma'_{\max}}{2} = 727.251 \cdot \text{kPa}$$

$$S_{\text{ut}} := 518.8 \text{ MPa} \quad \text{AISI 1040 Steel}$$

$$S_y := 353.4 \text{ MPa}$$

$$T_{\text{oper}} := 200$$

$$R_{\text{w}} := 0.99999$$

$$\text{load} := \text{"bending"}$$

$$\text{surface} := \text{"machined"}$$

$$S_{\text{eun}} := \begin{cases} \text{return } (0.5 \cdot S_{\text{ut}}) & \text{if } S_{\text{ut}} \leq 1400 \text{ MPa} \\ (700 \text{ MPa}) & \text{otherwise} \end{cases}$$

$$C_{\text{load}} := \begin{cases} \text{return } 1 & \text{if load = "bending"} \\ \text{return } 1 & \text{if load = "torsion"} \\ \text{return } 0.7 & \text{if load = "axial"} \end{cases}$$

$$C_{\text{size}} := \begin{cases} \text{return } \left[ 1.189 \cdot \left( \frac{d_1}{\text{m}} \right)^{-0.097} \right] & \text{if } 0.008 \text{ m} < d_1 \leq 0.250 \text{ m} \\ \text{return } 0.6 & \text{if } d_1 > 0.250 \text{ m} \end{cases}$$

$$A_{\text{w}} := \begin{cases} \text{return } 1.34 & \text{if surface = "ground"} \\ \text{return } 2.70 & \text{if surface = "machined"} \\ \text{return } 2.7 & \text{if surface = "cold_rolled"} \\ \text{return } 14.4 & \text{if surface = "hot_rolled"} \\ \text{return } 39.9 & \text{if surface = "forged"} \end{cases}$$

$$b_{\text{surf}} := \begin{cases} \text{return } (-0.085) & \text{if surface} = \text{"ground"} \\ \text{return } (-0.265) & \text{if surface} = \text{"machined"} \\ \text{return } (-0.265) & \text{if surface} = \text{"cold_rolled"} \\ \text{return } (-0.718) & \text{if surface} = \text{"hot_rolled"} \\ \text{return } (-0.995) & \text{if surface} = \text{"forged"} \end{cases}$$

$$A = 2.7$$

+

$$b = -0.265$$

$$C_{\text{surface}} := A \cdot \left( \frac{S_{\text{ut}}}{\text{ksi}} \right)^b$$

$$C_{\text{temp}} := \begin{cases} \text{return } 1 & \text{if } T_{\text{oper}} \leq 450 \\ [1 - 0.0032 \cdot (T_{\text{oper}} - 840)] & \text{otherwise} \end{cases}$$

$$C_{\text{reliability}} := \begin{cases} \text{return } 1.000 & \text{if } R = 0.50 \\ \text{return } 0.897 & \text{if } R = 0.90 \\ \text{return } 0.814 & \text{if } R = 0.99 \\ \text{return } 0.753 & \text{if } R = 0.999 \\ \text{return } 0.702 & \text{if } R = 0.9999 \\ \text{return } 0.659 & \text{if } R = 0.99999 \end{cases}$$

$$C_{\text{total}} := C_{\text{load}} \cdot C_{\text{size}} \cdot C_{\text{surface}} \cdot C_{\text{temp}} \cdot C_{\text{reliability}} = 1.012$$

$$S_e := C_{\text{total}} \cdot S_{\text{eun}} = 262.454 \cdot \text{MPa}$$

$$S_m := \begin{cases} \text{return } (0.75 \cdot S_{\text{ut}}) & \text{if load} = \text{"axial"} \\ (0.9 \cdot S_{\text{ut}}) & \text{otherwise} \end{cases}$$

$$S_m = 466.92 \cdot \text{MPa}$$

$$z := -3$$

$$b_{\text{surf}} := \frac{1}{z} \cdot \log \left( \frac{S_m}{S_e} \right) = -0.083$$

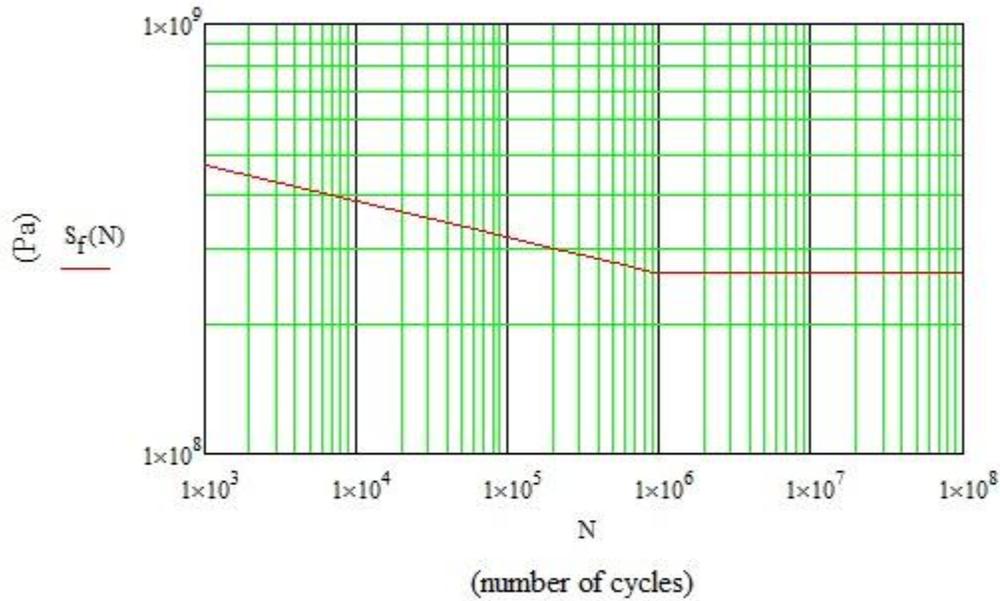
$$a_{\text{surf}} := \frac{S_m}{1000^{b_{\text{surf}}}} = 830.677 \cdot \text{MPa}$$

$$S_f = a \cdot N^b$$

Graphing the S-N diagram:

$$N_{\text{min}} = 10^3, 10^5 \dots 10^8$$

$$S_f(N) := \begin{cases} \text{return } a \cdot N^b & \text{if } N < 10^6 \\ S_e & \text{otherwise} \end{cases}$$



$$N = 10^6$$

Given

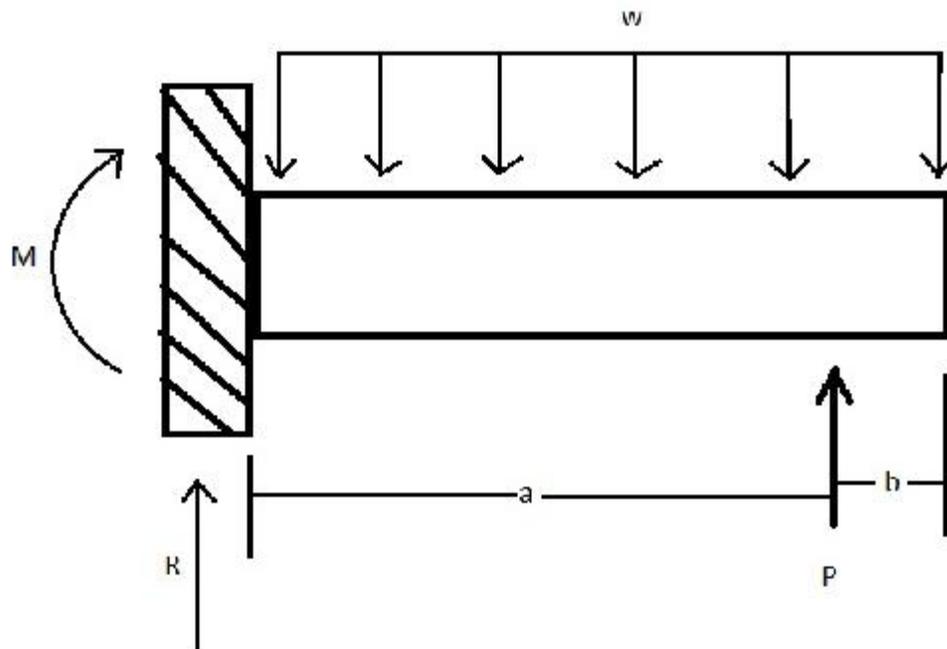
$$\sigma'_{\text{mean}} = a \cdot N^b$$

$$\text{Find}(N) = 4.625 \times 10^{36}$$

$$\text{Safety factor} := \frac{S_e \cdot S_{\text{ut}}}{\sigma'_{\text{alt}} \cdot S_{\text{ut}} + \sigma'_{\text{mean}} \cdot S_e} = 239.649$$

## Activator – Bending due to Leveling Slider

**Drawing:**



$w$  = weight of the bar

$P$  = force of the spring compressing due to the activator opening the gripper arms

$R$  = force reaction at the center of the activator

$M$  = moment reaction at the center of the activator

$a$  = distance from the center of the activator to the center of the leveling slider

$b$  = distance from the center of the leveling slider to the end of the leveling slider

$L$  = total length of the the beam ( $a + b$ )

$h$  = height of the bar

$d$  = depth of the bar

## Initial Values:



$$\rho_{\text{steel}} := 7850 \frac{\text{kg}}{\text{m}^3} \quad \text{density of steel}$$

$$E := 207\text{GPa} \quad \text{modulus of elasticity of steel}$$

$$\underline{\underline{G}} := 70\text{GPa} \quad \text{modulus of rigidity of steel}$$

$$a := 25.351425\text{mm}$$

$$b := 4.768375\text{mm}$$

$$\underline{\underline{L}} := a + b = 30.12\text{mm}$$

$$h := 12\text{mm}$$

$$d := 31\text{mm}$$

$$\underline{\underline{A}} := h \cdot d = 372\text{mm}^2 \quad \text{cross sectional area}$$

$$w := \rho_{\text{steel}} \cdot A \cdot g = 28.637 \frac{\text{N}}{\text{m}}$$

$$I_{zz} := \frac{d \cdot h^3}{12} = 4.464 \times 10^3 \text{mm}^4$$

$$P_{\text{max}} := 150\text{N}$$

### Solving for reaction forces and moments:

$$\underline{\underline{R}} := 0$$

Given

$$-w \cdot L + R + P_{\text{max}} = 0$$

$$\underline{\underline{R}} := \text{Find}(R) = -149.137\text{N}$$

$$M_1 := 0$$

Given

$$-M_1 + P_{\text{max}} \cdot a - w \cdot L \cdot \frac{L}{2} = 0$$

$$\underline{\underline{M_1}} := \text{Find}(M_1) = 3.79\text{N}\cdot\text{m}$$

## Singularity Functions:



$$S(x, z) := \text{if}(x \geq z, 1, 0)$$

$$x := 0, 0.005 \cdot L \dots L$$

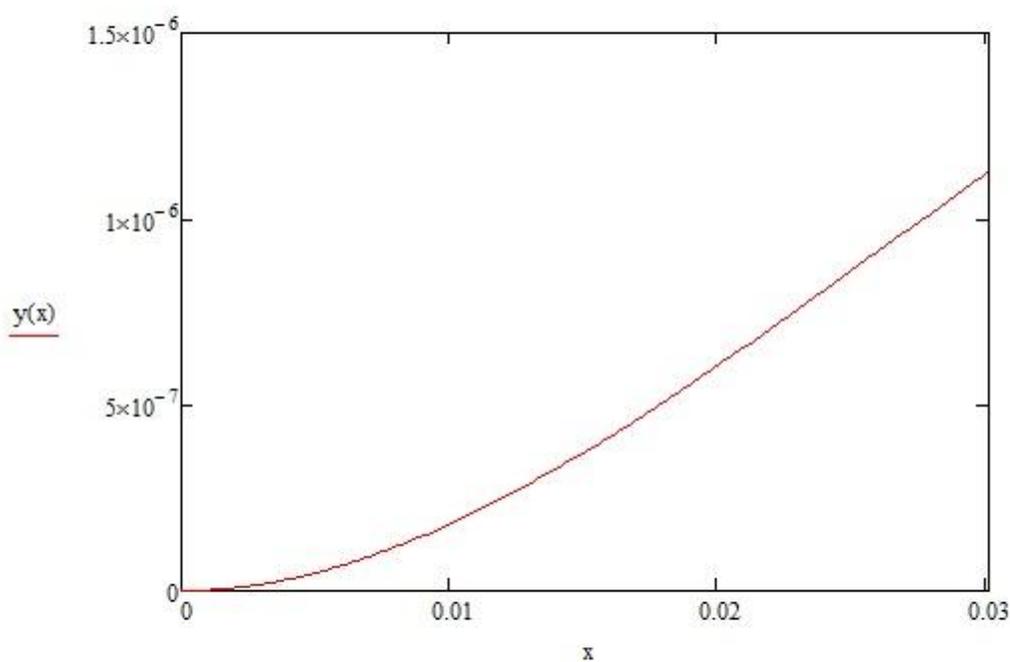
$$q(x) := -w \cdot S(x, 0)$$

$$V(x) := R \cdot S(x, 0) - w \cdot S(x, 0) \cdot (x - 0)^1 + P_{\max} \cdot S(x, a)$$

$$M(x) := M_1 \cdot S(x, 0) + R \cdot S(x, 0) \cdot (x - 0)^1 - \frac{w}{2} \cdot S(x, 0) \cdot (x - 0)^2 + P_{\max} \cdot S(x, a) \cdot (x - a)^1$$

$$\theta(x) := \frac{1}{E \cdot I_{zz}} \left[ M_1 \cdot S(x, 0) \cdot (x - 0)^1 + \frac{R}{2} \cdot S(x, 0) \cdot (x - 0)^2 - \frac{w}{6} \cdot S(x, 0) \cdot (x - 0)^3 + \frac{P_{\max}}{2} \cdot S(x, a) \cdot (x - a)^2 \right]$$

$$y(x) := \frac{1}{E \cdot I_{zz}} \left[ \frac{M_1}{2} \cdot S(x, 0) \cdot (x - 0)^2 + \frac{R}{6} \cdot S(x, 0) \cdot (x - 0)^3 - \frac{w}{24} \cdot S(x, 0) \cdot (x - 0)^4 + \frac{P_{\max}}{6} \cdot S(x, a) \cdot (x - a)^3 \right]$$



maximum deflection:

$$y(L) = 1.127 \times 10^{-3} \cdot \text{mm}$$

$$y(L) = 4.438 \times 10^{-5} \cdot \text{in}$$

+

## Appendix B: Standard Parts

### Bearings

All bearings selected are from SKF Group.

#### Outside gear arm bearing

<http://www.skf.com/skf/productcatalogue/Forwarder?action=PPP&lang=en&imperial=false&windowName=null&perfid=105001&prodid=1050010203>

Deep groove ball bearings, single row, unsealed							Tolerances, see also text		Radial internal clearance, see also text	
Product information							Recommended fits		Shaft and housing tolerances	
Principal dimensions			Basic load ratings		Fatigue load limit $P_u$	Speed ratings		Mass	Designation	
d	D	B	dynamic	static		Reference speed	Limiting speed			
mm			kN		kN	r/min		kg	-	
17	40	12	9,95	4,75	0,2	38000	24000	0,065	6203	

Technical drawing of a deep groove ball bearing showing dimensions:  $D = 40$ ,  $D_2 = 35$ ,  $B = 12$ ,  $d = 17$ ,  $d_1 = 24,5$ ,  $r_{1,2min} = 0,6$ .

Technical drawing of a deep groove ball bearing showing dimensions:  $D_{amax} = 35,8$ ,  $r_{amax} = 0,6$ ,  $d_{amin} = 21,2$ .

**Calculation factors**

$k_r$  0,025

$f_0$  13

## Inside gear arm bearing

<http://www.skf.com/skf/productcatalogue/Forwarder?action=PPP&lang=en&imperial=false&windowName=null&perfid=105001&prodid=1050010004>

Deep groove ball bearings, single row, unsealed								Product information	
Principal dimensions			Basic load ratings		Fatigue load limit	Speed ratings		Mass	Designation
d	D	B	C	C <sub>0</sub>	P <sub>u</sub>	Reference speed	Limiting speed		
mm			kN		kN	r/min		kg	-
20	42	12	9,95	5	0,212	38000	24000	0,069	6004

Technical drawing of a deep groove ball bearing showing dimensions: D=42, B=12, d=20, d<sub>1</sub>=27,2, D<sub>2</sub>=37,2, r<sub>1,2min</sub>=0,6.

Technical drawing of a deep groove ball bearing showing dimensions: D<sub>amax</sub>=38,8, r<sub>amax</sub>=0,6, d<sub>amin</sub>=23,2.

**Calculation factors**

k<sub>r</sub> 0,025  
f<sub>0</sub> 14

## Stationary gear bearing

<http://www.skf.com/skf/productcatalogue/Forwarder?action=PPP&lang=en&imperial=false&windowName=null&perfid=167031&prodid=1670310070>

<b>Angular contact thrust ball bearings, double direction, high-precision, BTM..A design, 30°, hybrid, preload A</b> Product information <span style="float: right;">Tolerances, 2344(00) design, BTM design Recommended fits, shafts, housings Shaft and housing tolerances</span>										
Principal dimensions				Basic load ratings		Fatigue load limit $P_u$	Attainable speeds when lubricating with grease oil-air		Mass	Designation
d	D	H	C	C	$C_0$					
mm				kN		kN	r/min		kg	-
70	110	36	-	30	64	2,75	9500	13000	1,05	BTM 70 A/HCP4CDBA

Technical drawing of the bearing showing dimensions:  $D = 110$ ,  $D_1 = 104$ ,  $H = 36$ ,  $d = 70$ ,  $d_1 = 88,6$ ,  $r_{1,2min} = 1,1$ ,  $r_{3,4min} = 0,6$ .

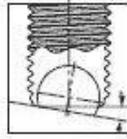
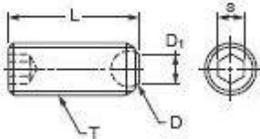
Technical drawing of the bearing showing dimensions:  $D_a = 104$ ,  $r_{bmax} = 0,6$ ,  $r_{a\ max} = 1$ ,  $d_{amin} = 82$ .

**Preload when unmounted [N]**  
250

## Fasteners

### Set screw

[http://www.catalogs.com/db/service?domain=amsp&command=productList&category=ref\\_no\\_table\\_9\\_56](http://www.catalogs.com/db/service?domain=amsp&command=productList&category=ref_no_table_9_56)



The ball can rotate up to 9° to securely fasten the object.

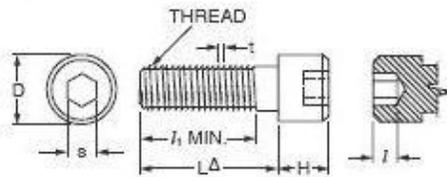


- **SCREW** – Carbon Steel, Black Oxide Finish; 304 Stainless Steel
- **BALL** – Carbon Steel, HRC 56...60; 440C Stainless Steel, HRC 55...60

Catalog Number		T	L	D	D <sub>1</sub>	s	Mass
Carbon Steel	Stainless Steel	Thread Size	Length			Hex Size	g
MKAS9MMC0406	MKAS9MMY0406		6				0.4
MKAS9MMC0410	MKAS9MMY0410	M4 X 0.7	10	2.5	2	2	0.7
MKAS9MMC0416	MKAS9MMY0416		16				1
MKAS9MMC0508	MKAS9MMY0508		8				0.8
MKAS9MMC0512	MKAS9MMY0512	M5 X 0.8	12	3	2.5	2.5	1.3
MKAS9MMC0520	MKAS9MMY0520		20				2.3
MKAS9MMC0610	MKAS9MMY0610		10				1.5
MKAS9MMC0616	MKAS9MMY0616	M6 X 1	16	4	3.2	3	2.5
MKAS9MMC0620	MKAS9MMY0620		20				3.4
MKAS9MMC0625	MKAS9MMY0625		25				4
MKAS9MMC0810	MKAS9MMY0810		10				2.5
MKAS9MMC0812	MKAS9MMY0812	M8 X 1.25	12	5.5	4.5	4	3.2
MKAS9MMC0820	MKAS9MMY0820		20				5.7
MKAS9MMC0825	MKAS9MMY0825		25				7.7
MKAS9MMC0830	MKAS9MMY0830		30				9
MKAS9MMC1012	MKAS9MMY1012		12				5
MKAS9MMC1016	MKAS9MMY1016	M10 X 1.5	16	7	6	5	7
MKAS9MMC1020	MKAS9MMY1020		20				9.5
MKAS9MMC1025	MKAS9MMY1025		25				11
MKAS9MMC1035	MKAS9MMY1035		35				16
MKAS9MMC1216	MKAS9MMY1216		16				10
MKAS9MMC1220	MKAS9MMY1220	M12 X 1.75	20	8.5	7.2	6	12.5
MKAS9MMC1230	MKAS9MMY1230		30				20
MKAS9MMC1240	MKAS9MMY1240		40				28
MKAS9MMC1620	MKAS9MMY1620		20				22
MKAS9MMC1625	MKAS9MMY1625	M16 X 2	25	12	10.7	8	28
MKAS9MMC1635	MKAS9MMY1635		35				41
MKAS9MMC1650	MKAS9MMY1650		50				48

## Socket head cap screws

[http://www.catalogds.com/db/service?domain=amsp&command=productList&category=ref\\_no\\_table\\_9\\_31](http://www.catalogds.com/db/service?domain=amsp&command=productList&category=ref_no_table_9_31)



• SCREW – 304 Stainless Steel • DIN 912

Priced Per 100 Pieces

Catalog Number	Thread Size	L <sup>Δ</sup> Length	t	D	H	s	l	I <sub>1</sub> Min.
*MD0912MXO016X003	M1.6	3	0.35	3	1.7	1.5	0.7	15
*MD0912MXO016X004		4						
*MD0912MXO016X005		5						
*MD0912MXO016X006		6						
*MD0912MXO016X008		8						
MD0912MXO020X004	M2	4	0.4	3.8	2	1.5	1	16
MD0912MXO020X005		5						
MD0912MXO020X006		6						
MD0912MXO020X008		8						
MD0912MXO020X010		10						
MD0912MXO025X006	M2.5	6	0.45	4.5	2.5	2	1.1	17
MD0912MXO025X008		8						
MD0912MXO025X010		10						
MD0912MXO025X012		12						
MD0912MXO030X008		8						
MD0912MXO030X010	M3	10	0.5	5.5	3	2.5	1.3	18
MD0912MXO030X012		12						
MD0912MXO030X016		16						
MD0912MXO040X008		8						
MD0912MXO040X010	M4	10	0.7	7	4	3	2	20
MD0912MXO040X012		12						
MD0912MXO040X016		16						
MD0912MXO040X025		25						
MD0912MXO040X030		30						

\*M1.6 material is 316 Stainless Steel.

Δ Screw is fully threaded unless L length is greater than I<sub>1</sub>.

Continued on the next page

## Square Tubing

<http://www.metricmetal.com/products/sq2395.htm>

SIZE mm	EIGHT kg/mm	EST. LBS. PER FT.
10 x 10 x 1.00	0.276	0.19
10 x 10 x 1.25	0.333	0.22
10 x 10 x 1.50	0.385	0.26
12 x 12 x 1.00	0.339	0.23
12 x 12 x 1.25	0.411	0.28
12 x 12 x 1.50	0.479	0.32
15 x 15 x 1.00	0.433	0.29
15 x 15 x 1.25	0.529	0.36
15 x 15 x 1.50	0.621	0.42
15 x 15 x 2.00	0.789	0.53
16 x 16 x 1.50	0.668	0.45
16 x 16 x 2.00	0.852	0.57
18 x 18 x 1.00	0.517	0.35
18 x 18 x 1.50	0.762	0.51
18 x 18 x 2.00	0.978	0.66
20 x 20 x 1.00	0.59	0.40
20 x 20 x 1.25	0.725	0.49
20 x 20 x 1.50	0.856	0.58
20 x 20 x 2.00	1.103	0.74
20 x 20 x 2.50	1.332	0.90
22 x 22 x 1.50	0.95	0.64
22 x 22 x 2.00	1.229	0.83
25 x 25 x 1.00	0.747	0.50
25 x 25 x 1.25	0.922	0.62
25 x 25 x 1.50	1.092	0.73
25 x 25 x 2.00	1.417	0.95
25 x 25 x 2.50	1.724	1.16
25 x 25 x 3.00	2.07	1.39
30 x 30 x 1.00	0.904	0.61
30 x 30 x 1.25	1.118	0.75
30 x 30 x 1.50	1.327	0.89
30 x 30 x 2.00	1.731	1.16
30 x 30 x 2.50	2.117	1.42
30 x 30 x 3.00	2.483	1.67
30 x 30 x 4.00	3.32	2.23
34 x 34 x 1.50	1.516	1.02
34 x 34 x 2.00	1.983	1.33
34 x 34 x 2.50	2.431	1.64
34 x 34 x 3.00	2.86	1.92
35 x 35 x 1.25	1.314	0.88

## Solenoid

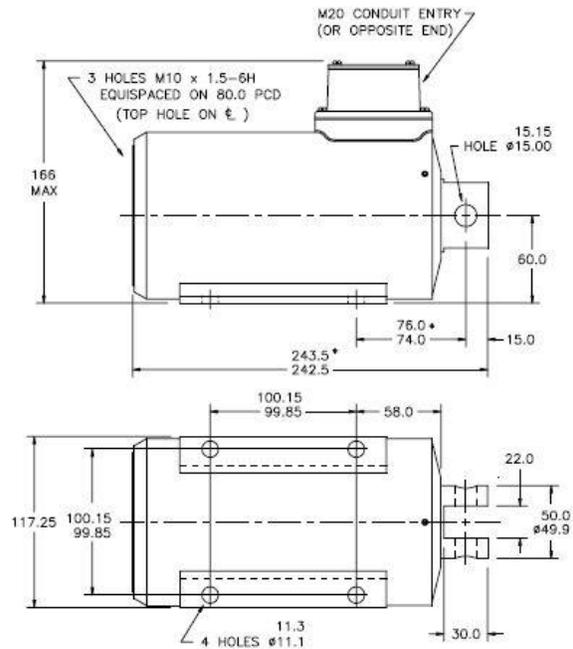
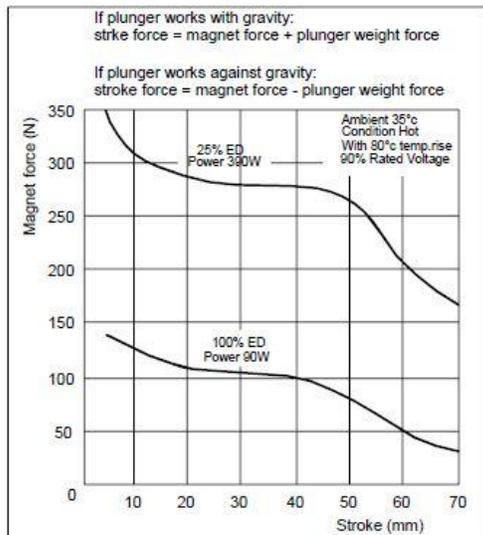
<http://www.mechtronics.co.uk/solenoids-tubular.html>

# Tubular Solenoid (Model M115)

**MT50205**



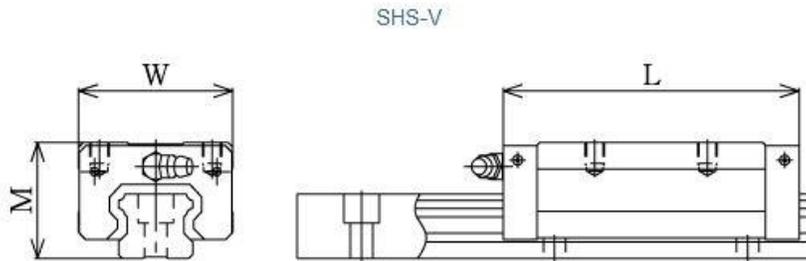
- High Force / Long Stroke
- Fully Enclosed Construction with Potted Coil
- Pull & Thrust Options
- Anti-Residual Disc as Standard
- Alternative Finishes
- Alternative Terminations Available
- Optional Return Springs



\* DIMENSIONS WITH SOLENOID ENERGISED. TOLERANCES (UNLESS STATED)  $\pm 0.25$ mm.

## Linear Motion Rail

[https://tech.thk.com/en/products/thk\\_cat\\_main\\_fourth.php?id=1103](https://tech.thk.com/en/products/thk_cat_main_fourth.php?id=1103)



Dimensional drawing

Model No.	Outer dimensions			Basic load rating						Detail specifications	CAD		Dimensional drawing
	Height M	Width W	Length L	Radial		Reverse radial		Side			2D (DXF)	3D	
				Dynamic rating C	Static rating C <sub>0</sub>	Dynamic rating C <sub>L</sub>	Static rating C <sub>0L</sub>	Dynamic rating C <sub>T</sub>	Static rating C <sub>0T</sub>				
mm	mm	mm	kN	kN	kN	kN	kN	kN	kN				
SHS 15V	24	34	64.4	14.2	24.2	14.2	24.2	14.2	24.2	○			
SHS 20V	30	44	79	22.3	38.4	22.3	38.4	22.3	38.4	○			
SHS 25V	36	48	92	31.7	52.4	31.7	52.4	31.7	52.4	○			
SHS 30V	42	60	106	44.8	66.6	44.8	66.6	44.8	66.6	○			
SHS 35V	48	70	122	62.3	96.6	62.3	96.6	62.3	96.6	○			
SHS 45V	60	86	140	82.8	126	82.8	126	82.8	126	○			
SHS 55V	70	100	171	128	197	128	197	128	197	○			
SHS 65V	90	126	221	205	320	205	320	205	320	○			

## Gears

<http://www.qtcgears.com/KHK/newgears/KHK216.html>



Specifications			
Precision grade	JIS B 1704 grade 4	Core hardness	HB300~320
Gear teeth	Gleason	Surface hardness	HRC55~60
Pressure angle	20°	Surface treatment	-
Helix angle	35°	Surface finish	Cut
Material	SCM415 (Alloy steel)	Datum reference surface for gear cutting	Bore
Heat treatment	Overall carburizing		

- Back surface of B7 type is machinable due to masking during carburization.

