Interactive Worked Examples Learning Strategy in the Assistment.com System

An Interactive Qualifying Project

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Ryan Svoboda

Shane Robert Gibbons

Emilia Holban

Professor Neil Heffernan, Advisor

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Abstract

A study comparing the learning support facilitated by interactive worked examples and scaffolding questions was made using the Assistments.org System. The problems used were chosen from questions from past examinations in the 8th grade Mathematics MCAS. After analyzing the data it was conclusive that learning occurred by using either interactive worked examples or scaffolding questions. It was inconclusive, however, that there is a difference between the two learning methods.

Acknowledgments

This project could not have existed without the support and guidance of Professor Neil Heffernan, the ASSISTment.org project and the ASSISTment team who worked on the system while this study was conducted. A special thank you also goes towards the students and teachers at Burncoat Middle School, Worcester, MA who used our experiment.

Introduction

This Interdisciplinary Qualifying Project had as goal the development of an experiment that would compare the learning effect of interactive worked examples versus the original system used by the ASSISTment.org project - scaffolding questions. The experiment was targeted towards students in 8'th grade, who represented the sole users of the ASSISTment.org project at the time of the experiment.

The Mathematical problems used were taken from MCAS past examinations. For each problem two different inputs into the Assistment system were constructed – one breaking down the problem into sub-questions, the "scaffolding strategy" that would guide the student towards the answer of the original question and another would guide the student through the step by step solution of the problem, adding at the end a very similar problem such that the student feels compelled to go through and read the interactive worked example.

Both strategies had as goal teaching the students how to solve similar problems that they were likely to encounter while taking the Massachusetts Comprehensive Assessment System examination (MCAS), a required exam at the end of 8th grade.

In order to test the difference between the teaching capability of scaffolding questions versus interactive worked examples, separate curriculums including the two strategies but the same problem sets were created. We performed a pretest and a posttest on the students who used the curriculums and concluded from the gathered data that there was no conclusive difference between the two methods, but in both cases there was evidence that learning took place.

Background

1.1. The Assistment Project Now

The Assistment Project is a program designed to both assist learning of examination material and to asses the progress of the students while they are learning. This program belonging to Worcester Polytechnic Institute was founded by Professor Neil Heffernan. It is an interactive tutoring system capable of instructing students while assessing the progress that they make. A typical 'Assistment' consists of a problem which the student may attempt to solve. If the student fails to submit the correct answer for the problem, they are then taken through a series of 'scaffolding' problems that attempt to break down the problem into simpler steps. The students can attempt to solve scaffolds on their own, or they can request as many hints as necessary to complete the step. Finally, after completing all of the scaffolds the student returns to the initial problem and attempts to answer it again

The system has had several upgrades and has gone through multiple upgrades during the years. This IQP project took place from 2005 to 2006 and used one of the first versions of the software. Since then many other tools have been added, aiding Analysis of Experiments, Curriculum Building, etc.

Now, the system contains tools for professors like periodically reporting to the teachers the progress of the students, analysis tools to better evaluate the improvement of the class, better tools for adding new problems, tools for creating new experiments that test teaching methods and a new tool for curriculum building(a curriculum is a collection of problems).

1.2. The MCAS

The Massachusetts Comprehensive Assessment System (MCAS) is a standardized test produced by the state of Massachusetts, designed to test and measure all public school students in Massachusetts. Satisfactory completion is required as a condition for eligibility for a high school diploma. The MCAS program is also used as a gage to see how well individual schools and districts are transferring knowledge to the students.

The MCAS program uses a series of tests in English Language arts, Mathematics, Science and Technology/Engineering, and History and Social Science. The Assistment system is primarily aimed at helping students learn the skills necessary to do well on the mathematical portion of the MCAS. Recently, the Assistment system has been experimenting with branching out to cover the other topics involved in the MCAS. Many of the problems used in the Assistment program are taken directly from previous tests, thus ensuring that the material covered in the program is relevant to the MCAS test.

1.3. The Old Assistment System

As mentioned above, this IQP was done in the old system, from 2005 to 2006. At the time curriculums were done by writing by hand XML files containing the names of the items and there was no Analysis tool.

The first encounter with the system was done through the intro page, which would include the login screen, the links towards new account creation for both new students and new teachers or towards pages that would provide more details about the system – Press releases, papers associated with the project, the entities founding the project or the people who work for the project or have done so in the past.

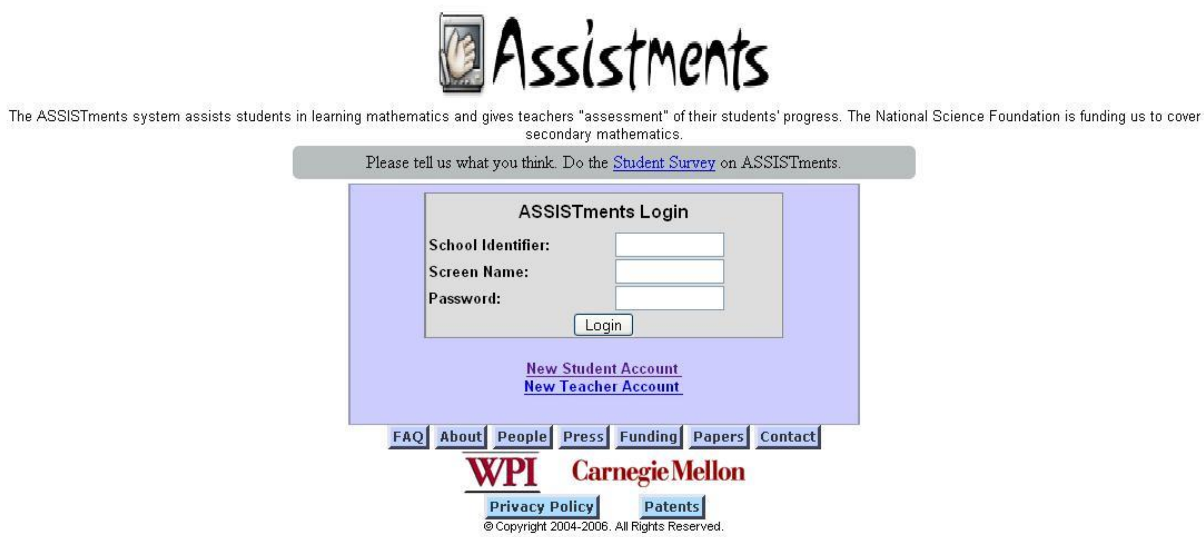


Fig 1. First Contact with the Assistment Project

In order to better assess students and to be able to use the Assistment Program in schools, each student needs to enroll in his own school and choose his teacher. . This is what the School Identifier in the above picture is used for. This would provide her with access to the assignments posted by the teacher and would facilitate the possibility of using the system as a school aid in both teaching and grading students. Having access to her own students, a teacher can keep track of their performance over time.

As a student, after signing up for an account, the next task is to join a class by looking up their teacher's name in the list of teachers from their school and then choose the period they belong to.

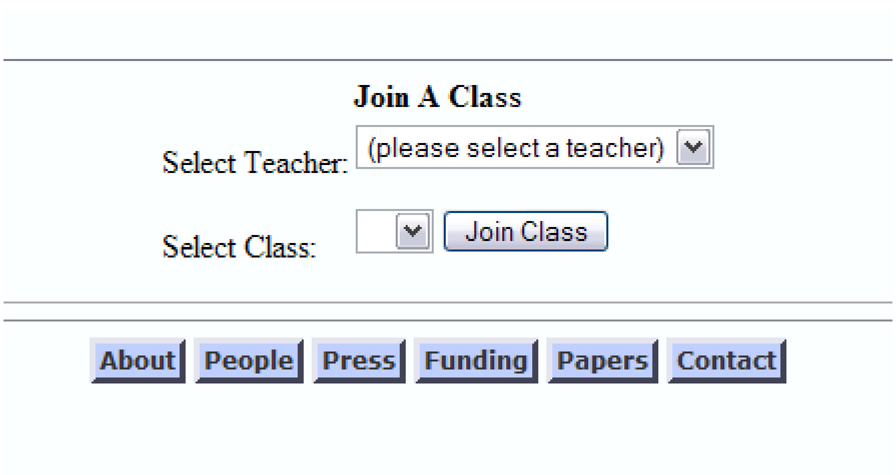


Fig 2. Choosing a class

The next step would be to start work by choosing out of the list of curriculums posted by the teacher whatever one you want or need to start.

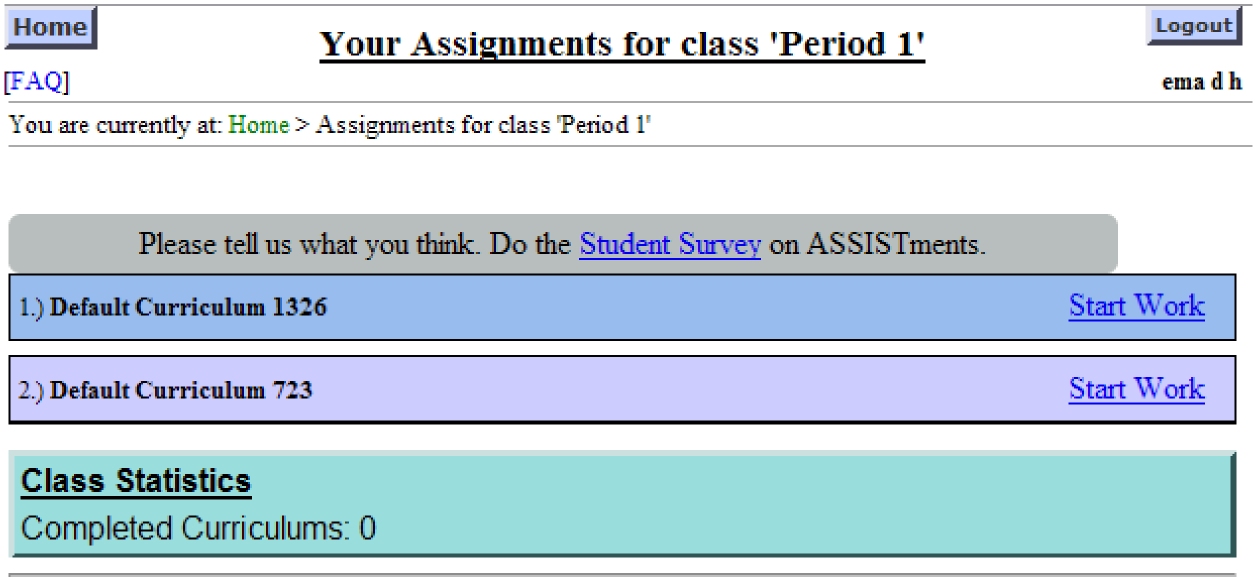


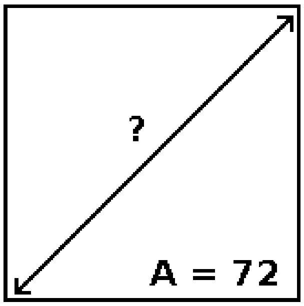
Fig3. First time student curriculum choice.

1.4. Structures of Problems

The main content of the system consists of Mathematical problems that are created under different styles – they support different type of input for answers (multiple choice or text box, for example) and different methods of helping students who are having trouble finding the answers. These methods are chosen by the person who built the specific problem - the problem can have hints, can have scaffoldings (a collection of sub-problems related to the original question) that lead the student to the right result. A combination of the strategies can be used as well. There are problems that contain hints for questions within the scaffolding. Pictures are supported and often used as aid for explaining problems. To better exemplify the notions of scaffolding questions and hints, an example of a problem on the Assistment system is presented below. This is a problem using only the Hints strategy. One can see the Hint button, that helps provide students with clues if they get stuck.

Assistment: (8190) Number Sense and Operations. Computing the square root(2003_Retest_10g_11)

Problem #8190



The formula, $d = \sqrt{(2A)}$, gives the length of the diagonal of a square television screen in terms of A, its area.

What is the length of the diagonal of a square television screen that has an area of 72 square inches?

- ○ 6 inches
- ○ 12 inches
- ○ 17 inches
- ○ 8 inches

Submit

Hint

Fig4. Typical Hints Problem in the Assistment System

Students can click on the Hint button several times, each time getting a new hint. For the above problem, these are the hints provided. Each new hint is delimited from the previous ones by a line.

This problem is straightforward. You need to replace A with 72 and then find d.
2A = 2 * 72 = 144
$\sqrt{144} = 12$
The final answer is **12 inches**.

Fig 5. Hint Sequence for the Given Example

To exemplify the scaffolding strategy, another problem will be used as example. This problem has both hints and scaffolding questions. If a student does not get the initial question correctly or uses hints in this initial situation, a set of questions guiding him towards the answer will appear.

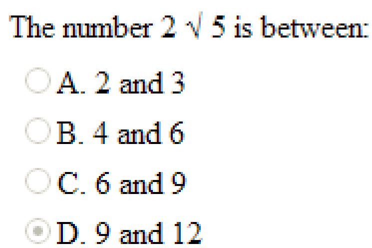


Fig 6. Problem Using Scaffolding Questions and Hints

By getting the answer wrong, a new set of questions will appear one at a time, each one with access to hints. The following figure shows the entire scaffolding tree of this example:

The number 2 √ 5 is between:

- A. 2 and 3
- B. 4 and 6
- C. 6 and 9
- D. 9 and 12

Hmm, no.

Let me break this down for you.

What are the two numbers closest to 5 that you know the square root of?

- A. 1 and 3
- B. 4 and 9
- C. 4 and 5
- D. 3 and 9

The square roots of these two numbers will give you a range in which the √ 5 will be in. What are the square roots of 4 and 9?

- A. 1 and 3
- B. 2 and 3
- C. 2 and 4
- D. 4 and 9

Problem #4096

Because the √ 5 is between 2 and 3, 2 √ 5 is must be between:

- 2 and 3
- 4 and 6
- 6 and 9
- 9 and 12

Submit

Hint

Fig 7. Example of Scaffolding Questions Structure

1.5. Other Systems

There are other available web based software systems designed to teach students how to solve mathematical problems. A lot of the problem types used by them are similar to the ones implemented in the Assistment System – scaffolding questions, examples, hints, use of similar problems to teach a certain concepts. Most of the software available is not free so it is more difficult to use them on a large scale, like the Assistmentcom Project.

2 Content

2.1 Background

There has been much research done in the field of worked examples in the past. Some of it varies greatly from the topics and format investigated here, while some is very similar. The effectiveness of worked examples has been demonstrated in a study that showed that this method shortened the time it took students to complete a three year mathematics course to only two years *(Zhu and Simon, 1987)*. It has also been shown that worked examples can be more effective then conventional problem solving for teaching both algebra *(Sweller and Cooper, 1985)* and geometry *(Paas, 1992)*.

There are some drawbacks to the use of worked examples as an instructional tool. It's been shown that when worked examples are used in combination with conventional problems students will often ignore the worked examples and only refer back to them when they encounter difficulty solving the conventional problems *(van Merrionboer and Paas, 1990)*. There are two main schools of thought on ways to solve this problem.

One strategy for ensuring that learners read and think about worked examples is the use of "completion problems" *(van Merrionboer and Kramer, 1990)*. Completion problems are worked examples with sections left blank that the learners need to fill in. This forces the learners to read and try to comprehend the examples. Our research does not make use of this method.

Another strategy for combating example skipping is the asking of questions about the example itself *(Sweller, Jeroen, van Merrienboer and Pass 1998)*. This is not to be confused with the use of worked examples and conventional problems in combination, as the questions asked in this case are about the specific example.

2.2 Problems with the traditional format

It has been observed that given a normally structured Assistment some students will have a tendency to "game" the problem (Feng et al, 2005). That is, they will request hints in rapid succession until they receive a "bottom-out hint", which reveals the correct answer to the student. This results in a quick progression through the Assistments with minimal knowledge retention.

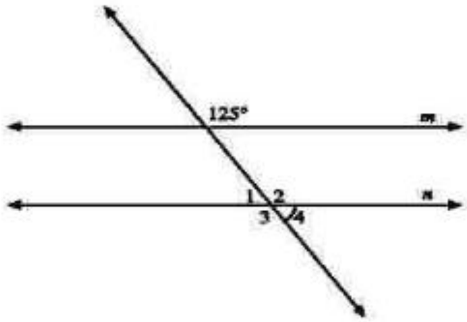
We keep the students focus on the worked example and the steps necessary to complete it by asking comprehension questions about the steps themselves. It is fairly common practice in worked example studies to group worked examples with relevant conventional problems. We felt that this was unnecessary, and that students should have the specific steps stressed, as opposed to simply presented. In our experiment we aim to prove that students retain enough from the worked examples to provide adequate knowledge to solve similar problems about the same topics.

2.3 Designing the Interactive Worked Examples

In this study we have proposed an alternative model that will we refer to as the Interactive Worked Example method. The format for this model is as follows: The student is first given a completely worked out problem, broken down into steps leading up to the solution. The student is instructed to study this problem and its solution until they feel they understand it. The student is then given a series (generally about equal in number to the number of steps that the problem is broken down into) of "comprehension questions" about the process of finding the solution to problem. An example of a question in the Worked Example format is shown in the figure below.

An example of such a question would be, "According to step 2, what does it mean when ***M*** is less than 0 in the slope-intercept form of the equation of a line?". These questions stress the general skills needed to solve the problem, and help to ensure that the students are fully reading and understanding the Interactive Worked Examples. This method of ensuring student participation varies from the "Completion Strategy" described by van Merrionboer and Kramer (1990) in which sections of the example are left blank for the learner to fill in. We felt that giving the full example and asking comprehension questions was the best way to ensure a complete understanding of the principles involved and the steps needed to solve a problem.

Assistment: (9003)



Lines m and n are parallel, what is the measure of angle 4?

Step 1) Since lines m and n are parallel and intersected by the same line, the corresponding angles are equal. This means that angle 2 is also 125 degrees.

Step 2) Angles 2 and 4 are supplementary angles because they are seperated by a single line. This means that they add up to 180 degrees.

Step 3) Since we know that angle 2 is 125 degrees, and that angles 2 and 4 add up to 180 degrees, we can solve for angle 4 like so:

Angle 2 + Angle 4 = 180

We now substitute 125 degrees for angle 2 and get:

125 + Angle 4 = 180

We now subtract 125 degrees from each side of the equation so that Angle 4 is all alone on one side. This gives us:

Angle 4 = 180 - 125

When we do the subtraction we see that angle 4 is equal to 55 degrees.

Figure 1

2.4 Benefits of the Interactive Worked Example System

Our worked example method combated gaming, at least to some degree, since we did not provide "bottom-out hints" but instead referred students back to the steps given for solving the problem. Students seemed to learn rather quickly that the gaming methods that may have worked for them in previous Assistments would not be applicable to this new format. In sessions that we observed, a large portion of students would take what seemed an appropriate amount of time reading the example steps before attempting to complete the comprehension questions.

There is some evidence (Mayer and Clark, 2002b) that Worked Examples can be more effective when broken into clearly labeled sub-goals. Our worked example model does exactly that, breaking problems into 3 – 5 clearly stated steps. Each step is numbered and has a very specific objective. These objectives are further stressed by the step-specific comprehension problems that follow.

The figure below shows a typical worked example problem that was included in our experiment. The original problem context is stated and then is followed by a series of steps which explain the typical way to solve this specific problem. We chose to work through a specific example over a general example to teach students how to apply the general principals to solving problems. The answer to the original problem is given to the student at the end of the steps. After the student has finished reading through the steps they are given a series of comprehension questions which will ask them about the general concepts behind each step of solving the original problem.

2.5 Benefits of comprehension questions

One strategy for ensuring that learners read and think about worked examples is the use of "completion problems" (van Merrionboer and Kramer, 1990). We have chosen to take a similar approach to this; after reading through the worked example, the student is asked questions about the general topics discussed in the problem.

This approach has two distinct benefits. The first is to combat skipping the problem, since the answers to these questions can be found in the steps of the worked examples, the student is encouraged to go back and reread the problem if they do not have a firm grasp of the subject matter. The second advantage applies to students who already understand the material. They are able to answer the comprehension questions easily, thus providing them with a brief review of the topic without forcing them to slowly step through a problem that they have already have a complete understanding of. Some example comprehension questions are shown in the figure below.

(Problem ID: 4750)

Which of the following statements are true about a regular hexagon?

Answers: (Interface Type: RADIO_BUTTON)

✗ None of the interior angles are equal

✗ All of the interior angles are less than 90 degrees

✓ **All of the interior angles are equal**

(Problem ID: 4757)

What does supplementary mean for angles?

Answers: (Interface Type: RADIO_BUTTON)

✓ **The angles add up to 180 degrees**

✗ The angles add up to 90 degrees

✗ The angles are both less than 90 degrees

Sample Comprehension Questions

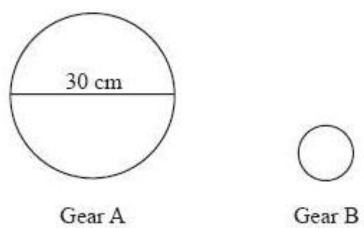
2.6 Content Walkthrough

Here are some Interactive Worked Example problems explained step by step.

2.6.1 Assistment #8703

The following Assistment demonstrates a typical Interactive Worked Example.

"Beta IWE 2004 - 2" (Problem ID: 8703)



The circles above represent the gears of a bicycle. The diameter of Gear A is 30 centimeters. The ratio of the diameter of gear A to the diameter of gear B is 3:1. What is the circumference, in centimeters of gear B?

Step 1) Since the ratio of Gear A's diameter to Gear B's diameter is 3:1, The diameter of Gear B is 1/3 of the Diameter of Gear A. This means the diameter of Gear B is 10 cm.

Step 2) The formula for circumference is Pi times the diameter. Since we now know the diameter of Gear B is 10 cm, we can calculate the circumference as 10*pi.

Please click "Hint" after you have finished reading the problem.

The question is asked in the same manner that a normal problem in the Assistment system is, however, it is directly followed by the steps one would take to solve the problem. The student is encouraged to read the problem, and then click on the “Hint” button to continue. When this button is clicked, the student is presented with the first comprehension question.

(Problem ID: 8704)
What does the ratio A:B mean?
Answers: (Interface Type: RADIO_BUTTON)
✓ **A divided by B**
✗ A minus B *You should read Step 1 more closely*
✗ A plus B *You should read Step 1 more closely*
✗ B divided by A *Not quite, you've got it backwards*

This comprehension question ensures that the student understands what the symbol for a ratio implies. There are also buggy messages that appear when the student selects the incorrect answer. This question is meant to correspond to step 1 of the solution process. When the student answers this question correctly, they move on to the next comprehension problem.

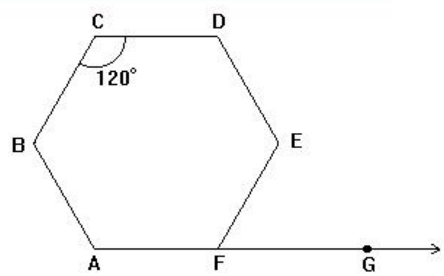
(Problem ID: 8705)
What is the formula for the circumference of a circle?
Answers: (Interface Type: RADIO_BUTTON)
✗ 2 Pi times diameter *Not quite*
✗ 2 times Pi *The answer should involve the diameter*
✗ 2 times diameter *2 is not the correct constant*
✓ **Pi times diameter**

This comprehension question asks the student for the formula of the circumference of a circle. The formula is also stated in the second step of the walkthrough. Buggy messages are again included to help the student if they select the incorrect answer. This is the final comprehension question, and when the student submits the correct answer they are allowed to move on to the next problem.

2.6.2 Assistment #4751

The following Assistment was a prototype for color based visual cues in the comprehension questions, in order to encourage the students to go back and reread the process to obtain the solution.

"IWE 21 2000 (regular hex)" (Problem ID: 4751)



This is a regular hexagon. How many degrees are in angle EFG?

Step 1) Regular indicates that all of the interior angles of this pentagon are equal. Since we know that one interior angle is 120 degrees, we know that all of them are 120 degrees.
Step 2) Since AFE is adjacent to EFG, it is supplementary. This means that AFE + EFG = 180 degrees. To find EFG, we simply subtract AFE from 180. So 180 - 120 = 60 degrees.

The keywords "regular" and "supplementary" and highlighted to show importance. These are the key concepts that a student needs to master to solve this problem.

(Problem ID: 4750)
Which of the following statements are true about a regular hexagon?
Answers: (Interface Type: RADIO_BUTTON)
✗ None of the interior angles are equal
✗ All of the interior angles are less than 90 degrees
✓ **All of the interior angles are equal**

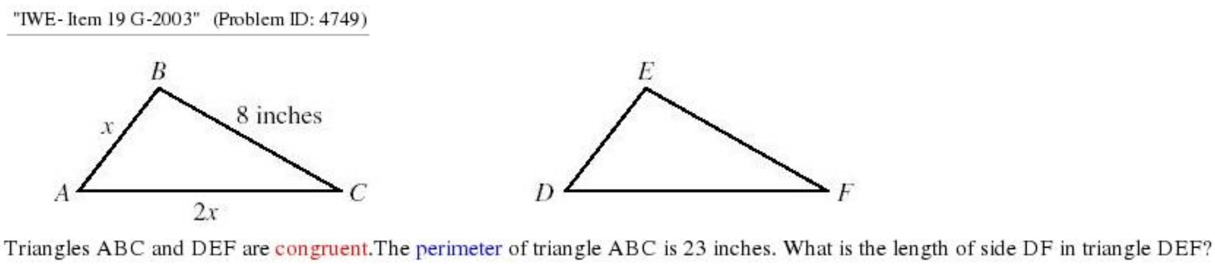
The first comprehension question asks the student about the concept of a regular polygon, in this case, a hexagon. The word "regular" is highlighted again in the same color to encourage the student to look back at the original steps to find the solution to this problem. When they answer correctly they move onto the next comprehension question.

(Problem ID: 4757)
What does supplementary mean for angles?
Answers: (Interface Type: RADIO_BUTTON)
✓ **The angles add up to 180 degrees**
✗ The angles add up to 90 degrees
✗ The angles are both less than 90 degrees

The second comprehension question asks the student about the meaning of supplementary. The word "supplementary" is again highlighted in the same color to encourage the student to look back at the original steps to find the solution to this problem. This is the final comprehension question and when answered correctly the student is allowed to move on to the next problem.

2.6.3 Assistment #4749

This Assistment uses the color based cues as well, to help the student learn about perimeters, and congruent triangles. This problem also includes algebraic equation solving.



In the main question the terms "congruent" and "perimeter" are highlighted, since these are the main concepts that this problem is testing. This Asisstment uses incremental comprehension based questioning. Instead of reading how to solve the entire problem at first, each step in solving it is broken down and then a question is asked after each part.

(Problem ID: 4745)

Step 1) The first step to solving this problem is to understand the meaning of the word congruent.

The length of each side of a congruent triangles has the same length as a corresponding side in the other triangle.

What are congruent triangles?

Answers: (Interface Type: RADIO_BUTTON)

✗ Triangles that have the different size sides

✗ Triangles that have the same size angles

✓ **Triangles that have the same size sides**

The first comprehension question provides an explanation of the term congruent, and then asks the student to select the answer which corresponds to this definition. When the student selects the correct answer they move on to the next part.

(Problem ID: 4746)

Step 2) The second step in this problem is to understand the meaning of the word perimeter.

The perimeter of a shape is the total length of all the sides of that shape.

What is the perimeter of a shape?

Answers: (Interface Type: RADIO_BUTTON)

✗ The total length of each side and angle

✓ **The total length of each side of a shape**

✗ The total number of degrees that the shape has in each angle

This comprehension question is similar to the first. It defines perimeter, and then asks the student to select the correct definition of the term.

(Problem ID: 4747)

Step 3) The perimeter of triangle ABC is 23. Since we know that the perimeter of a triangle is the total length of all of its sides, we can set up an equation.

The length of side AB = X
The length of side AC = 2X
The length of side BC = 8

By adding these values together we have the equation:
X+2X+8=23
By adding the like terms we get **3X+8=23**
To get the variable on a side by itself we can subtract 8 from both sides of the equation leaving us with **3X+8-8=23-8** or **3X=15**
To solve for X we divide both sides of the equation by 3, which gives us **X=5**

What geometry term did we learn that allowed us to set up and solve this equation?

Answers: (Interface Type: RADIO_BUTTON)

✗ Congruent

✓ **Perimeter**

✗ Perpendicular

This step of the problem solving process explains how to set up and solve an algebraic equation. It uses the definition of the term perimeter to show how to set up this equation, and then breaks down solving it into atomic steps. At the end the student is asked to idenitify which geometry term was used to set up the equation.

(Problem ID: 4748)

Step 4) Now that we know the value of X, we can plug it into our diagram.

We know that congruent triangles have sides that have equal lengths that correspond to each other. The length of side DF corresponds to the length of side AC.

The length of side DF is 10 inches.

What geometry term did we learn that we used to solve this problem?

Answers: (Interface Type: RADIO_BUTTON)

✓ **Congruent**

✗ Parallel

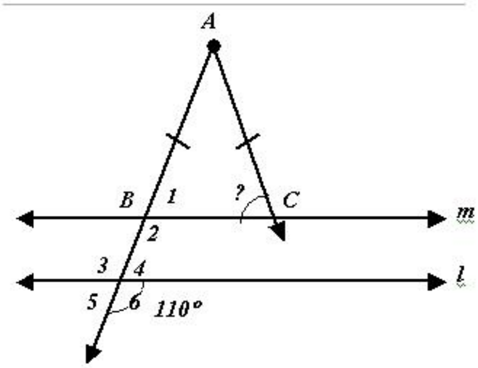
✗ Perimeter

The final step in this problem involves using the definition of the term congruent, which we learned in the previous questions. It also provides another rendering of the initial figure, but it substitutes in the value of X determined in the previous step. This shows the student a visual example of what the process is. The question then follows with an explanation of how to apply our knowledge of the term congruent to solve the problem. The student is then asked which term was used in this step, to ensure that they are reading the problem.

2.6.4 Assistment #4744

This Assistment is an example of an Interactive Worked Example. It provides a step by step process to solve this specific problem, then asks the students comprehension questions to ensure that they read and understood the material.

"IWE 33 G-2002 (Isosceles Angles)" (Problem ID: 4744)



In the figure shown, lines l and m are parallel, and triangle ABC is isosceles. What is the measure of angle ACB?

Step 1) You first must use the fact that parallel lines that intersect the same line, intersect it at the same angles. This means that angle 1 is equal angle 6, which is 110 degrees.
Step 2) By the definition of an isosceles triangle two of the angles are equal. The unkown angle and angle 1 must be the same since they are both on sides that are equal (indicated by the perpendicular marks on the lines). This means that the unknown angle has the same measure, 110 degrees.

The concepts introduced here involve parallel lines, isosceles triangles, corresponding angles, and supplementary angles. When the student is finished reading the explanation of how to solve the problem, they move onto the first comprehension question.

(Problem ID: 4742)

Which of the following statements about parallel lines intersecting the same line are true?

Answers: (Interface Type: RADIO_BUTTON)

✗ They angles they form with the line are always 90 degrees

✗ They cannot intersect the same line

✓ **The angles that they form with the line are equal**

This question asks the student the same question that was answered in step one of the walkthrough solution. If the student had read the question, or if they had previous knowledge about parallel lines, this question is simple and reinforces the knowledge.

(Problem ID: 4743)

Which of the following statements about isosceles triangles are true?

Answers: (Interface Type: RADIO_BUTTON)

✗ They have no equal angles

✗ They have interior angles that sum to 270 degrees

✓ **They have two equal angles**

The second and final comprehension question asks the students about isosceles triangles. This was explained in step 2 of the solution, so if the student has a problem with this question, they can always refer back to the original problem to find the answer. When they select the correct answer they are allowed to move onto the next problem.

3 Experiment

3.1 Hypothesis

We hypothesized that by presenting students with a completely worked out example they will learn more from it, instead of presenting the students with a problem to solve. We believe this for several reasons. Firstly, as described in Mayer and Clark (2002a) "Working memory has a limited capacity that becomes inefficient when having to retain even a few items, If the only way to build job relevant skills is to perform many practice exercises, working memory can become overloaded...Worked examples are more efficient for learning new tasks because they reduce the load in working memory".

3.2 Overview

To test our hypothesis we built a curriculum for students where each student was randomly chosen to either work through four worked examples of problems, or given four normal Assistment versions of the same problems. After the students were done with the first four problems they would move on to the transfer section, where all of them were given the same four problems presented in the standard Assistment format.

A randomly assigned curriculum was used to ensure that an unbiased population of students was either given the worked example problems or the standard Assistments. In our experiment 151 of the 309 students were randomly assigned to work on the worked examples, and 158 students were assigned to the normal Assistments. Since the experiment did not include a pretest before the random section, there was no data to show if the worked examples improved learning for each individual student, however, by not including a pretest, it eliminates the possibility of creating less accurate data due to overexposure of a single concept to the limited number of concepts covered by this experiment.

The items that each student were given were all taken directly from previous MCAS mathematics exams. Each student received the exact same eight problems; however, approximately half of the students received worked example versions of the first four problems, while the rest of the students received the standard version of the Assistments. This type of experiment allows us to receive data based on how well the students perform on the transfer section. By analyzing how well students who received worked example problems did on the transfer section compared to students who did not receive the worked example problems we can determine if there is a statistical significance in the ability to perform better on the concepts covered in our curriculum.

3.3 Realization of experiment

Our experiment was conducted in many different public schools in the Worcester region of Massachusetts. The curriculum was given to over three hundred students in the public school system. The students that were selected to be involved in our experiment we felt were a good representation of the public school system population. The ability of these students ranged from those in special education classes to those in honors classes.

The problems used were all take directly from previous 8th grade MCAS tests. All students should have been taught or at least exposed to these concepts before. Therefore this was not meant to be an introduction to new concepts, but a reinforcement of skills and techniques that the students' teachers should have taught them.

Many similar experiments are structured in such a way that each problem in the experimental conditions has one and only one matching problem in the post-test, or transfer section. We felt that such an organization tends to test for repetition of an identical series of steps with different number values being the only variation. In an attempt to determine what actual skills were retained well enough for students to apply them whenever necessary, we opted for a less direct problem mapping. In our format, each worked example taught one or more skills, which were present in one or more transfer items. This skill-based post-test mapping forced students to apply the skills in new combinations, which we felt were a more accurate gage of skill retention then identically formatted morphs. The skill mappings are shown in the figure below..

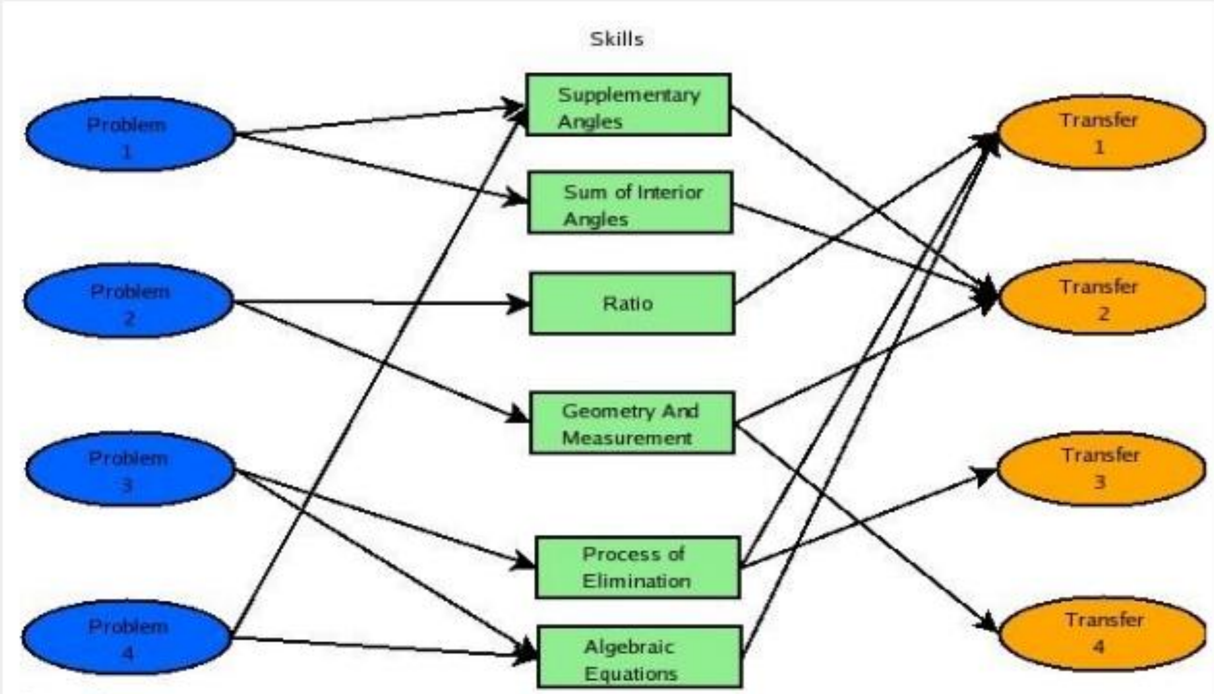


Figure 2

Skill Mappings

4 Results

Our experiment was run on over 300 8th grade students in the Worcester Massachusetts public school system over slightly more then a month. The curriculum was short enough that most students that participated in our study completed it in the single class session that it was assigned for. This class length varied from school to school and day to day, but was generally somewhere between 30 minutes to an hour.

4.1 Analysis

For this analysis, correct problems were scored with a 1, and incorrect problems were assigned a 0. Problems were deemed correct only if the student managed to choose the correct answer completely on their own, and without using the scaffolding questions. This amounted to questions only being counted as correct if students got them correct on their first try.

One hindrance in the analysis of the data collected was the inability to asses students' prior knowledge or ability because of the format that we chose, which included no pretest. An alternative approach to prior skill assessment might be to use date from the two experimental conditions themselves. This, however, is not applicable to our model, since the worked examples gather little to no information about the students knowledge of the topic that they are attempting to teach. therefore, instead of comparing gain in score between the experimental and control groups, we compared only the scores in the post-test section.

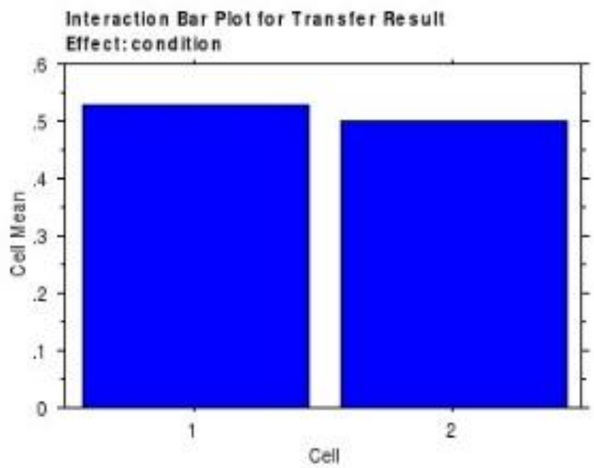


Figure 3

ANOVA analysis of Transfer Results

ANOVA analysis of the data, using the the experimental condition that each student had participated in as the factor and the transfer section scores as the dependent variable results in bar graph in the figure above.

In this graph, Cell 1 is the experimental worked example condition, and Cell 2 is the control condition consisting of standard Assistments. While the mean transfer score of students from the worked example condition is slightly higher, it is not a statistically significant difference.

The ANOVA table for the analysis is given below. The p-value of 0.3518 that is listed clearly indicates that there is no statistical significance to the post-test performance differences of students from the two conditions. This indicates that, at least in a test group this size, worked examples in the format that we have examined here provide no real advantage to learning over the conventional scaffolding model used by the Assistments project.

Due to the nature of the condition to post-test mapping (as illustrated in the figure below) that was used, learning can not be clearly evaluated on a skill-specific level. Since specific in-condition items are not directly matched with post-test items, but instead the two groups involve the same skill set, no evaluation can be made in more detail then a comparison of the post test scores from students in both conditions.

ANOVA Table for Transfer Result

	DF	Sum of Squares	Mean Square	F-Value	P-Value	Lambda	Power
condition	1	.217	.217	.868	.3518	.868	.146
Residual	1236	309.050	.250				

ANOVA Conditional Analysis

4.2 Limitations and Future Work

During our research the group observed several external variables that could have affected the results of the experiment. These variables could be, in future experiments, prevented or eliminated.

For example, some students were inclined to disregard the worked example, jump straight to the comprehension questions and, by trial and error, get to the right answers. Other students, after disregarding the given steps, would just skim the example for the answer and stop after finding it, without reading the step by step explanations. Both of these approaches resulted in a decrease in learning. This could be avoided in the future by changing the format of the answers from multiple choice to text / number field input, making trial and error a less effective strategy.

Another approach would be avoiding comprehension questions containing words that can be found in the worked example. For instance, if one of the steps contains "the sum of the interior angles in a triangle is 180 degrees", none of the comprehension questions should contain "sum of interior angles" in its text, as students might be tempted to skim the worked example and read only the part containing this phrase to get the answer.

Another limitation we encountered was the fact that some students lacked the basic knowledge necessary to understand the steps of the worked example. As an illustration, a worked example explaining how to find the circumference of a circle will result in little to no increase in learning if the student does not know what a radius is. This obstacle is impossible to eliminate, but its impact could be reduced by introducing more in-depth hints that explain basic notions needed to understand the worked example.

Another encountered problem was the lack of application in our approach. Worked examples have as result learning the steps necessary to solve certain problems and getting the students more and more familiar with mathematical notions. Without applying these learnt skills they have a great chance of being forgotten, so the overall increase in learning is low. What could be done is adding after worked examples conventional problems related in skills required.

The low number of students who took part in the experiment could be a reason why the data we have shows very little increase in learning by using worked examples versus conventional problems. Additionally, the 300 students who took part in the experiment were pupils from a very low number of schools. Many of them share the same teachers, have the same degree of knowledge. As the subjects are not randomly chosen, the results of the analysis only reflect the population of the involved schools and do not reflect how worked examples would affect learning of students in general. When the Assistments System becomes more widely spread, the experiment should be repeated on randomly chosen larger sample of students.

4.3 Conclusion

In conclusion, we have learned that Interactive Worked Examples did not have a statistically significant impact on knowledge retention for short term tests. These results do not prove that Interactive Worked Examples provide no merit over standard problems. Many of the students responded positively to the worked examples, some saying that they preferred having the material presented and "taught" to them, instead of feeling like they are taking a test. We recommend that further testing be done in this field, as there were many external factors involved that may have skewed our results.

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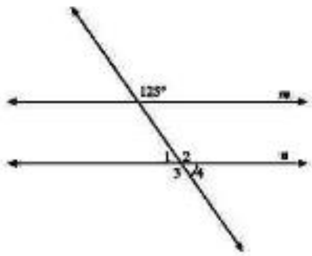
Appendix A – Interactive Worked Example Problems

IWE-2000-Spring-7



Lines m and n are parallel, what is the measure of angle 4?

Step 1) Since lines m and n are parallel and intersected by the same line, the corresponding angles are equal. This means that angle 2 is also 125 degrees.

Step 2) Angles 2 and 4 are supplementary angles because they are seperated by a single line. This means that they add up to 180 degrees.

Step 3) Since we know that angle 2 is 125 degrees, and that angles 2 and 4 add up to 180 degrees, we can solve for angle 4 like so:
Angle 2 + Angle 4 = 180
We now substitute 125 degrees for angle 2 and get:
125 + Angle 4 = 180
We now subtract 125 degrees from each side of the equation so that Angle 4 is all alone on one side. This gives us:
Angle 4 = 180 - 125
When we do the subtraction we see that angle 4 is equal to 55 degrees.

When two parallel lines are both intersected by the same line, which angles are equal?

- Adjacent angles
- All of the angles
- Complimentary angles
- Corresponding Angles

What is the name for two angles that sum 180 degrees?

- Alternate Interior
- Complementary
- Supplementary
- Vertical

To solve an equation for a variable, how should you try to re-arrange it?

- Get everything on one side
- Get that variable alone on one side

When you do something to one side of an equation, what should you do to the other side?

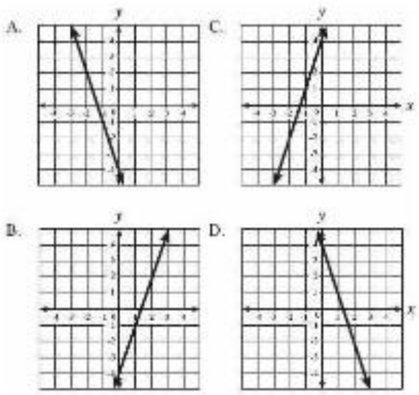
- Nothing
- The opposite of what you did to the first side
- The same thing

- m
- x
- y

What does it mean for the b value in the equation y = mx + b to be equal to 4?

- It has a slope of 4
- It intercepts the y-axis at y = 4
- The x-value is 4
- The y-value is 4

IWE-2004-14



Which graph above best represents $y = -3x + 4$?

Step 1) The equation for a line can be generalized as $y = mx + b$, (this is known as the slope-intercept form) where m is the slope and b is the point it intercepts the y axis. In this case, $m = -3$, and $b = 4$.

Step 2) Since m (the slope) is negative we know the line will go down as it goes right. This limits possible correct solutions to A and D.

Step 3) Since b (the y-intercept) is 4, the line must cross the y-axis at $y = 4$. Since we've already restricted possible correct answers to A and D, and D is the only one that fits this new criteria, D is the correct answer.

What is the slope-intercept equation for a line?

- $m = bx$
- $m = yx + b$
- $x = my + b$
- $y = mx + b$

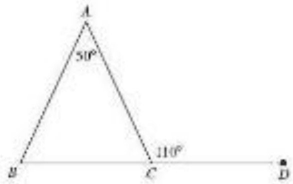
In the equation $y = mx + b$. What represents the slope of the line?

- b
- m
- x
- y

What does it mean for the b value in the equation $y = mx + b$ to be equal to 4?

- It has a slope of 4
- It intercepts the y-axis at $y = 4$
- The x-value is 4
- The y-value is 4

IWE-Control-2004-15



Triangle ABC is shown above. Points B, C and D are collinear. What is the measure of angle B?

- 120 degrees
- 50 degrees
- 60 degrees
- 70 degrees

What is the measure of angle ACB?

- 180 degrees
- 50 degrees
- 60 degrees
- 70 degrees

Submit

Angles ACB and ACD are supplementary.

Angles that are supplementary have a sum of 180 degrees.

180 degrees 110 degrees = 70 degrees

Angle ACB has a measure of 70 degrees.

What is the measure of angle B?

- 120 degrees
- 50 degrees
- 60 degrees
- 70 degrees

Submit

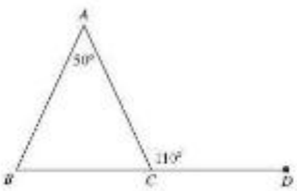
The sum of the interior angles of a triangle is 180 degrees.

We know 2 of the interior angles (70 degrees and 50 degrees).

This means the last angle, B, is 180 – (50 – 70).

The measure of angle B is 60 degrees.

IWE-2004-15



Triangle ABC is shown above. Points B, C and D are collinear. What is the measure of angle B?

Step 1) Angles ACB and ACD are supplementary. This means that 180 – ACD = ACB. Since ACD = 110 degrees, ACB = 70 degrees.

Step 2) The sum of the interior angles of a triangle is 180 degrees. We know 2 of the interior angles (70 degrees and 50 degrees). This means the last angle, B, is 180 – (50 + 70). This equals 60 degrees.

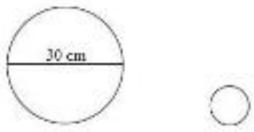
What does it mean if two angles are supplementary?

- The difference between their measures is 180 degrees
- The difference their measures them is 90 degrees
- The sum of their measures is 180 degrees
- The sum of their measures is 90 degrees

What is the sum of the measures of the interior angles of a triangle?

- 120 degrees
- 180 degrees
- 360 degrees
- 90 degrees

IWE-Control-2004-2



The circles above represent the gears of a bicycle. The diameter of Gear A is 30 centimeters. The ratio of the diameter of gear A to the diameter of gear B is 3:1. What is the circumference, in centimeters of gear B?

- 15 * Pi cm
- 30 * Pi cm
- 5 * Pi cm
- 10 * Pi cm

What is the diameter of Gear B?

- 15 cm
- 30 cm
- 5 cm
- 10 cm

Submit

Since the ratio of Gear A's diameter to Gear B's diameter is 3:1, The diameter of Gear B is 1/3 of the Diameter of Gear A.

This means the diameter of Gear B is 10 cm.

What is the formula of the circumferince of a circle?

- Pi * diameter / 2
- Pi * diameter^2
- Pi * diameter
- 2 * Pi * diameter

Submit

Pi * diameter

So what is the circumference of gear B?

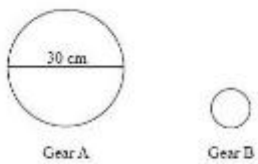
- 15 * Pi cm
- 30 * Pi cm
- 5 * Pi cm
- 10 * Pi cm

Submit

So the diameter of gear B is 10 cm and the formula of the circumference is Pi * Diameter

Pi * 10 = 10 * Pi.

IWE-2004-2



The circles above represent the gears of a bicycle. The diameter of Gear A is 30 centimeters. The ratio of the diameter of gear A to the diameter of gear B is 3:1. What is the circumference, in centimeters of gear B?

Step 1) Since the ratio of Gear A's diameter to Gear B's diameter is 3:1, The diameter of Gear B is 1/3 of the Diameter of Gear A. This means the diameter of Gear B is 10 cm.

Step 2) The formula for circumference is Pi times the diameter. Since we now know the diameter of Gear B is 10 cm, we can calculate the circumference as 10*pi.

Please click "Hint" after you have finished reading the problem.

What does the ratio A:B mean?

- A divided by B
- A minus B
- A plus B
- B divided by A

What is the formula for the circumference of a circle?

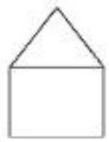
- 2 Pi times diameter
- 2 times Pi
- 2 times diameter
- Pi times diameter

IWE-Control-2000-19

Assistments

[Home]

Assistment: (9345) Geometry



In the figure above, the perimeter of the equilateral triangle is 24 inches. What is the area of the square?

- 24
- 32
- 64
- 8

Let me break this down for you.

What is the length of one side of the triangle?

- 12
- 24
- 6
- 8

[Submit]

An equilateral triangle is a triangle where the lengths of all the sides are equal.

The perimeter of a polygon is the sum of the length of all the sides.

Divide the perimeter by the number of sides to get the length of one side.

24 / 3 = 8

The length of one side of the triangle is 8.

What is the length of one side of the square?

- 12
- 24
- 64
- 8

[Submit]

One side of the square is the same as one side of the triangle.

Since they are the same line fragment they have the same length.

The length of one side of the square is 8.

What is the area of the square?

- 24
- 32
- 64
- 8

[Submit]

The formula for the area of a square is the length of a side squared.

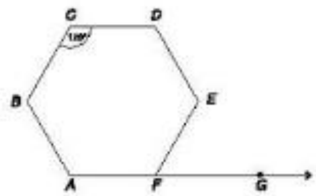
8 squared is 64.

The area of the square is 64.

[Done] [Hint] [More]

Assistments
[Home]

Assistment: (9334) Regular Hexagon



The hexagon depicted above is regular. What is the measure of angle EFG?

- 90 degrees
- 60 degrees
- 120 degrees
- 180 degrees

Let me break this down for you.

What is the measure of angle AFE?

- 60 degrees
- 120 degrees
- 180 degrees
- 720 degrees

Submit

The hexagon is regular.

Regular shapes have equal size angles.

All of the angles in the hexagon are the same size.

The measure of angle AFG is 120 degrees.

What is the measure of angle EFG?

- 120 degrees
- 60 degrees
- 180 degrees
- 90 degrees

Submit

Angle EFG is supplementary to Angle AFG.

Supplementary angles have a total measure of 180 degrees.

180 degrees - 120 degrees = 60 degrees

The measure of angle EFG is 60 degrees.

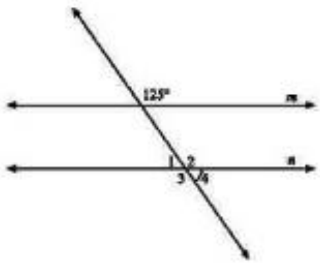
Done | Hint | More

IWE-Control-2000-Spring-7

Assistments

[Home]

Assistment: (9351)



Lines m and n are parallel, what is the measure of angle 4?

- 120 degrees
- 180 degrees
- 55 degrees
- 75 degrees

Let me break this down for you.

What is the measure of angle 2?

- 125
- 180
- 55
- 90

Submit

Lines m and n are parallel and intersected by the same line, therefore the corresponding angles are equal

This means that angle 2 is also 125 degrees.

Knowing that angles 2 and 4 are supplimentary, and that the measure of angle 2 is 125 degrees, what is the measure of angle 4?

- 120
- 180
- 55
- 75

Submit

Supplimentary angles add up to 180 degrees

This means angle 2 + angle 4 = 180

substituting values this gives you: angle 4 + 125 = 180

This means: angle 4 = 180 - 125

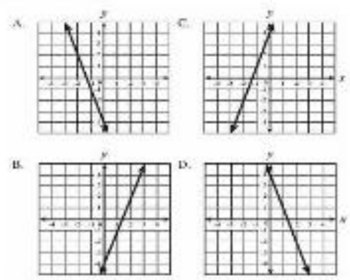
Angle 4 is 55 degrees. Choose 55.

Done Hint More

IWE-Control-2004-14

Assistments
[Home]

Assistment (9299)



Which graph below best represents $y = -3x + 4$?

- ○ A
- ○ B
- ○ C
- ○ D

Let me break this down for you.

What is the slope of the line described by the equation?

- ○ Positive
- ○ Negative

[Submit]

> This line is of the form $y = mx + b$
>
> In the equation $y = mx + b$, m represents the slope of the line
>
> if $y = -3x + 4$ is equivalent to $y = mx + b$, and m is the slope, this would mean that the slope is -3
>
> If the slope is -3, then the slope is negative. Select 'Negative'.

Since you know the slope is negative, which graphs can you eliminate as possible answers?

- ○ B, D
- ○ D
- ○ C, B
- ○ A

[Submit]

> You can eliminate any graphs with a positive slope.
>
> Graphs with positive slope go up and to the right.
>
> Graphs B and C go up and to the right.
>
> Graphs B and C can be eliminated. Check graphs B and C.

What is the Y-intercept of the described by the equation?

- ○ -3
- ○ 4
- ○ 7
- ○ 1

[Submit]

> The equation $y = -3x + 4$ is equivalent to $y = mx + b$
>
> In the equation $y = mx + b$, b is the y-intercept.
>
> If $y = -3x + 4$ is equivalent to $y = mx + b$, then $b = 4$. So the y intercept is 4. Choose 4.

Since we've restricted the possible answers to graphs A and D, and we now know the y-intercept, which graph must be the correct one?

- ○ D
- ○ A

[Submit]

> The y-intercept is the point where the line that is being graphed crosses the y axis.
>
> In graph D the line crosses the y axis at $y = 4$.
>
> D is the correct answer. Choose D.

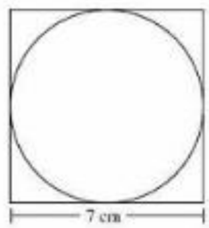
[Done] [Hint] [More]

IWE-Control-2004-25

Assistments

[Home]

Assistment: (9352)



The figure above shows a circle inscribed in a square. Which of the following is the best approximation of the circumference of the circle?

- ○ 28 cm
- ○ 10 cm
- ○ 14 cm
- ○ 22 cm

What is the diameter of the circle?

- ○ 14 cm
- ○ 10 cm
- ○ 4 cm
- ○ 7 cm

Submit

The diameter of the circle is the same as the length of one side of the square.

The square has a side-length of 7 cm

The diameter of the circle is 7 cm. Choose 7 cm.

Since you know the diameter of the circle, what is it's circumference?

- ○ 10 cm
- ○ 28 cm
- ○ 14 cm
- ○ 22 cm

Submit

The formula for circumference of a circle is pi * diameter.

Substituting values into the above equation, you get: Circumference = 7 * 3.14

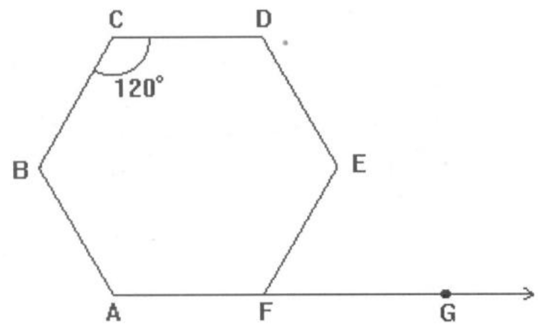
7 * 3.14 is approximately 22. So the answer is 22 cm. Choose 22 cm.

Done Hint More

Appendix B – Captures of Assistments Used

Problem ID: 4751

"IWE 21 2000 (regular hex)" (Problem ID: 4751)



This is a regular hexagon. How many degrees are in angle EFG?

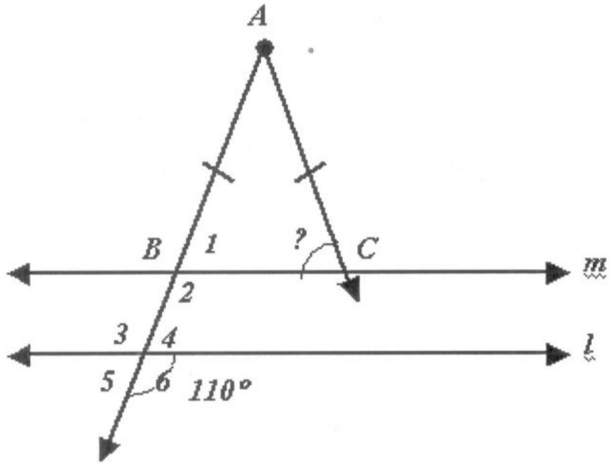
Step 1) Regular indicates that all of the interior angles of this pentagon are equal. Since we know that one interior angle is 120 degrees, we know that all of them are 120 degrees.
Step 2) Since AFE is adjacent to EFG, it is supplementary. This means that AFE + EFG = 180 degrees. To find EFG, we simply subtract AFE from 180. So 180 - 120 = 60 degrees.
Answers: (Interface Type: RADIO_BUTTON)

(Problem ID: 4750)
Which of the following statements are true about a regular hexagon?
Answers: (Interface Type: RADIO_BUTTON)
✗ None of the interior angles are equal
✗ All of the interior angles are less than 90 degrees
✓ **All of the interior angles are equal**

(Problem ID: 4757)
What does supplementary mean for angles?
Answers: (Interface Type: RADIO_BUTTON)
✓ **The angles add up to 180 degrees**
✗ The angles add up to 90 degrees *No, that is called*
✗ The angles are both less than 90 degrees

Problem ID: 4744

"IWE 33 G-2002 (Isosceles Angles)" (Problem ID: 4744)



In the figure shown, lines l and m are parallel, and triangle ABC is isosceles. What is the measure of angle ACB?

Step 1) You first must use the fact that parallel lines that intersect the same line, intersect it at the same angles. This means that angle 1 is equal angle 6, which is 110 degrees.
Step 2) By the definition of an isosceles triangle two of the angles are equal. The unkown angle and angle 1 must be the same since they are both on sides that are equal (indicated by the perpendicular marks on the lines). This means that the unknown angle has the same measure, 110 degrees.
Answers: (Interface Type: RADIO_BUTTON)

(Problem ID: 4742)
Which of the following statements about parallel lines intersecting the same line are true?
Answers: (Interface Type: RADIO_BUTTON)
✗ They angles they form with the line are always 90 degrees
✗ They cannot intersect the same line
✓ **The angles that they form with the line are equal**

(Problem ID: 4743)
Which of the following statements about isosceles triangles are true?
Answers: (Interface Type: RADIO_BUTTON)
✗ They have no equal angles
✗ They have interior angles that sum to 270 degrees
✓ **They have two equal angles**

Problem ID: 3730

"IWE 28 2001" (Problem ID: 3730)

Example Probability Question: How many different ways can 4 dominos be lined up?

Answers: (Interface Type: TEXT_FIELD)

✓ **no answer**

(Problem ID: 3728)

Step 1) The way to determine in how many different orders things can be arranged is by using something called ***factorial***. When you take the factorial of a number you multiply it by all of the numbers before it. This means that 3 factorial is 1x2x3=6, and 5 factorial is 1x2x3x4x5=120. We will be able to use factorial to solve this type of problem.

Before proceeding, let's make sure you understood this step. Which of the following is equal to 4 factorial?

Answers: (Interface Type: RADIO_BUTTON)

✗ 1+2+3+4 *That's incorrect. Please try again.*
✗ 3x4 *That's incorrect. Please try again.*
✗ 4x5x6 *That's incorrect. Please try again.*
✓ **1x2x3x4**

(Problem ID: 3729)

Step 2) Now that we know about factorial, we can use it to find the number of ways things can be arranged. The factorial of the number of objects you are arranging is the number of ways they can be organized. Since we are organizing 4 dominos in this example, 4 factorial, or 1x2x3x4 = 24 is the number of ways they can be organized.

Let's make sure you understood this step. If you have 12 objects, how would you find the number of ways they can be arranged?

Answers: (Interface Type: RADIO_BUTTON)

✓ **Take 12 factorial**
✗ Add up all the numbers before 12 *That's incorrect. Please try again.*
✗ Square 12 *That's incorrect. Please try again.*

Problem ID: 3703

"WorkedExample-Item26-1998" (Problem ID: 3703)

Assistment: MA/Work Example Item26-1998/Work Example Item26-1998-behavior0.xml

Let's try this type of problem:

You have to buy a book which is marked for $51.95. The rate of sales tax is 5%. How much do you have to pay?

You have to pay the 51.95 and the 5% of that price. 5% of 51.95 is 51.95 * 5/100 = 51.95 * 0.05 = 2.5975. Round that to 2.60, as you could never pay $2.5975.

So, in total, you have to pay 51.95 + 2.6 = 54.55

An easier way to calculate the 5% would be by first finding how much 10% of the given sum is and then dividing this result by 2.

10% of 51.95 is 51.95 * 10/100 = 5.195. When rounded, this is 5.2. 5% is half of the previous result, meaning 1/2 * 5.2 = 2.6

Now try it yourself:

Roberto bought skates marked for $69.95. The rate of sales tax is 5%. How much will Roberto have to pay to purchase the skates?

Answers: (Interface Type: TEXT_FIELD)

✓ **73.45**

✗ 73.44 *Looks like you rounded down when you should have rounded up.*

✗ 73.4975 *That is the exact decimal but you need round up the nearest cent. Would a store ever charge you $73.4975.*

(Problem ID: 3701)

Let's break this problem down. Sales tax is 5%. How much will Roberto pay for **sales tax** for the skates?

Answers: (Interface Type: TEXT_FIELD)

✓ **3.5**

✓ **3.50**

✗ 3.49 *That is very close but you rounded wrong. It is true that 5% of $69.95 equals 3.4975 but that should be rounded up to 3.50.*

✗ 7 *That is 10% but you need to find 5%*

Hint 1:
5% is half of 10% Find 10% of the price of the skates.
Hint 2:
To find 10%, you move the decimal point to the left one place.
Hint 3:
10% of $69.99 is $6.995 which rounds to $7.00.
Hint 4:
To find 5%, take half of 10% which was $7.00.Find half of $7.00.
Hint 5:
Half of $7.00 is $3.50.Type 3.50 in the answer box.

(Problem ID: 3702)

Good- the sales tax would be $3.50. So how much in total will Roberto have to pay to purchase the skates?

Answers: (Interface Type: TEXT_FIELD)

✓ **73.45**

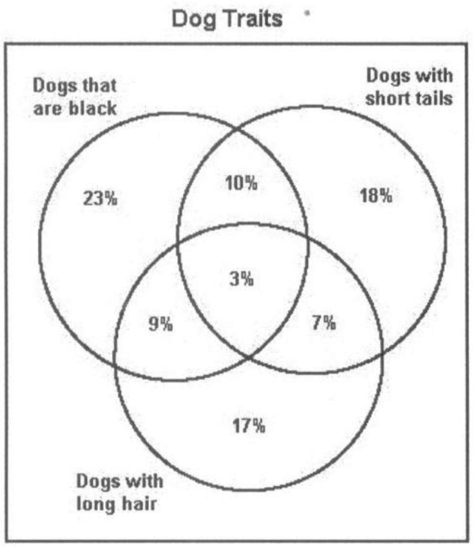
✗ 73.44 *Looks like you rounded wrong. You are off by only one cent.*

Hint 1:
Add $69.95 plus $3.50.
Hint 2:
The change is $0.95 and $0.50. $0.95 + $0.50 = $1.45.
Hint 3:
The dollars are $69 and $3. $69 + $3 = $72.
Hint 4:
Now add up the dollar amount and the change amount.
Hint 5:
The dollar amount is $72 and the change amount was $1.45. $72 + $1.45 = $73.45. Enter 73.45.

Problem ID: 3722

"WorkedExample-Item9D-2003" (Problem ID: 3722)

Example Venn Diagram Problem:



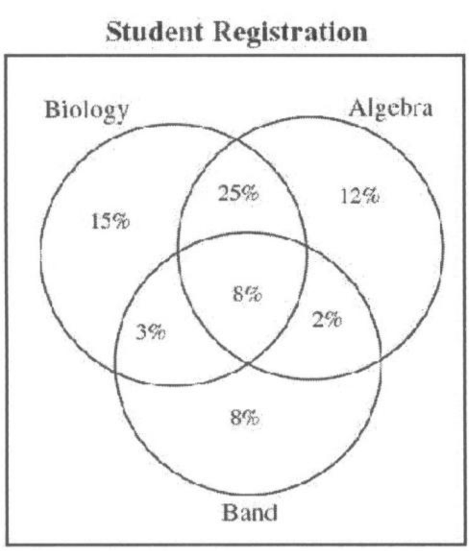
In a group of 1000 dogs, the previous diagram shows the percentages of dogs with various traits.
Out of the 1000 dogs, how many don't have a short tail and long hair?

Step 1) First, find the percentage of dogs that do have a short tail and long hair. To do this look at the area that the "short tail" and "long h circels overlap. The percentage indicated here is the percentage of dogs with both short tails *and* long hair. This is 7%.

Step 2) Now to find the percentage that do *not* have both a short tail and long hair, we must subtract the percentage that do 7% from 100%. 100 - 7 = 93. So the percentage of dogs that do not have short tails and long hair is 93%.

Step 3) Now using this percentage, we need to find out the actual number of dogs out of the total that have a short tail and long hair. The total number of dogs is 1000 so we multiply it by 0.93 to get 93% of it. This means that 930 dogs do not have short tails and long hair.

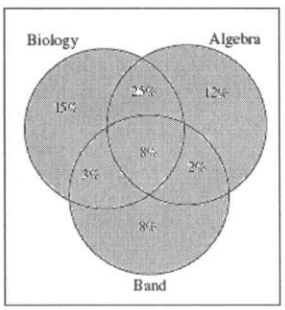
Now try to solve one on your own, using what you've learned from this example. Refer back to the example whenever you need to.



The diagram above shows a relationship among the percentages of students who chose to take Biology, Algebra or Band. If 900 students signed up to take courses, how many will not be taking Biology, Algebra or Band?
Answers: (Interface Type: TEXT_FIELD)
✓
✓ **243**

(Problem ID: 3719)
What is the total percentage of people taking Biology, Algebra or Band?
Answers: (Interface Type: TEXT_FIELD)
✓ **73%**
✓ **73**
✗ **38** *No- that is the percentage of students that were in 2 or 3 classes.*
✗ **100** *No. Add up the numbers and you will see they do not equal 100%. Some percentage of students did not take any of the three classes.*
✗ **100%** *No. Add up the numbers and you will see they do not equal 100%. Some percentage of students did not take any of the three classes.*
✗ **8** *No. 8% of students were in all three classes.*
✗ **35** *No- that is the percentage of students that were in just one of the three classes. You want to figure out the percentage of students that were in any of three.*
✗ **27** *That is the answer to the next question. Instead type 73.*
✗ **657** *You are so close. You have almost solved the whole problem. You figured out that the answer to this question is 73% so type that in. (By the way- 657 in the number of students that would be taking one of the three classes but the question asked how many would NOT be).*

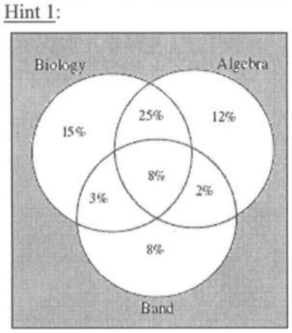
Hint 1:



Sum up all of the percentages shown in the diagram below.
Hint 2:
The total number of students taking Biology, Algebra or Band is the sum of all of the percentages. What is 15 + 25 + 8 + 2 + 3 + 12 + 8?
Hint 3:
The total number of students taking Biology, Algebra or Band is 73%. Type in 73.

(Problem ID: 3720)
Correct. Now you need to find out the percentage of students who did NOT sign up for Biology, Algebra or Band.
Answers: (Interface Type: TEXT_FIELD)
✓ **27**
✓ **27%**
✗ **657** *Close but you took 73% of 900 but that will tell the number of number of students that took would not have taken any of the classes. For this question, type 27.*
✗ **827** *Close 100%-73% = 23%. Type 23%*
✗ **8** *I am not sure why you said 8, but it is the case that you can see from the diagram that 8% of students took all three classes.*
✗ **73** *:Looks like you already gave that answer.*

Hint 1:



You need to determine what percentage of students is in the shaded part of the diagram given below.
Hint 2:

The total percent of students is 100%.The percent of students taking Biology, Algebra or Band is 73%.What is the percentage of students not taking Biology, Algebra or Band?
Hint 3:
What is 100 - 73?
Hint 4:
The percentage of students not taking Biology, Algebra or Band is 27%.Type in 27.

(Problem ID: 3721)

Right. Now you can find the number of students who will NOT be taking Biology, Algebra or Band. The total number of students is 900. What is 27% of 900?

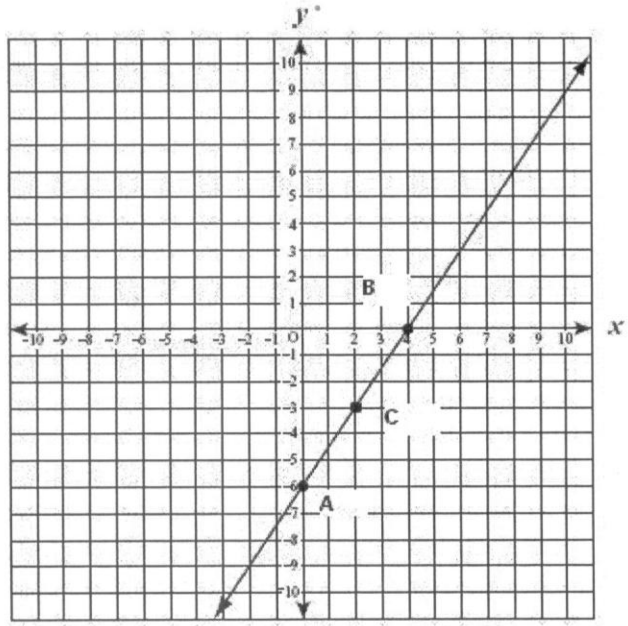
Answers: (Interface Type: TEXT_FIELD)

✓ **243.00**
✓ **243**
✗ 873 *To take 27% you need to multiply the percentage by 900, not substract.*
✗ 24300 *Very close, you just did the decimal arithmetic wrong. You need to move the decimal point two places to the left.*

Hint 1:
Turn 27% into a decimal and multiply by 900.
Hint 2:
27% is 0.27 in decimal.
Hint 3:
What is 900 * 0.27?
Hint 4:
900 * 0.27 = 243. Type in 243

Problem ID: 4734

"IWE 18 A-2003 (slope chart)" (Problem ID: 4734)



What is the slop of the line shown above?

Step 1) Choose any two points to find the slope between. Since this is a straight line it doesn't matter which ones you choose. In this example we'll pick A and B.
Step 2) Since slope is rise over run, or change in y divided by change in x, you must first compute the rise. This is the difference in y values between the two points you chose. In this example the rise is 6 - 0 = 6.

Step 3) Now you must compute run. This is the difference in x values. In this example it is 4.

Step 4) Now divide rise by run. This is the fraction 6/4 which simplifies to 3/2.
Answers: (Interface Type: RADIO_BUTTON)

(Problem ID: 4733)
What is slope defined as?
Answers: (Interface Type: RADIO_BUTTON)
✗ Run over Rise *Close, but not quite.*
✗ Rise times Run *That's the wrong operation.*
✓ **Rise over Run**

(Problem ID: 4735)

What is meant by "run"?

Answers: (Interface Type: RADIO_BUTTON)

✗ The difference in y values of points on a line *Not quite.*

✓ **The difference in x values of two points on the line**

✗ The product of the x and y values of a point *Perhaps you should look at the steps worked out above.*

(Problem ID: 4736)

What is meant by "rise"?

Answers: (Interface Type: RADIO_BUTTON)

✗ The difference in x values of two points on the line *Close, but not quite.*

✓ **The difference in y values of two points on the line**

✗ The sum of the y values of two points on a line *Maybe you should read the steps as they are worked out above.*

Problem ID: 3705

"IWE Item 20 2003" (Problem ID: 3705)

Let?s first work through an example, and then move on to the actual problem. What is 2/3 of 2 2/5?

Step 1) We first must make 2 2/5 into an improper fraction so that we can work with it more easily. To do this, we rewrite the whole number as a fraction with the same denominator as the fractional part. Thus, 2 becomes 10/5, because there are 10 1/5?s in the whole number 2. We now add this to the fractional part and get 12/5.

Step 2) Now, we must multiply the two fractions (2/3 and 12/5) together, because ?of? means multiply. To do this, we multiply the top by the top and the bottom by the bottom. This gives us (2*12)/(3*5) = 24/15. This is our final answer.

Now, let?s see if you understood why we took the steps we did, and what purpose each one served.

What's the first step in making 2 2/5 into an improper fraction?

Answers: (Interface Type: TEXT_FIELD)

✓ **make 2 into a fraction of the form 10/5**

(Problem ID: 3704)

What does ?of? mean in the above problem?

Answers: (Interface Type: TEXT_FIELD)

✓ **multiply**

Problem ID: 4890

Substitution and Order of Operations Example

Let's take this equation for example:

3(-4(2y + x) - y))

Find the value of this expression if x = 6 and y = -2.

To do a complicated expression easily, you need to break it down into simple steps according to the order of operations.

Some of you may remember the phrase:
'P'lease 'E'xcuse 'M'y 'D'ear 'A'unt 'S'ally
This is a usefull tool for remembering the order of operations: The first letter of each word stands for a mathematical operation in the order that they should be calculated.
'P'arenthesis 'E'xponents 'M'ultiplication 'D'ivision 'A'ddition 'S'ubtraction

Step 1: 'P'arenthesis The first step to simplifying an equation is to do the expressions inside of any parenthesis.

In this equation there are two sets of parenthesis; when this happens, you have to do all of the work in the innermost set of parenthesis before moving on to the next set of parenthesis.

The first step to solve this equation is to simplify the inner most parenthesis:

3(-4**(2y + x)** - y))

Substitute x = 6 and y = -2 into the expression in the parenthesis.

(2y + x) --> (2*-2 + 6) --> (-4 + 6) --> (2)

Now you have this equation: **3(-4(2) - y))**

There are still more parenthesis in the problem, so you must do the work inside them before doing any work on the outside.

Looking only at the parenthesis you have (-4(2) - y)
-Notice that when you have a number directly outside of a set of parenthesis, like -4(2), it indicates multiplication. -4(2) *means* -4*2.

Using the order of operations we learned above, we know that we must multiply before we do addition.
(-4*2 - y) First we must multiply: (-4*2 - y) = (-8 - y)
We now have to substitute in y = -2 into the expression.
(-8 - y) --> (-8 - -2) --> (-10)

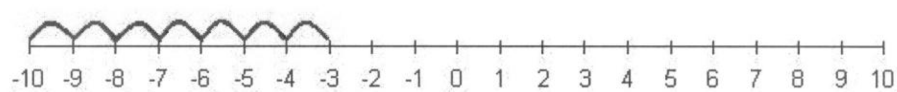
All we have left now is **3(-10)**
Once again, numbers directly outside of parenthesis indicate multiplication: 3(-10) = 3 * -10

The final step is just 3 * -10 = 30

Answers: (Interface Type: ALGEBRA_FIELD)
✓
✓
✓ **-1*34**
✗ 34 *Make sure you check your signs.*

(Problem ID: 4886)
According to the correct order of operations, you need to compute whatever is in the innermost parentheses first. What do you get when you substitute -3 for x and 7 for y in (x - y)?
Answers: (Interface Type: ALGEBRA_FIELD)
✓ **-10**
✗ 4 *No. You need to go to the left on the number line when you subtract.*
✗ 10 *No. You need to go to the left on the number line when you subtract.*
✗ -3-7 *You have to simplify your answer.*

Hint 1:
What is (-3 - 7)?
Hint 2:



Start on -3 and go 7 spaces to the left on the number line
Hint 3:
The answer is -10. Type in -10.

(Problem ID: 4887)
Good. Now the problem looks like this:
-2(x - 2(-10))
Let's get rid of the innermost parentheses by doing the next operation. What is -2(-10)?
Answers: (Interface Type: TEXT_FIELD)
✓ **20**
✗ -8 *Remember that when a number is on the outside of parenthesis you need to multiply it by the inside number.*
✗ -20 *Remember that a negative number multiplied by a negative number gives a positive number.*
✗ -12 *Remember that when a number is on the outside of parenthesis you need to multiply it by the inside number.*

Hint 1:
The -2(-10) means you should multiply what is inside the parentheses by whatever is outside the parentheses.
Hint 2:
What is -2 * -10?
Hint 3:
The answer is 20.

(Problem ID: 4888)
Right. Now the problem looks like this:-2**(x + 20)**. Compute what is inside the parentheses, by substituting x = -3 into (x + 20).

Answers: (Interface Type: TEXT_FIELD)
✓ **17**
✗ 23 *Remember that the 3 is negative.*
✗ 34 *Great, that's the whole thing. But we are looking only for the number that has to be inside the parenthesis.*

Hint 1:
What is (-3 + 20)?
Hint 2:
This can be rewritten as (20 - 3). What is (20 - 3)?
Hint 3:
The answer is 17. Type in 17.

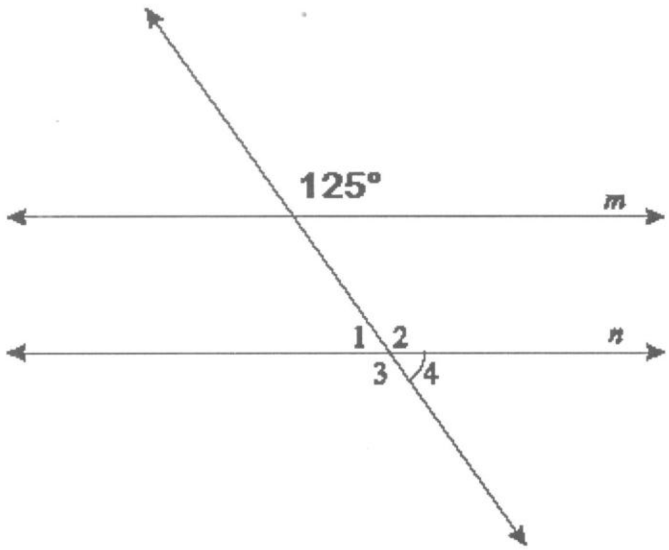
(Problem ID: 4889)

Now you can do the last operation. What is **-2(17)**?
Answers: (Interface Type: TEXT_FIELD)
✓ **-34**
✗ 15 *Remember that when a number is on the outside of parenthesis you need to multiply it by the inside number.*
✗ 34 *Remember that the 2 is negative. A negative number times a positive number gives you a negative number.*

Hint 1:
-2(17) means that you should multiply what is inside the parentheses by what is outside the parentheses.
Hint 2:
What is -2 * 17?
Hint 3:
The answer is -34. Type in -34.

Problem ID: 4714

"IWE 7-G-2000 (Alt Ext Angles)" (Problem ID: 4714)



Use the figure above to answer the following questions. Lines m and n are parallel. What is the measure of angle 4?

Step 1) Find angle s using the fact that it is **supplementary** to the 125 degree angle. Supplementary angles add up to 180. So, if we subtract 125 from 180, we get s.

Step 2) Since m and n are parallel, they intersect the third line at the same angles. This means that angle 4 has the same value as angle s.

Answers: (Interface Type: RADIO_BUTTON)

(Problem ID: 4713)

What does it mean for two angles to be supplimentary?

Answers: (Interface Type: RADIO_BUTTON)

✗ They add up to 125 degrees *No, just because 125 happens to be an angle in this figure does not mean that it is a reasonable answer.*

✓ **They add up to 180 degrees**

✗ They add up to 90 degrees *No, angles that add up to 90 degrees are called complimentary angles.*

(Problem ID: 4715)

If two parallel lines intersect the same line, what can be said about the angles that they intersect it at?

Answers: (Interface Type: RADIO_BUTTON)
✗ They are right angles
✗ They are supplementary
✓ **They are the same**

(Problem ID: 4737)

If two parallel lines intersect the same line, what can be said about the angles that they intersect it at?

Answers: (Interface Type: RADIO_BUTTON)
✗ They are right angles
✓ **They are equal**
✗ They are supplementary

"Worked_Example-Probability 37 - 2002 Morph" (Problem ID: 4885) [MA - 2002 - Spring - 37]

Let's work through a probability example together now:

Number of Children	Number of Families with this many children
0	72
1	147
2	245
3	125
4	57
5	36
6 or more	28

Now lets find out the probability if we pick a random family that they willl have at least 3 or more children.

The fisrt step in solving this problem is to determine which groups qualify for the situation of having 3 or more children. Which groups have at least 3 or more children? Good, now to find the probability you divide the number of people who qualify under the situation by the total number of people.

# of Families with 3 or more children
-- = The Probability that a family will have at least 3 or more children
Total # of Families

First we need to find out how many families have 3 or more children. To do this, add the total number of Families in each row from the table that has 3 or more children.

Number of Children	Number of Families with this many children
3	125
4	57
5	36
6 or more	28

** 125 + 57 + 36 + 28 = 246**

Now we need to find the total number of families in the sample

72 + 147 + 245 + 125 + 57 + 36 + 28 = 710

Now that we have the number of families with 3 or more children and the total number of families, we can divide them to find the probability that we will chose a family with at least 3 children or more.

246 The # of families with 3 or more children
---- = -- = **.346**
710 The Total Number of Families

Round .346 to the nearest hundreth and you get .35 which is the same as 35%

There is a 35% probability that a randomly selected family from this sample will have at least 3 or more children.
Answers: (Interface Type: TEXT_FIELD)
✓ **0.35**
✓ **.35**
✓ **35%**
✓ **35**
✗ 80 *To round to the nearest hundreth wound be .81 or 81%. Type in .81.*

(Problem ID: 4880) [MA - 2002 - Spring - 37]
The chart below shows the amount spent by customers at a department store on a typical business day.

Amount Spent	Number of Customers
$0	158
$0.01-$5.99	94
$6.00-$9.99	203
$10.00-$19.99	126
$20.00-$49.99	47
$50.00-$99.99	38
$100 and over	53

Based on the information in the chart, what is the probability that a customer entering the store on a typical day will spend **less than $20**? Express your answer as a decimal and round your answer to the nearest hundredth.
Answers: (Interface Type: TEXT_FIELD)

(Problem ID: 4881) [MA - 2002 - Spring - 37]
Let's assign some meaning to the event in this problem. Let's assume that event is the number of customers who will spend at most $20.

Which one of the following is **not** an outcome of an event "number of customers who will spend at most $20"?
Answers: (Interface Type: RADIO_BUTTON)

✗ Number of customers who will spend $20
✗ Number of customers who will spend $10
✓ **Number of customers who will spend $50**
✗ Number of customers who will spend $0

Hint 1:
At most $20 means $20 or less.

(Problem ID: 4882) [MA - 2002 - Spring - 37]

Probability of an event = number of time an event occurs / total number of outcomes

What is the number of times an event occurs? (What is the number of people that spend between zero dollars and $19.99)

Answers: (Interface Type: TEXT_FIELD)

✓ **581**

Hint 1:
What is the number of customers who will spend $20 or less?
Hint 2:
To find the total number of customers who will spend $20 or less, you will need to add the number of customers who will spend $0 to the number of customers who will spend $0.01 - $5.99 to the number of customers who will spend $6.00 - $9.99 to the number of customers who will spend $10.00 - $19.99
Hint 3:
158 + 94 + 203 + 126 = 581. Type 581 in the text field provided and hit the enter key to submit the answer.

(Problem ID: 4883) [MA - 2002 - Spring - 37]

What is the total number of outcomes?

Answers: (Interface Type: TEXT_FIELD)

✓ **719**

Hint 1:
What is the total number of customers?
Hint 2:
Use the given table to add up all the customers at a department store on a typical business day.
Hint 3:
158+94+203+126+47+38+53 = 719. Type 719 in the text box provided and hit the enter key to submit the answer.

(Problem ID: 4884) [MA - 2002 - Spring - 37]

So, what is the closest probability that a customer entering the store on a typical day will spend **at most** $20? Express your answer as a decimal and round it to the nearest hundredth.

Answers: (Interface Type: TEXT_FIELD)

✓ **81%**
✓ **81**
✓ **0.81**
✓ **.81**

Hint 1:
Use the formula for the probability of an event that I already gave you to find the solution.
Hint 2:
Divide the number of outcomes corresponding to the event by the total number of outcomes.

Problem ID: 3710

"IWE Item36 2002" (Problem ID: 3710)

Example Fraction Division Problem:

$$p \div \frac{1}{5}$$

Evaluate the above equation.

Answers: (Interface Type: TEXT_FIELD)

✓

✗ *That is the exact decimal but you need round up the nearest cent. Would a store ever charge you $73.4975.*

(Problem ID: 3706)

Step 1) When dividing two fractions, it is the same as multiplying the first one by the *inverse* of the second one. '
as:

$$\frac{p}{1} \text{ X } \frac{5}{1}$$

Answers: (Interface Type: RADIO_BUTTON)

✗ I don't understand this step *P can be thought of as the fraction P/1. This makes the above problem a matter of dividing two fractions. To accomplish this, you leave P/1 as it is, and flip the second fraction. You then change the division sign to a multiplication sign.*

✓ **I understand this step**

(Problem ID: 3707)

Step 2) Now, we multiply the two fractions together. To do this, we multiply the top of the first by the top of the
bottom of the second. This gives us the answer:

$$\frac{p}{1} \text{ X } \frac{5}{1} = \frac{5p}{1} = 5p$$

Answers: (Interface Type: RADIO_BUTTON)

✗ I don't understand this step *You multiply P by 5 to get the top. You multiply 1 by 1 get the bottom. This results in 5P/1. 5P/1 is the same us just 5P.*

✓ **I understand this step**

(Problem ID: 3708)

Now, using what you've learned, try one on your own. Refer back to the example whenever you need it.

$$p \div \frac{1}{10}$$

Which of the following is equivalent to the equation above?

Answers: (Interface Type: RADIO_BUTTON)

✗ $^{p}/_{10}$ *That is the exact decimal but you need round up the nearest cent. Would a store ever charge you $73.4975.*

✗ .01p *That is the exact decimal but you need round up the nearest cent. Would a store ever charge you $73.4975.*

✗ .1p *That is the exact decimal but you need round up the nearest cent. Would a store ever charge you $73.4975.*

✓ **10p**

(Problem ID: 3709)

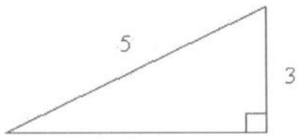
Default For a correct answer please Update

Answers: (Interface Type: TEXT_FIELD)

Problem ID: 372

"Worked Example - Item15-1998" (Problem ID: 3727)

Example Right Triangle Problem:



Pythagorean Theorem

$A^2 + B^2 = C^2$

You know two sides of a right triangle, shown below. Find the unknown length.

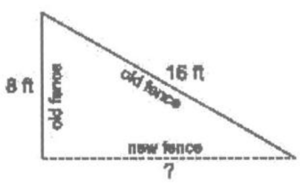
Step 1) We know that the sum of the squared lengths of the two shorter sides of a right triangle is equal to the square of the longest side, or $a^2+b^2=c^2$. Since we know one of the short sides, and the long side, we can plug those into the equation for a and c. This gives us: $3^2+b^2 = 5^2$.

Step 2) Let's simplify that equation. $5^2 = 5 \times 5 = 25$. $3^2 = 3 \times 3 = 9$. This means the equation can now be rewritten as: $9 + b^2 = 25$.

Step 3) Next, we want to get our unknown variable, *b* all alone on one side of the equation. To do that, we must subtract 9 from each side. This gives us $b^2 = 25 - 9$. This simplifies to $b^2 = 16$.

Step 4) Now we just have to get rid of that exponent on b so that we can see what it is. Since b is squared, to unsquare it we take the square root of both sides. This gives us b = 4. Thus, we know the length of the unkown side is 4.

Now, using what you've learned from the example, try one on your own. Refer back to the example whenever you need to.

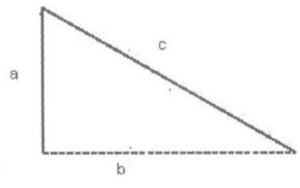


Manuel is planning a flower garden shaped like a right triangle.
He will use an old 16-foot fence for the longest side and an old 8-foot
fence for another side as shown. What is the best estimate of the amount of fencing he will need to the nearest whole number for the third side?

Answers: (Interface Type: TEXT_FIELD)

✓ **14**

(Problem ID: 3723)



What is the relationship between the sides of the right triangle shown above?

Answers: (Interface Type: RADIO_BUTTON)

✗ $c^2 + a^2 = b^2$ *No, this is not correct. You need to apply the Pythagorean theorem.*

✗ $b^2 + c^2 = a^2$ *No, this is not correct. You need to apply the Pythagorean theorem.*

✗ $a^2 - b^2 = c^2$ *No, this is not correct. You need to apply the Pythagorean theorem.*

✓ **$a^2 + b^2 = c^2$**

Hint 1:
This calls for the Pythagorean theorem. This theorem can be used to find the length of a side of a right triangle given the other 2 sides.
This is the Pythagorean theorem: $a^2+b^2=c^2$

(Problem ID: 3724)
Assume *x* is the new fence and the values of the old fences are as shown in the figure (refer to the figure in the main problem). Apply the Pythagorean equation to these values.
Answers: (Interface Type: RADIO_BUTTON)
✗ $16^2 + 8^2 = x^2$ *No, this is not correct. Please refer to the equation in the previous answer and apply it to the given values.*
✓ **$8^2 + x^2 = 16^2$**
✗ $8^2 - x^2 = 16^2$ *No, this is not correct. Please refer to the equation in the previous answer and apply it to the given values.*
✗ $x^2 + 16^2 = 8^2$ *No, this is not correct. Please refer to the equation in the previous answer and apply it to the given values.*

Hint 1:
This calls for the Pythagorean theorem. Please refer to the equation in the previous answer and apply it to the given values.
Hint 2:
$8^2+x^2=16^2$ is the correct choice.

(Problem ID: 3725)
Now what is the value of x^2 in the equation $8^2 + x^2 = 16^2$?
Answers: (Interface Type: TEXT_FIELD)
✓ **192**
✗ 14 *Great Job- that is the value for x but this question as was asking for x squared. Type in 192. The next question will ask for 14.*

Hint 1:
To find x^2 in the equation $8^2 + x^2 = 16^2$, we write the equation as: $x^2=16^2-8^2$
Hint 2:
To find x^2 you have to find out what you are adding to 8^2 to get 16^2. Calculate 16^2 which is 16 times 16 and 8^2 which is 8 times 8.
Hint 3:
$16^2 = 256$ and $8^2 = 64$
$x^2 = 256 - 64$
What is 256 - 64?
Hint 4:
256-64=192

(Problem ID: 3726)
Now we are ready to answer the final question. So x^2 is 192. What is the best estimate of x to the nearest whole number?
Answers: (Interface Type: TEXT_FIELD)
✓ **14**
✗ 13.8 *Good- but just round that to 14 please.*

Hint 1:
To find x consider: $10^2=100$ therefore x must be larger than 10.
Hint 2:
Let us try a larger number say, 13.
One way to multiply 13*13 is
13*10=130
13*3=39
130+39=169
Thus, 13*13=169
Hint 3:
Here we know that 192 is greater than 169, and so x is greater than 13.
Hint 4:
So try x = 14
Actually, 14*14 = 196, but you are looking for the best estimate. This is the closest to 192. Hence x=**14** so type 14.