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Integer-valued DEA super-efficiency based on directional distance function with an application of evaluating mood and its impact on performance

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Integer-valued DEA super-efficiency based on directional distance function with an application of evaluating mood and its impact on performance

Abstract: The conventional data envelopment analysis (DEA) assumes that the inputs and outputs are real values. However, in many real world instances, some inputs and outputs must be in integer values. While integer-valued DEA models have been proposed, the current paper develops an integer-valued DEA super-efficiency model. Super-efficiency DEA models are known to have the problem of infeasibility. Recent studies have shown that directional distance function (DDF) based super-efficiency does not seem to have the infeasibility issue. However, the existing DDF DEA approach cannot be directly modified to incorporate integer values under the concept of super-efficiency. The current paper thus modifies the DDF approach so that integer values can be incorporated under the concept of super-efficiency. Our proposed approach is then applied to evaluating mood and its impact on performance. We use both traditional methods as well as the new DEA model to calculate a set of scores for the constructs under the investigation. These analyses extend the application of the DEA method to judgment and decision making. In particular, the results show that the new DEA model is able to reveal subtle nuances such as the impact of mood on performance with a decision support system.

Keywords: Data envelopment analysis (DEA); Efficiency; Integer; Super-efficiency; Mood

1. Introduction

Data envelopment analysis (DEA) is a data-oriented technique originally introduced by Charnes, Cooper and Rhodes (1978) for measuring relative efficiency for peer decision making units (DMUs). These DMUs have multiple performance measures that are called inputs and outputs. A recent citation-based study by Liu et al. (2013) indicates that DEA literature's size will significantly eventually grow in the coming years in many application areas. Such a projected growth is accompanied with new methodological development in DEA along with new applications.

Standard DEA models assume real values for all inputs and outputs. However, there are situations when inputs and outputs have integer values. In fact, in many behavioral studies integer values representing an individual's perceptions of and/or reactions to a stimulus form a good portion of collected data. For example, the technology acceptance model (TAM) (Davis, 1989), a widely used acceptance theory in information system (IS) research is composed of integer values only. According to this theory a user's perception of ease of use and usefulness of a system can predict the user's willingness to use the system. All three variables in this model (ease of use, usefulness, and intention to use) are captured as subjective self-report measures on a Likert-type scale (e.g., Djamasbi et al., 2010). Similarly, behavioral studies often capture a person's affective state using Likert-type scales (e.g., Djamasbi et al., 2010; Djamasbi, 2007). For example, a person's positive mood is often reported by integer values that represent the person's "happy", "pleased", and/or "glad" feeling states (Djamasbi, 2007; Djamasbi et al., 2010; Elsbach and Barr, 1999).

Researchers have studied the issue of modeling integer inputs/outputs in DEA. Lozano and Villa (2006) propose the mixed integer linear programming (MILP) model to restrict the computed targets to integers. Kuosmanen and Kazemi Matin (2009) later improve Lozano and

Villa's (2006) model based upon a new axiomatic foundation for production models involving integers. One related issue is how to develop super-efficiency models when integer inputs and outputs are present. The concept of super-efficiency was first introduced by Andersen and Petersen (1993) to differentiate the performance of efficient DMUs. Super-efficiency refers to a situation when the DMU under evaluation is removed from the reference set. In general, the super-efficiency models work well under the assumption of constant returns to scale (CRS), but become infeasible for certain DMUs under the assumption of variable returns to scale (VRS) (Seiford and Zhu, 1999). This infeasibility restricts a broader use of super-efficiency method. While the infeasibility issue of super-efficiency under the standard DEA has been well studied (see, e.g., Tone, 2002; Liang et al., 2009; Du et al., 2010), super-efficiency of DEA models with integer values has received limited attention. Du et al. (2012) are the first and only study that develops an additive super-efficiency model with integer inputs and outputs. Although their super-efficiency model is always feasible (under VRS condition), their model requires a two-stage procedure where the efficient and inefficient DMUs are calculated in two different computer runs, and their approach is based upon non-radial DEA measures. Later Chen et al. (2012) incorporate undesirable factors into Du et al.'s (2012) integer-valued additive efficiency and super-efficiency models to cope with the special cases where production outputs are undesirable and restricted to integer values.

On the other hand, Ray (2008) develops a radial super-efficiency model based upon Nerlove-Luenberger (N-L) measure of directional distance function (DDF) (Chambers et al., 1996). The advantage of Ray's (2008) is that the model is based upon radial measures and does not pose a similar infeasibility problem in the conventional VRS super-efficiency models. The current paper therefore seeks to incorporate integer values into Ray's (2008) model. We show that Ray's (2008) approach cannot be directly modified to incorporate integer values, because the modified model can rate all DMUs (including inefficient DMUs) being efficient or on the frontier. Note that the DDF requires that the inputs decrease at the same rate as outputs increase to reach the DEA frontier. Such a requirement becomes problematic when inputs and outputs are integer valued under the concept of super-efficiency. We therefore modify this particular requirement by assuming that inputs and outputs of a DMU under evaluation can use different changing rates to reach the frontier constructed by the remaining DMUs under super-efficiency. Our integer DDF-based model does not suffer the problem of infeasibility (under the condition of VRS).

Unlike the model of Du et al. (2012), the two-stage calculation procedure is no longer needed in our approach. Our integer DDF-based measure of super-efficiency is capable of deriving a full ranking of efficient DMUs. Being able to distinguish the performance of efficient DMUs can not only provide decision makers with better insights into the performance of peer DMUs, but also help carry out further analysis for decision makings. For example, as demonstrated in our empirical application section, the newly developed model can be used to study whether decision makers in positive mood made more accurate judgments when using a Decision Support System (DSS).

The remainder of the paper is organized as follows. The next section briefly presents the concept of directional distance function (DDF) along with the DEA super-efficiency models with integer values. We use a numerical example to show that DDF-based super-efficiency with integer inputs and outputs will produce undesirable results. We then develop our integer DDF-based super-efficiency model that is always feasible (under the VRS condition). We then apply the newly developed model to examine several hypotheses that were formed based on research in the area of the effective usage of a Decision Support System (DSS). Conclusions are given in the last section.

2. Directional distance function and DEA models with integer values

Suppose we have *n* DMUs with *m* inputs and *s* outputs. Unit *j* is represented by $DMU_j\left(j=1,\ldots,n\right)$, whose *i*th input and *r*th output are denoted as $x_{ij}\left(i=1,\ldots,m\right)$ and $y_{rj}\left(r=1,\ldots,s\right)$, respectively. Then under the assumption of VRS, the production possibility set (PPS) formed from the above set of *n* DMUs is represented by

$$T=\left\{\left(x,y\right)|\, x_i\geq\sum_{j=1}^{n}\lambda_j x_{ij}, i=1,\ldots,m;\, y_r\leq\sum_{j=1}^{n}\lambda_j y_{rj}, r=1,\ldots,s;\sum_{j=1}^{n}\lambda_j=1, \lambda_j\geq 0, j=1,\ldots,n\right\}$$

Consider an input-output bundle or DMU_k $\left(x_k,y_k\right)$ and a reference input-output bundle $\left(g^x,g^y\right)$. Then based on *T*, the directional distance function (DDF) is defined as (Chambers et al., 1996):

$$D\left(x_k,y_k;g^x,g^y\right)=\max\beta:\left(x_k+\beta g^x,y_k+\beta g^y\right)\in T$$

The reference bundle $\left(g^x,g^y\right)$ can be chosen in an arbitrary way, which makes the DDF varies with reference to any specific DMU. Chambers et al. (1996) select $\left(-x_{ik},y_{rk}\right)$ for $\left(g^x,g^y\right)$, and obtain the standard DDF as

$$D\left(x_k,y_k\right)=\max\beta:\left(\left(1-\beta\right)x_k,\left(1+\beta\right)y_k\right)\in T \quad (1)$$

In DDF (1), each input is decreased and each output is increased simultaneously by the same proportion β.

Ray (2008) points out that the optimal value of β in (1) is the Nerlove-Luenberger (N-L) measure of technical inefficiency for the evaluated DMU, whose efficiency can be calculated as

$(1-\beta)$. Ray (2008) further develops a super-efficiency model

$$D_k(x_k, y_k) = \max \beta_k : ((1-\beta_k)x_k, (1+\beta_k)y_k) \in T_k \tag{2}$$

where $T_k = \left\{ (x, y) \middle| x_i \geq \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j x_{ij}, i = 1,\ldots,m; y_r \leq \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j y_{rj}, r = 1,\ldots,s; \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1,\ldots,n, j \neq k \right\}$.

We now suppose that part of the inputs and outputs are constrained to integer values. We follow the notations used in Kuosmanen and Kazemi Matin (2009), and denote the subsets of integer-valued, real-valued inputs and outputs by I^I, I^{NI}, O^I and O^{NI}, respectively. We assume that I^I and I^{NI}, as well as O^I and O^{NI} are mutually disjoint, and satisfy $I^I \cup I^{NI} = \{1, 2, \ldots, m\}$, $O^I \cup O^{NI} = \{1, 2, \ldots, s\}$. For any integer-restricted measure, its reference target with respect to the efficient frontier is also supposed to be an integer. Then, the VRS version of Kuosmanen and Kazemi Matin's (2009) input-oriented mixed integer linear programming (MILP) model is developed as

$$\begin{aligned}
& Eff_k^+ = \min \ \hat{\theta}_k \\
& s.t. \quad y_{rk} + s_r^+ = \sum_{j=1}^{n} \lambda_j y_{rj}, r \in O^I \cup O^{NI} \\
& \qquad \hat{\theta}_k x_{ik} - s_i^- = \sum_{j=1}^{n} \lambda_j x_{ij}, i \in I^{NI} \\
& \qquad \tilde{x}_{ik} - s_i^- = \sum_{j=1}^{n} \lambda_j x_{ij}, i \in I^I \\
& \qquad \hat{\theta}_k x_{ik} - \tilde{s}_i^- = \tilde{x}_{ik}, i \in I^I \\
& \qquad \tilde{x}_{ik} \in Z_+, i \in I^I \\
& \qquad \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1,\ldots,n \\
& \qquad s_r^+ \geq 0, s_i^- \geq 0, r \in O^I \cup O^{NI}, i \in I^I \cup I^{NI}, \tilde{s}_i^- \geq 0, i \in I^I
\end{aligned} \tag{3}$$

where $s_r^+, s_i^-, \tilde{s}_i^-$ represent slack variables, $\hat{\theta}_k$ represents the input-oriented efficiency for DMU_k, and $\tilde{x}_{ik} \in Z_+, i \in I^I$ is the integer-valued reference point for inputs from I^I. DMU_k is considered efficient if the optimal value for $\hat{\theta}_k$ equals one. If we remove the DMU_k under evaluation from the reference set, we get the super-efficiency version of (3). However, such super-efficiency model can be infeasible for some efficient DMUs.

Based upon (3), the super-efficiency version of (2) can be presented as

$$
\begin{aligned}
\max \quad & \beta_k \\
s.t. \quad & y_{rk} + \beta_k y_{rk} = \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j y_{rj} - s_r^+, \, r \in O^{NI} \\
& x_{ik} - \beta_k x_{ik} = \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j x_{ij} + s_i^-, \, i \in I^{NI} \\
& \tilde{x}_{ik} - s_i^I = \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j x_{ij}, \, i \in I^I \\
& \tilde{y}_{rk} + s_r^I = \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j y_{rj}, r \in O^I \\
& \tilde{x}_{ik} + s_i^- = x_{ik} - \beta_k x_{ik}, \, i \in I^I \\
& \tilde{y}_{rk} - s_r^+ = y_{rk} + \beta_k y_{rk}, r \in O^I \\
& \tilde{x}_{ik} \in Z_+, i \in I^I \\
& \tilde{y}_{rk} \in Z_+, r \in O^I \\
& \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j = 1, \, \lambda_j \geq 0, \, j = 1, \ldots, n, \, j \neq k \\
& s_r^+ \geq 0, s_i^- \geq 0, r \in O^I \cup O^{NI}, i \in I^I \cup I^{NI} \\
& s_i^I \geq 0, i \in I^I, s_r^I \geq 0, r \in O^I \\
& \beta_k \textit{ free in sign}
\end{aligned}
\tag{4}
$$

We next consider the numerical example presented in Table 1 where we have five DMUs with two integer-valued inputs and one integer-valued output.

<Insert Table 1 about here>

If we apply model (4) to the DMUs in Table 1, we have β_k^* = 0 for all five DMUs, indicating that all five DMUs are on the frontier. This is obviously an erroneous result since DMUs C and D are dominated by DMUs A and B, respectively. This problem is caused by using a unified changing rate β_k for both inputs and outputs, which leads to the same optimal solution between DMUs A and C (or B and D). In the next section, we will modify model (4) so that the frontier determined by the DMUs is correctly reflected in the super-efficiency model.

3. Integer DDF-based super-efficiency

From the numerical example in Table 1, we see that both the two inputs and the single output cannot be moved at the same rate to the frontier due to the fact that output level is already efficient. This indicates that different rates β_x and β_y should be used for inputs and outputs, respectively. If DMU *k* is efficient, inputs should be augmented and outputs should be contracted, which means $\beta_x \le 0$ and $\beta_y \le 0$. On the other hand, if DMU *k* is inefficient, inputs should be contracted and outputs should be augmented, which means $\beta_x \ge 0$ and $\beta_y \ge 0$. These conditions can be incorporated by enforcing $\beta_x \beta_y \ge 0$. Therefore, model (4) can be modified as

$$
\begin{aligned}
\max \quad & \beta_x + \beta_y \\
s.t. \quad & y_{rk} + \beta_y y_{rk} = \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j y_{rj} - s_r^+ , r \in O^{NI} \\
& x_{ik} - \beta_x x_{ik} = \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j x_{ij} + s_i^- , i \in I^{NI} \\
& \tilde{x}_{ik} - s_i^I = \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j x_{ij} , i \in I^I \\
& \tilde{y}_{rk} + s_r^I = \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j y_{rj} , r \in O^I \\
& \tilde{x}_{ik} + s_i^- = x_{ik} - \beta_x x_{ik} , i \in I^I \\
& \tilde{y}_{rk} - s_r^+ = y_{rk} + \beta_y y_{rk} , r \in O^I \\
& \tilde{x}_{ik} \in Z_+ , i \in I^I \\
& \tilde{y}_{rk} \in Z_+ , r \in O^I \\
& \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j = 1,\ \lambda_j \geq 0,\ j = 1, \ldots, n,\ j \neq k \\
& s_r^+ \geq 0, s_i^- \geq 0, r \in O^I \cup O^{NI}, i \in I^I \cup I^{NI} \\
& s_i^I \geq 0, i \in I^I, s_r^I \geq 0, r \in O^I \\
& \beta_x \beta_y \geq 0
\end{aligned} \tag{5}
$$

Due to the constraint of $\beta_x \beta_y \geq 0$, model (5) is a non-linear program. However, the following procedure will convert it into a mixed integer linear programming problem.

By introducing two binary integer variables, *w* and *z*, the nonlinear constraint $\beta_x \beta_y \geq 0$ can be transformed into a set of linear constraints as follows:

$$
\begin{aligned}
& -M(1-w) \leq \beta_x \leq Mw \\
& -Mz \leq \beta_y \leq M(1-z) \\
& w + z = 1 \\
& w \in \{0,1\},\ z \in \{0,1\}
\end{aligned}
$$

Where *M* is a sufficiently large number. Note that $w = 1$ and $z = 0$ signify $\beta_x \geq 0$ and $\beta_y \geq 0$, and $w = 0$ and $z = 1$ signify $\beta_x \leq 0$ and $\beta_y \leq 0$, respectively. Replacing $\beta_x \beta_y \geq 0$ with the above set of linear constraints along with the two binary integer variables, model (5) now becomes a mixed integer linear programming problem.

Note that $\beta_x^* \le 1$ and $\beta_y^* \ge -1$. We therefore define the super-efficiency score as $\dfrac{1-\beta_x^*}{1+\beta_y^*}$. The higher the score, the more efficient a DMU under evaluation is. Note also that when a DMU under evaluation is inefficient, this score is between 0 and 1. When a DMU under evaluation is efficient, this score is greater than 1. When $\beta_y^* = -1$, the score diverges to infinity and we allow the super-efficiency score of infinity.

Unlike the model of Du et al. (2012), the two-stage calculation procedure is no longer needed in our approach.

Note that non-zero slacks can still exist after the movements represented by β_x^* and β_y^* in model (5). We can incorporate a non-Archimedean infinitesimal with sum of slacks into the objective function of model (5) to reflect the optimal slack calculation process in the standard DEA model.

Table 2 reports the results when model (5) is applied to the numerical example in Table 1. The fourth column of Table 2 shows the reference point for a DMU under evaluation. Both DMUs A and B are efficient, and therefore their supper-efficiency scores are greater than 1. DMU E is inefficient, and therefore its supper-efficiency score is less than 1. Both DMUs C and D are on the integer frontier, and thus their super-efficiency scores are equal to 1. Note that under the standard DEA, C and D would be inefficient or dominated.

<Insert Table 2 about here>

We need to point out because of the integer-valued inputs and outputs, the DEA frontier is different from the one under the standard DEA model. Under the standard DEA, the frontier is

AB. Under the integer DEA, the frontier is expanded with other efficient points (that are dominated by AB under the standard DEA). In Figure 1, we have four points that are integer-efficient within the production possibility set (DMUs A and B, and other two squared points). Note that the concept of integer-efficient is different from the conventional DEA efficient. The projection of E is E' in Figure 1. Except for DMU E, all points in Figure 1 are on the integer DEA frontier. However, we note that DMU C is dominated by DMU A, and DMU D is dominated by DMU B. This dominance is reflected by the slack value in input 1 and input 2. In other words, DMUs C and D are weakly integer-efficient. A DMU is *integer-efficient* if and only if there is no integer point within the production possibility set that dominates the DMU, or $\beta_x^* = \beta_y^* = 0$ and all slacks are zero at optimality. Otherwise, if $\beta_x^* = \beta_y^* = 0$ and some slacks are positive, then a DMU is weakly integer-efficient.

<Insert Figure 1 about here>

We finally examine a situation when x_2 is not integer-valued in Table 1, namely, we assume only x_1 and y_1 are integer-valued. Table 3 reports the results.

<Insert Table 3 about here>

Under this situation, A, B and C are all efficient (C is weakly efficient) and their reference points are themselves. D' and E' are the reference points of D and E, respectively. Note that D' and E' are not exactly on the line segment between A and B. D' and E' are weakly efficient. The integer efficient points are A and B and the four squared points in between A and B in Figure 2, when x_1 and y_1 are integer-valued.

<Insert Figure 2 about here>

4. Application

Decision making models in the information systems (IS) literature often rely on a cognitive framework (e.g., Davis et al., 1989; Tao et al., 2007; Benbasat and Todd, 1996). Grounded in empirical findings of cognitive psychology in the past three decades, a number of IS studies have shown that including users' mild positive mood states in such models can provide a more complete picture of user behavior (Djamasbi et al., 2010; Djamasbi et al., 2009; Djamasbi 2007). These studies show that a user's mild positive mood state not only can impact the user's evaluation of a decision support system (Djamasbi et al., 2010; Djamasbi et al., 2009), but also can influence his or her effective usage of that system (Djamasbi, 2007). For example, users who were induced with a positive mood, compare to those whose mood was not manipulated, utilized more information that was provided by a DSS and made more accurate decisions (Djamasbi, 2007).

In this study we use our new proposed model to examine users' decision behavior. A task based on Holt et al.'s (1960) model of the production-scheduling problem is used. The production-scheduling problem is often used in decision making studies (e.g., Djamasbi et al., 2004; Lim et al., 2005; Djamasbi, 2007) because it is a managerially relevant problem which has been calibrated with actual data (Holt et al., 1956).

4.1 Data

Consistent with previous IS decision making studies that investigate the impact of mood on adoption and effective usage of decision support systems (e.g., Djamasbi, 2007; Djamasbi et al., 2010), we included users' positive mood in our investigations. To measure a user's positive mood, we used a previously validated 3-item Likert-type self-report questionnaire which asked the participants to rate how accurately a number of descriptors matched their current mood

(Djamasbi, 2007). The participants rated on a seven-point scale (with 1 denoting “strongly disagree”, 4 denoting “neutral”, and 7 denoting “strongly agree”) how the words “pleased”, “happy”, and “glad” described their mood. These words have been shown to reliably capture one’s mild positive affective states (Djamasbi, 2007; Djamasbi et al., 2010; Elsbach and Barr, 1999). It is customary to calculate a single composite mood score for each participant. As in prior studies (e.g., Djamasbi et al., 2010), we calculated a single mood score by averaging the items “happy”, “pleased”, and “glad” on the survey for each participant.

Decision behavior, as in prior studies (Djamasbi et al., 2004; Djamasbi, 2007) was operated as quality of judgments made using a DSS. Judgment is a cognitive process in which an individual comes to a conclusion about a future event which is represented through a set of data (Hammond, 1975; Djamasbi et al., 2004). In our case, participants had to make a judgment about the production level at a manufacturing plant based on a set of available data regarding the size of the work force, the inventory levels, and product demand. The production-scheduling decision was modeled through the following equation (Holt et al., 1956):

$$\mathrm{Pr}oduction \quad Decision = \beta_0 + \beta_1 * WF - \beta_2 * INV + \beta_3 * DCM + \beta_4 * DNM + \beta_5 * DTM + e \qquad (6)$$

where *WF* refers to workforce, *INV* to inventory, *DCM* to current demand, *DNM* to demand for the next month, *DTM* to demand for two months ahead. The coefficients values were β_0=148.5, β_1=1.005, β_2=0.464, β_3==0.464, β_4==0.239, and β_5==0.113. The last variable in the above equation is a randomly generated and normally distributed error term (*e*) which represents the uncertainty of the real world decision environments (Djamasbi, 2007; Djamasbi et al., 2004).

To calculate quality of judgments, we used the approach proposed by Lee et al. (2003). This approach assesses the accuracy of a judgment in terms of mean squared error (*MSE*) and decomposes it into three sub-error components: *bias*, *variability*, and *linear correspondence*:

$$MSE = bias + \text{var}\,iability\ + linear\ correspondence \tag{7}$$

Bias, in the above equation, represents the extent to which participants' judgments fall above or below the production decisions calculated with the formula in Equation (6). *Variability* represents the error attributable to the difference between the standard deviations of the decision values calculated via Equations (6) and those of the participants' judgments. *Linear correspondence* refers to deviation between decision values derived from Equation (6) and those decisions made by participants. This last term reflects decision makers' lack of achievement in matching their judgments with the optimal decision values generated by Equation (6).

The participants were 134 business students in a major university. As in prior studies (Djamasbi, 2007), each participant made 30 production decisions. The task is designed in a way so that decision makers who are new to the task can improve their judgments over the course of the trials by utilizing the provided feedback after each trial (Djamasbi, 2007; Lim et al., 2005; Remus et al., 1996). Because our participants were business students who had no prior experience with the task, they served as a suitable population for our experiment (Cooksey, 1996; Swieringa and Weick, 1982).

The overall judgment quality was assessed by examining participants' performance over all 30 trials (Djamasbi et al., 2004). As in prior research, to track improvement over the course of the trials we also tracked performance in blocks of trials (Djamasbi et al., 2004; Gillis, 1975). Prior research suggests that participants need roughly about fifteen trials to gain experience with the task and adopt a decision strategy (Remus and Katteman, 1987; Djamasbi et al., 2004). Thus, we examined performance in the first and last blocks, each of which consisted of fifteen trials.

4.2. Extending the application of DEA

A major objective of the paper is to show that DEA technique is applicable to many different problems. To achieve this objective, we selected a problem that is typically not examined using the DEA method. In the following sections, we first test our research question via a method that is typically used in the DSS literature to test the phenomenon under investigation and then we repeat the same test using the DEA scores.

4.2.1 Will higher mood scores lead to better performance (smaller MSE errors)?

Positive affect theory asserts that positive mood has a significant impact on a person's problem solving ability (for a review of this theory and related literature, see Isen, 2008). Grounded in this theory, a previous study shows that users induced with a positive mood, compare to those whose mood was not manipulated, made more accurate decisions using a DSS (Djamasbi, 2007). Because people who were induced with a positive mood had higher mood scores than those whose mood was not manipulated, the results of this previous study suggests a possible positive relationship between mood scores and performance. We test this assertion with the following regression model:

$$MSE_{1-30} = b_0 + b_1 * MSB \qquad (8)$$

where $MSE_{1\text{-}30}$ represents errors as calculated in Equation (7) for all the 30 trials in the task, and *MSB* refers to average mood scores before starting the task. The results showed that 4% (R^2 = 0.04) of the variation in error ($MSE_{1\text{-}30}$) was explained by mood scores. *MSB* had significant negative relationship with $MSE_{1\text{-}30}$ (b_1=-3449.59, t=-2.25, p=0.03), that is, the higher the average mood scores the smaller the $MSE_{1\text{-}30}$ values and thus the better performance.

Next we tested the same regression model by using the DEA technique for calculating the error values for all 30 trials. We developed the DEA scores based upon the *MSE* measures. We use model (5) to develop a composite error measure by using the bias, variability, and linear correspondence. Note that bias, variability, and linear correspondence are not integer values, and smaller values for all the three measures indicate better performance. Therefore, we treat them as DEA inputs in model (5). In this case, there are no DEA outputs, or we can assume there is one output with an equal value of 1 across all DMUs (students). The resulting super-efficiency score is called “DEA error score”. A larger “DEA error score” indicates smaller errors.

$$DEA - Error_{1-30} = b_0 + b_1 * MSB \quad (9)$$

The results showed that 4% (R^2 = 0.04) of the variation in the error in the above model was explained by users’ mood scores before starting the task. *MSB* had significant positive relationship with *DEA-Error*$_{1\text{-}30}$ (b_1=0.05, t=2.26, p=0.03). Because higher *DEA-Error*$_{1\text{-}30}$ values represent better performance, these results indicate that higher mood scores resulted in better performance.

Next we tested the regression model by using the DEA technique for calculating both the error values for all 30 trials and the mood scores before starting the task. We calculated a new set of DEA scores based upon the mood scores. While in Djamasbi et al. (2010), the average of three mood scores is used as a composite measure (assuming that the three mood scores are equally important), the new DEA model enables us to develop a composite mood score without the need for averaging and focusing on the best-mood frontier. Since a higher value for “glad score”, “happy score”, and “pleased score” indicates a better mood, we treat these three mood measures/scores as DEA outputs in model (5). In this case, there are no DEA inputs, or we can assume there is one input with an equal value of 1 across all DMUs (students). Note that all these three mood measures are integer factors in model (5). We call the resulting super-efficiency score as “DEA mood score”. A larger “DEA mood score” indicates better mood.

$$DEA - Error_{1-30} = b_0 + b_1 * DEA - MSB \quad (10)$$

where as before $DEA\text{-}Error_{1\text{-}30}$ is the DEA score for error in all trials and *DEA-MSB is the DEA* score representing users' mood before starting the task. The results showed that 3% (R^2 = 0.03) of the variation in $DEA\text{-}Error_{1\text{-}30}$ was explained by *DEA-MSB* and that higher *DEA-MSB* scores resulted in higher $DEA\text{-}Error_{1\text{-}30}$ values (b_1=0.23, t=2.01, p=0.04).

4.2.2 Do higher positive mood scores before the task lead to higher mood scores after the task?

Positive mood can act as an "emotional currency" improving one's ability to work under stressful situations (Aspinwall, 1998). Because complex cognitive tasks are stressful (Streufert and Streufert, 1978), decision makers may benefit from their positive mood. In other words, people with higher mood scores before completing a cognitively complex task (such as the planning task in our study) are likely to have higher mood scores after the task. We captured users' mood before and after the task with the same survey discussed in Section 4.1 of this paper. We created an average mood score for each participant to describe their mood before and after the task. We tested the relationship between the mood scores before and after the task through the following regression model:

$$MSA = b_0 + b_1 * MSB \tag{11}$$

where *MSA* and *MSB* refer to average mood scores after and before completing the task, respectively. The results show that 45% (R^2= 0.45) of variation in mood scores after the task are explained by mood scores before the task and that the two mood scores have a significant positive relationship (b_1=0.63, t=10.28, p=0.000).

Next we repeated the above analysis by using DEA mood scores. We calculated a DEA mood score for moods after the task as we calculated the DEA mood scores for moods before the task in Section 4.1. We treated the value for mood items ("glad score", "happy score", and “pleased score”) which were captured after the task as DEA outputs in model (5). As before, we assumed a single input with an equal value of 1 across all DMUs (students). As before, the three mood measures are integer factors in model (5). We called the resulting super-efficiency score as “DEA mood score after the task”. A larger “DEA mood score after the task” indicates better mood. We used the following regression model to test the relationship between DEA mood scores before and after the task:

$$DEA-MSA = b_0 + b_1 * DEA-MSB \tag{12}$$

In the above equation *DEA-MSB* represents users’ mood state before the task and *DEA-MSA* represents users’ mood after the task. The results confirmed that the two variables were significantly and positively correlated (R^2=0.41, b_1 =0.60, t=9.46, p=0.000).

4.2.3 Can performance in the first half of the task predict performance in the second half of the task?

The task used in this study requires decision makers to develop a decision strategy (Remus and Katteman, 1987; Djamasbi et al., 2004). High quality judgments depend on a user’s ability to form good decision strategies (Hammond, 1975; Remus and Katteman, 1987; Djamasbi et al., 2004). Prior studies suggest that it takes novice users about 15 trials to learn the task and build their decision policy (Remus and Katteman, 1987; Djamasbi et al., 2004). This suggests that better performance in the first 15 trials indicates the formation of a higher quality policy during this time. Thus, those who have a better performance in the first 15 trials are likely to have a better performance in the subsequent trials as well. To test this possibility we used the following regression model to examine the relationship between performance in the first and last block of the task. Note that the first and last block of the task, each consisted of 15 trials.

$$MSE_{16-30} = b_0 + b_1 * MSE_{1-15} \tag{13}$$

where $MSE_{1\text{-}15}$ and $MSE_{16\text{-}30}$ refer to performance (error) in the first and last block of trials in the task. The results showed a significant positive relationship between performance in the first and last blocks (R^2=0.19, b_1 =0.35, t=5.46, p=0.000) supporting the assertion that judgment quality in second block of the task can be predicted by judgment quality in the first block of the task.

Then we tested the same phenomenon with DEA scores. We performed two DEA analyses with the same inputs and outputs used in the previous section. The first DEA analysis contained two DEA applications of model (5). One application contained DMUs for the first 15 trials and the other contained the second 15 trails. Namely, we treated these two blocks of trails in separate DEA runs and obtained two sets of super-efficiency scores. We performed an F-test between the two sets of DEA scores and accepted the hypothesis that the variances of the two sets of DEA scores are equal. We then performed a t-test (assuming equal variances). The test indicated that the averages of the two sets of DEA scores were significantly different (p=0.03). The first trial block had a higher average DEA super-efficiency. However, we should point out that DEA is a relative measure and its results are valid only within a set of DMUs under consideration. Therefore, while the t-test indicates that the two trail blocks show a difference with respect to performance, we cannot use the average DEA scores to compare the performance difference. To compare the performance difference, we need to perform the second DEA analysis where the two blocks are evaluated together under the same model. Namely, we doubled the size of DMUs in our second DEA analysis. In this case, the average DEA super-efficiency scores of the first and second block trails were 0.69 and 0.74, respectively. This indicates that overall the decision makers in our study had a better performance in the second block of the task. These results are consistent with the prior studies that suggest decision makers use the first block of the task to learn the task (Remus and Katteman, 1987; Djamasbi et al., 2004).

4.2.4 Discussion

In this study we developed a new DEA. We then used the newly developed DEA as an alternative method for calculating scores for several different constructs, which were used to test our assertions. Our analyses supported our assertions by showing significant results for the constructs under the investigation regardless of whether they were obtained by traditional methods or by the new DEA method. Our results showed a strong positive correlation between 1) positive mood scores before the task and performance, 2) positive mood scores before and after the task, and 3) performance in the first and last block of the task. Additionally, the tested models yielded similar results when DEA scores were used. These results together extend the applicability of DEA to a domain that traditionally does not use the DEA method. Additionally, the results revealed that the new DEA method is sensitive enough to detect subtle nuances such as a user's mood and its impact on performance when using a DSS.

These results have both theoretical and practical implications. From a theoretical point of view the results extend the literature on decision making by showing a significant correlation between mood and performance scores, and by showing that the performance in the second half of the task can be predicted by performance in the first half of the task. The results also contribute to mood literature by showing that positive mood did indeed act as emotional currency in our study; those who had higher mood scores before the task had also higher mood scores after the task. From a practical point of view the results show that the new DEA is applicable to a new domain. Hence, the results extend the applicability of the DEA method to a new domain, namely judgment and decision making using a DSS.

5. Conclusions

The current paper develops an integer-valued super-efficiency model based upon directional distance function (DDF). Such a model is necessary for our application of evaluating mood and its impact on performance in judgment and decision making using a DSS. Such an information system research domain requires user information to be collected in a Likert-type scale which represents integer values in DEA models. The use of super-efficiency concept is also necessary to break ties in efficient units in a sense that our DEA scores are used in regressions. Although DDF-based super-efficiency model has been proposed in the literature, we show that the existing DDF approach cannot be directly modified to incorporate integer values under super-efficiency concept. The use of DDF is for the purpose of addressing the well-known infeasibility issue in super-efficiency.

Our empirical application successfully demonstrates the usability of our newly developed super-efficiency model. Our model offers an alternative approach for examining several key issues predicting a user's decision behavior when utilizing a DSS.

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Table 1. Numerical example

DMU	x_1	x_2	y_1
A	2	4	1
B	7	1	1
C	3	4	1
D	7	2	1
E	9	4	1

Table 2. Integer DDF-based super-efficiency

DMU	β_x^*	β_y^*	Projection	Super-efficiency
A	-0.5	0	$x_1 = 3$, $x_2 = 4$, $y_1 = 1$	1.5
B	-1	0	$x_1 = 7$, $x_2 = 2$, $y_1 = 1$	2
C	0	0	$x_1 = 3$, $x_2 = 4$, $y_1 = 1$	1
D	0	0	$x_1 = 7$, $x_2 = 2$, $y_1 = 1$	1
E	1/3	0	$x_1 = 6$, $x_2 = 2$, $y_1 = 1$	2/3

Table 3. When x_1 and y_1 are integer-valued

DMU	β_x^*	β_y^*	Projection	Super-efficiency
A	-0.5	0	$x_1 = 3$, $x_2 = 4$, $y_1 = 1$	1.5
B	-1	0	$x_1 = 7$, $x_2 = 2$, $y_1 = 1$	2
C	0	0	$x_1 = 3$, $x_2 = 4$, $y_1 = 1$	1
D	0.142857	0	$x_1 = 6$, $x_2 = 1.714286$, $y_1 = 1$	0.857143
E	0.444444	0	$x_1 = 5$, $x_2 = 2.222222$, $y_1 = 1$	0.555556

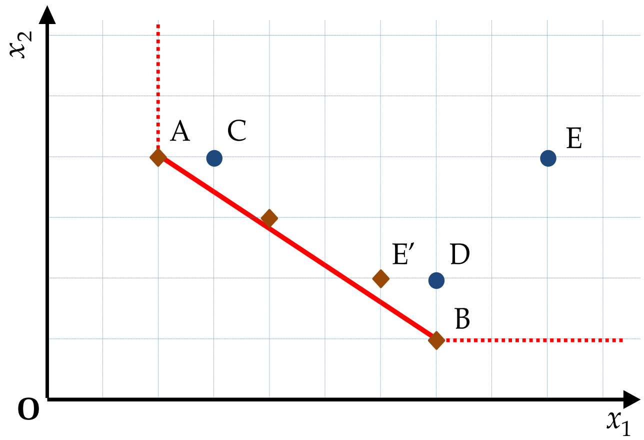

Figure 1. Integer DEA frontier

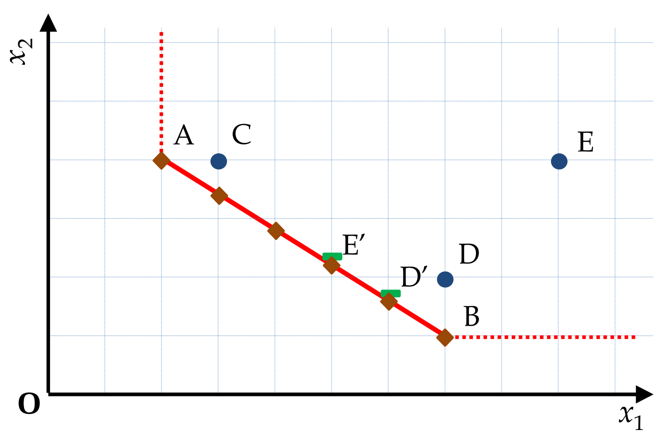

Figure 2. Integer DEA frontier when only x_1 and y_1 are integer-valued