## **COMPOUND HYDRO-KINETIC SUBMERSIBLE TURBINE**

A Major Qualifying Project Report

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## Abstract

This project investigates the concept of a compound hydrokinetic turbine, a device that can extract energy from flowing fluids such as ocean currents, channels or rivers. The compound turbine operates without delivering a reaction force to its supporting structure or mooring. Conceptually the device could also operate in air and thus be an alternative configuration to the conventional wind turbine. Also reviewed is availability of energy from ocean currents, the performance of airfoils developed for wind turbine use in water rather than air and various systems for mooring hydrokinetic devices in deep water.

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# List of Acronyms

BEM	Blade Element Momentum
HAWT	Horizontal Axis Wind Turbine
VAWT	Vertical Axis Wind Turbine
NACA	National Advisory Committee for Aeronautics
NREL	National Renewable Energy Laboratory

# Authorship

1. Introduction	ALL
1.1 Background	
1.1.1 Wind Turbine Design	Royds
1.1.2 Application of Wind Turbine Theory to Hydrokinetic Turbines	Perron
1.1.3 Turbine Power Generation and Optimization	Royds
1.1.4 Current Underwater Turbine Related Research and Projects	Perron
2. Methodology	
2.1 Research	McConnell
2.2 Prototype Design	McConnell
2.3 Prototype Manufacturing	Royds
2.4 Future Research and Development	McConnell
3. Compound Hydrokinetic Submersible Turbine	
3.1 Mooring Advantages	Perron
3.2 Rotation Speed Advantages	Perron
3.3 Energy Efficiency Advantages	Perron

## **Executive Summary**

The production of energy is a necessity in the world today, and will remain to be in the future. The desire to find renewable sources for this energy is longstanding, as there are both environmental benefits to this alternative energy, as well as an unlimited store of it. Hydrokinetic turbines are an emerging solution to this need, using the flow of water to create energy. The goal of this project was to design a unique hydrokinetic turbine that could be used to generate energy from ocean currents, and potentially function as an alternative design for a wind turbine.

The project included not only a design for a hydrokinetic turbine, but also investigation into the availability of energy from ocean currents around the world, the applicability of airfoils designed for wind turbines to an underwater environment, and mooring systems for deep sea applications. Preliminary research showed the large potential for power generation from several currents around the world, with the Gulf Stream off the coast of Florida being one. The team conducted analysis of theoretical airfoil performance in both air and water, and demonstrated that the performance of these airfoils designed for wind turbine applications was comparable in both environments. The design of the turbine was initiated using a MATLAB script that utilized blade element momentum theory to determine optimal blade twist. Using the result of this program, the blades and hub were modeled in SolidWorks. Commercial motors and prop adapters were purchased to allow assembly of a prototype of the design.

Although time did not allow for testing to be conducted, the team created a test stand for use in the wind tunnel as this proved to be a simpler option than finding a suitable underwater test location. The design of the test stand will allow for testing of this project to be completed in the future, as well as other projects that may require use of the wind tunnel. To allow for further investigation into the feasibility of the turbine design, the team suggests that the scale model be assembled and tested for its aerodynamic properties and power generation. Based on the results of these tests, improvements can be made to optimize the design, which could then move on to be tested in water. If the feasibility of this design is proven, this project can pave the way for a viable renewable energy production source for the future.

## **1. Introduction**

In modern society, energy is a vital part of our day-to-day lives. To cater to a significant increase in worldwide population, it is necessary to increase current methods of energy production. With the global population at an all-time high of just over 7 billion people with estimates of close to 10 billion people by 2050, energy demands can be expected to rise as well.<sup>1</sup> While current forms of energy production can sustain life now, there exists the possibility that they may not be able to accommodate such a large population in the future.

Currently, close to 80% of worldwide energy production is a product of non-renewable sources such as coal, oil, and natural gas.<sup>2</sup> While these are large sources of energy production, each has adverse effects on the environment and humanity. Energy produced from coal not only poses a risk to the health and wellbeing of coal miners, but also poses this same threat to the environment by creating harmful chemicals when burned. The act of oil extraction is also potentially harmful to the environment as is evident by the recent BP oil spill that occurred in the Gulf of Mexico.<sup>3</sup>

While these forms of energy are capable of providing power to the current population, coal energy, which has some of the largest reserves on earth, is beginning to diminish. The possibility exists that we may not only run out of power, but also cause destruction to the world we live in if we continue to use fossil fuels the way we do. This has initiated investigation into the possibility of alternative forms of energy production through sources such as wind, sun, and water.

Many projects have been completed over the past few decades to develop alternative forms of energy; however, none have been adopted on a large scale. Some of the largest wind turbine farms in the United States are located in the San Bernardino valley in California, but do not have the possibility to power large areas. Many issues have risen from large alternative energy farms taking up land, but not being able to produce comparable amounts of electricity to current forms of energy production. One underutilized energy source in the world remains to be the ocean.

<sup>&</sup>lt;sup>1</sup> Kunzig, Robert. "Special Series: 7 Billion." National Geographic. N.p., Jan. 2011. Web. 5 Apr. 2015

<sup>&</sup>lt;sup>2</sup> "FAQs: Renewable Energy." International Energy Agency. N.p., n.d. Web. 5 Apr. 2015.

<sup>&</sup>lt;sup>3</sup> Elliot, Debbie. "5 Years After BP Oil Spill, Effects Linger And Recovery Is Slow." *NPR*. NPR, 20 Apr. 2015. Web. 26 Apr. 2015.

With the ocean covering over 70% of the planet, the potential for energy production is large. The team will be exploring the idea of utilizing a compound hydrokinetic submersible turbine to generate energy from ocean currents. Our project will involve the design of a unique submersible turbine. In addition, the availability of energy from ocean currents around the world, airfoil performance underwater, and deep sea mooring systems will be investigated. With the development of a feasible submersible turbine, our project can contribute to viable energy source for the future.

## **1.1 Background**

Using wind as a source of energy is a practice that dates back centuries. Windmills for grinding grain have been reported as far back as the tenth century in Persia.<sup>4</sup> Wind was a major source of energy for transportation by boat, grinding grain, and pumping water. In the early 1900sattempts were made to design and build utility scale wind turbines.<sup>5</sup> In the 1960s, however, development of wind turbines was put on hold because petroleum became commercially available and inexpensive. In 1982, wind technology made a resurgence, with the production of wind farms, as a result of federal laws and incentives to produce clean energy in California.<sup>6</sup> Wind power has grown at an average rate of 28% per year from 1995 to 2012.<sup>7</sup> Recently, the generation of power from fluid flow has stimulated interest in extending the concept of wind turbines from air to water in the world's search for renewable energy.

## **1.1.1 Wind Turbine Design**

This section will review some of the basic components included in the design of a wind turbine, including those that are part of the structural design, as well as the electrical components used for power generation and control.

<sup>&</sup>lt;sup>4</sup> Nelson, Vaughn. *Wind Energy: Renewable Energy and the Environment, Second Edition.* CRC Press: Florida. 2014, 1.

<sup>&</sup>lt;sup>5</sup> Nelson, 6.

<sup>&</sup>lt;sup>6</sup> Nelson, 11.

<sup>&</sup>lt;sup>7</sup> Nelson, 13.

#### **Structural Design**

The majority of commercial wind turbines utilize a horizontal axis design.<sup>8</sup> This design differs from a vertical axis turbine in that the rotor and generator are located on the top of the support system/ tower rather than at the base, and the turbine blades are facing the fluid flow as opposed to being oriented transversely. The structural components of the horizontal axis wind turbine (HAWT) include a nacelle, rotor blades and hub, and a tower to support these components. The blades are attached to a hub and have a high lift to drag ratio. The hub connects to the nacelle, which houses the generating components of the turbine.<sup>9</sup>

#### **Power Generation Components**

The electrical components for the power generation are housed inside of the nacelle. Traditionally, the main rotor shaft connects to a gearbox, or a speed up drive, which takes the low rotational speed of the turbine and outputs a higher rotational speed to the generator.<sup>10</sup>

#### Controls

Wind turbines are often equipped with blade pitch and yaw controls. Simpler wind turbine designs may not have these components. The pitch control changes the pitch of the blades to prevent stalling in high wind speeds and to ensure that the coefficient of power remains as high as possible at all times.

The yaw control device is used to accommodate changing wind direction. It allows the turbine to turn from side to side depending on the direction from which the wind is coming. Like the pitch control device, the yaw device is included for optimization purposes, to make sure that the turbine will intercept as much wind as is available.<sup>11</sup>

<sup>&</sup>lt;sup>8</sup> Dixon, S.L., and C.A. Hall. "Chapter 10 - Wind Turbines". *Fluid Mechanics and Thermodynamics of Turbomachinery, Sixth Edition*. Butterworth-Heinemann. © 2010. Books24x7.

<sup>&</sup>lt;sup>9</sup> "How Do Wind Turbines Work?" *Energy.gov: Office of Energy Efficiency and Renewable Energy*. N.p., n.d. Web. 17 Dec. 2014.

<sup>&</sup>lt;sup>10</sup> "How Do Wind Turbines Work?"

<sup>&</sup>lt;sup>11</sup> "Anatomy of a Wind Turbine." Anatomy of a Wind Turbine. N.p., n.d. Web. 19 Dec. 2014.

#### 1.1.2 Application of Wind Turbine Theory to Hydrokinetic Turbines



the nacelle, and a secure attachment to the sea floor.

Wind turbines and hydrokinetic turbines operate on the same principle: converting the kinetic energy from a fluid stream into mechanical and then electrical power. Despite the similarities, there are special considerations to take into account when operating a turbine underwater; cavitation, increased reaction forces, placement, and mooring fall into this category.

Figure 1 shows the components of a horizontal-axis wind turbine.<sup>12</sup> A hydrokinetic turbine design would be similar, with a generator placed inside of a nacelle, and a set of blades attached to the hub that would draw energy from the surrounding fluid flow.

Implementation of an effective underwater turbine will require consideration of the effects of hydrodynamic forces on the blades, prevention of water from leaking into

 <sup>&</sup>lt;sup>12</sup> Hau, Erich. Wind Turbines. New York: Springer Berlin Heidelberg, 2006, 73.
 *Figure 1 Components of a horizontal-axis wind turbine*

#### Cavitation

Hydrokinetic turbines have shown declined performance over the years of their operation for numerous reasons. The effect of cavitation is a predominant issue that affects underwater turbine blades. According to Bernoulli's equation, an increase in flow velocity conversely results in a decrease in pressure. Cavitation occurs when flow velocity increases enough to drop the pressure of the local flow below its

vapor point. Vapor pressure is defined as the



Figure 2: Variation of efficiency with respect to cavitation factor

pressure at which a liquid will turn into vapor at a given temperature. When a liquid turns to vapor, it boils and creates bubbles. The bubbles are formed in locations of low pressure, and are carried downstream by the flow. When the bubbles reach a location with a higher pressure than their structure can accommodate, they suddenly collapse. Liquid water rushes from all sides to fill the void, and collides in the center of the space. The pressure at the center of this void can reach up to 7,000 atm.<sup>13</sup> Bubbles are created and formed thousands of times every second at high blade velocities, and the resulting shockwaves erode the material of blades.

High cavitation factors lead to increased suction on the back-end of a blade and reduce the performance of the turbine overall. Figure 2 shows the effect of cavitation on turbine efficiency.<sup>14</sup>

## Hydro-dynamic Forces on Blades

 <sup>&</sup>lt;sup>13</sup> Pardeep Kumar and R. P. Saini. "Study of cavitation in hydro turbines – A review," *Renewable and Sustainable Energy Reviews*. India: Indian Institute of Technology, July 2009.
 <sup>14</sup> Ibid.

Blade design is a critical factor in the production of a hydrokinetic horizontal-axis turbine. These machines are subject to intense dynamic fluid forces.<sup>15</sup> Water is over 800 times denser than air,

and because of this, a turbine will experience larger forces on its blades when placed in a flow of water. It is important for the blades of an underwater turbine to be able to withstand the environment that it will operate in.

A flow of water produces greater drag and hydrodynamic load than a flow of air moving at the same velocity. A similarly rated underwater turbine can produce up to four times as much



Figure 3: Blade bending stress on the pressure face (left) and suction face (right) at various blade angles corresponding to V=1.7m/s

energy per year as a wind turbine of the same size, because of the higher density of the fluid flow. At the same time, the turbine experiences greater stress. Figure 3 shows the distribution of stress on the front and back of turbine blades.<sup>16</sup> It is difficult to perform maintenance on underwater turbines because they are often placed in extreme locations in order to farm the best currents, and the blades are subject to more collisions with debris than wind turbines. The phenomenon of cavitation is also present in underwater applications, and can rapidly erode the blades of a marine turbine. The extreme operating conditions of underwater turbines emphasize the need to construct blades out of strong material.

Generally, marine turbine blades are constructed from fiber reinforced polymer composites. Fiber composite materials offer excellent strength-to-weight ratios and stiffness-to-weight ratios, improve fatigue resistance and damping properties, and can be easier to manufacture in complex shapes.<sup>17</sup>

<sup>&</sup>lt;sup>15</sup> Michael R. Motley and Ramona B. Barber. "Passive control of marine hydrokinetic turbine blades," *Composite Structures*. Washington: University of Washington, April 2014.

<sup>&</sup>lt;sup>16</sup> Motley and Barber.

<sup>&</sup>lt;sup>17</sup> Motley and Barber.

#### **Ocean Currents**

Ocean currents can provide significant sources of energy, yet they are presently underutilized. Water is driven in large quantities mainly by wind and solar heating of the waters near the equator.<sup>18</sup> Water density and salinity also affect flow in more minor amounts. These currents have relatively constant velocity, move large amounts of water, and are predictable.



Figure 4: Major Ocean Currents around the World <sup>19</sup>

The total power in ocean currents worldwide is estimated to be approximately 5,000 GW. It is estimated that harvesting 1/1000<sup>th</sup> of the total energy available from the Gulf Stream, which borders the east coast of the United States, would supply Florida with 35% of its electrical needs.<sup>20</sup>

#### Mooring

An underwater turbine will need to be moored in an ideal location to harvest the energy from ocean currents. There are more options for mooring a turbine underwater than there are for constructing a turbine in the air because of buoyancy. An underwater turbine can float in the

<sup>&</sup>lt;sup>18</sup> Minerals Management Service. "Ocean Current Energy Potential on the U.S. Outer Continental Shelf," U.S. Department of the Interior. 2006, 2.

<sup>&</sup>lt;sup>19</sup> Ocean Currents Map. Digital Image. NOAA. N.p., n.d. Web. < http://www.adp.noaa.gov/currents\_map.html>

<sup>&</sup>lt;sup>20</sup> Minerals Management Service, 3.

location of optimal fluid flow easily due to buoyancy. To achieve a similar effect with wind turbines, one would have to attach the machine to a balloon to hover in the air.

Below, two options for mooring a marine turbine are shown.<sup>21 22</sup> Mooring depends on the location of the current, the layout of the sea floor, and the design intent.



It is possible to suspend a turbine from the ocean surface by attaching a buoy to the nacelle. Anchoring the turbine by way of a flexible tether is also possible, but it is then necessary to counteract the

Figure 6: Seabed Mooring

Figure 5: Tethered Mooring

torque produced by the rotation of the blades. The simplest

method is to attach a column into the sea floor and mount the turbine on the column, as seen in the top image.Sea floor mounting becomes impractical or impossible within currents like the Gulf Stream where the prevailing depth can 2400 feet

#### **1.1.3 Turbine Power Generation and Optimization**

The following sections outline the principles governing the power generation of a turbine as well as the optimization of this power.

## **Power Generation**

A German physicist named Albert Betz derived an equation for the maximum power that can be extracted from the wind. The formula is shown below:

$$P_{Betz} = \frac{1}{2} C_{p,Betz} \rho v_1^3 A$$

<sup>&</sup>lt;sup>21</sup> Bottom Mounted Axial Turbines. Digital Image. *Wikipedia*. N.p., n.d. Web.

<sup>&</sup>lt;a href="http://en.wikipedia.org/wiki/Tidal\_stream\_generator#/media/File:Bottom\_Mounted\_Turbines.png">http://en.wikipedia.org/wiki/Tidal\_stream\_generator#/media/File:Bottom\_Mounted\_Turbines.png</a>>.

<sup>&</sup>lt;sup>22</sup> Cable Tethered Turbine. Digital Image. *Wikipedia*. N.p., n.d. Web.

<sup>&</sup>lt;a href="http://en.wikipedia.org/wiki/Tidal\_stream\_generator#/media/File:Cable\_Tethered\_Turbine.png">http://en.wikipedia.org/wiki/Tidal\_stream\_generator#/media/File:Cable\_Tethered\_Turbine.png</a>>.

This is known as Betz's Law and it is applicable to all Newtonian fluids. It is based off of the idea that after wind passes through a turbine is loses speed due to kinetic energy extraction by the turbine. If the turbine were to extract all of the energy, the wind would stop moving after passing through the turbine. It would then block any new wind from passing through. Betz derived  $C_{P,Betz}$ , the maximum power coefficient that would allow the turbine to function. In the above equation,  $\rho$  is the density of air,  $v_1$  is the velocity of the wind, and A is the cross sectional area through which the wind moves. This formula does not take into account the efficiency of the wind turbine or tool being used to extract the power, but simply gives the theoretical maximum power that a turbine would be able to output.

To determine the maximum power output for a specific wind turbine, a similar version of the same formula is used. A turbine cannot operate at the ideal, maximum $C_p$ , and losses from the mechanical transmission and electrical generation must also be taken into account. Because of this, the equation for power output uses a  $C_p$  particular to the turbine as well as the total efficiency of the turbine. The efficiency of the turbine depends on the efficiency of the gear box and generator, and the coefficient of power depends on the blade pitch angle and the tip speed ratio. <sup>2324</sup> The tip speed ratio,  $\lambda$ , is equal to  $\frac{\omega_r R}{\omega_1}$  where  $\omega_r$  is the rotational speed of the turbine and R is the radius of the blades.

#### **Optimization of Power**

Optimization of the power that a turbine can extract will require not only optimization of the parameters that will affect the power output, but also control of these parameters. Under optimum conditions, according to Betz, the maximum value of  $C_p$  is .593. In reality this does not occur. Most turbines will operate at an efficiency of about 75 to 80% of  $C_p$ . To obtain a high value of  $C_p$  requires a blade design that is very smooth coupled with a tip speed ratio above 10.<sup>25</sup> Although this is not possible, there are methods of optimizing blade design to produce an optimal

<sup>23</sup> Dixon, Hall

<sup>&</sup>lt;sup>24</sup> Kusiak, Andrew, and Haiyang Zheng. "Optimization of Wind Turbine Energy and Power Factor with an Evolutionary Computation Algorithm."*Energy* 35.3 (2010): 1324-332. Web.

<sup>&</sup>lt;sup>25</sup> Dixon, Hall

power output. One such method is the blade element momentum method which is discussed in more detail in Chapter 3 of this report.

Even if the design of a turbine is optimized, if it is not placed in an optimum environment, then its full potential for power generation will not reached. An optimum environment would consist of a relatively fast and reliable fluid speed, whether it be air or water. The controls of the turbine can ensure that its performance is maximized with the given environment. The yaw control ensures that the turbine will be facing the fluid flow at all times. The pitch control ensures that the optimum rotational speed is reached and that the blades do not stall.

## **1.1.4 Current Underwater Turbine Related Research and Projects**

Underwater turbines are an emerging technology, but there has been recent research into the idea by several organizations. Some of this research is outlined in the following sections.

## Tethered Undersea Kite (TUSK) – WPI Major Qualifying Project

In May of 2014, a team of WPI students completed a design for a Tethered Undersea Kite (TUSK) for power generation in the ocean. The intent of the project was "to design a rigid-wing underwater kite with an attached turbine that extracts power from an ocean current, and to test a viable scale model of the system". <sup>26</sup>The project approached undersea energy generation through exploitation of currents and tidal flows. The kite was designed to fly in a figure eight pattern at high speed and move through water in order to generate power.<sup>27</sup>



Figure 7: TUSK Design

<sup>&</sup>lt;sup>26</sup> Aye-addo, Nyansafo, James O'connor, and Randy Perez. "Design of a Scale Model Tethered Undersea Kite for Power Generation." Diss. Worcester Polytechnic Institute, 2014. Web. <a href="http://www.wpi.edu/Pubs/E-project/Available/E-project-050114-105311/unrestricted/TUSK\_Project\_Final\_AOP.pdf">http://www.wpi.edu/Pubs/E-project/Available/E-project-050114-105311/unrestricted/TUSK\_Project\_Final\_AOP.pdf</a>>.

<sup>&</sup>lt;sup>27</sup> "Looking for Tomorrow's Power Source? Go Fly a (Underwater) Kite." N.p., n.d. Web. 19 Dec. 2014.

## Florida Atlantic University

A team of researchers at Florida Atlantic University has been conducting experimental testing of submersible turbine generators. This project intends to explore the emerging technology of hydrokinetic turbines by mooring a turbine in the Gulf Stream, 13 miles off shore, to collect energy from a steady ocean current. The turbine will be placed at a depth of 100 to 160 feet below the surface where it can collect energy with 10 foot blades. <sup>28</sup> This project is the first of its kind to be placed in federal waters.



Figure 8: A small scale research turbine from the Florida Atlantic University

## **Verdant Power**

Verdant Power is a company that has placed submersible turbines in New York City's East River, a tidal strait that runs between Manhattan and Queens. Verdant Power's turbines are capable of supplying power to 20-30 homes at peak hours, and have been used to power a supermarket and parking garage near their installation location. The current model uses a 16 foot diameter rotor that spins at 40 revolutions per minute, changing direction depending on the direction of the tide. <sup>29</sup> Verdant has had several difficulties with the implementation of this technology, with several models breaking after installation, but they have since refined the design and are moving continuing to move forward. They have also conducted research to show that the turbines will not have any negative environmental impacts.

 <sup>&</sup>lt;sup>28</sup> Sanders, Kerry. "Researchers to Test Ocean Turbine Generators Off Florida Coast." *NBC News*. N.p., n.d. Web. 19 Dec. 2014.
 <sup>29</sup> Montopoli, Brian. "Powering the Future: Underwater Turbines Harness River Power." *CBSNews*. CBS Interactive, n.d. Web.

<sup>18</sup> Dec. 2014.



Figure 9: Verdant Power Submersible Turbine

#### **Sea Generation**

In 2008, the company Sea Generation installed a major underwater turbine in Northern Ireland's Strangford Narrows a few hundred meters from the shore. Because of its proximity to the coast, they can moor in water that is relatively shallow. This also simplifies the problems with maintenance, cabling, and mooring for deep sea turbines. The currents of the Strangford Narrows move the turbine's twin 52 foot diameter blades. The blades spin 10 to 20 times per minute and generate 1.2MW of tidal energy. The blades can also retract to above the surface for easy maintenance. <sup>30 31</sup> In addition to this turbine, the company has plans to



create a farm of underwater turbines to provide power to thousands of homes.

Figure 10: Sea Gen Marine Turbine

<sup>&</sup>lt;sup>30</sup> Mone, Gregory. "WORLD'S LARGEST UNDERWATER TURBINE INSTALLED." Popular Science. N.p., n.d. Web.

<sup>&</sup>lt;sup>31</sup> Sea Generation Ltd - Strangford Lough. N.p., n.d. Web. 19 Dec. 2014.

## 2. Methodology

The goal of this project is to implement turbine technology, known to work above ground, in an underwater setting. The major requirement for this project is to design a compound hydrokinetic submersible turbine capable of generating continuous power when placed in a constant current. However, since the timeframe for this project makes it impractical to attempt a full scale model, the team developed the following objectives for this project in addition to designing a full scale model of the turbine:

- Research submersible turbine technology currently under development or in production
- Design both a full scale model and a scaled prototype model capable of being tested by the team's available resources
- Manufacture a scaled prototype model capable of generating power
- Define future areas of research and development for the design of a compound hydrokinetic submersible turbine

## 2.1 Research

In order to begin the development of our compound hydrokinetic submersible turbine, the team first began conducting research on current methods employed in an above ground setting in order to gain a basic understanding of power generation from wind turbines. This information would then allow the team to apply principles used in wind turbines to turbines used underwater.

At this point in the research, the team had uncovered a large amount of data relating to the basic design, function, and problems of wind turbines, primarily HAWTs. This primary focus on HAWTs was not a decision based on any criteria, but was closely related to the design that was envisioned for the turbine to be designed in this project. The summary of the information relating to HAWTs was presented in the background chapter of this report. Once the team was able to understand the basic principles behind the operation of a HAWT, we researched methods used to optimize the power output of a HAWT.

A summary of the information relating to wind turbine power optimization is presented in the background, but will also be reviewed later in this chapter for the optimization of our

submersible turbine design. Understanding the methods used to optimize the power output of a HAWT, we researched the ability to apply wind turbine theory to underwater applications. In our preliminary research, we were not concerned with finding information specifically relating to submersible turbine design and performance, because it was known that information on these topics was limited.

A summary of the information relating to the adaptation of current wind turbine design models to hydrokinetic turbines is presented in the background chapter, but like power optimization, this information will be reviewed again in the design process for our submersible turbine. Once the team was confident that we would be able to use similar models in both above ground and underwater settings, we began to research current submersible turbines in production or in design. As stated before, our design would be similar to that for a HAWT, however for this part of the research phase, we included all options in our search.

A summary of the information relating to current turbine projects and research as well as barriers and problems for underwater power generation can be found in the background chapter of this report. As a final point of interest in our preliminary research, the team researched underwater currents throughout the world in order to develop parameters that we would use to design our submersible turbine for this project.

A summary of the parameters relating to ocean currents and fluid properties in the design of our hydrokinetic submersible turbine is located in the background chapter of this report. Overall, through research, the team was able to gain an understanding of power generation from turbines, wind turbine design and optimization, submersible turbine design and optimization, and design parameters for the development of our concept. While there was limited information on the ability to use models developed for wind turbine optimization, we were able to find that these theories are based on principles of fluid mechanics. From this assumption, it was clear that the formulas applied for use in air would be viable in any other fluid as well using appropriate assumptions for the different environment. Once the team had enough information to start the design process, we put together a plan of action for developing our hydrokinetic submersible turbine.

## 2.2 Prototype Design

The team put together the following list of components necessary to design or purchase in order to develop a prototype model for this project. As stated before, the main focus of this project was to develop a functioning prototype turbine capable of generating power using available testing resources. In order to develop this prototype the team would need to complete the following tasks:

- Define design parameters for concept turbine
- Select or design an airfoil to be used based on design parameters
- Design an turbine blade based on the airfoil selected
- Design or purchase a generator system based on design parameters
- Design or purchase a turbine housing based on purchased and created components
- Design or purchase a mounting system for the turbine

#### **2.2.1 Design Parameters**

After looking at data collected from our background research on ocean currents and properties, the team developed a set of design parameters. These design parameters would be inputs for the calculations that we would need to design or select various components of the turbine. In order to determine this set of parameters, the team selected a future testing location for a full scale model. Using information relating to the consistency, depth, surroundings, location, and proximity to land of various global currents, the team selected a location within the Gulf Stream in the Florida Straits. A major factor in the selection of this location came from the abundance of research completed on this specific section of the Gulf Stream flow. While the team was able to find information regarding other currents around the world, some assumptions would have to be made based on missing data for the variability in ocean conditions based on both time and location. The selection of this current yielded the following design parameters:

Fluid: Water Fluid Temperature: 20°C Fluid Salinity: 34.9 g/kg Fluid Density: 1036 kg/m<sup>3</sup> Fluid Velocity: 1.5 m/s

After selection of this data for our primary design parameters, the team was presented with a conceptual prototype and full scale design parameters that we would use to develop the model. The parameters of the turbine design are as follows:

Full Scale Main Blade Radius: 5 m Full Scale Hub Radius: 1 m Prototype Main Blade Radius: 1 m Prototype Hub Radius: 0.2 m Number of Blades: 3

The next parameter came from previous research completed on blade element theory. It has been observed that the optimal tip speed ratio for a 3 bladed rotor is  $4.19^{32}$ . In order to accommodate expected losses of about 20%, the team selected a slightly higher value for use in our design. The parameter for blade design tip speed ratio is the following:

Blade Tip Speed Ratio: 5

Lastly, in order to select proper generator system components, the team computed the maximum idealized power that could be created by a turbine using the parameters defined above. Using equations developed from Bernoulli's equations, the theoretical maximum power created by a turbine using the Betz power coefficient limit is defined as<sup>33</sup>

$$P_{Betz} = \frac{1}{2} C_{P,Betz} \rho v_1^3 A \tag{1}$$

<sup>&</sup>lt;sup>32</sup> Ragheb, Magdi. Optimal Rotor Tip Speed Ratio. Rep. N.p., 11 Mar. 2014. Web. 18 Dec. 2014.

<sup>&</sup>lt;sup>33</sup> Gundtoft, Soren. Wind Turbines. Rep. University College of Aarhus, June 2009. Web. 18 Dec. 2014.

Therefore,

$$P_{Betz} = \frac{1}{2} \left( \frac{16}{27} \right) (1036) (1.5^3) (\pi 5^2) = 81367.25 \text{ W} = 81.37 \text{ kW}$$

This calculation would yield a final parameter:

Power Generation: 81.37 kW

#### 2.2.2 Airfoil Selection

Because our project is simply a proof of concept, the team was not concerned with going through the extensive process of designing an airfoil for our specific design intentions. We were able to find that several other projects for underwater turbines that have been completed, or are in development, have made the assumption that blades optimized for wind turbine power production should yield similar results for other fluid environments. The team understood that these airfoil shapes may not be the best for water, but in order to put together a prototype model in the allotted time, we decided to use a similar approach. We put together a list of programs that have already developed these optimized airfoils. These programs are through the National Renewable Energy Laboratory in the United States of America, the Aeronautical Research Institute in Sweden, Risø National Laboratory in Denmark, and the Delft University of Technology in the Netherlands.

In order to select an airfoil to construct a prototype blade design, the group used the software XFLR5 to complete an analysis of the different airfoil shapes that we found in our background research. In order to determine the parameters to be used for the XFLR5 analysis, the team computed some basic parameters of the expected fluid flow using simple blade element theory. In blade element theory, a blade is divided into sections as shown in the figure below.



Figure 11: Blade Element Theory - Velocities and Angles

Using the parameters defined above and selecting 20 sections, the group was able to determine the local Reynolds number for each section. This process was completed using simple geometry from the previous figure to determine the induced flow velocity at each section along the blade. For these calculations, it was assumed that the axial interference factor (a) and the tangential inference faction (a') were zero. The results of this process are listed in the table below.

Section	r [m]	r/R	u [m/s]	w [m/s]	Re
1	0.5	0.1	0.75	1.677051	1.60E+06
2	0.725	0.145	1.0875	1.852743	1.76E+06
3	0.95	0.19	1.425	2.068967	1.97E+06
4	1.175	0.235	1.7625	2.314391	2.20E+06
5	1.4	0.28	2.1	2.580698	2.46E+06
6	1.625	0.325	2.4375	2.862063	2.73E+06
7	1.85	0.37	2.775	3.154461	3.00E+06
8	2.075	0.415	3.1125	3.455091	3.29E+06
9	2.3	0.46	3.45	3.761981	3.58E+06
10	2.525	0.505	3.7875	4.073715	3.88E+06
11	2.75	0.55	4.125	4.389262	4.18E+06
12	2.975	0.595	4.4625	4.707856	4.48E+06

13	3.2	0.64	4.8	5.028916	4.79E+06
14	3.425	0.685	5.1375	5.352	5.10E+06
15	3.65	0.73	5.475	5.676762	5.41E+06
16	3.875	0.775	5.8125	6.002929	5.72E+06
17	4.1	0.82	6.15	6.330284	6.03E+06
18	4.325	0.865	6.4875	6.658653	6.34E+06
19	4.55	0.91	6.825	6.987891	6.66E+06
20	4.775	0.955	7.1625	7.317883	6.97E+06
Tip	5	1	7.5	7.648529	7.28E+06

Table 1: XFLR5 Reynold's Number Calculations

Using this data, the team was able to set up XFLR5 to be able to analyze different airfoil shapes using the given Reynolds numbers. Using default settings in XFLR5, we ran the simulation for angles of attack between 0-15 degrees to see results for coefficients of lift and drag over that range. The ratio of these coefficients known as the "glide ratio" is the main interest of this part of the analysis. Typically the highest value of this ratio will provide the maximum lift and therefore maximum power. A sample of the analysis conducted on a S805A airfoil shape is shown in the figure below.



Figure 12: XFLR5 Results for CL/CD vs. Alpha for S805A Airfoil

Using this data, the team was able to select a design angle of attack, design coefficient of lift, and design coefficient of drag that we would be able to use as parameters for the blade design using the blade element momentum (BEM) theory that will be discussed in the next section.

#### 2.2.3 Blade Design

Once the group chose an airfoil shape, the next step would be to determine the blade design for both the small and large turbines. While the values for each of the blade design parameters would be different for each turbine, the method remains the same. The method used to create the blade design is the BEM method. This iterative method uses a few simple equations to determine the proper twist along the blade span. Using a MATLAB script provided by our advisor, the team determined parameters for the design of a blade for our turbine. The script, provided in the appendix, has six steps to output the necessary parameters. The table below illustrates the steps used in the script.

Step	Action
1	Set a and a' with zero values
2	Evaluate the flow angle $(\phi)$
3	Evaluate the local angle of incidence ( $\alpha$ )
4	Evaluate a and a'
5	Check convergence of a and a', repeat steps 2-5 until convergence is achieved
6	Calculate local forces (Torque, Power, Axial Force)
	Table 2: BEM Method

The formulas used to determine these parameters are the following:

$$\phi = \tan^{-1} \frac{R}{r \cdot J} \cdot \left(\frac{1 - a}{1 + a}\right) \tag{2}$$

$$\boldsymbol{\alpha} = \boldsymbol{\phi} - \boldsymbol{\beta} \tag{3}$$

$$\lambda = \frac{Z \cdot l \cdot C_L}{8 \cdot \pi \cdot r} \tag{4}$$

$$a = \frac{1}{1 + \frac{1}{\lambda} \sin \phi \cdot \tan \phi}$$

$$a' = \frac{1}{\frac{1}{\lambda}\cos\phi - 1}\tag{6}$$

Sample results from this script are shown in the figure below.

	Radius 0.2300	a 0.3452	a_p 0.0106	phi 8.0266	beta 4.0266	lambda 0.0104	i 11.0000	j 12.0000
	Radius 0.2500	a 0.4242	a_p 0.0097	phi 6.5166	beta 2.5166	lambda 0.0095	i 12.0000	j 19.0000
N =								
23	35.6200							
tord	4 =							
	0.0445							
powe	er =							
	6.6732							
Axia	al Force =							
	0.2590							
		Figure	13: Samp	le Matlab	BEM Meth	nod Resul	ts	

After this information is put together, the blades can be modeled using the selected airfoil shape in a computer aided design (CAD) program. Once one blade is modeled, it can be rotated 360 degrees around an elliptical dome shape to create the hub and blades of the turbine. An example of the output of this process is shown in Figure 14.

(5)



Figure 14: Example Turbine Blade Design

## 2.3 Prototype Manufacturing

To complete manufacturing of the design, the team needed to order additional off the shelf parts for a generator and shaft adapter. Utilizing internet searches, the team found several options for each as shown in the tables below.

The team chose to search for a DC motor that could act as a generator to allow the turbine to produce electricity. In searching for motors, the team considered three primary constraints. These were price, size, and a parameter called the back EMF. The back electromotive force, or back EMF, is the voltage output due to the motors rotations. It is a ratio relating the voltage output per RPM. A motor with a higher back EMF will produce more voltage per RPM. Table 3: Motor SelectionTable 3 shows some of the options that were explored, the first being the one that was chosen.

Motor	Manufacturer	Body	Back EMF	Price
		Diameter		
+	NMB Technologies	.803in.	.002155 V/rmp	\$3.76000
Ţ	Corporation	(20.4mm)		
À	Portescap	.866in.	.0016439 V/rpm	\$77.48000
K		(22.0mm)		
+	NMB Technologies	.953in.	.0009339 V/rpm	\$4.66000
	Corporation	(24.2mm)		

#### Table 3: Motor Selection

The team explored options for couplers to connect the shaft of the motor to the propeller. After searching several manufacturers, we chose to redirect our search to prop adapters for rc aircraft because the sizes available were small enough for the 2mm shaft that the motor had. Table 4 shows some of the prop adapters that were looked at, the first being the one the team chose.

Coupler	Manufacturer	Price
	E-flite	Need
	E-flite	\$3.00
	E-flite	\$3.00

Table 4: Coupler Selection

## 2.4 Future Research and Development

Due to the time constraints of this project, our team was not able to complete the project goals as set forth in the beginning of this project. In order for the completion of these goals, our team set forth a list of tasks that would need to be completed in order to achieve these goals. First, our team recommends that prototype parts be purchased. It is necessary for motors, couplers, shafts, hub connectors, and any other necessary parts to be purchased if not being manufactured. These parts will be utilized in a prototype hydrokinetic turbine to test the feasibility of each part for use in a finalized design.

We recommend that the prototype turbine blades, nacelles, etc. be created via 3D printing for relatively cheap and quick results. The 3D printed parts should be designed to accommodate any parts that are not being custom designed. Once the custom parts have been manufactured, all parts are to be combined and tested via the test stand that our group has created. The turbine test stand is intended to be used in the wind tunnel located in the basement of Higgins Laboratory. The stand allows the turbine's aerodynamic capabilities to be easily tested in the wind tunnel, because testing the turbine in air is simplistic and more feasible than testing the turbine in water. Reynold's number calculations will provide the information necessary to replicate similar flow in wind to an ocean current.



Figure 15: Test Stand



Figure 16: Test Stand Pivot

Based on the results of testing, it is possible to design improved editions of the turbine in order to meet project requirements. Prototype parts such as motors can be replaced in favor of superior performance. The base airfoil for the blades and the blade design base on blade element theory can be refined after testing to produce proper aerodynamic characteristics.

After prototyping and testing, a finalized list of parts can be chosen. The final model will be comprised of optimum parts and a proven design. The final model can be tested via the wind tunnel as the prototype, or it is possible that an apparatus can be designed to test the model underwater. This will require extensive research into underwater test stand design and flow generation. The results can be analyzed similarly to the prototype design.

## 3. Compound Hydro-kinetic Submersible Turbine

With the basics of hydrokinetic turbines established, our group introduced the notion of a compound hydrokinetic turbine. A compound hydro-kinetic submersible turbine is an innovative design that focuses on improving the common deficiencies that plague conventional hydro-kinetic turbines. This compound device consists of primary turbine that is free to rotate on its support axis, and attached to each blade tip of the primary turbine is a smaller, energy generating secondary turbine. While the primary turbine generates no energy, the secondary turbines contain generators and experience increased fluid velocity. This unique design enables simplified mooring techniques, increased fluid flow velocity, and more efficient energy collection. Figure 17Figure 17: Compound Turbine Design illustrates this design.



Figure 17: Compound Turbine Design

## **3.1 Mooring Advantages**

Typically, a turbine support structure experiences a reaction torque that arises from the rotation of its blades. This subjects the structure to high stresses that can degrade material over time. An important advantage of the compound turbine structure is that the net reaction torque of the primary turbine is canceled by the torques of the secondary turbines. This is similar to how many

helicopters fly; the primary blade on a helicopter will spin the body of the helicopter in the opposite direction of its rotation, unless there is a tail rotor to cancel out the reaction torque of the primary blade. By canceling the net reaction torque of the hydro-kinetic turbine, it is possible to simplify the mooring techniques. Instead of anchoring the turbine to the sea floor which poses problems of costly deep-sea installation and maintenance, the turbine can be attached to buoys and suspended into the fluid flow via a tether. Without net reaction torque, the tether will not twist and tangle. A tethered turbine can be floated to optimal locations in the fluid flow, or to the surface for easy maintenance. This introduces the possibility of an active stabilization and control system for the turbine body, which could allow the compound turbine to be "driven" to the most efficient locations in the fluid flow.

## **3.2 Rotation Speed Advantages**

As the rotation speed of a turbine increases, the energy output increases exponentially. The secondary turbines mounted on a compound turbine will experience rotational velocities of much greater magnitudes than the average single turbine. The increase in velocity of the secondary turbines is a function of the tip speed ratio of the primary turbine. For example, with a tip speed ratio of five, which is common for horizontal axis turbines, the velocity seen at the secondary turbines is equal to five times the velocity experienced by the primary turbine. The higher rotational velocity of the secondary turbines could eliminate the need for complex and maintenance-intensive gearboxes that are prevalent in commercial horizontal axis turbines. Without the need for speedup drives the turbine could instead operate with an alternator or direct drive generator and enjoy increased efficiency and reliability. Coupled with the simplified mooring, the turbine can generate energy more efficiently than other unique designs, like the Hydro Kite (discussed earlier). While the Hydro Kite also produces increased flow speeds, it suffers from large drag forces on its tether due to its sweeping, figure-eight pattern of motion.

## **3.3 Energy Efficiency Advantages**

A compound turbine is likely to be more energy efficient than a conventional horizontal axis turbine. A conventional turbine loses energy through the rotating a flow past its blades to produce energy. The compound design is advantageous because the primary blade is free spinning, and also spins the secondary turbines which are used to generate electricity. A free spinning blade will not apply rotation to the fluid downstream because it is not being used to generate electricity. This reduces the net loss of energy required in spinning the blades. Also, a single turbine will lose energy in the form of blade tip vortices, which spiral downstream in paths behind the rotor. The compound design takes advantage of blade tip vortices because each secondary turbine is mounted at a blade tip and would utilize the vortex to spin the secondary blades. A compound design has significant potential to be more efficient than a singulars design.

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## Appendix A: BEM MATLAB code

This code illustrates the iterative process of the blade element momentum theory.

```
% Compound Turbine Blade Element Method
% Computes
clc;
clf;
clear all;
close all;
% Specify Parameters %
                          % Number of Blades
Z=3;
cx1=7.5;
                          % velocity, m/sec
R=0.25;
                         % Tip radius, m
r h=0.03;
                         % Radius of Hub
c h=0.05;
                         % chord @ hub
dr=0.02;
                         % length of blade element
                         % Tip speed ratio
J=5;
                         % Section Lift Coefficient
Cl=0.4;
                         % Section Drag Coefficient
Cd=Cl*0.01;
alpha=Cl/.1;
                         % Section angle of attack
                         % pi
pi=3.1416;
rho=1.2;
                         % density of air kg/m^3
%rho=998.0;
                         % density of water kg/m^3
                         % Blade Taper Ratio tip-chord/root-chord
BTR=1.0;
omega=(cx1*J)/R;
nr=ceil((R-r_h)/dr);
                         % Angular Velocity
%m=1:1:nr;
                         % allocate r
r=r h:dr:R;
l=zeros(0,nr);
                         % allocate l
a=zeros(0, length(r)); % allocate a
a_p=zeros(0, nr); % allocate a_prime
phi=zeros(0, nr); % allocate phi
beta=zeros(0,nr);
                       % allocate beta
% allocate lambda
% allocate d_tau
lambda=zeros(0,nr);
d_tau=zeros(0,nr);
CF=0.001;
                         % convergence factor
%% Iterate over length of blade %%
for i=1:length(r)
% Compute a, a prime %
```

```
conv=1;
a(i)=0;
a p(i)=0;
a last=0;
a p last=0;
%l(i)=c h*(1-((r(i)-r h)/(R-r h)))*BTR;
l(i)=c h;
j=1;
    while conv >= CF
        phi(i) = at and (R/(r(i)*J)*(1-a(i))/(1+a p(i)));
        beta(i)=phi(i)-alpha;
        lambda(i) = (Z*l(i)*Cl) / (8*pi*r(i));
        a(i) = (lambda(i) * cotd(phi(i))) / (sind(phi(i)) + lambda(i) * cotd(phi(i)));
        a p(i)=lambda(i)/(cosd(phi(i))-lambda(i));
        conv=abs(a last-a(i)+a p last-a p(i));
        a last=a(i);
        a p last=a p(i);
        X=[r(i) a(i) a p(i) phi(i) beta(i) lambda(i) i j];
        if j>20
           break
        end
        j=j+1;
        if conv < CF
            break
        end
    end
    %disp(j);
    disp('
            Radius a ap phi beta lambda i
j');
    disp(X);
d tau(i)=0.5*rho*Z*l(i)*power(omega,2)*power(R,4)*power(((1+a p(i))/(cosd(phi
(i))),2)*power((r(i)/R),3)*Cl*sind(phi(i))*dr;
    F(i)=0.5*rho*Z*l(i)*R*power(cx1,2)*power((1-
a(i))/(sind(phi(i))),2)*Cl*cosd(phi(i))*dr;
end
```

```
N=omega/2*pi
torq=sum(d_tau)
power=omega*torq
Axial Force=sum(F(i))
```