EXPECTED MAXIMUM DRAWDOWNS UNDER CONSTANT AND STOCHASTIC VOLATILITY

BY

SUHILA NOURI

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

PROFESSIONAL MASTERS DEGREE IN FINANCIAL MATHEMATICS

WORCESTER POLYTECHNIC INSTITUTE

2006

APPROVED BY LUIS J. ROMAN

____

TABLE OF CONTENTS

Abstract

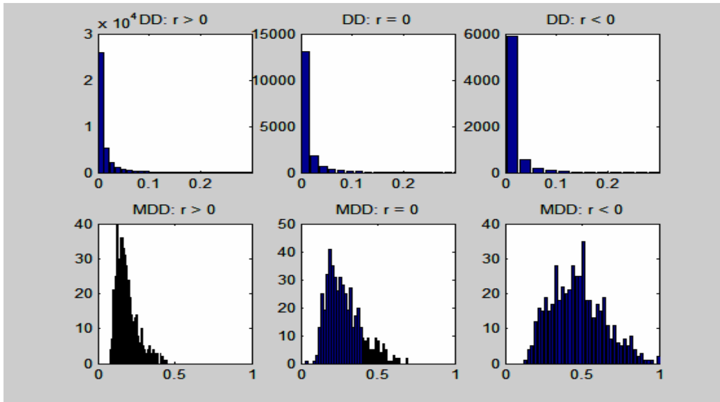
The maximum drawdown on a time interval [0, T] of a random process can be defined as the largest drop from a high water mark to a low water mark. In this project, expected maximum drawdowns are analyzed in two cases: maximum drawdowns under constant volatility and stochastic volatility. We consider maximum drawdowns of both generalized and geometric Brownian motions. Their paths are numerically simulated and their expected maximum drawdowns are computed using Monte Carlo approximation and plotted as a function of time. Only numerical representation is given for stochastic volatility since there are no analytical results for this case. In the constant volatility case, the asymptotic behavior is described by our simulations which are supported by theoretical findings. The asymptotic behavior can be logarithmic for positive mean return, square root for zero mean return, or linear for negative mean return. When the volatility is stochastic, we assume it is driven by a mean-reverting process, in which case we discovered that if one uses the effective volatility in the formulas obtained for the constant volatility case, the numerical results suggest that similar asymptotic behavior holds in the stochastic case.

Introduction

Quantifying risk is a primary concern of any investor. If the standard deviation of returns for a manager is large enough to produce a loss during some time period, that manager will experience drawdowns. Many consider a manager's drawdowns to be a better measure of risk than simply considering the volatility of returns or a return/risk measure such as the Shape ratio [1]. Also, taking drawdowns as a description of a manager's historical performance has the distinct quality of referring to a physical reality. It is known that the Commodity Futures Trading Commission (CFTC) has a mandatory disclosure regime that requires futures traders to disclose as a part of their performance their "worst peak-to-valley drawdown" [6]. Particularly in hedge funds, estimating drawdown and maximum drawdown is imperative for estimating the probability of reaching a stop-loss that may set off large liquidations and of reaching the high water mark prior to the end of the year that will result in a performance fee [7].

A drawdown is defined as change in value of a portfolio from any established peak (high water mark) to the subsequent trough (low water mark). A high water mark has occurred if it is higher than any previous net asset value and is followed by a loss. A low water mark has occurred if it is the lowest net asset value between two high water marks. A maximum drawdown of a portfolio is the largest drop from a high water mark to a low water mark. Even though a manager can only have one maximum drawdown, it is informative to look at the distribution from which the maximum drawdown was drawn. If one considers several managers, all with the same or similar track records and return characteristics, it is feasible to see what their distributions of worst drawdowns look like

[1]. Simulated drawdowns (DD) and corresponding maximum drawdowns (MDD) can be seen below, with mean return .2, 0, and -.2, sigma .3, and time interval 1.



This project is structured as follows: Chapter one will cover the theoretical and empirical results of drawdowns and expected maximum drawdowns under the assumption of constant volatility. Chapter two will reveal what happens to expected maximum drawdowns when the volatility function is taken to be stochastic. Chapter three will show numerical examples of expected maximum drawdowns under the cases of constant and stochastic volatility. Finally, Chapter four concludes the theoretical and empirical findings reported in this project and contains recommendations for future work.

Chapter 1: Drawdowns and Maximum Drawdowns under Constant Volatility

1.1 Empirical Derivations

Of the various parameters that can influence drawdowns: length of track record, mean return, volatility of returns, skewness, and kurtosis, only the first three have great effect [1]. The possibility of experiencing a drawdown of any size is significantly independent of the amount of time the manager has been in the business, and so length of track record does not greatly effect the distribution of drawdowns. Mean returns, however, do effect the distribution. Higher mean returns lead to smaller expected drawdown. Volatility of returns also has significant influence over drawdowns; the greater the volatility, the greater the expected drawdown. Skewness and kurtosis do not greatly affect drawdown. A possible reason for this is that drawdowns result from adding together a sequence of returns and by the central limit theorem, even if the distribution of returns is highly skewed or contains fat tails, their sum produces a relatively normal distribution of returns [1].

Analysis of historical data suggest that the relationship between mean return, volatility of returns, and drawdown can be empirically described by

$$DD/\sigma = f(\mu/\sigma)$$

where σ is the standard deviation of returns and μ is the mean return [1]. And so, a manager's drawdowns divided by the volatility of returns can be written as a function of that manager's Sharpe ratio. When viewing the shape of this function, the curve indicates that volatility matters more than mean return. If a manager's mean return is doubled and the volatility is held constant, the expected drawdown per unit of volatility is lowered by less than half. Likewise, if the volatility is doubled but the mean return is held constant, expected maximum drawdown will more than double per unit of volatility. However, if one is only concerned with the size of drawdowns instead of drawdown per unit volatility, the empirical function can be rewritten as

$$DD = \sigma f(\mu/\sigma).$$

Viewing the relationship between drawdowns and returns in this form indicates some other properties. For one, if both mean return and volatility are doubled, and so the modified Sharpe ratio is not changed, expected maximum drawdowns will exactly double. Second, if volatility is doubled, expected maximum drawdowns will more than double. And third, mean return would have to be more than doubled to make up for doubling the volatility. This relationship also shows that two managers can have the same volatility of returns but different expected drawdowns if their mean returns are different. Also, two managers with the same modified Sharpe ratio can have different expected drawdowns if their volatilities of returns are different.

Maximum drawdowns, as in drawdowns, are greatly affected by the mean return and the volatility of returns, but not skewness or kurtosis. However, unlike the case with drawdowns, the likelihood of a manager experiencing a larger drawdown than he ever has before increases with each day. An increase in the length of track record will shift the maximum drawdown distribution to the left. Higher returns will generate smaller maximum drawdowns. However, higher volatility will increase the possibility of large maximum drawdowns.

1.2 Concerns with Using Drawdown as a Statistical Measure

Drawdowns lose value as the estimate of a manager's quality due to some limitations. There is a relationship between drawdown and two important statistics, mean and variability. Any portfolio with a long-run positive return is expected to "drift" upward in time. When a positive expected return rises from a stochastic or partly stochastic process, the upward move will have some random variation that will cause the portfolio's value to decline below its previously obtained peak. This fall is a drawdown. A drawdown will be smaller if the upward drift is steeper or the variability of the process is lower. Because of this, drawdown is a function of mean and variability. Without knowledge of this function, however, or the return generating process, one cannot understand what the level of the drawdown relates to. This means a raw drawdown offers little information as a statistic and even less as a predictive one [6].

Since a maximum drawdown is one number determined by a series of data, there is a large error associated with it. Because of this, the generation of a future return from historical maximum drawdowns carries a high chance of error. Large errors in statistical measures can be resolved by averaging. In this project, however, a series of maximum drawdowns are simulated.

Besides maximum drawdowns being error-prone, however, there are two other adjustments that should be made. One is that drawdowns must be compared on the same time interval. All things held equal, drawdowns are larger the greater the frequency of the time interval. So any investment that is managed daily will be at a disadvantage over one managed weekly or monthly. Secondly, managers with long track records will experience larger maximum drawdowns. They have been in the business longer and have overcome more difficulties in the market than a newcomer.

Thus, to improve drawdowns as a statistic, the track record of a manager, the error, measurement interval, and volatility must all be taken into account. Also, the return generating process must be known. It can be argued that all of these disadvantages to drawdowns are too much to correct and it is better to focus on mean and volatility. But when using drawdowns, it is important to use them with these things in mind and with the understanding of the underlying process rather than simply historical record.

1.3.1 Theoretical Findings of a Standard Brownian Motion

Drawdowns have been studied in a variety of disciplines, among them are: mathematics, physics, and management. The simplest theoretical formulation of drawdowns is that of downfalls in a standard Brownian motion. Results can be obtained for maximum drawdowns in this case. Let $B = (B_t)_{0 \le t \le 1}$ be of standard Brownian motion on a probability space $(\Omega, F, \mathbf{P})$ where $B_0 = 0$, $E[B_t] = 0$, and $E[B_t^2] = t$, then the maximum drawdown is defined as:

$$MDD = \sup_{0 \le t \le t' \le 1} (B_t - B_{t'}).$$

It can be seen that MDD describes the maximum downfall in the trajectories of the Brownian motion on the time interval [0, 1]. It can be seen that the following equivalent definitions hold:

$$MDD = \sup_{0 \le t \le 1} (B_t - \inf_{t \le t' \le 1} B_{t'})$$

$$\sup_{0 \le t' \le 1} (\sup_{0 \le t \le t'} B_t - B_{t'}).$$

In [6], the authors R. Douady, A.N. Shiryaev, and M. Yor showed that the distribution of maximum drawdowns for a standard Brownian motion is the same as that of

$\sup_{0 \leq t \leq 1} |B_t|$.

Moreover, the expectation is

$$E[MDD] = \text{sqrt}(\pi/2) \quad \text{(Note: sqrt} \equiv \text{square root)}$$

(where t = 1, otherwise the result is sqrt(π/(2t)) and the distribution function $F_D(x) = \mathbf{P}\{MDD \leq x\}$ is given by

$$F_{MDD}(x) = 1 - 1/\text{ sqrt}(2\pi) \sum_{k=-\infty}^{\infty} \int_{-x}^{x} [e^{-(y+4kx)^2/2} - e^{-(y+2x+4kx)^2/2}]dy.$$

1.3.2 Theoretical Findings of a Generalized Brownian Motion

Maximum drawdowns are defined as a measure of risk that can be defined for more general stochastic processes. It is common to assume that the value of a portfolio follows a "generalized Brownian motion." Let B_t be a standard Brownian motion as defined in the previous section. Then, X_t is a generalized Brownian motion if it is of the form

$$X_t = \sigma B_t + \mu t \quad 0 \leq t \leq T$$

for the given constants $\mu \in \mathbf{R}$, the drift rate, and σ, the diffusion parameter, greater than zero.

The high, H, and low, L, of X_t are given by

$$H = \sup_{t \in [0,T]} X_t \qquad L = \inf_{t \in [0,T]} X_t$$

and the maximum drawdown is defined by

$$MDD(T; \mu, \sigma) = \sup_{t \in [0,T]} [\sup_{s \in [0,t]} X_s - X_t].$$

The distribution function for MDD is

$$G_{MDD}(h) = 2\sigma^4 \sum_{n=1}^{\infty} \frac{\theta_n \sin\theta_n}{\theta^4\theta_n^2 + \mu^2h^2 - \sigma^2\mu h} \exp\{-\frac{\mu h}{\sigma^2}\} \left(1 - \exp\{-\frac{\sigma^2\theta_n^2 T}{2h^2}\} \exp\{-\frac{\mu^2 T}{2\sigma^2}\}\right)$$
$$+M$$

where, for $n \geq 1$, θ_n is the positive solution of the eigenvalue condition

$\tan\theta_n = (\sigma^2/\mu h)\theta_n$

and M is

$= 0$	$\mu < \sigma^2/h$
$= (3/e)(1 - \exp\{-\mu^2 T/2\sigma^2\})$	$\mu = \sigma^2/h$
$= \dfrac{2\sigma^4\eta \sinh\eta \exp\{-\mu h/\sigma^2\}}{\sigma^4\eta^2 - \mu^2 h^2 + \sigma^2\mu h} (1 - \exp\{-\mu^2 T/2\sigma^2\} \exp\{\sigma^2\eta^2 T/2h^2\})$	$\mu > \sigma^2/h$

where η is the unique positive solution of

$\tanh\eta = (\sigma^2/\mu h)\eta.$

Using the identity $E[MDD] = \int_0^\infty dh\, G_{MDD}(h)$, it is determined that

$E[MDD] = (2\sigma^2/\mu)\, Q_{MDD}(\mu\ \mathrm{sqrt}[T/(2\sigma^2)])$

where $Q_{MDD}(x)$ is defined to be

$Q_p(x)$	$\mu > 0$
$\gamma\ \mathrm{sqrt}(2x)$	$\mu = 0$
$-Q_n(x)$	$\mu < 0$

where $\gamma = \mathrm{sqrt}(\pi/8)$ and Q_p and Q_n are functions whose exact values are known and tabulated. Their asymptotic behavior is given by

$Q_p(x) \rightarrow$	$\gamma\ \mathrm{sqrt}(2x)$	$x \rightarrow 0$
	$\frac{1}{4}\log(x) + 0.49088$	$x \rightarrow \infty$
$Q_n(x) \rightarrow$	$\gamma\ \mathrm{sqrt}(2x)$	$x \rightarrow 0$
	$x + \frac{1}{2}$	$x \rightarrow \infty.$

The general result obtained by Malik Magdon-Ismail, Amir F. Atiya, Amrit Pratap, and Yaser S. Abu-Mostafa in [6], is that

$E[MDD] = (2\sigma^2/\mu)\, Q_{MDD}(\alpha^2).$

where $\alpha = \mu$ sqrt$[T/(2\sigma^2)]$. Notice that if T is one, μ equals zero, and σ equals one, as is the case in standard Brownian motion, these results give:

$$E[MDD] = \text{sqrt}(\pi/2)$$

which agrees with the findings of Douady, Shiryaev, and Yor.

1.4 The Asymptotic Behavior Expected Maximum Drawdowns

The distribution and asymptotic behavior of maximum drawdowns for a generalized Brownian motion were also analyzed in [7]. Those results show that the asymptotic behavior for expected maximum drawdowns as T tends to infinity are:

$$E[MDD] = 2\sigma^2/\mu\ Q_p(\mu^2T/(2\sigma^2)) \rightarrow \sigma^2/\mu(0.63519 + 0.5\log(T) + \log(\mu/\sigma) \quad \text{if } \mu > 0$$

$$= 1.2533\sigma\ \text{sqrt}(T) \quad \text{if } \mu = 0$$

$$= -2\sigma^2/\mu\ Q_n(\mu^2T/(2\sigma^2)) \rightarrow \mu T - \sigma^2/\mu \quad \text{if } \mu < 0.$$

1.5 Geometric Brownian Motion

Theoretical results can also be obtained for maximum drawdowns of a geometric Brownian motion. In this case maximum drawdowns follow the process

$$dX_t = \mu X_t dt + \sigma X_t dB_t.$$

Taking the log transformation X_t^ = log X_t and using Ito's formula, one obtains

$$dX_t\hat{} = (\mu - ½\sigma^2)dt + \sigma X_t dB_t.$$

This result is the same as the generalized Brownian motion case with altered parameters. The result for expected maximum drawdowns then becomes:

$$E[MDD] = 2\sigma^2/(\mu - ½\sigma^2)\ Q_p((\mu - ½\sigma^2)^2T/(2\sigma^2)) \quad \text{if } \mu > 0$$

$$= 1.2533\sigma\sqrt{T} \quad \text{if } \mu = 0$$

$$= -2\sigma^2/(\mu - \tfrac{1}{2}\sigma^2)\, Q_n(\mu^2 T/(2\sigma^2)) \qquad \text{if } \mu < 0.$$

Chapter 2: Expected Maximum Drawdowns under Stochastic Volatility

It is no longer sufficient to use the Black-Scholes model to explain modern market occurrences [3]. This is especially true since the 1987 crash. The common practice, both in analytical and practical application, has been to change the volatility function from being constant to stochastic. A stochastic process is more complex since the volatility is the driving process but cannot be seen. Moreover, the volatility tends to be mean-reverting. This means it fluctuates at high levels for a time period and then fluctuates at a low level for a similar amount of time. The Black-Scholes model relies on a lot of assumptions that do not necessarily hold true, one being constant volatility. A well known problem between predicted European option prices determined by Black-Scholes and options traded in the market, the smile curve, can be resolved by stochastic volatility models. This shows that using stochastic volatility models resolves a problem in one area where the constant model failed. In this work maximum drawdowns are considered only when the volatility is a function of some special cases of mean-reverting processes.

2.1 Mean-Reverting Stochastic Volatility Models

A mean-reverting process refers to the time it takes for a process to return to the mean level of its long-run distribution. A mean-reverting process in financial modeling means the linear pullback term in the drift of the volatility process. Letting $\sigma_t = f(Y_t)$, where f is a positive function, the mean-reverting stochastic volatility means the stochastic differential equation for Y_t resembles:

$$dY_t = \alpha(m - Y_t)dt + \ldots\ dZ_t$$

where Z_t is a Brownian motion correlated with (W_t), α is the rate of mean reversion, and m is the long-run mean level of Y. The drift term pulls Y to m, so σ_t is pulled to the mean value of f(Y) with respect to the invariant distribution of Y.

The Ornstein-Uhlenbeck and Cox-Ingersoll-Ross Processes

One mean-reverting process used is the Ornstein-Uhlenbeck process. This process is defined as the solution of

$$dY_t = \alpha(m - Y_t)dt + \beta\ dZ_t.$$

The process is Gaussian given in the terms of its starting value y by

$$Y_t = m + (y - m)e^{-\alpha t} + \beta\int_0^t e^{-\alpha(t-s)}\ dZ_s$$

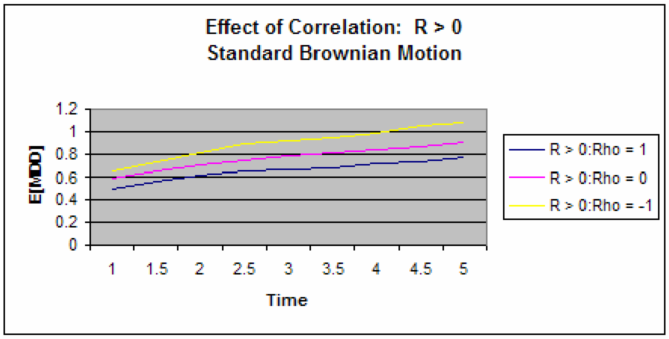
where Y_t is normally distributed with mean $m + (y - m)e^{-\alpha t}$ and variance $\beta^2/[2\alpha(1 - e^{-\alpha t})]$. As t increases to infinity, its long-run distribution becomes normally distributed with parameters $(m, \beta^2/(2\alpha))$. The second Brownian motion (Z_t) is usually correlated with the Brownian motion (W_t) from the stochastic differential equation

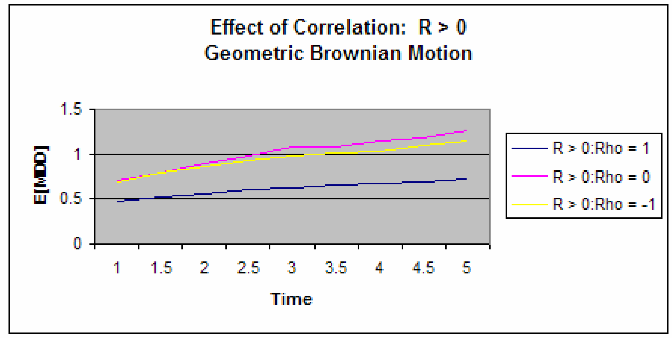
$$dX_t = \mu X_t\ dt + \sigma_t X_t\ dW_t.$$

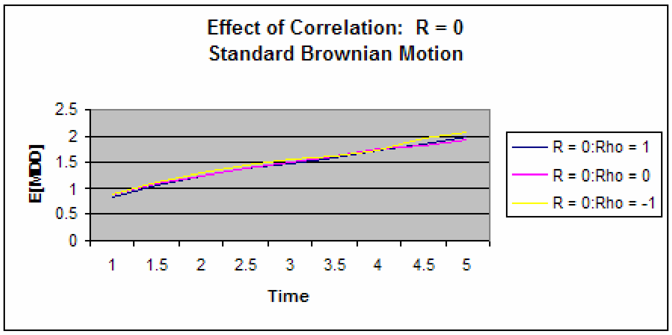
Let $\rho\ \varepsilon\ [-1, 1]$ denote the instantaneous correlation coefficient defined by

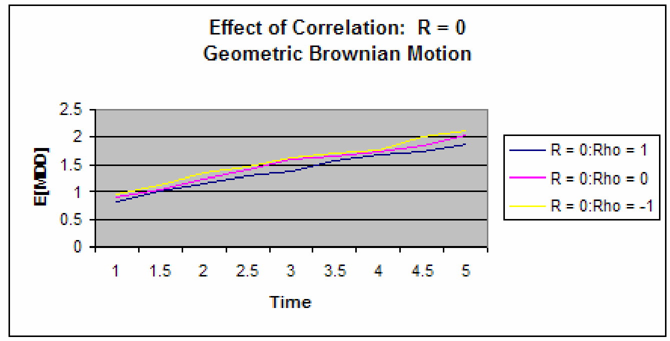
$$d\langle W, Z\rangle_t = \rho dt.$$

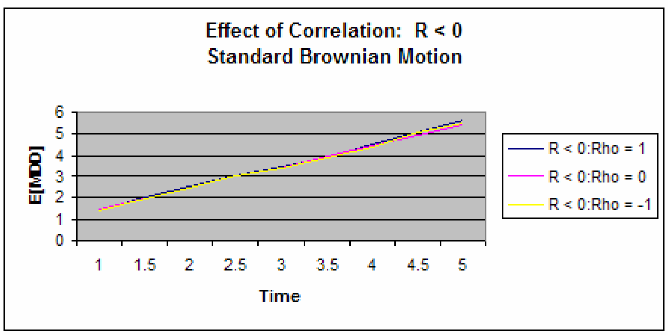
In empirical studies it is often found that ρ is less than zero. Asset prices tend to decrease when volatility increases, causing a negative correlation between the two. As can be seen in the plots below, correlation has lesser effect when it is negative as opposed to when it is greater than or equal to zero.

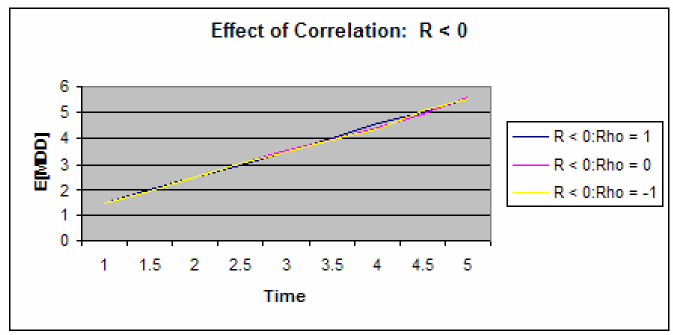












The Ornstein-Uhlenbeck process is one of the processes that often drive sigma and is the one implemented in this project.

The Cox-Ingersoll-Ross process is another mean-reverting model used and is defined by

$$dYt = \kappa(m' - Y_t)dt + \upsilon \text{ sqrt}(Y_t)\, dZ_t.$$

Recall that for the OU process, the asymptotic behavior of Y_t is the same for either $t \rightarrow \infty$ or $\alpha \rightarrow \infty$. This means that the problem can be rescaled for different times and a fixed time T and one can compare the asymptotic behavior as α tends to infinity.

2.2 The Stochastic Volatility Model

In the stochastic case of determining expected maximum drawdowns, we will assume a mean-reverting OU process. The driving process for the return on a portfolio becomes

$$dX_t = \mu X_t dt + \sigma_t X_t dW_t$$

where

$$\sigma_t = f(Y_t)$$

$$dY_t = \alpha(m - Y_t)dt + \beta dZ_t\hat{}.$$

The rate of mean-reversion, α, indicates how quickly the driving volatility process, Y_t, tends back to its equilibrium. Y_t is Gaussian with mean $(m + (y - m)e^{-\alpha t})$ and variance $[\beta^2/(2\alpha)](1 - e^{-\alpha t})$. What is of interest is the asymptotic behavior of X_t and how it affects the drawdowns of the process X_t. First one finds an initial distribution Y_0 so that for any $t > 0$, Y_t has the same distribution. This would be its invariant distribution. So, as t tends to infinity, Y_t is Gaussian with mean m and variance $\beta^2/(2\alpha)$. When using the OU process, this invariant distribution is determined by the density

$$\Phi(y) = 1/\text{sqrt}(2\pi\upsilon^2) \exp[-(y - m)^2/2\upsilon^2]$$

where

$$\upsilon^2 = \beta^2/(2\alpha).$$

From the fact that

$$Y_t \sim N(m + (y_0 - m)e^{-\alpha t}, \upsilon^2(1 - e^{-2\alpha t}))$$

or from

$$E[(Y_t - m)(Y_s - m)] = \upsilon^2 e^{-\alpha|t-s|}$$

with s = 0, it can be seen that with υ fixed the limits $t \to \infty$ and $\alpha \to \infty$ are the same in terms of distributions. Therefore when t tends to infinity,

$$1/t \int_0^t g(Y_s)\, ds \approx \langle g \rangle$$

where $\langle g \rangle$ denotes $\int_{-\infty}^{\infty} g(y)\Phi(y)dy$. $\langle g \rangle$ is the average of g respect to the invariant distribution Φ. The uniqueness of the invariant distribution and the correlation property are the primary characteristics of an ergodic process. Denote the effective volatility, which is the volatility function averaged against the invariant distribution, by

$$\bar{\sigma}^2 \approx \langle f^2 \rangle.$$

In the empirical findings of this thesis, the volatility function is taken to be exp(y). So the effective volatility becomes

$$\int_{-\infty}^{\infty} e^{2y}\Phi(y)dy.$$

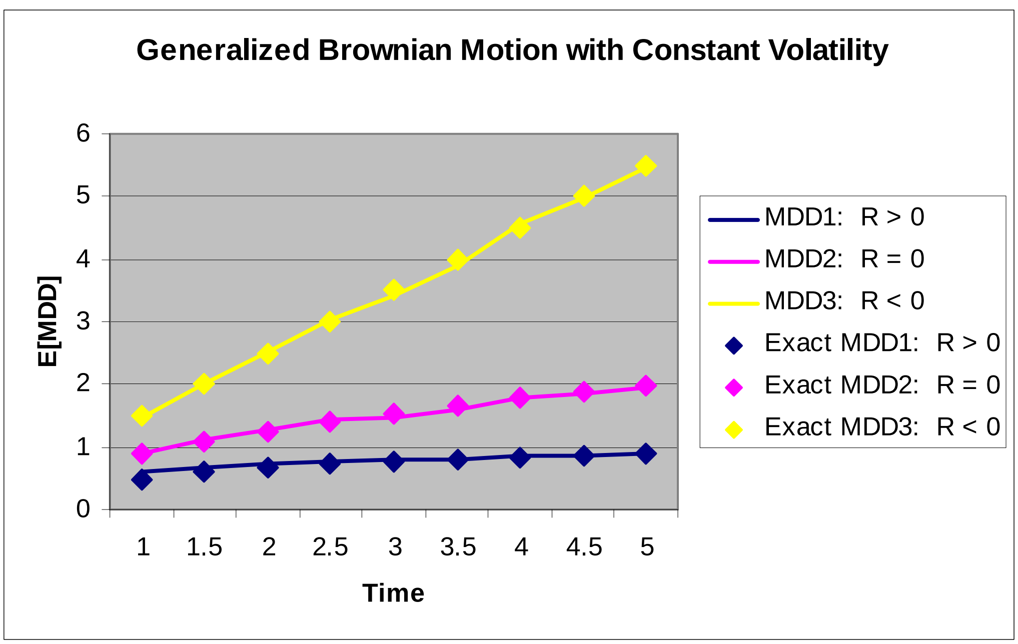
Solving this integral yields an effective volatility of

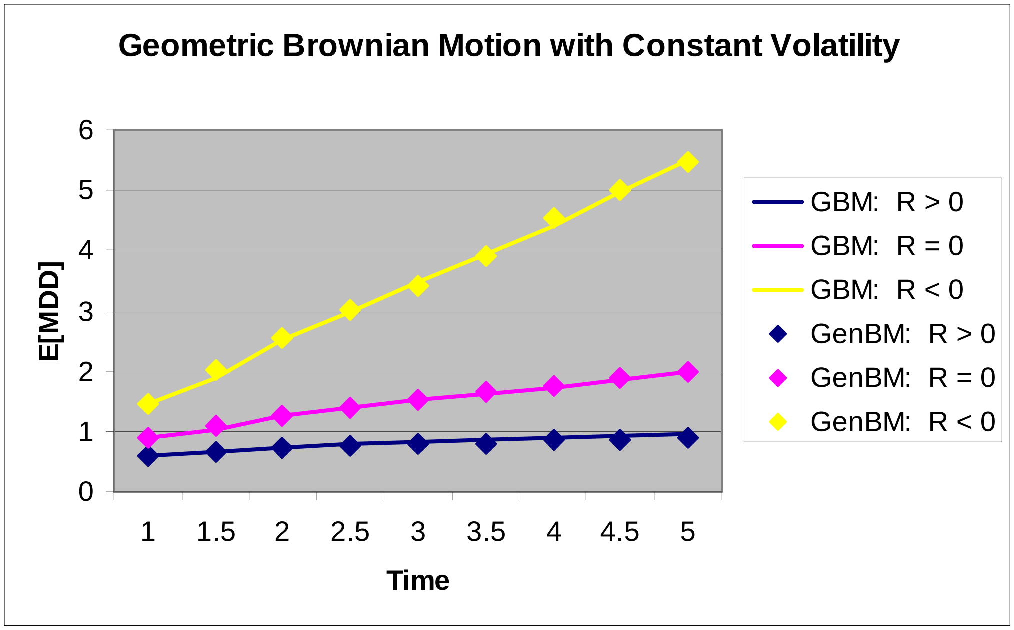
$$\bar{\sigma}^2 = e^{2(m + \upsilon^2)}.$$

Chapter 3: Numerical Results

3.1 Constant Volatility

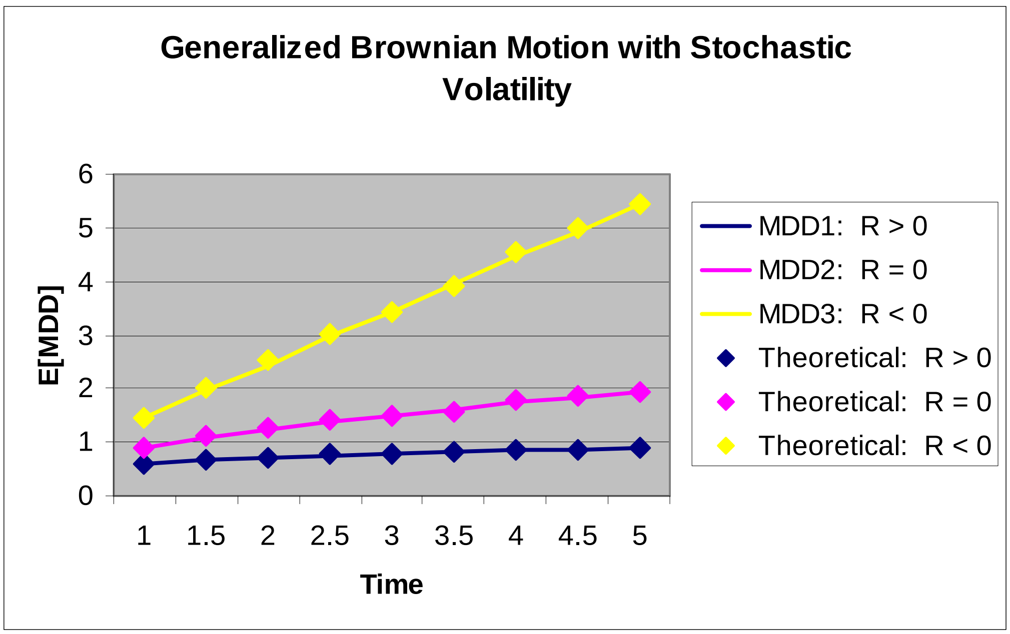
In this section we use the code developed in [8] to compare the numerical simulations to the theoretical asymptotic behavior of expected maximum drawdowns. The theoretical results were tested numerically in Matlab by simulating returns using finite difference schemes for the processes involved and computing expected values via Monte Carlo methods. The stock path is determined to follow a generalized or geometric Brownian motion through a switch statement in the code. The volatility is kept constant at .3 and the return is .2, 0, or -.2 to observe the asymptotic behavior for positive, zero, and negative returns. Drawdowns and maximum drawdowns are stored in arrays. The code is run for varying values of time that is incremented by a half from 1 to 5 and the maximum drawdowns at each time T is plotted against the theoretical value obtained from the theoretical formulas stated in Chapter one. Below is the graph for the expected maximum drawdowns for generalized Brownian motion under constant volatility and then for geometric Brownian motion under constant volatility. In the generalized Brownian motion plot, MDD stands for the numerical generation of expected maximum drawdowns and Exact MDD represents the theoretical calculation of expected maximum drawdowns for the given mean and volatility. In the plot for geometric Brownian motion (GBM), expected maximum drawdowns are plotted for generalized Brownian motion (GenBM), as well, for comparison.

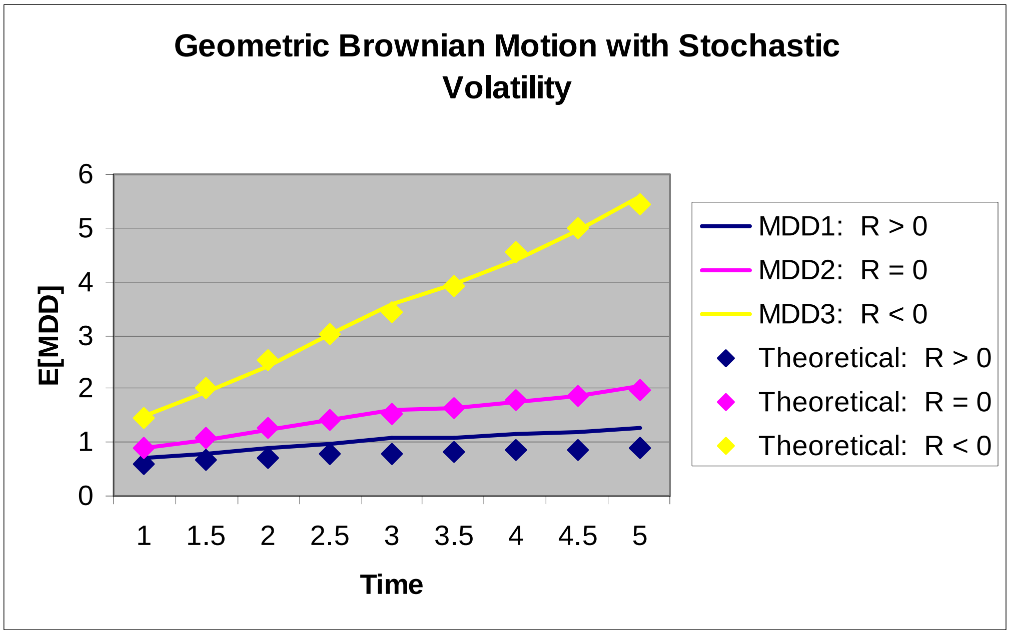




3.2 Stochastic Volatility

A limitation exists in the stochastic case since no theoretical results are currently known. However, in this section we show that if one uses the effective volatility in place of the volatility in the formulas obtained for the constant volatility case, then similar numerical results are obtained in determining expected maximum drawdowns. Just as in the constant volatility case, returns are generated via finite difference schemes and the expected values are computed via Monte Carlo approximations. The stock path is determined to follow a generalized or geometric Brownian motion through switch statements in the code. The volatility function is $\sigma_x = \exp(x)$, and so the effective volatility function is as it was defined in Section 2.2 and the return is .2, 0, or -.2 to cover the asymptotic behavior for positive, zero, and negative returns. As in the constant case, drawdowns and maximum drawdowns are stored in arrays. The code is run for varying values of time incremented by a half from 1 to 5 and the expected maximum drawdowns at each time T is plotted. Below is the graph for the expected maximum drawdowns for generalized Brownian motion with stochastic volatility and then for geometric Brownian motion with the same volatility function. "MDD" stands for the numerical expected maximum drawdowns generated. These expected maximum drawdowns with stochastic volatility are plotted against the theoretical results obtained in Chapter one, but using the effective volatility instead. It can be seen that the theoretical results hold even in the stochastic case.





Chapter 4: Conclusion and Future Work

Maximum drawdowns have been shown to reflect a physical reality of risk in the performance of a portfolio manager. They relate mean return and variability to yield a measurement of risk in return. They provide such a good insight into performance that the CFTC requires a disclosure of maximum drawdowns of futures traders. When considering maximum drawdowns, it is imperative to take into account the mean return, variability of returns, and length of track record. Each parameter effects the distribution of maximum drawdowns, and so it is important when comparing expected maximum drawdown results to keep these parameters in mind.

It has been shown that when the volatility is taken to be constant, expected maximum drawdowns follow an asymptotic behavior depending on whether the mean return is positive, negative, or zero. Though there are no such analytical results for expected maximum drawdowns of stochastic volatility, results can be obtained through numerical simulations. These experiments suggest that the same asymptotic behavior holds in the stochastic case. But the crucial aspect to account for is the effective volatility. This brings up the intriguing problem of analytically showing what the asymptotic behavior of expected maximum drawdowns is under stochastic volatility. In this project, expected maximum drawdowns were numerical simulated under the mean-reverting process of Ornstein-Uhlenbeck and an exponential volatility function. Another interesting experiment would be to conduct more simulations with other mean-reverting processes and alternate volatility functions. Partial results considering other volatility functions and further analysis on the effect of correlation can be found in [8].

References

1. Burghardt, Galen, Ryan Duncan, and Lianyan Liu. "Understanding Drawdowns." Carr Futures 6 May 2003.
2. Douady, R., A. N. Shiryaev, and M. Yor. "On Probability Characteristics of 'Downfalls' in a Standard Brownian Motion." Theory Probab. Appl. 44 No. 1 (1998): 29-38.
3. Fouque, Jean Pierre, George Papanicolaou, and Ronnie Sircar. Derivatives in Financial Markets with Stochastic Volatility. Cambridge Univesity Press 2000.
4. Harding, David, Georgia Nakou, and Ali Nejjar. "The Pros and Cons of 'Drawdown' as a Statistical Measure of Risk for Investments." AIMA Journal April 2003.
5. Lopez de Prado, Marcos Mailoc and Achim Peijan. "Measuring Loss Potential of Hedge Fund Strategies." The Journal of Alternative Investments Summer 2004.
6. Magdon-Ismail, Malik, Amir F. Atiya, Amrit Pratap, and Yaser S. Abu-Mostafa. "On the Maximum Drawdown of a Brownian Motion." J. Appl. Prob. 41 (2004): 147-161.
7. "Maximum Drawdown." Cutting Edge 2004: n. pag. Online. Internet. October 2004. Available: www.risk.net.
8. Roman, Luis and Shankar Subramaniam. "Effects of Stochastic Volatility in the Distribution of Drawdowns and Maximum Drawdowns." To be submitted.