2016

Worcester Polytechnic Institute

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ECONOMIC SCENARIO GENERATOR

Economic Scenario Generator

A Major Qualifying Project
submitted to the faculty of
WORCESTER POLYTECHNIC INSTITUTE
in partial fulfilment of the requirements for the
degree of Bachelor of Science

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Date:
04/27/2016

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Abstract

Economic volatility is unpredictable, but investors try their best to prepare for the worst possibilities. A simulation of this fluctuation may be captured using mathematical models. An Economic Scenario Generator (ESG) uses a mathematical procedure to simulate, not predict, the returns for assets. We built our own ESG, using Excel, that is designed to include unlikely economic catastrophes. Our ESG simulated the returns of 10 exchange traded funds (ETFs) based on the historical returns of these ETFs. The final simulated returns of our ESG provide scenarios that may help investors prepare for various economic conditions.

Acknowledgements

Our team would like to acknowledge the following individuals for their valuable contributions to the success of this Major Qualifying Project:

- ➢ Professor Jon Abraham for his direction, support, and advice.
- ➢ Professor Barry Posterro for his knowledge, direction and guidance

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Executive Summary

Most individuals and companies would agree that being confident in their investments is advantageous. Although it is currently impossible to predict the future, it is possible to simulate it. An economic scenario generator (ESG) is a model that simulates asset returns for a group of correlated assets. The parameters of the model are developed using maximum likelihood estimation on historical return data. Through a process known as regime switching, an ESG is able to switch between different scenarios that may include a growing economy, a falling economy, and an economic crash. Our ESG uses parameters to generate simulations that are representative of the real world.

Background

There are two types of Economic Scenario Generators (ESGs), market consistent ESGs and real world ESGs. A market consistent ESG generates returns that are consistent with market prices. A real world ESG generates returns that are consistent with actual data and real world expectations, and this is the type of ESG we created. Our ESG transforms random numbers using an inverse transform method and then correlates them to achieve the desired distribution and covariance structure. The parameters and covariance structure used were calculated based on historical Exchange Traded Funds (ETFs) data. ETFs track groups of commodities and assets and can be traded similarly to common stocks.

Since the economy goes through different periods of volatility, a proper ESG should portray this in its simulations. Our ESG will go through multiple states of volatility during different stages of the simulation to model the real world economy going through periods of

varying volatility. These states are referred to as regimes. A Markov Chain approach is used to model the transitions between regimes. Depending on the regime the ESG is currently in, there is a probability of either stay in that regime or switch into one of the others. The inverse transform method is used to sample random numbers for probability distributions, which determine the regime the ESG is in.

Once this is completed, parameters can be calibrated to emulate the historical ETF data through Maximum Likelihood Estimation (MLE). This method analyzes data to estimate the parameters of the distribution, and produces the parameters that give the highest probability, or likelihood, of seeing the data that was observed.

Our ESG uses a covariance matrix constructed using the covariance of the ETF data, which can then be decomposed using Cholesky Decomposition. Covariance refers to a statistical relationship involving dependence used to measure how much two random variables change together. Given matrix A is positive definite it can be decomposed into a product of a unique lower triangular matrix L and its transpose L^T, where $A = L \times L^T$. For the ESG, matrix A is the covariance matrix of the historical data. The matrix A can be decomposed to get L and L^T. Then, L^T is multiplied by the random numbers so that they will have the same covariance structure as the historical ETF data.

Methodology

The exchange traded funds (ETF) and exchange traded notes (ETN) we utilized for our simulations include: SPY, IWM, TLT, HYG, GLD, EFA, VXX, OIL, FEZ, and EEM, because they are well established and have at least five years of historical data. For all ten ETFs we collected five years of daily returns and then took the natural log of these returns to get log returns.

Taking the values from the ETFs, we used MLE to calculate the mean and volatility of Regime 1 and Regime 2 of all ten ETFs by following the steps used by Mary Hardy in her paper "A Regime Switching Model of Long Term Stock Returns." Since we had the parameters for the first two regimes, we had to create the parameters for Regime 3 and adjust the other regimes accordingly. The first regime simulates a healthy economy with a positive mean and low volatility; the second simulates a falling economy with a lower mean and higher volatility, and the third regime simulates an economic crash with a low mean and very high volatility. In order to implement the regime switching process, we generated a random number that determined which regime the ten returns would be in.

We used Excel version 2007 to generate the random numbers needed to simulate the returns. Real world stock markets are often simulated using a lognormal distribution. In order for the random numbers we generated to be lognormally distributed we took the natural log of the returns from the ETF data. Then the random numbers are made lognormally distributed when they are multiplied by the covariance matrix and then added to the mean.

To construct a covariance matrix we used the data from the ten ETFs and plugged them into the Excel function COV(array1, array2), which created a symmetric 10 by 10 matrix. We then decomposed this matrix to get the lower (L) and upper (L^T) matrices. The upper matrix (L^T) was then multiplied by the set of random numbers and then added to the mean, to get the same correlation as the ETF data. These correlated lognormally distributed random numbers give us our simulated returns.

Results

The goal of this project was to create a real world economic scenario generator (ESG) with three regimes that is well-defined, understandable and reproducible. By using MLE on the ETF data that we collected, we found that 74.33% of the data could be classified as Regime 1 and 25.67% of the data could be classified as Regime 2. Regime 3's parameters were not based on the data that we collected so we did not include it in MLE. We set the probability of landing in Regime 3 extremely low at 0.5%, which altered the probability of being in Regime 1 to 74.2% and in Regime 2 to 25.3%.

We ran the ESG three different times with 100 scenarios, 1000 scenarios and 4000 scenarios. As we expected, we found that our ESG became more accurate as the number of scenarios increased. We defined more accurate as minimizing the difference between the parameters found from the data and the simulated returns. This holds true for all parameters: mean, standard deviation, covariance, and the probability of being in a regime. The greatest difference in the 4000 simulations run in any regime between the means is 0.000696 and the greatest difference in any regime between the standard deviations is 0.039725.

1.0 Introduction

Most individuals and companies would agree that being confident in their investments is advantageous. It would be extremely valuable to them to have the ability to determine what investments have too great a risk. Although it is not possible to predict the future, it is possible to simulate it. Through the use of thousands of simulations that are based on historical data, investors can make more informed choices regarding their economic decisions (Moudiki, 2014).

An economic scenario generator (ESG) is a model that receives parameters and outputs economic simulations. To ensure the parameters have the highest probability of representing the historical data, they are calculated using maximum likelihood estimation (MLE). Through a process known as regime switching, an ESG is able to switch between different scenarios that may include a growing economy, a falling economy, and an economic crash ("Economic Scenario Generator," 2014). These processes are explained in detail in 2.0 Background.

Our project used ten Exchange-Traded Funds (ETFs), however the ESG that we created is customizable for the number and variety of investment opportunities that exist in the world today. Our ESG uses parameters to generate simulations that are representative of the real world. One of our goals was to create a program in Microsoft Excel that was not only well defined and understandable, but able to be reproduced. Our procedure is discussed in 3.0 Methodology.

The ESG we created was able to deliver simulations that were accurate to three decimal places and became increasingly accurate as the number of simulations increased. We defined accurate as minimizing the difference between the parameters found from the data and the

simulated returns. Comparisons between the three regimes, how our ESG is different from other ESGs, and additional outcomes are discussed in 4.0 Results.

As we completed our project, several things came to our attention that would be beneficial for our ESG. As we would like to see these enhancements completed we have given an outline for them, in 5.0 Recommendations, for anyone that would like to continue our project in the future.

2.0 Background

An Economic Scenario Generator (ESG) utilizes parameters, that are calibrated to historical data, as inputs to create simulations of possible scenarios that could occur given this data. Creating an ESG not only requires knowledge in a programming language, but an understanding of several mathematical processes which are discussed in this chapter.

2.1 Economic Scenario Generator

There are two types of Economic Scenario Generators (ESGs), market consistent ESGs and real world ESGs, that each have their own applications. A market consistent ESG generates returns that are consistent with market prices. A real world ESG generates returns that are consistent with actual data and real world expectations (Moudiki, 2014). The real world ESG scenarios reflect the possible states of the economy, which usually includes risk premium and the calibration of volatilities and correlations based on historical data. Market consistent scenarios can help us calculate market prices today, while real world scenarios can show us what the world might look like tomorrow ("Economic Scenario Generator," 2014).

These two ESGs are mainly used by insurance companies and banks. Life insurance companies use ESGs as part of their Asset Liability Management (ALM) process, property and casualty insurance companies use it as part of their Dynamic Financial Analysis (DFA) process, and banks use it as part of their Balance Sheet Management (BSM) process (Blum & Dacorogna). ESGs are one of the main components of the DFA, ALM and BSM processes, which are methods used to evaluate and model the financial risks and benefits of a company, by creating large quantities of computer simulations. These processes are different from other

methods of actuarial analysis because they do not analyze their components separately, but as a complete picture. Specifically, they can be used to measure the benefits and levels of risk associated with current assets, and determine what other options exist to help minimize these risks and maximize the benefits (Kaufmann, Gadmer, & Klett, 2001).

2.2 Exchange Traded Funds Data

Simulations must be based on historic data. The data required to produce simulations depends on the desired results. For example, in order to simulate stock market returns, past years of stock market returns must be used. Likewise, the return period being simulated must be the same period for the historic data that was pulled; if daily returns are being simulated, the ESG must analyze daily return data. Exchange Traded Funds (ETFs) are a source of historic data. They track groups of commodities and assets and can be traded similarly to common stocks. Stocks are generally modeled using a lognormal distribution and because of their similarity to common stocks, ETFs can also be modeled using a lognormal distribution. An ESG takes the parameters that define the historical data's distribution as inputs and uses them to produce simulations from random numbers.

2.3 Random Number Generator

Most Random Number Generators (RNGs) rely on the use of a formula and are therefore considered pseudo-random numbers. They must pass several rigorous tests to be considered suitable for professional use. Two of the most notable tests for these pseudo-random numbers are the DIEHARD test and the standards set by the National Institute of Standards and Technology (NIST)(Rotz, Falk, & Joshee, 2004).

Microsoft Excel's RNG has had many problems in the past. The original RNG, in Excel versions up to 2003, provided approximately one-million pseudo-random numbers before it started repeating. Microsoft's 2003 edition of Excel tried to implement a Wichmann-Hill Generator. The Wichmann-Hill random number process involves generating three random numbers on [0,1], summing them, and then using the decimal part of the sum as the pseudo-random number. With the implementation of this process Excel's RNG would have provided approximately ten-trillion pseudo-random numbers before repeating. However, because Microsoft did not properly implement this process, their RNG produced negative pseudo-random numbers. Although still an improvement from their previous version, Microsoft's 2003 RNG did not pass the standards set by institutions such as NIST or the DIEHARD test (McCullough, 2008). With the release of Microsoft's 2007 version of Excel the problem of creating negative random numbers was fixed. In addition to fixing this problem, Microsoft added the function RANDOMIZE which allows the user to seed the random number generator so their result can be reproduced. The 2007 version of Excel has passed both the DIEHARD test and the standards set by the NIST (Microsoft, 2016).

An ESG uses random numbers by transforming them to the desired distribution using the covariance from the ETF data. In the next sections we will explore the processes to transform the random numbers into simulated returns.

2.4 Markov Chains – Regime Switching

One of the fundamental concepts of an ESG is that it simulates the real world. Since the economy goes through different periods of volatility, an ESG should portray this in its simulations as well. One way to do this is through the use of multiple states of volatility. The

ESG will go through these states during different stages of the simulation to model the real world economy going through periods of varying volatility. These states are referred to as regimes (Anton, Grobe, Rorres, & Grobe, 1994).

Markov Chains are used to model the transitions between regimes. Depending on the regime the ESG is currently in, there is a probability of either staying in that regime or switching into one of the others.

"If a Markov chain has k possible states, which we label as 1, 2,…, k, then the probability that the system is in state i at any observation after it was in state j at the preceding observation is denoted by p_{ij} and is called the transition probability from state j to state i. The matrix $P=[p_{ij}]$ is called the transition matrix of the Markov Chain (Ross, 2013)."

Movement between regimes in the Markov Chain process is determined through a discrete application of the Inverse Transform Method. For example below, Table 1: Transition Matrix, is the transition matrix that holds the sample probabilities of switching between two regimes.

	Starting in Regime 1	Starting in Regime 2
Ending in Regime 1	0.80	0.65
Ending in Regime 2	0.20	0.35

Table 1: Transition Matrix

In the example below in Table 2: Regime Switching Example, we start in Regime 1 and the first random number is 0.76998. Since this number is in the range of 0 to 0.80, we stay in Regime 1. The next random number picks up where the last regime left off, so we again start in Regime 1. The next random number is 0.82837 which falls in the range of 0.80 to 1, therefore

we switch into Regime 2. For the following two numbers the process would repeat except it would use the probabilities in the right column of the transition matrix because we are now starting in Regime 2. This process is repeated until the desired number of simulations is reached.

Starting in Regime	Random Number	Ending in Regime
1	0.76998	1
1	0.82837	2
2	0.21792	2
2	0.57101	1
...	...	...

Table 2: Regime Switching Example

2.5 Inverse Transform Methods

The Inverse Transform Method is used to convert uniform random numbers to outcomes based on the underlying probability distribution(s) of the variable under consideration (for instance, determining the changes in regime or determining the daily returns). To use the Inverse Transform Method, take the inverse of the desired cumulative distribution function (CDF) and then plug in numbers that are uniformly distributed on (0,1). A useful definition given by S.M. Ross is: "Let U be a uniform (0,1) random variable. For any continuous distribution function F the random variable X defined by $X=F^{-1}(U)$ has distribution F. [$F^{-1}(u)$ is defined to be that value of x such that F(x)=u.]" (Ross, 2013)

For example ("The Inverse Transform Algorithm", 2016): Let the density be defined as

$f(x) = \lambda e^{-\lambda x}$ and the CDF defined as $F(x) = 1 - e^{-\lambda x}$. Set $F(x) = u$ and solve for x:

$$1 - e^{-\lambda x} = u$$
$$e^{-\lambda x} = 1 - u$$
$$x = \frac{-\log(1 - u)}{\lambda}$$

Since the ETF return data is lognormally distributed, the random numbers go through an

inverse lognormal transformation in order to be used in the Markov Chain process. The formula

for this is: $Y = (X * \sigma) + \mu$. Where X is a random number generated by Excel, σ is the standard

deviation, and μ is the mean. Once this is completed, parameters can be calibrated to emulate

the ETF data through Maximum Likelihood Estimation.

2.6 Maximum Likelihood Estimation

Maximum Likelihood Estimation (MLE) is used to determine the parameters of an

unknown distribution based on given data. As its name implies, it outputs the parameters that

give the highest probability, or likelihood, of seeing the data that was observed (Hardy, 2001).

For Example: "Suppose the weights of randomly selected American female college

students are normally distributed with unknown mean μ and standard deviation σ. A random

sample of ten American female college students yielded the following weights (in pounds):

115 122 130 127 149 160 152 138 149 180

Based on the definitions given above, identify the likelihood function and the maximum

likelihood estimator μ; the mean weight of all American female college students.

The solution would be:

The probability density function of X$_i$ is: $f(x_i, \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x_i - \mu)^2}{2\sigma^2}}$ where $-\infty < x < \infty$.

The parameter space is $\Omega = \{(\mu, \sigma): -\infty < \mu < \infty \; and \; 0 < \sigma < \infty\}$. Therefore, the likelihood function is:

$$L(\mu, \sigma) = \sigma^{-n} 2\pi^{-n/2} e^{\frac{-1}{2\sigma^2} \sum_{i=1}^{n}(x_i - \mu)^2}, \text{ where } -\infty < \mu < \infty \text{ and } 0 < \sigma < \infty.$$

It can be shown, upon maximizing the likelihood function with respect to μ, that the maximum likelihood estimator of μ is:

$$\mu' = \frac{1}{n}\sum_{i=1}^{n} x_i \text{ where } x_i = \text{average}.$$

Based on the given sample, a maximum likelihood estimate of μ is:

$$\mu' = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{10}(115 + \cdots + 180) = 142.2 \; lbs.$$

Note that the only difference between the formulas for the maximum likelihood estimator and the maximum likelihood estimate is that the estimator is defined using capital letters, to denote that its value is random, and the estimate is defined using lowercase letters, to denote that its value is fixed and based on an observed sample" ("Maximum Likelihood Estimation," 2016). Note, too, that the maximum likelihood estimate of the mean weight of all American female college students based on this sample is simply the sample mean of the observations, not a completely unexpected result.

2.7 Covariance Matrix

Covariance refers to a statistical relationship involving dependence used to measure how much two random variables change together. Covariance for a sample is defined as:

$$Cov(X, Y) = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{n - 1}$$

A covariance matrix depicts the covariance of an array of random variables relative to each other. The matrix contains the variance of each random variable on the diagonal and the covariance between each pair of random variables in the other positions. It is symmetric, this is because the covariance between X and Y, and between Y and X are the same (Law & Kelton, 1991). Additionally, it can easily be shown that the covariance between a random variable X and itself, *Cov(X, X)* reduces to *Var(X)*. Thus, the variance-covariance matrix is described as:

$$Variance - Covariance\ Matrix = \begin{bmatrix} Var[X_1] & Cov[X_1, X_2] & \cdots & Cov[X_1, X_n] \\ Cov[X_2, X_1] & Var[X_2] & & Cov[X_2, X_n] \\ \vdots & \vdots & \ddots & \vdots \\ Cov[X_n, X_1] & Cov[X_n, X_2] & \cdots & Var[X_n] \end{bmatrix}$$

The variance-covariance matrix constructed using the covariance of the historic ETF data, can then be decomposed using Cholesky Decomposition.

2.8 Cholesky Decomposition

Every symmetric, positive definite matrix A can be decomposed into a product of a unique lower triangular matrix L and its transpose L^T, where $A = L \times L^T$ and L is called the Cholesky factor of A, as shown below:

$$A = LL^* = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & \cdots & 0 \\ l_{21} & l_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ l_{n1} & l_{n2} & \cdots & l_{nn} \end{pmatrix} \begin{pmatrix} l_{11} & l_{21} & \cdots & l_{n1} \\ 0 & l_{22} & \cdots & l_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & l_{nn} \end{pmatrix}$$

For Example:

$$\begin{pmatrix} 25 & 15 & -20 \\ 15 & 45 & 6 \\ -20 & 6 & 26 \end{pmatrix} = \begin{pmatrix} 5 & 0 & 0 \\ 3 & 6 & 0 \\ -4 & 3 & 1 \end{pmatrix} * \begin{pmatrix} 5 & 3 & -4 \\ 0 & 6 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

For the ESG, matrix A is constructed with the covariance between the ETF data. Then matrix A is decomposed to get L and L^T. If the set of random numbers is oriented horizontally, they are multiplied by L. If the set of random numbers is oriented vertically they are multiplied by L^T. This multiplication results in a set of random numbers that have the covariance structure of A (Burden & Faires, 1997).

3.0 Methodology

The goal of this project was to develop a real world economic scenario generator. In order to meet our goal, we identified several mathematical processes - shown in Table 3: Gantt Chart - that needed to be implemented. This chapter explains our procedure in detail.

Weeks	1 to 10	6 to 9	11 to 15	14 and 15	16 to 22	23 to 28	29
Dates	9/3/2015-11/9/2015	10/5/2015-11/2/5	11/3/2015-12/11/15	1/1/2016-1/18/2016	1/19/2016-3/4/2016	3/14/2016-4/20/2016	4/26/2016
Research							
ETF Data							
Cholesky - Covariance Matrix							
Inverse Transform - Log Normal Random Numbers							
Markov Chain - Transition Matrices							
MLE - Mary Hardy							
Regimes							
Tweaking Excel							
Writing Report							
Presentation							

Table 3: Gantt Chart

3.1 ETF Data

The exchange traded funds (ETF) and exchange traded notes (ETN) we utilized for our simulations include: SPY, IWM, TLT, HYG, GLD, EFA, VXX, OIL, FEZ, and EEM. These ten were chosen because they are well established, capture a wide variety of markets and have at least five years of historical data, which is essential for the accurate approximation of the covariance matrix. We found the data for these ETFs on the NASDAQ website. Table 4: Summary of ETFs, below is a summary of the underlying assets that each ETF represents.

ETF/ETN	SPY	IWM	TLT	HYG	GLD
Underlying Index/ Commodity	S&P 500	Russel 2000	Barclays U.S. 20+ Year Treasury Bonds	Markit iBoxx USD Liquid High Yield	Gold bullions spot price
Features of the Index	Largest 500 U.S. companies	Smallest 2000 companies in the Russel 3000 index of small-cap equities	U.S. Treasury Bonds that will not reach maturity for twenty or more years	High yield corporate bonds for sale in the U.S.	Bars of gold with a purity of 99.5% or higher
ETF/ETN	EFA	VXX	OIL	FEZ	EEM
Underlying Index/ Commodity	MSCI EAFE	S&P 500 VIX Short-Term Futures	S&P GSCI Crude Oil Total Return	EURO STOXX 50	MSCI Emerging Markets
Features of the Index	Large-cap and medium-cap equities	CBOE Volatility Index which measures the volatility of S&P 500 futures	Returns of oil futures contracts with West Texas Intermediate	50 of the largest and most liquid Eurozone stocks	Medium-cap and large-cap equities from emerging markets

Table 4: Summary of ETFs

For all ten ETFs we collected five years of daily returns. We then took the natural log of each return and calculated the mean and standard deviation of each, which is shown in Table 5: ETF Data, below.

	EFA	VXX	OIL	FEZ	EEM	HYG	TLT	IWM	SPY	GLD
μ	0.0002	-0.0033	-0.0005	0.0001	0.0002	0.0001	0.0001	0.0005	0.0005	0.0001
σ	0.0139	0.0399	0.0211	0.0184	0.0160	0.0065	0.0100	0.0146	0.0110	0.0112

Table 5: ETF Data

3.2 Maximum Likelihood Estimation

Taking the values from the ETFs, and assuming a two-regime model, we used MLE to calculate the mean and volatility of each regime of every ETF. We started by calculating the parameters for Regimes 1 and 2. We followed the steps used by Mary Hardy in her paper "A Regime Switching Model of Long Term Stock Returns" to complete MLE. We first used MLE to solve for the transition probabilities, mean and volatility of the SPY data. We used the invariant

probability formula to determine transition probabilities. Let $\rho_{x,y}$ be the probability of switching from regime x to regime y. The invariant formula has two parts:

$$\pi_1 = \frac{\rho_{2,1}}{\rho_{1,2}+\rho_{2,1}} \text{ and } \pi_2 = \frac{\rho_{1,2}}{\rho_{1,2}+\rho_{2,1}}.$$

After this initial step, the remainder of MLE is solved using a recursive formula (Hardy, 2001).

The recursive formula can be broken up into two parts. The numerator depends on which regime you are in and which regime you are switching to. The denominator is the sum of all possible numerators and remains the same. We assumed the probabilities were normally distributed and used the normal distribution formula $\phi\left(\frac{X-\mu_1}{\sigma_1}\right)$ or $\phi\left(\frac{X-\mu_2}{\sigma_2}\right)$ for our calculations.

$\mu_1 = Mean\ of\ Regime\ 1$ $\quad\quad\quad\quad\quad\quad$ $\mu_2 = Mean\ of\ Regime\ 2$

$\sigma_1 = Volatility\ of\ Regime\ 1$ $\quad\quad\quad\quad\quad$ $\sigma_2 = Volatility\ of\ Regime\ 2$

$X = Observation$

The denominator of the recursive formula is: $\rho_1 \times \phi\left(\frac{X-\mu_1}{\sigma_1}\right) + \rho_2 \times \phi\left(\frac{X-\mu_2}{\sigma_2}\right)$. The numerator of the recursive formula is one of the following, depending on the current regime. Numerator 1: $\rho_1 \times \phi\left(\frac{X-\mu_1}{\sigma_1}\right)$, Numerator 2: $\rho_2 \times \phi\left(\frac{X-\mu_2}{\sigma_2}\right)$ (Hardy, 2001).

After completing MLE for the SPY data, we used the transition probabilities we found to then complete MLE for all 10 ETFs. Since we had the parameters for the first two regimes, we had to create the parameters for Regime 3 and adjust the other regimes accordingly.

3.3 Markov Chain

In order to simulate a realistic changing economy, we created three regimes that had different levels of volatility. The first regime represents a healthy economy with a positive mean and low volatility, the second represents a falling economy with a lower mean and higher

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volatility, and the third regime represents an economic crash with a low mean and very high volatility.

We used MLE to find the transition probabilities, mean, and volatility for Regime 1 and Regime 2. We created the parameters for Regime 3 in a somewhat arbitrary fashion; by making the mean twice that of Regime 2 and the volatility 1.5 times that of Regime 2. Additionally, we made the probability of switching into Regime 3 very small so that it would only occur about 0.5% of the time. The mean and volatilities for the regimes can be found below in Table 6: Regime Parameters, and the regime transition probabilities can also be found below in Table 7: Regime Switching Probabilities.

	Regime 1 Mean	Regime 1 Volatility	Regime 2 Mean	Regime 2 Volatility	Regime 3 Mean	Regime 3 Volatility
SPY	0.000801	0.007506	-0.000511	0.018425	-0.001022	0.027638
VXX	-0.006785	0.025173	0.002812	0.056696	0.005623	0.085045
EFA	0.000644	0.008994	-0.000599	0.020469	-0.001198	0.030704
OIL	0.000040	0.014657	-0.001746	0.031241	-0.003493	0.046861
FEZ	0.000571	0.012594	-0.000936	0.027337	-0.001871	0.041006
EEM	0.000286	0.011384	0.000002	0.025918	0.000005	0.038877
HYG	0.000158	0.003279	-0.000183	0.011019	-0.000365	0.016528
TLT	-0.000054	0.012103	0.000252	0.007386	0.000504	0.011079
IWM	-0.000435	0.024310	0.000816	0.010224	0.001631	0.015336
GLD	-0.000341	0.014743	0.000385	0.008076	0.000771	0.012114

Table 6: Regime Parameters

	Starting in Regime 1	Staring in Regime 2	Starting in Regime 3
Ending in Regime 1	0.9920	0.0206	0.1250
Ending in Regime 2	0.0074	0.9790	0.1250
Ending in Regime 3	0.0006	0.0004	0.7500

Table 7: Regime Switching Probabilities

In order to implement the regime switching process, we created a random number that determined which regime the ESG would be in. For each simulated daily return, a random number determines which regime and therefore which parameters the lognormally distributed

return would have. For example, if we begin in Regime 1 and generate a random number between 0 and 0.992 then the model would remain in Regime 1, between 0.992 and 0.9994 it would switch to Regime 2, and between 0.9994 and 1 it would switch to Regime 3. These numbers are based on the transition probabilities found using MLE and slightly adjusted to account for the addition of Regime 3. After the creation and calibration of the regime switching process, additional random numbers were needed to be transformed into simulated returns.

3.4 Inverse Transform Methods

We used Excel version 2007 to generate the random numbers needed to simulate the returns. Real world stock markets are often simulated using a lognormal distribution. In order for the random numbers we generated to be lognormally distributed we took the natural log of the returns from the ETF data (Sharpe). Since we calibrated the parameters for the ESG using MLE over these lognormal returns, application of these parameters also results in lognormally distributed data. Therefore, to complete our simulation all we had to do was apply the covariance structure and add the appropriate means to each resulting value. The final value shares the parameters of the historical data and has a lognormal distribution.

3.5 Covariance and Cholesky Decomposition

To construct a covariance matrix, we used the data from the ten ETFs. Using the Excel function COV(array1, array2) we found the covariance of each ETF to each other based on regime, which created a symmetric 10 by 10 matrix of covariances for each regime. We then decomposed each matrix using Cholesky Decomposition to get the lower (L) and upper (L^T) matrices. We multiplied the random numbers by matrix L^T based on regime and then added the appropriate mean to each. This gave the random numbers the desired covariance structure, we

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then added the means so they would reflect the historical data. These correlated lognormally

distributed random numbers are the results of the ESG simulation.

4.0 Results

The goal of this project was to create a real world economic scenario generator (ESG) with three regimes that is well-defined, understandable and reproducible. Having successfully completed the projected, our key findings are discussed below.

4.1 Results from MLE

Completing the MLE allowed us to find the probabilities of switching between Regime 1 and Regime 2. The results are shown below in Table 8: Initial Regime Switching Probabilities.

	Ending in Regime 1	Ending in Regime 2
Starting in Regime 1	0.9920	0.0206
Starting in Regime 2	0.0074	0.9790

Table 8: Initial Regime Switching Probabilities

Since Regime 3 was not based on the data that we collected, we did not include it in MLE. We created the probability of switching to Regime 3 and adjusted the other probabilities accordingly. The final daily regime switching probabilities that we used in our ESG are shown in Table 7: Regime Switching Probabilities.

4.2 Comparing Parameters to Simulations

By using MLE on the ETF data that we collected, we found that 74.3268% of the data could be classified as Regime 1 and 25.6731% of the data could be classified as Regime 2. Regime 3's parameters were not based on the data that we collected so we did not include it in MLE. It was our intention that Regime 3 simulate an economic crash with high levels of volatility and a high negative mean. However economic crashes are not common and it is for this reason that we set the probability of landing in Regime 3 extremely low at 0.5%. In order to account for

the addition of Regime 3, we altered the probability of being in Regime 1 to 74.2% and in

Regime 2 to 25.3%.

After 4000 simulations, we compared the results of how many times a return ended in a

regime with the probability that they were supposed to be in that regime. We expected the

difference between these results to be extremely low. These results are shown below in Table

9: Regime Probabilities.

	Parameter	Simulated	Difference
Regime 1	74.20%	76.47%	-2.27%
Regime 2	25.30%	23.31%	1.99%
Regime 3	0.50%	0.22%	0.28%

Table 9: Regime Probabilities

Additionally, we compared the simulated covariance matrix for each regime with the

covariance matrix we made using the ETF data. Table 10: Covariance Difference Matrices,

below shows the differences between the simulated covariance matrix and the covariance

matrix made using the data. The greatest difference in any regime is 3.9188 $E^{(-5)}$.

Regime 1	SPY	VXX	EFA	OIL	FEZ	EEM	HYG	TLT	IWM	GLD
SPY	3.7383E-08	2.3510E-07	3.6232E-08	9.7633E-08	1.0397E-08	8.7012E-09	2.9319E-09	1.3174E-08	1.2839E-08	1.9769E-07
VXX	2.3510E-07	1.5135E-06	5.0719E-07	1.2947E-07	9.4580E-07	4.9206E-07	6.5276E-08	3.9011E-07	3.6566E-07	1.4276E-07
EFA	3.6232E-08	5.0719E-07	8.1121E-08	2.5064E-07	7.4984E-08	1.8082E-08	1.8243E-08	2.9124E-08	2.0491E-09	2.1674E-07
OIL	9.7633E-08	1.2947E-07	2.5064E-07	5.0054E-07	5.6931E-07	2.4177E-08	7.3676E-09	1.5801E-08	1.1252E-07	1.9195E-07
FEZ	1.0397E-08	9.4580E-07	7.4984E-08	5.6931E-07	6.0579E-08	6.8371E-09	1.1533E-08	1.8090E-09	5.4838E-08	3.1980E-07
EEM	8.7012E-09	4.9206E-07	1.8082E-08	2.4177E-08	6.8371E-09	8.2601E-08	5.5224E-08	1.3814E-08	4.6154E-09	2.1163E-07
HYG	2.9319E-09	6.5276E-08	1.8243E-08	7.3676E-09	1.1533E-08	5.5224E-08	5.6929E-09	7.0175E-08	1.7052E-08	7.0109E-08
TLT	1.3174E-08	3.9011E-07	2.9124E-08	1.5801E-08	1.8090E-09	1.3814E-08	7.0175E-08	9.1368E-09	1.6265E-07	1.0410E-07
IWM	1.2839E-08	3.6566E-07	2.0491E-09	1.1252E-07	5.4838E-08	4.6154E-09	1.7052E-08	1.6265E-07	2.8786E-08	2.5292E-07
GLD	1.9769E-07	1.4276E-07	2.1674E-07	1.9195E-07	3.1980E-07	2.1163E-07	7.0109E-08	1.0410E-07	2.5292E-07	6.7383E-08
Regime 2	SPY	VXX	EFA	OIL	FEZ	EEM	HYG	TLT	IWM	GLD
SPY	1.0517E-06	8.5468E-07	1.1449E-06	1.8100E-06	1.5010E-06	1.4589E-06	7.9401E-07	4.1428E-07	1.3021E-06	5.4036E-07
VXX	8.5468E-07	4.4840E-07	1.4275E-06	4.7085E-06	1.8109E-06	1.5319E-06	7.8989E-07	3.6906E-07	6.3638E-07	1.7135E-06
EFA	1.1449E-06	1.4275E-06	1.5607E-06	2.2667E-06	2.0205E-06	1.9015E-06	1.0741E-06	8.7540E-07	1.3593E-06	6.3894E-07
OIL	1.8100E-06	4.7085E-06	2.2667E-06	3.6070E-06	2.4729E-06	3.1043E-06	1.1106E-06	2.7747E-07	2.2065E-06	1.0953E-06
FEZ	1.5010E-06	1.8109E-06	2.0205E-06	2.4729E-06	2.6957E-06	2.5411E-06	1.4265E-06	9.9539E-07	2.0462E-06	6.5106E-07
EEM	1.4589E-06	1.5319E-06	1.9015E-06	3.1043E-06	2.5411E-06	2.2183E-06	1.0095E-06	1.0991E-06	1.8563E-06	6.1507E-07
HYG	7.9401E-07	7.8989E-07	1.0741E-06	1.1106E-06	1.4265E-06	1.0095E-06	6.0665E-07	5.1901E-07	1.0570E-06	1.2258E-07
TLT	4.1428E-07	3.6906E-07	8.7540E-07	2.7747E-07	9.9539E-07	1.0991E-06	5.1901E-07	2.5615E-07	5.5990E-07	4.9690E-07
IWM	1.3021E-06	6.3638E-07	1.3593E-06	2.2065E-06	2.0462E-06	1.8563E-06	1.0570E-06	5.5990E-07	1.4541E-06	5.9479E-07
GLD	5.4036E-07	1.7135E-06	6.3894E-07	1.0953E-06	6.5106E-07	6.1507E-07	1.2258E-07	4.9690E-07	5.9479E-07	5.4929E-07
Regime 3	SPY	VXX	EFA	OIL	FEZ	EEM	HYG	TLT	IWM	GLD
SPY	7.7528E-07	1.2852E-05	1.2669E-06	3.1545E-06	1.5016E-06	1.8310E-06	4.3063E-07	1.8983E-07	3.6511E-06	1.6773E-06
VXX	1.2852E-05	2.1841E-05	1.0781E-05	4.1928E-06	1.4963E-05	7.2348E-07	2.3679E-05	6.7708E-06	2.9299E-05	3.9188E-05
EFA	1.2669E-06	1.0781E-05	1.1682E-06	2.5569E-06	1.1426E-06	3.0432E-06	9.7434E-08	2.1724E-06	5.1483E-06	3.1909E-06
OIL	3.1545E-06	4.1928E-06	2.5569E-06	1.7116E-05	8.1640E-06	5.0845E-06	2.7554E-06	2.0561E-06	3.7896E-06	1.5078E-05
FEZ	1.5016E-06	1.4963E-05	1.1426E-06	8.1640E-06	2.4766E-06	3.6254E-06	1.1510E-06	2.6799E-06	6.1593E-06	3.6061E-06
EEM	1.8310E-06	7.2348E-07	3.0432E-06	5.0845E-06	3.6254E-06	4.4735E-06	1.6197E-06	4.8219E-07	7.7462E-06	4.1802E-06
HYG	4.3063E-07	2.3679E-05	9.7434E-08	2.7554E-06	1.1510E-06	1.6197E-06	3.6707E-06	2.5499E-06	1.1406E-06	5.5079E-07
TLT	1.8983E-07	6.7708E-06	2.1724E-06	2.0561E-06	2.6799E-06	4.8219E-07	2.5499E-06	3.0882E-06	7.4255E-07	2.5417E-06
IWM	3.6511E-06	2.9299E-05	5.1483E-06	3.7896E-06	6.1593E-06	7.7462E-06	1.1406E-06	7.4255E-07	1.0408E-05	1.4000E-07
GLD	1.6773E-06	3.9188E-05	3.1909E-06	1.5078E-05	3.6061E-06	4.1802E-06	5.5079E-07	2.5417E-06	1.4000E-07	3.9062E-06

Table 10: Covariance Difference Matrices

4.3 Increasing Simulation Size Improves Accuracy

We ran the ESG three different times with 100 scenarios, 1000 scenarios and 4000 scenarios. We found that our ESG became more accurate as the number of scenarios increased. We defined more accurate as minimizing the difference between the parameters found from the data and the simulated returns. This holds true for all parameters: mean, standard deviation, covariance, and the probability of being in a regime.

We expected the smallest simulation size of 100 to have the greatest differences between its parameters and simulated results. In Table 11: Trial 1, we can see that the greatest difference in any regime between the means is less than 1% at 0.009401. The greatest difference in any regime between the standard deviations is 0.037023.

Trial 1:Regime 1	SPY	VXX	EFA	OIL	FEZ	EEM	HYG	TLT	IWM	GLD
μ_1 Parameter	8.0146E-04	-6.7853E-03	6.4391E-04	3.9994E-05	5.7115E-04	2.8644E-04	1.5815E-04	-5.3531E-05	-4.3459E-04	-3.4095E-04
μ_1 Simulation	-2.2469E-04	2.6156E-03	-2.7859E-04	-1.5344E-03	-5.1177E-04	5.0438E-04	-4.7242E-05	1.7665E-04	1.2204E-03	6.4267E-04
Differences	1.0262E-03	-9.4008E-03	9.2250E-04	1.5744E-03	1.0829E-03	-2.1795E-04	2.0540E-04	-2.3018E-04	-1.6549E-03	-9.8362E-04
σ_1 Parameter	7.5063E-03	2.5173E-02	8.9943E-03	1.4657E-02	1.2594E-02	1.1384E-02	3.2787E-03	1.2103E-02	2.4310E-02	1.4743E-02
σ_1 Simulation	1.8521E-02	5.2267E-02	2.2350E-02	2.8695E-02	2.8795E-02	2.5441E-02	1.1271E-02	1.3281E-02	2.4041E-02	1.4274E-02
Differences	1.1015E-02	2.7094E-02	1.3356E-02	1.4038E-02	1.6201E-02	1.4057E-02	7.9918E-03	1.1777E-03	-2.6945E-04	-4.6878E-04
Trial 1: Regime 2	SPY	VXX	EFA	OIL	FEZ	EEM	HYG	TLT	IWM	GLD
μ_2 Parameter	-5.1083E-04	2.8116E-03	-5.9887E-04	-1.7463E-03	-9.3569E-04	2.4718E-06	-1.8264E-04	2.5197E-04	8.1571E-04	3.8531E-04
μ_2 Simulation	-5.3157E-05	3.9887E-03	-2.8735E-05	-1.1222E-03	-9.7028E-05	8.2846E-04	1.6274E-05	2.6487E-04	1.5105E-03	7.4481E-04
Differences	-4.5767E-04	-1.1771E-03	-5.7013E-04	-6.2410E-04	-8.3866E-04	-8.2599E-04	-1.9892E-04	-1.2903E-05	-6.9480E-04	-3.5950E-04
σ_2 Parameter	1.8425E-02	5.6696E-02	2.0469E-02	3.1241E-02	2.7337E-02	2.5918E-02	1.1019E-02	7.3859E-03	1.0224E-02	8.0758E-03
σ_2 Simulation	1.8528E-02	5.2484E-02	2.2370E-02	2.8669E-02	2.8821E-02	2.5485E-02	1.1275E-02	1.3287E-02	2.4090E-02	1.4286E-02
Differences	1.0313E-04	-4.2129E-03	1.9010E-03	-2.5723E-03	1.4844E-03	-4.3305E-04	2.5607E-04	5.9015E-03	1.3866E-02	6.2100E-03
Trial 1: Regime 3	SPY	VXX	EFA	OIL	FEZ	EEM	HYG	TLT	IWM	GLD
μ_3 Parameter	-1.0217E-03	5.6232E-03	-1.1977E-03	-3.4925E-03	-1.8714E-03	4.9436E-06	-3.6529E-04	5.0394E-04	1.6314E-03	7.7061E-04
μ_3 Simulation	-2.2303E-03	1.2089E-02	-3.4678E-03	-3.1184E-03	-3.5089E-03	2.7130E-04	1.8586E-03	2.0067E-03	-7.2045E-04	1.6015E-03
Differences	1.2086E-03	-6.4659E-03	2.2701E-03	-3.7412E-04	1.6376E-03	-2.6635E-04	-2.2239E-03	-1.5028E-03	2.3519E-03	-8.3086E-04
σ_3 Parameter	2.7638E-02	8.5045E-02	3.0704E-02	4.6861E-02	4.1006E-02	3.8877E-02	1.6528E-02	1.1079E-02	1.5336E-02	1.2114E-02
σ_3 Simulation	8.2236E-03	4.8021E-02	1.2767E-02	2.9802E-02	1.7527E-02	1.4059E-02	1.3043E-02	1.3414E-02	1.5956E-02	2.1852E-02
Differences	-1.9414E-02	-3.7023E-02	-1.7937E-02	-1.7059E-02	-2.3478E-02	-2.4818E-02	-3.4854E-03	2.3352E-03	6.2038E-04	9.7381E-03

Table 11: Trial 1

We expected that the second largest simulation size of 1000 would have similar differences between the parameters and simulated results. In Table 12: Trial 2, we can see that the greatest difference in any regime between the means is 0.002939. The greatest difference in any regime between the standard deviations is 0.038420. This was consistent with our expectations.

Trial 2:Regime 1	SPY	VXX	EFA	OIL	FEZ	EEM	HYG	TLT	IWM	GLD
μ_1 Parameter	8.0146E-04	-6.7853E-03	6.4391E-04	3.9994E-05	5.7115E-04	2.8644E-04	1.5815E-04	-5.3531E-05	-4.3459E-04	-3.4095E-04
μ_1 Simulation	8.0876E-04	-6.7728E-03	6.3496E-04	2.5010E-05	5.6409E-04	2.8213E-04	1.5487E-04	-4.9464E-05	-4.2941E-04	-3.3585E-04
Differences	-7.2969E-06	-1.2478E-05	8.9495E-06	1.4985E-05	7.0533E-06	4.3023E-06	3.2822E-06	-4.0677E-06	-5.1706E-06	-5.0991E-06
σ_1 Parameter	7.5063E-03	2.5173E-02	8.9943E-03	1.4657E-02	1.2594E-02	1.1384E-02	3.2787E-03	1.2103E-02	2.4310E-02	1.4743E-02
σ_1 Simulation	7.2692E-03	3.4996E-02	9.8798E-03	1.8084E-02	1.3806E-02	1.1503E-02	4.1030E-03	8.7001E-03	1.0095E-02	1.0171E-02
Differences	-2.3709E-04	9.8235E-03	8.8549E-04	3.4265E-03	1.2118E-03	1.1887E-04	8.2428E-04	-3.4032E-03	-1.4215E-02	-4.5720E-03
Trial 2: Regime 2	SPY	VXX	EFA	OIL	FEZ	EEM	HYG	TLT	IWM	GLD
μ_2 Parameter	-5.1083E-04	2.8116E-03	-5.9887E-04	-1.7463E-03	-9.3569E-04	2.4718E-06	-1.8264E-04	2.5197E-04	8.1571E-04	3.8531E-04
μ_2 Simulation	-5.8366E-04	2.9485E-03	-6.8080E-04	-1.7191E-03	-1.0122E-03	-8.7514E-05	-1.8591E-04	2.2459E-04	7.0783E-04	3.7111E-04
Differences	7.2830E-05	-1.3691E-04	8.1939E-05	-2.7182E-05	7.6530E-05	8.9986E-05	3.2609E-06	2.7384E-05	1.0788E-04	1.4196E-05
σ_2 Parameter	1.8425E-02	5.6696E-02	2.0469E-02	3.1241E-02	2.7337E-02	2.5918E-02	1.1019E-02	7.3859E-03	1.0224E-02	8.0758E-03
σ_2 Simulation	1.8711E-02	5.2455E-02	2.2531E-02	2.8972E-02	2.8722E-02	2.5781E-02	1.1253E-02	1.3276E-02	2.4273E-02	1.4220E-02
Differences	2.8580E-04	-4.2413E-03	2.0619E-03	-2.2689E-03	1.3846E-03	-1.3765E-04	2.3458E-04	5.8905E-03	1.4049E-02	6.1447E-03
Trial 2: Regime 3	SPY	VXX	EFA	OIL	FEZ	EEM	HYG	TLT	IWM	GLD
μ_3 Parameter	-1.0217E-03	5.6232E-03	-1.1977E-03	-3.4925E-03	-1.8714E-03	4.9436E-06	-3.6529E-04	5.0394E-04	1.6314E-03	7.7061E-04
μ_3 Simulation	-1.2089E-03	8.5621E-03	-1.1507E-03	-3.5807E-03	-1.5970E-03	-1.1103E-04	-2.5601E-04	6.9301E-04	7.4995E-04	4.6757E-04
Differences	1.8723E-04	-2.9389E-03	-4.7013E-05	8.8208E-05	-2.7443E-04	1.1597E-04	-1.0928E-04	-1.8907E-04	8.8146E-04	3.0304E-04
σ_3 Parameter	2.7638E-02	8.5045E-02	3.0704E-02	4.6861E-02	4.1006E-02	3.8877E-02	1.6528E-02	1.1079E-02	1.5336E-02	1.2114E-02
σ_3 Simulation	8.4987E-03	4.6624E-02	1.3484E-02	2.6889E-02	1.9174E-02	1.4467E-02	1.3737E-02	1.0908E-02	1.6236E-02	2.2141E-02
Differences	-1.9139E-02	-3.8420E-02	-1.7220E-02	-1.9973E-02	-2.1832E-02	-2.4410E-02	-2.7915E-03	-1.7078E-04	9.0042E-04	1.0027E-02

Table 12: Trial 2

We expected the largest simulation size of 4000 to have the smallest differences between the parameters and simulated results. In Table 13: Trial 3, we can see that the greatest difference in any regime between the means is 0.000696. The greatest difference in any regime between the standard deviations is 0.039725. This finding shows that the mean of our results became more accurate as the simulation size increased while the standard deviation became slightly less accurate, which is logical because with more trials there is more opportunity for outliers.

Trial 3: Regime 1	SPY	VXX	EFA	OIL	FEZ	EEM	HYG	TLT	IWM	GLD
μ_1 Parameter	8.0146E-04	-6.7853E-03	6.4391E-04	3.9994E-05	5.7115E-04	2.8644E-04	1.5815E-04	-5.3531E-05	-4.3459E-04	-3.4095E-04
μ_1 Simulation	7.9409E-04	-6.7436E-03	6.3282E-04	4.5950E-05	5.5412E-04	2.7572E-04	1.5772E-04	-5.8486E-05	-4.4700E-04	-3.3348E-04
Differences	7.3715E-06	-4.1695E-05	1.1091E-05	-5.9552E-06	1.7026E-05	1.0712E-05	4.3269E-07	4.9549E-06	1.2414E-05	-7.4680E-06
σ_1 Parameter	7.5063E-03	2.5173E-02	8.9943E-03	1.4657E-02	1.2594E-02	1.1384E-02	3.2787E-03	1.2103E-02	2.4310E-02	1.4743E-02
σ_1 Simulation	7.2619E-03	3.4959E-02	9.8686E-03	1.8115E-02	1.3788E-02	1.1491E-02	4.1000E-03	8.7064E-03	1.0065E-02	1.0194E-02
Differences	-2.4435E-04	9.7860E-03	8.7426E-04	3.4576E-03	1.1943E-03	1.0710E-04	8.2121E-04	-3.3969E-03	-1.4245E-02	-4.5487E-03
Trial 3: Regime 2	SPY	VXX	EFA	OIL	FEZ	EEM	HYG	TLT	IWM	GLD
μ_2 Parameter	-5.1083E-04	2.8116E-03	-5.9887E-04	-1.7463E-03	-9.3569E-04	2.4718E-06	-1.8264E-04	2.5197E-04	8.1571E-04	3.8531E-04
μ_2 Simulation	-4.6572E-04	2.6740E-03	-5.3353E-04	-1.7573E-03	-8.5254E-04	6.4965E-05	-1.7290E-04	2.5971E-04	8.8537E-04	3.8338E-04
Differences	-4.5114E-05	1.3762E-04	-6.5336E-05	1.1072E-05	-8.3154E-05	-6.2494E-05	-9.7409E-06	-7.7439E-06	-6.9658E-05	1.9281E-06
σ_2 Parameter	1.8425E-02	5.6696E-02	2.0469E-02	3.1241E-02	2.7337E-02	2.5918E-02	1.1019E-02	7.3859E-03	1.0224E-02	8.0758E-03
σ_2 Simulation	1.8726E-02	5.2621E-02	2.2568E-02	2.8781E-02	2.8842E-02	2.5822E-02	1.1291E-02	1.3290E-02	2.4241E-02	1.4176E-02
Differences	3.0081E-04	-4.0755E-03	2.0991E-03	-2.4603E-03	1.5050E-03	-9.6654E-05	2.7276E-04	5.9042E-03	1.4017E-02	6.1004E-03
Trial 3: Regime 3	SPY	VXX	EFA	OIL	FEZ	EEM	HYG	TLT	IWM	GLD
μ_3 Parameter	-1.0217E-03	5.6232E-03	-1.1977E-03	-3.4925E-03	-1.8714E-03	4.9436E-06	-3.6529E-04	5.0394E-04	1.6314E-03	7.7061E-04
μ_3 Simulation	-8.1841E-04	5.6279E-03	-8.4826E-04	-2.8203E-03	-1.6298E-03	5.9634E-05	-2.7113E-04	3.5122E-04	2.3277E-03	6.1120E-04
Differences	-2.0325E-04	-4.7256E-06	-3.4947E-04	-6.7219E-04	-2.4154E-04	-5.4690E-05	-9.4159E-05	1.5272E-04	-6.9631E-04	1.5941E-04
σ_3 Parameter	2.7638E-02	8.5045E-02	3.0704E-02	4.6861E-02	4.1006E-02	3.8877E-02	1.6528E-02	1.1079E-02	1.5336E-02	1.2114E-02
σ_3 Simulation	8.1174E-03	4.5320E-02	1.3146E-02	2.7590E-02	1.8704E-02	1.4050E-02	1.3804E-02	1.0846E-02	1.5420E-02	2.1023E-02
Differences	-1.9520E-02	-3.9725E-02	-1.7558E-02	-1.9272E-02	-2.2302E-02	-2.4827E-02	-2.7238E-03	-2.3270E-04	8.3914E-05	8.9097E-03

Table 13: Trial 3

4.4 Details about Program

These three runs also allowed us to measure the run time. The ESG takes approximately eight minutes per thousand scenarios to run. We were also able to discover that Excel limited the number of scenarios we were able to run. Excel has enough space to run approximately 4100 scenarios. This made exploring the accuracy of our ESG for more than 4000 scenarios difficult.

5.0 Recommendations

Based on our results, which are given in the previous section, we recommend the following for consideration in future projects.

MATLAB
Rewrite the code using MATLAB. This will allow for the ESG to be run on a supercomputer and the code will be simpler to distribute. It will also be useful to have access to MATLAB's library of functions, which is constantly updated.

Super Computer
Use a supercomputer to run the simulations in parallel. This will decrease the run time and increase the number of simulations that are able to be run, therefore making the results more accurate.

Output to txt File
Send output to txt file. This will allow for the number of runs to be unlimited because it will not run out of space. Additionally, simulation data could be more easily exported to other software for parsing.

User Interface
Create a user interface. This will allow for the ESG to be used and run by someone who has not worked on the project or does not understand the processes.

Automatic Results Checker
Include in the code a results checker. This could be a difference matrix of the parameter-matrix subtracted by the simulated-matrix. This would allow the user to know right away if their results are within a reasonable error.

Regime 3
Make regime 3 more accurate. This could mean using parameters based on historical data from economic crashes.

Works Cited

Anton, H., Grobe, E. M., Rorres, C., & Grobe, C. A. (1994). *Student Solutions Manual [to Accompany] Elementary Linear Algebra, Applications Version* (7th ed.): Wiley.

Blum, P., & Dacorogna, M. (2004). Encyclopedia of Actuarial Science: John Wiley & Sons, LTD.

Burden, R. L., & Faires, J. D. (1997). *Numerical Analysis* (Sixth ed.): Brooks/Cole Publishing Company.

Economic Scenario Generator. (2014). Retrieved from http://www.barrhibb.com/documents/downloads/SP28936_MA_Economic_Scenario_Generator.pdf

Hardy, M. R. (2001). A Regime-Switching Model of Long-Term Stock Returns. *North American Actuarial Journal, 5*(2), 12.

Kaufmann, R., Gadmer, A., & Klett, R. (2001). Introduction to Dynamic Financial Analysis. *Astin Bulletin, 31*(1), 38.

Law, A. M., & Kelton, W. D. (1991). *Simulation Modeling and Analysis* (Second ed.): McGraw Hill, Inc.

Maximum Likelihood Estimation. (2016). Retrieved from https://onlinecourses.science.psu.edu/stat414/node/191

McCullough, B. D. (2008). Microsoft Excel's 'Not the Wichmann-Hill' random number generators: Elsevier B.D.

Microsoft. (2016). Description of the RAND Function in Excel. Retrieved from https://support.microsoft.com/en-us/kb/828795

Moudiki, T. (2014). ESGToolkit, Tools for Economic Scenario Generators.

Ross, S. M. (2013). *Simulation*: Academic Press.

Rotz, W., Falk, E., & Joshee, A. (2004). A Comparison of Random Number Generators Used in Business- 2004 Update. Washington DC: Ernst & Young LLP.

Sharpe, M. J. Lognormal Model for Stock Prices. San Diego, California: University of California, San Diego.

The Inverse Transform Algorithm. (2016). Mizzou University of Missouri