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Design, Implementation and Assessment of the Effectiveness of a 4th Grade Mathematics After-hours MCAS Program at Adams Street School

An Interactive Qualifying Project Report

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WORCESTER POLYTECHNIC INSTITUTE

in partial fulfillment of the requirements for the

Degree of Bachelor of Science

by

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Preface

The goal of this project was to help improve education in Worcester County. If we look at the statistics for the Massachusetts Comprehensive Assessment System (MCAS) examination, it is evident that there is much left to be done as many students are still found to be of unsatisfactory competence.. Hence, that is what I decided to focus my efforts on.

There was a need for Mathematics help at Adams Street School on Adams Street in Worcester. The Principal, Mrs. Betty Army, was planning to implement an after-hours mathematics help program to help raise their MCAS mathematics scores. I volunteered to help out with that. It turned out to be a golden opportunity for both the school and me, as I feel we have both benefited greatly from this experience.

The purpose of this report is to allow anyone who wants to implement a similar after-hours program in preparation for the MCAS examination in 4th grade mathematics to see the sort of strategies they might adopt, the kind of mistakes they should avoid and the sort of assessments they should make. It is essentially intended to be a rudimentary manual for such a program.

I hope that my work will in some small way benefit the Worcester Public School system.

M.V.S. Chandrashekhar

Acknowledgements

First and foremost, I would like to thank my Advisor Dr. John Goulet and Mrs. Melissa Schachterle, the teacher at Adams Street that I was under. I would also like to thank Mrs. Betty Army, principal of Adams Street School. The children I worked with really were a joy to instruct and they did put a lot of hard work and time into making this project a success and I would like to thank them, for without them, this project would have come to naught. I would also like to extend my regards to the staff from TouchMath who proved to be very friendly and helpful, explaining their methods to me in a very clear manner. Finally, I would like to thank anyone else who was involved in making this project a success.

INTRODUCTION

On Thursday the 4th of February, we met with Mrs. Betty Army and Mrs. Melissa Schachterle at the Adams Street School in Worcester, MA regarding possible teaching opportunities for the completion of my Interactive Qualifying Project (IQP) requirement at WPI. The meeting was a success and many ideas regarding the specifics of the project were discussed. Some of these included assisting with an after-hours mathematics help program and working with Mrs. Schachterle in preparing her 4th grade science class for the spring administration of the standardized Massachusetts Comprehensive Assessment System (MCAS) examination.

In our discussion, we learned that there were academic improvements that needed to be made. The standard of the students, as measured by their performance in the MCAS exam, is up to par in certain areas, but sorely lacking in others. This was as explained by Mrs. Betty Army. The figure below (Figure 1) shows the performance of the 4th grade students at Adams Street School in mathematics and science for the Spring 1999 administration of the test. There certainly was room for improvement in every area.

Mathematics	School	0	22	56	22	0	230	18
	District	9	19	46	26	0	231	2,029
	State	12	24	44	19	0	235	77,007
Science & Technology	School	0	39	61	0	0	234	18
	District	7	36	45	12	0	236	2,030
	State	10	46	36	8	0	240	77,003

Figure 1: The mathematics scores of students at Adams Street School for the Spring 1999 administration of the MCAS.

In particular, the mathematics portion of the test was singled out as requiring extra attention. In order to improve this situation, Mrs. Army planned to institute an after-hours Mathematics help program for weak students. The children were divided up into small groups of about 6 students each so that they would receive ample attention that cannot always be offered to them in a traditional classroom setting. It was hoped that these sessions would improve their mathematics test scores. There were certain problems associated with this program as many students take the bus to school, so after-hours transport were a concern. Parents who were unwilling to involve their children in these programs also pose a problem. However, preliminary responses from parents were encouraging and the program is now on track. The primary administrators of this program have to be certified teachers, so I played a secondary assistant role in the instruction of these students.

As far as the specific academic goals of this program go, it was highlighted that the students in this school have a lot of difficulty in understanding abstract concepts such as sequencing and pre-algebraic techniques (see Chapter 3 for examples of such problems). One possible measure that was proposed was the use of hands-on kits that would allow the children to experience first hand the concepts they are uncomfortable with, such as sequencing, for which Lego blocks could be used. The school does not have many facilities to this end, but there were avenues through which these kits were obtained for use in the classroom.

With respect to the science portion of the test, the performance of the school has been satisfactory, but they would like to see further improvement, as there are still many weak students. The state allows students that do not meet a certain minimum standard in the MCAS the right to be tutored one-on-one by a certified teacher. However, it was decided that I would be more effective if I concentrated solely on the mathematics after-hours program.

In order to track the children's progress, we planned to assess where they stood at the start of the program. A pretest was administered to identify the individual student's weaknesses. Throughout the course of the term, the students' weaknesses were focused on as mentioned above. The areas of emphasis changed with the progress of the students and their performance in class.

Mrs. Schachterle mentioned that many of these students come from "difficult" backgrounds and as such are very harsh upon themselves. Many of them almost expected to fail the test before taking it and thus required a lot of encouragement to get them through it and improve their scores. It is known that a positive state of mind enhances performance in school. Positive feedback included small rewards such as candy or small toys upon improvement in a certain area. When the children saw that they were making progress, they were more motivated to work towards the exam and felt more confident about it.

The IQP culminated in the students taking the MCAS early this June and although the actual results may take a while to come in, feedback from the students was used to assess their performance and the effectiveness of our efforts. This takes place throughout the course of this report, as the reader will remark.

My efforts involved me being an assistant in the classroom and in the preparation and administration of the aforementioned tests. All our work was be documented and recommendations were made at the end so that any other schools that might want to make use of this system will know what kind of difficulties to expect. The gearing of classes and help sessions towards MCAS type problem solving through a hands-on approach to abstract concepts did raise test scores and improved the standard of education in Adam's Street School.

Background

1.History of the MCAS

The MCAS was implemented in response to the Massachusetts Education Reform Law of 1993 which included changes in the hiring of educational personnel, implementation of education program, local governance within the school, management of school finance, governance at a state level, student guidelines as well as technical changes.[1]

This law required that the MCAS be designed to test all public school students across the Commonwealth of Massachusetts, including students with disabilities and those with limited English Proficiency. It also specifies that the test be administered in at least grades 4, 8 and 10. The performance of the students will be gauged by the standards laid out in the Massachusetts *Curriculum Frameworks*. The law also details the various levels at which scores are to be reported, namely at the individual, school and district levels. Furthermore, it calls for the MCAS to serve as one basis of accountability for students, schools and districts (for example, starting 2001, grade 10 students must pass the grade 10 tests as one condition to be eligible for a high school diploma).

2.Overview of the MCAS

The Massachusetts Comprehensive Assessment System tests participants on their knowledge of the English Language Arts, Mathematics, Science & Technology and

History and Social Science (grades 8 and 10), based on standards and objectives specified in the Massachusetts *Curriculum Frameworks,* which was modified in 1993 in response to the Reform Law of 1993. There are also plans to incorporate testing of the learning standards in the Foreign Languages Curriculum Framework in the future.

The test is generally administered in the spring of every year and participation is compulsory for all students at public schools in grades 4, 8 and 10. There is no way a student may be exempted from taking the test so as to ensure that all Massachusetts public school students have the opportunity to learn the subject material as specified in *the Curriculum Frameworks*. For students with disabilities who meet certain criteria will be allowed to use special accommodations as necessary. Work is currently being done to facilitate this process for disabled students as well as for students with limited English proficiency who have attended school in the United States for three or fewer years.

There are 4 types of questions that appear on the MCAS. These are multiple choice questions, short-answer questions, open-response questions and writing prompts. Multiple-choice questions are used in all areas o the test. Students are presented with 4 possible answers and they must select one answer which they deem to be correct. Short-answer questions appear only in the Mathematics section of the test and students are asked to generate a short solution that is numeric in nature. The open-response questions are used in all parts of the test as well. Students are asked to provide a one or two paragraph response using figures as appropriate. Finally, the writing prompts are used in

[1] The entire document detailing the Education Reform Law of 1993 can be viewed online at http://www.doe.mass.edu/mcas/1098facts.html (see References).

the English language arts portion of the test and participants are requested to write a composition based on a writing prompt, which may be in the form of a reading passage.

3. Reporting of Test Scores

Test scores are reported for individual students on a four-tier benchmark. The 4 levels in order from most proficient to least proficient are Advanced, Proficient, Needs Improvement and Failing.

MCAS scores are reported on a scale of 200-280. The following tables (Figure 2 and Figure 3) show the definitions of these levels.[2]

Performance Level	Description
Advanced	Students at this level demonstrate a comprehensive and in-depth understanding of rigorous subject matter and provide sophisticated solutions to complex problems.
Proficient	Students at this level demonstrate a solid understanding of challenging subject matter and solve a wide variety of problems.
Needs Improvement	Students at this level demonstrate a partial understanding of subject matter and solve some simple problems.
Failing	Students at this level demonstrate a minimal understanding of subject matter and do not solve even simple problems.

Figure 2: Definitions of the proficiency levels.

[2] For content-specific definitions, refer to the following website: http://www.doe.mass.edu/mcas/mcaspld.html (see References).

Performance Level	Scaled Score Interval
Advanced	260 to 280
Proficient	240 to 259
Needs Improvement	220 to 239
Failing	200 to 219

Figure 3: Threshold scores for the various proficiency levels.

Once these definitions were made, standards had to be set to establish what each of these categories would really mean. To this end, over 200 individuals, both educators and non-educators, who, according to their area of content expertise, served on one of 12 standard setting panels. This process was conducted in August 1998. On panel was set up for each grade level for each subject area. The English Language Arts panel was subdivided into Reading and Writing panels to yield the total of 12. The make-up of the panels was as follows; 79% of the panelists were classroom teachers, school administrators or college and university faculty, and 21% were non-educators including scientists, engineers, writers, attorneys, and government officials.

The reported scores are used for various purposes. Perhaps the most important use of the MCAS results is in the improvement of learning and teaching standards. Parents and students will be able to use the results to monitor students' progress. On a more global level, local educators will use results to help identify strengths and weaknesses in curriculum and instruction. In the near future, the Board of Education will establish standards for districts that improve or fail to improve student academic performance as is detailed in the Education Reform Law.

Also, as has already been stated, beginning with graduating class of 2003, the 10th grade MCAS examination must be passed in order for the student to be eligible for a high school diploma.

4. Guidelines for Participation in the MCAS

All public school students must participate in the MCAS including those with disabilities and those who are not proficient in the English language. This is to provide all students with the opportunity to learn the material required on the examination.

Parents cannot legally refuse their children the right to take the MCAS. This is detailed in Massachusetts General Laws chapter 76, Sections 2 and 4, which establishes penalties for truancy as well as for inducing unlawful absence of a minor from school. In addition, school discipline codes generally define local rules for school attendance and penalties for unauthorized absence from school or from a required part of the school day.

5. Specific Exam Objectives for the Mathematics Section of the MCAS

<u>Background on the Mathematics Section</u>

The Mathematics section of the Massachusetts Comprehensive Assessment System (MCAS) is based exclusively on the learning standards described in the Massachusetts *Mathematics Curriculum Framework* (1996). These learning standards were developed in collaboration with teachers, school and district administrators, mathematicians, college faculty, parents, and representatives of business and community organizations across the state. They were developed to address concerns of clarity, accessibility, consistency, and mathematical accuracy, and to be aligned with the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics.*

The *Mathematics Curriculum Framework* identifies expectations for student learning, organized by content. Since MCAS is administered at grades 4, 8, and 10, mathematics questions for each of the tested levels focus on the learning standards specified for that grade level, plus the standards identified at all preceding grade levels. Consequently, students will be required to demonstrate cumulative content knowledge and mathematical thinking skills, e.g., grade 8 students will be tested on all learning standards identified in the *Framework* from kindergarten through grade 8.

Content Knowledge and Mathematical Thinking Skills to be Assessed

The MCAS Mathematics Assessment is designed to assess two fundamental dimensions of learning: content knowledge, and thinking skills in using and applying mathematics.

Content Knowledge

The following 4 content strands have been identified by the *Framework* as the foundation for the MCAS mathematics assessment and its reporting categories.

-Number Sense

-Patterns, Relations, and Functions

-Geometry and Measurement

-Statistics and Probability

The table below (figure 4) shows the approximate distribution of MCAS questions by content strand for grade 4.

Content Strand	*Grade 4*
Number Sense	35%
Patterns, Relations, and Functions	20%
Geometry and Measurement	25%
Statistics and Probability	20%

Figure 4. The distribution of MCAS questions by content strand.

Mathematical Thinking

In addition to content knowledge, students will be expected to demonstrate problem-solving and mathematical communication and reasoning skills, as well as skill at making connections between math content and its real-world application. For the purposes of the MCAS Assessment, these skills are grouped into three major areas:

-conceptual under-standing

-procedural knowledge

-problem solving.

Conceptual Understanding.

Questions in this area assess student skills in labeling, verbalizing, and defining concepts; recognizing and generating examples and counter-examples; using models, diagrams, charts, and symbols to represent concepts; translating from one mode of representation to another; and comparing, contrasting, and integrating concepts.(See Chapter 4 for examples from the Statistics and Probability content strand).

Procedural Knowledge.

Questions in this area assess student skills related to executing procedures and verifying results; explaining reasons for steps in procedures; recognizing correct and incorrect procedures; developing new procedures, or extending or modifying familiar ones; and recognizing situations in which a procedure is appropriate, necessary, or correctly applied.

Problem Solving.

Questions in this area assess student skills in selecting appropriate mathematical concepts and procedures for both real-life and mathematical problem situations and appropriately applying these concepts and procedures; selecting and using appropriate problem-solving strategies; and verifying and generalizing solutions.

Mathematical Thinking Skill	*Grade 4*
Conceptual Understanding	40%
Procedural Knowledge	40%
Problem Solving	20%

Figure 5: Distribution of MCAS questions by mathematical thinking skill.

All questions on the Mathematics Assessment test both knowledge of learning standards from one or more *Mathematics CurriculumFramework* content strands and one or more mathematical thinking skills.

<u>Excerpts from the Frameworks Detailing 4th Grade Mathematics Requirements</u>

Number Sense

Number Sense and Numeration

Students engage in problem solving, communicating, reasoning and connecting to

-construct number meanings by using manipulatives and other physical materials to represent concepts of numbers in the real world

-demonstrate an understanding of our numeration system by relating counting, grouping and place value concepts.

-interpret the multiple uses of numbers by taking real-world situations and translating them into numerical statements.

Concepts of Whole Number Operations

Students engage in problem solving, communicating, reasoning and connecting to

-model and discuss a variety of problem situations to help students move from the concrete to the abstract.

-relate the mathematical language and symbolism of operations to problem situations

-identify a variety of problem structures that can be represented by a single operation.

-know when to use the operations of addition, subtraction, multiplication and division and also describe their relationships.

Fractions and Decimals

Students engage in problem solving, communicating, reasoning and connecting to

-demonstrate an understanding of the basic concepts of fractions, mixed numbers and decimals.

-use models to relate fractions to decimals, find equivalent fractions, and explore operations on fractions and decimals.

-apply fractions and decimals to problem situations.

Estimation

Students engage in problem solving, communicating, reasoning, and connecting to describe the strategies used in exploring estimation.

-determine when an estimate is appropriate.

-apply estimation when working with quantities, measurement, and computation.

-use estimation to check solutions to determine if the results of computational problems make sense.

Whole Number Computation

Students engage in problem solving, communicating, reasoning, and connecting to

-model, explain, and develop proficiency with basic facts and algorithms.

-use calculators in appropriate computational situations.

Patterns, Relations and Functions

Patterns and Relationships

Students engage in problem solving, communicating, reasoning, and connecting to

-identify, describe, extend, and create a wide variety of patterns.

-represent and describe mathematical relationships.

-explore the use of variables and open sentences to express relationships.

-use patterns and relationships to analyze mathematical situations.

Algebra/Mathematical Structures

Students engage in problem solving, communicating, reasoning, and connecting to

-discover how to form, then write, number sentences for real problems.

-investigate and describe ways to find missing components in number sentences.

-demonstrate through hands-on activities, an understanding of maintaining balances in number sentences.

-explain the use of variables in number sentences.

-explore and demonstrate an understanding of commutative properties for addition and multiplication.

Geometry and Measurement

Geometry and Spatial Sense

Students engage in problem solving, communicating, reasoning, and connecting to

-describe, model, draw, and classify shapes.

-investigate and predict the results of combining, subdividing, and changing shapes.

-develop spatial sense.

-use geometric ideas to develop numerical ideas.

-recognize and appreciate geometry in the world.

Statistics and Probability

Statistics and Probability

Students engage in problem solving, communicating, reasoning, and connecting to

-collect, organize, and describe data.

-construct, read, and interpret displays of data.

-formulate and solve problems that involve collecting and analyzing data.

-explore and describe the concepts of chance.

Piaget's Theory on Learning

Jean Piaget was a French behavior scientist from the early 20th century. Many learning theories are very caught up in the dichotomy between nature and nurture. His theory, however, managed to achieve a balance between the two. There were other scientists who had proposed similar learning mechanisms, but Piaget stands alone in the magnitude of his research over 60 years and for having generated a very coherent and cogent theory.

This school of thinking believes in "Progressivism-Cognitive Development". Nature and nurture are both central to this idea. This is an interactionist viewpoint. Mental development is seen as the product of interaction between the child and the environment. This idea was first explored by Plato, then other learning theorists including Jean Piaget. The child is not seen purely as a product of his own nature, nor is he seen as being completely controlled by the environment. He is viewed as a scientist, explorer and inquirer and is critically instrumental in constructing and organizing the environment and his own development. Motivation for intellectual development comes from within the child.

The child builds up schemata (concept structures) from the world around him, and these are modified as he grows. Learning takes place through interaction with the environment and the people around him. He assimilates and then accommodates new ideas to modify his schemata[3].

Piaget's theory will form the basis for our mode of instruction, which will mainly be hands-on and interactive. I will come back to his theory and its implication on education in Chapter 6.

Procedure

In this project, we hoped to improve MCAS scores at Adams Street School. As we saw in the Introduction, the scores were very low. This was one aspect of education that was improved at a local school here in Worcester. These children will thus, in a small way, be able to better serve society in the future. Moreover, since the instruction was conducted in the subject of mathematics, the students will hopefully be better versed in these skills, which are essential to their effective functioning in today's society.

There were 4 steps in the accomplishment of this task. Firstly, we administered a pretest to judge where the students stood. Then, the MCAS-oriented instruction began in earnest. Throughout the course of the instruction, assignments were given to gauge the progress

[3] For a more comprehensive treatment of this theory, please refer to Wadsworth (see References).

of the students. This culminated in the administration of the MCAS examination in May 2000. Then we commenced our statistical analysis of the results from which we gleaned whether or not the measures implemented had any impact on the students' performance. Following this, there was an assessment of the efficacy of our efforts and recommendations were be made to wrap things up.

The pretest was designed in collaboration with Mrs. Schachterle for mathematics. One test was administered. The standard of the test was tied to that of the actual MCAS examination. It seemed likely at the time that the students would not fare too well in this examination, as they had not had too much MCAS specific training. This is indeed what happened. These figures gave us the starting point for our analysis. This was done as soon as my CORI clearance came through (see below for details).

Instruction began upon administration of the test. This was based on the MCAS objectives discussed in the Literature Review. Each skill was addressed and the students had to demonstrate sufficient proficiency in each skill in order to be able to move on. Certain compromises had to be made, however, due to time constraints. This proficiency was measured by the administration of assignments and observations throughout the course of the term. These assignments, like the pretest, were designed in collaboration with the relevant instructor. If a student did not demonstrate sufficient proficiency, he or she received special attention in that area. This continued until the student demonstrated a satisfactory level of proficiency. If a large proportion of the class fared poorly in the test, it was construed as inadequate instruction in class. We thus reviewed the relevant material at a class level rather than on an individual level. Instruction was carried out throughout the course of the term until the administration of the actual MCAS. These included terms D and E according to our calendar.

As was mentioned in the Introduction, many students at Adams Street School have difficulty with abstract concepts such as sequencing and patterns as Mrs. Schechterle reiterated, time and again. As such, special attention was given to these concepts. A hands-on approach was utilized to this end. For example, pattern block kits that are available in the classroom were used. These kits allowed students to construct patterned structures to help them understand patterns and sequences, as experience is an excellent teacher.

The actual MCAS was administered in May. The present policy is such that the results are not released until the November of the present year, which would exceed our timeline. In order to circumvent this problem, we administered a mock MCAS examination (which I helped design). This served two purposes: one was to prepare the students for the actual exam and give them the confidence they need. The other utility in this was to give us a good indication of how the students would fare in the actual exam and also give us the figures we needed in our assessment of this program.

The next step was to analyze the results that we obtained. This was done in the form of statistical and observational analysis. The pretest scores for each individual skill (or content strand) were correlated with the final Mock MCAS examination for any statistically significant improvements.

Finally, we had to assess what had been done, so any enhancements in the students' performance were noted. Certain skills were not learnt as well as expected, so the mode of attack for that skill was rethought and recommendations were made on what we judged to have gone wrong. All the materials that were used in class were preserved for future reference. The correlation and assessment was done over Term A term of the academic year 2000-2001.

Logistics

Thus far, we have only considered the academic side of this project. There is, however, a logistic side that is crucial to the success of this project. The school is located on Adams Street in Worcester, which is about 5 minutes by car from WPI. It is not, however, within walking distance. Thus I had to take a car everyday. I do not own a car, so I had to get rides from acquaintances. If that was not possible on a certain day for some reason, I had to hire a taxi to get to and from the school. Hence taxi fares and gasoline costs comprised the bulk of the cost of the project (see Appendix 1).

There was also the issue of the CORI clearance, which had to be obtained before I could carry out work at the school for any extended period of time. The CORI is a clearance that is required for any employee or worker in a Massachusetts school. This is a very important security issue as the schools want to ensure the safety of their students. I filed for my clearance during early December and it came through in mid March.

The mathematics after-hours program has many issues associated with it by virtue of the fact that it will be carried out after school. As was mentioned in the Introduction, transport for the children is a fairly big issue. Many of the parents of the children in this school come from lower income groups and hence do not own cars. The only way these children might be able to get home is if they live close enough to the school that they can walk the distance home. There are also many parents who are unwilling to send their children to this program for many reasons. These factors might affect participation in the program. Parents must thus be strongly encouraged to send their children to this program and find ways of alternative transport as this would prove extremely beneficial to their children in the MCAS examination, which is quickly becoming a very important test.[4]

Through our project, we hope to impact, in at least some small way, the scores on this test and thereby, the standard of education at Adams Street School. Through the continuous assessment and tutoring of the students at this school, we hope to improve their scores and provide a blueprint that other schools might be able to use to improve their quality of education by raising their MCAS scores.

[4] It is now compulsory for 10^{th} graders to pass the 10^{th} grade MCAS examination in order to be able to graduate from high school. This might extend to lower classes soon.

Chapter 1: Preliminary Testing and Analysis

Instruction to the students began on the first day of D-term 2000 and went on till June 20^{th} 2000, well into the summer. The work was conducted during the after–hours mathematics program at Adams Street School with the 4^{th} grade class. We met 3 times a week on Tuesdays, Wednesdays and Thursdays from 2:00-3:30pm.

I was set up to work with 4 students, who for purposes of confidentiality, will only be referred to as A,B,C and D. A and B were female and C and D were male. A, B and C were from the fourth grade class at Adams Street School and were 11 years old. Student D, however, was from the 3^{rd} grade class, and later on proved to be a useful subject in our study. He was 10 years old. A, B and C are all from very underprivileged families, if they have families at all, Some of them live with foster parents or at shelters. Student D hails from a slightly more affluent middle class family. He was grouped with the fourth graders as his level of understanding was on par with the rest of the students I was working with. He did not receive the pretest as he joined the after-hours class a week after I began instruction, but let it be said that his questionable understanding of the subject matter warranted his attending the help program.

The first day, we administered the pretest that has already been mentioned. To this end, the students were asked to do Lessons 1 through 10 in the "Soaring Scores on the MCAS in Mathematics" book (See References). These lessons covered a good portion of the concepts that have been discussed in the Introduction.

Despite the fact that there were hints after every question, the students still had great difficulty in answering them and did very badly. The results were either 0 or close to it. The actual numbers are presented at the end of this section.

In the following section, we will analyze each question for what the children were being tested, the common errors and the concept problems behind them. The question types referred to all come from the MCAS question definitions in the Introduction.

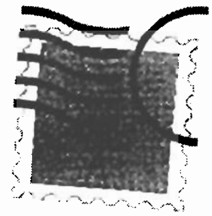
Lesson 1: Number Sense and Numeration

Question 1 deals with simple addition and multiplication. The students are asked to take information from a chart and use it to solve the problem. It is a multiple choice question.

Robert bought 3 rolls of stamps and 4 books of stamps. How many stamps did he buy in all?

- Ⓐ 34
- Ⓑ 304
- ● 340
- Ⓓ 3,400

Stamps	
Rolls	100 stamps
Books	10 stamps



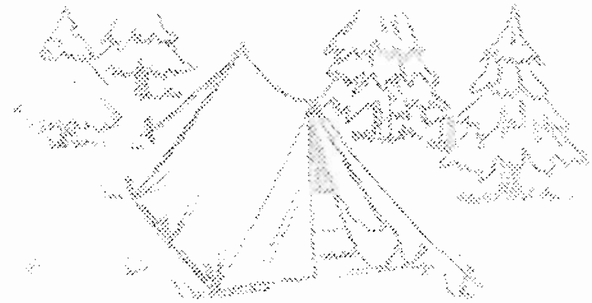
HINT *The chart gives information that is needed to solve the problem. Use the chart to find the number of stamps in a roll and to find the number of stamps in a book.*

340

All 3 of the test takers got this question right, but what was disturbing was the method by which they did it. They were all very uncomfortable with multiplication and would add up a number 3 times instead of multiplying by 3. They wrote their work by the side of the question, from which I was able to make these conclusions. The correct answer is choice C, which gives a total of 340.

Question 2 is a short-response question.

There were 190,675 members in the nature club. If ten thousand more people join, the club will have its greatest number of members ever. How many members would that be?



Answer:

HINT

Change ten thousand into a numeral. Think about how one thousand would be written as a numeral. Next, find the total number of members if ten thousand members join. What operation can you use?

This question deals with place values and simple addition. Only one of the students got this one correct. However, they were all able to identify that addition was the operation to use. Also, they were able to convert 10000 from the word form to the numerical form, They had trouble writing the addition equation and placing the number 10000 in the correct spot. They would write

$$\begin{array}{l} \ \ 190675 \\ \underline{+10000\quad\ } \end{array}$$

which is clearly wrong, indicating a severe problem in identifying place values. The correct answer in this case is 200,675.

Lesson 2: Concepts of whole Number Operations

Question 1 in this section is a short response question a deals with simple one step division.

Find the quotient.

$9\overline{)65}$

Answer:

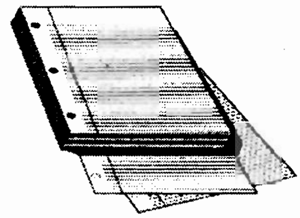
HINT!

To find the quotient, you need to divide. What steps do you need to do to divide? You can also draw a picture. How many would be in each group if you divided 65 into 9 equal groups?

None of the students were able to get this one correct. They all wrote down 8 as the answer, indicating that they have a vague conception of what division is, but were not too sure how to go about doing it. Instead of multiplying and getting the value which gives the number closest to the dividend that is *less* than the dividend, they wrote down the number that was closest to but greater than the dividend, which is a very flawed concept. The correct answer in this case is 7.

Question 2 is a multiple choice question and deals with choice of operations as applied to a real world situation.

Mr. Reed bought 6 notebooks. Each notebook costs \$1.79. Which answer choice shows how to find the total cost of the notebooks Mr. Reed bought?



- ● \$1.79 + 6
- Ⓑ $\$1.79 \times 6$
- Ⓒ $\$1.79 \div 6$
- Ⓓ \$1.79 − 6

HINT!

Decide whether the problem is about combining or separating amounts. Decide whether equal amounts are being combined or separated. Which operation matches these actions?

All the students got this question wrong. They all obviously had trouble in choosing the operation to use. One student, B, however, did manage to figure out that \$1.79 had to be added 6 times, but could not make the connection that this was a multiplication operation. The correct answer is B.

Lesson 3: Fractions and Decimals.

Question 1 is a short response question and deals with comparing the sizes of 2 fractions.

1. Make the number sentence true by filling in the circle with >, <, or =.

$\frac{2}{3} \bigcirc \frac{3}{5}$

Answer: ______________________

HINT!

Note the denominators in the fractions you are comparing. Find a common denominator. Rename each fraction using a common denominator.

Student A was the only one who managed to correctly solve this problem and it seemed like it was a guess as later on, when I asked her to explain how she had done it, she was unable to explain to me. Essentially, the students had no conception of fractions at all and how to visualize them. They had to learn to convert fractions to the same denominator for easy comparison. They also had to learn to visualize fractions as parts of a pie. The correct answer is 2/3>3/5.

Question 2 is a multiple-choice question.

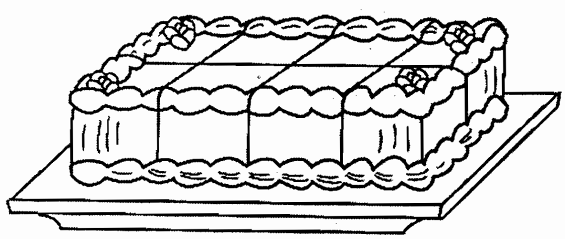
2. Manuel ate 2 pieces of the cake. Dina ate 1 piece of the cake. Darrell ate 3 pieces of the cake. Which fraction names the part of the cake Dina and Darrell ate?

Ⓐ $\frac{1}{2}$

Ⓑ $\frac{2}{8}$

Ⓒ $\frac{4}{4}$

Ⓓ $\frac{4}{6}$



The numerator in a fraction means the number of parts. The denominator means the number of parts in the whole. Use a fraction to name the part of the cake that Dina and Jamie ate. Find the fraction in the answer choices.

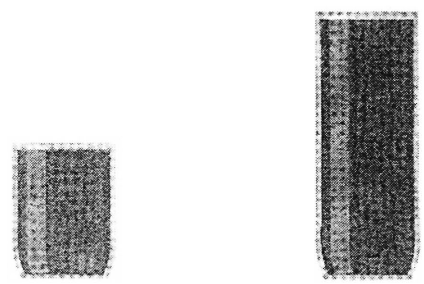
In this question, the students should have been able to identify that the whole consisted of 8 parts by looking at the picture. They should have then been able to add the number of pieces that Dina and Darrell ate to get the fraction 4/8=1/2. They were, however, unable to do this, indicating that they had trouble relating fractions to real life problems. Also, as we have seen, they had trouble with division, so the simplification of four-eighths to one-half would have been a problem.

Lesson 4: Estimation

The first question in this section was a multiple choice question dealing with simple estimation.

Which is the BEST estimate of the amount of water in Glass B?

Ⓐ 350 milliliters
Ⓑ 655 milliliters
Ⓒ 700 milliliters
Ⓓ 3,450 milliliters



Glass A
345 milliliters

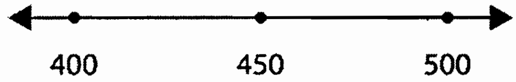
Glass B

HINT! *Use the amount given for Glass A to help you find the amount in Glass B. How much more would Glass B hold than Glass A?*

None of the students got this problem correct. All of them had major trouble with the concept of estimation. One student, student A was able to see that Glass B was double of Glass A and proceeded to add 345 to 345 to get 690, but could not make the connection that this was *approximately* equal to 700. She labeled the question as being "too weird".

The next question in this section was of the short-response type and deals with rounding off.

Which hundred is the number 465 nearest to?



Answer: ______________________________

Think of where the number 465 would be positioned on the number line. Decide which hundred comes just before 465 and which hundred comes just after 465. Then decide which of these hundreds the number 465 is nearest to.

All the students wrote down 450 as the answer. This shows that they are able to visualize the number line well and estimate which point the number is closest to, but they were not able to understand the question asking them to find the nearest *hundred*. This is another sign of a lack of understanding of place values.

Lesson 5: Whole Number Computation

Mrs. James bought snack-sized cereal to bring on the family camping trip. Ea
package had 6 boxes of cereal. There were 4 people in the family. Mrs. James wanted each person to have 1 box for each of the 9 days of the trip. How many packages should she have bought?

Ⓐ 6
Ⓑ 13
Ⓒ 19
Ⓓ 36

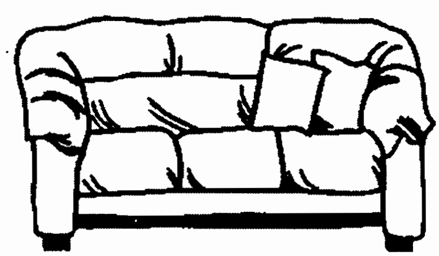
Decide how you can show the different parts of the problem. You might draw a picture. Pay attention to the different words in the problem, such as ***package, box, each of the 9 days,*** *and so on.*

Question 1 of Lesson 5 is a multiple choice question and deals with extracting information from a "word" problem and how to process it. It also involves multiplication and division.

None of the students were able to carry out the required operations. They all simple indicated that they were stuck by margin of the page. They had trouble with taking the required information from the problem and then from that, deciding which operations to use. The correct answer for this question is 6 packages.

The other problem on this section was of the short-response variety and it was on the subject of simple computations in the context of a real-life situation.

A sofa originally cost $1,003. Then the sofa went on sale and the price was changed to $295 less. What was the price of the sofa on sale?



Answer: ______________________________

What does the word less tell you about how the starting price and the sale price compare? What operation can you use to find the sale price?

Not a single one of the test-takers even attempted this problem. They simply left it blank highlighting "word" problems as a serious trouble area. The correct answer is $1003-$295=$708 .

Lesson 6: Patterns and Relationships.

This is a short response question dealing with simple arithmetic patterns.

1. Write the RULE to find the next number in this pattern.

14, 29, 44, 59, ____

Answer:

HINT!

To find the rule for the pattern, compare the first number with the second number. What was done to the first number to get the second number? What operation could be used?

2 of the 3 students answered this question correctly, but the other one was definitely on the right track too. All of the students correctly identified that the rule was that each number was the previous number increased by 15, but the third student wrote down 74, the actual number, instead of the rule, which was why she got it wrong. However, she has a problem with subtraction and basic operations as right by the side, she wrote down check marks to count between the numbers and she probably used the result to find the next number in the pattern. This is, of course, a perfectly valid way of solving the problem, but points to a deeper problem of difficulty with simple arithmetic operations. This question was, however, very encouraging, hinting at the children's great potential.

The next question is a multiple choice question dealing with the distributive law. This is a very content-based question and I believe that the students had probably never encountered such relationships in class before.

2. Which is equivalent to 32×11?

Ⓐ $320 \times 10 + 1$

Ⓑ $(32 \times 10) + (32 \times 1)$

Ⓒ $352 \div 11$

Ⓓ $32 + 11$

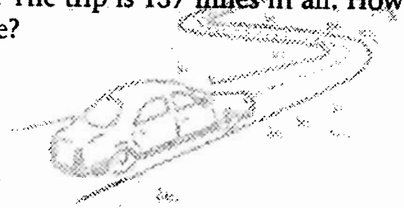
HINT! *Find the solution to the question. Which of the choices has a solution that is equivalent?*

None of the students managed to get this answer, but we cannot really draw any conclusions from it as it is a very content-oriented question, as opposed to a skills-oriented question. Incidentally, the correct answer is B.

Lesson 7: Algebra and Mathematical Structure

Write a number sentence to show how you could solve the problem below. Be sure to include the answer in your response.

Peter has driven 89 miles on a trip, so far. The trip is 157 miles in all. How many more miles does Peter need to drive?



Answer: ____________________________

HINT! *How might you show the miles Peter drove so far? How would you show the miles in all? What do you need to find? What operation can help you find the missing number?*

The first problem in this lesson is shown above. It is a short response question dealing with data extraction and choice of operations. None of the students were able to do this, reinforcing the fact that they had a lot of trouble with “word” problems. The correct answer in this case is 157-89=required distance.

Question 2 of this section is a multiple choice question on the subject of mathematical structure. In particular, it pertains to the inverse relationship between multiplication and division.

If $\triangle \times \square = 64$, which of the following is TRUE?

Ⓐ $64 - \square = \triangle$

Ⓑ $64 - \square = 0$

Ⓒ $\square \div \triangle = 64$

Ⓓ $64 \div \square = \triangle$

HINT

Think of how the operations are related to one another. How is multiplication related to division? What is the difference between multiplication and subtraction? You can also replace the shapes with numbers. Use the numbers to check each choice.

Again, none of the students were able to solve this problem correctly. This points again to a weak understanding of multiplication and division and how they are related. The correct answer in this case is D.

1. Use your pattern blocks to answer this question.

a. Find a pattern block that has 4 right angles. Draw the block in the space below.

b. Find a pattern block that has only acute angles. Draw the block in the space below.

c. Find a pattern block that has 2 obtuse angles and 2 acute angles. Draw the block in the space below. Draw a circle around each obtuse angle.

HINT!

Compare the angles on each of the pattern blocks. How are they alike and different? To find a block with right angles, think about what you know about right triangles. Then think of how acute angles are different from obtuse angles.

Lesson 8:Geometry and Spatial Sense

This was the only question in this section and it is of the open-response type from the MCAS. It pertains to interpreting descriptions of shapes and visualizing them in real life. The students get a number of shaped blocks referred to as "pattern blocks" by the question and they have to select and draw the appropriate shape from the pile. None of the students was able to even begin to answer the question, highlighting a sore need for explanation of abstract concepts. When asked after the test if they understood what acute and obtuse angles were, they were able to reply, so it was not their content strand that was lacking, but rather a gap in reasoning that needed to be bridged. The correct answers for this problem required identifying and drawing a square, triangle and a rhombus for parts *a, b* and *c* respectively.

Lesson 9: Measurement

The first question in this section was of the multiple-choice type. This again deals with a "word" problem and involves data extraction. It also deals with adding time.

1. After the school day ends at 3:00, Lori will go to soccer practice for 50 minutes Then she will ride the school bus home for 15 minutes. At what time will Lori get home?

Ⓐ 3:50

Ⓑ 4:05

Ⓒ 4:15

Ⓓ 5:05

HINT! *You can draw a clock face. You can start at the time school ends and count on to show time passing for soccer practice and the bus ride.*

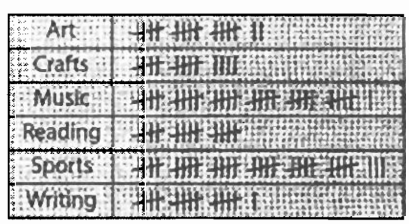
Again, the students were very pressed for answers and could not even figure out that they had to add for this problem, let alone do the temporal addition. All the students left this question blank. We have another problem area here. The correct answer in this case is B.

The next question is a short-response question that deals with the concept of area as applied to a rectangle. None of the students were able to answer it correctly. One student, student C, wrote down the formula for the *perimeter* of the figure, which was a very grave conceptual error. It seemed that the children had not been instructed very well in this concept and were thus presumed to have a content strand missing.

Lesson 10: Statistics and Probability

There was only one question in this section and it was of the multiple-choice variety and concerns simple data extraction, reading charts and comparison. 2 of the students solved this problem correctly, which shows that they have a rudimentary understanding of counting and comparison, so statistics are something we were able to concentrate less on. The correct answer in this case is B.

1. The students took a survey of favorite hobbies. These were the results.



Which statement about the survey is TRUE?

Ⓐ Sports and writing received the greatest number of votes.

Ⓑ Sports and music together received more votes than the rest of the hobbies combined.

Ⓒ Reading received more votes than writing.

Ⓓ Arts and crafts combined received less votes than reading and writing combined.

HINT *The words greatest and less are clues that numbers are being compared. What does the word combined mean?*

Results and Conclusions

The table below (Figure 6) shows the results of the pretest and classifies the results by student and question number. As we can see, the results were very poor and the students have a lot of work to do to improve their MCAS mathematics scores. They also confirmed our belief that the students were having trouble with more abstract concepts as Mrs. Schachterle had originally pointed out. Figures 7 and 8 give the breakdown according to question type and content strand.

	Student A	Student B	Student C
Lesson 1:1	O	O	O
Lesson 1:2	O	X	X
Lesson 2:1	X	X	X
Lesson 2:2	X	X	X
Lesson 3:1	X	O	X
Lesson 3:2	X	X	X
Lesson 4:1	X	X	X
Lesson 4:2	X	X	X
Lesson 5:1	X	X	X
Lesson 5:2	X	X	X
Lesson 6:1	O	X	X
Lesson 6:2	X	X	X
Lesson 7:1	X	X	X
Lesson 7:2	X	X	X
Lesson 8:1	X	X	X
Lesson 9:1	X	X	X
Lesson 9:2	X	X	X
Lesson 10:1	X	O	X
Total/18	3	3	1

Figure 6: Results of the pretest by student and question number. Correct responses are marked with an "O" and wrong answers are marked with an "X".

	Student A	Student B	Student C	Average %
Number\10	2	2	1	16.66667
Pattern\4	1	0	0	8.333333
Geometry\5	0	0	0	0
Statistics\1	0	1	0	33.33333

Figure 7: Results of the pretest by student and content strand.

	Student A	Student B	Student C	Average %
Multiple\9	2	2	1	18.51852
Short\8	1	1	0	0.083333
Open\3	0	0	0	0

Figure 8: Results of pretest by student and question type.

We now knew what areas of mathematics the students are lacking in and we planned our course of action from that. It seemed that the only things that the students were comfortable with were simple addition and subtraction. Even multiplication and division seemed to be too difficult from them. Data extraction and analysis from "word" problems were also found to be lacking. Abstract content subjects such as area, perimeter and time arithmetic needed a lot of work as well. Based on this, we planned our course of action. Mrs. Schachterle would teach the required subjects during regular class time and I worked with the weaker students during the after-hours program to reinforce the concepts taught in class.

The children's grasp of the subject matter was clearly below what one would normally expect from a typical fourth grader. According to Piaget's theories (see introduction and later chapters), the children's intellectual development was slowed down at some point, and this can be attributed to many factors which will be discussed in a later chapter. As such, we decided to begin teaching them as though they were younger children, learning all these concepts from scratch.

Plan of Action

In this section, we will discuss our course of action, listing out the topics that we covered in chronological order. We chose these specific fields to better prepare the children for the MCAS examination.

1. Number Sense

   Since basic addition, subtraction, multiplication and division are the basis for all other mathematical concepts, the children were drilled in these for a few weeks. We used a variety of hands-on tools to help the children understand these fundamental ideas. These will be discussed in Chapter 2.

2. Patterns, Relations and Functions

   With the understanding they got from the four basic mathematical operations, the students were instructed in more abstract concepts that will eventually form the basis for their understanding of algebra. They were given many problems and were stepped through the problem-solving process. More information on this is available in Chapter 3.

3. Geometry and Measurement

   This section was very abstract, so they were given a lot of hands-on tools to help them understand these ideas. This is discussed in more detail in Chapter 4.

4. Statistics and Probability

   This is the final content group in the MCAS curriculum. This proved to be a very difficult topic to treat due to its level of abstraction and more will be mentioned about it in Chapter 4.

The course of action listed above only mentions the content strands mentioned in the Framework, but we did continuously stress the mathematical thinking skills that are so essential to our cause. In the succeeding chapters, we will discuss in detail what we did, showing examples of problems, explanations and methods we used to help the children learn better.

Chapter 2: Improving Number Sense and Numeration

In this chapter, we will be discussing the techniques we used to try to help the students understand basic arithmetic better. We placed a lot of emphasis on this section as it is absolutely crucial that they understood this in order to make any meaningful progress in other areas of mathematics.

We will first present our course of action followed by an analysis of our results.

1. <u>Drilling the students in basic addition and subtraction with borrowing and carrying.</u>

The students were given worksheets with a list of simple addition and subtraction problems. Below are a few sample problems (Figure 9) that they were given. The teacher, Mrs. Schachterle had already gone through the carrying and borrowing concepts in class and it seemed the students understood it, but still needed help with the mechanical aspects of such calculations.

10000-686=	445+782=
3456+4567=	776-123=
1234-999=	7752+654=

Figure 9: Sample mathematics drill problems for addition and subtraction.

Essentially, I was more of a facilitator for these exercises than an actual instructor as I was helping the students solve the problems when they needed help. Occasionally, the students were stumped by borrowing from consecutive zeroes, for example,

10000

-9834

which can be a rather involved calculation. In such a case, I would go through the question with them, helping them through every step. Also, I stressed the importance of place value, constantly reminding the children that it is the **final** units digits that must line up, and not the **first** digit. Eventually, the students did become more comfortable with it and from then on, I did not see any mistakes in the understanding of place value.

According to Piaget's theory, young children build up their understanding of the world through hands-on experience that modify their schemata, which are concept structures. Thus, in order to drive the concept of place value home, Mrs. Schachterle came up with a game she called "Powers of Ten" in which the children are given blocks that represent the various powers of ten. For example, a unit block would be 1, a strip of 10 blocks would be 10, a square of 100 blocks would be 100 and so on. Then the children start off with 0 blocks and roll 2 dice to see how much they are going to add to their total, so they have to add correctly and figure out the sum. Their incentive to play was that the winner would be the one with the highest total at the end. Another variation of this game that was used is when the children start off with a square of 100 and roll the dice to see how much they are to *subtract* from the total. The winner is the first one to get to 0. (Figure 10)

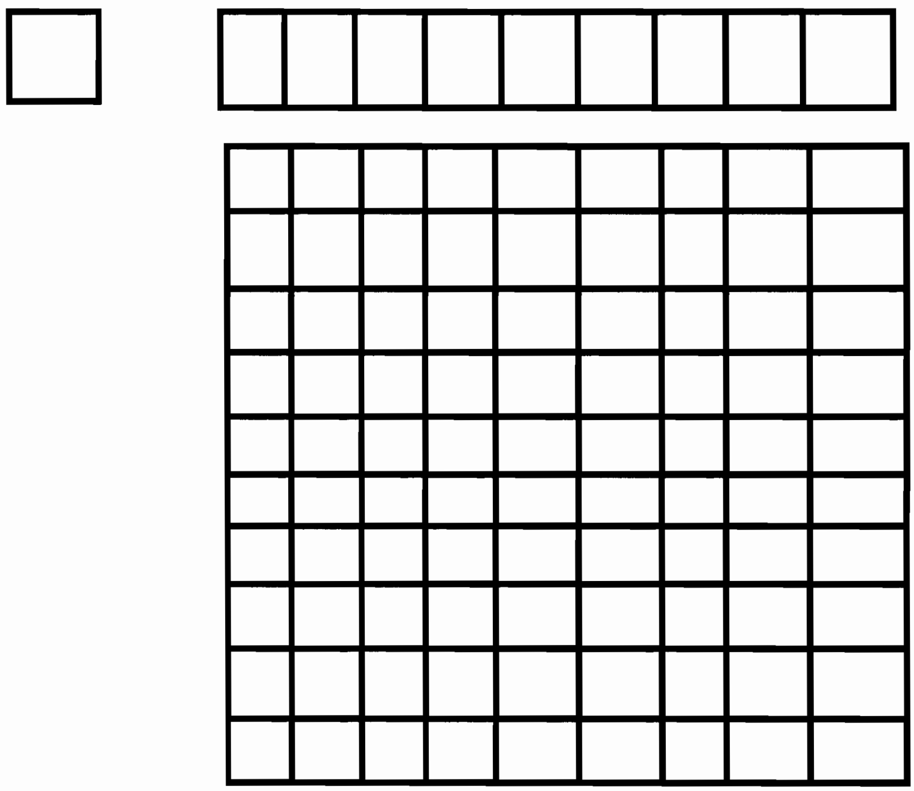


Figure 10: Illustration of the blocks in "Powers of Ten".

The children enjoyed the game very much, and it proved to be a very useful tool in teaching them about place values and simple addition and subtraction. Occasionally, during the course of my stint at Adams Street, I would get the children to play this game to keep them motivated, as they got restless very often.

Another technique that we experimented with was TouchMath, which involves identifying key touch points on each numeral to help students add and subtract. The only prerequisites are the ability to count forwards and backwards. The reasoning behind it is that TouchMath provides support for learners of all ages and learning styles as it capitalizes on tactile (touching the key points with a pencil tip), visual (observing the points while counting) and aural (counting aloud) learning patterns. We did not stress this too much as it would have taken too long for the students to adapt to a new method of adding and subtracting. The students did, however, enjoy it very much and were captivated by this new way of doing math. TouchMath has a lot of potential and educators should keep it in mind for possible incorporation into the curriculum (See Appendix B for a more detailed discussion on TouchMath).

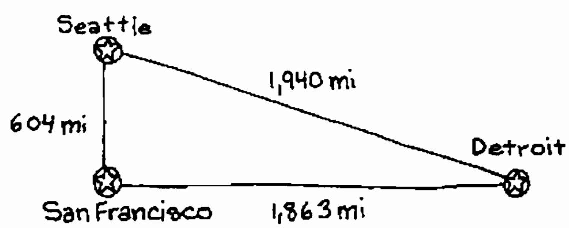
2. <u>Applying the computational skills to "word" problems</u>.

As we can see from the MCAS question from Chapter 1 and the Objectives presented in the Introduction, we can see that the MCAS places a huge emphasis on "word" problems and more concept based calculations as opposed to the more mechanical rote-based calculations we had been dealing with until this point.

In order to help them with this, the students were given a number of problems of this kind. I had an answer key with a step-by-step solution process to help guide the students through these problems. I have presented a few of these problems (Figure

3. What do we need to find? Show on the picture of your truck what we need to calculate.

Write a story problem that this picture would help you to solve.



86

Chicago is 440 km from Cleveland. Charlene's dad went to Chicago on a business trip and returned to Cleveland the next day. How far did he drive in all?

87

There are 2 shipments weighing 450 kg and 535 kg waiting at the warehouse to be shipped. Both shipments are headed for Boston. A truck that can carry a maximum of 1,000 kg arrives to pick up the 2 shipments. How much could a third shipment weigh after the 2 shipments have been loaded onto the truck?

88

Figure 11: A few sample "word" problems pertaining to simple mathematical operations.

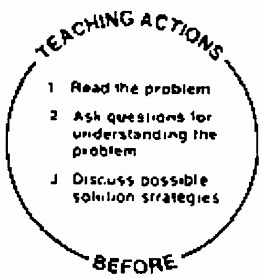
The children would then begin to solve the question on their papers and volunteer to come up and write up their solution. One common mistake in the solution of this problem was that the children would add up all three numbers, namely, 535, 450 and 1000 to get the final answer. When that happened, I would ask the relevant student to go back to the blackboard and re-think what he or she had done. I stressed words like "How much *more* can we place in the truck?" and gave them hints like "Think of finding the last shipment as though you were *taking away* space from the truck". Such prompts proved to be very useful and the children did eventually understand the solution.

To cement the concept, I asked verbal questions such as the following to check if their thought processes were going the right way.
Extension question: Suppose I had a bag which could hold 20 books. I first put in 5 books and then another 5. how many more could I fit into the bag?

Then I would ask them what steps they would have to take to solve the problem, and they would reply, "First find the number of books you put in *altogether*." They would then talk about how much space was *left* in the bag and that first addition and then subtraction would be the operations to use.

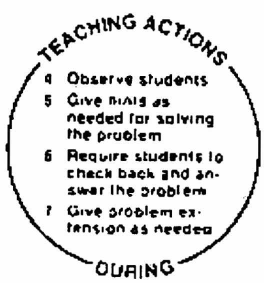
The emphasis was on the problem solving process rather than on the mechanics of the addition and subtraction as that was the area they were sorely lacking in.

Furthermore, I made sure that I guided them through the problem instead of solving it for them. If they were completely stumped on a problem, I would solve the problem for them, explaining the steps along the way, and then give them a similar problem, asking them to solve it themselves. This strategy proved to be very effective



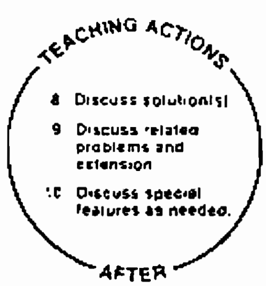
Understanding the Problem

- How much do the 2 shipments weigh? (450 kg and 535 kg)
- How much will the truck hold? (1,000 kg)



Planning a Solution

- What is the combined weight of the 2 shipments? (450 kg + 535 kg = 985 kg)
- Which operation would you use to find out how much more weight the truck can carry? (subtraction)



Finding the Answer

Choose the Operations

$$\begin{array}{r} 450 \\ +\underline{535} \\ 985 \end{array} \rightarrow \begin{array}{r} 1{,}000 \\ -\underline{985} \\ 15 \end{array}$$

A third shipment could weigh 15 kg.

Figure 12: Guide to solution strategy.

3. Improving multiplication and division skills

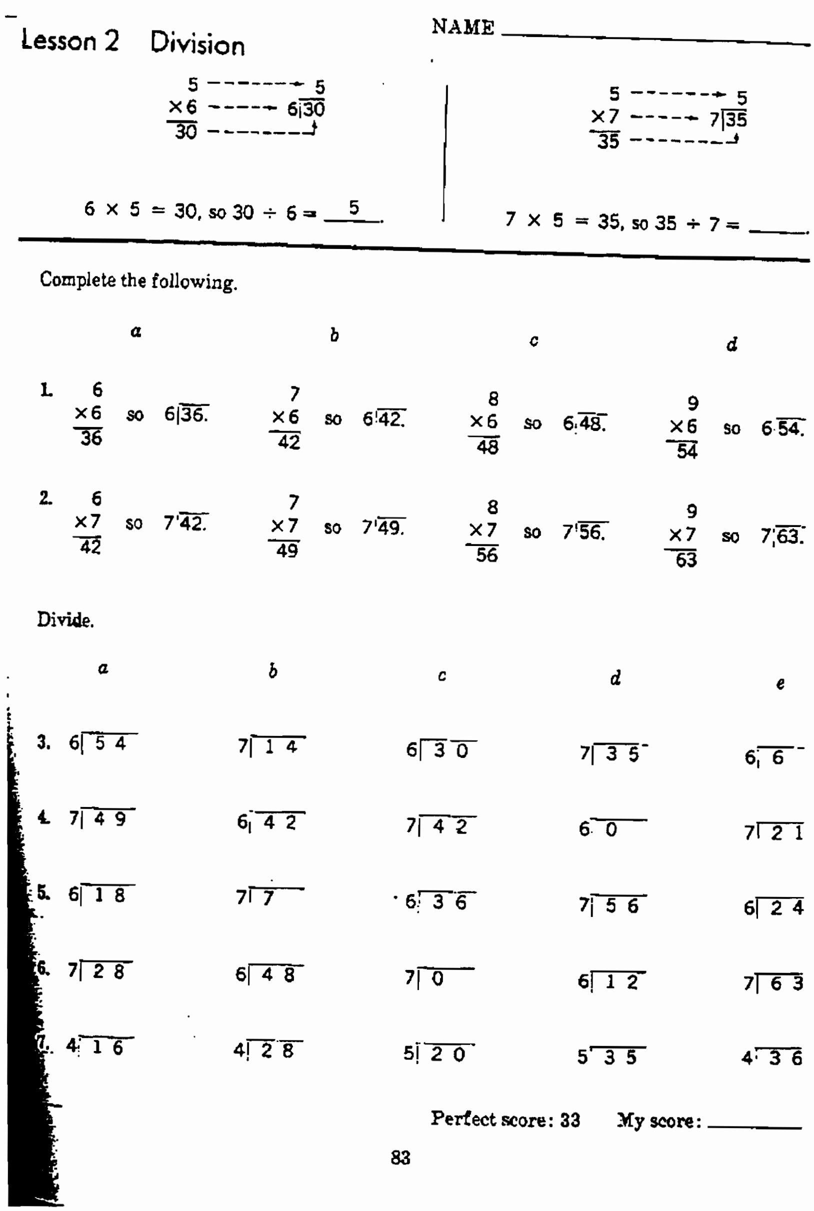
Throughout the course of stages 1 and 2, the children showed a serious ineptitude for multiplication and division, so I decided that this would be a good time to work on that. I did essentially the same thing I did for stage 1, giving them many drill problems, a few of which are presented in Figure 13.

I also tried to show them the inverse relationship between multiplication and division by giving them the worksheet below. I also used the analogy of how addition and subtraction are inverse operations of each other to guide them through this difficult concept. To give them a hands-on experience with the 2 operations, we used the "Powers of Ten" blocks again. They would, for example, be given 3 strips of ten and be asked to see how many there are in total. Another example would be to give them 9 unit blocks and ask them to put them into 3 equal groups. In order to illustrate the concept of remainder, a similar exercise but with say, 11 blocks into 4 groups was used. This helped the children visualize the operations a little better.

It seemed that at the end of the multiplication and division sections, the students were fairly confident in short multiplications and divisions. They were, still, however, very unsure about long division, being unable to carry out the mechanical steps necessary. Perhaps more drill problems would have helped, but we did not have enough time before the MCAS.

4. <u>Fractions and decimals</u>

This was a very challenging topic as fractions are rather abstract concepts to visualize. When first asked what they thought fractions were, the students did not really have an answer, so I knew that a lot of work was required.



Lesson 2 Division

NAME ______________________

5 → 5; ×6 → 6)30; 30	5 → 5; ×7 → 7)35; 35
6 × 5 = 30, so 30 ÷ 6 = 5.	7 × 5 = 35, so 35 ÷ 7 = ____.

Complete the following.

	a	b	c	d
1.	6 × 6 = 36 so 6)36.	7 × 6 = 42 so 6)42.	8 × 6 = 48 so 6)48.	9 × 6 = 54 so 6)54.
2.	6 × 7 = 42 so 7)42.	7 × 7 = 49 so 7)49.	8 × 7 = 56 so 7)56.	9 × 7 = 63 so 7)63.

Divide.

	a	b	c	d	e
3.	6)54	7)14	6)30	7)35	6)6
4.	7)49	6)42	7)42	6)0	7)21
5.	6)18	7)7	6)36	7)56	6)24
6.	7)28	6)48	7)0	6)12	7)63
7.	4)16	4)28	5)20	5)35	4)36

Perfect score: 33 My score: ______

83

Figure 13: Sample worksheet for multiplication and division drill.

The strategy I employed for teaching them about fractions and decimals was to draw out pizza pies on the blackboard. First, I started out with fractions, telling them about halves, thirds and quarters in terms of pizza slices. Then I moved on to explain that the decimal point was an extension of the normal number and we could represent fractions or parts of a whole in decimal notation.

The model I used to help them visualize decimals was the currency system. The children were very familiar with the division of a dollar into 100 cents and the decimal notation used to record it. So, I used that as a starting point to describe decimals to them. I once again stressed the importance of place value and how when writing decimal numbers for addition or subtraction, the decimal points should always be lined up.

I had difficulty explaining the concept of equivalent fractions to them. I tried to draw out slices of pizza, cut one slice in half and show them how the 2 new pieces made up an original slice i.e. that 2/16=1/8. The children saw it with respect to a pizza pie, but had a lot of trouble relating it to problems, such as those in Lesson 3 of the pretest. The first question in the pretest required comparison of two fractions with different denominators. The children were very uncomfortable with multiplying the numerators and denominators by a certain number to get a common denominator. They did not understand why that was being done.

Since I could not get them to understand the concepts of equivalent fractions, I taught them very mechanical ways of doing those problems such as blindly converting denominators to common numbers, to add numerators if the denominators are common etc.

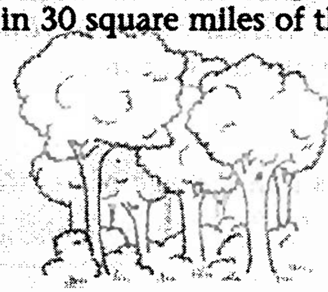
This was not the most effective way of improving the children's test scores. However, with the amount of time we were working with, we could not afford to spend more time on this topic with the students as we had many other topics to cover.

5. Estimation

The students already had a rather firm grip on estimation, but did not have the terminology for it, so I told them about the different kinds of questions that could be asked. One example of such a question is given below.

A scientist counted 103 trees in 1 square mile of a nature park. Which is the BEST estimate of the number of trees in 30 square miles of the park?

Ⓐ 300
Ⓑ 1,300
Ⓒ 3,000
Ⓓ 30,000



They were very comfortable with taking 100 as a good estimate of 103 and then multiplying by 30 to get the answer. The were also taught about the terminology "Round off to the nearest *hundred*" etc., which they picked up fairly quickly.

I also went through the problem from the pretest pertaining to the doubling and explained to the students that the had to pick the *nearest* answer and not the exact one when it came to such problems. We did not need to spend much time on this topic as it was very straightforward and the students had no trouble picking it up.

After these sections were completed, it seemed to me that the children had a firmer grip on basic mathematical operations. Fractions and long division still left a lot to be desired, but it seems to me that the gains far outweighed the topics that were not learnt. Also note that throughout the course of instruction, "word" problems were stressed and were the focus of our efforts.

I assigned the students a number of assignments that were related to the material covered in this material. They did these during the after-hours program and I was there to give them assistance as and when they needed it. The students really did show a marked improvement in their understanding of the subject matter, which contributed to their better performance on the MCAS.

Chapter 3: Improving Patterns, Relations and Functions

There are two major content sub-strands in this section of the MCAS, namely Patterns and Relations (sub-strand 1) and Algebra/Mathematical structures (sub-strand 2). Many MCAS problems in these sections are very similar in nature and are repeated year after year. Hence, I decided to adopt a very examination-oriented approach to this section, explaining the concepts as I went along. This is justified as many of these rely on the skills discussed in Chapter 1 and learning can be effective through a problem-oriented approach if the fundamentals are strong.

I identified 6 major kinds of problems that could be asked

1. Number patterns- the student is given a sequence of numbers which have a relationship e.g. 1,2,3,4,5,6,7_ and they have to fill in the last number, which in this case is 8. (sub-strand 1)
2. Question formulation- the student is given a series of data and information and is asked to make up a problem based on that data. (sub-strand 2)
3. Evaluating symbols- the student is given a piece of information telling them that so many symbols equal so much and they are asked what each symbols is worth. e.g. Brian has 7 check marks. He has 21 points. What does each check mark stand for? (sub-strand 2)

4. Shape patterns- the student is given a sequence of shapes and is asked to fill in the rest of the sequence. This problem type is very similar to problem type 1.(sub-strand 1).
5. Balancing problems- the student is described a relationship between 2 sides and is asked a question about them to balance the 2 sides. A variation on this is to identify a lack of balance. These are closely related to problem type 3. (sub-strand 2)
6. Commutative and distributive law- the student is asked to identify the equality of 2 different equations that are written in different ways e.g. 3X4=4X3 or 3X4=3X1+3X3. They have to recognize the commutative and distributive nature of addition and multiplication. This does not, however, apply to division and subtraction (distributive law holds for subtraction, but the 4th grade MCAS does not deal with it). (sub-strand 1).

We worked with these of problems for a while, and I will illustrate one kind of problem for each. Them I will proceed to describe how I went about treating each kind and the difficulties I faced. We did not, however, have time to go through extra assignments, so we do not have any data for this portion of the instruction. I will, however, sum up my findings and observations at the end of the chapter, which will give the reader insight into the progress of the students at this point.

<u>Problem Type 1: Number Patterns</u>

There is a large archive of MCAS problems of this type available online and in many reference books. One such problem is shown below.

What is the next number in the pattern below?

0, 1, 3, 6, 10, ____

A. 20

B. 15

C. 14

D. 26

This was a rather tricky problem for the children, but once they knew there had to be a pattern, they started looking for it. They would come up to the board and start writing their ideas down. It was very encouraging when they figured out the answer without much prompting at all. The correct answer is B.

Mrs. Schachterle later told me that they had done many such number pattern problems during regular class time. This reinforces the philosophy that practice makes perfect in mathematics.

Problem Type 2: Question Formulation

These questions are very involved and require a lot of creativity on the part of the student to actually make his or her own questions. For this question type, the students tended to formulate very simplistic addition questions. This was acceptable as the students were now armed with the ability to tackle the MCAS examination. As we recall, one of the initial problems that the students faced was data extraction. Since they were able to grasp the basics of this skill, progress was made.

Below is a sample problem and some solutions that the students presented.

Example: Write a question that can be answered using the data in this story. Then find the answer.

There are 7 red stripes and 6 white stripes on the American flag. Mrs. Michael has 4 flags.

Solution by Student A: How many are there in all?

7+6=13

13+13+13+13=52

Solution by Student B: How many white stripes are there in all?

6X4=24

As we can see, student B was rather uncomfortable with multiplication, but she was able to carry out effective data extraction and decide what operations to use. As mentioned before, the students did not have much trouble with this problem type as they had the freedom to make it as simple as possible.

<u>Problem Type 3: Evaluating Symbols</u>

This was another extremely abstract topic and serves as preparation for algebra. The students had to be able to identify what a symbol means in a certain context. Below is a sample problem we did in class that illustrates this.

Example: John had 3 X's written on his hand. This means that he scored 33 points. What does each X stand for?

We first established that the X's were only symbols for something John had won. This was also more accessible to the students as "points" and "games" were part of their schemata and they could apply their understanding to the problem. They were immediately able to identify that each X was 11 points since 3X11=33. In this situation, they made the connection that division is the inverse of multiplication without realizing it, but when it came to more abstract algebraic examples, they were rather lost. It took a lot of effort in trying to get the students to understand that symbols, no matter what they are called are just a representation for numbers, like a name written down on a piece of paper is like for a person. We did many drill problems of this type (and number 5, which is closely related). This concept did eventually sink in, surprisingly, as we shall see in chapter 5.

Problem Type 4: Shape Patterns

These problems are very similar in nature to problem type 1, but involve abstract symbols rather than numbers. The children did not really have much trouble with this problem type as they saw it more as a puzzle than as work, which is always a good mindset to have. For children, they seem to learn so much better when they are playing than when they are "studying".

One example of such a problem is:

Example: What is the next shape in the following pattern?

□ X ∇ O X ∇ □ O □ X ∇

A. ∇

B. X

C. O

D. □

This is a typical MCAS sort of pattern and is fairly straightforward. The students were able to solve the question without any difficulties. They also enjoyed drawing the shapes on the board, which also helped them to learn better.

Problem Type 5: Balancing Problems

For this problem type, the students have to be able to fill in the "blanks" in an equation. For the sample problem shown below, the students were asked to do the computations on the left hand side, which are very straightforward. The difficult part was to get them to fill in the right hand side. The number concept that we are dealing with here is the inverse relationship between multiplication and division. Hence, I asked the students to decide what number multiplied by 5 gives 60. They then referred to their times tables and were able to make the connection. One student, student D, who was a little more advanced than the other children in his learning, was able to identify immediately that the box must have been 12 as the second pair of parentheses gives us 5 which is also on the right hand-side. I did not, however, teach the other children to solve the problem that way as I wanted to give them a general all-encompassing method to solve every kind of problem of this variety.

Which number belongs in the box to make the number sentence below true?

$$(15 - 3) \times (2 + 3) = \square \times 5$$

A. 5

B. 15

C. 12

D. 30

Problem Type 6:Commutative and Distributive Law

This was perhaps the easiest problem type to deal with as the children already had an intuitive idea of what commutative law is. They knew that interchanging order for addition and multiplication and division is acceptable and will result in the same answer. For the distributive law, Mrs. Schachterle went through the relationship during regular class time and the students were very comfortable with it. In order to reinforce these ideas, I gave them the following 2 problems.

Example: Suppose $\nabla X\square=75$, complete the following number sentence $\square X\nabla=?$.

The students were all sure that the answer was 75. They managed to identify the answer without too much effort.

Example: Which one of the following does 25X5 equal to?

A. 25X(5+5)

B. 5X(5+5)

C. (25X3)+(25X2)

D. 2X(5X5)

The students were able to identify correctly, with relative ease, that the answer was C. This skill can be attributed to the drilling done in class by Mrs. Schachterle.

Now that I have described how we went about treating each individual question type and my observations, I would like to mention another tool that was *invaluable* in our efforts to help the children learn better: the Math Blaster PC software by Havas Interactive[5]. This software presents mathematical patterns, problems and concepts in the form of very enjoyable games that the children can play. The interface is very user friendly and the characters are very cartoon-like, making them very endearing to the children. There would be days when I would just leave the children to play the Math Blaster games, and they would be content doing that. I would then observe them and it was interesting to see how much they learned from just exploring the game on their own.

The ability to just explore the various mouse controls and game options was itself a challenge for the children, one which they overcame easily. Furthermore, the games presented in the software were rather involved when it came to patterns etc. but the students had no trouble solving them, as they wanted to *win*. It made me realize that if I

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[5] For more information on the software, please visit http://www.havasint.com.

made anything like a game for the children, they would be more willing to learn, which would help the overall assimilation of knowledge.

Final Thoughts

For this portion of instruction, the children were very enthusiastic and receptive to me, which could be due to a variety of reasons. One is that this subject matter is a little more appealing to them, with a more puzzle-like mood to it as opposed to the more mechanical operations from Chapter 2. Another possibility is that by this time the children had adapted to my style of teaching and had become more comfortable with me. I believe that the Math Blaster software also helped the children learn better. Evidently, circumstances were very fortunate during this period of instruction and the children were very cooperative.

Chapter 4: Improving Geometry, Measurement, Statistics and Probability

For the above 2 content strands, we decided to merge instruction as both are very graphic topics and there are not that many different kinds of problems. Furthermore, there are only 3 sub strands in all between these two topics, so a detailed treatment of one chapter per content strand would not be justified.

In each topic, there are a few concepts that are essential, and these are the ones I covered during the after-hours program. I also relied on Mrs. Schachterle's instruction during regular class time so that the students would first learn the concept from the teacher, and I would then reinforce and provide drill.

The concepts discussed in the MCAS can be summarized into the following categories. I treated each of these in class and explained the concepts to the children. I also gave them certain problems on them. I will provide samples of problems for each concept I covered. Again, time constraints did not allow for assignments to be given out. We shall again provide observations and analysis for each topic treated.

Parallel and Orthogonal Lines

These are very simple ideas and were treated by the use of string and rulers. The string was cut into 10 inch length pieces and laid out on the table. I then told them that if the 2 pieces of string would not intersect even if I stretched them out, then they were parallel. The students had already covered this in class and were very comfortable with that explanation. I let them play with the string for a while and then asked them to come up to the board to draw a pair of parallel lines. They were able to do that very well.

I used a similar sort of approach to explain right angles and orthogonality to them. They referred to them as "square angles" as they had been taught in class. This too they were good with.

The reason they were so comfortable with this is that they were able to relate it to their everyday experience such as right angles and parallel lines on tables, blackboards etc, and they were able to develop the abstract side of their schemata through experience. Furthermore, the use of yet another hands-on tool allowed them to experience these concepts and cement them in their mind.

Here is an example of a very simplistic problem I used to reinforce the concept:

Example: Which of the following has 2 square angles and 2 pairs of parallel lines

A. Square

B. Triangle

C. Circle

D. Star

I asked them to come up to the board to draw the 4 figures. They were then asked to describe the features of each shape and if they corresponded to the description in the problem. They were able to identify the correct solution (choice A) very easily.

Acute and Obtuse Angles

Mrs. Schachterle went through these concepts in class. Acute angles were described as being "narrow" angles and obtuse angles were described as being "wide" angles. I brought out the pattern blocks and got the students to classify each angle into one of the 2 categories. That was very helpful. After that, I gave them the following problem to help them cement the concept.

Example: Draw a triangle that has 1 obtuse angle and 2 acute angles.

I intentionally specified a triangle to help guide the students through their thinking process, as it reduces one level of abstraction. When a triangle was specified, they were immediately able to draw it out, as they understood how the angles were supposed to look. However, in problems where the shapes were not specified, they were not able to identify the solution. I spent some time on it, but was not able to get them to understand it completely. Again, due to time constraints, I had to move on.

Shapes

This is a very content-oriented topic as it involves a lot of terminology. Mrs. Schachterle went through the main definitions in class and the students were able to identify the shapes that I presented to them on the board. I could not adopt a hands-on approach to this topic, as I did not have any shapes for the more complex shapes like trapezoid, kite, rhombus etc.

Terminology list: Rhombus, Kite, Trapezoid, Parallelogram, Quadrilateral, Rectangle, Square, Triangle, Circle, Equilateral, Isosceles, Pentagon, Hexagon, Heptagon, Octagon.

For the octagon, I was able to relate the shape to that of a stop sign, so the children were able to understand it well. However, to distinguish between the various quadrilaterals proved to be a challenge. As we shall see in Chapter 5, they did not manage to master these concepts at all.

All I could do for this section was to ask the students to go up to the board and draw specific shapes. They were fairly competent in the triangles and basic quadrilaterals, but had trouble distinguishing between the other shapes. I would also ask them to identify shapes I would draw on the board, and that was as much as I could deal with this topic owing to time constraints.

Measuring

This was a very easy topic to teach. The children had been measuring with rulers from second grade, if not earlier and it was not a challenge to teach it, as they already understood it very well. To reinforce the concept, I basically got them to measure a variety of objects in inches as well as centimeters.

Example: How long is the following line to the nearest centimeter?

This also helped them with their estimation skills, as they had to round off to the nearest unit.

Area and Perimeter

We did not get to spend enough time on this topic, as we had to move on to the Probability and Statistics content strand. However, I did try to explain the concepts to them, focusing on intuitive understanding of the concepts. This is not an ideal replacement for actual experience, but our situation prompted it.

To illustrate area, I drew a random figure on the board and drew a grid over it. I then showed them that the area is the number of squares covering the figure. I then gave them the formula for the area of a rectangle.

After that, I presented the following problem.

Example: What is the area of a rectangle with length 6 ft and width 7 ft.?

They were immediately able to multiply 6 by 7 to get 42. However, it was an immediate short-term kind of knowledge. They did not retain it and did not really understand the concept.

For perimeter, I described that it was the length around an object. Suppose one took a piece of string and put it round its edges, and then measured its length, that would be the perimeter. I then gave them the following question:

Example: Suppose there was a rectangle with length 6 and width 2, what would its perimeter be?

I got them to go to the board to draw out the rectangle and do out the problem on the board. They were able to do it at that time by adding the length and width twice to get the answer which is 16.

I provided a few more simple examples, which they were able to solve immediately, but they seemed to be doing it mechanically, instead of really thinking about it. This explains why they forgot these concepts once they left the classroom. (See Chapter 5).

<u>Time</u>

The children already knew how to tell time, so it was relatively easy to treat the topic of time arithmetic. I told them that it was exactly like adding ordinary numbers except that when they saw more than 60 minutes on the minutes side, that they had to add 1 to the hours side and subtract 60 from the minutes. This was very clear to them. Subtraction was little trickier though as borrowing for time involves the number 60. I told them that if the upper minute number was smaller, to add 60 to it and subtract one from the hour number. I then went through an example problem. The were fairly clear with this as they already knew how to read time and do simple time arithmetic (such as the time left to going home etc.). There was enough time to give them drill problems and they were comfortable with these ideas.

The following is one of the sample problems I gave them.

Jonathan just finished his homework. The time is shown on the clock below. He did his homework for 1 hour, 50 minutes. What time did he start his homework?

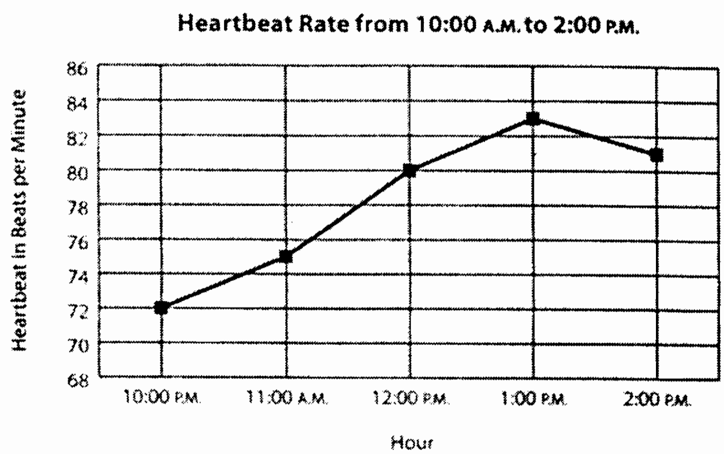
Ⓐ 3:25
Ⓑ 4:00
Ⓒ 4:25
Ⓓ 7:05

The correct answer is A. They initially had trouble with this question, but were able to work through it with a little guidance

Reading Charts and Extracting Data

There really was no way of going through this topic other than to do drill problems. Many different kinds of problems can be asked. Hence I went through many problems to get the students into thinking in the statistical mode. Below is a sample problem:

Chen measured his heartbeat over several hours. He made a line graph to show the data.



Between which hours did Chen's heartbeat rate decrease?

Ⓐ 10:00 A.M. and 11:00 A.M.
Ⓑ 11:00 A.M. and 12:00 P.M.
Ⓒ 12:00 P.M. and 1:00 P.M.
Ⓓ 1:00 P.M. and 2:00 P.M.

The students were very confident with these questions as a lot of them are very graphical. Thus, the students saw them as puzzles and enjoyed doing them. The correct answer to the above question is D.

Mean

Initially, the students were very apprehensive about this concept. They had a rough idea of what an average was, but were not sure about how to solve the problems, nor could the explain to me satisfactorily what the physical meaning was. I initially told them to add up all the numbers and divide by the number of figures. Then I gave them a number of problems on averages. They were not able to solve them.

I then decided to relate averages to something their schemata had assimilated. I used the example of sand piles. Initially, we have piles of sands of different heights next to each other, and when we even the piles out, the height of the new pile would be the *mean /average* height. This clicked immediately in them and they were able to solve the questions with greater ease. (Figure). As we see in Chapter 5, their conception of this idea definitely improved.

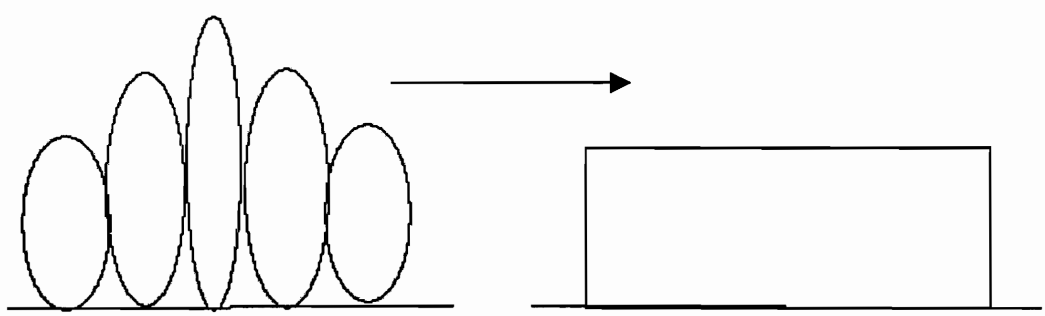


Figure: Illustrating the explanation for averaging. The various piles of sand are smoothed out to form one uniform pile.

Example: John bought 4 books. They cost $1.50, $2.00, $2.50, $3.00. What was the average cost of his books?

Answer: (1.50+2.00+2:50+3.00)/4=$2.25

Probability

I cannot stress enough the importance of relating mathematical concepts to real life phenomena that the children can understand. When I replaced the word probability with chance and used the example of dice, the children became very enthusiastic and responsive to calls to solve probability problems. They were able to solve the questions very easily and managed to arrive at the answers with little help.

According to Piaget, the reason why they assimilated these ideas so easily was because they had already developed their schemata of chance through board games such as Monopoly and Chutes and Ladders, which involve dice or spinners, which have random outcomes. Also games such as Scrabble in which the players have to pick out random letters from a bag improve their conception of probability as they realize the more copies of a letter there are in the bag, the better their chance of picking it.

Example: There are 10 balls in a bag. 4 of them are red. If I picked out a ball at random, what is the probability of me picking out a red ball?

Solution: There are 4 red balls out of 10, so the probability must be 4./10.

The students managed to solve the above problem without any prompting.

Final Thoughts

These content strand were dealt with very close to the actual administration of the MCAS, so we were in a hurry to cover all the topics. Hence, we had to compromise in certain areas. However, some topics like probability and measurement were very encouraging as the students had already developed their own schemata through the course of their lives, allowing them to make the abstraction from real life to mathematics.

Chapter 5: The Mock MCAS Examination- Results and Conclusions

After the course of our instruction, about 1 week, before the actual MCAS examination, I decided that I would give a mock MCAS examination. Mrs. Schachterle suggested the MCAS 4th grade mathematics examination that was administered during the spring of 1999. A copy of the examination with the answer key is attached in Appendix C. 4 students namely, A, B, C and D participated in the mock examination.

The exam was administered on Tuesday the 20th of May 2000 during the after-hours program. Due to time constraints, the mock exam was organized in one session instead of the standard two. This proved to be sufficient time for the children, however, as they passed in their exams about an hour and a half after they were handed out, and they did not complain.

I then collected the exams back and proceeded to correct them and interpret the results. Below, I shall a give a question-by-question analysis of only the Session 1 questions as the Session 2 results essentially mirror those of Session 1[6]. This will be followed by each student's results by problem number, content strand and question-type. Finally, I shall carry out an analysis of my findings and then evaluate the program.

[6] Please refer to the questions in Appendix C when reading the question-by-question analysis as this will make the explanations much clearer.

Problem 1

This was a multiple-choice question under the Number Sense/Whole Number Computation sub strand. All the students got this question correct, indicating that certain basic concepts have been firmly planted in their minds.

Problem 2:

This was a multiple-choice question in the Geometry and Measurement/Measurement sub strand. Only 2 of the 4 students got this question correct, indicating a lack of understanding in measurement and estimation. All the students were required to do was to see which one of the choices was between one a two pounds, so this problem can be interpreted 2 ways: that there is a deficiency in multiplication, or that there is a deficiency in value judgments. Either way, this was cause for concern as it was a rather simple problem.

Problem 3:

This was a multiple-choice question of the Statistics and Probability content strand. Everyone got this question correct, showing that they had a good grasp of simple chart reading and data analysis.

Problem 4:

This was another multiple-choice question from the Statistics and Probability content strand. Again, all 4 students solved this problem correctly, illustrating their strength in basic data extraction.

Problem 5:

This question was a multiple-choice question of the Patterns, Relations and Functions/Algebra/Mathematical structures sub strand. All the students got this question right, highlighting that they really do understand the commutative property very well.

Problem 6:

This question was of the multiple-choice variety from the Statistics and Probability content strand. Only students C and D managed to solve this one correctly, showing that there is a problem in symbol evaluation. Again, this was an abstract concept that should have had more time put into it.

Problem 7.

This was yet another multiple-choice question from the Patterns, Relations and Functions/Patterns and Relationships sub strand. All the students managed to solve this problem and it can be attributed to the fact that "blocks" and "streets" are real concrete things with which they are familiar. Their schemata of these concepts are very well developed, so they had no difficulty in answering it.

Problem 8:

This multiple-choice question was from the Number Sense/Estimation content sub-strand. Only Student C got this one correct, showing that there is a distinct lack of understanding in estimation. Perhaps it was unclear how many "slices" were in the garden, or maybe the students just did not know to multiply is unclear as it was a multiple-choice question and none of the students showed any work.

Problem 9:

This multiple-choice question from the Patterns/Relations and Functions/Patterns and Relations sub strand was a rather encouraging problem. It was of the Balancing Type identified in Chapter 3. 3 out of the 4 students got it correct, which was rather surprising as this was a rather abstract problem requiring some rather involved thinking processes. This could again be explained by the fact that weights are something the children are used to dealing with and can do calculations pertaining to them fairly quickly.

Problem 10:

This was of the Open-response type and was of the Number Sense/Concepts of Whole Number Operations sub-strand. Students C and D scored ¾ for this and Student B scored 1.5/4. Student A had no idea how to solve the problem and left it blank. However, those who did attempt the problem managed to fill in correctly the rest of the table presented in the problem. Students C and D would have obtained full points if not for silly mistakes. The only reason student B only got 1.5 was because she did not even attempt the other parts, but judging from her performance and participation during the after-hours program, it could attributed to her eagerness to finish early. The results from this question were very encouraging

Problem 11:

This is a short-response question from the Geometry and Measurement/Measurement sub strand. All the students got this question correct as children are taught to use rulers from the time they are in 2^{nd} grade, if not earlier. This again reiterates the point that familiarization, experience and practice are the best teachers.

Problem 12:

This was a short-response question from the Number Sense/Whole Number Computation sub strand. Only Student D solved this question correctly, which realized my fears that the children were not too comfortable with long division and multiplication.

Problem 13:

This open-response question from the Geometry and Measurement/Geometry and Spatial Sense sub strand proved to be a real problem for most of the students. Only Student D had any conception of what was happening. Student A managed to identify that a rhombus has four equal sides. This is rather perplexing as Mrs. Schachterle went through all the shapes in class and I reviewed them during the after-hours classes. These questions are very content-based, so the only possible explanation is that the children did not pay much attention during class and hence did not retain the information for very long.

Problem 14:

This was a multiple-choice question from the Geometry and Measurement/Geometry and Spatial Sense sub strand. All the students did well on this problem. Their success could probably be attributed to the guided nature of multiple-choice questions. They are not left cold searching for answers on their own, so they feel much more comfortable with them. This is of the same sub-strand as the previous problem and deals with similar concepts, but the results were so vastly different that the analysis is rather tricky.

Problem 15:

This multiple-choice question was of the Statistics and Probability flavor. All the students except for student B got this one correct. The success in this question can again be attributed to the fact the shirts and pants are within the sphere of the children's schemata and they do not have any trouble relating them to a problem.

Problem 16:

This multiple-choice question was of the Number Sense/Number Sense and Numeration sub strand. Student B was the only one who got it wrong. Again, the guided nature of this problem allowed the children to try out every answer to check which one does not belong. This also shows that they are rather competent in simple addition as they would have had to do the addition for every case to see if they added up to 452.

Problem 17:

This was an open-response question of the Patterns, Relations and Functions/Patterns and Relationships sub strand. Students B,C and D did very well on this question, but Student A did not even know where to start as she just wrote a few random numbers into the spaces and left it as such. The fact that the children did so well in this question can be explained by the fact that they had done many such pattern problems in the Math Blaster software discussed in Chapter 3. This is again, very encouraging, as the notation used in the problem definition is rather abstract.

This concludes our question-by-question analysis of Session 1 of the Mock MCAS examination.

Results

The following 3 Figures will give our results. The students' scores are broken down by problem, question type and content strand.

	Student A	Student B	Student C	Student D	Average%
Number\18	6	4	12.5	13	49.3
Pattern\11	6	9	11	11	49.3
Geometry\14	7	4	4	11	46.4
Statistics\11	4	3	8	9	54.5

Figure 14: Scores by Content Strand

	Student A	Student B	Student C	Student D	Average%
Multiple\29	17	12	22	25	65.5
Short\5	3	2.5	4.5	5	75.0
Open\20	3	5.5	9	14	39.4

Figure 15: Scores by question type.

Question	Student A	Student B	Student C	Student D
1	O	O	O	O
2	O	X	X	O
3	O	O	O	O
4	O	O	O	O
5	O	O	O	O
6	X	X	O	O
7	O	O	O	O
8	X	X	O	X
9	O	X	O	O
10	0\4	1.5\4	3\4	3\4
11	O	O	O	O
12	X	X	X	O
13	1\4	X	0\4	3\4
14	O	O	O	O
15	O	X	O	O
16	O	X	O	O
17	0\4	4\4	3\4	4\4
18	O	X	O	O
19	O	X	X	X
20	X	X	O	O
21	O	X	O	O
22	X	O	X	O
23	O	O	O	O
24	X	O	O	O
25	O	X	O	O
26	X	X	O	O
27	0\4	0\4	2\4	2\4
28	O	0.5	O	O
29	X	X	0.5	O
30	O	O	O	O
31	2\4	0\4	O	2\4
32	X	X	O	O
33	X	X	O	O
34	X	O	X	O
35	O	X	O	O
36	X	X	X	X
37	X	X	X	X
38	X	O	O	O
39	O	O	O	O
Total\54	23	20	35.5	44

Figure 16: Scores by problem number.

The percentages given above are the total number correct for each student divided by the total for that section. We then took the average percentage for all 4 students.

Statistical Analysis of the Results

I will now compare the results of the Pretest in Chapter1 with the final Mock Examination Results. I will compare results by content strands and take the averages of the students' scores for each content strand. The tables below (Figures 17 and 18) show the percentage of correct answers before and after instruction.

	Pretest	Mock
Number	16.7	49.3
Pattern	8.3	81.8
Geometry	0.0	46.4
Statistics	33.3	54.5

Figure 17 : Comparison of scores before and after instruction with respect to content strands.

	Pretest	Mock
Multiple	18.5	65.5
Short	8.3	75.0
Open	0.0	39.4

Figure 18: Comparison of scores before and after instruction with respect to question types.

There was definitely a marked improvement in all areas, especially in the short question category. The students were still very uncomfortable with the open-response question type, as shown by the low score of 40%, but that is still a large improvement over the 0% we saw initially.

In terms of content strands, the biggest improvement and highest score were in the Patterns, Relations and Functions section, which showed a marked improvement. The content strand areas show fairly low scores, but this was due to the fact that the open-response question type brought the overall scores down. This shows that the students were beginning to get comfortable with the concepts, but needed some sort of guidance that the shorter questions provided. Hence, I decided to provide a content strand analysis without the open-response questions. The table below (Figure 19) summarizes these results. Then, we can draw a more meaningful set of conclusions.

	Student A	Student B	Student C	Student D	Average %
Number\14	5	2.5	9.5	10	48.2
Pattern\7	6	5	7	7	89.2
Geometry\6	4.5	4	5	6	81.3
Statistics\7	4	3	6	7	71.4

Figure 19: Scores by content strand excluding open response scores.

If this were the actual MCAS examination, only students C and D would have passed. Evidently, the other 2 students show extremely sub-par competence.

Here, we can clearly see the source of all our problems. Despite the fact that the students had developed sufficient proficiency in the concept areas, they still lacked number sense and mechanical ability and that was what was bringing their scores down.

As we had observed in Chapter 3, the students really enjoyed doing Patterns, Relations and Functions type problems, and this is clearly reflected in the scores with and without the open response questions.

Their guided sense of Geometry and Measurement is also commendable, although when left out in the cold, they seem rather lost. This could be explained by the fact that there were words that they did not understand in the open-response Geometry and Measurement questions such as quadrilateral etc. Also, the concepts of area and volume seemed to be somewhat hazy, even during class time. We did not, have enough time, unfortunately, to drill the students in these areas even more. These two reasons can explain the poor performance in the overall geometry section.

Finally, the Statistics and Probability section was rather encouraging as the open-response question involved them drawing a histogram, something they must have been very uncomfortable with, as we did not spend much time on actually drawing out charts etc.

On the whole, we can see that our efforts were fruitful to a certain extent. The students made considerable progress through the course of their study. However, some of our strategies were misguided and we shall discuss these issues in the following section.

Evaluation and Recommendations for Change

I only spent about three and a half months working with the children and this is certainly not enough time to institute a miracle turnaround. However, with my short stint at Adams Street School in collaboration with the 4th Grade teacher, Mrs. Schachterle, we certainly showed the impact that an after-hours mathematics help program, properly implemented, can have.

The biggest issue that we faced was in getting the children to learn Number Sense concepts such as estimation, long multiplication, division etc. This was the portion of the test that dragged them down in terms of scores. The best way to rectify this would be to spend more time on drilling the students with more mechanical problems and use more hands-on techniques to get them to learn these concepts better. Maybe TouchMath should be instituted as a standard part of every curriculum (see Appendix B). Due to our tight schedule, however, we were unable to provide them with ample practice for the actual MCAS, and it shows in the scores.

Fractions were another weak point and could be improved by using more hands-on tools such as cutting round pieces of cardboard into slices and allow the students to play with them rather than just drawing pictures on the board. As Piaget's theories have stressed, experience is the best way to modify schemata and these concepts of fractions, multiplication and division are but schemata (i.e. concept structures).

Other weak areas were the concepts of area, volume and perimeter. It was very difficult for us to implement hands-on teaching methods, as some of these would have been rather messy. One idea would be to use the displacement of water to teach volume. Another would be to use string to measure the perimeters of shapes. These should be done during regular class time and the after-hours program should provide more drill in problems.

Another moot point was the students' grasp of the terminology used to describe shapes, such as "quadrilateral", "parallel" etc. The only solution to this would be to expose them to these words throughout the course of the academic year to familiarize them with it. It seems that these words are only used while teaching that specific section.

For statistics, the students should be made to prepare practical charts more often. This was done during the regular class time, but it seems that they were still very reluctant to draw the graphs on the Mock Examination. The only way I can think of to help this situation would be to provide more drill in drawing graphs.

The test area the students were most afraid of was the open-response question type. This was due to the fact that their schemata were not developed enough to deal with such involved problems. The only way for them to become comfortable with the open response questions would be to instruct them from the start of the academic year in order for them to develop their skills enough to be confident of tackling the more involved problems.

The students' learning was definitely influenced by our teaching and the mode of instruction (blackboard vs. hands-on). However, these are not the only factors that contribute to their progress. I shall now proceed to discuss other factors behind their intellectual development.

Other Observations and Conclusions

Observations

From the results above, it is clear that there is a big difference between Students A and B and C and D. Student D is obviously more proficient than all the other students. This can be explained by the fact that he has more support at home from his mother. He comes from a single-parent middle class home with a steady income. His mother is very involved in his education and despite the fact that he is younger than the other students, he was able to do well on the Mock Exam. Student B's parents are involved, but do not support her enough at home.

From Mrs. Schachterle's dealings with Student B's parents, it seems that they do not stress the importance of homework and putting in time into her schoolwork. Evidently, if the parents are not that concerned about their children's education, the student will not be motivated enough to learn.

Student A did not even have a stable foster home until towards the end of the course of instruction. The lack of a stable home really contributed to her poor performance as she did strike me as a very intelligent and capable student. She was able to understand concepts fairly quickly and do the practice drill problems when they were given to her. She did, however, do better than Student B did.

Student C has a stable home, but his parents are very traditional and stern. This may not be an explanation for his performance (he is a very bright child and could certainly have done much better), but it is something worth considering. They are, however, very supportive of his education. Perhaps a little more freedom would have allowed him to explore the world and learn a little more. Another possible explanation would be that he simply did not have enough preparation for the examination, a point that has already been mentioned.

Conclusions

It seem that drilling the students with more problems, and working through them, explaining the concepts to them, (as opposed to rote drilling) and simple spending more time with them on their problems would prove to be the best way to improve their grades.

Furthermore, a more practice-oriented teaching method would allow the students to pick up concepts quicker as not all students can grasp abstract ideas easily.

Another important factor is for parents and teachers have to work hand in hand for the child to be able to learn properly. New and novel teaching methods can be useful, but only if there is sufficient support from the parents. The children must be motivated enough to pursue their education as the "motivation for learning and development is primarily internal[7]". If the children do not have a stable home, they will not be motivated to work. Also, if their parents do not pay enough attention to their education, the students will lose the drive to learn, as they do not feel a comforting security from their parents' support. Too much attention can also be undesirable, as the children might begin to look upon learning as a chore, which is also bad. A happy medium would be the best solution.

[7] Page 4 Wadsworth (See References).

Chapter 6: Piaget's Ideas and Recommendations Applied to American Education

This entire report was based on Piaget's theorems and how the students must in the end, learn the concepts themselves. All instructors can do is to provide appropriate stimuli to encourage the students to build their schemata. Barry Wadsworth, expounding upon Piaget's ideas says " Number, length and area concepts cannot be constructed only from hearing about them or reading about them.....it must be constructed from active exploration". This was a flaw in parts of our instruction as we were unable to provide enough real-world stimuli to the students in certain areas due to time constraints. We can, however, learn from our mistakes and ensure that in future implementations we do not neglect these topics. Hopefully, future programs will have a longer time frame to work with.

Piaget also stresses the need for giving students more independence in a very broad sense. This does not mean that they be allowed to do whatsoever they wish, but be allowed to explore certain ideas in whatsoever manner they wish. To this end, educators must make available to the students the appropriate tools such as pattern blocks, Lego kits, interesting computer software, playing cards etc. He criticizes traditional authoritative teaching methods that many American schools adopt. The teacher is seen as a "dictator" and the children are to do whatever they are told. They are not allowed any freedom of thought. For example, for teaching multiplication, the teacher lays out the rules of addition, carrying, adding zeroes etc, and the students are to follow these rules blindly without exploration. In contrast to this, a Piagetian teaching method would involve handing out blocks (as we did in our instruction), allowing the students to play with them and explore the mathematical relationships between them. A more modern version of this would be to provide them with interesting software in a game format (e.g. MathBlaster), which would be something novel to them, and would make them want to explore.

His ideas are centered around the processes of mental disequilibration (a state of "mental limbo") and accommodation to recover the equilibrium. The disequilibration leads to a desire on the students part to achieve equilibrium. In other words, their curiosity must be piqued. In order to stimulate this, instructors should surprise students with phenomena that throw them off guard and challenge their present ideas. For example, in science, making a feather and a penny drop in a vacuum would really surprise many students and would encourage them to search for the explanation themselves. This would be preferable to just talking about such a theoretical situation and explaining the concept of gravity to them.

Critical thinking, problem solving should not be separated from intellectual development. Piaget postulates that students, if stimulated in the right way will develop these skills automatically. He again reiterates that traditional classroom methods cannot accomplish this and will only lead to rote learning.

Another important aspect of his theory is how he describes the fallacy of grouping children in classes by age. The fallacy lies in the fact the different children develop at different rates. They might not even develop various skills in the same order. Hence, in order for effective learning to take place, children should be grouped according to ability and intellectual maturity. One important reason for this is that if there are some students who are more advanced and others who are less so, the less advanced students will feel inadequate when they end up learning slower than the more advanced ones, and this could lead to a mental block, as they convince themselves that they are unable to learn and may become unreceptive to learning. The students construe the teacher's fast pace as a lack of respect for their abilities and tend to become very uncooperative. "Respect begets respect," says Wadsworth in his book (see References).

The implications of the above paragraph could be very revolutionary. Almost all American public schools group children according to age, and the standards are decided by a central state board although this has been changing as of late. As such, there are many students who are not advanced enough that are stuck in a higher grade level than they should be. This is where the after-hours program help out. They pick out the students needy of more help and give them the instruction at the level they need. Furthermore, the after-hours programs should be labeled as *help* programs as opposed to "remedial" programs as that would make the students feel inadequate again.

Teachers should also be understanding of the children's background and how some of them might have had their development slowed down due to trauma or just lack of opportunity. They should not be reprimanded when they fail to learn as quickly as the others might be. I was very understanding with the students when they failed to grasp concepts immediately. I had to be very patient and explain things to them slowly. I also did not try to impose my authority on them. Rather, I tried to be a friend who was helping them out. Again, respect begets respect.

Piaget's theories cannot be reduced to a set of rules that one can step through to improve learning at their school. It is simply one way of looking at learning and the educator must mold these ideas to his own needs.

This is exactly what I have done in this project. I had a goal: to improve the students' MCAS scores. I then developed a set of strategies to achieve that, keeping Piaget's ideas at the back of my mind. Despite the fact that we only had a few short months to carry out our work, we managed to improve the scores of students who had originally done very badly. There were some strategies that were not so efficient, but I hope that future educators looking to improve their school's mathematics scores will be able to learn from my mistakes and apply some of my findings and recommendations.

It is my wish that public education in the United States will be reformed, keeping the idea that "respect begets respect" in mind . Perhaps my work will contribute in some way to improving education in this country.

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3.Curriculum: Foundations, Principles and Issues 3rd Edition

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Web Resources

1. www.doe.mass.edu/mcas/

   This is the Massachusetts Department of Education MCAS website.

2. www.havasint.com

   This is the Havas Interactive website for the MathBlaster software package.

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1. The New Flexible Math Meets Parental Rebellion

   Author: Anemona Hartocollis

   The New York Times, Tuesday, April 27 2000

2. Information on TouchMath

   TouchMath Inc.

Appendix A: Sample Assignments

This section gives some of the sample assignments the students were given during the Number Sense instruction period.

Problem Solving

Solve each problem.

1. How many pieces of rope 4 meters long can be cut from 15 meters of rope? How much rope will be left over? ________ pieces can be cut. ________ meters will be left over.	1.
2. Dale has 51¢. Magic rings cost 8¢ each. How many magic rings can he buy? How much money will he have left? Dale can buy ________ magic rings. He will have ________ ¢ left.	2.
3. Maria has a board that is 68 inches long. How many pieces 9 inches long can be cut from this board? How long is the piece that is left? ________ pieces can be cut. ________ inches will be left.	3.
4. 28 liters of water was used to fill buckets that hold 5 liters each. How many buckets were filled? How much water was left over? ________ buckets were filled. ________ liters of water was left over.	4.
5. It takes 8 minutes to make a doodad. How many doodads can be made in 60 minutes? How much time would be left to begin making another doodad? ________ doodads can be made. ________ minutes would be left.	5.

Perfect score: 10 My score: ________

These problems have the numbers omitted. Identify the operation or operations needed to solve each problem.

1. Rita ran a different number of miles on Monday, Tuesday, Wednesday, and Thursday. How far did she run?

2. Last week Tim bought a can of tennis balls. Tennis balls were on sale this week, so he bought another can. How much money did he save by buying the can on sale?

101

Use this table to find the average number of points scored by a player on the Dragon's basketball team.

Heidi	7
Crescenda	12
Ellen	11
Deana	6

102

Felipe and Luis sold blackberries. Felipe sold 2 gallons (gal) for $2.00 each, and Luis sold 3 gal for $1.75 each. How much money did they make together?

103

NAME ____________________

PRE-TEST—Division

Divide.

	a	*b*	*c*	*d*	*e*
1.	$3\overline{)26}$	$5\overline{)47}$	$6\overline{)49}$	$8\overline{)62}$	$9\overline{)77}$
2.	$2\overline{)28}$	$4\overline{)48}$	$3\overline{)93}$	$7\overline{)84}$	$6\overline{)96}$
3.	$5\overline{)67}$	$8\overline{)93}$	$9\overline{)97}$	$6\overline{)87}$	$7\overline{)97}$
4.	$3\overline{)186}$	$4\overline{)236}$	$7\overline{)161}$	$9\overline{)425}$	$8\overline{)612}$
5.	$3\overline{)369}$	$4\overline{)840}$	$2\overline{)964}$	$5\overline{)696}$	$7\overline{)898}$

Perfect score: 25 My score: __________

Name

Estimate
Divide
Multiply
Subtract
Compare
Bring Down

$3\overline{)72}$ $7\overline{)84}$ $4\overline{)76}$

$4\overline{)96}$ $2\overline{)78}$ $6\overline{)72}$

$3\overline{)78}$ $4\overline{)56}$ $6\overline{)43}$

$5\overline{)44}$ $3\overline{)28}$ $6\overline{)24}$

Appendix B: TouchMath

This technique relies on key touch points on each number to facilitate addition, subtraction and multiplication. All the students have to be able to do is to count forward, backward and know their times tables. They also have to memorize the touch points on the numbers. These are all fairly reasonable prerequisites for 4^{th} graders.

There are two kinds of touch points: single touch points and double touch points. Single touch points are counted once whereas double touch points are counted twice. The figure below illustrates these.(Figure B-1)

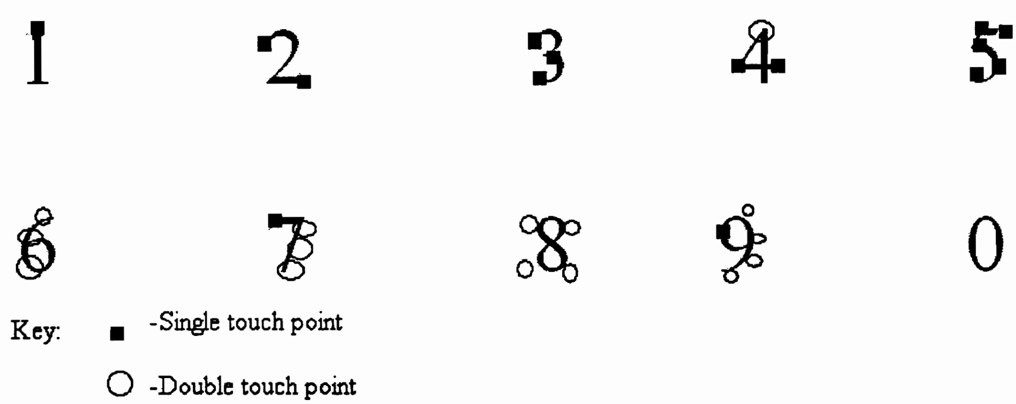


Figure B-1: Figure illustrating the various touch points on the numerals.

<u>Addition</u>

The student starts and taps out each touch point with his pencil tip while he counts out aloud, giving 1 tap for each single touch point and 2 for each double touch point. He simply counts in sequence and goes down the right hand side of the addition problem, counting in sequence (i.e. 1,2,3,4...etc.). He then carries on to the next place value. If there is a carry over, he writes it down above the column and continues as before.

Subtraction

The student again starts on the right-most digit. He then starts from the number on top and counts backwards, counting for every touch point on the bottom digit, tapping his pencil tip as he does so. He records the number and moves on to the next place value.

The borrowing process has to be done before this counting is done, though. This can be done by comparing the upper and lower digits and if the lower digit is larger, then "borrow" 1 from the next place value and add 10 to the present place value. Then continue as before.

Multiplication

The student must know his times tables to be able to do this. He takes the multiplier, and this becomes the functioning times table. Then, he counts in multiples of the multiplier for the first digit on the right again tapping out each touch point. He then records the answer and moves on to the next place value.

If there is a carry over, he must add it over to the multiple-counting result. This can be done by using the touch points as mentioned above. For multipliers greater than 9, he must understand the concept of moving the second digit multiplication to the left by one place value.

Conclusion

According to research, there are 3 kinds of learning styles: visual, auditory and tactile. Each student has his or her own preference and TouchMath supports all 3 styles. The novelty of TouchMath itself proved to be enough for the students to be fascinated with it. This is a technique that is consistent with Piaget's ideas on stimulating interest in order to facilitate teaching.

TouchMath[1] also produces workbooks, flashcards and other such products that make use of this technique they developed. They are all very graphical in nature, which will appeal very much to the younger students.

[1] For more information on TouchMath, contact 1-800-888-9191 during regular office hours.

Appendix C: Mock MCAS Examination from Spring 1999

Session 1, Multiple-choice Questions

Use the information in the box below to answer question 1.

Maria is playing a game with a number cube. The numbers 1–6 are on the cube. She tosses the number cube five times.

1. In this game, what is the HIGHEST total she could make in five tosses?

   A. 17

   B. 21

   C. 25

✔ D. 30

Reporting Category/Substrand for Item 1: ***Number Sense*/Whole Number Computation (p. 118)**

2. A bag of pretzels weighs between one and two pounds. Which of these could be the weight of the pretzels? (1 pound = 16 ounces)

   A. 10 ounces

   B. 14 ounces

✔ C. 24 ounces

   D. 36 ounces

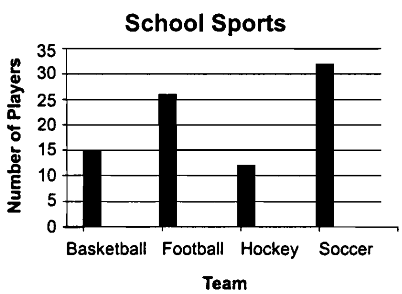
Reporting Category/Substrand for Item 2: ***Geometry and Measurement*/Measurement (p. 120)**

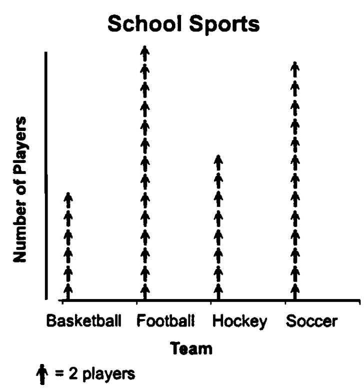
Use the information in the chart below to answer question 3.

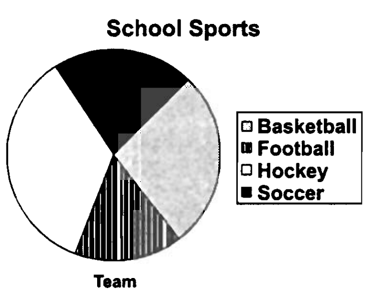
School Sports

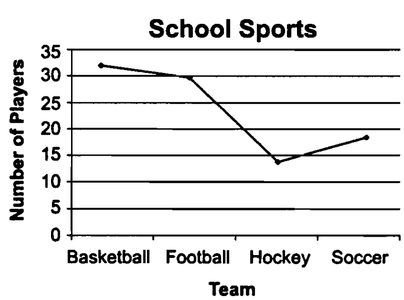
Team	Number of Players
Basketball	12
Football	28
Hockey	16
Soccer	26

3. Barbara decided to make a graph of the number of players on her school's sports teams. Which graph correctly shows the number of players on each team?

A. 

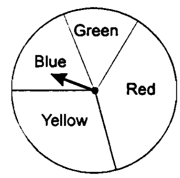
✔ B. 

C. 

D. 

*Reporting Category/*Substrand for Item 3: ***Statistics and Probability/NA*** **(p. 120)**

4. This is a spinner for a game.



Which color are you most likely to spin?

A. blue

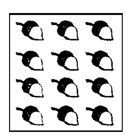
B. green

C. yellow

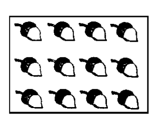
✔ D. red

*Reporting Category/*Substrand for Item 4: ***Statistics and Probability*/NA (p. 120)**

5. Melvin collected acorns from the yard. First he placed them like this:



Then he placed them like this:



Which number sentence shows the TWO ways Melvin placed his acorns?

✔ A. $3 \times 4 = 4 \times 3$

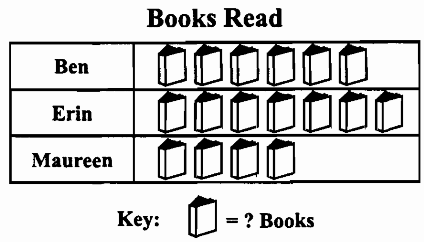
B. 3×4

C. $3 \times 4 > 4 \times 3$

D. $3 \times 4 \geq 4 \times 3$

*Reporting Category/*Substrand for Item 5: ***Patterns, Relations, and Functions*/Algebra/Mathematical Structures (p. 119)**

6. Ben's teacher made this graph to show how many books Ben, Erin, and Maureen have read so far this year. She wants them to figure out how many books each  stands for.



Ben knows that he has read 18 books. How many books does each  stand for?

A. 2

✔ B. 3

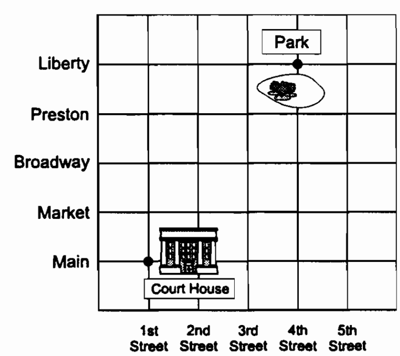
C. 5

D. 6

*Reporting Category/*Substrand for Item 6: ***Statistics and Probability*/NA (p. 120)**

Use the information and the map to answer question 7.

The Wheelchair Club has planned a race for wheelchair users. The race will start at the Court House and end at the Park. The race must travel along the streets shown on the map.



7. The club would like the route to be 11 blocks long. Which route is 11 blocks long?

✔ A. (1st, Main) to (1st, Preston) to (3rd, Preston) to (3rd, Market) to (4th, Market) to (4th, Liberty)

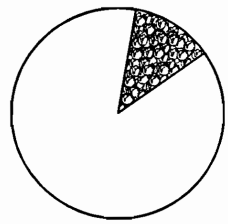
B. (1st, Main) to (5th, Main) to (5th, Liberty) to (4th, Liberty)

C. (1st, Main) to (1st, Liberty) to (3rd, Liberty) to (3rd, Market) to (4th, Market) to (4th, Liberty)

D. (1st, Main) to (3rd, Main) to (3rd, Broadway) to (4th, Broadway) to (4th, Liberty)

*Reporting Category/*Substrand for Item 7: ***Patterns, Relations, and Functions*/Patterns and Relationships (p. 119)**

8. A gardener is planting a flower bed as shown.



The gardener has planted 85 tulips so far. What is the BEST ESTIMATE for the total number of tulips that will be in the garden?

A. 150–200

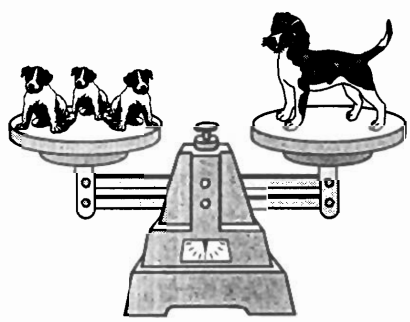
B. 300–400

✔ C. 650–750

D. 1200–1300

Reporting Category/Substrand for Item 8: ***Number Sense*/Estimation (p. 118)**

9. A mother dog and her three puppies weigh a total of 32 pounds. The mother dog's weight equals the weight of the three puppies together.



What is the weight of the mother dog?

A. 4 pounds

B. 8 pounds

C. 12 pounds

✔ D. 16 pounds

Reporting Category/Substrand for Item 9: ***Patterns, Relations, and Functions*/Patterns and Relationships (p. 119)**

...u stion

...t below to answer question 10.*

...ee...	1	2	3	4	5	6	7
...mount	$1.50	$3.00	$4.50	$6.00			

...e pattern for weeks 5, 6, and 7 in your Student Answer Booklet.
...e... or words to describe the pattern.

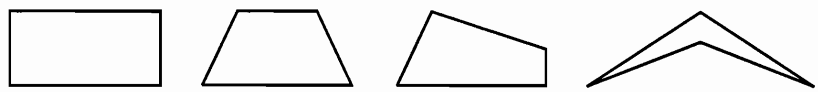
... pattern in the chart to write a story problem about money. Be sure your problem
... estion.

...ite a number sentence to go with your story problem. Be sure to include the answer
... ur number sentence.

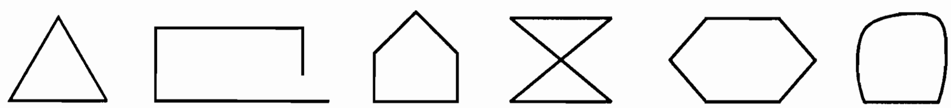
Reporting Category/Substrand for Item 10: **Number Sense/Concepts of Whole Number Operations (p. 118)**

Session 1, Open-response Question

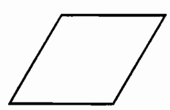
13. These shapes are quadrilaterals.



These shapes are not quadrilaterals.



a. Write a complete definition for a quadrilateral. Be sure to tell all the important ideas about what makes a quadrilateral.

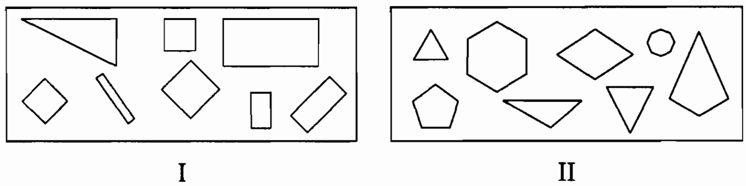
b. This is a rhombus: 

A rhombus is a special kind of quadrilateral. Explain what makes it different from other quadrilaterals.

*Reporting Category/*Substrand for Item 13: ***Geometry and Measurement/*****Geometry and Spatial Sense (p. 119)**

Session 1, Multiple-choice Questions

Use the sets below to answer question 14.



14. What rule did Ray use to sort the shapes above into two groups?

A. I = three sides
II = four sides

B. I = big shapes
II = small shapes

✔ C. I = shapes with right angles
II = shapes without right angles

D. I = shapes with four or more angles
II = shapes with less than four angles

*Reporting Category/*Substrand for Item 14: ***Geometry and Measurement/*Geometry and Spatial Sense (p. 119)**

15. What is the GREATEST number of different outfits you can make with 2 pairs of pants and 5 shirts? (Each outfit must have exactly one pair of pants and one shirt.)

A. 5

B. 7

✔ C. 10

D. 25

*Reporting Category/*Substrand for Item 15: ***Statistics and Probability/*NA (p. 120)**

16. Which does NOT equal 452?

✔ A. 3 hundreds, 5 tens, 12 ones

B. 3 hundreds, 15 tens, 2 ones

C. 4 hundreds, 5 tens, 2 ones

D. 4 hundreds, 4 tens, 12 ones

*Reporting Category/*Substrand for Item 16: ***Number Sense/*Number Sense and Numeration (p. 117)**

Session 1, Open-response Question

17. These are input-output tables. Each table has a different rule. When a number **n** is put in, it is changed by the rule so that a different number comes out. Table 1 has been completed for you.

a. Complete Tables 2 and 3 in your Student Answer Booklet.

Table 1

Input	n	8	1	5	9	21
Output	n + 5	13	6	10	14	26

Input-Output Rule: n + 5

Table 2

Input	n	2		9	7
Output	n x 9	18	54		63

Input-Output Rule: n x 9

Table 3

Input	n	36	16	8	
Output		9		2	7

Input-Output Rule: ______________

b. Write an input-output rule for Table 3 using the letter **n**.

c. Use a new rule to make up your own input-output table. Complete Your Table in your Student Answer Booklet. Be sure to include your rule using the letter **n**. (You may NOT use the rules from Tables 1, 2, or 3.)

Your Table

Input	n				
Output					

Input-Output Rule: ______________

*Reporting Category/*Substrand for Item 17: ***Patterns, Relations, and Functions/*****Patterns and Relationships (p. 119)**

Session 2, Multiple-choice Questions

18. There are 60 pieces of art paper and 42 children. If each child gets one piece of art paper, how many pieces will be left for another project?

A. 9

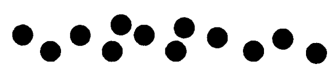
✔ B. 18

C. 27

D. 42

Reporting Category/Substrand for Item 18: ***Number Sense*/Whole Number Computation (p. 118)**

Use the picture of marbles below to answer question 19.



19. There were 12 marbles on the floor. Lisa picked up $\frac{1}{2}$ of the 12 marbles and Tom picked up $\frac{1}{4}$ of the 12 marbles. How many marbles were picked up?

A. 2

B. 5

✔ C. 9

D. 10

Reporting Category/Substrand for Item 19: ***Number Sense*/Fractions and Decimals (p. 118)**

20. Marco is practicing for track and field day. If he practices for 360 minutes, what would you do to find out how many HOURS Marco practiced?

A. Add 60 to the total number of minutes.

B. Subtract 60 from the total number of minutes.

C. Multiply the total number of minutes by 60.

✔ D. Divide the total number of minutes by 60.

Reporting Category/Substrand for Item 20: ***Number Sense*/Concepts of Whole Number Operations (p. 118)**

21. Which number fits ALL of these clues?
    - a multiple of 3
    - an even number
    - a multiple of 8
    - not a multiple of 9

✔ A. 48

B. 54

C. 63

D. 72

*Reporting Category/*Substrand for Item 21: ***Number Sense/*****Number Sense and Numeration (p. 117)**

22. Your lunch time begins at 12:40 P.M. If your lunch time is 35 minutes long, what time does it end?

A. 12:05 P.M.

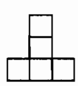
B. 1:10 P.M.

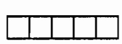
✔ C. 1:15 P.M.

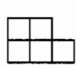
D. 1:30 P.M.

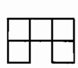
*Reporting Category/*Substrand for Item 22: ***Geometry and Measurement/*****Measurement (p. 120)**

23. Which shape CAN be folded to form an OPEN box?

✔ A. 

B. 

C. 

D. 

*Reporting Category/*Substrand for Item 23: ***Geometry and Measurement/*****Geometry and Spatial Sense (p. 119)**

24. What number does n stand for in the sentence below?

$$(8 + 2) + 6 = 8 + (n + 6)$$

✔ A. 2

B. 6

C. 8

D. 16

*Reporting Category/*Substrand for Item 24: ***Patterns, Relations, and Functions/*Algebra/Mathematical Structures (p. 119)**

The chart below shows the number of student lunches ordered for one week at Wilson Elementary School. Use the chart to answer questions 25 and 26.

School Lunches Ordered at Wilson Elementary School

Day of the Week	**Number of Lunches Ordered**
Monday	75
Tuesday	25
Wednesday	50
Thursday	100
Friday	150

25. What is the total number of school lunches ordered during the week?

A. 150

B. 250

C. 300

✔ D. 400

*Reporting Category/*Substrand for Item 25: ***Statistics and Probability/*NA (p. 120)**

26. How many more students ordered school lunches on Friday than on Monday?

A. 55

B. 65

✔ C. 75

D. 85

*Reporting Category/*Substrand for Item 26: ***Statistics and Probability/*NA (p. 120)**

Session 2, Open-response Question

Use the information in the chart to answer question 27.

School Lunches Ordered at Wilson Elementary School

Day of the Week	**Number of Lunches Ordered**
Monday	75
Tuesday	25
Wednesday	50
Thursday	100
Friday	150

27. a. Look at the grid in the answer space for question 27 in your Student Answer Booklet. Using this grid, draw a BAR GRAPH to show the school lunch data. Be sure to correctly label your graph.

b. After studying your graph, write two different possible reasons for Friday's lunch count.

*Reporting Category/*Substrand for Item 27: ***Statistics and Probability*/NA (p. 120)**

Session 2, Short-answer Questions

28. Compute:

$$\begin{array}{r} 3{,}972 \\ 4{,}826 \\ +8{,}321 \\ \hline \end{array}$$

Correct Answers: 17119 *or* 17,119

*Reporting Category/*Substrand for Item 28: ***Number Sense/*Whole Number Computation (p. 118)**

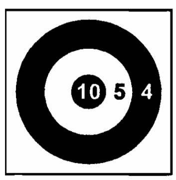
29. Compute:

$789 \div 9 =$

Correct Answers (any of the following):
87R6, 87.6, 87.7, $87\frac{2}{3}$, *or* $87\frac{6}{9}$

*Reporting Category/*Substrand for Item 29: ***Number Sense/*Whole Number Computation (p. 118)**

Use the target pictured below to answer question 30.



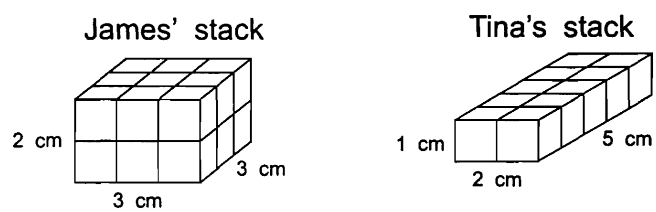
30. Teri tossed bean bags at this target. She scored 29 points in the FEWEST possible number of tosses. Write a number sentence that shows the sum of points Teri scored in each of these tosses.

Correct Answers (any of the following or their equivalents):
$10 + 10 + 5 + 4 = 29$
$10(2) + 5 + 4 = 29$
$10 + 10 + 5 + 4 = n = 29$

*Reporting Category/*Substrand for Item 30: ***Patterns, Relations, and Functions/*Algebra/Mathematical Structures (p. 119)**

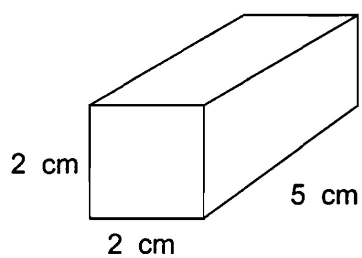
Session 2, Open-response Question

31. James and Tina made these stacks of centimeter cubes.



a. Whose stack has more centimeter cubes? Explain or show how you know you are right.

b. Another student named Amanda has 20 centimeter cubes in all. She has found the box shown below.

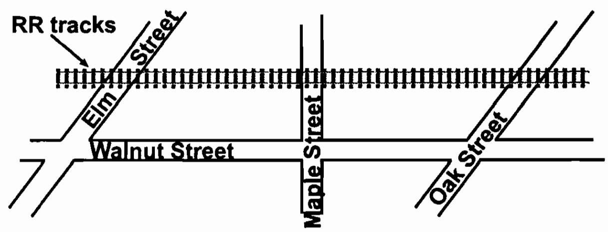


Will this box hold her 20 centimeter cubes? Explain why it WILL or WILL NOT hold 20 centimeter cubes.

*Reporting Category/*Substrand for Item 31: ***Geometry and Measurement/*Measurement (p. 120)**

Session 2, Multiple-choice Questions

Use the picture below to answer question 32.



32. Which street is parallel to the railroad tracks?

    A. Oak Street

    ✔ B. Walnut Street

    C. Maple Street

    D. Elm Street

*Reporting Category/*Substrand for Item 32: ***Geometry and Measurement/*****Geometry and Spatial Sense (p. 119)**

33. Shawn spent $8.69 to have his watch repaired and $2.15 on library fines. Which is the BEST ESTIMATE of the amount of money he spent?

    A. about $9

    B. about $10

    ✔ C. about $11

    D. about $12

*Reporting Category/*Substrand for Item 33: ***Number Sense/*****Estimation (p. 118)**

34. A coin is flipped six times. The results are heads, tails, heads, tails, tails, and tails. If the coin is tossed again, what is the chance that the result will be tails?

    A. 0 chance

    ✔ B. 1 out of 2

    C. 1 out of 3

    D. 2 out of 2

*Reporting Category/*Substrand for Item 34: ***Statistics and Probability/*****NA (p. 120)**

35. What number belongs in each shape to make both of the following number sentences true?

○ × ○ = 36
○ + □ = 13

✔ A. ○ = 6
□ = 7

B. ○ = 8
□ = 5

C. ○ = 7
□ = 6

D. ○ = 9
□ = 4

*Reporting Category/*Substrand for Item 35: ***Patterns, Relations, and Functions/*Algebra/Mathematical Structures (p. 119)**

36. Which numeral represents 1 ten, 7 ones, and 9 tenths?

A. 0.179

B. 1.79

✔ C. 17.9

D. 179.0

*Reporting Category/*Substrand for Item 36: ***Number Sense/*Fractions and Decimals (p. 118)**

37. How could Haley check this division problem?

$$\begin{array}{r} 329\text{ R2} \\ 8\overline{)2634} \end{array}$$

A. Multiply 2634 by 2; then add 8.

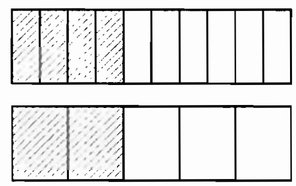
✔ B. Multiply 8 by 329; then add 2.

C. Divide 8 by 329; then add 2.

D. Divide 329 by 2; then add 8.

*Reporting Category/*Substrand for Item 37: ***Number Sense/*Concepts of Whole Number Operations (p. 118)**

Use the diagram below to answer question 38.



38. $\frac{4}{10} = \frac{\square}{\square}$

A. $\frac{1}{5}$

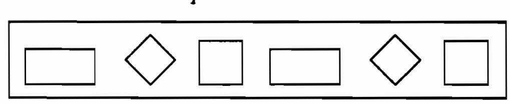
✔ B. $\frac{2}{5}$

C. $\frac{3}{5}$

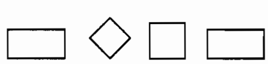
D. $\frac{4}{5}$

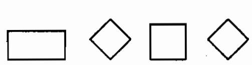
*Reporting Category/*Substrand for Item 38: ***Number Sense/*****Fractions and Decimals (p. 118)**

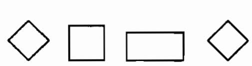
Use the pattern in the box below to answer question 39.

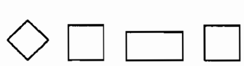


39. What are the next four figures in the pattern above?

✔ A. 

B. 

C. 

D. 

*Reporting Category/*Substrand for Item 39: ***Patterns, Relations, and Functions/*****Patterns and Relationships (p. 119)**

Appendix D

Budget Summary Request Form

Title: **Design, Implementation and Assessment of the Effectiveness of a 4th Grade Mathematics After-hours MCAS Program at Adams Street School**

1. Type of Project: IQP

2. Dvision: Education and Technology

3. Faculty Advisor's Initials

4. Project Registration No.

51-JAG-0001

5. STUDENTS	BOX	MAJOR	YEAR	TERMS AND UNITS
M.V.S. Chandrashekhar	275	ECE	2001	C,D,A '00 1/3 each

6. Total Cost of Project

$200.00-Gasoline and taxi fare for transport to and from the school.

7. Total amount of support requested:

$150.00

8. Total amount of support from all other sources:

$50.00 from self

9. Will any funds in addition to the above be required later in the project? No.

10. Brief statement of objectives and procedures.

Design, Implementation and Assessment of the Effectiveness of a 4th Grade Mathematics After-hours MCAS Program at Adams Street School in Worcester. A hands on, exam-oriented mode of instruction will be employed.

11. I have reviewed the project proposal and budget detail which are attached to this Budget Request for Approval.

Project Advisor's Signature

John A. Goulet

Professor John A. Goulet, Major Advisor