Design and Analysis of a Neck-Support-Incorporated Helmet for Reducing the Risk of Concussion in Ice Hockey

A Major Qualifying Project Proposal to be submitted to the faculty of Worcester Polytechnic Institute in partial fulfillment of the requirements for the Degree of Bachelor of Science

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Abstract

The goal of this project is to reduce the risk of concussion for ice hockey players by incorporating neck support into the current helmet design. This will provide an additional restoring moment during impact that will reduce the acceleration of the head. Computational analysis and mathematical modelling were used to distinguish which preliminary design features should be used in the prototype. The neck support will utilize smart fluids that exhibit shear thickening in response to force that will provide the restoring moment to reduce the acceleration of the head upon impact; while also allowing the player full range of motion when not experiencing an impact. The prototype and an unmodified helmet will be tested on a head-form equipped with accelerometers in an air cylinder impact test. Comparing the test results of the prototype to those of the unmodified helmet will determine if the neck support improves the helmet's ability to reduce the acceleration of the head upon impact. Additionally, head injury assessment functions were researched and the Head Impact Power (HIP) equation was chosen as a means of determining whether the prototype reduces risk of concussion.

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1. Introduction

An estimated 1.6 to 3.8 million sports-related concussions occur in the United States each year [1]. According to a medical journal review by Thurman and Guerrero, the most severe concussions have caused more than 50,000 deaths and another 70,000-90,000 permanent disabilities in a year [2] . Concussions can be debilitating and present physical, cognitive, emotional, and sleep related symptoms that can last months after the concussion occurred [1]. Permanent cognitive and memory deficits are among the devastating consequences of incurring repeated concussions [3]. All athletes involved in a contact sport are at risk for concussion [4]. The detrimental effects of concussions along with the high incidence of sport-related concussions have become public knowledge and a top concern of anyone involved with a contact sport. For this reason concussions are referred to as "a silent epidemic" [5].

According to a study published by the National Athletic Trainers Association, ice hockey has the highest incidence of concussions for males involved in contact sports [4]. This is due to the aggressive nature of the sport as well as the high speeds, up to 30 mph, ice hockey players are able to reach [6]. The force experienced by the player during an impact is directly related to the sudden change in the player's velocity and acceleration. When ice hockey players get shoved into the boards or into other players, they experience higher forces than most other athletes simply due to their higher initial speeds [6]. As concussions have become one of the top concerns of many people involved with contact sports, rules and regulations regarding hockey protective equipment have become stricter.

Most sports have specific safety equipment that athletes are required to wear to protect them from injury. Many contact sports require that all players wear a helmet that meets regulations set specifically for the intended sport. Unfortunately, the required helmets are mainly designed to prevent skull fracture and do not do much to prevent concussions. Many organizations have provided resources for discovering better ways to protect athletes [1]. The research conducted by these organizations has provided knowledge on ways to improve the identification and treatment of sport-related concussions. Resources were also contributed to developing better protective gear that would hopefully reduce the chance of concussion. Despite these efforts, the incidence of sport-related concussions is still alarmingly high and even growing in some demographics [1]. Understanding the biomechanics of a concussion helps explain why wearing a helmet has minimal effect on preventing concussions.

Although diagnosis of a concussion can be difficult, the definition of a concussion was established with consensus during the 4th International Conference on Concussion in Sport [1]. In short, the definition that was established stated that a concussion is a brain injury and is defined as a complex pathophysiological process affecting the brain, induced by biomechanical forces. Along with this definition, common heuristics of the nature of a concussive head injury were also agreed upon as useful guidelines for diagnosis. One of these guidelines explains that a concussion can be caused by a direct impact to the head, or an impact elsewhere on the body that has an impulsive force transmitted to the head [1]. Generally helmets are designed to prevent skull fracture and reduce direct focal external transfers of force, while having minimal, if any, effect on rotational accelerations [7]. Since rotational accelerations are the primary underlying mechanism of concussions, this explains why external padding secured on the head, like a helmet, has minimal effect on preventing concussions [7]. This raises the question, "Is there a better way to protect athletes from concussion than traditional safety gear?"

Recently a study was conducted to discover whether there is a correlation between neck strength and risk of concussion. During this study, athletic trainers working at high schools that participated in the National High School Sports-Related Injury Surveillance Study measured the neck strength of all students in school-sponsored soccer, basketball, or lacrosse using both a hand-held dynamometer and a hand-held tension scale. These athletes, distributed throughout 25 states, were monitored for concussion by tracking the athletic trainers' weekly submissions of exposure and injury data to the National High School Sports-Related Injury Surveillance Study online data collection tool. After two academic years, it was concluded that for every one pound increase in neck strength, odds of sustaining a concussion decreased by five percent [7]. This makes sense since the moment provided by a strong neck can minimize the effects of an impact by reducing the change in acceleration. A helmet incorporating neck support that simulates and enhances the restoring moment provided by a strong neck would be able to reduce the change in acceleration of the head during an impact. In theory, this type of helmet would decrease the potential for concussion.

2. Background

Before attempting to create a device that will reduce the chance of concussions, one must fully understand what constitutes a concussion as well as the criteria for diagnosing the severity of a concussion. Discovering the mechanisms in which concussions generally occur in hockey will provide essential information for developing protective head gear. In order to develop protective gear that can feasibly be worn by hockey players, it is important to identify the standards and regulations hockey equipment must meet. Exploring the hockey equipment currently available will provide baselines from which improvements can be made. Additionally, materials that could potentially be utilized in the design as well as testing mechanisms that could be used to evaluate the current and modified designs were also researched. This chapter provides the findings of the concussion, hockey, materials, and testing mechanisms research that was conducted.

2.1 Defining Concussions

Some medical experts define a concussion as an immediate loss of consciousness with a period of amnesia after a hit to the head [8, 9]. Other experts define a concussion as brain trauma which may result in cognitive, somatic, emotional and sleep disturbances, which can occur regardless of whether there was loss of consciousness [9]. Experts agree that all concussions can be described as temporary disruptions of brain function due to a direct or indirect impact (i.e. "whiplash") that results in an abrupt change in the acceleration of the head. Because symptoms of concussions can often be misinterpreted, some concussions go undiagnosed [10].

Even though neurologists and physicians cannot agree upon every post-concussion symptom, there are scales for determining the severity of a concussion. One of the scales commonly used is the post-traumatic amnesia (PTA) scale, which bases the severity of the traumatic brain injury (TBI) on the duration of the post-traumatic amnesia. The loss of consciousness (LOC) scale bases the severity of the concussion on the duration of the loss of consciousness. Although the predictive validity of these scales is well-established, each may be influenced by factors unrelated or indirectly related to the TBI [11]. Since the vast majority of concussions are not severe and occur without loss of consciousness or post-traumatic amnesia, TBI may be present even if the indicators previously used for the scales are not present. Since there is no brain scan or blood test to definitively diagnose a concussion, symptom-based scales are relied upon. Relying on a single indicator scale could lead to mild concussions going undiagnosed. Because of the shortcomings of single indicator scales, the Mayo clinic developed a classification system that distinguishes the clinical characteristics of the least and the most severe TBIs. The Mayo classification system uses multiple indicators to classify TBIs as: a moderate-severe TBI in which a TBI definitely occurred; a mild TBI in which a TBI probably occurred; or a Symptomatic TBI in which it is possible that a TBI occurred. The details of the Mayo TBI Severity Classification system are shown in Figure 2-1.

Table 1. Mayo TBI Severity Classification System

A. Classify as Moderate-Severe (Definite) TBI if one or more of the following criteria apply:
   1. Death due to this TBI
   2. Loss of consciousness of 30 minutes or more
   3. Post-traumatic anterograde amnesia of 24 hours or more
   4. Worst Glasgow Coma Scale full score in first 24 hours <13 (unless invalidataed upon review, e.g., attributable to intoxication, sedation, systemic shock)
   5. One or more of the following present:
      - Intracerebral hematoma
      - Subdural hematoma
      - Epidural hematoma
      - Cerebral contusion
      - Hemorrhagic contusion
      - Penetrating TBI (dura penetrated)
      - Subarachnoid hemorrhage
      - Brain Stem Injury

B. If none of Criteria A apply, classify as Mild (Probable) TBI if one or more of the following criteria apply:
   1. Loss of consciousness of momentary to less than 30 minutes
   2. Post-traumatic anterograde amnesia of momentary to less than 24 hours
   3. Depressed, basilar or linear skull fracture (dura intact)

C. If none of Criteria A or B apply, classify as Symptomatic (Possible) TBI if one or more of the following symptoms are present:
   - Blurred vision
   - Confusion (mental state changes)
   - Dazed
   - Dizziness
   - Focal neurologic symptoms
   - Headache
   - Nausea

Figure 2-1: Mayo TBI Severity Classification [11]

In order to determine the severity or grade of a concussion, neuropsychological testing needs to be done [12]. Recent modifications have been made in the evaluation of concussion severity to better assess the full range of concussion severities. Doctors manage each case individually and determine the presence and severity of a concussion based on multiple tests and scientific evidence [13-15]. The Academy of Sports Medicine and the American Academy of Neurology developed guidelines in order to diagnose and manage Sport-Related Concussions specifically, as shown in Table 2-1 [16, 17].

Table 2-1: Guidelines of Management in Sports-Related Concussion [13, 16]

MARK	**FIRST TIME CONCUSSION**	**SECOND TIME CONCUSSION**
Ranking 1: no loss of consciousness, brief period of confusion, mental symptoms for <15 min	Remove player from sport Examine the player for 5 min If in 15 minutes symptoms are not present, player may return to play	Allow player to play in 1 week timeframe if symptoms have subsided
Ranking 2: no loss of consciousness, brief period of confusion, sporadic mental symptoms for > 15 min	Remove player from sport for rest of day Examine symptoms of player and look for intracranial lesions Allow player to play within a 1 week timeframe	Allow player to play after 2 weeks if symptoms have subsided
Ranking 3: any sort of consciousness lost (place, date, etc.)	Neurological examination in hospital until post-concussive symptoms stabilize Allow player to play in a week if unconsciousness lasted seconds Allow player to play in 2 weeks if unconsciousness lasted 1-6 minutes	Do not allow player to play until all symptoms have been cleared and absent for 1 month

The American Academy of Pediatrics has developed measuring tools that determine sports-related concussions' severity and have concluded that a single test cannot suffice for the accurate determination of a concussion's severity. In the event of potentially severe head trauma, there are seven main assessment tools for diagnosing a concussion. Among the seven concussion assessment tools, four of them are especially relevant to hockey concussion injuries. The pros and cons of these four assessments are shown in Table 2-2.

Table 2-2: Pros and Cons of Concussion Assessment Tools

TOOL	DESCRIPTION	CONS	PROS
GSC (Glasgow Coma Scale):	Used onsite at time of concussion; ranks three levels of response: *(Eye opening)* Score: 1-5 *(Verbal Response)* Score: 1-5 *(Motor Response)* Score: 1-6 <u>Severity of injury classified as:</u> **Severe:** GCS 3-8 (no lower than 3) **Moderate:** GCS 9-12 **Mild:** GCS 13-15[18]	Might create confusion between concussed and non-concussed subjects (history of patient)	Fast (1-2 min); Can determine severity of a severe brain injury
HITS (Head Impact Telemetry System)	The first system to measure impact of players in real time. Used by live sensors which send information to a computer registering it in a 3-D graph of the head. Receptor computer can be located within 150 yards from player. The sensors are able to detect duration, magnitude, direction and location of up to 100 hits. Mainly designed for when a player experiences a hit of 10G's or higher. [19]	ONLY used in sports with helmets; Correlation of data with symptoms can be misleading	Live monitoring of impact; Detects and record all of the impacts that might cause concussion Good scale measuring system
SAC (Standardized Assessment of Concussion)	SAC is an onsite test that measures functions such as: *Orientation*: day, date, month, year, time *Immediate memory*: recall of five words in three separate trials *Neurologic*: Loss of consciousness (occurrence, duration), Strength, Amnesia (either retrograde or anterograde), Sensation, Coordination, Delayed Recall, Maneuvering and Concentration Each is attributed a value as found in **Error! Reference source not found.**, this is given a score out of 30, the higher the score, the more severe concussion [20, 21]	Correlation of data with symptoms can be misleading; Useless if conducted more than 48 hours after time of injury Cannot assess cerebral function	Measures orientation, memory, focus; Intuitive operating system; Short (5-7 min)
SCAT2 (Sport Concussion Assessment Tool)	Mainly focuses on testing cognitive skills affected by concussion. Does not determine concussion degree or athlete's recovery or return to play status. [22]	Long (15-20 min); Requires a professional to conduct; No score or scale; Not very reliable due to weight of symptoms	Testing of cognitive skills affected by concussion

With the improved categorization of concussions, doctors are better able to prescribe appropriate rehabilitation regimens. Follow-up assessments during the athlete's rehabilitation must be conducted to accurately determine when a player can safely participate in his or her sport again after sustaining a concussion. There are eight main follow-up assessments given at different intervals to track the patient's recovery [19]. Four of the follow-up assessments also stand out as particularly relevant to hockey concussion injuries. The pros and cons of these assessments are shown in Table 2-3.

Table 2-3: Pros and Cons of Follow-Up Concussion Assessment [13]

TOOL	DESCRIPTION	CONS	PROS
ImPACT (Immediate Post-Concussion Assessment Cognitive Test)	Conducted using software when an athlete no longer has symptoms (24-72 hours post-injury) *Measures*: player symptoms, verbal and visual memory, processing speed, and reaction time Gives a summary of measurements; can determine if player should return to play [23]	Long (20 min); Positive and negative rate can be false; No scale to determine recovery	Able to diagnose multiple areas of neurocognitive function; Correlated MRI tests; No professional needed
DTI or Diffusion MRI (Diffusion tension imaging)	Provides mapping on how molecules have spread out in biological tissue after a concussion. This mainly sees water molecule diffusion in the brain segment and it is an in-vivo, non-invasive testing mechanism. It can show molecular interaction with other macromolecules, with fibrous tissue, with membranes among others [24]	Cost; Long time to complete; No complete diagnosis	Can determine if white brain matter is affected; Great image; No invasion of any kind

fMRI (Functional magnetic resonance imaging)	Uses MRI technology to measure brain action by indicating changes in blood flow patterns; relies on neuronal activation coupling. It mainly detects and uses blood-oxygen-level dependent (BOLD) to compare results. It specializes in detecting brain activity and interaction with spinal cord due to change in blood flow. It provides high resolution images where notable change on circulation can be shown if area is affected [25]	Cost; Long time to complete; Can affect blood vessel activation	Can detect constant abnormalities in brain function; Often used as clinical validation tool to assess brain functionality; No invasion of any kind
MRS (Magnetic resonance spectroscopy)	Technique to measure metabolic variations of brain strokes, tumors, disorders, Alzheimer's, depressions and concussions affecting the brain functionality. It is used to measure intramyocellular lipid content (IMCL). It uses MRI technology which is able to send signals based on H+ (hydrogen protons) in order to get dimensions of the brain in x, y, z coordinates and determine the concentrations of molecules in certain areas[26]	Cost; Long time to complete; Limitation in diagnosis	Ability to measure brain metabolism; Delivers information on brain function recovery time; No invasion of any kind

2.2 Injuries in hockey

Athletes playing contact sports, such as hockey, are at risk for sustaining a concussion. Multiple organizations have done studies to understand the frequency and cause of concussions. Wilcox et al. performed a study on occurrences of concussions in contact sports. The study evaluated eight sports and compiled data on typical injuries. They looked at all concussions, excluding concussions due to whiplash injury, spinal cord injury, facial bone fractures, or soft tissue injuries. This study found that hockey had the greatest incidence of concussions for males, and tae kwon do has the greatest incidence rate of concussions for females [6]. According to the 2008-2010 NCAA Men's and Women's Ice Hockey Rules and Interpretations, body checking is allowed in men's ice hockey, but is not allowed in women's ice hockey. Lack of checking may contribute to tae kwon do having the greatest incidence rate of concussions in female sports.

Hockey is different than other contact sports because players move at higher rates of speed on a playing area of solid ice [27]. Hockey players can skate at speeds of up to 30 mph and can slide at maximum rates of 15 mph. Contacting physical obstacles at such high speeds results in abrupt deceleration causing the player to experience higher impact forces. A study by Denny-Brown and Russell, regarding the acceleration and deceleration of the players' body and specifically their head, determined that in order for a concussion to occur, acceleration and deceleration must be present [28].

A study was performed on men's and women's National Collegiate Athletic Association Division I ice hockey teams to analyze the magnitude and frequency of head impacts during games. This study determined the distribution of the mechanisms of impact and concluded that for both men's and women's collegiate ice hockey, the most frequent impact mechanism was contact with another player. The impact mechanism that generated the greatest-magnitude head accelerations was contact with the ice though the frequency of this type of impact was low [6]. The distribution of impact mechanisms is shown in Table 2-4.

Table 2-4: Impact Mechanisms in Collegiate Ice Hockey [6]

	Head Impact % of total impacts, (Number of Impacts of that Type)		Head-Impact Frequency per Game By Impact Mechanism	
Impact Mechanism	**Men (n=270)**	**Women (n=242)**	**Men**	**Women**
Contact with another player	50.4 % (136)	50 % (121)	0.464	0.208
Contact with ice	7 % (19)	11.2 % (27)	0.104	0.106
Contact with boards/glass	31.1 % (84)	17.3 % (42)	0.349	0.095
Contact with stick	1.9 % (5)	2.9 % (7)	Not Provided, because incidence rate was insignificant	
Contact with goal	0.4 % (1)	0 % (0)		
Contact with puck	0.4 % (1)	0.8 % (2)		
Indirect Contact	4.4 % (12)	15.3 % (37)	0.087	0.1
Celebrating	4.4 % (12)	2.5 % (6)	0.08	0.073

The peak linear and rotational accelerations generated by the impact mechanisms are shown in Table 2-5.

Table 2-5: Resultant Peak Linear and Rotational Acceleration of Head Impacts Greater than 20g Sustained by Collegiate Ice Hockey Players for Each Injury Mechanism (95% Confidence Interval)

Mechanism	Linear Acceleration (g)	Rotational Acceleration (rad/[s.sup.2])
Men		
Contact with another player	28.0 (26.4, 29.7)	2901.8 (2514.5, 3348.7)
Contact with ice	40.1 (31.8, 50.5)	3454.9 (2590.2, 4608.4)
Contact with boards	32.1 (29.7, 34.7)	3350.4 (2995.9, 3746.8)
Indirect contact	31.5 (26.4, 37.8)	2873.8 (1949.8, 4235.7)
Celebrating	25.9 (23.6, 28.4)	2056.3 (1707.9, 2475.7)
Women		
Contact with another player	27.9 (26.3, 29.6)	2323.0 (2031.6, 2656.9)
Contact with ice	35.2 (30.9, 40.0)	2318.9 (1644.2, 3270.4)
Contact with boards	26.8 (25.8, 27.9)	1859.5 (1587.0, 2178.8)
Indirect contact	29.5 (25.6, 34.0)	1861.3 (1387.1, 2497.6)
Celebrating	23.3 (20.1, 27.0)	923.3 (675.2, 1262.5)

Source: Head Impact Mechanisms In Collegiate Ice Hockey[29]

A seven-year study was performed by the Canadian Medical Association Journal (CMAJ) to research and provide statistics regarding concussions in the National Hockey League (NHL).

The CMAJ worked with the NHL to determine two major variables in hockey: concussion and time loss. The goals of this study were to determine the rates and trends of concussions as well as the post-concussion signs, symptoms, physical examination findings and time between the injury and return to play. This evaluation was performed between the 1997-1998 season and the 2003-2004 season. Results showed 559 physician-diagnosed concussions throughout the seven seasons with an average of 80 per year. The game rate recorded 5.8 concussions per 100 players per season and overall, an average of 1.8 concussions per 1000 game player-hours. Of these 559 concussions, physician regulated recovery time averaged about six days per concussion. Of the instances, 69% missed ten or less days of unrestricted play and 31% missed more than ten days [30-33]. Statistics regarding positions of players experiencing concussions were highlighted and are displayed in Table 2-6.

Table 2-6: Percent of Concussions for Each Position

POSITION	PLAYERS ON THE ICE AT ONCE	% OF RECORDED CONCUSSIONS
CENTERMEN	1	30.5%
DEFENSEMEN	2	31.4%
WINGERS	2	33.6%
GOALIES	1	4.5%

From the data shown in Table 2-6, centermen, defensemen and wingers recorded approximately the same percent of concussions. By factoring in the amount of players on the ice at one time, researchers found that centermen experienced concussions twice as much as defensemen and wingers.

Detailed data was presented indicating common post-concussion symptoms. The percent occurrence of headaches, dizziness, nausea, neck pain, low energy or fatigue, blurred vision, amnesia, and loss of consciousness were all post-concussion symptoms. The distribution of these statistics is shown in Table 2-7.

Table 2-7: Occurrence of Post-Concussion Symptoms

SYMPTOM	% OCCURRENCE
Headache	71 %
Dizziness	34 %
Nausea	24 %
Neck Pain	23 %
Low energy or fatigue	22 %
Blurred vision	22 %
Amnesia	21 %
Loss of consciousness	18 %

Of the 559 concussions occurring during the seven-year period, 13 % of post-concussion neurologic examinations were abnormal [33].

Many athletes in contact sports experience multiple concussions throughout their participation, which raises additional concerns. Research showed that football players who had endured multiple concussions were at an increased risk and earlier onset of memory impairment, including mild cognitive impairment, and Alzheimer's dementia. There was also a news release in 2009 about a case of chronic traumatic encephalopathy in a former NHL player. The news release encouraged researchers to study concussions further in order to better protect athletes in potentially harmful situations [33-35].

Le Bihan et al. recently performed a study that evaluated the incidence rates of concussion in junior hockey in comparison to the previously mentioned study of the NHL [35]. Neurosurgical Focus evaluated two teams of junior ice hockey players during one regular season. Junior ice hockey players range in age from approximately 16-21 years old. Overall, this study was not able to observe all 36 regular season games, but the procedure for collecting data used six licensed physicians, and 16 non-physician observers, such as kinesiologists, certified ice hockey coaches, physical therapists, massage therapists, chiropractors and former junior ice hockey players. The overall results of this study were 21 concussions observed in 52 games. This rate can be quantified as 21.52 concussions per 1000 athlete exposures [35]. This study shows that not only are concussions a problem in the NHL, but they are a problem early on with teenagers in junior ice hockey.

Hutchison et al. held a study from 1998-2000 with players of ages 15 to 20 in Canadian Amateur Hockey leagues to find the rate of concussions occurring in hockey. This study used 272 participants in its first year of study and 283 in the second year of study; of these participants, 115 participated in both year one and year two. Results of this study showed that over this two-year period, 379 concussions were reported. Of the 379 reported, 90% of them occurred during a game, 7.9% occurred during practice, and 2.1% occurred at other times [32]. High rate of concussion is clearly a concern in all levels of ice hockey.

Many experts agree that ice hockey is a dangerous sport and that players are susceptible to concussions during play. Concussions in hockey affect not only the player injured but also the entire team who must play without key players. Since concussions commonly cause detrimental lasting effects, sustaining multiple concussions could cut a player's career short. Preventing concussions will enhance the sport by allowing good players to participate for longer, making team dynamics less erratic.

Monitoring athletes during play has been a topic for discussion in concussion detection. Multiple products are available and patented that will sense if conditions have occurred that could potentially cause a concussion. For instance, a sensor pad was created for use in football helmets. This sensor analyzes impacts that players have encountered and quantifies the data for observers. This specific sensor pad lines helmets with a five-point sensor pad made by polymer film. It uses a CC2530 series system-on-chip transceiver and microcontroller made by Texas Instruments. The sensor uses a wireless RF communicator. Data is stored in an onboard memory unit capable of recording 40 alerts. After the data is quantified, if a predefined threshold is exceeded, a wireless receiver is triggered and indicates that a potentially harmful impact has occurred [36]. This sensor is currently being used by 19 college football teams and is working its way into youth and high school leagues.

Multiple patents have also been filed on the topic of helmets that incorporate concussion indicators and force detection devices. In 1995, a patent was filed called “Sports helmet capable of sensing linear and rotational forces.” This design was specifically created to detect not only impact on the body, but also to observe both linear and rotational impacts. Accelerometers are present in this design and sense three orthogonally oriented linear forces. When the device senses an impact exceeding the limits previously specified, an electrical signal is sent to a lamp or LED on the sidelines indicating that a potentially harmful impact has occurred [37].

A patent titled “Concussion Indicator” was filed in 2013 to monitor the acceleration in a helmet. The sensor can be applied to either the inner or outer portion of the helmet depending on the athlete’s preference. When the sensor is mounted to the outside of the helmet, indicators can be shown to observers. If the sensor is mounted on the inside of the helmet, the player must remove the helmet before visualizing the indicator. One of the unique qualities of this design is that different indicators signify different degrees of concussions that could have occurred. By visualizing the indicator, observers can identify the intensity of a potential concussion [38].

Research shows that sensors currently used are designed to monitor accelerations and calculate the force experienced by athletes. The sensors indicate whether maximum thresholds have been reached and if there was a chance that a concussion occurred. The main objective of sensors currently on the market is to sense whether or not a concussion has occurred. No research was found on how sensors can be used to prevent concussions from occurring.

2.3 Head Injury Criterion

There are many ways to analyze risk of injuries to the head. One common and versatile method is the Head Injury Criterion (HIC). The HIC is an equation based on the head's acceleration and time over which the acceleration occurs. The result from the equation is an integer that can help determine the likelihood or severity of a head injury. The equation for the HIC is as follows:

Equation 1: $$HIC = \left(\frac{1}{t_2 - t_1} * \int_{t_1}^{t_2} \hat{a}(t) * dt\right)^{2.5} * (t_2 - t_1)$$

Equation 2: $$\hat{a} = a/g$$

Where $\hat{a}$ is the unit-less, normalized acceleration of the head with respect to gravity, g (9.8 m/s^2), and t is time measured in seconds. HIC therefore has units of seconds [39]. Many studies have been completed trying to find at what HIC head injuries will occur. The head injuries of concern are usually surface contusions and concussions [40]. Shear stress concentration and motion of the brain within the skull are known causes of these injuries and directly related to head acceleration with respect to a period of time. This is why the HIC is such a useful tool in quantifying the chance of a head injury and its severity.

An HIC of 200 seconds is commonly considered the threshold at which a concussion may occur [41]. When testing protective equipment (i.e. helmets, seat belts, etc.), HIC values below 200 seconds must be achieved consistently before the design is considered safe to be on the market. However, since each situation and person is different, the HIC does not provide definitive proof of concussion, but rather it provides an indication of the probability that a concussion occurred. There are incidences in which the HIC is under 200 seconds but a head injury did occur, as well as incidences in which the HIC is over 200 seconds without a head injury occurring [41, 42]. An HIC of around 240 seconds indicates a 50 % probability of concussion and an HIC around 485 seconds corresponds to 95 % probability [43]. The HIC is frequently used as a standard when testing equipment, since it has been shown to fairly accurately predict how well safety equipment will reduce the risk of concussion.

2.4 Head Impact Power

Another method to analyze the risk of injury to the head is with the Head Impact Power (HIP) equation. While the HIC takes into account only the resultant linear acceleration of the head at the center of gravity, the HIP uses both the linear and angular accelerations of the head at the center of gravity [43]. This yields a more accurate prediction than the HIC at the cost of using a more complicated equation. The equation can be seen below:

Equation 3: $$HIP = m_h a_x(t) \int a_x(t)dt + m_h a_y(t) \int a_y(t)dt + m_h a_z(t) \int a_z(t)dt + I_x \alpha_x(t) \int \alpha_x(t)dt + I_y \alpha_y(t) \int \alpha_y(t)dt + I_z \alpha_z(t) \int \alpha_z(t)dt$$

Where m_h is the mass of the head and I_i are the moments of inertia of the human head about the corresponding axis. The $a_x(t)$, $a_y(t)$, and $a_z(t)$, are the linear acceleration components and $\alpha_x(t)$, $\alpha_y(t)$, and $\alpha_z(t)$ are the angular acceleration components all as functions of time. Since the HIP is a time-dependent function, the maximum value obtained is used as the HIP value [42].When Newman et. al. developed the HIP; its ability to predict concussion risk was compared to other head injury assessment functions, including *Maximum linear acceleration, Maximum linear acceleration with dwell times, the Severity Index (SI), the Head Injury Criterion (HIC), and Angular and Linear acceleration GAMBIT equation.* The SI, is the current NOCSAE

(National Operating Committee on Standards for Athletic Equipment) standard and incorporates average acceleration with time duration, with a limiting value of 1200. The GAMBIT (Generalized Acceleration Model for Brain Injury) uses angular and linear acceleration in the equation $G_{\max(t)} = \sqrt{\left(\frac{a_{res}(t)}{250}\right)^2 + \left(\frac{\propto_{res}(t)}{25000}\right)^2}$, where $a_{res}(t)$ and $\propto_{res}(t)$ are the instantaneous translational and rotational acceleration respectively. Utilizing game video and the associated medical records of twelve NFL head to head impacts, Newman et al. was able to create full-scale laboratory reconstruction of the incidences with helmeted Hybrid III dummies. Each dummy was equipped with nine linear accelerometers placed strategically around the head. For each reconstructed incidence, all six head injury assessment functions were calculated for each player involved in the impact. The results of the calculations as well as whether a MTBI was reported for each case is shown in Table 2-8.

Table 2-8: Head Injury Assessment Function Results for Each Player Involved in the Impact [43]

Case No. 1 = tackler 2 = tackled	Reported MTBI 0=no 1=yes	A_{max} (m/s²)	a_{max} (rad/s²)	SI	HIC	GAMBIT	HIP (kW)
07-2	0	596	6265	121	93	0.35	6.7
38-2	1	1162	9678	743	554	0.60	23.3
39-2	1	1263	5729	663	521	0.55	19.8
48-2	0	562	5855	157	130	0.32	9.7
57-2	1	758	5786	255	207	0.38	12.1
59-2	0	807	5035	207	138	0.38	8.0
69-2	1	595	4168	181	130	0.25	9.0
71-2	1	1211	5434	655	510	0.52	24.0
77-2	1	788	5128	272	185	0.37	13.2
84-2	1	804	9244	317	225	0.49	17.6
92-2	1	1054	8877	706	508	0.48	21.6
98-2	1	893	7548	366	301	0.46	18.3
07-1	0	489	2832	65	51	0.23	3.4
38-1	0	588	5205	158	127	0.32	6.6
39-1	0	431	4184	61	43	0.18	3.3
48-1	0	310	2817	45	37	0.17	2.6
57-1	0	317	3937	51	37	0.20	4.0
59-1	0	314	1950	32	28	0.14	1.8
69-1	0	371	2593	83	50	0.17	3.6
71-1	1	1005	5555	519	433	0.45	19.3
77-1	0	342	2563	68	53	0.17	4.4
84-1	0	442	3036	98	77	0.22	4.6
92-1	0	586	6070	218	164	0.33	8.3
98-1	0	827	4487	245	187	0.38	10.4

Source: A New Biomechanical Assessment of Mild Traumatic Brain Injury Part 2 – Results and Conclusions [45]

Based on the 24 cases in Table 2-8, univariate logistic regressions were performed for each head injury assessment function. The concussion probability curves that were generated permitted the determination of the specific values of each head injury assessment function that corresponded to significant concussion probabilities. From the probability curve for the HIP, a value of 12.79 kW corresponded to a 50% chance of concussion and an HIP of 20.88 kW corresponded with a 95% chance that a concussion occurred [43]. These values are only preliminary and require additional testing.

Logistic regression analysis revealed which of the head injury assessment functions were most reliable. In regression analysis, the significance (p-value) is often used to determine if an independent variable should be included in the model. According to Newman et al. $p \leq .25$ is used as the threshold for the inclusion of an independent variable; the lower the p-value the higher the significance of an independent variable. Similarly, the -2 Log Likelihood Ratio (-2LLR) indicates whether adding the independent variable to the constant has improved the model. A zero value of the -2LLR indicates an exact fit of the regression model to the data, ergo a smaller -2LLR value indicates a higher significance. Newman et al. compared the p-value and -2LLR values of each head injury assessment function, shown in Table 2-9, to distinguish the best concussion predictor.

Table 2-9: Results from Logistic Regression Analysis [43]

	A_{max}	α_{max}	SI	HIC_{15}	GAMBIT	HIP
Significance P-value	0.011	0.029	0.024	0.020	0.013	0.008
-2LLR	18.059	20.676	18.195	19.347	18.031	14.826

Source: A New Biomechanical Assessment of Mild Traumatic Brain Injury Part 2 – Results and Conclusions [45]

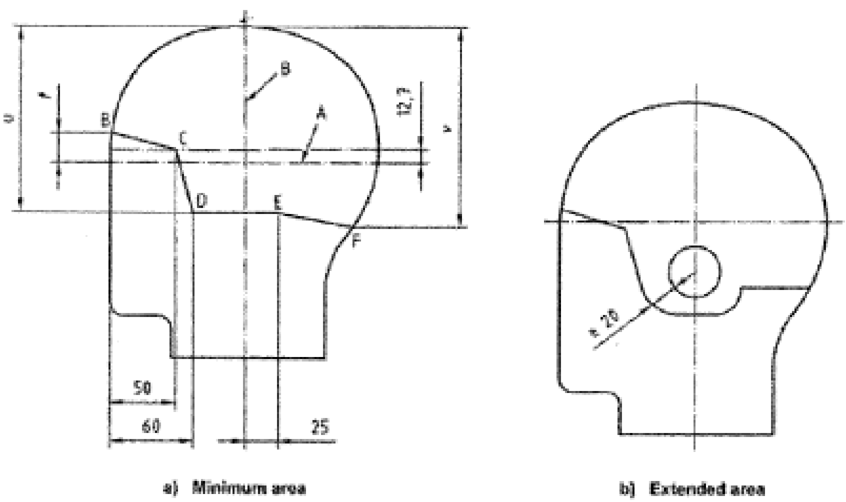
The HIP equation proved to be the most significant variable, signifying it is the most reliable predictor of concussion out of the head injury assessment functions utilized in this study. A more recent study by Marjoux et. al. concluded that an HIP of 24 kW and of 30 kW corresponded to 50% and 95% risk of concussion, respectively.

2.5 Helmet Standards

With hockey being a high contact sport, protective equipment and contact rules are a necessity to reduce the number of injuries. The importance of regulated hockey equipment ensures that each issued item of protective equipment offers a baseline of protection. Hockey equipment is regulated by the Hockey Equipment Certification Council (HECC), a non-profit organization. All of USA Hockey, NCAA, and the National Federation of State High School

Association (NFHS) must wear gear that is HECC approved [44]. The HECC uses the assessment standards set forth by the American Society for Testing and Materials International (ASTM), which is the standard in America.

The ASTM F1045 standard states proper testing methods as well as the minimum requirements. The standard also defines the proper specifications for head, which are also found in the ASTM F2220 standard. Figure 2-2 shows the minimum helmet coverage requirements for proper fitting based on the circumference of the head and helmet. It is very important that the helmet fits and is tested properly, which is described in the standard.



Headform code letter	Size: inside circumference of helmet mm	Distance mm		
		t	u	v
A	500	24	123	132
E	540	25	128	140
J	570	27	130	145
M	600	28	132	151

NOTE 1—A-Reference plane, B-Coronal plane

FIG. 5 Type 1–Area of Coverage

Figure 2-2: F2220 Specifications for Head forms, Area of Coverage [16]

The Testing Methods include impact and drop testing and a shock absorption test. The impact requirement states that the helmet must remain intact, meaning that it must have no visible cracks in the helmet while withstanding impact accelerations up to 300 g's [44]. The chinstrap also needs to up hold standards. It has to have a separation force from the helmet from between 50 and 500 N. Also, while exerting 109 N the chinstrap must not exceed one inch of displacement [45]. Each of these tests must be executed using ambient, hot, and cold temperatures to ensure that the helmet can withstand all forces during game play. After proper certification that the helmet meets all requirements by the ASTM F1045 standard, the HECC will then place their label of approval on the helmet. This label is not to be altered or taken off , or the equipment certification becomes void [44].

A study by Robert Edward Wall, attempted to answer the question of what standard to use in the National Hockey League (NHL) due to it being an international league. The three standards that were compared were the ASTM, Canadian Standards Association (CSA) and International Organization Standards (ISO). This study showed that not one single standard would shine over the others. In fact, each standard had an area where it performed better than another, making it a difficult comparison. Most importantly, it was concluded that the helmets tested performed relatively the same when based on peak acceleration measurements, but there were differences during multiple impacts. Wall suggests the possibility of combining the standards to create one single standard that can be accepted worldwide [46].

2.6 In Play Regulations

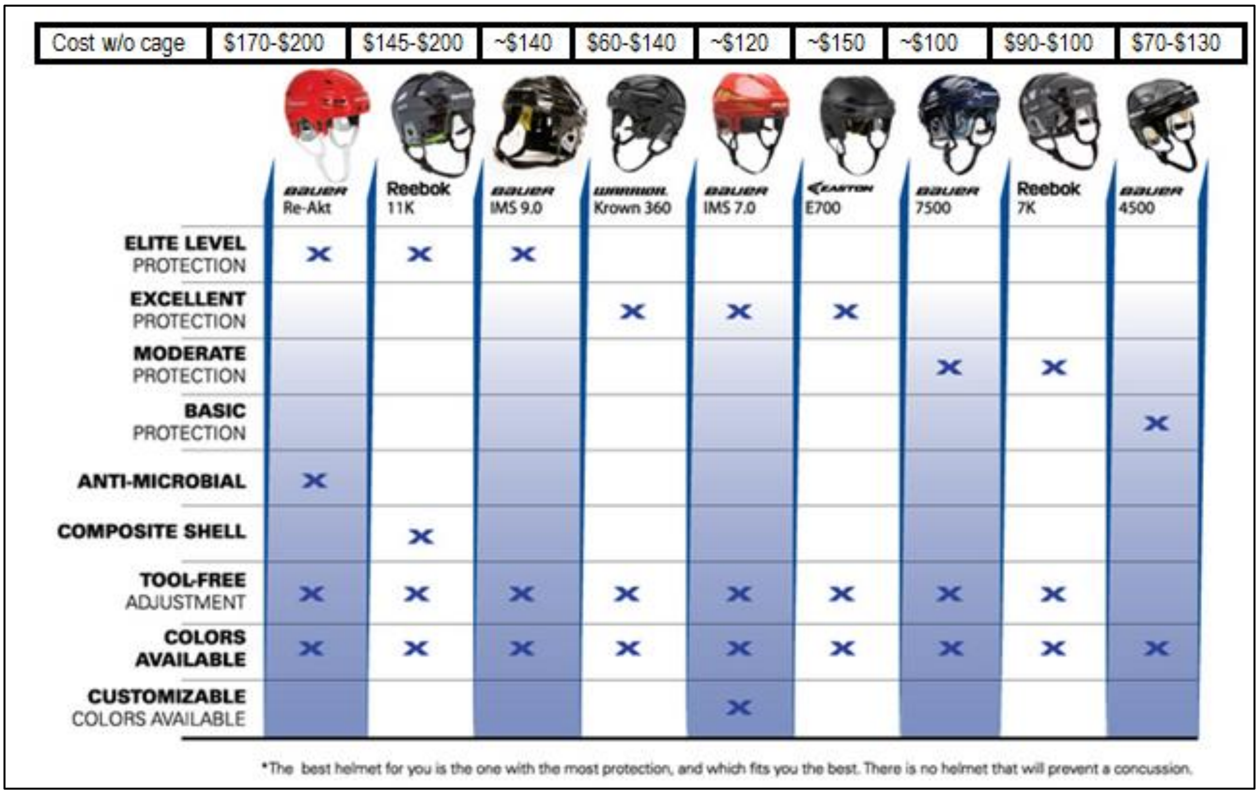
Regulations during play are also set in place to aid in reducing the number of severe injuries. There are many different sets of rules based on age and location. The lower the age, the more regulations developed for play and more equipment requirements. The NHL offers the least amount of regulated protection for its players due to it being played by the most advanced athletes. In the NHL, hitting or checking from behind or contacting a player's head during a hit or check results in penalties or possible ejection from the game. At the NHL level of play, a helmet is the only headgear required [47]. Since the NHL is essentially an international league, it has not adopted one set of standards for its protective headgear; generally ASTM or CSA certified equipment is used.

The collegiate level is regulated by the National Collegiate Athletic Association (NCAA). Players must wear HECC approved helmet and face mask that is securely fastened. The NCAA also requires the use of a mouth guard. Penalties can arise if a player is checked from behind, charged, boarded, or undergoes contact to the head as mentioned in the NCAA 2014 rulebook. These regulations were put in place to help reduce injury and frequency of concussions.

Despite the implementation of rules and regulations, the occurrence of concussion is still higher than one would hope. Only further implementing rules, regulations and more advanced protective equipment can promote a reduction in the rate and severity of concussions that may arise from playing hockey.

2.7 Current Protective Equipment in Ice Hockey

Modern hockey helmets can be classified by level of protection. There are helmets specifically designed for beginners, for professional players, and for many levels in between [48]. The equipment guide on PureHockey.com, shown in Figure 2-3, classifies the *Reebok 11k* helmet, the *Bauer Re-Akt* Helmet, and the *Bauer IMS 9.0* Helmet as offering "Elite Level Protection", which is the highest level of protection. Reebok achieved the elite level protection of the 11k helmet by designing it with "a better fit equals more safety" in mind [49]. While most modern hockey helmets offer length-wise adjustment, and some advanced helmets offer length-and width-wise adjustment, the 11k helmet provides the only 360 degree adjustment available [49]. The 11k helmet accomplishes the 360 degree fit by utilizing Reebok's Microdial II Anchoring system, which wraps the Expanded Polypropylene (EPP) foam, foam commonly used for impact absorption in helmets, around the unique shapes of the player's head and locks the helmet into place [49]. This system eliminates gaps and pressure points to provide a more protective and comfortable fit. The composite subshell of this helmet makes it Reebok's lightest fully adjustable helmet.



Cost w/o cage	$170-$200	$145-$200	~$140	$60-$140	~$120	~$150	~$100	$90-$100	$70-$130
	Bauer Re-Akt	Reebok 11K	Bauer IMS 9.0	Warrior Krown 360	Bauer IMS 7.0	Easton E700	Bauer 7500	Reebok 7K	Bauer 4500
ELITE LEVEL PROTECTION	x	x	x						
EXCELLENT PROTECTION				x	x	x			
MODERATE PROTECTION							x	x	
BASIC PROTECTION									x
ANTI-MICROBIAL	x								
COMPOSITE SHELL		x							
TOOL-FREE ADJUSTMENT	x	x	x	x	x	x	x	x	
COLORS AVAILABLE	x	x	x	x	x	x	x	x	x
CUSTOMIZABLE COLORS AVAILABLE					x				

*The best helmet for you is the one with the most protection, and which fits you the best. There is no helmet that will prevent a concussion.

Figure 2-3: Helmet Comparison

Source: Adapted from Pure Hockey Webpage[50]

The Bauer helmets have many features that contribute to their classification as "Elite protection." Both Bauer models utilize Vertex foam and Poron XRD liners for impact management, as well as dual-density ear covers with clear protective film to eliminate abrasion

[51]. The Vertex foam has the same density as EPP foam but is lighter and provides improved, high- and low- energy impact protection [51]. The Poron XRD foam is made up of urethane molecules that are flexible until placed under high pressure at which time the molecules momentarily stiffen [52]. It has been shown to absorb 90% of the energy of a high-force impact. Poron XRD is also very lightweight and breathable. The Vertex foam is used on areas of the helmet proven to experience less impact, while Poron XRD is placed in the areas where the majority of impacts occur [51]. The Bauer helmets also feature memory foam temple pads that provide maximum comfort and a snug fit. Bauer products also employ MICROBAN, which offers antimicrobial protection to resist odors and mildew.

The Bauer Re-Akt helmet is marketed as the first hockey helmet to offer protection against all types of hits [53]. Whereas all certified hockey helmets are required to protect against high-energy linear impacts, the Bauer Re-Akt also protects against low-energy linear impacts, and rotational impacts. Rotational impacts have been shown to cause serious head injuries [54]. The Bauer Re-Akt helmet achieves this optimal protection by utilizing Bauer's SUSPEND-TECH liner system. Upon impact the SUSPEND-TECH liner remains with the head, ensuring the placement of pads is maintained, while the shell with its interior liner rotate to absorb and deflect the forces of the impact [53]. This system is advertised as being able to minimize the movement of the head during impacts, which would greatly reduce the likelihood of a concussion [54].

The current padding inside most hockey helmets is an expanded polypropylene and vinyl nitrile. These two paddings have shown to have very similar effects on the risk of injury as concluded in a study on the effects of impact management materials in ice hockey helmets on head injury criteria [4]. All three of the models mentioned above offer tool-free adjustment to make fitting the helmets to the player's head quick and easy [50]. Many experts agree that the proper fitting of the helmet and cage is as important for protection as the helmet's design [50]. Regardless of the impact absorbance technology or stability features incorporated in a helmet, if the helmet does not fit properly, it will not protect a player's head sufficiently [50].

Despite all the features and protective measures, hockey helmets still seem underwhelming compared to the top rated football helmets. When comparing the interior of a hockey helmet to the interior of a football helmet, as seen in Figure 2-4, it is apparent how much more cushioning is available in the football helmet [55]. Considering hockey is right after football as the sport responsible for the most concussions, the lack of padding in hockey helmets compared to football helmets is puzzling. Perhaps the huge difference in helmet interiors is due to hockey having different conditions and mechanisms in which concussions occur than those in football.

Another possibility could be that football manufacturers have been improving their designs in response to Virginia Techs five-point STAR (Summation of Tests for the Analysis of Risk) rating system that was first implemented in 2011. Virginia Tech tests football helmets and awards the helmet one to five stars depending on its ability to reduce the risk of head injury and concussion. The head of the biomedical engineering department at Virginia Tech, Dr. Duma, led meetings with scientists and football helmet manufacturers to discuss improving head protection and providing the science behind the methodology of the STAR rating system. The STAR rating system makes consumers aware of which football helmets reduce the risk of concussion the most, motivating the manufacturers to strive for the five-star mark, the highest rating awarded by the Virginia Tech helmet ratings. Each year more of the newly released football helmets are achieving the five-star rating. In the past two years, Virginia Tech has begun lab and rink testing and analysis to develop an analogous STAR rating system for hockey helmets. The hope is that this rating system will have a similar impact on hockey helmets by motivating and informing hockey helmet manufacturers on improving the protective ability of their hockey helmets [55-57].

Hockey vs. Football: Inside a current hockey helmet (left), alongside a top-rated football helmet. (Jake Smith of Virginia Tech)

Figure 2-4: Comparison of Hockey Vs Football Helmet

Source: NCHL [58]

In addition to helmets, face protection is an important factor in preventing serious injuries considering pucks can travel up to 100 miles per hour. Rules requiring face protection vary from league to league. All face protectors connect to the player’s helmet and fall into one of three categories. The most common facial protection for amateur players is the full cage, which consists of metal bars running vertically and horizontally across the player’s face [59]. The full cage offers full protective coverage, great ventilation, and the most durability [59]. The cage is very affordable and requires little to no maintenance [59]. However, some players feel that the wire cage is distracting while playing [59].

The second option for facial protection is the full shield, which consists of an impact-resistant plastic covering the eyes and mouth with breathable holes at the bottom of the mask

[59]. The full shield offers the same amount of protection as the cage without the distraction of wires running through the player's line of sight. The downside of full shields is that more maintenance is required than with the cage [59]. Usually, anti-fog solution must be applied to the surface of the shield before each game to limit the amount of fog that occurs during play [59]. Most shields come with an anti-scratch coating that must also be applied to the mask to improve its durability [59]. Even with proper maintenance, typically the full shield still will not last as long as a cage would [59].

The last option is called a visor or a half shield and is for hockey players over the age of 18 years old that are in a league that does not require full facial protection [59]. Half shields are made of high impact-resistant, transparent plastic that covers the top half of the face stopping at the bottom of the nose [60]. The half shield provides the least inhibited vision, with its transparent plastic offering excellent straight ahead and peripheral vision [60]. The half shield does not tend to fog up as much as the full shield but, still experiences some fog issues [59]. The half shield is more flexible than the full shield, making it slightly less durable [59]. This option provides the least protection because it leaves the mouth, jaw, chin, and the bottom of the nose vulnerable to injury [59].

Innovative fog-free technology has been developed and implemented in hockey face shields. Avision Ahead Bould Hockey Shield Company claims their newest Elite Masks are fog-free and scratch resistant on both sides of the shield; they even offer a money back guarantee. Avision Ahead worked with hockey players from the University of Denver to create their new elite masks [61]. The elite masks passed impact testing with results 115% greater than what is required for certification [61]. The features of the elite mask were accomplished by making the replaceable, injection-molded convex lens with 100% fog-free inside and hard coated outside [61]. The full shield has a straight pro-style lower edge so that vision is not distorted. If the

Avision Ahead elite masks are as anti-fog and scratch resistant as their website claims, this could make the maintenance requirements of the full shield closer to that of the cage.

As of now, wearing the helmet face cage or visor is optional for NHL players [62]. The IIHF, Hockey Canada, and USA Hockey require players whom are women or under the age of 18 to wear full face masks [62]. IIHF and Hockey Canada also require at least a visor be worn by players not mandated to use full facial protection, which covers the remaining players that don't fall into the above categories [62]. Many hockey players complain that the face cage/visor impacts their field of vision, which explains why many NHL players choose not to wear them.

Hockey Canada requires and USA Hockey recommends that neck laceration protectors are used for all positional players. This is because, although neck laceration injuries are rare, when a neck laceration does occur it can be very serious and even deadly. There are three main types of neck protectors available for non-goalie players [63]. The most common is the strap style neck protector, which provides the least amount of coverage [63]. The next style is the "Strap Yoke" which offers a bit more protection than the strap. Both of these types of neck protection are usually made of ballistic nylon or a similar material. The least common neck protector is the Turtleneck; it offers the most coverage and is usually made of 100% Kevlar or Armortex with abrasion resistant properties [63]. Figure 2-5 shows each of the neck laceration protector styles along with the percentage of players who wear each [63]. However, since laboratory testing of neck laceration protectors may not represent actual on-ice mechanisms of injury, their effectiveness is undetermined. A study done by the Mayo Clinic showed that players have experienced lacerations while wearing each type of neck protector available. According to this study, 27% of the neck laceration incidences reported occurred while the player was wearing a neck protector. All the neck protectors currently available are intended for laceration prevention, meaning their purpose is to prevent cuts and scrapes to the area covered.

So, the neck protectors do not protect against the impact of a puck or stick to the neck, and do not provide any support against whiplash.

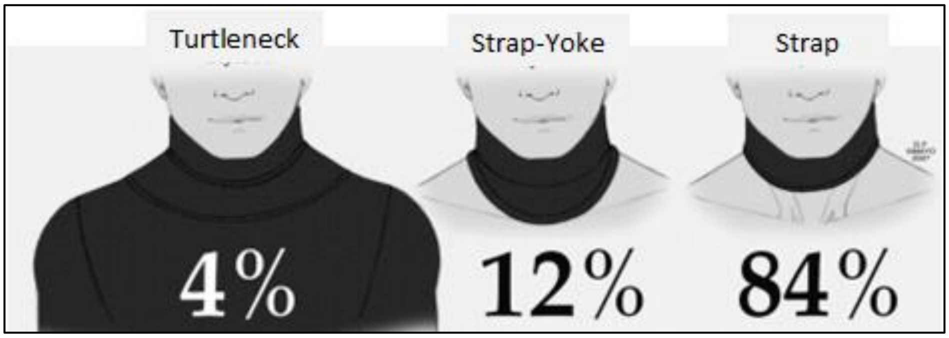


Figure 2-5: Types of Neck Laceration Protectors and Percentage of Players who Wear Each [63]

2.8 Testing Methods

There are numerous ways to test how a helmet protects against impact forces. Three very common impact tests are the drop weight impact test, pendulum impact test and air cylinder impact test. Similar forces can be exerted on the helmet as a result of each testing method but, depending on the desired impact, one test may be better suited than another.

2.8.1 Drop Weight Impact Test

A drop weight impact test involves dropping a weight on the device in order to simulate a desired impact force. Gravity, height of the drop, and the mass of the object being dropped are the factors that change the force of the impact. The impact force from this test is linear and unidirectional. The drop is guided by rails during the free fall stage to assure a straight down impact [64]. The assumption has to be made that the rails are frictionless in order to calculate the impact velocity through conservation of energy. The initial potential energy can be calculated before the drop and that energy will equal the final kinetic energy at the moment of impact.

Figure 2-6: Drop Weight Impact Test

Source: Drop Test [65]

2.8.2 Pendulum Impact Test

A pendulum impact test is similar to a drop weight impact test in that it also uses conservation of energy to determine the impact velocity. Instead of dropping a weight vertically onto the device to be tested, the weight is swung from a set height on a stiff arm as a pendulum. This allows for a horizontal impact on the device to be tested. A horizontal impact may be preferred over a vertical impact due to the rotational accelerations that could result in addition to linear accelerations. Generally, a pendulum impact test is used to break a specimen. Having broken the specimen, the pendulum swings back to a height lower than the starting point. The energy it took to break the specimen can then be calculated [66]. This test can be modified, however, by the use of a catch mechanism in order to just apply an impact force to a device.

Figure 2-7: Pendulum Impact Test

Source: Pendulum Impact [66]

2.8.3 Air Cylinder Impact Test

An air cylinder uses compressed air to deliver a controlled linear force [67]. Most air cylinders have specific forces that can be exerted for different amounts of air pressure. The force that the cylinder can exert also depends on the size of the bore or any attachments to the end of it. Air cylinders are useful for impact testing since, for the same air pressure, the force will always be the same. Since the capabilities of an air cylinder are known at the time of purchase, very few calculations are needed to assure the correct force will be applied to the device being tested. These devices can be used to apply both linear and rotational forces to the device being tested like the pendulum impact device. The main drawback is that this device cannot run on gravity, like the two previously mentioned, and needs to be powered by compressed air.

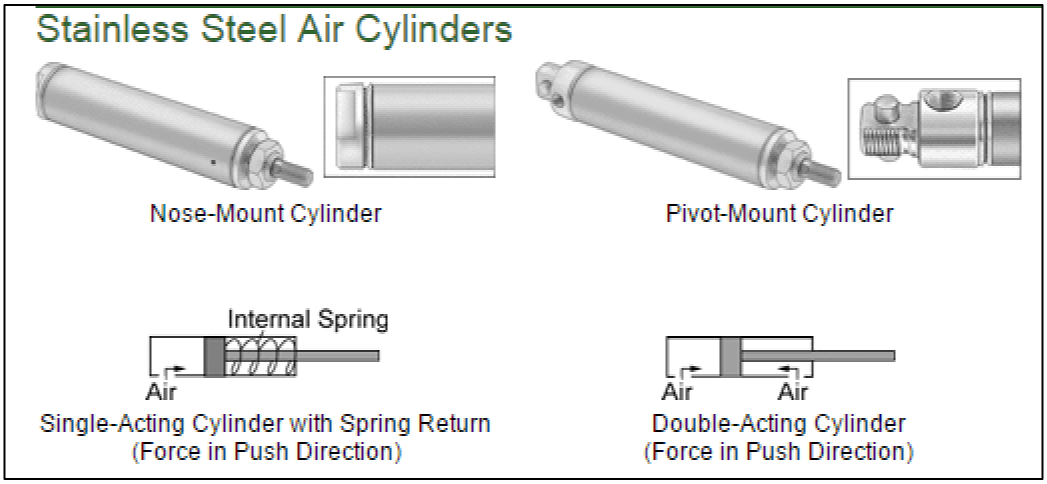


Figure 2-8: Air Cylinders

Source: McMaster Carr [67]

2.9 Smart Materials

Smart fluids are versatile materials with many possible applications in engineering that have properties that respond to different stimuli, such as forces, electrical fields, and magnetic fields. Non-Newtonian fluids have viscosities that change in response to shear rate. As a comparison, "normal," Newtonian fluids flow continuously under shear. A common example of a non-Newtonian fluid is Oobleck. Oobleck is a suspension of cornstarch in water that has a shear rate dependent viscosity that increases with increasing shear rate. Oobleck can become so viscous in response to a time-dependent force that it transforms into an elastic solid. Once the shear force is removed Oobleck returns to its original, low-viscosity state.

Extensive research was done on different smart fluids in a Major Qualifying Project from 2014 [68]. Specifically, that project group focused on fluids that demonstrated shear thickening, or increasing viscosity when a shear stress is applied. The goal of their project was to use shear-thickening fluids in a device to slow down a football player's head during an impact. Their research led them to focus on Polyethylene Glycol (PEG) and Oobleck as possibilities to use inside of the device they designed. These two smart fluids both demonstrate shear thickening and other similar physical properties. Because these fluids' viscosities increase depending on shear rate, they were used in the football helmet device to reduce the acceleration of a player's head during a hit. Two other fluids that were mentioned in this MQP report for possible application in the device were electro rheological and magneto rheological fluids.

The properties of Oobleck can be modified by adding glucose or cooking it until it becomes a gel. Adding glucose to the suspension increases the viscosity that can be reached when a force is applied [69]. Cooking the cornstarch and water suspension increases the viscosity of the fluid both before and after a shear stress is applied. The longer it is cooked the greater its viscosity due to evaporation of the water and additional swelling of the cornstarch molecules. There are countless combinations of modifications that can be made to a cornstarch and water suspension allowing for specific, desired traits to be achieved.

Polyethylene Glycol (PEG) is a polymer that has dilatant properties which means its viscosity increases when shearing is present. PEG has the same molecular structure as Polyethylene Oxide (PEO) and Polyoxyethelyne. Yet each of these polymers has different physical properties mainly due to their differing molecular masses. This polymer is labeled as PEG when the molecular mass is less than 20,000 g/mol and PEO when the molecular mass is greater than 20,000 g/mol. POE, however, can refer to the polymer of any molecular mass [2].

Smart fluids are not limited to reacting to shear stresses. Electro rheological (ER) and magneto rheological (MR) fluids are two sophisticated smart fluids that react to electric and magnetic fields respectively. The responses can occur within a few milliseconds but do differ depending on the fluid. ER fluids have a much weaker response than MR fluids and are generally unusable unless enhanced [3]. MR fluids can be used without any additional enhancements due to their stronger effects. MR fluids are also not as easily affected by contamination as ER fluids making them much more useful in many applications [4].

Table 2-10: Viscosity and Price of Various Smart Fluids

Material	Viscosity (η) at 25^{o} C	Price per gram without water
PEG - 400	70 cP	$0.028
PEO	12-50 cP	$7.74
Cornstarch Suspension	400-53,000cP*	$0.0026
Glucose as an additive	N/A	$0.0011
Polyanaline	Dependent on applied voltage	$12.36

*The viscosity of the cornstarch and water suspension is a range due to the effects cooking and glucose can have on it. Any value within this range can be obtained.

3. Methodology

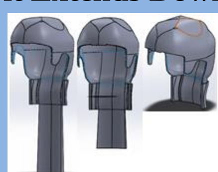
With the knowledge acquired from the background research, the project goal and objectives were more fully defined. A plan for achieving the project goals and objectives was devised and followed throughout the design process. This plan involved identifying design variables, generating the variations available for each variable and developing a method for evaluating how well each variation would contribute to achieving the project goals and objectives. The variations were assessed against critical design criteria. Assessing the variations against the design criteria involved research, engineering intuition, dynamic calculations, computational analysis and preliminary performance testing. This section describes the process of defining project objectives, and design development.

3.1 Project Goal and Objectives

With the nearly four million estimated sports-related concussions a year, athletes risk suffering the devastating effects of sustaining a concussion, each time they play the game they love. Ice hockey players are especially at risk due to the higher speeds obtained and the hard ice playing environment. Numerous studies have concluded that there is a high risk of concussion for ice hockey players at all levels of play. The concerns and effects of concussions have lead league officials to seek equipment that better protects players from concussions.

The goal of this project is to reduce the risk of concussion for ice hockey players by incorporating neck support into the current helmet design to provide an additional restoring moment during impact that will reduce the acceleration of the head. The addition of a neck support that simulates and enhances the restoring moment provided by a strong neck should reduce the risk of concussion. This is based on the finding that for every one pound increase in neck strength, as measured by a hand-held dynamometer and tension scale, the odds of sustaining a concussion decrease by five percent [7]. The ambition of the neck support incorporated helmet is to generate HIP values less than 24 kW when subject to a force typically experienced during a hockey game. Achieving HIP values below 24 kW will result in a less than 50% chance of sustaining a concussion. In order to achieve our goal, we established the variables involved with incorporating the neck support and developed options for each variable. The variables we established along with options for each variable are shown in Table 3-1. To determine which option would be used we devised a list of determining criteria.

Table 3-1: Options for Each Design Variable

Variables	Options
Material	Oobleck -Different concentrations -Cooked vs uncooked… Borax and PVA MR fluids
Pattern	One solid Vertical strips Horizontal strips
Amount of Overlap with Helmet 	Covering back of head entirely Up to lower back of head Barely overlapping with helmet
How Far it Extends Down the back 	In line with shoulders To mid-shoulder blade To bottom of shoulder blade
Amount of Wrap Around 	Just on back of neck To beneath the ears All the way around
Method of Implementation	Velcro around neck Make it adhesive to skin Memory forming materials Ear-muff mechanism Flat spring

In addition to achieving the main goal, the hope is that the modified helmet could be feasibly worn during an ice hockey game without imposing any significant limitations that a typical helmet would not impose. Although it is not the main focus of the project, in order to make implementing the design feasible, we developed the following objectives.

Feasibility Objectives:

1. The players' range of motion while wearing the modified helmet should not be decreased by more than 4% of their range of motion with the current helmet.
2. The player is able to remove the modified helmet in no more than an extra 5 seconds compared to the removal time of a current hockey helmet.
3. The design shall not incorporate any extrusions that will negatively affect player comfort or safety.

3.2 Designing the Neck-Support-Integrated-Helmet

A process for determining the best option for each of the previously identified variables was created. With these feasibility objectives in mind the first three determining criteria for evaluating the options for each variable were created as shown in the Table 3-2. In addition the main goal and feasibility objectives, cost, availability, and ease of implementation were also determining factors. Table 3-2 shows the complete list of determining criteria that was established, along with how each option's ability to meet the criteria will be assessed. The questions in the Table 3-2 will be assessed for each option and compared to determine the best option for each variable.

Table 3-2: Determining Criteria

Determining Criteria	Questions to Assess Each Option
Affordable?	How much will it cost to implement?
Available?	Do we have access to the required materials? How easy will it be obtain all the required materials? How much time will it take to obtain all the required materials?
Easy to implement?	Is there a plan for implementing this option? If so, how many steps will it take to get the option implemented?
Reduces HIP?	Does computational analysis predict this option will help reduce the HIP? Do calculations predict this option will help reduce HIP? Does engineering intuition predict this option will help reduce HIP?
Allows Full Range of Motion?	Does this option provide the flexibility necessary to allow full range of motion? How many degrees of freedom are potential affected by this option?
Easy to Use?	Will this option require additional steps to equip the helmet? It so, how many additional steps does this option require?
Is it comfortable and Safe?	Will this option cause parts to be protruding off of helmet? If so, how many? Will this option utilize hard materials that can injure someone upon impact more so than a typical helmet? Will this option lead to sharps corners or parts on the helmet?

Before deciding on the specifics of the helmet modification, a baseline hockey helmet was chosen. It was important that the baseline helmet be commonly used so that the results of testing could be extended to typical hockey situations. In order to ensure that the helmet purchased was a popular one, the choice of helmet was limited to those presented in the equipment guide on PureHockey.com (see in Figure 2-3). In addition, it was required that the helmet offer good protection before any modifications; thus, any observed improvements can be attributed to the modification and not an unsafe baseline helmet. Ideally, either the Bauer IMS 9.0 or the Reebok 11k would be used since these helmets are classified as elite protection. The Bauer Re-Akt helmet was quickly eliminated from consideration since the SUSPEND-TECH liner could make it infeasible to modify the helmet.

Two helmets were required so that one could remain unmodified to test as a control and the other to modify for comparison to the controlled results. Reusing the control helmet by modifying it after it has been tested could produce inaccurate results, since the integrity of the helmet might diminish from enduring multiple impacts. Based on the need for two helmets and budgetary restraints, two Bauer IMS 7.0 helmets were purchased; this was the helmet that offered the best protection out of the affordable options. Utilizing a cage was determined to be necessary for testing so that the dummy head would not be impacted directly. One of the helmets was purchased with a cage that could be transferred to whichever helmet is being tested at the time.

Once the helmets were purchased, options for each variable were evaluated against the determining criteria. Each variable needs to be determined before building a prototype, since time and budget constraints will only allow for the fabrication of a single prototype.

3.2.1 Evaluating the Options for the Smart Fluids

The first variable that was evaluated was the smart material that would be used for the neck support. The material choice is a critical variable that will have a huge influence on whether the prototype meets the main goal of lowering the HIP. Although, the other neck support variables will contribute, an appropriate choice for the material is crucial for creating a device that will successfully reduce the HIP. There are two main requirements the material of the neck support must meet:

1) The material must be able to provide a restoring moment against the force of an impact to reduce the acceleration of the head.
2) The material must provide the player with uninhibited use of his or her full range of motion.

At first glance, these requirements seem somewhat contradictory, but a smart fluid that exhibits shear thickening in response to stimuli should be capable of performing both requirements. The materials identified as potential candidates for the neck support were ER fluids, MR fluids, cross-linked polymers, and Oobleck. First, the availability, affordability, and ease of implementation of each option were considered, enabling the elimination of ER and MR fluids from consideration, based on their cost and complicated implementation. Then cross-linked polymers were eliminated since they break when exposed to high shear force. Therefore, the material of the neck support was determined to be Oobleck.

The fact that Oobleck has a low resting-state viscosity and exhibits shear thickening in response to a large shear force rate, makes it a very suitable material for fulfilling both the material requirements. However, the standard two part cornstarch to one part water Oobleck concentration may make it prone to settling at the bottom of the capsules used to enclose it due to its low resting viscosity. This could impede its ability to meet the first requirement, since the material must remain distributed throughout the vertical length of the neck support in order to provide sufficient restoring moment, as illustrated in the Figure 3-1.

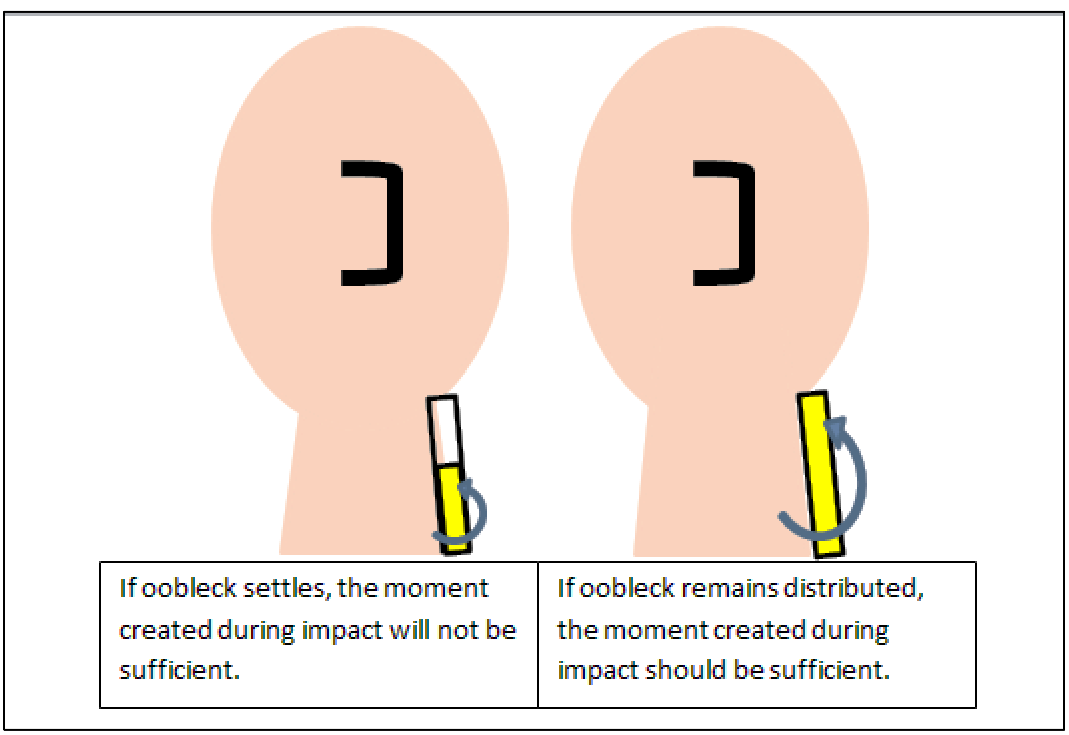


Figure 3-1: Illustration of the Effect of Oobleck Settling at the Bottom of the Neck Support

As discussed in the background section, there are several modifications of the Oobleck creation process that result in variations of Oobleck that display different properties. Experimentations with these modifications were conducted to determine which would produce Oobleck with properties that best achieve the material requirements. The properties of interest include resting viscosity, and the relationship between viscosity and shear rate.

In order to determine the viscosity properties of the Oobleck produced from each variation experiment, balls of different mass will be dropped through a volume of Oobleck. The time it takes for the balls to move through the Oobleck will be utilized to obtain the viscosity properties of each. The complete procedure for determining the viscosity of Oobleck is shown in Appendix: D-2.

I. Oobleck Creation Variation Experiments

The Oobleck creation variation experiments include varying concentration, microwaving, boiling, and stove-top cooking of the Oobleck. The first experiment was to compare uncooked Oobleck to Oobleck cooked using a 1000 Watt microwave for differing amounts of time. The microwave seemed to be too aggressive of an option since a difference of ten seconds resulted in a completed gelled over solid. The second experiment involved cooking the Oobleck in plastic bags in hot water. The procedure used for this experiment is shown in Appendix: D-2. Only subtle changes in initial viscosity were observed from this experiment. This method is much more difficult than using a microwave to cook the Oobleck. It is necessary to mix up the Oobleck in the bags periodically while cooking to assure the texture stays consistent. Additionally, the bags need to be kept away from the sides of the pot and up off the bottom by use of a steaming rack to keep the plastic from melting. Increasing the concentration of cornstarch to water was attempted but the shear thickening properties of the Oobleck made mixing difficult. The next experiment involved cooking the Oobleck directly in a pan on the stove top to evaporate the water out of the suspension.

One cup of water was mixed with one cup of cornstarch in a small pan. Once the suspension was uniform, it was cooked over heat three (low-medium) on a gas stove. The mixture was stirred constantly during the cooking process until it started to form a paste and become very thick. Once the mixture no longer had flowing fluid left, it was removed from the heat and then taken out of the pan to help stop the cooking process. Once cooled, the Oobleck had the texture and viscosity of Play Doh but had lost the desirable shear thickening properties it had when it was a liquid.

This experiment was repeated using one cup of water mixed with one cup of cornstarch and one tablespoon of white sugar, glucose. Glucose has been shown to increase the viscosity of liquid Oobleck. However, once it was cooked and then cooled, this modified Oobleck had no discernable difference between the stove-top cooked Oobleck without glucose.

It is believed that the desired paste-like substance with shear-thickening properties was not achieved due to the amount of heat retained in the cooked Oobleck. It took over 30 minutes for the mixture to cool and during that time more of the water had evaporated. This turned the paste, observed at the end of the cooking period, into crumbly dough. This dough was easily manipulated but did not have any of the shear thickening properties necessary to reduce the accelerations of the head during impact. Adding water back to the dough was attempted in order to make a paste however, the shear-thickening properties were not recovered. Table 3-3 shows the results from experimenting with various modifications to the Oobleck creation process.

Table 3-3: Results from Modifications in the Creation of Oobleck

Cornstarch to Water Concentration	Modification Method and Duration		Initial Viscosity	Shear Thickening Exhibited*	Comments
2:1	Unmodified				
2:1	Microwave	20 sec			Slightly gelled
2:1		30 sec	Like solid	No	Entirely gelled over
2:1	Plastic bags in boiling water	1 min	Unnoticeable difference from uncooked	Yes	
		5 min	Unnoticeable difference from uncooked	Yes	
		10 min	Unnoticeable difference from uncooked	Yes	
		15 min	Slightly higher than uncooked	Somewhat	
1:1	Stove Top		Like Play-Doh	No	Like Play Doh
1:1:1	Stove Top with 1 tablespoon of glucose		Like Play-Doh	No	Like Play Doh

***Viscosity vs shear force graph in Appendix E**

The desired resting viscosity is one that permits full range of motion but also ensures the Oobleck remains distributed throughout the vertical length of neck. A resting viscosity similar to the viscosity of Play Doh would make the Oobleck capable of staying distributed throughout the vertical length of the neck. The challenge is to create an Oobleck with a resting viscosity similar to that of Play Doh that retains its shear thickening properties.

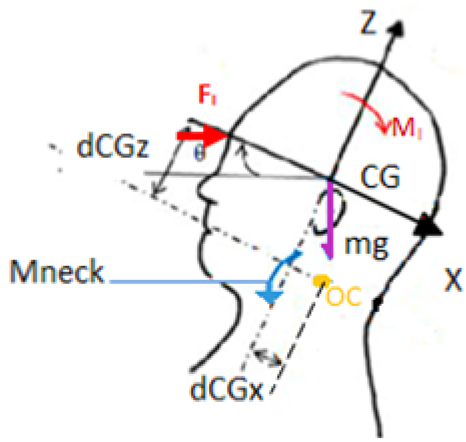
II. Calculations for Determining Necessary Dampening Coefficient

The shear thickening to force relationship, formally called a constitutive model, of each variation of Oobleck helps distinguish which variation of Oobleck should be used. First, the dampening coefficient necessary for providing a sufficient restoring moment upon impact was calculated. In order to perform the necessary viscosity calculations, a full understanding of how concussions typically occur in hockey had to be obtained and modelled mathematically. Since player-to-player collisions are the most common impact mechanism during ice hockey games, a player skating at top speeds into a stationary player was modelled [6]. The average weight of a professional ice hockey player is 210 pounds force plus 30 pounds force of equipment [70]. This means the average mass of a professional hockey player with equipment is 109 kg. Donaldson et al. studied the accelerations of elite skaters instructed to skate as fast they could, starting from a stand-still [10]. Data were collected after a specified duration, and the average of the elite skaters' accelerations was 4.375 m/s^2 [8]. Considering a worst-case scenario, in which the player hitting into the stationary player transfers the entire force to the impacted player, the obtained values were used in the following equation to determine a typical force experienced by an ice hockey player.

Equation 4: $$\boldsymbol{F_{impact} = m_{player} * a_{player}}$$

Where, F_{impact} is the force experienced by the player being impacted, m_{player} is the average mass of an equipped professional hockey player, and a_{player} is the average acceleration of an elite skater. This provided a force of 476 N, which would be used to test the helmets.

Using Figure 3-2 below as a free body diagram and rearranging the sum of moments equations provided the following differential equation:



F_I	Impact Force (i.e. Pressure * Area) through Center of Gravity
M_I	Moment from Impact
OC	Occipital Condyle; Point of Rotation
Mg	Gravitational Force (i.e. mass times gravitational constant)
CG	Center of Gravity of the head
M_{neck}	Restoring Moment from neck
θ	Angle of Rotation
d_{CGz}	Distance from Center of Gravity to OC along z-axis
d_{CGx}	Distance from Center of Gravity to OC along x-axis

Figure 3-2: Free Body Diagram of Head During Impact

Equation 5:

$$\frac{d^2}{dt^2}\theta_k + \frac{k_{damp}}{I_y}\cdot\frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y}\cdot\theta_k = \frac{\left(Pressure\cdot Area_{Bore}\cdot d_{CGx} + m_{head}\cdot g\cdot d_{CGz}\right)\cdot\sin\left(\Omega_F\cdot t\right) - \left(Pressure\cdot Area_{Bore}\cdot d_{CGz} + m_{head}\cdot g\cdot d_{CGx}\right)\cdot\cos\left(\left(\Omega_F\cdot t\right)\right)}{I_y}$$

Where, $I_y = 233\ kg * cm^2$, and is the moment of inertia about the center of gravity of the human head [28]

$k_{necks} = 50\ \frac{N*m}{rad}$, and is the spring constant that has been used to model the response of the human neck during impact [72]

$k_{damp} = 5\ \frac{s*N*M}{rad}$, and is the dampening coefficient that has been used to model the response of the human neck during impact [72]

$d_{CGz} = 55\ mm$ and $d_{CGx} = 13\ mm$, and are the distance from the head's center of gravity to the point about which the head rotates (the Occipital Condyle (OC)) along the z- and x- axis respectively [72].

Solving the above differential equation provided the equations for the angular displacement, velocity, and acceleration of the center of gravity of the player's head, shown with corresponding graphs below (complete calculation can be seen in Appendix: C-1.)

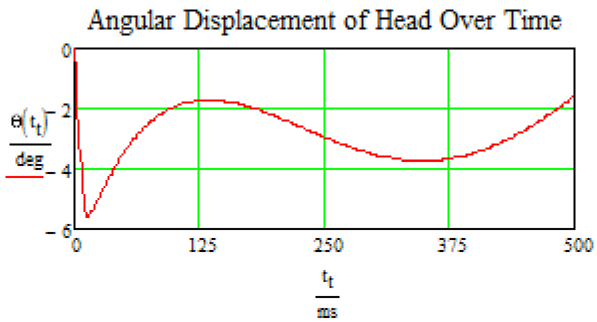


Figure 3-3: Graph of Angular Displacement of the Head versus Time, F = 476 N

Equation 6: $\theta(t) = c_1 * e^{r_1 * t} + c_2 * e^{r_2 * t} + A * \sin(\Omega_F * t) + B * \cos(\Omega_F * t)$,

Where, $\theta(t)$ is the angular displacement of the center of gravity of the head as a function of time,

t is time in seconds,

Ω_F is the forcing frequency and was estimated using graphs, and

$C_1, C_2, A, B, r_1, \& \ r_2$ are all constants that were solved for using initial value conditions (the calculations of these constants can be seen in Appendix C.)

Through differentiation the equations for angular velocity and acceleration were determined:

Angular Velocity:

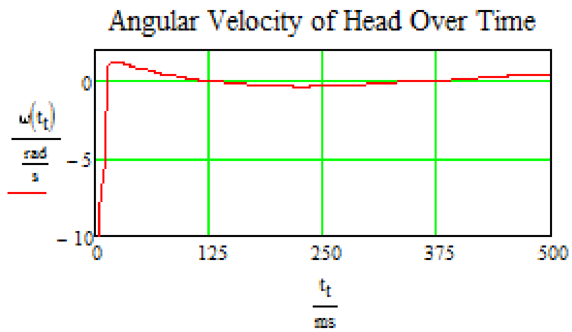


Figure 3-4: Graph of Angular Velocity of Head versus Time, F = 476 N

Equation 7: $\omega(t) = c_1 * r_1 * e^{r_1 * t} + c_2 * r_2 * e^{r_2 * t} + A * \Omega_F * \cos(\Omega_F * t) - B * \Omega_F * \sin(\Omega_F * t)$

Angular Acceleration:

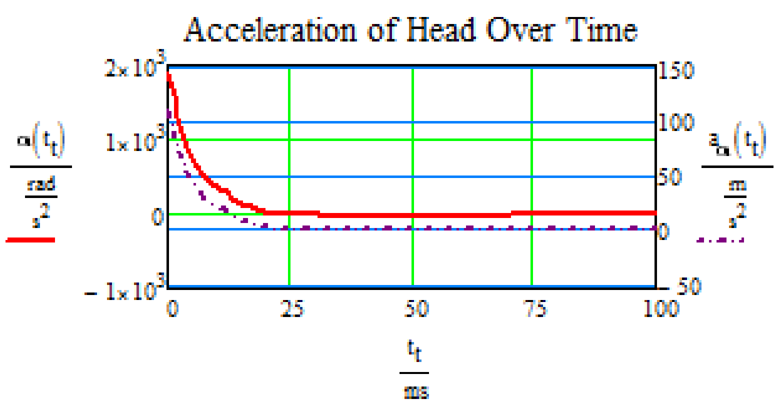


Figure 3-5: Graph of Acceleration of Head versus Time, F = 476 N

Equation 8: $\alpha(t) = c_1 * r_1^2 * e^{r_1 * t} + c_2 * r_2^2 * e^{r_2 * t} - A * \Omega_F^2 * \sin(\Omega_F * t) - B * \Omega_F^2 * \cos(\Omega_F * t)$

Unfortunately, when this information was entered into the HIP equation, it only produced a value of 1.077 kW, way below the 50% concussion likelihood HIP value of around 24 kW. Therefore, initial assumptions were reexamined. It was concluded that the low HIP value was probably because the force used in the calculations was determined from a standing start, static view point. A more accurate force was then acquired using the change in momentum equation.

Again, considering a worst-case scenario of two players skating their fastest at 30 mph and hitting head on (causing one to come to a complete stop), provides the initial momentum and the final velocity of one of the players. Rearranging the impulse equals change in momentum formula and plugging in the known variables allowed the impact force to be calculated as seen in the equations below (complete calculations can be seen in Appendix: C-2).

Equation 9: $\boldsymbol{Impulse = \Delta momentum}$

Equation 10: $\boldsymbol{Impulse = F * t}$ **Equation 11:** $\boldsymbol{\Delta momentum = m_{hp} * (v_i - v_f)}$

Equation 12: $\boldsymbol{F = \frac{m_{hp} * (v_i - v_f)}{t} = 1.217 * 10^5 N}$

Where, F is the impact force

t is the estimated time duration of impact

m_{hp} is the average mass of an equipped hockey player

v_i & v_f are initial and final velocity, respectfully

This force generated an extremely large acceleration and HIP value, indicating that the worst-case scenario that was modelled may have been too extreme.

In an attempt to obtain a more realistic value for the force capable of producing a concussion in a hockey player, background research was consulted for head accelerations that have been obtained from sensors located in helmets of athletes. Provided in the study Newman et al., conducted for developing the HIP were the maximum linear accelerations sensed in the heads of NFL players who had collided head to head with another player along with whether either player sustained a concussion. Averaging the accelerations of the players who had sustained a concussion generated an acceleration of 953.3 m/s^2. Using this acceleration and the typical mass of a human head in the force equals mass times acceleration equation provided a force of $4.195 * 10^3$ N (Complete calculations can be seen in Appendix: C-3).

Utilizing this force to solve for new constants in the angular displacement, velocity and acceleration equations generated the following graphs:

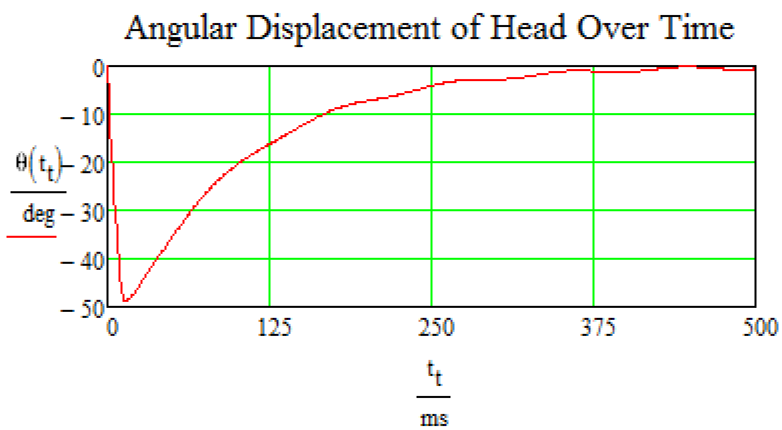


Figure 3-6: Graph of Angular Displacement of Head versus Time F = 4.195*10^3 N

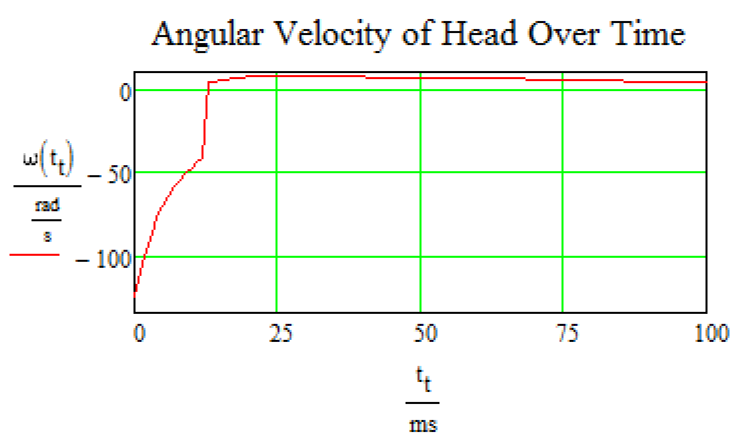


Figure 3-7: Graph of Angular Velocity of Head versus Time, F = 4.195*10^3 N

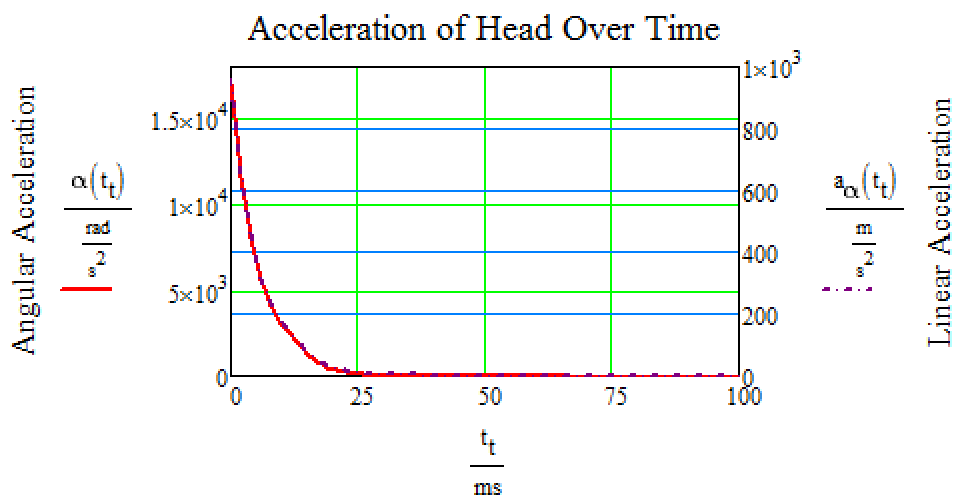


Figure 3-8: Graph of Acceleration of Head versus Time F = 4.195*10^3 N

This acceleration generated an HIP of 60 kW which is twice the 30 kW HIP value that corresponds to 95% concussion risk, but is still an obtainable value in certain situations. However, a force that would generate an HIP value that is more typical of an ice hockey player was still desired. Also provided in the study conducted by Newman et al. was the peak acceleration corresponding to a 50% chance of concussion. So this acceleration of 761.5 m/s^2 was multiplied by the mass of the human head to obtain a force of 3.35 *10^3 N. Using this force to solve for new constants in the angular displacement, velocity, and acceleration equations generated the following graphs (complete calculations can be seen in Appendix: C-4).

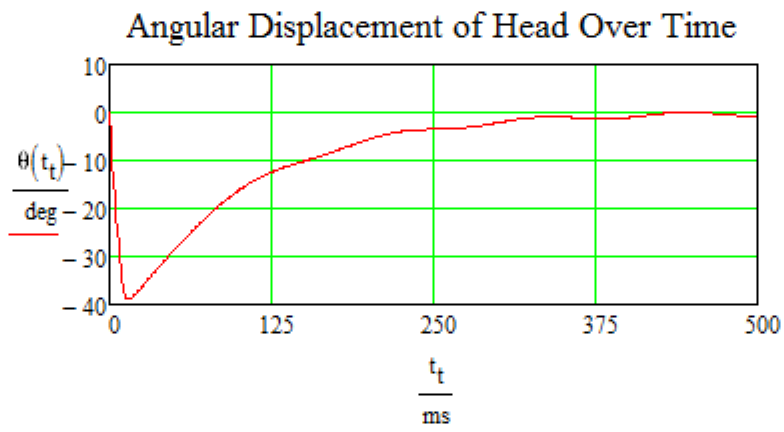


Figure 3-9: Graph of Angular Displacement of Head versus Time, F=3.35*10^3 N

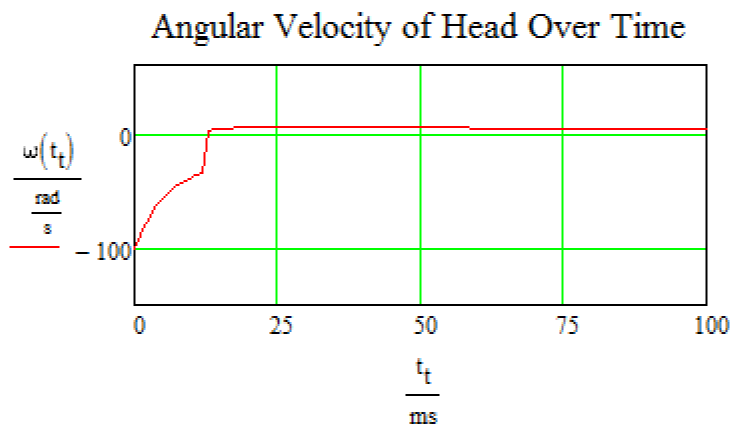


Figure 3-10: Graph of Angular Velocity of Head versus Time, F = 3.35*10^3 N

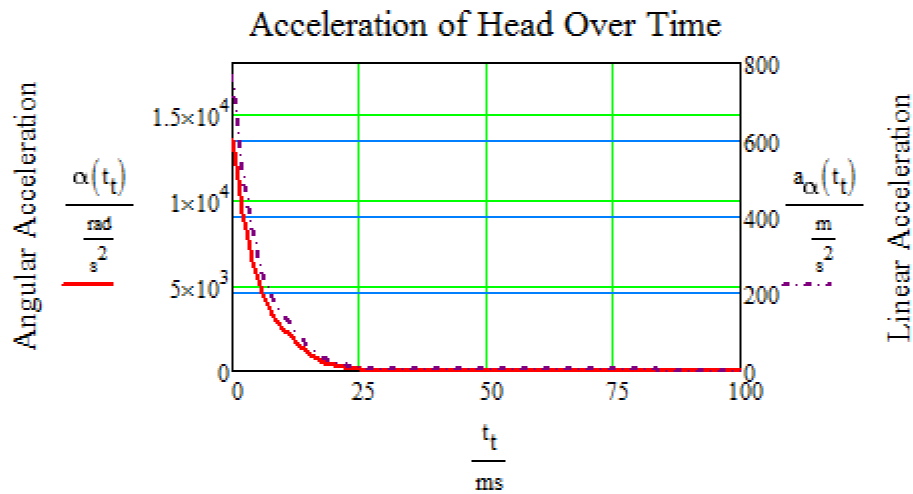
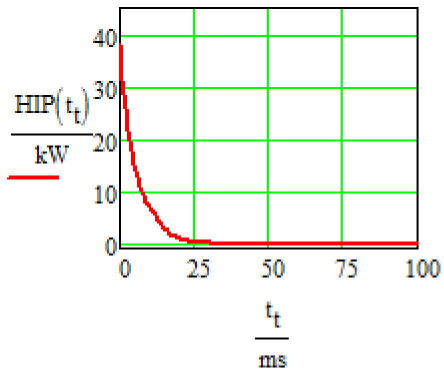


Figure 3-11: Graph of Acceleration of Head versus Time F = 3.35*10^3 N

Using the acceleration equation generated by a force of 3.35*10^3 N produces a reasonable HIP value of about 38 kW, as seen in Figure 3-12, below. This HIP indicates that there is over a 95% concussion risk.

$$HIP(t_t) := m_{head} \cdot a_x(t_t) \cdot \int_{0ms}^{30ms} a_x(t_t)\, dt_t + m_{head} \cdot a_z(t_t) \cdot \int_{0ms}^{30ms} a_z(t_t)\, dt_t + I_y \cdot \alpha(t_t) \cdot \int_{0ms}^{30ms} \alpha(t_t)\, dt_t$$



$$HIP(0ms) = 38.07 \cdot kW$$

Figure 3-12: HIP Value Corresponding to a Force of 3.35*10^3 N

This force generates a realistic HIP value indicating very high risk of concussion. However, this force cannot be achieved using the air cylinder already purchased for the test rig. The exploration of alternative test methods is discussed in Section 3.3Test Set-Up and Procedure. The solution found from the exploration of alternative test methods was to scale-down the mass of the head and the tension in the neck proportionately to the ratio between the realistic force of 3.35*10^3 N and the small, maximum force the air cylinder is able to deliver . The largest force that could be achieved using the air cylinder was calculated by multiplying the area of the air cylinder bore by 100 psi (the maximum pressure available). The maximum force that can be generated using the air cylinder is 786 N. Dividing the realistic force of 3.35*10^3 N by the maximum force achievable provided a scaling factor of 4.26. To determine the validity of the scaling down test rig solution a mathematical model in which the average mass of a human head, the spring and dampening coefficients used for modelling the human neck and the moment of inertia were divided by the scaling factor. Once equations utilizing the scaled-down values were created a variable representing the dampening coefficient of the neck support was added (see below).

Equation 13 Scaled-Down Differential Equation Including a Dampening Coefficient of the Neck Support

$$I_y \cdot \frac{d^2}{dt^2}\theta_k + \left(k_{damp} + k_{oobleck}\right)\cdot\frac{d}{dt}\theta_k + k_{necks}\cdot\theta_k =$$

$$\frac{\left(P\cdot Area_{Bore}\cdot d_{CGx} + m_{head}\cdot g\cdot d_{CGz}\right)\cdot\sin\left(\Omega_F\cdot t\right) - \left(P\cdot Area_{Bore}\cdot d_{CGz} + m_{head}\cdot g\cdot d_{CGx}\right)\cdot\cos\left(\Omega_F\cdot t\right)}{I_y}$$

Where P = 100 psi, and is the maximum available pressure,

$Area_{Bore} = 11cm^2$, and is the cross-section area of the air cylinder bore

M_{head}, I_y, k_{damp}, and k_{necks} are the values listed previously divided by the scaling factor

Solving the differential equations with $k_{oobleck} = 0\frac{m^2*kg}{s}$ generated the following angular displacement, velocity, and acceleration graphs.

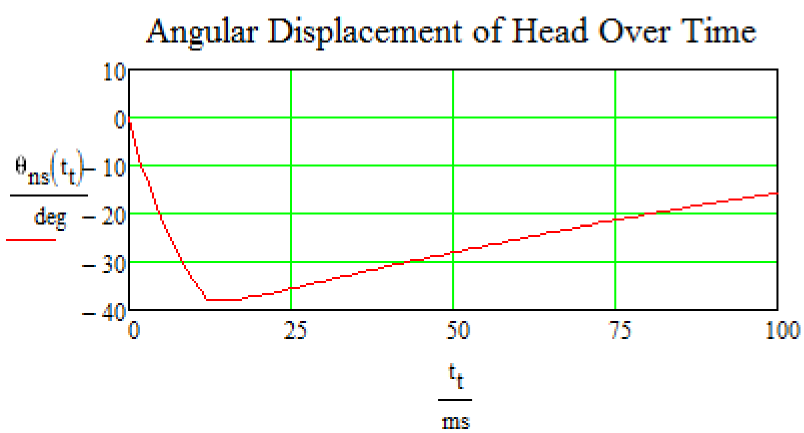


Figure 3-13: Graph for Angular Displacement from Scaled-Down Values and Dampening Coefficient = 0 m^2*kg/s

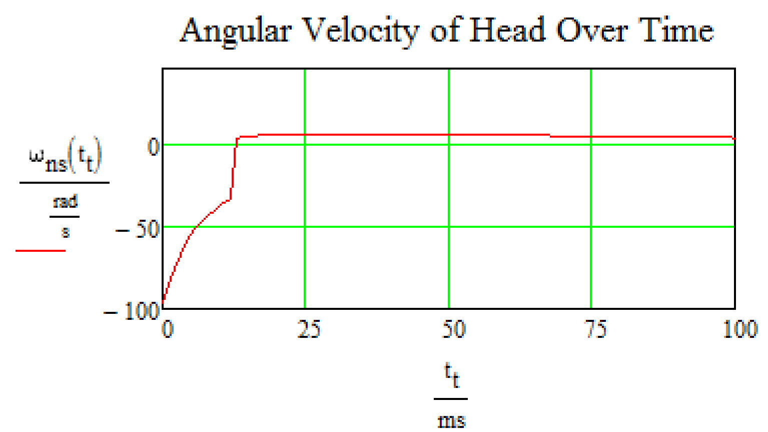


Figure 3-14: Graph for Angular Velocity from Scaled-Down Values and Dampening Coefficient = 0 m^2*kg/s

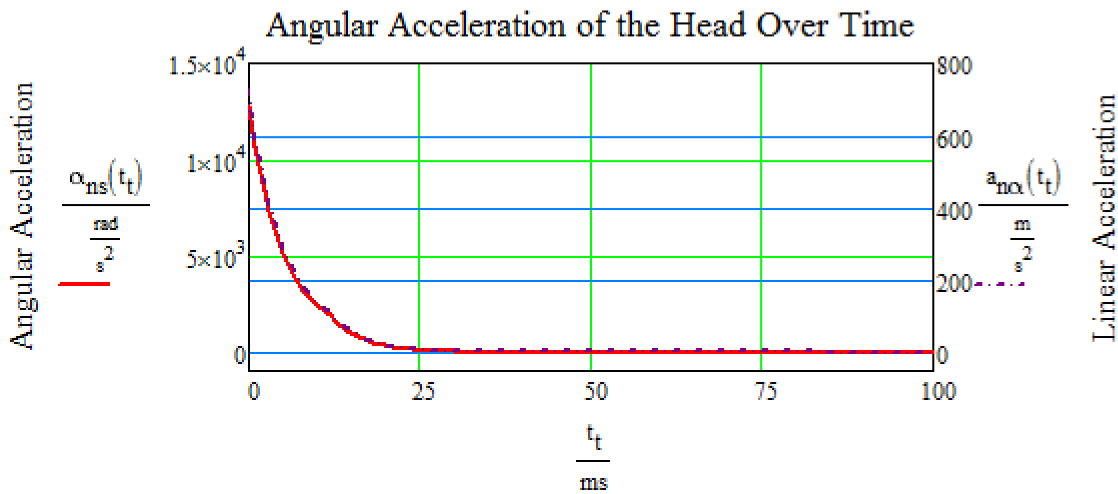
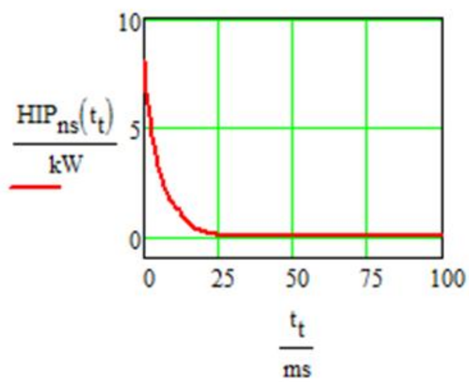


Figure 3-15: Graph for Angular Acceleration from Scaled-Down Values and Dampening Coefficient = 0 m^2*kg/s

$$HIP_{ns}(t_t) := m_{head} \cdot a_{nx}(t_t) \cdot \int_{0ms}^{30ms} a_{nx}(t_t)\, dt_t + m_{head} \cdot a_{nz}(t_t) \cdot \int_{0ms}^{30ms} a_{nz}(t_t)\, dt_t + I_y \cdot \alpha_{ns}(t_t) \cdot \int_{0ms}^{30ms} \alpha_{ns}(t_t)\, dt_t$$



$$HIP_{ns}(0ms) = 8.032 \cdot kW$$

$$HIP_{ns}(0ms) \cdot ScalingFactor = 34.242\, kW$$

Figure 3-16: Graph and Equation for the HIP Generated from the Scaled-Down Values and Dampening Coefficient = 0 m^2*kg/s

As shown in Figure 3-16, the scaled-down version of the impact generated an HIP value of 8 kW. Multiplying this scaled-down HIP by the scaling factor produces an HIP value of 34 kW very similar to the HIP generated from the mathematical model utilizing the realistic force. This HIP value just slightly exceeds the 30 kW value that corresponds to 95% risk of concussion.

The calculations were done in MathCad so the dampening coefficient could be changed and the equations and HIP value would automatically update (complete calculations can be seen in Appendix: C-5 and Appendix: C-6.) First $k_{oobleck} = k_{damp} = 1.173.\frac{m^2 * kg}{s}$, the scaled-down dampening coefficient of the neck was tried. Multiplying the HIP produced, by the scaling factor, generated an HIP value of 16.305 kW which is below the 24 kW threshold corresponding to 50 % risk of concussion. In order to determine the smallest dampening coefficient still capable of producing HIP values below 24 kW, the dampening coefficient was set equal to varying amounts until a dampening coefficient that generated an HIP value just below 24 kW was found. The dampening coefficient necessary for generating an HIP less than 30 kW, corresponding to 95 % concussion risk, was also determined. How much the different dampening coefficients were able to reduce the HIP was quantified by calculating the HIP reduction percentage using the following equation:

$$\%HIP_{Reduction} = \frac{HIP_{no\ neck\ support} - HIP_{neck\ support}}{HIP_{no\ neck\ support}} * 100\%$$

Where, $HIP_{no\ neck\ support} = 34.242\ kW$, and is the HIP generated when dampening coefficient of neck support equals 0 $\frac{m^2 * kg}{s}$

$HIP_{neck\ support}$ is the HIP generated by the dampening coefficient

Table 3-4 lists the values guessed for the dampening coefficients along with the corresponding HIP values that were generated, and the HIP reduction percentage.

Table 3-4: Determining the Smallest Dampening Coefficient Capable of Reducing Risk of Concussion to below 50%

$k_{oobleck}$ $(\frac{m^2*kg}{s})$	HIP (kW)	% HIP Reduction from HIP from no neck support
0 (No neck support)	34.242	N/A
$k_{damp} = 1.173$	16.305	52.4 %
1	17.667	47.4 %
.75	20.095	40.2 %
.5	23.302	30.7 %
.45	24.071	28.4 %
.47	23.757	29.3 %
.46	23.913	28.8 %
.25	27.733	17.5 %
.2	28.829	14.2 %
.18	29.293	12.8 %
.15	30.016	10.7 %
.16	29.771	11.4 %

As shown in Table 3-4, in order to reduce the chance of concussion to less than 95% (i.e. less than a 30 kW HIP value), a dampening coefficient of .16 $\frac{m^2*kg}{s}$ is necessary. In order to generate an HIP below 24 kW, indicating a concussion risk less than 50 %, a 28.8 % HIP reduction is necessary. The dampening coefficient capable of this percentage reduction was found to be .46 $\frac{m^2*kg}{s}$, as indicated by the green shading above. This means in order to achieve the project goal, the neck support must induce a dampening coefficient of at least .46 $\frac{m^2*kg}{s}$.

3.2.2 Evaluating Material Options for Oobleck Capsules

The material and method for encapsulating the Oobleck also had to be determined. The Oobleck has to be enclosed in liquid-tight capsules that will be sewn into a fabric-like material that fits around the neck. The capsule material has to endure impacts without rupturing and be flexible enough so that it does not interfere with the properties of the Oobleck. The material for enclosing the Oobleck was chosen based on durability, resistance to leakage, impact characteristics, availability and price. The first affordable option tested was thick, powder-free nitrile gloves. These gloves are flexible, abrasion resistant, and meant to be a barrier between skin and the chemicals or biohazards being handled [73]. The gloves provide a leak-proof barrier between the Oobleck and the neck support fabric. To utilize the gloves as Oobleck capsules, the fingers were cut off and filled with Oobleck. The fingers should be short and thin enough so that the Oobleck will not all settle at the bottom but rather remain distributed throughout the length of the finger. These fingers would then be sewn into the fabric of the neck support. A few options for sealing the capsules were tested.

First, a finger from the nitrile glove was filled to capacity using a funnel while still allowing room to be able to tie a knot to seal it. The Oobleck used was roughly 2.25:1 cornstarch concentration. First, the capsule was dropped on the ground. When no signs of cracks or leaks were present, we submitted it to the next test involving a 50th percentile male jumping on it. The capsule appeared to retain its integrity. For the final test, a collegiate softball player threw the Oobleck capsule as hard as possible at a wall. The capsule was thoroughly examined and no leaks or tears were present.

Although a simple knot seemed to secure the Oobleck sufficiently, other sealing options were tested to determine if there was an option that did not create a protrusion (knot) on the capsule. Oobleck was funneled into another finger, filling it almost entirely while leaving just enough of an opening to cover it in super glue. The opening was pushed and held closed until the super glue dried. Then the finger was dropped and Oobleck started leaking out.

So another glove was made the same way but had an additional step of folding the glued seam over and gluing it to itself to reinforce the seal. This finger withstood being dropped on the floor but ruptured when thrown by the collegiate softball player at the wall. It is believed that the integrity of the glove was compromised by the hardening of the glue. The glue seam created a stiff edge that inhibited the nitrile's flexibility forcing it to rupture when hit hard enough. Therefore, it was decided to just use a knot to seal the capsules.

3.2.3 Neck Support Enclosure Material

After researching potential materials, neoprene was chosen to fabricate the neck support that will hold the capsules of Oobleck within it. Neoprene is a synthetic rubber used in many applications due to its flexibility, durability, and resistance to breaking down in water [74]. Some of its uses are wet suits, waders, mouse pads, elbow and knee pads, insulated can holders, and orthopedic braces. Neoprene can be purchased as is, with fabric laminated on one side, or with fabric laminated on both sides.

Due to its flexibility, durability, and water resistance, neoprene was chosen for the fabric of the neck support. This material can be sewn using a sewing machine and can be put under tension to help keep the shape of the neck support. Additionally, its water resistance is helpful in case any leakage occurs with fluid holders in the neck support. It will not add a noticeable amount of protection to the player but will be soft, light, and form fitting for comfort and mobility.

3.2.4 Evaluating the Options for the Neck Support Pattern

Choosing the right pattern in which the Oobleck is arranged in the neck support is also a very important decision since different patterns may help or hinder the material's ability to achieve the project's goals and objectives. Designing the neck support pattern involves determining the orientation of the Oobleck capsules, how many capsule-filled pockets should be in the neoprene, and how many capsules should be in each pocket. First, the orientation of the Oobleck capsule was considered. Free body diagrams (see Figure 3-17) were created for vertical and horizontal orientation of the capsules. From the free body diagrams, it became apparent that a vertical orientation would be necessary to ensure that the material provides a sufficient restoring moment in response to a force.

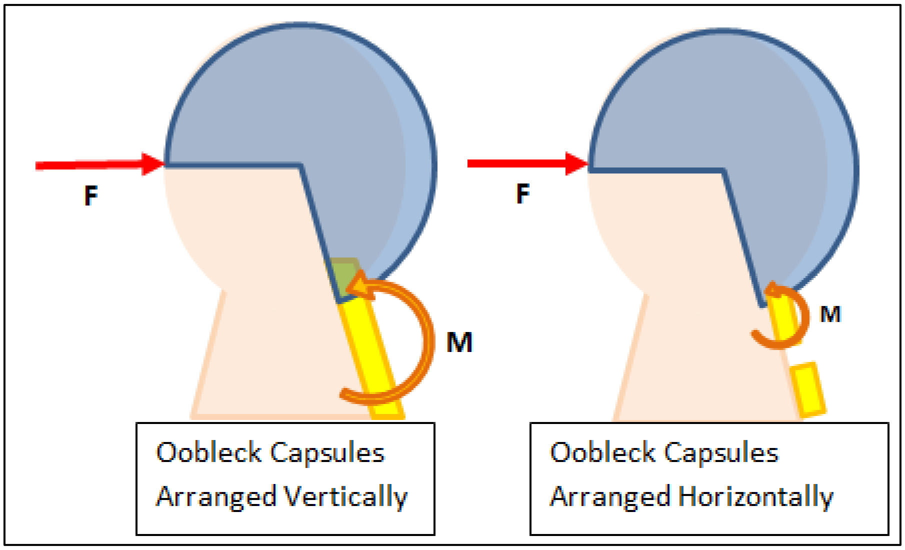


Figure 3-17: Free Body Diagrams for Horizontally and Vertically Aligned Oobleck Capsules

In order to determine how many pockets to use, the decision on how far around the neck the neck support should wrap needed to be made. Wrapping all the way around the neck was eliminated from consideration due to a high potential of reducing the player's range of motion and comfort. Wrapping it around to right beneath each ear was the option chosen since it would provide a restoring moment from more angles than a neck support just covering the back of the neck. Once this decision was made, the corresponding length around the dummy's neck was measured as roughly six inches. After measuring the diameter of the Oobleck capsules, simple division was used to determine that the maximum amount of pockets that could fit was six. Based on the measurements it was decided that two pockets would run along the back of the neck and then two pockets would be on each side of the neck support shown in Figure 3-18. The pockets of the neck support will fasten close using Velcro so that after the initial testing, capsules can be examined for leakage. This will also permit varying the amount of capsules in the pockets so that additional tests can be performed to discover how many capsules per pocket would be optimal.

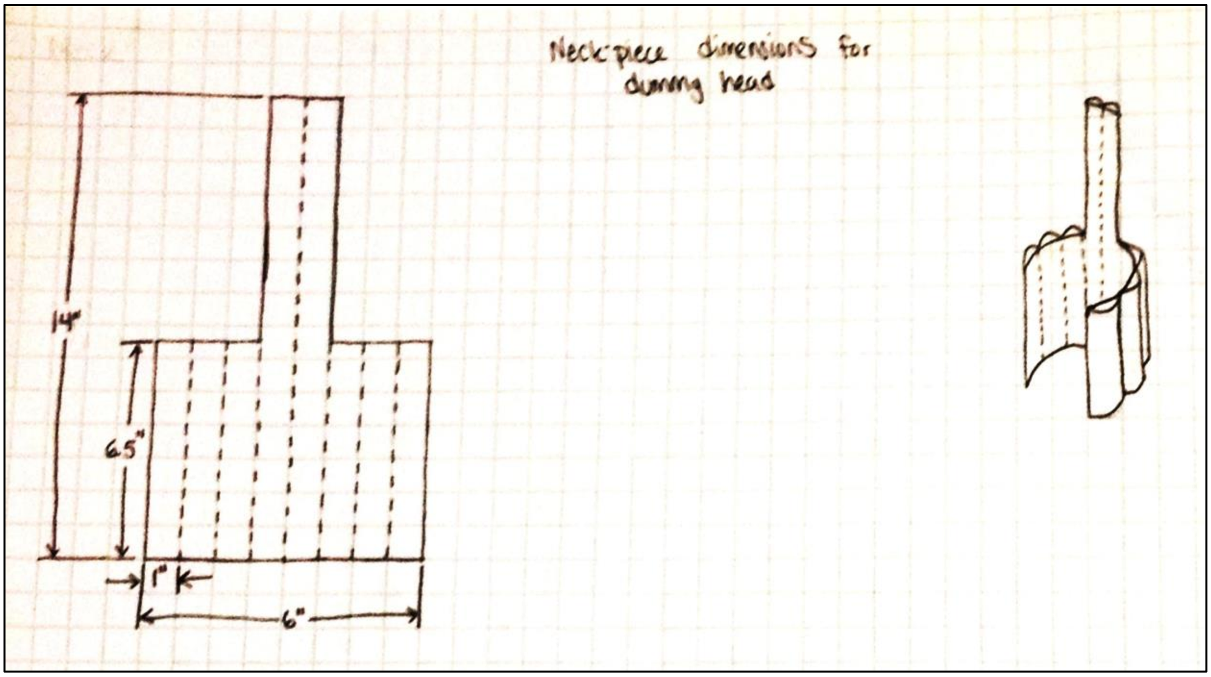


Figure 3-18: Dimensioned Sketch of Neck Support Pattern

3.2.5 Evaluating the Options for Implementation Methods

Determining how to ensure that the neck support form-fits to the neck is a challenge. Ideas were brainstormed and narrowed down to the most feasible ideas. One design idea is to incorporate the mechanism found within flexible ear muffs. This ear muff mechanism would be sewn into the top and bottom of the neck support fabric and then be pushed around the neck. This idea may be accompanied by the use of two torsion springs to ensure the top of the back of the neck support is against the back of the neck beneath the helmet. This method, along with other feasible implementation methods, are evaluated in Table 3-5.

Table 3-5: Determining Method of Implementation to best meet Determining Criteria

Variable: Implementation Method	Options	Adhesion to skin	Memory Forming Materials	Ear muff mechanism	Torsion Spring
Available?	Do we have access to the required materials for this option?	Yes	Potentially	Yes	Yes
	How easy will it be to obtain all the required materials for this option?	Fairly easy once appropriate material is determined	Not sure	Very easy	Very easy
	How much time will it take to obtain all the required materials for this option?	Depending on finding the right material and shipping	Not sure	2-10 business days depending on shipping	2-10 business days depending on shipping
Affordable?	How much will it cost to implement this option?	Not sure	Probably a lot	$14 at most	$12 at most
Easy to Implementation?	Is there a plan for implementing this option?	Yes	No	Yes	Somewhat
	If so, how many steps will it take to get the option implemented?	At least 2	At least 3	At least 3	At least 4
Easy to Use?	Will this option require additional steps to equip?	Yes	No	Yes	No
	How many extra steps will need to be followed to equip?	1		1	
Comfortable and Safe?	Will this option necessitate the use of dangerous materials or protruding parts?	No	Potentially	No	Potentially

*Additional Considerations: Will skin adhesive be reusable? Is there a memory forming material that remains somewhat flexible?

3.2.6 Evaluating the Options for How Much Overlap there is Between the Helmet and the Neck Support and How Far Down the Back the Neck Support Should extend.

The evaluation of how much the helmet should overlap with the neck support is shown in Table 3-6. This decision came down to which option would best be able to reduce the HIP value during impact, according to computational analysis. As shown in Table 3-6 , when assessing how far down the back the neck support should extend, the option of "extend to bottom of shoulder blades," was eliminated based on the fact that it would probably interfere with other padding worn by hockey players. The HIP value produced from the computational analysis of each option will be the determining factor between the remaining options.

Table 3-6: Determining How Much Overlap between the Helmet and Neck Support There Should be to Best Meet the Determining Criteria

Variable: How Much Overlap with Helmet	**Options**	**Covering back of head entirely**	**Barely overlapping with helmet**	**In between**
Easy to Implementation?	Is there a plan for implementing this option?	yes	yes	yes
	If so, how many steps will it take to get the option implemented?	At least 4	At least 4	At least 4
Reduces HIP?	What HIP Value was obtained in Computational Analysis?	TBD	TBD	TBD

Table 3-7: Determining How Far Down the Back the Neck Support Should Extend to Best Meet the Determining Criteria

Variable: How Far Down Back	Options	In line with shoulders	Down to mid-shoulder blade	Down to bottom of shoulder blade
Easy to Implementation?	Is there a plan for implementing this option?	yes	no	no
	If so, how many steps will it take to get the option implemented?	At least 2	At least 3	At least 3
Reduces HIP?	What HIP Value was obtained in Computational Analysis?	TBD	TBD	TBD
Easy to Use?	Will it interfere with other padding worn by hockey players?	Not	Possibly	Probably

3.2.7 Preliminary Computational Analysis

Certain variables required computational analysis to determine whether the option would reduce the HIP. In order to perform computational analysis, geometric models were created in SolidWorks. Figure 3-19 shows a SolidWorks model of a potential design with the neck support wrapping almost entirely around the neck and extending down the back to below the shoulder blades. To create the hockey helmet in SolidWorks, pictures of the front, side, and bottom view of the Bauer IMS 7.0 helmet were inserted on their corresponding planes and scaled appropriately. The spline tool was used to trace the pictures and the surfacing tools transformed the traces into three-dimensional surfaces. The surfaces were then trimmed, extruded and knitted to create a model of the hockey helmet. Then the helmet was brought into an assembly and new parts were created in the assembly to create the neck support part.

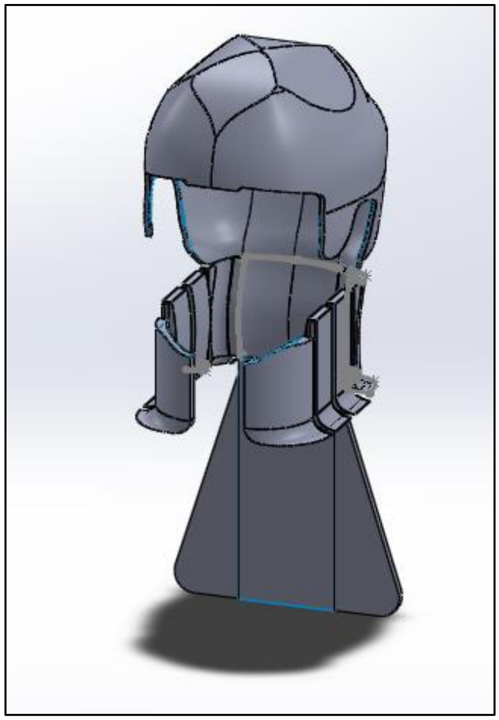


Figure 3-19: SolidWorks Model of a Potential Design

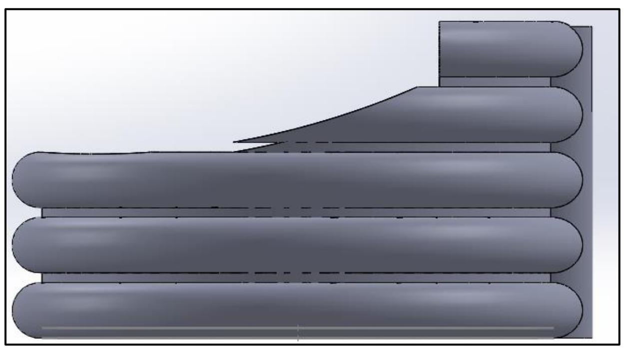


Figure 3-20: One of the SolidWorks Models with a Horizontal-Pattern Neck Support

The computational analysis is still in progress and will be completed by the rest of the team. ANSYS workbench was utilized for its static and dynamic loading tools to simulate the force experienced during a head-to-head impact in hockey. The results of the computational analysis will be used for determining the following two variables:

1. How much overlap there should be between the helmet and the neck support.
2. How far down the back the neck support should extend.

Models that are identical except for differing amounts of overlap between the helmet and neck support will undergo static and dynamic load analysis. The results will be analyzed and used to calculate the HIP value for each model. The model that generates the smallest HIP value will be used as the base model for assessing the second variable. This base model will be adapted to incorporate varying lengths of neck support extension. Static and dynamic load analysis, using the same loading conditions as the ones used for analyzing the first variable, will be performed on each of these models. Again, the results will be analyzed and used to calculate the HIP. The acceleration of the center of gravity of the head as a function of time will have to be obtained in order to calculate the HIP value. The model that produces the smallest HIP value will be used to develop the prototype.

At this point, only preliminary analysis has been done to get a basic idea of the stresses and accelerations experienced. The computational analyses done so far has mainly served as a learning experience to explore the various analytical features ANSYS offers for impact analysis and to trouble shoot any problems that arise. First static loading then dynamic loading conditions were applied to the simplest form of the model, as shown in Figure 3-21.

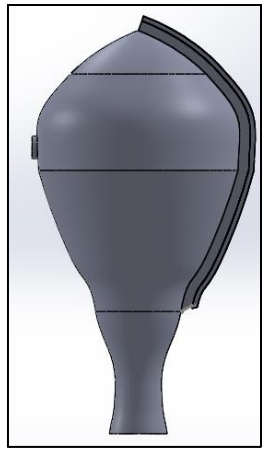


Figure 3-21: Simplified SolidWorks Model for ANSYS Analysis

Through preliminary trials a procedure for the static structural analysis in ANSYS was developed and is shown in Appendix: D-3.

In each ANSYS scenario, all parts were assigned the same material for simplicity. Polyethylene material was chosen because it accurately depicted the material of the hockey helmet shell and was one of the only quasi-appropriate materials that had all the information for the necessary properties saved in the ANSYS material database. Since this inaccurately depicts the head, neck support, and padding as the same material as the helmet's hard shell, the results must not be taken as representative of reality but rather used exclusively as means of comparison between the models. As long as this inaccuracy is reflected in all models tested, and the variable of interest is the only difference between the models results will still provide insight into which option for each variable being tested should be used. As ANSYS WorkBench analysis skills are further developed the possibility of reflecting the various materials of each component during analysis may arise. However, at this point it is not considered essential so whether the varying materials end up reflected in analysis will depend on the complexity of doing so, and the time available.

The simplified models were first subject to a static structural test, one in which a force of 416.738 N was applied to a point located on the same horizontal axis as the center of gravity. This simulated static structural test was repeated applying an acceleration of 1938 m/s^2 instead of the force. Each of these values was determined based off the initial mathematical model of the impact test. The stress was computed in these models to identify points of weakness and key areas where support needed to be added. The value of the results of the applied acceleration analysis were called into questions, since applying an acceleration may force an acceleration on a point rather than provide what acceleration would naturally occur in response to an applied force. Therefore, analysis via applied acceleration was considered not suitable for this application, since the result necessary for calculating the HIP is the acceleration.

The first model analyzed in ANSYS was representative of the current hockey helmet on the market. This includes a head, back padding and outer shell. The helmet and shell stop at where the head connects to the neck. A point on the bottom of the neck was fixed and the force was applied to a point on the model that lies on the same horizontal axis as the center of gravity. This model is shown in Figure 3-22.

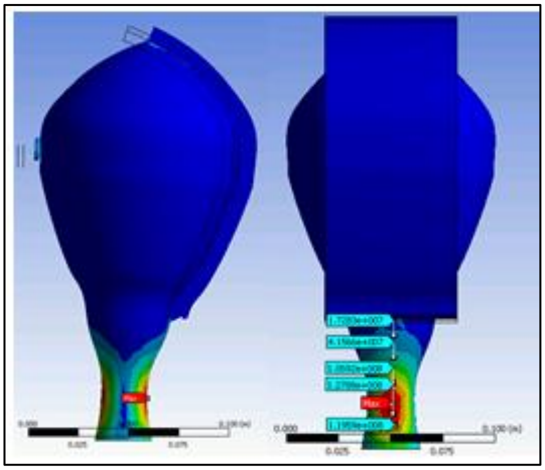


Figure 3-22: Computational Static Analysis of Unmodified Helmet

The second model that has been analyzed portrayed a helmet that incorporates a neck support. In this scenario, the outer shell remains at the same length as the first model, while the inner helmet support is extended further down the neck. The force was applied to the same location as the first analysis, and the same points on the model were fixed. Model two is shown in Figure 3-23.

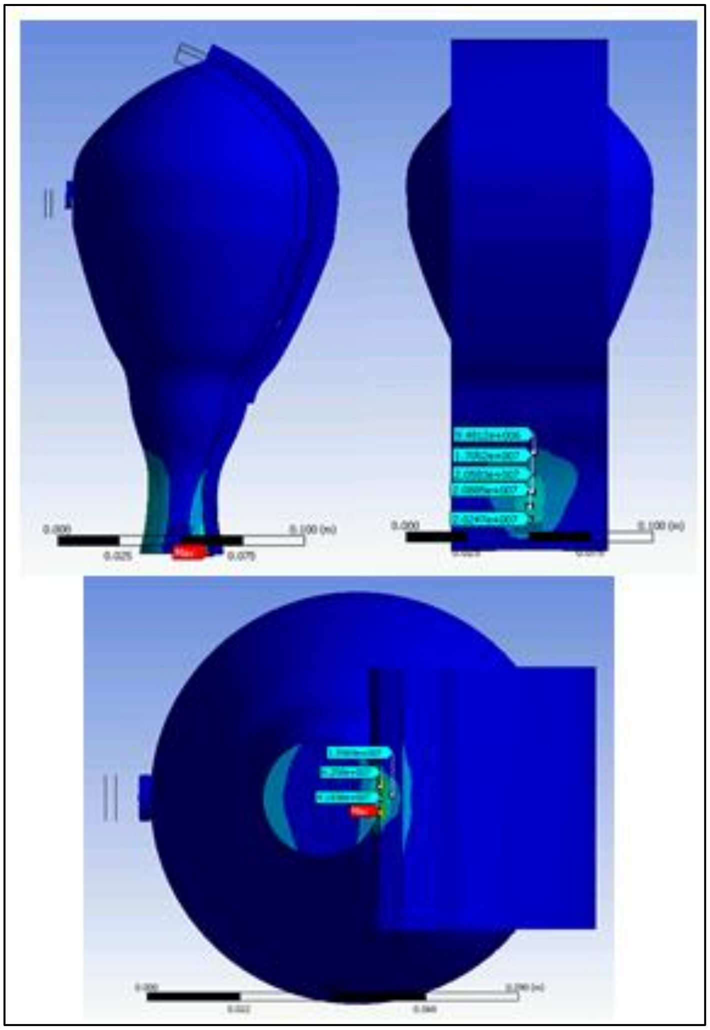


Figure 3-23: Computational Static Analysis of Head with Neck Support Incorporated Helmet

The third model is just of a head. The head model was tested in order to compare the differences between having no helmet, the current helmet, and additions to the helmet. Model three is shown in Figure 3-24.

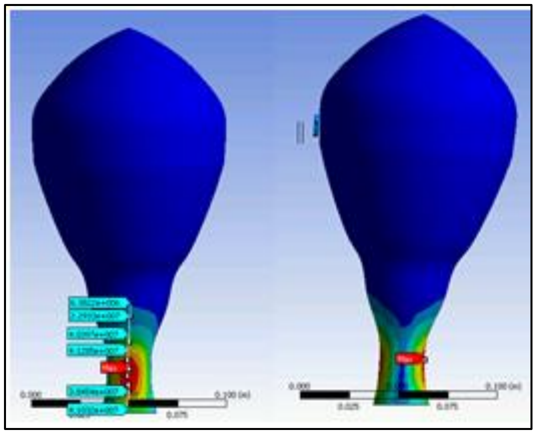


Figure 3-24: Computational Static Analysis of Head without Helmet

The three models used for static analysis were then subjected to dynamic analysis in ANSYS. In each model, a point on the neck was fixed and a constant force was applied to the impact point on the front of the head (the point that is on the same horizontal axis as the center of gravity). Through this preliminary analysis a procedure for running a dynamic analysis was developed and is shown in Appendix: D-4 .

A better understanding of the capabilities of ANSYS was obtained from this preliminary computational analysis. The best practices for efficient and valuable analysis were also discovered as a result of running the preliminary analysis. The ability to identify values for maximum stress, acceleration, and total deformation of a model were obtained. The method of probing specific points on the model to determine acceleration or stress at the point was discovered and utilized to obtain average values for areas of interest. For instance, in each acceleration model, a series of points were probed vertically along the back of the neck and head and averaged to provide an average acceleration experienced along the back of the neck. The values of each model were then compared to determine the extent to which a longer neck support influences the acceleration. It was also realized that in order to obtain an equation for the acceleration as a function of time, (which is necessary for the HIP calculation,) the acceleration data needs to be exported from ANSYS to excel or similar software that can then generate a best-fit equation for the acceleration versus time.

ANSYS recorded maximum values for deformation and stresses. The locations of the stresses and deformations varied depending on each model. Because the stresses were measured at different locations, and not at the same point of the neck, the values do not necessarily make an equivalent comparison. Based on this it was determined that for additional computational analysis it would be necessary to identify the same point on each model to obtain the results of. The values recorded in ANSYS as maximum and minimum stress and maximum deformation in static structural analysis are shown in Table 3-8.

Table 3-8: ANSYS Stress and Deformation Values

	F= 416.738 N			a= 1938 m/s^2	
	Max Deformation	**Max Stress**	**Min Stress**	**Max Deformation**	**Max Stress**
Model 1	1.5549e-002 m	4.2262e007 Pa occurs on back of helmet	1292.9 Pa	6.6096e-002 m	1.3274e008 Pa occurs on back of helmet
Model 2	1.1076e-002 m	7.7372e007 Pa on back of padding	368.5 Pa	3.4922e-002 m	2.4594e008 Pa on back of padding
Model 3	2.0533e-002 m	4.245e007 Pa on head	832.2 Pa	5.5849e-002 m	1.1551e008 Pa on head

The yield strength of polyethylene is $2.6 * 10^7$ Pa, as shown in Table 3-8 all the maximum stresses experienced according to the analysis exceed this yield strength [75]. After solving all three models dynamically the maximum values were recorded for stress, acceleration, and total deformation (see Table 3-9).

Table 3-9: ANSYS Analysis Maximum Stress, Maximum Acceleration, and Total Deformation Resulting from F = 416 N

	F= 416.738 N		
Model	**Max Stress**	**Max Acceleration**	**Total Deformation**
Model 1	1.0024e007 Pa	3.7996e005 m/s^2	1.3077e-003 m
Model 2	1.0024e007 Pa	3.7994e005 m/s^2	1.3077e-003 m
Model 3	9.2417e006 Pa	3.0343e005 m/s^2	6.0139e-003 m

Screenshots from the preliminary structural and dynamic analysis can be seen in Appendix: A.

4. Test Set-Up and Procedure

Testing is necessary in order to determine if the project goals and objectives are met. Since the main goal of this project was to reduce the HIP during an impact, an impact test must be conducted. During the impact test the acceleration of the head as a function of time must be obtained in order to calculate the HIP value. Additionally, the testing procedure used on our prototype must also be conducted on an unmodified helmet so that comparisons can be made. Then, even if our goal of reducing the HIP to below 24 kW is not achieved, whether our prototype is an improvement compared to current hockey head gear can still be determined. In addition to evaluating how well the prototype meets the project goal, assessments must be conducted for determining how well it meets the feasibility objectives. This chapter describes the set-up and procedure for the impact tests as well as the feasibility objectives assessments.

4.1 Assessing Feasibility Objectives

If our helmet accomplishes our main goal of lowering the HIP value, then we will check to see if it achieves the feasibility design objectives. The feasibility design objectives were:

1. The players' range of motion should not be decreased by more than 4% of their range of motion with the modified helmet.
2. The player is able to remove the modified helmet in no more than an extra 5 seconds compared to the removal time of a current hockey helmet.
3. The design shall not incorporate any extrusions that will negatively affect player comfort or safety.

4.1.1 Range of Motion Assessment

A range of motion test is essential to ensure the helmet will not inhibit a player's ability to play the game. This test will be similar to one that a physical therapist would perform. The use of a goniometer will allow us to measure various angles that will determine the subject's range of motion. Flexion and extension, side-bending and rotation of the head will all be tested, and a measurement, usually in the form of degrees, will be taken while wearing the standard helmet and then the modified helmet. Flexion is defined as the chin to chest motion and extension is defined as the motion of looking up to the ceiling. When measuring flexion/extension, the ear is used as the axis, the nose is the point to which to measure, while one arm stays perpendicular to the floor. When the subject moves their head in the given direction, it will give a maximum degree of movement. Side bending uses the largest spinal bump as the axis, the middle of the head for one arm and the other arm stays perpendicular to the floor. Lastly, the rotational measurement can be taken by looking down on the head and aligning one arm with the nose, while having the second arm perpendicular to the arm initially in line with the nose. Each measurement will be in the form of degrees. Table 4-1 will be used to record the range of motion of each participant. Values from each participant will be averaged and the results of the current helmet and the modified helmet will be compared.

Table 4-1: Chart to Compare Results of Range of Motion Test

	No Helmet	Current Helmet	No Helmet vs. Current	Modified Helmet	No helmet vs. Modified	Current vs. Modified
Flexion						
Extension						
Side Bend (right)						
Side Bend (left)						
Rotation (left)						
Rotation (right)						

There should be calculations that display the differences between each test, which are indicated by the grayed columns. This will display the percentage of the loss of range of motion.

4.1.2 Ease of Use Assessment

Protective equipment must be easy to use so that players will want to use it. Also, how to properly use and equip the gear must be intuitive, so that players will not equip it incorrectly and put themselves at risk of injury. Hockey players are frequently seen taking off their helmets while they are on the bench to cool off. So, hockey players will be more likely to consider using our modified helmet if they are able to equip and remove it easily and quickly.

In order to assess the ease of use of our prototype, we will time how long it takes each member of our group to put on the unmodified helmet. Each member will be timed putting on the helmet three times. The average of each member's first attempt will provide a score indicating how easy it is to equip. The average of all the trials of every member will provide a score indicating how quickly one is able to equip the helmet. Including the additional trials will account for the decrease in equipping time that may occur as players become more accustom to equipping it. This process will be repeated for removing the unmodified helmet (as opposed to equipping). The average of all the scores will provide an overall ease of use score for the unmodified helmet.

This procedure will be repeated for the modified helmet. Scores for equipping the unmodified helmet will be compared to the scores for equipping the modified helmet. Likewise, the scores for removing each of the helmets will be compared as well as the overall ease of use score of each helmet. If the scores for the modified helmet are no more than five additional seconds than the scores achieved by the unmodified helmet, then it can be concluded that our prototype accomplished the ease of use feasibility objective. Table 4-2 will be utilized while completing the ease of use assessment.

Table 4-2: Ease of Use Assessment Form

	Trial	Participant 1	Participant 2	Participant 3	Participant 4	Ease of Use score	Quickness score	Overall Ease of Use Score
Equipping Unmodified Helmet	**1**							
	2							
	3							
Removing Unmodified Helmet	**1**							
	2							
	3							
Equipping Modified Helmet	**1**							
	2							
	3							
Removing Modified Helmet	**1**							
	2							
	3							
Comparison of Scores (How many additional seconds for modified helmet?)								

4.1.3 Comfort Assessment

Comfort testing is an important concept when it comes to sports-related equipment; if it is not comfortable, then players will not want to use it. A preliminary test with the unaltered hockey helmet will be performed and then repeated with modified helmet. To test comfort, the hockey helmet will be worn while moving and while being stationary and a comfort score will be assigned. The motion testing will determine the thermal effects, which means the amount of heat the device encapsulates. The test subjects will provide a score of one to five, with five being best, on the thermal and overall comfort, as well as the ease of use. They will also comment on whether they would consider wearing the device while playing a sport.

Table 4-3: Comfort Assessment Form

	1	2	3	4	5
Does it get too warm when worn during activity?					
Is it easy to use?					
Is it comfortable to wear?					

4.2 Developing the Test Rig

Considering the force of 476 N that was determined from the average mass and acceleration of hockey players used in force equals mass times acceleration, it was determined that an air cylinder impact test would be the best choice for testing the helmets. This test is the most controlled method and requires the least amount of space. A single-acting, spring-return cylinder with a bore diameter of 1.5 inches from McMaster Carr was purchased for the test set-up. The diameter of the bore was used to determine the necessary pressure using the following equation. The pressure had to be less than 100 psi since that is the maximum amount of pressure available in the labs.

$$\textbf{Equation 14}: Force = Pressure * Area$$

$$\textbf{Equation 15}: Pressure = \frac{Force}{\frac{\pi}{4} * D_{bore}^2}$$

Where D_{bore}^2 is the squared diameter of the bore of the air cylinder in meters-squared.

With the chosen air cylinder the equation yielded a necessary pressure of around 60 psi which is well below the 100 psi available. Using additional properties of the air cylinder found on the product information section of its website, calculations were done to determine the necessary stroke length based on the duration of time for which the air cylinder should remain in contact with the helmet during the impact test. Based on the calculations, which can be seen in Appendix C, we purchased the four-inch stroke length option for the air cylinder since it would provide additional length than the necessary length to leave room for error.

4.2.1 Developing the Structure of the Test Rig

Once the appropriate air cylinder was chosen, a test rig for conducting the helmet impact test was devised. The design of the test rig is essential for accurate testing of the prototype and unmodified helmet. It was important to be able to administer a regulated impact force. The head-form that would wear the helmet was salvaged from a previous MQP and the rest of the rig was designed and created around the head-form and air cylinder. To ensure accuracy and repeatability of the tests, the test mechanism had to keep every component secured to each other in some way. There were a few specifications that were defined that were important for designing the test rig.

- Needs to be rigid; headpiece and impact device must be connected
- Have the ability to rotate the head piece
- Have ability to adjust the height of the impact device
- Must be small and light enough to transport

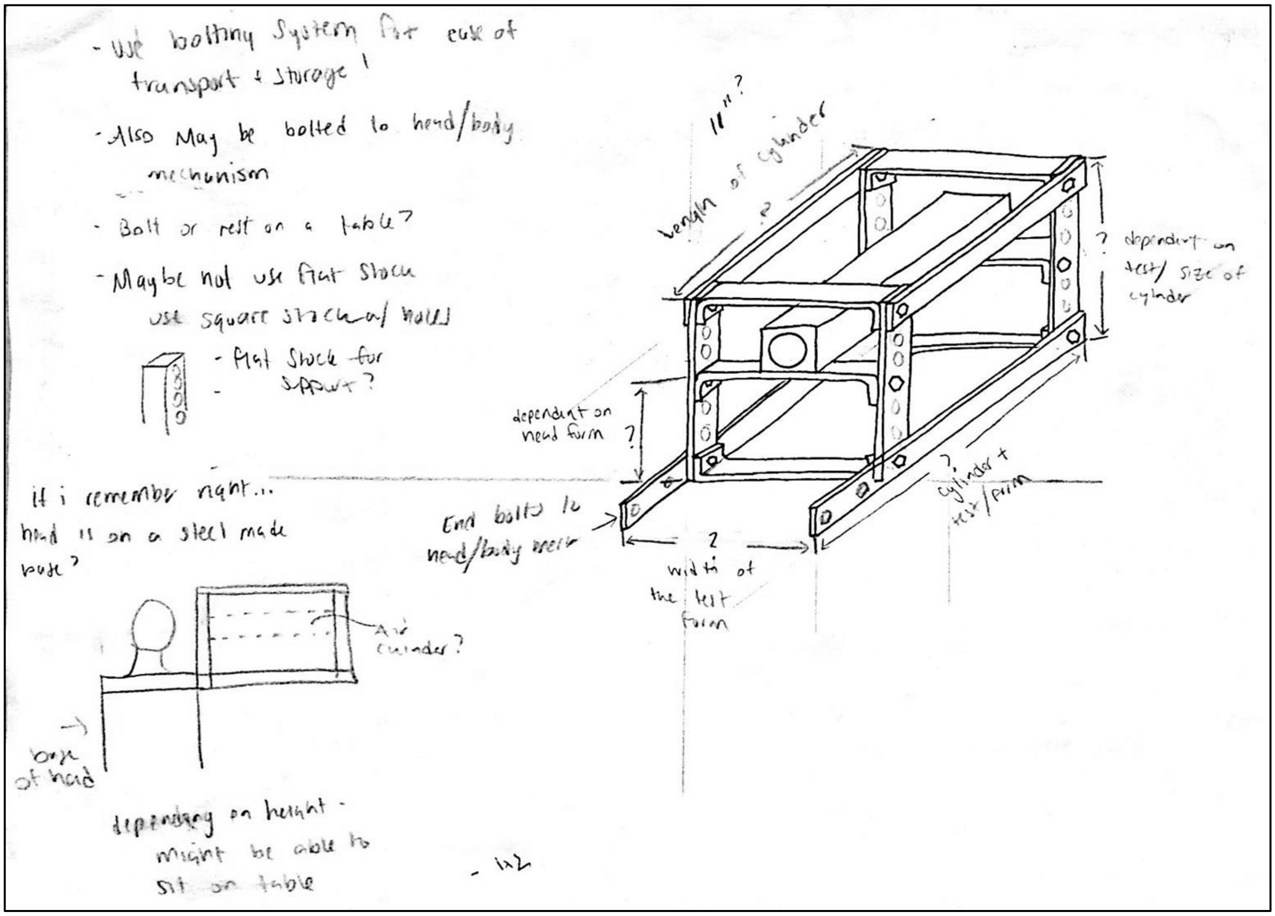


Figure 4-1: Initial Sketch of Test Set-Up

A pneumatic air cylinder was attached to a rigid metal structure and supplies the amount of force needed to impact the head form. The initial test set-up, shown below, exhibits most of the specifications listed above. The use of a perforated metal allows for the air cylinder to be height adjusted for various impacts. The metal will also allow for a bolt together feature that will offer easy disassembly if need be. The initial design also allowed the metal structure to be bolted to the head form.

After performing some research on available parts, it was found that a perforated steel angle frame would be suitable for this application since it offers support from two directions. A reiteration of the design was then modeled in SolidWorks as shown in Figure 4-2.

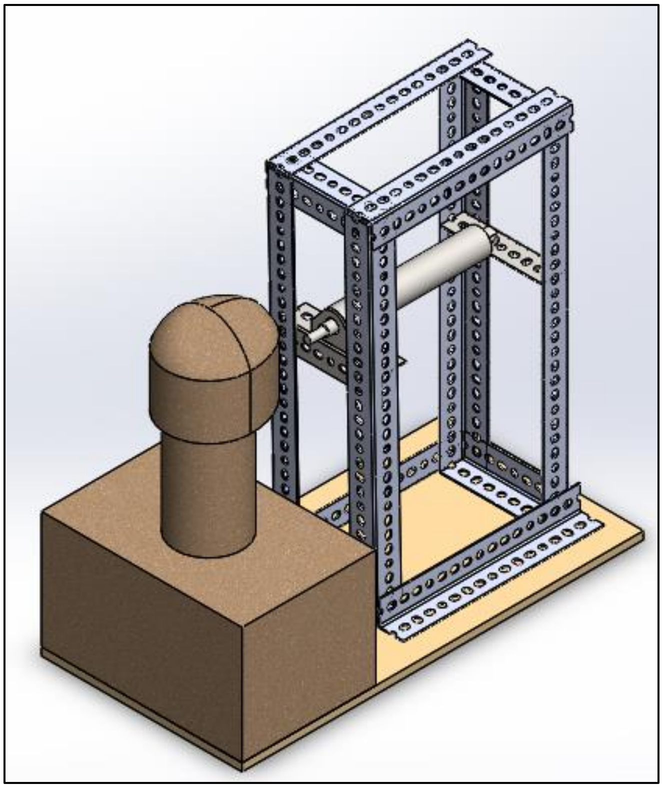


Figure 4-2: SolidWorks Reiteration of Test Rig

The base of this test set-up was a ¼ inch thick piece of plywood that provides stability and ensures the base is more rigid than the head-form so that the base remains still while the head-form rotates upon impact. Calculations were executed to ensure that a ¼ inch thick piece of plywood would be strong enough to hold and transport the rest of the test set-up without bending too much (see Appendix: C-7.) The metal is held together by nuts and bolts as well as corner braces to ensure that it stays square and rigid. It also is bolted down to the plywood through the use of the perforated angle iron. The air cylinder is bolted to two cross bars that allow for height adjustment for different impacts. Ensuring that the air cylinder is level is essential for administering a straight-on impact. The use of extra washers to prop up the front side of the bracket was needed to level the cylinder.

The dimensions of the metal structure are 12”x 24” x 6”. The head stands about 19 inches off the board so the height of 24 inches on the metal structure will cover an impact at the top of the head. The small cross bars on the structure are 6 inches, which makes the structure slightly wider than the cylinder itself. The length of 12 inches was slightly long but the placement allows the vertical (24”) pieces to be adjusted to the proper length of the air cylinder. The metal was cut precisely so that the holes align properly. The air cylinder has foot brackets that are 9.5 inches apart, which means the bars holding the cylinder to the structure are that far apart and set into the rectangular structure.

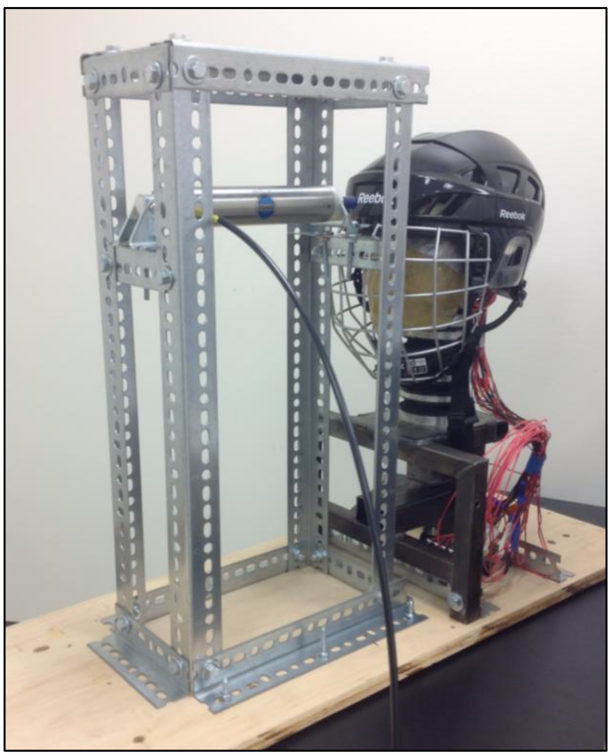


Figure 4-3: Finished Test Set-up

After constructing the metal test structure, the head-form was altered so that it could mount to the piece of plywood. Unnecessary metal on the bottom of the head-form was cut off and leveled so that holes could be drilled to allow the head-form to be bolted to the perforated angle frame at the required height. This allows for an easy on and off application of the head-form. All the components were placed on the plywood in their appropriate places and the holes were traced and then drilled. Then all the components were bolted together securely.

The head of the dummy device also needed to be altered. The helmets that were bought were slightly big so an inner shell was created to offset the difference in the helmet to head size. This was created using Dow GREAT STUFF expanding insulation foam. A plastic bag was used to cover the head to ensure that the foam would not stick to the head itself. Then the head was covered in about an inch of the foam. Once the foam set and dried, it was then shaved down to ensure a snug fit when using the helmet. Since the shaved foam was susceptible to deterioration when rubbing it, a couple coats of spray clear coat were used to inhibit this issue.

4.2.2 Pneumatic Circuit Connecting Air Supply to Air Cylinder

The pneumatic circuit consists of the air supply, the air tank, the air cylinder, hose, and a solenoid switch. The air supply allows for a maximum of 100 psi output. A hose with quick connect fittings connects the air supply to the tank. Attached to the air tank is a pressure gauge that indicates the air pressure being delivered to the cylinder. The air tank also has an output that is controlled by a valve. A quarter inch tube, with male quick connect fittings of ¼" and a 1/8'' NPT, connect the output valve of the air tank to input port on the solenoid switch (each equipped with the corresponding female NPT fittings). Another strip of the quarter inch tubing connects the 1/8" NPT female fitting of the output port on the solenoid switch to the 1/8" NPT female fitting on the input port of the air cylinder. A LabView program was created to monitor the air pressure entering the switch. Once a pressure of 60 psi is detected, the switch will be triggered manually to release the air into the air cylinder. The complete LabView program is shown in Appendix: B 1 and Appendix: B 2. Utilizing the switch ensures the release of pressure is instantaneous which reduces the presence of a pressure gradient.

4.2.3 Impact Testing Methods

For completeness, both helmets will be impacted on four locations:

1. On the cage through the head's observed center of gravity
2. On the back of the helmet through the head's observed center of gravity
3. On the side of the helmet just above the ear slot
4. On the front of the helmet above the cage

Each of these locations will be impacted three times at the predetermined pressure of 60 psi. Accelerometers in the head will provide the accelerations to a program that will output acceleration as a function of time. The acceleration as a function of time will be used to calculate the HIP value. The complete acceleration acquisition program can be seen in Appendix: B-1and Appendix: B-2. The averages of HIP values at each location of the unmodified helmet will be compared to the averages of the corresponding HIP values of the modified helmet. The comparison of HIP values of our modified helmet to the unmodified helmet will help conclude whether our design is an improvement to the hockey helmets' ability to reduce the risk of concussion.

4.2.4 Rethinking the Test Set-Up

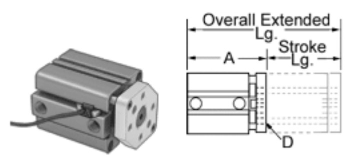
The 476 N force that the test set-up was designed to generate results in an HIP value that is way too low. This means that the force would not likely result in concussion and may not be enough to trigger the shear thickening response of the Oobleck. However, this was not discovered until after the test rig was built and ready to use. The pressure necessary to produce the more realistic force of 3.35 * 10^3 N was calculated using Equation 15: $Pressure = \frac{Force}{\frac{\pi}{4} * D_{bore}^2}$. This indicated that a pressure of 426 psi was required, which is exceeds the available 100 psi. Thus, buying a new air cylinder with a larger bore size was considered. McMaster Carr had a few compact extruded-aluminum, switch-ready air cylinders with four-inch bore diameters that would be capable of providing the necessary force using less than 100 psi (see Figure 4-4). However, they range in price from $140-$240, which would use the majority of the remaining budget. Another drawback is that the test rig would have to be redesigned to withstand the greater force delivered by the four-inch bore cylinder.

For additional information and specifications view More About Compact Extruded-Aluminum Switch-Ready Air Cylinders.

Bore Size	Retracted Lg. (A)	3/16"-1" Stroke Lengths	Each	1 1/16"-2" Stroke Length	Each	2 1/16"-3" Stroke Lengths	Each	Replacement Seal Kits	Each
4"	2.95" + Stroke Length	62245K621	$186.24	62245K622	$206.34	62245K623	$231.11	62245K918	$20.09

100V AC/5V, 12V, 24V DC Reed Switch	62245K91	Each	$17.98
100V AC/12V, 24V DC Reed Switch with LED	62245K92	Each	22.94
24V DC, 3 Wire Solid State NPN Switch	62245K94	Each	48.22
24V DC, 3 Wire Solid State PNP Switch	62245K95	Each	48.22

Nonrotating Cylinders



With 1 1/4" to 4" Bore – Shown with Switch

Bore Size	Force @ 100 psi, lbs.	Port Size, Female	Standard Cylinder Rod, Thread × Dp. (B)	(C) Standard Cylinder	(C) Nonrotating Cylinder	Nonrotating Cylinder Mounting Plate Holes, Thread × Dp. (D)	(E)	Mounting Holes (F)
½"	13	M5•	8-32 × 0.213	0.98"	0.98"	6-32 × 0.24"	0.87"	8-32
¾"	36.5	M5•	10-32 × 0.276	1.42"	1.42"	8-32 × 0.27"	1.42"	8-32▲
1"	58.5	M5•	¼"-28 × 0.394	1.57"	1.57"	8-32 × 0.33"	1.57"	¼"-20
1¼"	93.5	⅛" NPT	5/16"-24 × 0.524	1.77"	1.75"	10-24 × 0.33"	1.34"	¼"-20
2"	256	¼" NPT	½"-20 × 0.693	2.52"	2.51"	5/16 -18 × 0.48"	1.97"	5/16"-18
3¼"	703	⅜" NPT	⅝"-18 × 0.957	3.86"	3.85"	½ -13 × 0.55"	3.03"	½"-13
4"	1,108	⅜" NPT	¾"-16 × 1.063	4.61"	4.54"	½ -13 × 0.55"	3.70"	½"-13

•10-32 for nonrotating cylinders. ▲¼"-20 for nonrotating cylinders.

Figure 4-4: Compact Extruded-Aluminum, Switch-Ready Air Cylinder Options to Provide Necessary Force

In order to explore potentially cheaper options, the feasibility of using a hammer impact test was considered. Assuming the initial speed of the hockey player was 30 mph, the required energy of the hammer was calculated:

Equation 16: $\boldsymbol{Energy} = \frac{1}{2} * \boldsymbol{m_{head}} * \boldsymbol{v_{hp}^2} = \mathbf{395.7\ J}$

The hammer in the impact test would have to generate this energy in order to simulate the impact of a hockey player. The distance from the pivot point to the center of mass of the hammer was chosen to be 1.2 meters (denoted as L_{hammer}) so that it would be a manageable length that would not require a space with high ceilings. Using the free body diagram below, the necessary mass of the hammer (m_{hammer}) was determined.

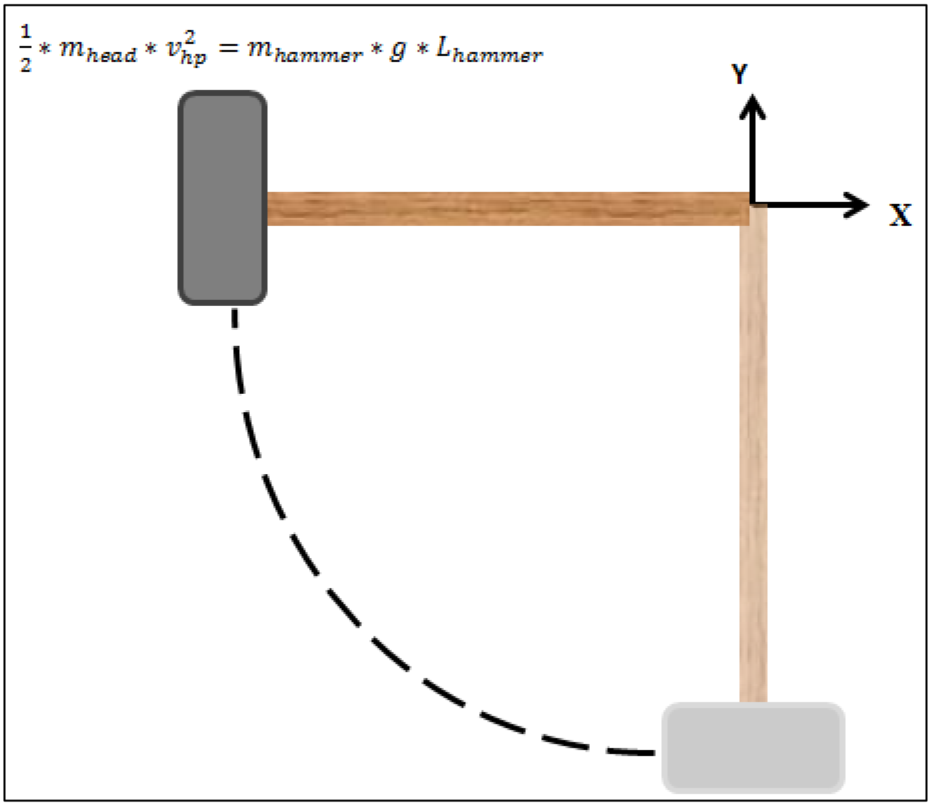


Figure 4-5: Free Body Diagram of Potential Hammer Impact Test

Equation 17: $\boldsymbol{Potential\ Energy\ of\ Hammer} = \mathbf{395.7\ J} = \boldsymbol{m_{hammer}} * \boldsymbol{g} * \boldsymbol{L_{hammer}}$

$$m_{hammer} = \frac{Potential\ Energy\ of\ Hammer}{L_{hammer} * g} = 33.6\ kg$$

This indicates that a hammer weighing 59.3 pounds force would be required. If this test device was used it would require much strength to set up and transport. If a location providing more space becomes available to use for testing then the length of the hammer could be increased which would decrease the weight of the hammer. However, in order to make the hammer a more manageable weight of around 35 pounds force the length of the hammer would need to be 2.5 m. This makes finding a location large enough to accommodate this kind of test set up very difficult.

Ideally, the air cylinder and test rig that are already set-up would be used. Currently, we are attempting to scale down the mass of the head form, the tension in the neck, and the viscosity of the Oobleck to be proportional to the small, maximum force the air cylinder is capable of. Then the acceleration generated by the scaled down set-up would be entered into the HIP equation, and the HIP value would be multiplied by the scaling factor for proof of concept.

Calculations were performed to model the scaled-down impact test. The maximum force (i.e. force generated at 100 psi,) that the air cylinder can produce is 786 N. A scaling factor of 4.26 was determined by dividing the realistic force of around $3.35 * 10^3$ N by the 786 N force possible. The mass, spring constant, dampening coefficient, and moment of inertia of the head were all divided by the scaling factor. The angular displacement, velocity, and acceleration equations and graphs generated can be seen in Figure 3-13, Figure 3-14, and Figure 3-15, respectively. The scaled-down model generated an HIP value just above the 30 kW threshold corresponding to 95% risk of concussion, as shown in Figure 3-16.

Based on the results of scaling down the mathematical model, scaling down the head-form mass and the neck tension proportionately should provide suitable, proof of concept results.

Based on this and the fact that the test rig for the smaller force is already constructed, the scaled-down test solution was chosen.

In order to actualize the scaled down testing solution the head-form has to be modified to reflect the scaled-down values calculated for in the mathematical model. The current head-form will be un-bolted from the base plate on the neck form and a lighter head form will be fabricated to be bolted in its place. The tension in the current neck will be adjusted until the desired reduction in tension is achieved.

5. Conclusions and Recommendations

This chapter starts with the accomplishments made for this project to date. It then summarizes the progress that has been made on this project. This chapter concludes with a summary of the steps completed and the steps that need to be taken in order to complete this project.

5.1 Accomplishments Contributed to Project

Since I needed to complete this MQP in two terms and my team will continue it in a third term, I tried to contribute as much as possible to the team's success in order to compensate for not being able to complete the project with them. The greatest accomplishment that I contributed to this project was performing all the calculations that have been completed thus far. This includes the mathematical models of the impact, determining the necessary stroke length of the air cylinder, determining the necessary thickness of plywood for use as the base of the test rig, and calculating the dampening coefficient the neck support must induce in order to achieve the project goal. These calculations informed the group's decision to utilize an air cylinder impact test and on which air cylinder should be purchased. Additionally, the calculations for determining the necessary dampening coefficient will help the team determine what viscosity of Oobleck should be used in the neck support.

Modeling the hockey helmet and most of the neck support variations in SolidWorks were some of the other accomplishments I contributed to this project. This includes creating the various simplified versions of the neck-support-incorporated helmet that were imported into ANSYS for analysis. Some of the other accomplishments include collaborating with the team on constructing the test rig, testing the nitrile gloves as Oobleck capsules, and developing the neck support pattern.

5.2 Summary of Project Progress

In summary, research on concussions, hockey injuries, hockey equipment standards, testing methods and materials was conducted. Design criteria were established and options for the design variables were identified and evaluated against design criteria. The availability, affordability, and contribution to achieving the project goals of each option were determining factors in distinguishing which option to use for each variable. How well the option would contribute to achieving the project goals and objectives was predicted through calculations and computational analysis.

Experiments with modifications in the Oobleck creation process were conducted and allowed for the elimination of certain modifications from consideration. Calculations were performed to determine what dampening coefficient would be desirable. Mathematical models were created to identify the force that needed to be applied to simulate a hockey impact, which helped identify an appropriate testing mechanism. Plans for the test rig were fabricated and the best features of each were combined in the final design. Materials for constructing the test rig were purchased and modified as necessary during construction. The dummy head-form was salvaged from a previous MQP and modified to fit our test rig. LabView programs to run the test and obtain the acceleration of the head were written.

5.3 Steps to Complete Project

The test rig needs to be scaled down to match the small force it was designed around, in order to be ready for testing. A scaling factor was determined by dividing the realistic impact force by the largest force the air cylinder can produce. The average mass of the human head is 4.4 kg, dividing this by the scaling factor generates 1.032 kg, the mass the dummy head should be [72]. A new head-form with this desired mass will be fabricated. Dividing the spring and dampening constants, that are used in modeling the response of a human neck, by the scaling factor produces constants of 11.7 N*m/rad and 1.17 m^2*kg/sec, respectively. The tension in the dummy neck must also be adjusted accordingly. Once the test rig is entirely scaled down, then the unmodified helmet can be tested. The unmodified helmet will also be subject to the assessments for the feasibility objectives.

The modifications of the Oobleck creation process will be repeated to produce large samples of Oobleck. Each of the Oobleck samples generated will be subject to the sphere-dropping procedure to obtain each sample’s viscosity to shear rate equation. This will help determine at what shear rate the Oobleck acts more like a solid than a liquid. Comparing the results of this procedure to the desired dampening coefficient of the neck support will establish which Oobleck creation modification should be used.

The plans for the prototype are complete but the actual fabrication needs to be completed. After the prototype is complete, it will be tested using the same procedure used for the unmodified helmet. The head accelerations generated during the test will be entered into the HIP equation. The HIP value obtained from each helmet will be compared and conclusions will be drawn. The HIP value produced by the prototype will be compared to the 24 kW or less project goal that was set. Alterations to the prototype may be done before retesting.

Once impact testing is complete, the tests designed for the feasibility objectives must be done on the prototype. Each member of the group must complete the range of motion, comfort, and ease of use feasibility tests for the prototype. Once this testing is done, analysis of the results will be performed.

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Appendix A: **Screenshots from ANSYS analysis of SolidWorks Models**

A-1: Screenshots from ANSYS Static Structural Analysis

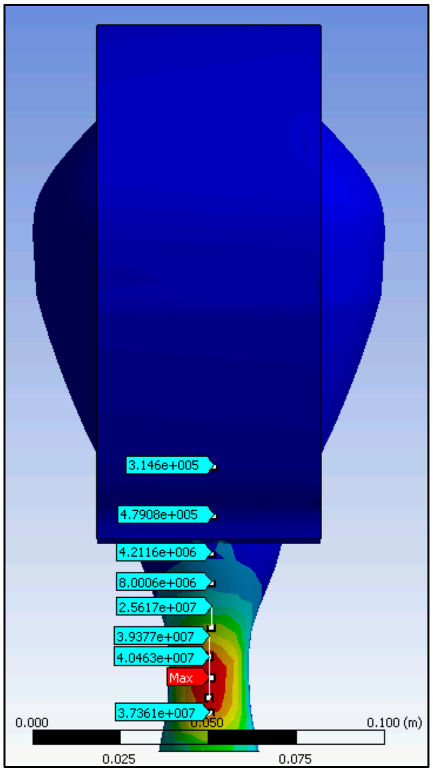


Figure 6-1: Model 1 Stress from Force

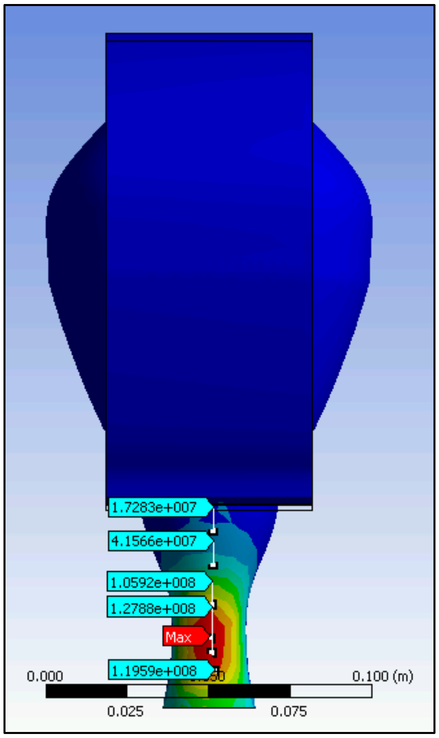


Figure 6-2: Model 1 Stress from Acceleration

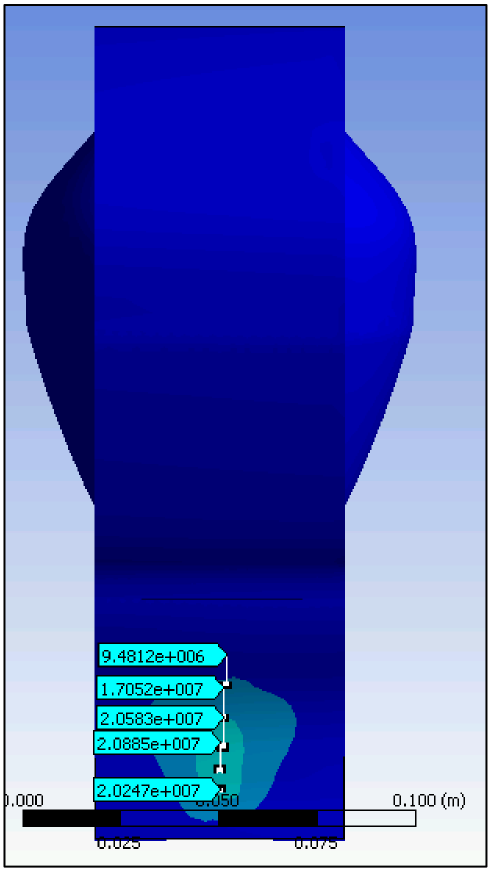


Figure 6-3: Model 2 stress from force

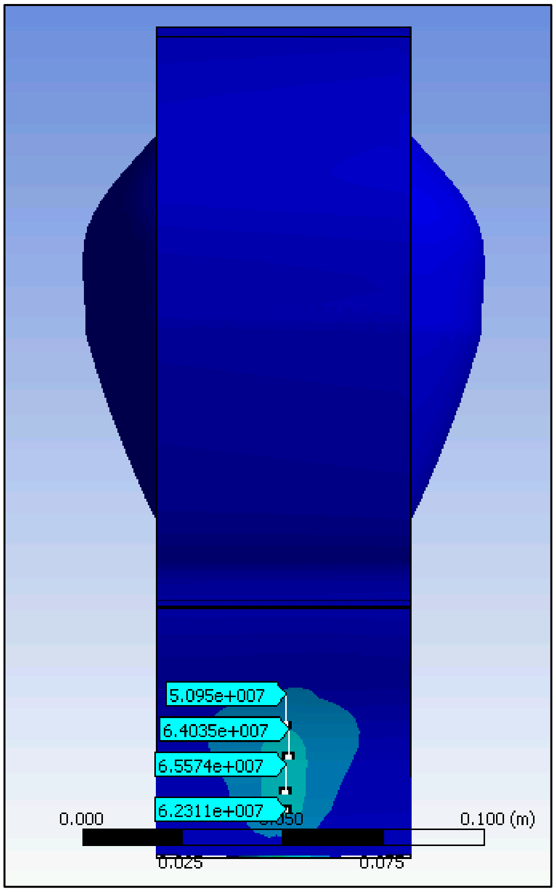


Figure 6-4: Model 2 stress from acceleration

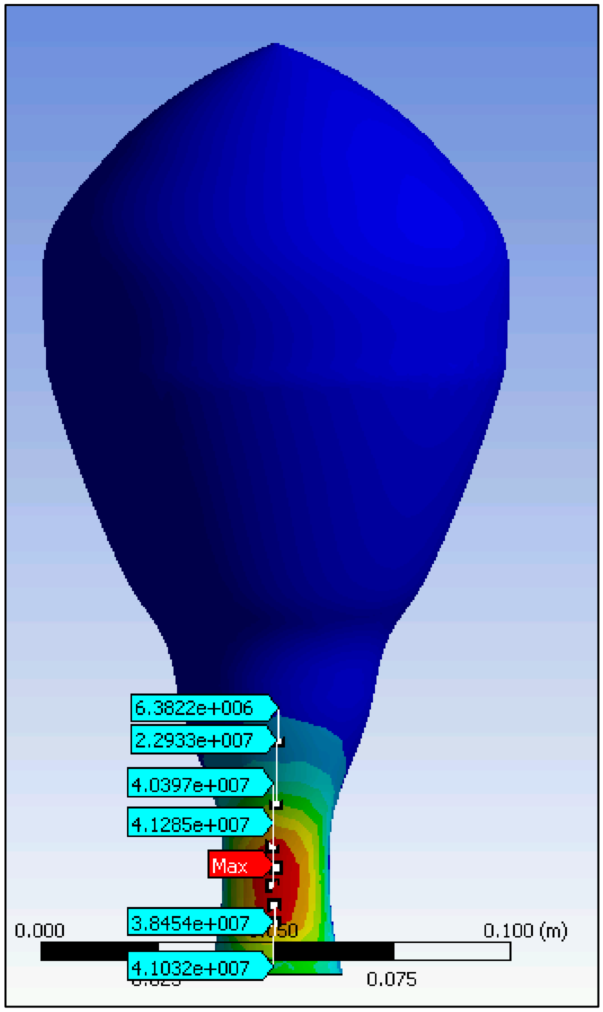


Figure 6-5: Model 3 stress from force

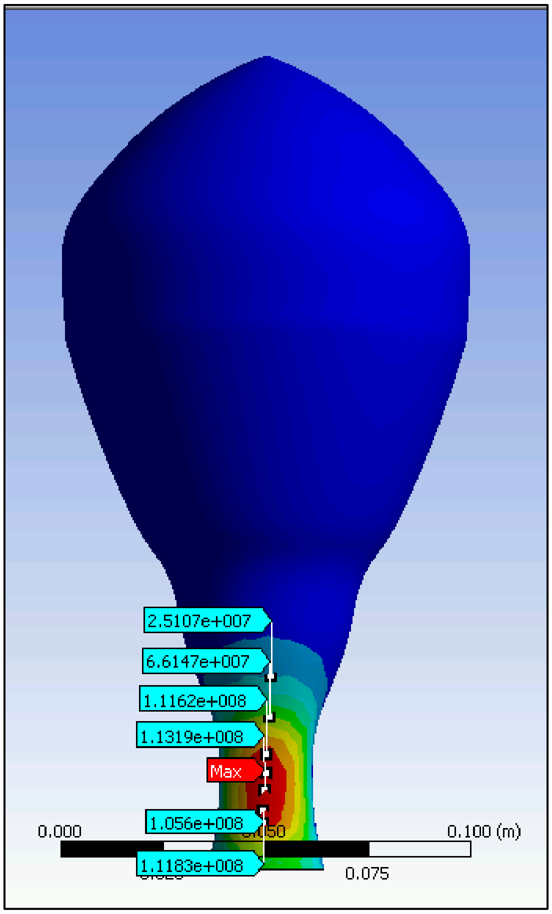


Figure 6-6: Model 3 stress from acceleration

A-2: Screenshots from ANSYS Dynamic Impact Analysis

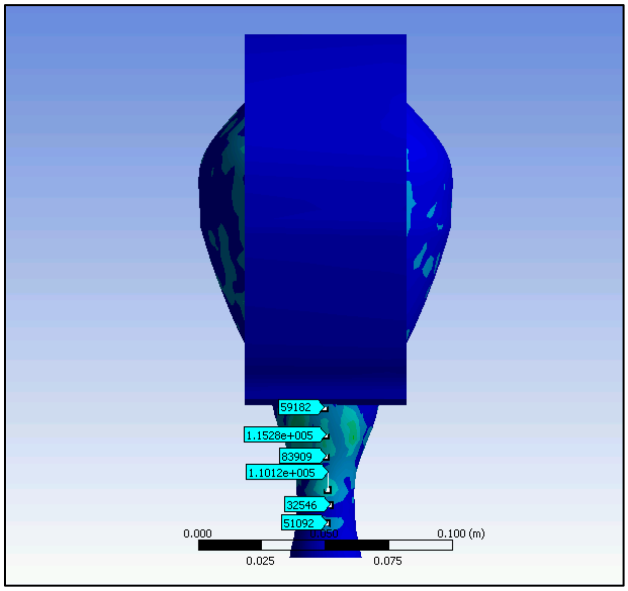


Figure 6-7: Model 1 Probed Acceleration Values

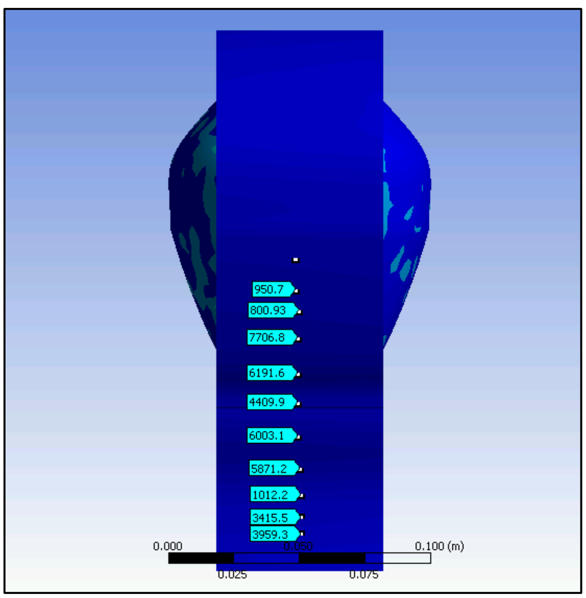


Figure 6-8: Model 2 Probed Acceleration Values

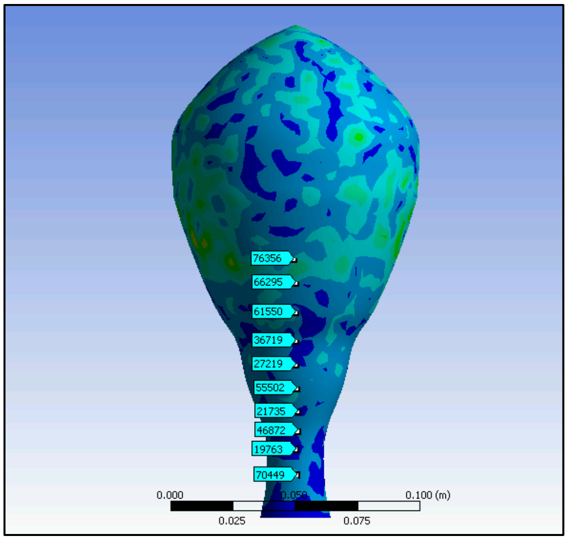
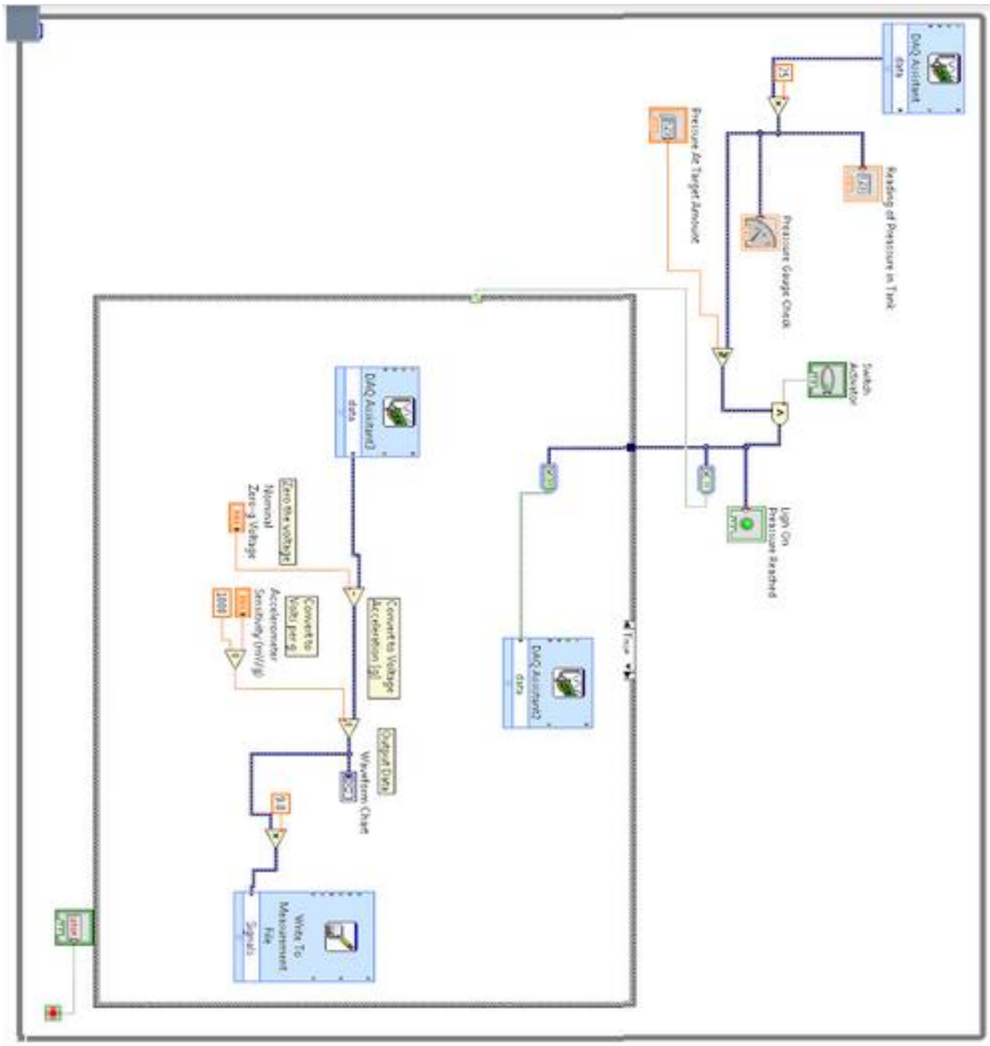


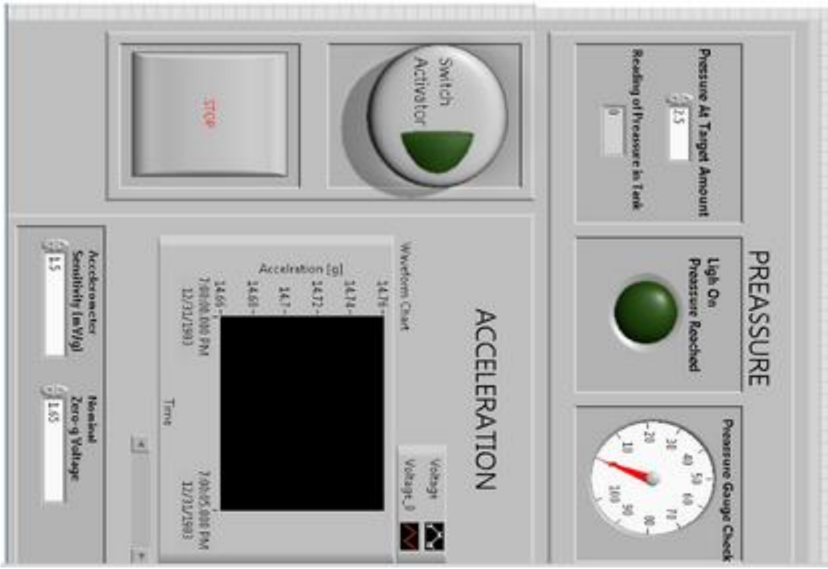
Figure 6-9: Model 3 Probed Acceleration Values

Appendix B: **Programs**

B-1 Screen-Shot LabView Program Back-End



B-2 Screen-Shot of LabView Program User Interface



Appendix C: **Calculations**

C-1: Original Calculations

Original Calculations

Average Weight of Professional Hockey Player $W_p := 210\,lbf = 934.127\,N$ $\quad W_{equipment} := 30\,lbf$

$W_{hp} := W_p + W_{equipment} = 1.068 \times 10^3\,N$

Speed of fast skater $v_{hp} := 30\,mph = 13.411\,\frac{m}{s}$ $\quad$ Mass $m_{hp} := \frac{W_{hp}}{g} = 108.862\,kg$

Average acceleration of an elite hockey player trying to speed up as fast as possible $A_{hps} := 4.375\,\frac{m}{s^2}$

Force from player to stationary player impact $F_I := m_{hp} \cdot A_{hps} = 476.272 \cdot N$

Determining Necessary Pressure and Stroke Length

$F_I = Pressure \cdot Area$

Volume of rod of Air cylinder w/ 1.5 in bore, 4 in stroke length

Stroke Length | Diameters | Cross-sect. Area

$L_{rod} := 4in$ $\quad D_{rod} := .44in$ $\quad Area_{rod} := \frac{D_{rod}^2}{4} \cdot \pi = 0.152 \cdot in^2$

$D_{Bore} := 1.5in$ $\quad Area_{Bore} := \frac{\pi}{4} \cdot \left(D_{Bore}^2\right) = 1.767 \cdot in^2$

$Volume_{rod} := Area_{rod} \cdot L_{rod} = 0.608 \cdot in^3$ $\quad Pressure := \frac{F_I}{Area_{Bore}} = 60.589 \cdot psi$

Stainless Steel rods Type 304 $\quad \delta_{ss} := .00803 \frac{kg}{cm^3} = 8.03 \times 10^3 \frac{kg}{m^3}$

Mass of Rod $\quad m_{rod} := \delta_{ss} \cdot Volume_{rod} = 0.08 kg$

Approximate mass of air cylinder $\quad m_{ac} := Area_{Bore} \cdot 11in \cdot \delta_{ss} = 2.558 kg$

Acceleration of Rod $\quad a_{rod} := \frac{F_I}{m_{rod}} = 5.951 \times 10^3 \frac{m}{s^2}$

$$761.5 \frac{m}{s^2} \cdot m_{head} = 3.351 \times 10^3 N$$

Moments of Inertia from Chalmers, Applied Mechanics, Master's Thesis 2010

$I_x := 204.117 kg \cdot cm^2$ $\quad I_y := 232.888 kg \cdot cm^2$ $\quad I_z := 150.832 kg \cdot cm^2$

Distance from Occipital Condyle (OC, point about which head rotates) to frankfort line (x-axis in reference frame) frankfort line is imaginary line connecting the upper margin of the aditory meatus (AM, external ear canal) to the lower orbital margin (cavity containing eyeball) Chalmers et al.

$d_{AMx} := 8mm$ $\quad d_{AMz} := 35mm$

Distance from OC to center of gravity (CG) from Chalmer et al.

$d_{CGx} := 13mm$ $\quad d_{CGz} := 55mm$

$$d_{CG} := \sqrt{d_{CGx}^2 + d_{CGz}^2} = 56.515 \cdot mm$$

$m_{head} := 4.4kg$ }from Chalmers et al.

Impulse equals Change in momentum equation $\quad Pressure \cdot Area_{Bore} \cdot t_I = \left(m_{hp} + m_{rod}\right) v_f$

Head | Player

Change in Velocity $\quad \Delta v_{Ih} := \frac{Pressure \cdot Area_{Bore} \cdot t_I}{\left(m_{head} + m_{rod}\right)} = 1.276 \frac{m}{s}$ $\quad \Delta v_{Ip} := \frac{Pressure \cdot Area_{Bore} \cdot t_I}{\left(m_{hp} + m_{rod}\right)} = 0.052 \cdot \frac{m}{s}$

Initial Acceleration $\quad a_{hi} := \frac{Pressure \cdot Area_{Bore}}{\left(m_{head} + m_{rod}\right)} = 106.31 \frac{m}{s^2}$ $\quad a_{rp} := \frac{F_I}{\left(m_{rod} + m_{hp}\right)} = 4.372 \frac{m}{s^2}$

Initial Angular Acceleration $\quad \alpha_{hi} := \frac{a_{hi}}{d_{CGz}} = 1.933 \times 10^3 \frac{1}{s^2}$

set-up/initial distance between head and air cylinder $d_i := 3.75\text{in}$

Time before impact $t_{BI} := \left(\frac{2 \cdot d_i}{a_{rod}}\right)^{.5} = 5.658 \cdot ms$

Velocity of rod right before impact $v_{rodBI} := a_{rod} \cdot t_{BI} = 33.67 \frac{m}{s}$

Impulse equals change in momentum $F_I \cdot t_I = (m_{rod} \cdot v_{rodBI}) - (m_{rod} + m_{head}) \cdot v_{rhc}$

$$(Pressure \cdot Area_{Bore} - k \cdot \theta) \cdot t_I = (m_{rod} \cdot v_{rodBI}) - (m_{rod} + m_{head}) \cdot v_{rhc}$$

Velocity of rod and head combined $v_{rhi} := \frac{(m_{rod} \cdot v_{rodBI})}{(m_{rod} + m_{head})} = 0.601 \frac{m}{s}$

Angular velocity of head after initial impact $\omega_{hi} := \frac{-v_{rhi}}{d_{CGz}} = -10.936 \cdot \frac{rad}{s}$

$k_{necks} := 50 \frac{N \cdot m}{rad}$ }from http://www.cs.ucla.edu/~dt/papers/siggraph06/siggraph06.pdf

$k_{damp} := .1s \cdot k_{necks} = 5\,s \cdot \frac{N \cdot m}{rad}$

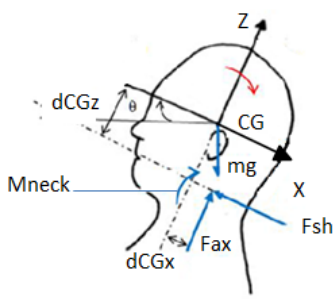
Damping Force

$F_{damp}(\theta) = k_{damp} \cdot \left(\frac{d}{dt}\theta\right)$

Spring Force

$F_{spring}(\theta) = k_{necks} \cdot \theta$

Peak moment of neck from Chalmers et al. $M_n := 4.78N \cdot m$



$\theta_k := 0deg, 1deg .. 95deg \qquad t := .000sec, .001sec .. t_I$

$$\Sigma F_x = Pressure \cdot Area_{Bore} \cdot \cos(\theta) + m_{head} \cdot g \cdot \cos\left(\frac{\pi}{2} - \theta\right) - F_{sh} = (m_{rod} + m_{head}) \cdot a_{rhx}$$

$$\Sigma F_z = Pressure \cdot Area_{Bore} \cdot \sin(\theta) - m_{head} \cdot g \cdot \sin\left(\frac{\pi}{2} - \theta\right) + F_{ax} = (m_{head}) \cdot a_{rhz}$$

$$\Sigma M_{OC} = (Pressure \cdot Area_{Bore} \cdot \sin(\theta)) \cdot d_{CGx} - (Pressure \cdot Area_{Bore} \cdot \cos(\theta)) \cdot d_{CGz} - m_{head} \cdot g \cdot d_{CGx} \cdot \sin\left(\frac{\pi}{2} - \theta\right) + m_{head} \cdot g \cdot d_{CGz} \cdot \cos\left(\frac{\pi}{2} - \theta\right) - k_{damp} \cdot \frac{d}{dt}\theta_k - k_{necks} \cdot \theta = I_y \cdot \alpha_I$$

$$I_y \cdot \frac{d^2}{dt^2}\theta_k + k_{damp} \cdot \frac{d}{dt}\theta_k + k_{necks} \cdot \theta_k = \left[(Pressure \cdot Area_{Bore} \cdot d_{CGx}) + m_{head} \cdot g \cdot d_{CGz}\right] \cdot \sin(\theta_k) - (Pressure \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}) \cdot \cos(\theta_k)$$

$$\frac{d^2}{dt^2}\theta_k + \frac{k_{damp}}{I_y} \cdot \frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y} \cdot \theta_k = \frac{(Pressure \cdot Area_{Bore} \cdot d_{CGx} + m_{head} \cdot g \cdot d_{CGz}) \cdot \sin(\Omega_F \cdot t) - (Pressure \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}) \cdot \cos((\Omega_F \cdot t))}{I_y}$$

Natural Angular Frequency

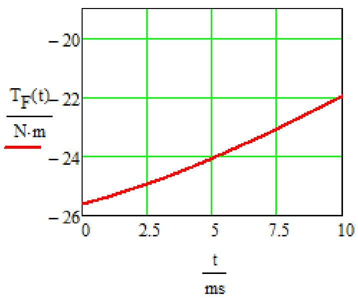
$$\Omega_n := \sqrt{\frac{k_{necks}}{I_y}} = 46.335\,\frac{1}{s}$$

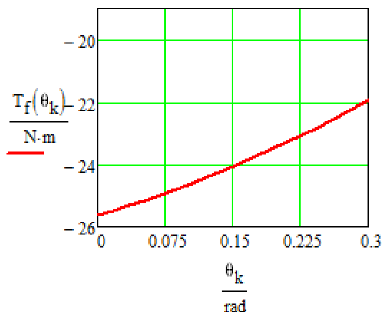
Finding Forcing angular frequency Ω.F

$\Omega_F := 30\,\frac{rad}{s}$ Kept trying different Ω.F until the graph of T.F and T.f looked the same

$$T_F(t) := \left[\left(Pressure \cdot Area_{Bore} \cdot d_{CGx}\right) + m_{head} \cdot g \cdot d_{CGz}\right] \cdot \sin\left(\Omega_F \cdot t\right) \ldots + \left[\left(-Pressure \cdot Area_{Bore} \cdot d_{CGz}\right) + m_{head} \cdot g \cdot d_{CGx}\right] \cdot \cos\left(\Omega_F \cdot t\right)$$

$$T_f(\theta_k) := \left[\left(Pressure \cdot Area_{Bore} \cdot d_{CGx}\right) + m_{head} \cdot g \cdot d_{CGz}\right] \cdot \sin\left(\theta_k\right) \ldots + \left[\left(-Pressure \cdot Area_{Bore} \cdot d_{CGz}\right) + m_{head} \cdot g \cdot d_{CGx}\right] \cdot \cos\left(\theta_k\right)$$





when T= -21J, θ.k = .1449 rad, t=4.83ms
when T=-20.001J, θ.k=.2295rad, t=7.65ms

$T_F(4.83ms) = -24.129\,J$ $T_f(.1449rad) = -24.129\,J$

$T_F(7.65ms) = -23.014\,J$ $T_f(.2295rad) = -23.014\,J$

Complimentary Solution (Left side of equation)

$$I_y \cdot \frac{d^2}{dt^2}\theta_k + k_{damp} \cdot \frac{d}{dt}\theta_k + k_{necks} \cdot \theta_k$$

$$\frac{d^2}{dt^2}\theta_k + \frac{k_{damp}}{I_y} \cdot \frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y} \cdot \theta_k = 0$$

$$r_1 := \frac{-\frac{k_{damp}}{I_y} + \sqrt{\left(\frac{k_{damp}}{I_y}\right)^2 - 4 \cdot \frac{k_{necks}}{I_y}}}{2} = -10.515\,\frac{1}{s}$$

$$r_2 := \frac{-\frac{k_{damp}}{I_y} - \sqrt{\left(\frac{k_{damp}}{I_y}\right)^2 - 4 \cdot \frac{k_{necks}}{I_y}}}{2} = -204.18\,\frac{1}{s}$$

$$\theta_I(t) = c_1 \cdot e^{r_1 \cdot t} + c_2 \cdot e^{r_2 \cdot t}$$

Particular Solution (right side of equation)

$$\left[\left(Pressure \cdot Area_{Bore} \cdot d_{CGx}\right) + m_{head} \cdot g \cdot d_{CGz}\right] \cdot \sin\left(\theta_k\right) - \left(Pressure \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right) \cdot \cos\left(\theta_k\right)$$

Guess

$$A \cdot \sin(\Omega_F \cdot t) + B \cdot \cos(\Omega_F \cdot t)$$

$$\omega_k(t) = A \cdot \Omega_F \cdot \cos(\Omega_F \cdot t) - B \cdot \Omega_F \cdot \sin(\Omega_F \cdot t)$$

$$\alpha_k(t) = -A \cdot \Omega_F^{2} \cdot \sin(\Omega_F \cdot t) - B \cdot \Omega_F^{2} \cdot \cos(\Omega_F \cdot t)$$

$$\frac{d^2}{dt^2}\theta_k + \frac{k_{damp}}{I_y} \cdot \frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y} \cdot \theta_k = -A \cdot \Omega_F^{2} \cdot \sin(\Omega_F \cdot t) - B \cdot \Omega_F^{2} \cdot \cos(\Omega_F \cdot t) + \frac{k_{damp}}{I_y} \cdot \left(A \cdot \Omega_F \cdot \cos(\Omega_F \cdot t) - B \cdot \Omega_n \cdot \sin(\Omega_F \cdot t)\right) + \frac{k_{necks}}{I_y} \cdot \left(A \cdot \sin(\Omega_F \cdot t) + B \cdot \cos(\Omega_F \cdot t)\right)$$

Rearrange

$$\left(-A \cdot \Omega_F^{2} - \frac{k_{damp}}{I_y} \cdot B \cdot \Omega_F + \frac{k_{necks}}{I_y} \cdot A\right) \cdot \sin(\Omega_F \cdot t) + \left(-B \cdot \Omega_n^{2} + \frac{k_{damp}}{I_y} \cdot A \cdot \Omega_F + \frac{k_{necks}}{I_y} \cdot B\right) \cdot \cos(\Omega_F \cdot t) = \frac{\begin{pmatrix} Pressure \cdot Area_{Bore} \cdot d_{CGx} \cdots \\ + m_{head} \cdot g \cdot d_{CGz} \end{pmatrix} \cdot \sin(\theta_k) - \begin{pmatrix} Pressure \cdot Area_{Bore} \cdot d_{CGz} \cdots \\ + m_{head} \cdot g \cdot d_{CGx} \end{pmatrix} \cdot \cos(\theta_k)}{I_y}$$

$$-A \cdot \Omega_F^{2} - \frac{k_{damp}}{I_y} \cdot B \cdot \Omega_F + \frac{k_{necks}}{I_y} \cdot A = \left[\frac{(Pressure \cdot Area_{Bore} \cdot d_{CGx}) + m_{head} \cdot g \cdot d_{CGz}}{I_y}\right]$$

$$-B \cdot \Omega_F^{2} + \frac{k_{damp}}{I_y} \cdot A \cdot \Omega_F + \frac{k_{necks}}{I_y} \cdot B = \frac{-Pressure \cdot Area_{Bore} \cdot d_{CGz} - m_{head} \cdot g \cdot d_{CGx}}{I_y}$$

$$\left(\frac{k_{necks}}{I_y} - \Omega_F^{2}\right) \cdot B + \frac{k_{damp}}{I_y} \cdot \Omega_F \cdot A = \frac{-Pressure \cdot Area_{Bore} \cdot d_{CGz} - m_{head} \cdot g \cdot d_{CGx}}{I_y}$$

$$\left(\frac{k_{necks}}{I_y} - \Omega_F^{2}\right) \cdot A - \frac{k_{damp}}{I_y} \cdot \Omega_F \cdot B = \frac{\left[(Pressure \cdot Area_{Bore} \cdot d_{CGx}) + m_{head} \cdot g \cdot d_{CGz}\right]}{I_y}$$

$$\left(k_{necks} - I_y \cdot \Omega_F^{2}\right) \cdot A - k_{damp} \cdot \Omega_F \cdot B = \left[(Pressure \cdot Area_{Bore} \cdot d_{CGx}) + m_{head} \cdot g \cdot d_{CGz}\right]$$

$$B_c = \frac{\left[(Pressure \cdot Area_{Bore} \cdot d_{CGx}) + m_{head} \cdot g \cdot d_{CGz}\right] - \left(k_{necks} - I_y \cdot \Omega_F^{2}\right) \cdot A}{-(k_{damp} \cdot \Omega_F)}$$

$$\left(k_{necks} - I_y \cdot \Omega_F^{2}\right) \cdot \left[\frac{\left[(Pressure \cdot Area_{Bore} \cdot d_{CGx}) + m_{head} \cdot g \cdot d_{CGz}\right] - \left(k_{necks} - I_y \cdot \Omega_F^{2}\right) \cdot A}{-(k_{damp} \cdot \Omega_F)}\right] + k_{damp} \cdot \Omega_F \cdot A = -Pressure \cdot Area_{Bore} \cdot d_{CGz} - m_{head} \cdot g \cdot d_{CGx}$$

$$\left(k_{necks} - I_y \cdot \Omega_F^{2}\right) \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGx} + m_{head} \cdot g \cdot d_{CGz}\right) - \left(k_{necks} - I_y \cdot \Omega_F^{2}\right)^2 \cdot A - (k_{damp} \cdot \Omega_F)^2 \cdot A = -(k_{damp} \cdot \Omega_F) \cdot \left(-Pressure \cdot Area_{Bore} \cdot d_{CGz} - m_{head} \cdot g \cdot d_{CGx}\right)$$

$$-\left(k_{necks} - I_y \cdot \Omega_F^{2}\right)^2 \cdot A - (k_{damp} \cdot \Omega_F)^2 \cdot A = (k_{damp} \cdot \Omega_F) \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right) - \left(k_{necks} - I_y \cdot \Omega_F^{2}\right) \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGx} + m_{head} \cdot g \cdot d_{CGz}\right)$$

$$A_c \cdot \left[-\left(k_{necks} - I_y \cdot \Omega_F^{2}\right)^2 - (k_{damp} \cdot \Omega_F)^2\right] = (k_{damp} \cdot \Omega_F) \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right) - \left(k_{necks} - I_y \cdot \Omega_F^{2}\right) \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGx} + m_{head} \cdot g \cdot d_{CGz}\right)$$

$$A_c := \frac{(k_{damp} \cdot \Omega_F) \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right) - \left(k_{necks} - I_y \cdot \Omega_F^{2}\right) \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGx} + m_{head} \cdot g \cdot d_{CGz}\right)}{-\left[\left(k_{necks} - I_y \cdot \Omega_F^{2}\right)^2 + (k_{damp} \cdot \Omega_F)^2\right]} = -0.161$$

$$B_c := \frac{\left[(Pressure \cdot Area_{Bore} \cdot d_{CGx}) + m_{head} \cdot g \cdot d_{CGz}\right] - \left(k_{necks} - I_y \cdot \Omega_F^{2}\right) \cdot A_c}{-(k_{damp} \cdot \Omega_F)} = -0.088$$

Solving for c.1 and c.2 using initial values

Using angular displacement at time 0 equals 0 deg
and angular velocity at time 0 equals initial angular velocity solved for above (ω.hi)

$$\theta_I(t) = \left(c_1 \cdot e^{r_1 \cdot t} + c_2 \cdot e^{r_2 \cdot t}\right) + \left(A_c \cdot \sin(\Omega_F \cdot t) + B_c \cdot \cos(\Omega_F \cdot t)\right)$$

$$\theta_I(0ms) = (c_1 + c_2) + (B_c \cdot \cos(0)) = 0deg \qquad c_1 = -(B_c \cdot \cos(0)) - c_2$$

$$\omega_{hi} = -10.936 \frac{1}{s}$$

$$\omega_I(t) = \left(c_1 \cdot r_1 \cdot e^{r_1 \cdot t} + c_2 \cdot r_2 \cdot e^{r_2 \cdot t}\right) + \left(A_c \cdot \Omega_F \cdot \cos(\Omega_F \cdot t) - B_c \cdot \Omega_F \cdot \sin(\Omega_F \cdot t)\right)$$

$$\omega_I(0ms) = (c_1 \cdot r_1 + c_2 \cdot r_2) + (A_c \cdot \Omega_F \cdot \cos(0)) = \omega_{hi}$$

$$\left[-(B_c \cdot \cos(0)) - c_2\right] \cdot r_1 + c_2 \cdot r_2 + A_c \cdot \Omega_F \cdot \cos(0) = \omega_{hi}$$

$$-(B_c \cdot r_1 \cdot \cos(0)) - c_2 \cdot r_1 + c_2 \cdot r_2 + A_c \cdot \Omega_F \cdot \cos(0) = \omega_{hi}$$

$$c_2 \cdot (r_2 - r_1) = \omega_{hi} - A_c \cdot \Omega_F \cdot \cos(0) + B_c \cdot r_1 \cdot \cos(0)$$

$$c_{2a} := \frac{(\omega_{hi} - A_c \cdot \Omega_F + B_c \cdot r_1)}{(r_2 - r_1)} = 0.027 \qquad c_{1a} := -(B_c \cdot \cos(0)) - c_{2a} = 0.062$$

For better results used initial acceleration instead of velocity

Using initial angular acceleration and initial angular displacement as initial conditions

$$\alpha_a(t) = c_1 \cdot r_1^2 \cdot e^{r_1 \cdot t} + c_2 \cdot r_2^2 \cdot e^{r_2 \cdot t} - A_c \cdot \Omega_F^2 \cdot \sin(\Omega_F \cdot t) - B_c \cdot \Omega_F^2 \cdot \cos(\Omega_F \cdot t)$$

$$\alpha_a(0ms) = c_1 \cdot r_1^2 + c_2 \cdot r_2^2 - B_c \cdot \Omega_F^2 = \alpha_{hi}$$

$$\left[-(B_c \cdot \cos(0)) - c_2\right] \cdot r_1^2 + c_2 \cdot r_2^2 - B_c \cdot \Omega_F^2 \cdot \cos(0) = \alpha_{hi}$$

$$-r_1^2 \cdot (B_c \cdot \cos(0)) - r_1^2 \cdot c_2 + c_2 \cdot r_2^2 = \alpha_{hi} + B_c \cdot \Omega_F^2$$

$$c_2 := \frac{\alpha_{hi} + B_c \cdot \Omega_F^2 + r_1^2 \cdot (B_c)}{(r_2^2 - r_1^2)} = 0.044$$

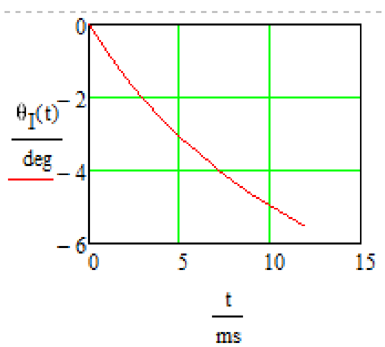
$$c_1 := -B_c - c_2 = 0.044$$

Necessary Stroke Length:

Complete Solution

$$\theta_I(t) := \left(c_1 \cdot e^{r_1 \cdot t} + c_2 \cdot e^{r_2 \cdot t}\right) + \left(A_c \cdot \sin(\Omega_F \cdot t) + B_c \cdot \cos(\Omega_F \cdot t)\right)$$

$$\theta_I(0ms) = 0$$

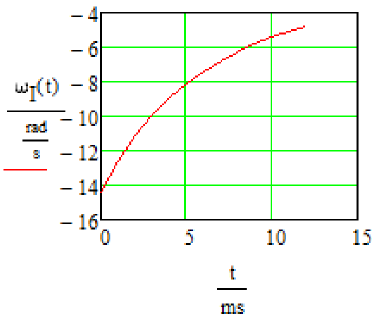


$$\theta_{EI} := \theta_I(t_I) = -5.551 \cdot deg$$

Distance cylinder is in contact with head

$$d_{EI} := \sqrt{2 \cdot d_{CG}^2 - 2 \cdot d_{CG}^2 \cdot \cos(\theta_{EI})} = 0.215 \cdot in$$

$$\omega_I(t) := c_1 \cdot r_1 \cdot e^{r_1 \cdot t} + c_2 \cdot r_2 \cdot e^{r_2 \cdot t} + A_c \cdot \Omega_F \cdot \cos(\Omega_F \cdot t) - B_c \cdot \Omega_F \cdot \sin(\Omega_F \cdot t)$$

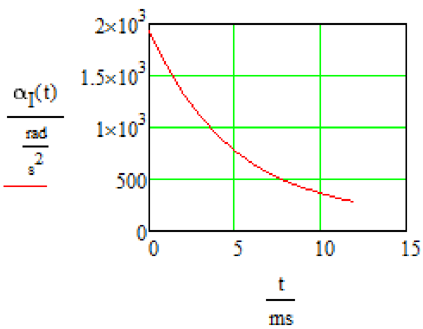


$\omega_I(0ms) = -14.354 \frac{1}{s}$

$\omega_{EI} := \omega_I(t_I) = -4.783 \cdot \frac{rad}{sec}$

$\omega_{hi} = -10.936 \frac{1}{s}$

$$\alpha_I(t) := c_1 \cdot r_1^2 \cdot e^{r_1 \cdot t} + c_2 \cdot r_2^2 \cdot e^{r_2 \cdot t} - A_c \cdot \Omega_F^2 \cdot \sin(\Omega_F \cdot t) - B_c \cdot \Omega_F^2 \cdot \cos(\Omega_F \cdot t)$$



$\alpha_{EI} := \alpha_I(t_I) = 289.303 \cdot \frac{rad}{s^2}$

$\alpha_I(0ms) = 1.933 \times 10^3 \frac{1}{s^2}$

$\alpha_{hi} = 1.933 \times 10^3 \frac{1}{s^2}$

After Impulse (i.e. after air cylinder is not in contact with head/helmet

After impulse ie after cylinder stops

Angle traveled through during impulse $\theta_{EI} = -5.551 \cdot deg$

Velocity $\omega_{EI} = -4.783 \frac{1}{s}$ $v_A := \omega_{EI} \cdot d_{CG} = -0.27 \frac{m}{s}$

$t_a := t_I, (t_I + .001s) .. .50s$ $\theta_{AI} := \theta_{EI}, \theta_{EI} + .25rad .. 2rad$

New Forcing Frequency

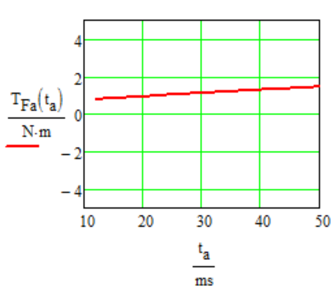
$\Omega_{Fa} := \frac{\theta_{EI}}{t_I} = -8.073 \frac{1}{s}$

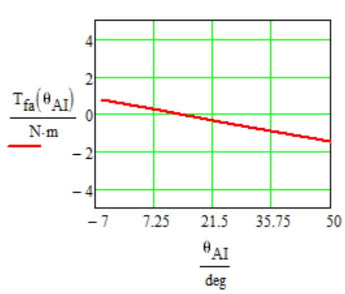
$T_{Fa}(t_a) := -m_{head} \cdot g \cdot d_{CGz} \cdot \sin(\Omega_{Fa} \cdot t_a) + (m_{head} \cdot g \cdot d_{CGx}) \cdot \cos(\Omega_{Fa} \cdot t_a)$

$T_{fa}(\theta_{AI}) := -(m_{head} \cdot g \cdot d_{CGz}) \cdot \sin(\theta_{AI}) + (m_{head} \cdot g \cdot d_{CGx}) \cdot \cos(\theta_{AI})$

$T_{Fa}(t_I) = 0.788 J$

$T_{fa}(\theta_{EI}) = 0.788 J$





$$-B_2\cdot\Omega_{Fa}^2 + \frac{k_{damp}}{I_y}\cdot A_2\cdot\Omega_{Fa} + \frac{k_{necks}}{I_y}\cdot B_2 = \left(\frac{-m_{head}\cdot g\cdot d_{CGx}}{I_y}\right)$$

$$-A_2\cdot I_y\cdot\Omega_F^2 - k_{damp}\cdot B_2\cdot\Omega_F + k_{necks}\cdot A_2 = m_{head}\cdot g\cdot d_{CGz}$$

$$\left(k_{necks} - I_y\cdot\Omega_F^2\right)\cdot A_2 - k_{damp}\cdot\Omega_F\cdot B_2 = m_{head}\cdot g\cdot d_{CGz}$$

$$-B_2\cdot I_y\cdot\Omega_F^2 + k_{damp}\cdot A_2\cdot\Omega_F + k_{necks}\cdot B_2 = -m_{head}\cdot g\cdot d_{CGx}$$

$$\left(k_{necks} - I_y\cdot\Omega_F^2\right)\cdot B_2 + k_{damp}\cdot\Omega_F\cdot A_2 = -m_{head}\cdot g\cdot d_{CGx}$$

$$B_2 = \frac{-m_{head}\cdot g\cdot d_{CGx} - k_{damp}\cdot\Omega_{Fa}\cdot A_2}{\left(k_{necks} - I_y\cdot\Omega_{Fa}^2\right)}$$

$$\left(k_{necks} - I_y\cdot\Omega_{Fa}^2\right)\cdot A_2 - k_{damp}\cdot\Omega_{Fa}\cdot\left(\frac{-m_{head}\cdot g\cdot d_{CGx} - k_{damp}\cdot\Omega_{Fa}\cdot A_2}{k_{necks} - I_y\cdot\Omega_{Fa}^2}\right) = m_{head}\cdot g\cdot d_{CGz}$$

$$\left(k_{necks} - I_y\cdot\Omega_{Fa}^2\right)^2\cdot A_2 + k_{damp}\cdot\Omega_{Fa}\cdot m_{head}\cdot g\cdot d_{CGx} + \left(k_{damp}\cdot\Omega_{Fa}\right)^2\cdot A_2 = \left(k_{necks} - I_y\cdot\Omega_{Fa}^2\right)\cdot\left(m_{head}\cdot g\cdot d_{CGz}\right)$$

$$\left[\left(k_{necks} - I_y\cdot\Omega_{Fa}^2\right)^2 + \left(k_{damp}\cdot\Omega_{Fa}\right)^2\right]\cdot A_2 + k_{damp}\cdot\Omega_{Fa}\cdot m_{head}\cdot g\cdot d_{CGx} = \left(k_{necks} - I_y\cdot\Omega_{Fa}^2\right)\cdot\left(m_{head}\cdot g\cdot d_{CGz}\right)$$

Sum of Moments After Impulse

$$M_{OCa} = \left(m_{head}\cdot g\cdot d_{CGz}\right)\cdot\sin(\theta) - \left(m_{head}\cdot g\cdot d_{CGx}\right)\cdot\cos(\theta) - k_{necks}\cdot\theta - k_{damp}\cdot\frac{d}{dt}\theta = I_y\cdot\left(\frac{d^2}{dt^2}\theta\right)$$

Complimentary Solution

$$\frac{d^2}{dt^2}\theta_a + \frac{k_{damp}}{I_y}\cdot\frac{d}{dt}\theta_a + \frac{k_{necks}}{I_y}\cdot\theta_a = \frac{\left(m_{head}\cdot g\cdot d_{CGz}\right)\cdot\sin\left(\Omega_{Fa}\cdot t_a\right) - \left(m_{head}\cdot g\cdot d_{CGx}\right)\cdot\cos\left(\Omega_{Fa}\cdot t_a\right)}{I_y}$$

$$\theta_{aI}\left(t_a\right) = c_3\cdot e^{r_1\cdot\left(t_a\right)} + c_4\cdot e^{r_2\cdot\left(t_a\right)}$$

Particular Solution

Guess

$$A_2\cdot\sin\left(\Omega_{Fa}\cdot t\right) + B_2\cdot\cos\left(\Omega_{Fa}\cdot t\right)$$

$$\omega_k(t) = A_2\cdot\Omega_{Fa}\cdot\cos\left(\Omega_{Fa}\cdot t\right) - B_2\cdot\Omega_{Fa}\cdot\sin\left(\Omega_{Fa}\cdot t\right)$$

$$\alpha_k(t) = -A_2\cdot\Omega_{Fa}^2\cdot\sin\left(\Omega_{Fa}\cdot t\right) - B_2\cdot\Omega_{Fa}^2\cdot\cos\left(\Omega_{Fa}\cdot t\right)$$

$$\frac{d^2}{dt^2}\theta_k + \frac{k_{damp}}{I_y}\cdot\frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y}\cdot\theta_k = -A\cdot\Omega_{Fa}^2\cdot\sin\left(\Omega_{Fa}\cdot t\right) - B\cdot\Omega_{Fa}^2\cdot\cos\left(\Omega_{Fa}\cdot t\right) + \frac{k_{damp}}{I_y}\cdot\left(A\cdot\Omega_{Fa}\cdot\cos\left(\Omega_{Fa}\cdot t\right) - B\cdot\Omega_{Fa}\cdot\sin\left(\Omega_{Fa}\cdot t\right)\right) + \frac{k_{necks}}{I_y}\cdot\left(A\cdot\sin\left(\Omega_{Fa}\cdot t\right) + B\cdot\cos\left(\Omega_{Fa}\cdot t\right)\right)$$

Rearrange

$$\left(-A_2\cdot\Omega_{Fa}^2 - \frac{k_{damp}}{I_y}\cdot B_2\cdot\Omega_{Fa} + \frac{k_{necks}}{I_y}\cdot A_2\right)\cdot\sin\left(\Omega_{Fa}\cdot t\right) + \left(-B_2\cdot\Omega_{Fa}^2 + \frac{k_{damp}}{I_y}\cdot A_2\cdot\Omega_{Fa} + \frac{k_{necks}}{I_y}\cdot B_2\right)\cdot\cos\left(\Omega_{Fa}\cdot t\right) = \frac{\left(m_{head}\cdot g\cdot d_{CGz}\right)\cdot\sin\left(\theta_k\right) - \left(m_{head}\cdot g\cdot d_{CGx}\right)\cdot\cos\left(\theta_k\right)}{I_y}$$

$$-A_2\cdot\Omega_{Fa}^2 - \frac{k_{damp}}{I_y}\cdot B_2\cdot\Omega_{Fa} + \frac{k_{necks}}{I_y}\cdot A_2 = \left(\frac{m_{head}\cdot g\cdot d_{CGz}}{I_y}\right)$$

$$\left(k_{necks} - I_y \cdot \Omega_F^2\right) \cdot A_2 - k_{damp} \cdot \Omega_F \cdot B_2 = m_{head} \cdot g \cdot d_{CGz}$$

$$-B_2 \cdot I_y \cdot \Omega_F^2 + k_{damp} \cdot A_2 \cdot \Omega_F + k_{necks} \cdot B_2 = -m_{head} \cdot g \cdot d_{CGx}$$

$$\left(k_{necks} - I_y \cdot \Omega_F^2\right) \cdot B_2 + k_{damp} \cdot \Omega_F \cdot A_2 = -m_{head} \cdot g \cdot d_{CGx}$$

$$B_2 = \frac{-m_{head} \cdot g \cdot d_{CGx} - k_{damp} \cdot \Omega_{Fa} \cdot A_2}{\left(k_{necks} - I_y \cdot \Omega_{Fa}^2\right)}$$

$$\left(k_{necks} - I_y \cdot \Omega_{Fa}^2\right) \cdot A_2 - k_{damp} \cdot \Omega_{Fa} \cdot \left(\frac{-m_{head} \cdot g \cdot d_{CGx} - k_{damp} \cdot \Omega_{Fa} \cdot A_2}{k_{necks} - I_y \cdot \Omega_{Fa}^2}\right) = m_{head} \cdot g \cdot d_{CGz}$$

$$\left(k_{necks} - I_y \cdot \Omega_{Fa}^2\right)^2 \cdot A_2 + k_{damp} \cdot \Omega_{Fa} \cdot m_{head} \cdot g \cdot d_{CGx} + \left(k_{damp} \cdot \Omega_{Fa}\right)^2 \cdot A_2 = \left(k_{necks} - I_y \cdot \Omega_{Fa}^2\right) \cdot \left(m_{head} \cdot g \cdot d_{CGz}\right)$$

$$\left[\left(k_{necks} - I_y \cdot \Omega_{Fa}^2\right)^2 + \left(k_{damp} \cdot \Omega_{Fa}\right)^2\right] \cdot A_2 + k_{damp} \cdot \Omega_{Fa} \cdot m_{head} \cdot g \cdot d_{CGx} = \left(k_{necks} - I_y \cdot \Omega_{Fa}^2\right) \cdot \left(m_{head} \cdot g \cdot d_{CGz}\right)$$

$$A_2 := \frac{\left(k_{necks} - I_y \cdot \Omega_{Fa}^2\right) \cdot \left(m_{head} \cdot g \cdot d_{CGz}\right) - k_{damp} \cdot \Omega_{Fa} \cdot m_{head} \cdot g \cdot d_{CGx}}{\left[\left(k_{necks} - I_y \cdot \Omega_{Fa}^2\right)^2 + \left(k_{damp} \cdot \Omega_{Fa}\right)^2\right]} = 0.035$$

$$B_2 := \frac{-m_{head} \cdot g \cdot d_{CGx} - k_{damp} \cdot \Omega_{Fa} \cdot A_2}{k_{necks} - I_y \cdot \Omega_{Fa}^2} = 0.017$$

Solving for c.3 and c.4 using initial conditions

$$\theta_{aI}(t_a) = c_3 \cdot e^{r_1 \cdot (t_a)} + c_4 \cdot e^{r_2 \cdot (t_a)} + A_2 \cdot \sin\left[\Omega_F \cdot (t_a)\right] + B_2 \cdot \cos\left[\Omega_F \cdot (t_a)\right]$$

$$\theta_{aI}(t_I) = c_3 \cdot e^{r_1 \cdot (t_I)} + c_4 \cdot e^{r_2 \cdot (t_I)} + A_2 \cdot \sin\left[\Omega_F \cdot (t_I)\right] + B_2 \cdot \cos\left[\Omega_F \cdot (t_I)\right] = \theta_{EI}$$

$$\omega_{aI}(t_a) = c_3 \cdot r_1 \cdot e^{r_1 \cdot (t_a)} + c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_a)} + A_2 \cdot \Omega_F \cdot \cos\left[\Omega_F \cdot (t_a)\right] - B_2 \cdot \Omega_F \cdot \sin\left[\Omega_F \cdot (t_a)\right]$$

$$\omega_{aI}(t_I) = c_3 \cdot r_1 \cdot e^{r_1 \cdot (t_I)} + c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_I)} + A_2 \cdot \Omega_F \cdot \cos\left[\Omega_F \cdot (t_I)\right] - B_2 \cdot \Omega_F \cdot \sin\left[\Omega_F \cdot (t_I)\right] = \omega_{EI}$$

$$c_3 = \frac{\theta_{EI} - A_2 \cdot \sin\left[\Omega_{Fa} \cdot (t_I)\right] - B_2 \cdot \cos\left[\Omega_{Fa} \cdot (t_I)\right] - c_4 \cdot e^{r_2 \cdot (t_I)}}{e^{r_1 \cdot (t_I)}}$$

$$c_3 = \frac{\omega_{EI} - A_2 \cdot \Omega_{Fa} \cdot \cos\left[\Omega_{Fa} \cdot (t_I)\right] + B_2 \cdot \Omega_{Fa} \cdot \sin\left[\Omega_{Fa} \cdot (t_I)\right] - c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_I)}}{r_1 \cdot e^{r_1 \cdot (t_I)}}$$

$$\frac{\omega_{EI} - A_2 \cdot \Omega_{Fa} \cdot \cos\left[\Omega_{Fa} \cdot (t_I)\right] + B_2 \cdot \Omega_{Fa} \cdot \sin\left[\Omega_{Fa} \cdot (t_I)\right] - c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_I)}}{r_1} = \theta_{EI} - A_2 \cdot \sin\left[\Omega_{Fa} \cdot (t_I)\right] - B_2 \cdot \cos\left[\Omega_{Fa} \cdot (t_I)\right] - c_4 \cdot e^{r_2 \cdot (t_I)}$$

$$\omega_{EI} - A_2 \cdot \Omega_{Fa} \cdot \cos\left[\Omega_{Fa} \cdot (t_I)\right] + B_2 \cdot \Omega_{Fa} \cdot \sin\left[\Omega_{Fa} \cdot (t_I)\right] - c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_I)} = r_1 \cdot \theta_{EI} - r_1 \cdot A_2 \cdot \sin\left[\Omega_{Fa} \cdot (t_I)\right] - r_1 \cdot B_2 \cdot \cos\left[\Omega_{Fa} \cdot (t_I)\right] - c_4 \cdot r_1 \cdot e^{r_2 \cdot (t_I)}$$

$$-c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_I)} + c_4 \cdot r_1 \cdot e^{r_2 \cdot (t_I)} = r_1 \cdot \theta_{EI} - r_1 \cdot A_2 \cdot \sin\left[\Omega_{Fa} \cdot (t_I)\right] - r_1 \cdot B_2 \cdot \cos\left[\Omega_{Fa} \cdot (t_I)\right] - \left[\omega_{EI} - A_2 \cdot \Omega_{Fa} \cdot \cos\left[\Omega_{Fa} \cdot (t_I)\right] + B_2 \cdot \Omega_{Fa} \cdot \sin\left[\Omega_{Fa} \cdot (t_I)\right]\right]$$

$$\alpha_{aI}(t_I) = r_1^2 \cdot \theta_{EI} - r_1^2 \cdot A_2 \cdot \sin[\Omega_{Fa} \cdot (t_I)] - r_1^2 \cdot B_2 \cdot \cos[\Omega_{Fa} \cdot (t_I)] - r_1^2 \cdot c_4 \cdot e^{r_2 \cdot (t_I)} + c_4 \cdot r_2^2 \cdot e^{r_2 \cdot (t_I)} - A_2 \cdot \Omega_{Fa}^2 \cdot \sin[\Omega_{Fa} \cdot (t_I)] - B_2 \cdot \Omega_{Fa}^2 \cdot \cos[\Omega_{Fa} \cdot (t_I)] = \alpha_{EI}$$

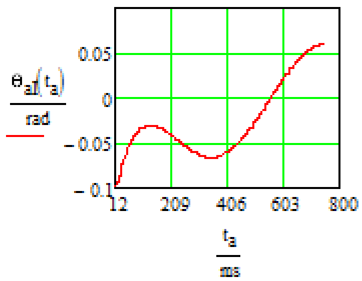
$$-r_1^2 \cdot c_4 \cdot e^{r_2 \cdot (t_I)} + c_4 \cdot r_2^2 \cdot e^{r_2 \cdot (t_I)} = \alpha_{EI} + A_2 \cdot \Omega_{Fa}^2 \cdot \sin[\Omega_{Fa} \cdot (t_I)] + B_2 \cdot \Omega_{Fa}^2 \cdot \cos[\Omega_{Fa} \cdot (t_I)] - r_1^2 \cdot \theta_{EI} + r_1^2 \cdot A_2 \cdot \sin[\Omega_{Fa} \cdot (t_I)] + r_1^2 \cdot B_2 \cdot \cos[\Omega_{Fa} \cdot (t_I)]$$

$$c_4 := \frac{\alpha_{EI} + A_2 \cdot \Omega_{Fa}^2 \cdot \sin[\Omega_{Fa} \cdot (t_I)] + B_2 \cdot \Omega_{Fa}^2 \cdot \cos[\Omega_{Fa} \cdot (t_I)] - r_1^2 \cdot \theta_{EI} + r_1^2 \cdot A_2 \cdot \sin[\Omega_{Fa} \cdot (t_I)] + r_1^2 \cdot B_2 \cdot \cos[\Omega_{Fa} \cdot (t_I)]}{\left[r_2^2 \cdot e^{r_2 \cdot (t_I)} - r_1^2 \cdot e^{r_2 \cdot (t_I)}\right]} = 0.086$$

$$c_3 := \frac{\theta_{EI} - A_2 \cdot \sin[\Omega_{Fa} \cdot (t_I)] - B_2 \cdot \cos[\Omega_{Fa} \cdot (t_I)] - c_4 \cdot e^{r_2 \cdot (t_I)}}{e^{r_1 \cdot (t_I)}} = -0.183$$

Determining at what time head returns to zero

$$\theta_{aI}(t_a) := c_3 \cdot e^{r_1 \cdot (t_a)} + c_4 \cdot e^{r_2 \cdot (t_a)} + A_2 \cdot \sin[\Omega_{Fa} \cdot (t_a)] + B_2 \cdot \cos[\Omega_{Fa} \cdot (t_a)]$$



$\theta_{raI} := \theta_{aI}(t_I) = -5.551 \cdot deg$ $\theta_{EI} = -5.551 \cdot deg$

$\theta_{aI}(.502s) = -1.489 \cdot deg$

$\theta_{aI}(2ms) = -3.633 \cdot deg$

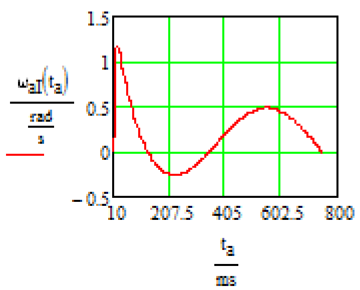
Guess $t_g := 350ms$

Given

$$\left[c_3 \cdot e^{r_1 \cdot (t_g)} + c_4 \cdot e^{r_2 \cdot (t_g)} + A_2 \cdot \sin[\Omega_{Fa} \cdot (t_g)] + B_2 \cdot \cos[\Omega_{Fa} \cdot (t_g)] = 0deg\right]$$

$\text{Find}(t_g) = 8.727\,s$

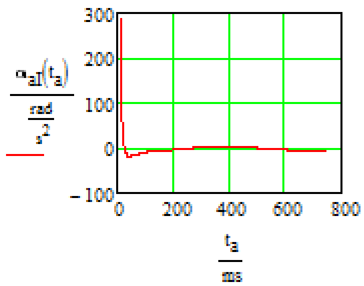
$$\omega_{aI}(t_a) := c_3 \cdot r_1 \cdot e^{r_1 \cdot (t_a)} + c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_a)} + A_2 \cdot \Omega_{Fa} \cdot \cos[\Omega_{Fa} \cdot (t_a)] - B_2 \cdot \Omega_{Fa} \cdot \sin[\Omega_{Fa} \cdot (t_a)]$$



$\omega_{EI} = -4.783 \cdot \frac{rad}{sec}$

$\omega_{raI} := \omega_{aI}(t_I) = 0.018\, \frac{1}{s} \cdot rad$

$$\alpha_{aI}(t_a) := c_3 \cdot r_1^{2} \cdot e^{r_1 \cdot (t_a)} + c_4 \cdot r_2^{2} \cdot e^{r_2 \cdot (t_a)} - A_2 \cdot \Omega_{Fa}^{2} \cdot \sin\left[\Omega_{Fa} \cdot (t_a)\right] - B_2 \cdot \Omega_{Fa}^{2} \cdot \cos\left[\Omega_{Fa} \cdot (t_a)\right]$$



$$\alpha_{EI} = 289.303 \, \frac{1}{s^2}$$

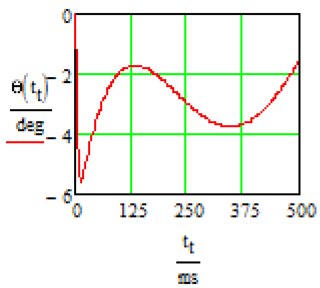
$$\alpha_{aI}(t_I) = 289.303 \, \frac{1}{s^2}$$

$$\alpha_{constant} := \alpha_{aI}(t_I) - \alpha_{EI} = 0 \, \frac{1}{s^2}$$

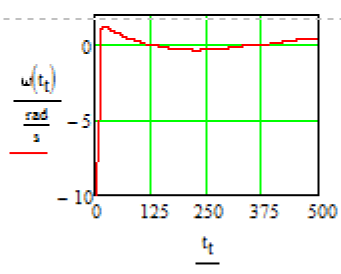
Before and After Impulse:

$t_t := 0s, .001s .. .5s$

$$\theta(t_t) := \begin{cases} c_1 \cdot e^{r_1 \cdot t_t} + c_2 \cdot e^{r_2 \cdot t_t} + A_c \cdot \sin(\Omega_F \cdot t_t) + B_c \cdot \cos(\Omega_F \cdot t_t) & \text{if } 0s \le t_t \le t_I \\ c_3 \cdot e^{r_1 \cdot (t_t)} + c_4 \cdot e^{r_2 \cdot (t_t)} + A_2 \cdot \sin\left[\Omega_{Fa} \cdot (t_t)\right] + B_2 \cdot \cos\left[\Omega_{Fa} \cdot (t_t)\right] & \text{if } t_I < t_t \le .5s \end{cases}$$



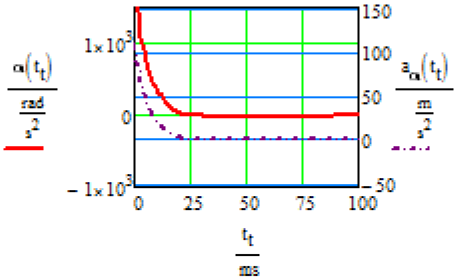
$$\omega(t_t) := \begin{cases} \left(c_1 \cdot r_1 \cdot e^{r_1 \cdot t_t} + c_2 \cdot r_2 \cdot e^{r_2 \cdot t_t} + A_c \cdot \Omega_F \cdot \cos(\Omega_F \cdot t_t) - B_c \cdot \Omega_F \cdot \sin(\Omega_F \cdot t_t)\right) & \text{if } 0s \le t_t \le t_I \\ \left(c_3 \cdot r_1 \cdot e^{r_1 \cdot t_t} + c_2 \cdot r_2 \cdot e^{r_2 \cdot t_t} + A_2 \cdot \Omega_{Fa} \cdot \cos(\Omega_{Fa} \cdot t_t) - B_2 \cdot \Omega_{Fa} \cdot \sin(\Omega_{Fa} \cdot t_t)\right) & \text{if } t_I < t_t \le .5s \end{cases}$$



$$\omega(t_I) = -4.783 \, \frac{1}{s}$$

$$\alpha(t_t) := \begin{cases} \left(c_1 \cdot r_1^{\,2} \cdot e^{r_1 \cdot t_t} + c_2 \cdot r_2^{\,2} \cdot e^{r_2 \cdot t_t} - A_c \cdot \Omega_F^{\,2} \cdot \sin(\Omega_F \cdot t_t) - B_c \cdot \Omega_F^{\,2} \cdot \cos(\Omega_F \cdot t_t)\right) & \text{if } 0ms \le t_t \le t_I \\ \left[c_3 \cdot r_1^{\,2} \cdot e^{r_1 \cdot (t_t)} + c_4 \cdot r_2^{\,2} \cdot e^{r_2 \cdot (t_t)} - A_2 \cdot \Omega_{Fa}^{\,2} \cdot \sin[\Omega_{Fa} \cdot (t_t)] - B_2 \cdot \Omega_{Fa}^{\,2} \cdot \cos[\Omega_{Fa} \cdot (t_t)]\right] & \text{if } t_I < t_t \le .5s \end{cases}$$

$$a_\alpha(t_t) := \alpha(t_t) \cdot d_{CG} \qquad \alpha(0ms) = 1.933 \times 10^3 \frac{1}{s^2} \cdot rad$$



$$a_\alpha(0ms) = 109.239 \frac{m}{s^2}$$

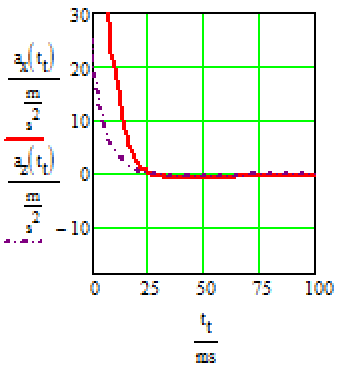
$$\frac{Pressure \cdot Area_{Bore}}{m_{rod} + m_{head}} = 106.31 \frac{m}{s^2}$$

$$\frac{93.021 \frac{m}{s^2}}{d_{CGz}} = 1.691 \times 10^3 \frac{1}{s^2}$$

Calculating HIP

HIP

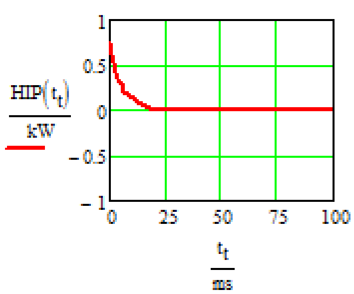
$$a_x(t_t) := \alpha(t_t) \cdot d_{CGz} \qquad a_z(t_t) := \alpha(t_t) \cdot d_{CGx}$$



$$a_{xmax} := a_x(0ms) = 106.31 \frac{m}{s^2}$$

$$a_{zmax} := a_z(0ms) = 25.128 \frac{m}{s^2}$$

$$HIP(t_t) := m_{head} \cdot a_x(t_t) \cdot \int_{0ms}^{25ms} a_x(t_t)\, dt_t + m_{head} \cdot a_z(t_t) \cdot \int_{0ms}^{25ms} a_z(t_t)\, dt_t + I_y \cdot \alpha(t_t) \cdot \int_{0ms}^{25ms} \alpha(t_t)\, dt_t$$



$$HIP(0ms) = 0.774 \cdot kW$$

HIP Too Low

C-2: Calculations; Using Impulse Equals Change in Momentum to Find Force

Using Impulse equals change in momentum to calculate force

Average Weight of Professional Hockey Player $W_p := 210lbf = 934.127N$ $W_{equipment} := 30lbf$

$W_{hp} := W_p + W_{equipment} = 1.068 \times 10^3 N$

Speed of fast skater $v_{hp} := 30mph = 13.411\frac{m}{s}$ Mass $m_{hp} := \frac{W_{hp}}{g} = 108.862kg$

Average acceleration of an elite hockey player trying to speed up as fast as possible $A_{hps} := 4.375\frac{m}{s^2}$

Typical Duration of Impact $t_I := .012s$ }Article stated hockey collision were all under 15ms

Impulse $Impulse = F_I \cdot t_I$ Impulse equals change in momentum $m_{hp} \cdot \Delta v = F_I \cdot t_I$

Change in Momentum $(m_{1i} \cdot v_{1i} + m_{2i} \cdot v_{2i}) = m_{1f} \cdot v_{1f} + m_{2f} \cdot v_{2f}$

$m_{1i} = m_{1f} = m_{2i} = m_{2f} = m_{hp}$

$m_{hp} \cdot (v_{1i} + v_{2i}) = m_{hp} \cdot (v_{1f} + v_{2f})$ Energy $E_I := \frac{1}{2} m_{hp} \cdot v_{hp}^2 = 9.79 \times 10^3 J$

Worst Case scenario both players speeding at top speeds directly into eachother, one comes to complete stop:

$v_{1i} = -v_{2i} = v_{hp}$ $v_{1f} := 0\frac{m}{s}$

$$F_I := \frac{m_{hp} \cdot (v_{hp} - v_{1f})}{t_I} = 1.217 \times 10^5 N$$

Complimentary Solution (Left side of equation)

$$I_y \cdot \frac{d^2}{dt^2}\theta_k + k_{damp} \cdot \frac{d}{dt}\theta_k + k_{necks} \cdot \theta_k$$

$$\frac{d^2}{dt^2}\theta_k + \frac{k_{damp}}{I_y} \cdot \frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y} \cdot \theta_k = 0$$

$$r_1 := \frac{-\frac{k_{damp}}{I_y} + \sqrt{\left(\frac{k_{damp}}{I_y}\right)^2 - 4 \cdot \frac{k_{necks}}{I_y}}}{2} = -10.515\frac{1}{s}$$

$$r_2 := \frac{-\frac{k_{damp}}{I_y} - \sqrt{\left(\frac{k_{damp}}{I_y}\right)^2 - 4 \cdot \frac{k_{necks}}{I_y}}}{2} = -204.18\frac{1}{s}$$

$$\theta_I(t) = c_1 \cdot e^{r_1 \cdot t} + c_2 \cdot e^{r_2 \cdot t}$$

Particular Solution (right side of equation)

$$\left[\left(\text{Pressure}\cdot\text{Area}_{\text{Bore}}\cdot d_{CGx}\right) + m_{head}\cdot g\cdot d_{CGz}\right]\cdot\sin(\theta_k) - \left(\text{Pressure}\cdot\text{Area}_{\text{Bore}}\cdot d_{CGz} + m_{head}\cdot g\cdot d_{CGx}\right)\cdot\cos(\theta_k)$$

Guess

$$A\cdot\sin(\Omega_F\cdot t) + B\cdot\cos(\Omega_F\cdot t)$$

$$\omega_k(t) = A\cdot\Omega_F\cdot\cos(\Omega_F\cdot t) - B\cdot\Omega_F\cdot\sin(\Omega_F\cdot t)$$

$$\alpha_k(t) = -A\cdot\Omega_F^{2}\cdot\sin(\Omega_F\cdot t) - B\cdot\Omega_F^{2}\cdot\cos(\Omega_F\cdot t)$$

$$\frac{d^2}{dt^2}\theta_k + \frac{k_{damp}}{I_y}\cdot\frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y}\cdot\theta_k = -A\cdot\Omega_F^{2}\cdot\sin(\Omega_F\cdot t) - B\cdot\Omega_F^{2}\cdot\cos(\Omega_F\cdot t) + \frac{k_{damp}}{I_y}\cdot\left(A\cdot\Omega_F\cdot\cos(\Omega_F\cdot t) - B\cdot\Omega_n\cdot\sin(\Omega_F\cdot t)\right) + \frac{k_{necks}}{I_y}\cdot\left(A\cdot\sin(\Omega_F\cdot t) + B\cdot\cos(\Omega_F\cdot t)\right)$$

Rearrange

$$\left(-A\cdot\Omega_F^{2} - \frac{k_{damp}}{I_y}\cdot B\cdot\Omega_F + \frac{k_{necks}}{I_y}\cdot A\right)\cdot\sin(\Omega_F\cdot t) + \left(-B\cdot\Omega_n^{2} + \frac{k_{damp}}{I_y}\cdot A\cdot\Omega_F + \frac{k_{necks}}{I_y}\cdot B\right)\cdot\cos(\Omega_F\cdot t) = \frac{\begin{pmatrix}F_I\cdot d_{CGx} \cdots \\ + m_{head}\cdot g\cdot d_{CGz}\end{pmatrix}\cdot\sin(\theta_k) - \begin{pmatrix}F_I\cdot d_{CGz} \cdots \\ + m_{head}\cdot g\cdot d_{CGx}\end{pmatrix}\cdot\cos(\theta_k)}{I_y}$$

$$-A\cdot\Omega_F^{2} - \frac{k_{damp}}{I_y}\cdot B\cdot\Omega_F + \frac{k_{necks}}{I_y}\cdot A = \left[\frac{\left(F_I\cdot d_{CGx}\right) + m_{head}\cdot g\cdot d_{CGz}}{I_y}\right]$$

$$-B\cdot\Omega_F^{2} + \frac{k_{damp}}{I_y}\cdot A\cdot\Omega_F + \frac{k_{necks}}{I_y}\cdot B = \frac{-F_I\cdot d_{CGz} - m_{head}\cdot g\cdot d_{CGx}}{I_y}$$

$$\left(\frac{k_{necks}}{I_y} - \Omega_F^{2}\right)\cdot B + \frac{k_{damp}}{I_y}\cdot\Omega_F\cdot A = \frac{-F_I\cdot d_{CGz} - m_{head}\cdot g\cdot d_{CGx}}{I_y}$$

$$\left(\frac{k_{necks}}{I_y} - \Omega_F^{2}\right)\cdot A - \frac{k_{damp}}{I_y}\cdot\Omega_F\cdot B = \frac{\left[\left(F_I\cdot d_{CGx}\right) + m_{head}\cdot g\cdot d_{CGz}\right]}{I_y}$$

$$\left(k_{necks} - I_y\cdot\Omega_F^{2}\right)\cdot A - k_{damp}\cdot\Omega_F\cdot B = \left[\left(F_I\cdot d_{CGx}\right) + m_{head}\cdot g\cdot d_{CGz}\right]$$

$$B_c = \frac{\left[\left(F_I\cdot d_{CGx}\right) + m_{head}\cdot g\cdot d_{CGz}\right] - \left(k_{necks} - I_y\cdot\Omega_F^{2}\right)\cdot A}{-\left(k_{damp}\cdot\Omega_F\right)}$$

$$\left(k_{necks} - I_y\cdot\Omega_F^{2}\right)\cdot\left[\frac{\left[\left(F_I\cdot d_{CGx}\right) + m_{head}\cdot g\cdot d_{CGz}\right] - \left(k_{necks} - I_y\cdot\Omega_F^{2}\right)\cdot A}{-\left(k_{damp}\cdot\Omega_F\right)}\right] + k_{damp}\cdot\Omega_F\cdot A = -F_I\cdot d_{CGz} - m_{head}\cdot g\cdot d_{CGx}$$

$$\left(k_{necks} - I_y\cdot\Omega_F^{2}\right)\cdot\left(-F_I\cdot d_{CGx} + m_{head}\cdot g\cdot d_{CGz}\right) - \left(k_{necks} - I_y\cdot\Omega_F^{2}\right)^{2}\cdot A - \left(k_{damp}\cdot\Omega_F\right)^{2}\cdot A = -\left(k_{damp}\cdot\Omega_F\right)\cdot\left(-F_I\cdot d_{CGz} - m_{head}\cdot g\cdot d_{CGx}\right)$$

$$-\left(k_{necks} - I_y\cdot\Omega_F^{2}\right)^{2}\cdot A - \left(k_{damp}\cdot\Omega_F\right)^{2}\cdot A = \left(k_{damp}\cdot\Omega_F\right)\cdot\left(F_I\cdot d_{CGz} + m_{head}\cdot g\cdot d_{CGx}\right) - \left(k_{necks} - I_y\cdot\Omega_F^{2}\right)\cdot\left(F_I\cdot d_{CGx} + m_{head}\cdot g\cdot d_{CGz}\right)$$

$$A_c\cdot\left[-\left(k_{necks} - I_y\cdot\Omega_F^{2}\right)^{2} - \left(k_{damp}\cdot\Omega_F\right)^{2}\right] = \left(k_{damp}\cdot\Omega_F\right)\cdot\left(\text{Pressure}\cdot\text{Area}_{\text{Bore}}\cdot d_{CGz} + m_{head}\cdot g\cdot d_{CGx}\right) - \left(k_{necks} - I_y\cdot\Omega_F^{2}\right)\cdot\left(\text{Pressure}\cdot\text{Area}_{\text{Bore}}\cdot d_{CGx} + m_{head}\cdot g\cdot d_{CGz}\right)$$

$$A_c := \frac{(k_{damp} \cdot \Omega_F) \cdot (F_I \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}) - (k_{necks} - I_y \cdot \Omega_F^2) \cdot (F_I \cdot d_{CGx} + m_{head} \cdot g \cdot d_{CGz})}{-\left[(k_{necks} - I_y \cdot \Omega_F^2)^2 + (k_{damp} \cdot \Omega_F)^2\right]} = -41.032$$

$$B_c := \frac{\left[(F_I \cdot d_{CGx}) + m_{head} \cdot g \cdot d_{CGz}\right] - (k_{necks} - I_y \cdot \Omega_F^2) \cdot A_c}{-(k_{damp} \cdot \Omega_F)} = -18.504$$

Solving for c.1 and c.2 using initial values

$$\theta_I(t) = \left(c_1 \cdot e^{r_1 \cdot t} + c_2 \cdot e^{r_2 \cdot t}\right) + \left(A_c \cdot \sin(\Omega_F \cdot t) + B_c \cdot \cos(\Omega_F \cdot t)\right)$$

$$\theta_I(0ms) = (c_1 + c_2) + (B_c \cdot \cos(0)) = 0deg \qquad c_1 = -(B_c \cdot \cos(0)) - c_2$$

$$\alpha_a(t) = c_1 \cdot r_1^2 \cdot e^{r_1 \cdot t} + c_2 \cdot r_2^2 \cdot e^{r_2 \cdot t} - A_c \cdot \Omega_F^2 \cdot \sin(\Omega_F \cdot t) - B_c \cdot \Omega_F^2 \cdot \cos(\Omega_F \cdot t)$$

$$\alpha_a(0ms) = c_1 \cdot r_1^2 + c_2 \cdot r_2^2 - B_c \cdot \Omega_F^2 = \alpha_{hi}$$

$$\left[-(B_c \cdot \cos(0)) - c_2\right] \cdot r_1^2 + c_2 \cdot r_2^2 - B_c \cdot \Omega_F^2 \cdot \cos(0) = \alpha_{hi}$$

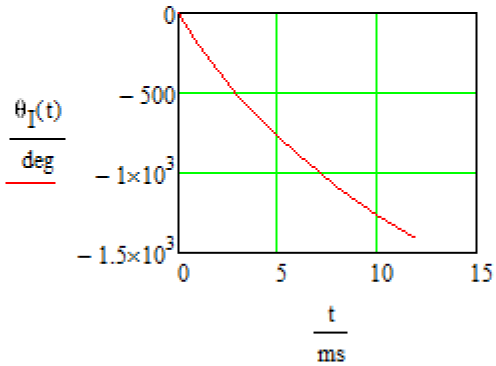
$$-r_1^2 \cdot (B_c \cdot \cos(0)) - r_1^2 \cdot c_2 + c_2 \cdot r_2^2 = \alpha_{hi} + B_c \cdot \Omega_F^2$$

$$c_2 := \frac{\alpha_{hi} + B_c \cdot \Omega_F^2 + r_1^2 \cdot (B_c)}{(r_2^2 - r_1^2)} = 11.426$$

$$c_1 := -B_c - c_2 = 7.078$$

Complete Solution

$$\theta_I(t) := \left(c_1 \cdot e^{r_1 \cdot t} + c_2 \cdot e^{r_2 \cdot t}\right) + \left(A_c \cdot \sin(\Omega_F \cdot t) + B_c \cdot \cos(\Omega_F \cdot t)\right)$$



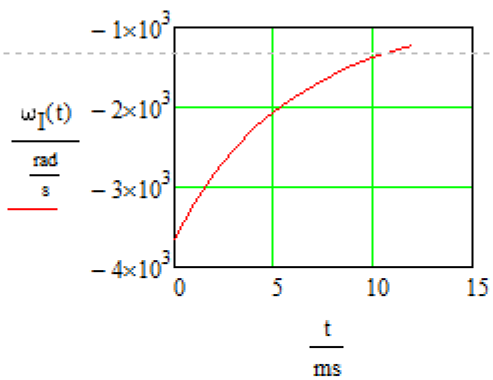
$$\theta_I(0ms) = 0$$

$$\theta_{EI} := \theta_I(t_I) = -1.406 \times 10^3 \cdot deg$$

Distance cylinder is in contact with head

$$d_{EI} := \sqrt{2 \cdot d_{CG}^2 - 2 \cdot d_{CG}^2 \cdot \cos(\theta_{EI})} = 1.285 \cdot in$$

$$\omega_I(t) := c_1 \cdot r_1 \cdot e^{r_1 \cdot t} + c_2 \cdot r_2 \cdot e^{r_2 \cdot t} + A_c \cdot \Omega_F \cdot \cos(\Omega_F \cdot t) - B_c \cdot \Omega_F \cdot \sin(\Omega_F \cdot t)$$

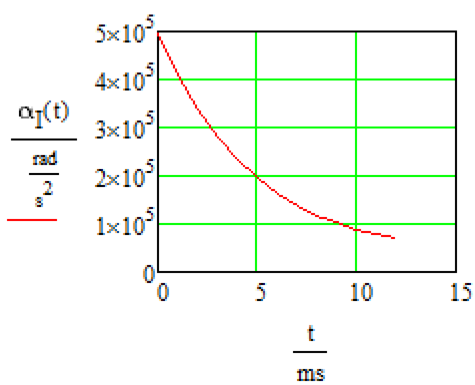


$$\omega_I(0ms) = -3.638 \times 10^3 \frac{1}{s}$$

$$\omega_{EI} := \omega_I(t_I) = -1.223 \times 10^3 \cdot \frac{rad}{sec}$$

$$\omega_{hi} = -174.792 \frac{1}{s}$$

$$\alpha_I(t) := c_1 \cdot r_1^2 \cdot e^{r_1 \cdot t} + c_2 \cdot r_2^2 \cdot e^{r_2 \cdot t} - A_c \cdot \Omega_F^2 \cdot \sin(\Omega_F \cdot t) - B_c \cdot \Omega_F^2 \cdot \cos(\Omega_F \cdot t)$$



$$\alpha_{EI} := \alpha_I(t_I) = 7.038 \times 10^4 \cdot \frac{rad}{s^2}$$

$$\alpha_I(0ms) = 4.938 \times 10^5 \frac{1}{s^2}$$

$$\alpha_{hi} = 4.938 \times 10^5 \frac{1}{s^2}$$

After Cylinder leaves contact with head/helmet

Sum of Moments After Impulse

$$M_{OCa} = (m_{head}\cdot g\cdot d_{CGz})\cdot\sin(\theta) - (m_{head}\cdot g\cdot d_{CGx})\cdot\cos(\theta) - k_{necks}\cdot\theta - k_{damp}\cdot\frac{d}{dt}\theta = I_y\cdot\left(\frac{d^2}{dt^2}\theta\right)$$

Complimentary Solution

$$\frac{d^2}{dt^2}\theta_a + \frac{k_{damp}}{I_y}\cdot\frac{d}{dt}\theta_a + \frac{k_{necks}}{I_y}\cdot\theta_a = \frac{(m_{head}\cdot g\cdot d_{CGz})\cdot\sin(\Omega_{Fa}\cdot t_a) - (m_{head}\cdot g\cdot d_{CGx})\cdot\cos(\Omega_{Fa}\cdot t_a)}{I_y}$$

$$\theta_{aI}(t_a) = c_3\cdot e^{r_1\cdot(t_a)} + c_4\cdot e^{r_2\cdot(t_a)}$$

Particular Solution

Guess

$$A_2\cdot\sin(\Omega_{Fa}\cdot t) + B_2\cdot\cos(\Omega_{Fa}\cdot t)$$

$$\omega_k(t) = A_2\cdot\Omega_{Fa}\cdot\cos(\Omega_{Fa}\cdot t) - B_2\cdot\Omega_{Fa}\cdot\sin(\Omega_{Fa}\cdot t)$$

$$\alpha_k(t) = -A_2\cdot\Omega_{Fa}^{2}\cdot\sin(\Omega_{Fa}\cdot t) - B_2\cdot\Omega_{Fa}^{2}\cdot\cos(\Omega_{Fa}\cdot t)$$

$$\frac{d^2}{dt^2}\theta_k + \frac{k_{damp}}{I_y}\cdot\frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y}\cdot\theta_k = -A\cdot\Omega_{Fa}^{2}\cdot\sin(\Omega_{Fa}\cdot t) - B\cdot\Omega_{Fa}^{2}\cdot\cos(\Omega_{Fa}\cdot t) + \frac{k_{damp}}{I_y}\cdot(A\cdot\Omega_{Fa}\cdot\cos(\Omega_{Fa}\cdot t) - B\cdot\Omega_{Fa}\cdot\sin(\Omega_{Fa}\cdot t)) + \frac{k_{necks}}{I_y}\cdot(A\cdot\sin(\Omega_{Fa}\cdot t) + B\cdot\cos(\Omega_{Fa}\cdot t))$$

Rearrange

$$\left(-A_2\cdot\Omega_{Fa}^{2} - \frac{k_{damp}}{I_y}\cdot B_2\cdot\Omega_{Fa} + \frac{k_{necks}}{I_y}\cdot A_2\right)\cdot\sin(\Omega_{Fa}\cdot t) + \left(-B_2\cdot\Omega_{Fa}^{2} + \frac{k_{damp}}{I_y}\cdot A_2\cdot\Omega_{Fa} + \frac{k_{necks}}{I_y}\cdot B_2\right)\cdot\cos(\Omega_{Fa}\cdot t) = \frac{(m_{head}\cdot g\cdot d_{CGz})\cdot\sin(\theta_k) - (m_{head}\cdot g\cdot d_{CGx})\cdot\cos(\theta_k)}{I_y}$$

$$A_2 := \frac{(k_{necks} - I_y\cdot\Omega_{Fa}^{2})\cdot(m_{head}\cdot g\cdot d_{CGz}) - k_{damp}\cdot\Omega_{Fa}\cdot m_{head}\cdot g\cdot d_{CGx}}{\left[(k_{necks} - I_y\cdot\Omega_{Fa}^{2})^2 + (k_{damp}\cdot\Omega_{Fa})^2\right]} = -2.35\times 10^{-5}$$

$$B_2 := \frac{-m_{head}\cdot g\cdot d_{CGx} - k_{damp}\cdot\Omega_{Fa}\cdot A_2}{k_{necks} - I_y\cdot\Omega_{Fa}^{2}} = 8.227\times 10^{-6}$$

$$-A_2 \cdot \Omega_{Fa}^{2} - \frac{k_{damp}}{I_y} \cdot B_2 \cdot \Omega_{Fa} + \frac{k_{necks}}{I_y} \cdot A_2 = \left(\frac{m_{head} \cdot g \cdot d_{CGz}}{I_y} \right)$$

$$-B_2 \cdot \Omega_{Fa}^{2} + \frac{k_{damp}}{I_y} \cdot A_2 \cdot \Omega_{Fa} + \frac{k_{necks}}{I_y} \cdot B_2 = \left(\frac{-m_{head} \cdot g \cdot d_{CGx}}{I_y} \right)$$

$$-A_2 \cdot I_y \cdot \Omega_F^{2} - k_{damp} \cdot B_2 \cdot \Omega_F + k_{necks} \cdot A_2 = m_{head} \cdot g \cdot d_{CGz}$$

$$\left(k_{necks} - I_y \cdot \Omega_F^{2} \right) \cdot A_2 - k_{damp} \cdot \Omega_F \cdot B_2 = m_{head} \cdot g \cdot d_{CGz}$$

$$-B_2 \cdot I_y \cdot \Omega_F^{2} + k_{damp} \cdot A_2 \cdot \Omega_F + k_{necks} \cdot B_2 = -m_{head} \cdot g \cdot d_{CGx}$$

$$\left(k_{necks} - I_y \cdot \Omega_F^{2} \right) \cdot B_2 + k_{damp} \cdot \Omega_F \cdot A_2 = -m_{head} \cdot g \cdot d_{CGx}$$

$$B_2 = \frac{-m_{head} \cdot g \cdot d_{CGx} - k_{damp} \cdot \Omega_{Fa} \cdot A_2}{\left(k_{necks} - I_y \cdot \Omega_{Fa}^{2} \right)}$$

$$\left(k_{necks} - I_y \cdot \Omega_{Fa}^{2} \right) \cdot A_2 - k_{damp} \cdot \Omega_{Fa} \cdot \left(\frac{-m_{head} \cdot g \cdot d_{CGx} - k_{damp} \cdot \Omega_{Fa} \cdot A_2}{k_{necks} - I_y \cdot \Omega_{Fa}^{2}} \right) = m_{head} \cdot g \cdot d_{CGz}$$

$$\left(k_{necks} - I_y \cdot \Omega_{Fa}^{2} \right)^{2} \cdot A_2 + k_{damp} \cdot \Omega_{Fa} \cdot m_{head} \cdot g \cdot d_{CGx} + \left(k_{damp} \cdot \Omega_{Fa} \right)^{2} \cdot A_2 = \left(k_{necks} - I_y \cdot \Omega_{Fa}^{2} \right) \cdot \left(m_{head} \cdot g \cdot d_{CGz} \right)$$

$$\left[\left(k_{necks} - I_y \cdot \Omega_{Fa}^{2} \right)^{2} + \left(k_{damp} \cdot \Omega_{Fa} \right)^{2} \right] \cdot A_2 + k_{damp} \cdot \Omega_{Fa} \cdot m_{head} \cdot g \cdot d_{CGx} = \left(k_{necks} - I_y \cdot \Omega_{Fa}^{2} \right) \cdot \left(m_{head} \cdot g \cdot d_{CGz} \right)$$

$$\theta_{aI}(t_a) = c_3 \cdot e^{r_1 \cdot (t_a)} + c_4 \cdot e^{r_2 \cdot (t_a)} + A_2 \cdot \sin[\Omega_F \cdot (t_a)] + B_2 \cdot \cos[\Omega_F \cdot (t_a)]$$

$$\theta_{aI}(t_I) = c_3 \cdot e^{r_1 \cdot (t_I)} + c_4 \cdot e^{r_2 \cdot (t_I)} + A_2 \cdot \sin[\Omega_F \cdot (t_I)] + B_2 \cdot \cos[\Omega_F \cdot (t_I)] = \theta_{EI}$$

$$\omega_{aI}(t_a) = c_3 \cdot r_1 \cdot e^{r_1 \cdot (t_a)} + c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_a)} + A_2 \cdot \Omega_F \cdot \cos[\Omega_F \cdot (t_a)] - B_2 \cdot \Omega_F \cdot \sin[\Omega_F \cdot (t_a)]$$

$$\omega_{aI}(t_I) = c_3 \cdot r_1 \cdot e^{r_1 \cdot (t_I)} + c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_I)} + A_2 \cdot \Omega_F \cdot \cos[\Omega_F \cdot (t_I)] - B_2 \cdot \Omega_F \cdot \sin[\Omega_F \cdot (t_I)] = \omega_{EI}$$

$$c_3 = \frac{\theta_{EI} - A_2 \cdot \sin[\Omega_{Fa} \cdot (t_I)] - B_2 \cdot \cos[\Omega_{Fa} \cdot (t_I)] - c_4 \cdot e^{r_2 \cdot (t_I)}}{e^{r_1 \cdot (t_I)}}$$

$$c_3 = \frac{\omega_{EI} - A_2 \cdot \Omega_{Fa} \cdot \cos[\Omega_{Fa} \cdot (t_I)] + B_2 \cdot \Omega_{Fa} \cdot \sin[\Omega_{Fa} \cdot (t_I)] - c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_I)}}{r_1 \cdot e^{r_1 \cdot (t_I)}}$$

$$\frac{\omega_{EI} - A_2 \cdot \Omega_{Fa} \cdot \cos[\Omega_{Fa} \cdot (t_I)] + B_2 \cdot \Omega_{Fa} \cdot \sin[\Omega_{Fa} \cdot (t_I)] - c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_I)}}{r_1} = \theta_{EI} - A_2 \cdot \sin[\Omega_{Fa} \cdot (t_I)] - B_2 \cdot \cos[\Omega_{Fa} \cdot (t_I)] - c_4 \cdot e^{r_2 \cdot (t_I)}$$

$$\omega_{EI} - A_2 \cdot \Omega_{Fa} \cdot \cos[\Omega_{Fa} \cdot (t_I)] + B_2 \cdot \Omega_{Fa} \cdot \sin[\Omega_{Fa} \cdot (t_I)] - c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_I)} = r_1 \cdot \theta_{EI} - r_1 \cdot A_2 \cdot \sin[\Omega_{Fa} \cdot (t_I)] - r_1 \cdot B_2 \cdot \cos[\Omega_{Fa} \cdot (t_I)] - c_4 \cdot r_1 \cdot e^{r_2 \cdot (t_I)}$$

$$-c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_I)} + c_4 \cdot r_1 \cdot e^{r_2 \cdot (t_I)} = r_1 \cdot \theta_{EI} - r_1 \cdot A_2 \cdot \sin[\Omega_{Fa} \cdot (t_I)] - r_1 \cdot B_2 \cdot \cos[\Omega_{Fa} \cdot (t_I)] - [\omega_{EI} - A_2 \cdot \Omega_{Fa} \cdot \cos[\Omega_{Fa} \cdot (t_I)] + B_2 \cdot \Omega_{Fa} \cdot \sin[\Omega_{Fa} \cdot (t_I)]]$$

$$c_4 \cdot \left[r_1 \cdot e^{r_2 \cdot (t_I)} - r_2 \cdot e^{r_2 \cdot (t_I)}\right] = r_1 \cdot \theta_{EI} - r_1 \cdot A_2 \cdot \sin[\Omega_{Fa} \cdot (t_I)] - r_1 \cdot B_2 \cdot \cos[\Omega_{Fa} \cdot (t_I)] - [\omega_{EI} - A_2 \cdot \Omega_{Fa} \cdot \cos[\Omega_{Fa} \cdot (t_I)] + B_2 \cdot \Omega_{Fa} \cdot \sin[\Omega_{Fa} \cdot (t_I)]]$$

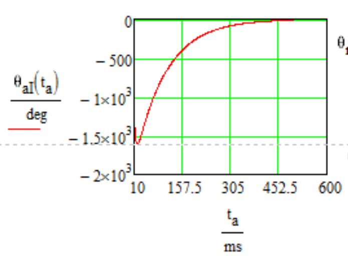
$$c_4 := \frac{r_1 \cdot \theta_{EI} - r_1 \cdot A_2 \cdot \sin[\Omega_{Fa} \cdot (t_I)] - r_1 \cdot B_2 \cdot \cos[\Omega_{Fa} \cdot (t_I)] - [\omega_{EI} - A_2 \cdot \Omega_{Fa} \cdot \cos[\Omega_{Fa} \cdot (t_I)] + B_2 \cdot \Omega_{Fa} \cdot \sin[\Omega_{Fa} \cdot (t_I)]]}{\left[r_1 \cdot e^{r_2 \cdot (t_I)} - r_2 \cdot e^{r_2 \cdot (t_I)}\right]} = 88.665$$

$$c_3 := \frac{\omega_{EI} - A_2 \cdot \Omega_{Fa} \cdot \cos[\Omega_{Fa} \cdot (t_I)] + B_2 \cdot \Omega_{Fa} \cdot \sin[\Omega_{Fa} \cdot (t_I)] - c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_I)}}{r_1 \cdot e^{r_1 \cdot (t_I)}} = -36.527$$

$$\frac{d^2}{dt^2}\theta_k + \frac{k_{damp}}{I_y} \cdot \frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y} \cdot \theta_k = -A \cdot \Omega_{Fa}^{\ 2} \cdot \sin(\Omega_{Fa} \cdot t) - B \cdot \Omega_{Fa}^{\ 2} \cdot \cos(\Omega_{Fa} \cdot t) + \frac{k_{damp}}{I_y} \cdot (A \cdot \Omega_{Fa} \cdot \cos(\Omega_{Fa} \cdot t) - B \cdot \Omega_{Fa} \cdot \sin(\Omega_{Fa} \cdot t)) + \frac{k_{necks}}{I_y} \cdot (A \cdot \sin(\Omega_{Fa} \cdot t) + B \cdot \cos(\Omega_{Fa} \cdot t))$$

Complete Solution

$$\theta_{aI}(t_a) := c_3 \cdot e^{r_1 \cdot (t_a)} + c_4 \cdot e^{r_2 \cdot (t_a)} + A_2 \cdot \sin[\Omega_{Fa} \cdot (t_a)] + B_2 \cdot \cos[\Omega_{Fa} \cdot (t_a)]$$

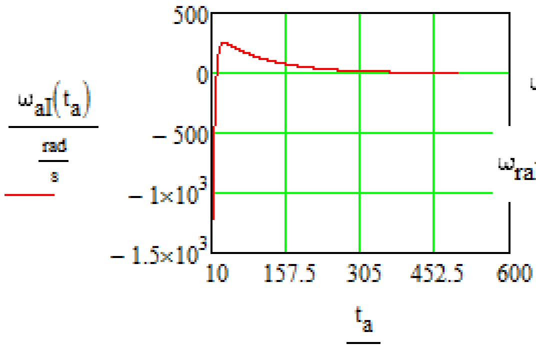


$$\theta_{raI} := \theta_{aI}(t_I) = -1.406 \times 10^3 \cdot deg \quad \theta_{EI} = -1.406 \times 10^3 \cdot deg$$

$$\theta_{aI}(.502s) = -10.673 \cdot deg$$

$$\theta_{aI}(17ms) = -1.592 \times 10^3 \cdot deg$$

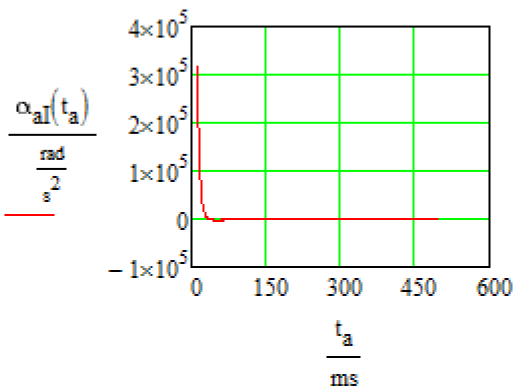
$$\omega_{aI}(t_a) := c_3 \cdot r_1 \cdot e^{r_1 \cdot (t_a)} + c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_a)} + A_2 \cdot \Omega_{Fa} \cdot \cos\left[\Omega_{Fa} \cdot (t_a)\right] - B_2 \cdot \Omega_{Fa} \cdot \sin\left[\Omega_{Fa} \cdot (t_a)\right]$$



$$\omega_{EI} = -1.223 \times 10^{3} \cdot \frac{rad}{sec}$$

$$\omega_{raI} := \omega_{aI}(t_I) = -1.223 \times 10^{3} \frac{1}{s} \cdot rad$$

$$\alpha_{aI}(t_a) := c_3 \cdot r_1{}^2 \cdot e^{r_1 \cdot (t_a)} + c_4 \cdot r_2{}^2 \cdot e^{r_2 \cdot (t_a)} - A_2 \cdot \Omega_{Fa}{}^2 \cdot \sin\left[\Omega_{Fa} \cdot (t_a)\right] - B_2 \cdot \Omega_{Fa}{}^2 \cdot \cos\left[\Omega_{Fa} \cdot (t_a)\right]$$



$$\alpha_{EI} = 7.038 \times 10^{4} \frac{1}{s^2}$$

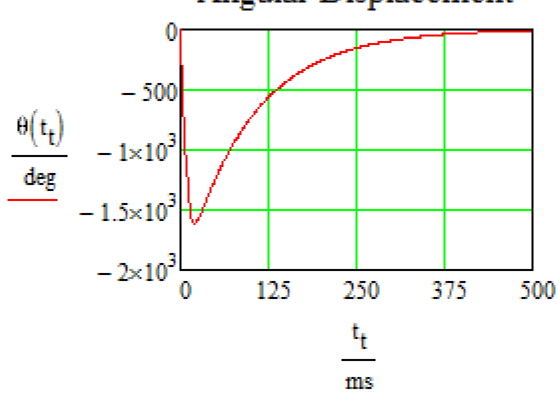
$$\alpha_{aI}(t_I) = 3.154 \times 10^{5} \frac{1}{s^2}$$

$$\alpha_{constant} := \alpha_{aI}(t_I) - \alpha_{EI} = 2.45 \times 10^{5} \frac{1}{s^2}$$

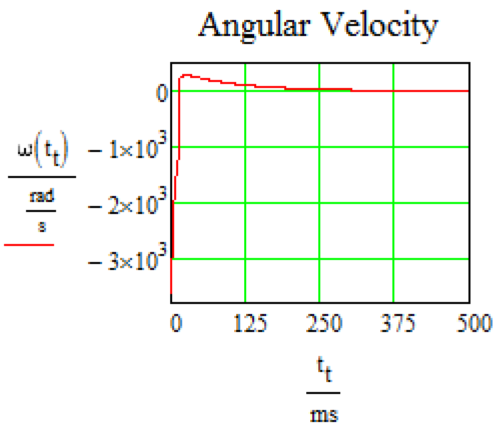
$$t_t := 0s, .001s .. .5s$$

$$\theta(t_t) := \begin{cases} c_1 \cdot e^{r_1 \cdot t_t} + c_2 \cdot e^{r_2 \cdot t_t} + A_c \cdot \sin(\Omega_F \cdot t_t) + B_c \cdot \cos(\Omega_F \cdot t_t) & \text{if } 0s \le t_t \le t_I \\ c_3 \cdot e^{r_1 \cdot (t_t)} + c_4 \cdot e^{r_2 \cdot (t_t)} + A_2 \cdot \sin\left[\Omega_{Fa} \cdot (t_t)\right] + B_2 \cdot \cos\left[\Omega_{Fa} \cdot (t_t)\right] & \text{if } t_I < t_t \le .5s \end{cases}$$

Angular Displacement



$$\omega(t_t) := \begin{cases} \left(c_1 \cdot r_1 \cdot e^{r_1 \cdot t_t} + c_2 \cdot r_2 \cdot e^{r_2 \cdot t_t} + A_c \cdot \Omega_F \cdot \cos(\Omega_F \cdot t_t) - B_c \cdot \Omega_F \cdot \sin(\Omega_F \cdot t_t)\right) & \text{if } 0s \le t_t \le t_I \\ \left(c_3 \cdot r_1 \cdot e^{r_1 \cdot t_t} + c_2 \cdot r_2 \cdot e^{r_2 \cdot t_t} + A_2 \cdot \Omega_{Fa} \cdot \cos(\Omega_{Fa} \cdot t_t) - B_2 \cdot \Omega_{Fa} \cdot \sin(\Omega_{Fa} \cdot t_t)\right) & \text{if } t_I < t_t \le .5s \end{cases}$$



$$\omega(t_I) = -1.223 \times 10^3 \frac{1}{s}$$

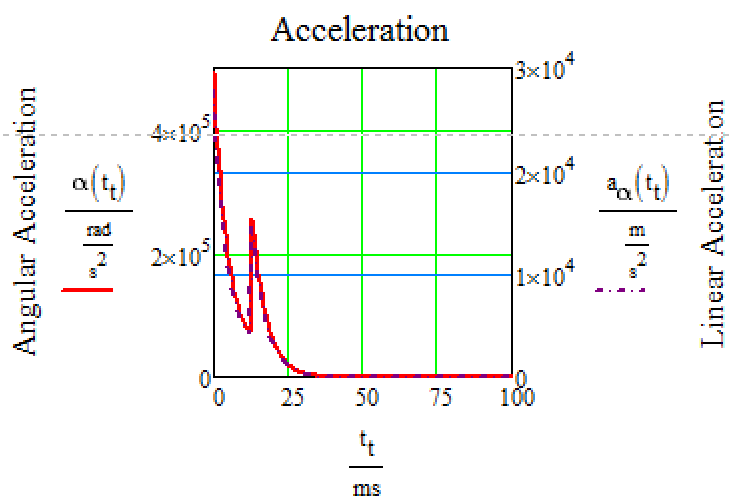
$$\omega(0s) = -3.638 \times 10^3 \frac{1}{s}$$

$$\alpha(t_t) := \begin{cases} \left(c_1 \cdot r_1^{\,2} \cdot e^{r_1 \cdot t_t} + c_2 \cdot r_2^{\,2} \cdot e^{r_2 \cdot t_t} - A_c \cdot \Omega_F^{\,2} \cdot \sin(\Omega_F \cdot t_t) - B_c \cdot \Omega_F^{\,2} \cdot \cos(\Omega_F \cdot t_t)\right) & \text{if } 0ms \le t_t \le t_I \\ \left[c_3 \cdot r_1^{\,2} \cdot e^{r_1 \cdot (t_t)} + c_4 \cdot r_2^{\,2} \cdot e^{r_2 \cdot (t_t)} - A_2 \cdot \Omega_{Fa}^{\,2} \cdot \sin\left[\Omega_{Fa} \cdot (t_t)\right] - B_2 \cdot \Omega_{Fa}^{\,2} \cdot \cos\left[\Omega_{Fa} \cdot (t_t)\right]\right] & \text{if } t_I < t_t \le .5s \end{cases}$$

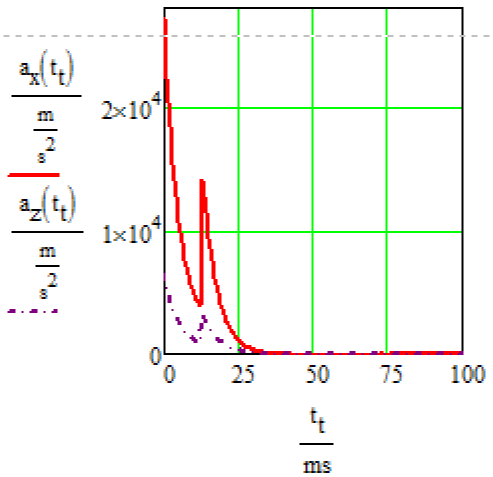
$$a_\alpha(t_t) := \alpha(t_t) \cdot d_{CG}$$

$$\alpha(0ms) = 4.938 \times 10^5 \frac{1}{s^2} \cdot rad$$

$$a_\alpha(0ms) = 2.791 \times 10^4 \frac{m}{s^2}$$



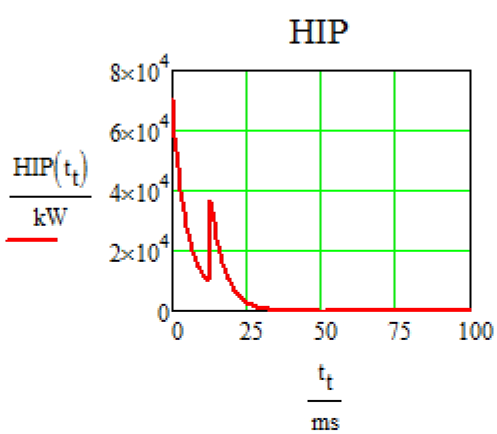
$$\frac{Pressure \cdot Area_{Bore}}{m_{rod} + m_{head}} = 2.716 \times 10^4 \frac{m}{s^2}$$



$$a_{xmax} := a_x(0ms) = 2.716 \times 10^4 \frac{m}{s^2}$$

$$a_{zmax} := a_z(0ms) = 6.419 \times 10^3 \frac{m}{s^2}$$

$$HIP(t_t) := m_{head} \cdot a_x(t_t) \cdot \int_{0ms}^{25ms} a_x(t_t)\, dt_t + m_{head} \cdot a_z(t_t) \cdot \int_{0ms}^{25ms} a_z(t_t)\, dt_t + I_y \cdot \alpha(t_t) \cdot \int_{0ms}^{25ms} \alpha(t_t)\, dt_t$$



$$HIP(0ms) = 7.05 \times 10^4 \cdot kW$$

HIP WAY too Large!!

C-3: Force Calculated from Average Maximum Head-Acceleration of Concussed NFL Players in Newman et al. Study

Average Max Acceleration of Concussed NFL players in Newman et al. study

$$A_{critical} := \frac{1162\frac{m}{s^2} + 1263\cdot\frac{m}{s^2} + 758\frac{m}{s^2} + 595\frac{m}{s^2} + 1211\frac{m}{s^2} + 788\frac{m}{s^2} + 804\frac{m}{s^2} + 1054\frac{m}{s^2} + 893\frac{m}{s^2} + 1005\frac{m}{s^2}}{10} = 953.3\frac{m}{s^2}$$

$$F_I := m_{head}\cdot A_{critical} = 4.195\times 10^3\,N$$

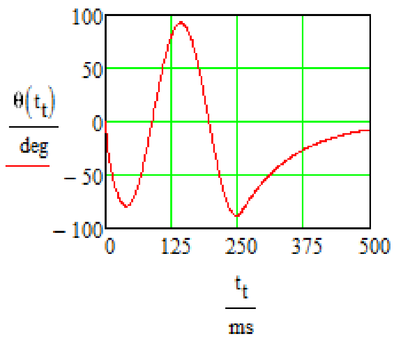
$$F_I\cdot t_I = m_{hp}\cdot(v_{hp} - v_f)$$

$$v_f := -\left(\frac{F_I\cdot t_I}{m_{hp}} - v_{hp}\right) = 3.779\frac{m}{s}$$

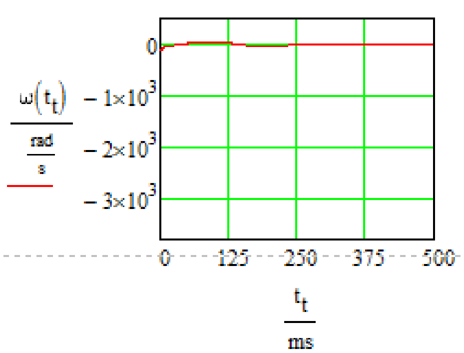
$$E_I := \frac{1}{2}\cdot m_{head}\cdot v_f^2 = 31.411\,J$$

$$t_t := 0s, .001s .. .5s$$

$$\theta(t_t) := \begin{cases} c_1\cdot e^{r_1\cdot t_t} + c_2\cdot e^{r_2\cdot t_t} + A_c\cdot\sin(\Omega_F\cdot t_t) + B_c\cdot\cos(\Omega_F\cdot t_t) & \text{if } 0s \le t_t \le t_I \\ c_3\cdot e^{r_1\cdot(t_t)} + c_4\cdot e^{r_2\cdot(t_t)} + A_2\cdot\sin[\Omega_{Fa}\cdot(t_t)] + B_2\cdot\cos[\Omega_{Fa}\cdot(t_t)] & \text{if } t_I < t_t \le .5s \end{cases}$$



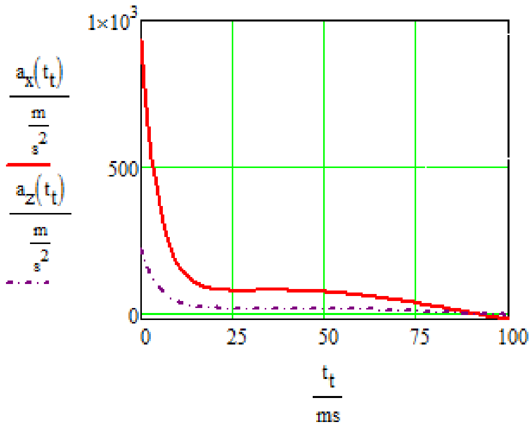
$$\omega(t_t) := \begin{cases} \left(c_1\cdot r_1\cdot e^{r_1\cdot t_t} + c_2\cdot r_2\cdot e^{r_2\cdot t_t} + A_c\cdot\Omega_F\cdot\cos(\Omega_F\cdot t_t) - B_c\cdot\Omega_F\cdot\sin(\Omega_F\cdot t_t)\right) & \text{if } 0s \le t_t \le t_I \\ \left(c_3\cdot r_1\cdot e^{r_1\cdot t_t} + c_2\cdot r_2\cdot e^{r_2\cdot t_t} + A_2\cdot\Omega_{Fa}\cdot\cos(\Omega_{Fa}\cdot t_t) - B_2\cdot\Omega_{Fa}\cdot\sin(\Omega_{Fa}\cdot t_t)\right) & \text{if } t_I < t_t \le .5s \end{cases}$$



$$\omega(t_I) = 3.471\frac{1}{s}$$

$$\omega(0s) = -125.541\frac{1}{s}$$

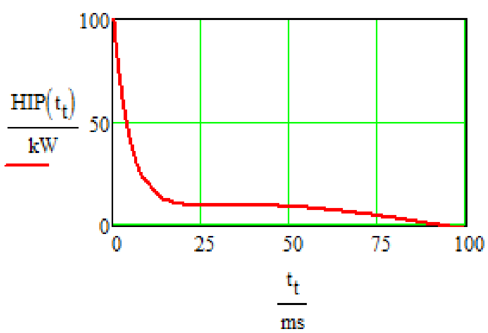
$a_x(t_t) := \alpha(t_t)\cdot d_{CGz}$ $\qquad a_z(t_t) := \alpha(t_t)\cdot d_{CGx}$



$a_{xmax} := a_x(0ms) = 936.27\,\frac{m}{s^2}$

$a_{zmax} := a_z(0ms) = 221.3\,\frac{m}{s^2}$

$$HIP(t_t) := m_{head}\cdot a_x(t_t)\cdot\int_{0ms}^{85ms} a_x(t_t)\,dt_t + m_{head}\cdot a_z(t_t)\cdot\int_{0ms}^{85ms} a_z(t_t)\,dt_t + I_y\cdot\alpha(t_t)\cdot\int_{0ms}^{85ms}\alpha(t_t)\,dt_t$$

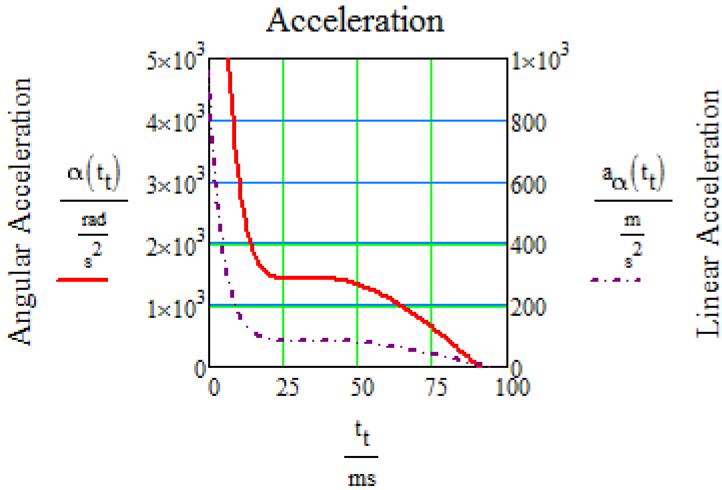


$HIP(0ms) = 108.445\cdot kW$

$$\alpha(t_t) := \begin{cases} \left(c_1 \cdot r_1^2 \cdot e^{r_1 \cdot t_t} + c_2 \cdot r_2^2 \cdot e^{r_2 \cdot t_t} - A_c \cdot \Omega_F^2 \cdot \sin(\Omega_F \cdot t_t) - B_c \cdot \Omega_F^2 \cdot \cos(\Omega_F \cdot t_t)\right) & \text{if } 0ms \le t_t \le t_I \\ \left[c_3 \cdot r_1^2 \cdot e^{r_1 \cdot (t_t)} + c_4 \cdot r_2^2 \cdot e^{r_2 \cdot (t_t)} - A_2 \cdot \Omega_{Fa}^2 \cdot \sin[\Omega_{Fa} \cdot (t_t)] - B_2 \cdot \Omega_{Fa}^2 \cdot \cos[\Omega_{Fa} \cdot (t_t)]\right] & \text{if } t_I < t_t \le .5s \end{cases}$$

$$a_\alpha(t_t) := \alpha(t_t) \cdot d_{CG}$$

$$\alpha(0ms) = 1.702 \times 10^4 \frac{1}{s^2} \cdot rad$$



$$a_\alpha(0ms) = 962.068 \frac{m}{s^2}$$

$$\frac{Pressure \cdot Area_{Bore}}{m_{rod} + m_{head}} = 936.27 \frac{m}{s^2}$$

HIP Value too high to realistically reflect Hockey collision

C-4: Calculating Force from Maximum Head Acceleration Corresponding to 95% Concussion Risk According to Newman et al.

Finding Force from average mass of head and the maximum head acceleration found to correspond to 95% concussion risk according to Newman et al.

$$A_{critical} := 761.5 \frac{m}{s^2}$$ }Amax corresponding to 95% chance of concussion from Newman et al. Development of HIP

$$F_I := m_{head} \cdot A_{critical} = 3.351 \times 10^3 \, N$$

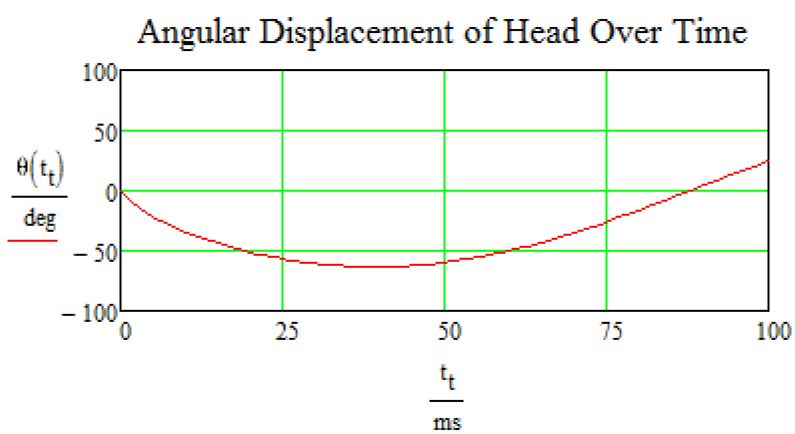
$$F_I \cdot t_I = m_{hp} \cdot (v_{hp} - v_f)$$

$$v_f := -\left(\frac{F_I \cdot t_I}{m_{hp}} - v_{hp}\right) = 5.717 \frac{m}{s}$$

$$E_I := \frac{1}{2} \cdot m_{head} \cdot v_{hp}^2 = 395.693 \, J$$

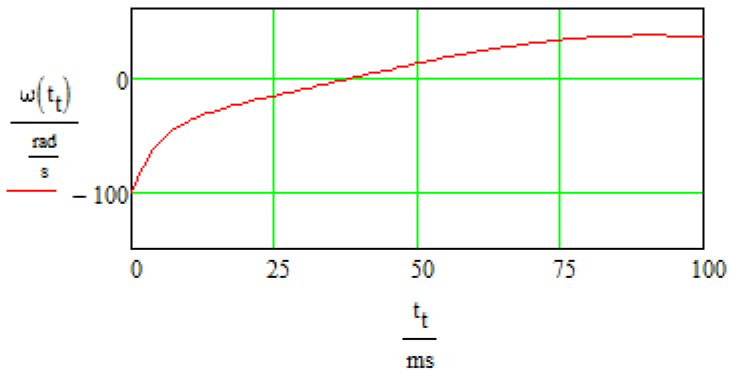
$$t_t := 0s, .001s .. 10s$$

$$\theta(t_t) := \begin{cases} c_1 \cdot e^{r_1 \cdot t_t} + c_2 \cdot e^{r_2 \cdot t_t} + A_c \cdot \sin(\Omega_F \cdot t_t) + B_c \cdot \cos(\Omega_F \cdot t_t) & \text{if } 0s \le t_t \le t_I \\ c_3 \cdot e^{r_1 \cdot (t_t)} + c_4 \cdot e^{r_2 \cdot (t_t)} + A_2 \cdot \sin[\Omega_{Fa} \cdot (t_t)] + B_2 \cdot \cos[\Omega_{Fa} \cdot (t_t)] & \text{if } t_I < t_t \le .5s \end{cases}$$



$$\omega(t_t) := \begin{cases} \left(c_1 \cdot r_1 \cdot e^{r_1 \cdot t_t} + c_2 \cdot r_2 \cdot e^{r_2 \cdot t_t} + A_c \cdot \Omega_F \cdot \cos(\Omega_F \cdot t_t) - B_c \cdot \Omega_F \cdot \sin(\Omega_F \cdot t_t)\right) & \text{if } 0s \le t_t \le t_I \\ \left(c_3 \cdot r_1 \cdot e^{r_1 \cdot t_t} + c_2 \cdot r_2 \cdot e^{r_2 \cdot t_t} + A_2 \cdot \Omega_{Fa} \cdot \cos(\Omega_{Fa} \cdot t_t) - B_2 \cdot \Omega_{Fa} \cdot \sin(\Omega_{Fa} \cdot t_t)\right) & \text{if } t_I < t_t \le .5s \end{cases}$$

Angular Velocity of Head Over Time



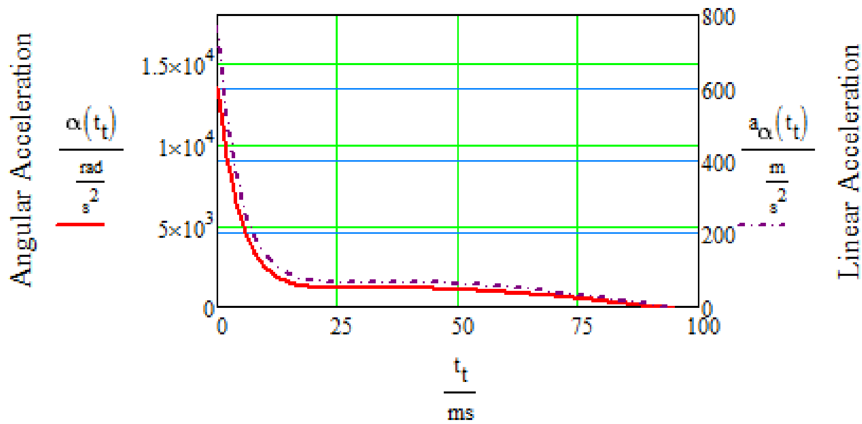
$$\omega(t_I) = 2.859 \frac{1}{s}$$

$$\omega(0s) = -100.306 \frac{1}{s}$$

$$\alpha(t_t) := \begin{cases} \left(c_1 \cdot r_1^{2} \cdot e^{r_1 \cdot t_t} + c_2 \cdot r_2^{2} \cdot e^{r_2 \cdot t_t} - A_c \cdot \Omega_F^{2} \cdot \sin(\Omega_F \cdot t_t) - B_c \cdot \Omega_F^{2} \cdot \cos(\Omega_F \cdot t_t)\right) & \text{if } 0ms \le t_t \le t_I \\ \left[c_3 \cdot r_1^{2} \cdot e^{r_1 \cdot (t_t)} + c_4 \cdot r_2^{2} \cdot e^{r_2 \cdot (t_t)} - A_2 \cdot \Omega_{Fa}^{2} \cdot \sin[\Omega_{Fa} \cdot (t_t)] - B_2 \cdot \Omega_{Fa}^{2} \cdot \cos[\Omega_{Fa} \cdot (t_t)]\right] & \text{if } t_I < t_t \le .5s \end{cases}$$

$$a_\alpha(t_t) := \alpha(t_t) \cdot d_{CG} \qquad \alpha(0ms) = 1.36 \times 10^{4} \frac{1}{s^2} \cdot rad$$

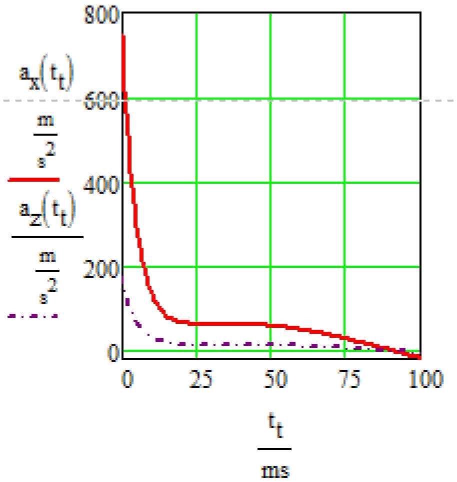
Acceleration of Head Over Time



$$a_\alpha(0ms) = 768.504 \frac{m}{s^2}$$

$$\frac{Pressure \cdot Area_{Bore}}{m_{rod} + m_{head}} = 747.896 \frac{m}{s^2}$$

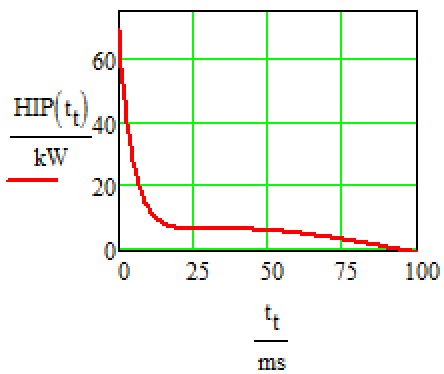
$$a_x(t_t) := \alpha(t_t) \cdot d_{CGz} \qquad a_z(t_t) := \alpha(t_t) \cdot d_{CGx}$$



$$a_{xmax} := a_x(0ms) = 747.896 \frac{m}{s^2}$$

$$a_{zmax} := a_z(0ms) = 176.775 \frac{m}{s^2}$$

$$HIP(t_t) := m_{head} \cdot a_x(t_t) \cdot \int_{0ms}^{90ms} a_x(t_t)\, dt_t + m_{head} \cdot a_z(t_t) \cdot \int_{0ms}^{90ms} a_z(t_t)\, dt_t + I_y \cdot \alpha(t_t) \cdot \int_{0ms}^{90ms} \alpha(t_t)\, dt_t$$



$$HIP(0ms) = 69.487 \cdot kW$$

Calculating necessary pressure

$F_I = Pressure \cdot Area$ $\qquad F_I = 753.245\,lbf$

Volume of rod of Air cylinder w/ 1.5 in bore, 4 in stroke length

$L_{rod} := 4in$ $\qquad D_{rod} := .44in$ $\qquad Area_{rod} := \frac{D_{rod}^{2}}{4} \cdot \pi = 0.152 \cdot in^{2}$

$D_{Bore} := 1.5in$ $\qquad Area_{Bore} := \frac{\pi}{4} \cdot \left(D_{Bore}^{2}\right) = 1.767 \cdot in^{2}$

$Volume_{rod} := Area_{rod} \cdot L_{rod} = 0.608 \cdot in^{3}$ $\qquad Pressure := \frac{F_I}{Area_{Bore}} = 426.249 \cdot psi$ } Larger than maximum available

---

Stainless Steel rods Type 304 $\qquad \delta_{ss} := .00803 \frac{kg}{cm^{3}} = 8.03 \times 10^{3} \frac{kg}{m^{3}}$

Mass of Rod $\qquad m_{rod} := \delta_{ss} \cdot Volume_{rod} = 0.08\,kg$

Acceleration of Rod

$a_{rod} := \frac{F_I}{m_{rod}} = 4.186 \times 10^{4} \frac{m}{s^{2}}$

approximate mass of air cylinder

$m_{ac} := Area_{Bore} \cdot 11in \cdot \delta_{ss} = 2.558\,kg$

$Pressure \cdot Area_{Bore} \cdot t_I = \left(m_{hp} + m_{rod}\right) v_f$

Considering larger air cylinder

$F_I = Pressure \cdot Area$ $F_I = 753.245\,lbf$

Volume of rod of Air cylinder w/ 3.5 in bore, 4 in stroke length

$L_{rod} := 4in$ $D_{rod} := .44in$ $Area_{rod} := \frac{D_{rod}^2}{4} \cdot \pi = 0.152 \cdot in^2$

$D_{Bore} := 3.5in$ $Area_{Bore} := \frac{\pi}{4} \cdot \left(D_{Bore}^2\right) = 9.621 \cdot in^2$

$Volume_{rod} := Area_{rod} \cdot L_{rod} = 0.608 \cdot in^3$ $Pressure := \frac{F_I}{Area_{Bore}} = 78.291 \cdot psi$

Stainless Steel rods Type 304 $\delta_{ss} := .00803 \frac{kg}{cm^3} = 8.03 \times 10^3 \frac{kg}{m^3}$

Mass of Rod $m_{rod} := \delta_{ss} \cdot Volume_{rod} = 0.08\,kg$

Acceleration of Rod

$a_{rod} := \frac{F_I}{m_{rod}} = 4.186 \times 10^4 \frac{m}{s^2}$

approximate mass of air cylinder

$m_{ac} := Area_{Bore} \cdot 11in \cdot \delta_{ss} = 13.926\,kg$

$Pressure \cdot Area_{Bore} \cdot t_I = \left(m_{hp} + m_{rod}\right) v_f$

Considering Hammer Test

Considering a hammer impact test

$L_{hammer} := 1.5m \qquad L_{hh} := .051m$

$L_{hammer} = 4.921\,ft \qquad L_{handle} := L_{hammer} - L_{hh} = 1.449\,m$

$$m_{hammer} := \frac{E_I}{L_{hammer} \cdot g} = 26.9\,kg$$

$m_{hammer} \cdot g = 59.303 \cdot lbf$

Considering a hammer impact test

$L_{hammer} := 2.5m \qquad L_{hh} := .051m$

$L_{hammer} = 8.202\,ft \qquad L_{handle} := L_{hammer} - L_{hh} = 2.449\,m$

$$m_{hammer} := \frac{E_I}{L_{hammer} \cdot g} = 16.14\,kg$$

$m_{hammer} \cdot g = 35.582 \cdot lbf$

C-5: Calculations for Scaled-Down Test Set Up

Average Weight of Professional Hockey Player $\quad W_p := 210lbf = 934.127\,N \qquad W_{equipment} := 30lbf$

$W_{hp} := W_p + W_{equipment} = 1.068 \times 10^3\,N$

Speed of fast skater $\quad v_{hp} := 30mph = 13.411\,\frac{m}{s} \qquad$ Mass $\quad m_{hp} := \frac{W_{hp}}{g} = 108.862\,kg$

Average acceleration of an elite hockey player trying to speed up as fast as possible $\quad A_{hps} := 4.375\,\frac{m}{s^2}$

Force from player to stationary player impact $\quad F_I := m_{hp} \cdot A_{hps} = 476.272 \cdot N$

$F_I = Pressure \cdot Area$

Volume of rod of Air cylinder w/ 1.5 in bore, 4 in stroke length

Stroke Length — Diameters — Cross-sect. Area

$L_{rod} := 4in$ $\quad D_{rod} := .44in$ $\quad Area_{rod} := \frac{D_{rod}^2}{4} \cdot \pi = 0.152 \cdot in^2$

$D_{Bore} := 1.5in$ $\quad Area_{Bore} := \frac{\pi}{4} \cdot \left(D_{Bore}^2\right) = 1.767 \cdot in^2$

$Volume_{rod} := Area_{rod} \cdot L_{rod} = 0.608 \cdot in^3$ $\quad Pressure := \frac{F_I}{Area_{Bore}} = 60.589 \cdot psi$

Stainless Steel rods Type 304 $\quad \delta_{ss} := .00803 \frac{kg}{cm^3} = 8.03 \times 10^3 \frac{kg}{m^3}$

Mass of Rod $\quad m_{rod} := \delta_{ss} \cdot Volume_{rod} = 0.08\,kg$

Approximate mass of air cylinder $\quad m_{ac} := Area_{Bore} \cdot 11in \cdot \delta_{ss} = 2.558\,kg$

Acceleration of Rod $\quad a_{rod} := \frac{F_I}{m_{rod}} = 5.951 \times 10^3 \frac{m}{s^2}$

Scaling Factor

set-up/initial distance between head and air cylinder $d_i := 3.75\text{in}$

Time before impact $t_{BI} := \left(\frac{2 \cdot d_i}{a_{rod}}\right)^{.5} = 12.214 \cdot \text{ms}$

Velocity of rod right before impact $v_{rodBI} := a_{rod} \cdot t_{BI} = 15.597 \frac{m}{s}$

Impulse equals change in momentum $F_I \cdot t_I = (m_{rod} \cdot v_{rodBI}) - (m_{rod} + m_{head}) \cdot v_{rhc}$

$$(\text{Pressure} \cdot \text{Area}_{Bore} - k \cdot \theta) \cdot t_I = (m_{rod} \cdot v_{rodBI}) - (m_{rod} + m_{head}) \cdot v_{rhc}$$

Velocity of rod and head combined $v_{rhi} := \frac{(m_{rod} \cdot v_{rodBI})}{(m_{rod} + m_{head})} = 1.219 \frac{m}{s}$

Angular velocity of head after initial impact $\omega_{hi} := \frac{-v_{rhi}}{d_{CGz}} = -22.159 \cdot \frac{rad}{s}$

Maximum Force Achievable by this Air Cylinder $\text{Pressure} := 100\text{psi}$

$$F_{maxac} := \text{Area}_{Bore} \cdot 100\text{psi} = 786.066\,\text{N}$$

Acceleration of Rod

$$a_{arod} := \frac{F_I}{m_{rod}} = 5.951 \times 10^3 \frac{m}{s^2}$$

$$\text{Pressure} \cdot \text{Area}_{Bore} \cdot t_I = (m_{hp} + m_{rod}) v_f$$

$$a_{rod} := \frac{a_{arod}}{\text{ScalingFactor}} = 1.277 \times 10^3 \frac{m}{s^2}$$

approximate mass of air cylinder

$$m_{ac} := \text{Area}_{Bore} \cdot 11\text{in} \cdot \delta_{ss} = 2.558\,\text{kg}$$

$$\text{ScalingFactor} := \frac{3.663 \cdot 10^3 \text{N}}{F_{maxac}} = 4.66$$

Moments of Inertia from Chalmers, Applied Mechanics, Master's Thesis 2010

$I_{fx} := 204.117 kg \cdot cm^2$ $\quad I_{fy} := 232.888 kg \cdot cm^2$ $\quad I_{fz} := 150.832 kg \cdot cm^2$

$$I_y := \frac{I_{fy}}{ScalingFactor} = 49.977\, kg \cdot cm^2$$

Distance from Occipital Condyle (OC, point about which head rotates) to frankfort line (x-axis in reference frame) frankfort line is imaginary line connecting the upper margin of the aditory meatus (AM, external ear canal) to the lower orbital margin (cavity containing eyeball) Chalmers et al.

$d_{AMx} := 8mm$ $\quad d_{AMz} := 35mm$

Distance from OC to center of gravity (CG) from Chalmer et al.

$d_{CGx} := 13mm$ $\quad d_{CGz} := 55mm$

$$d_{CG} := \sqrt{d_{CGx}^2 + d_{CGz}^2} = 56.515 \cdot mm$$

$m_{ahead} := 4.4kg$ }from Chalmers et al.

$$m_{head} := \frac{m_{ahead}}{ScalingFactor} = 0.944\, kg$$

$$\Delta v_{Ih} := \frac{Pressure \cdot Area_{Bore} \cdot t_I}{(m_{head} + m_{rod})} = 9.209 \frac{m}{s} \qquad \Delta v_{Ip} := \frac{Pressure \cdot Area_{Bore} \cdot t_I}{(m_{hp} + m_{rod})} = 0.087 \cdot \frac{m}{s}$$

$k_{fnecks} := 10\,\frac{N \cdot m}{rad}$ }from wang et al.
http://ac.els-cdn.com.ezproxy.wpi.edu/S0021929012006896/1-s2.0-S0021929012006896-main.pdf?_tid=98b9acdc-86ad-11e4-9849-00000aab0f6b&acdnat=1418904267_6b62bd72fb6dd655802ac5760d087ddc

$k_{fdamp} := .1s \cdot k_{fnecks} = 1\,s \cdot \frac{N \cdot m}{rad}$ } From chalmers et al.

$$k_{necks} := \frac{k_{fnecks}}{ScalingFactor} = 2.146\,\frac{1}{s}\,\frac{m^2 \cdot kg}{s}$$

$$k_{damp} := \frac{k_{fdamp}}{ScalingFactor} = 0.215\,\frac{m^2 \cdot kg}{s}$$

critical damping $k_{cd} := 2m\sqrt{m_{head} \cdot k_{necks}} = 2.847\,\frac{N \cdot m \cdot s}{rad}$

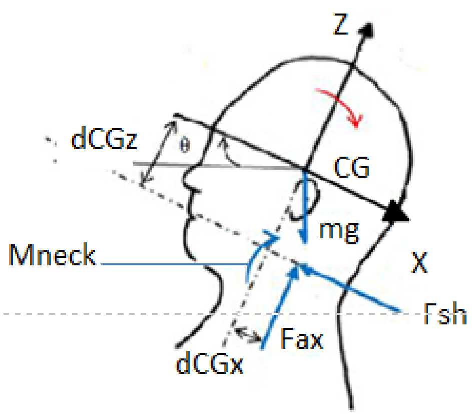
Damping Force

$$F_{damp}(\theta) = k_{damp} \cdot \left(\frac{d}{dt}\theta\right)$$

Spring Force

$$F_{spring}(\theta) = k_{necks} \cdot \theta$$

Peak moment of neck from Chalmers et al. $M_n := 4.78N \cdot m$

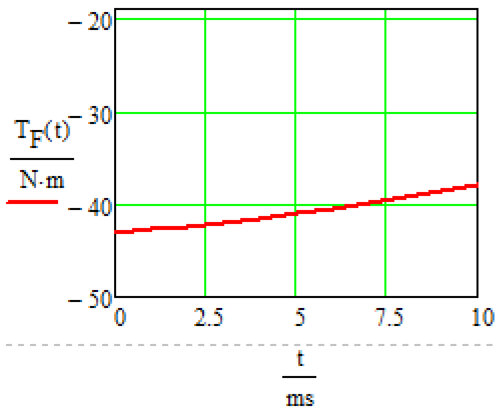


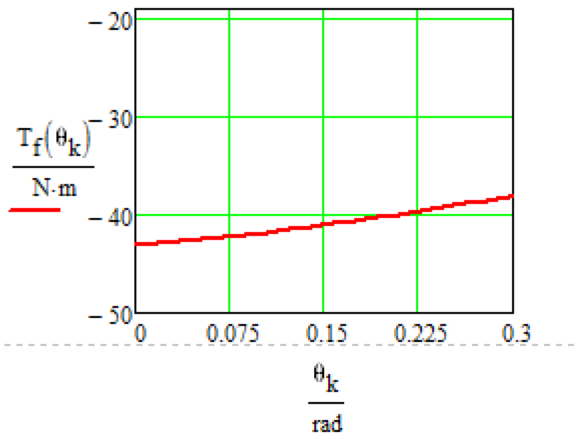
Finding Forcing angular frequency Ω.F

$\Omega_F := 30\,\frac{rad}{s}$ Kept trying different Ω.F until the graph of T.F and T.f looked the same

$$T_F(t) := \left[\left(Pressure \cdot Area_{Bore} \cdot d_{CGx}\right) + m_{head} \cdot g \cdot d_{CGz}\right] \cdot \sin\left(\Omega_F \cdot t\right) \ldots$$
$$+ \left[\left(-Pressure \cdot Area_{Bore} \cdot d_{CGz}\right) + m_{head} \cdot g \cdot d_{CGx}\right] \cdot \cos\left(\Omega_F \cdot t\right)$$

$$T_f\left(\theta_k\right) := \left[\left(Pressure \cdot Area_{Bore} \cdot d_{CGx}\right) + m_{head} \cdot g \cdot d_{CGz}\right] \cdot \sin\left(\theta_k\right) \ldots$$
$$+ \left[\left(-Pressure \cdot Area_{Bore} \cdot d_{CGz}\right) + m_{head} \cdot g \cdot d_{CGx}\right] \cdot \cos\left(\theta_k\right)$$





when T= -21J, θ.k = .1449 rad, t=4.83ms
when T=-20.001J, θ.k=.2295rad, t=7.65ms

$T_F(4.83ms) = -41.112\,J$ $T_f(.1449rad) = -41.112\,J$

$T_F(7.65ms) = -39.542\,J$ $T_f(.2295rad) = -39.542\,J$

Complimentary Solution (Left side of equation)

$$I_y \cdot \frac{d^2}{dt^2}\theta_k + k_{damp} \cdot \frac{d}{dt}\theta_k + k_{necks} \cdot \theta_k$$

$$\frac{d^2}{dt^2}\theta_k + \frac{k_{damp}}{I_y} \cdot \frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y} \cdot \theta_k = 0$$

$$r_1 := \frac{-\frac{k_{damp}}{I_y} + \sqrt{\left(\frac{k_{damp}}{I_y}\right)^2 - 4 \cdot \frac{k_{necks}}{I_y}}}{2} = -15.853 \frac{1}{s}$$

$$r_2 := \frac{-\frac{k_{damp}}{I_y} - \sqrt{\left(\frac{k_{damp}}{I_y}\right)^2 - 4 \cdot \frac{k_{necks}}{I_y}}}{2} = -27.087 \frac{1}{s}$$

$$\theta_I(t) = c_1 \cdot e^{r_1 \cdot t} + c_2 \cdot e^{r_2 \cdot t}$$

Particular Solution (right side of equation)

$$\left[\left(Pressure \cdot Area_{Bore} \cdot d_{CGx}\right) + m_{head} \cdot g \cdot d_{CGz}\right] \cdot \sin(\theta_k) - \left(Pressure \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right) \cdot \cos(\theta_k)$$

Guess

$$A \cdot \sin(\Omega_F \cdot t) + B \cdot \cos(\Omega_F \cdot t)$$

$$\omega_k(t) = A \cdot \Omega_F \cdot \cos(\Omega_F \cdot t) - B \cdot \Omega_F \cdot \sin(\Omega_F \cdot t)$$

$$\alpha_k(t) = -A \cdot \Omega_F^2 \cdot \sin(\Omega_F \cdot t) - B \cdot \Omega_F^2 \cdot \cos(\Omega_F \cdot t)$$

$$\frac{d^2}{dt^2}\theta_k + \frac{k_{damp}}{I_y} \cdot \frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y} \cdot \theta_k = -A \cdot \Omega_F^2 \cdot \sin(\Omega_F \cdot t) - B \cdot \Omega_F^2 \cdot \cos(\Omega_F \cdot t) + \frac{k_{damp}}{I_y} \cdot \left(A \cdot \Omega_F \cdot \cos(\Omega_F \cdot t) - B \cdot \Omega_n \cdot \sin(\Omega_F \cdot t)\right) + \frac{k_{necks}}{I_y} \cdot \left(A \cdot \sin(\Omega_F \cdot t) + B \cdot \cos(\Omega_F \cdot t)\right)$$

Rearrange

$$\left(-A \cdot \Omega_F^2 - \frac{k_{damp}}{I_y} \cdot B \cdot \Omega_F + \frac{k_{necks}}{I_y} \cdot A\right) \cdot \sin(\Omega_F \cdot t) + \left(-B \cdot \Omega_n^2 + \frac{k_{damp}}{I_y} \cdot A \cdot \Omega_F + \frac{k_{necks}}{I_y} \cdot B\right) \cdot \cos(\Omega_F \cdot t) = \frac{\begin{pmatrix} Pressure \cdot Area_{Bore} \cdot d_{CGx} \cdots \\ + m_{head} \cdot g \cdot d_{CGz} \end{pmatrix} \cdot \sin(\theta_k) - \begin{pmatrix} Pressure \cdot Area_{Bore} \cdot d_{CGz} \cdots \\ + m_{head} \cdot g \cdot d_{CGx} \end{pmatrix} \cdot \cos(\theta_k)}{I_y}$$

$$-A \cdot \Omega_F^2 - \frac{k_{damp}}{I_y} \cdot B \cdot \Omega_F + \frac{k_{necks}}{I_y} \cdot A = \left[\frac{\left(Pressure \cdot Area_{Bore} \cdot d_{CGx}\right) + m_{head} \cdot g \cdot d_{CGz}}{I_y}\right]$$

$$-B \cdot \Omega_F^2 + \frac{k_{damp}}{I_y} \cdot A \cdot \Omega_F + \frac{k_{necks}}{I_y} \cdot B = \frac{-Pressure \cdot Area_{Bore} \cdot d_{CGz} - m_{head} \cdot g \cdot d_{CGx}}{I_y}$$

$$\left(\frac{k_{necks}}{I_y} - \Omega_F^2\right) \cdot B + \frac{k_{damp}}{I_y} \cdot \Omega_F \cdot A = \frac{-Pressure \cdot Area_{Bore} \cdot d_{CGz} - m_{head} \cdot g \cdot d_{CGx}}{I_y}$$

$$\left(\frac{k_{necks}}{I_y} - \Omega_F^2\right) \cdot A - \frac{k_{damp}}{I_y} \cdot \Omega_F \cdot B = \frac{\left[\left(Pressure \cdot Area_{Bore} \cdot d_{CGx}\right) + m_{head} \cdot g \cdot d_{CGz}\right]}{I_y}$$

$$\left(k_{necks} - I_y \cdot \Omega_F^2\right) \cdot A - k_{damp} \cdot \Omega_F \cdot B = \left[\left(Pressure \cdot Area_{Bore} \cdot d_{CGx}\right) + m_{head} \cdot g \cdot d_{CGz}\right]$$

$$B_c = \frac{\left[\left(Pressure \cdot Area_{Bore} \cdot d_{CGx}\right) + m_{head} \cdot g \cdot d_{CGz}\right] - \left(k_{necks} - I_y \cdot \Omega_F^{\,2}\right) \cdot A}{-\left(k_{damp} \cdot \Omega_F\right)}$$

$$\left(k_{necks} - I_y \cdot \Omega_F^{\,2}\right) \cdot \left[\frac{\left[\left(Pressure \cdot Area_{Bore} \cdot d_{CGx}\right) + m_{head} \cdot g \cdot d_{CGz}\right] - \left(k_{necks} - I_y \cdot \Omega_F^{\,2}\right) \cdot A}{-\left(k_{damp} \cdot \Omega_F\right)}\right] + k_{damp} \cdot \Omega_F \cdot A = -Pressure \cdot Area_{Bore} \cdot d_{CGz} - m_{head} \cdot g \cdot d_{CGx}$$

$$\left(k_{necks} - I_y \cdot \Omega_F^{\,2}\right) \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGx} + m_{head} \cdot g \cdot d_{CGz}\right) - \left(k_{necks} - I_y \cdot \Omega_F^{\,2}\right)^2 \cdot A - \left(k_{damp} \cdot \Omega_F\right)^2 \cdot A = -\left(k_{damp} \cdot \Omega_F\right) \cdot \left(-Pressure \cdot Area_{Bore} \cdot d_{CGz} - m_{head} \cdot g \cdot d_{CGx}\right)$$

$$-\left(k_{necks} - I_y \cdot \Omega_F^{\,2}\right)^2 \cdot A - \left(k_{damp} \cdot \Omega_F\right)^2 \cdot A = \left(k_{damp} \cdot \Omega_F\right) \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right) - \left(k_{necks} - I_y \cdot \Omega_F^{\,2}\right) \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGx} + m_{head} \cdot g \cdot d_{CGz}\right)$$

$$A_c \cdot \left[-\left(k_{necks} - I_y \cdot \Omega_F^{\,2}\right)^2 - \left(k_{damp} \cdot \Omega_F\right)^2\right] = \left(k_{damp} \cdot \Omega_F\right) \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right) - \left(k_{necks} - I_y \cdot \Omega_F^{\,2}\right) \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGx} + m_{head} \cdot g \cdot d_{CGz}\right)$$

$$A_c := \frac{\left(k_{damp} \cdot \Omega_F\right) \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right) - \left(k_{necks} - I_y \cdot \Omega_F^{\,2}\right) \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGx} + m_{head} \cdot g \cdot d_{CGz}\right)}{-\left[\left(k_{necks} - I_y \cdot \Omega_F^{\,2}\right)^2 + \left(k_{damp} \cdot \Omega_F\right)^2\right]} = -6.478$$

$$B_c := \frac{\left[\left(Pressure \cdot Area_{Bore} \cdot d_{CGx}\right) + m_{head} \cdot g \cdot d_{CGz}\right] - \left(k_{necks} - I_y \cdot \Omega_F^{\,2}\right) \cdot A_c}{-\left(k_{damp} \cdot \Omega_F\right)} = 0.7$$

Solving for c.1 and c.2 using initial values

$$\theta_I(t) = \left(c_1 \cdot e^{r_1 \cdot t} + c_2 \cdot e^{r_2 \cdot t}\right) + \left(A_c \cdot \sin\left(\Omega_F \cdot t\right) + B_c \cdot \cos\left(\Omega_F \cdot t\right)\right)$$

$$\theta_I(0ms) = \left(c_1 + c_2\right) + \left(B_c \cdot \cos(0)\right) = 0deg \qquad c_1 = -\left(B_c \cdot \cos(0)\right) - c_2$$

$$\alpha_a(t) = c_1 \cdot r_1^{\,2} \cdot e^{r_1 \cdot t} + c_2 \cdot r_2^{\,2} \cdot e^{r_2 \cdot t} - A_c \cdot \Omega_F^{\,2} \cdot \sin(\Omega_F \cdot t) - B_c \cdot \Omega_F^{\,2} \cdot \cos(\Omega_F \cdot t)$$

$$\alpha_a(0ms) = c_1 \cdot r_1^{\,2} + c_2 \cdot r_2^{\,2} - B_c \cdot \Omega_F^{\,2} = \alpha_{hi}$$

$$\left[-\left(B_c \cdot \cos(0)\right) - c_2\right] \cdot r_1^{\,2} + c_2 \cdot r_2^{\,2} - B_c \cdot \Omega_F^{\,2} \cdot \cos(0) = \alpha_{hi}$$

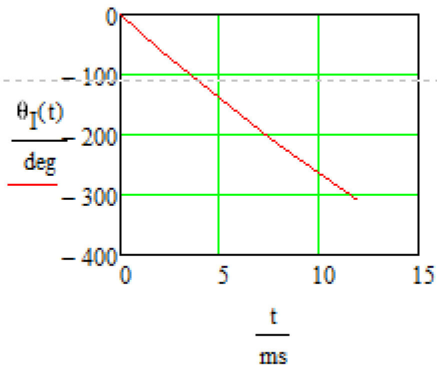
$$-r_1^{\,2} \cdot \left(B_c \cdot \cos(0)\right) - r_1^{\,2} \cdot c_2 + c_2 \cdot r_2^{\,2} = \alpha_{hi} + B_c \cdot \Omega_F^{\,2}$$

$$c_2 := \frac{\alpha_{hi} + B_c \cdot \Omega_F^{\,2} + r_1^{\,2} \cdot \left(B_c\right)}{\left(r_2^{\,2} - r_1^{\,2}\right)} = 30.598$$

$$c_1 := -B_c - c_2 = -31.299$$

Complete Solution

$$\theta_I(t) := \left(c_1 \cdot e^{r_1 \cdot t} + c_2 \cdot e^{r_2 \cdot t}\right) + \left(A_c \cdot \sin(\Omega_F \cdot t) + B_c \cdot \cos(\Omega_F \cdot t)\right)$$



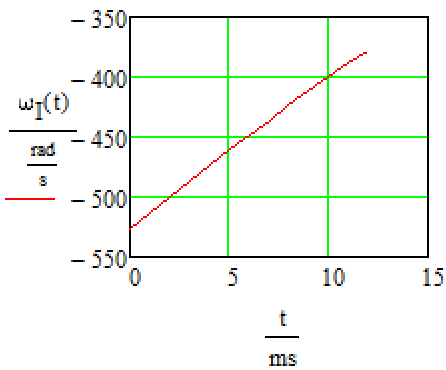
$$\theta_I(0ms) = -1.554 \times 10^{-15}$$

$$\theta_{EI} := \theta_I(t_I) = -309.178 \cdot deg$$

Distance cylinder is in contact with head

$$d_{EI} := \sqrt{2 \cdot d_{CG}^{\,2} - 2 \cdot d_{CG}^{\,2} \cdot \cos(\theta_{EI})} = 1.91 \cdot in$$

$$\omega_I(t) := c_1 \cdot r_1 \cdot e^{r_1 \cdot t} + c_2 \cdot r_2 \cdot e^{r_2 \cdot t} + A_c \cdot \Omega_F \cdot \cos(\Omega_F \cdot t) - B_c \cdot \Omega_F \cdot \sin(\Omega_F \cdot t)$$

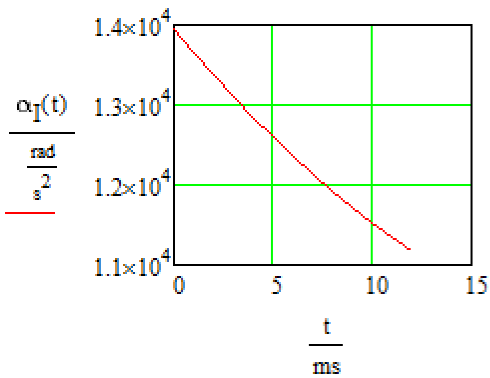


$$\omega_I(0ms) = -526.989 \frac{1}{s}$$

$$\omega_{EI} := \omega_I(t_I) = -377.89 \cdot \frac{rad}{sec}$$

$$\omega_{hi} = -22.159 \frac{1}{s}$$

$$\alpha_I(t) := c_1 \cdot r_1^{\,2} \cdot e^{r_1 \cdot t} + c_2 \cdot r_2^{\,2} \cdot e^{r_2 \cdot t} - A_c \cdot \Omega_F^{\,2} \cdot \sin(\Omega_F \cdot t) - B_c \cdot \Omega_F^{\,2} \cdot \cos(\Omega_F \cdot t)$$



$$\alpha_{EI} := \alpha_I(t_I) = 1.118 \times 10^4 \cdot \frac{rad}{s^2}$$

$$\alpha_I(0ms) = 1.395 \times 10^4 \frac{1}{s^2}$$

$$\alpha_{hi} = 1.395 \times 10^4 \frac{1}{s^2}$$

After impulse ie after cylinder stops

Angle traveled through during impulse $\theta_{EI} = -309.178 \cdot deg$

Velocity $\omega_{EI} = -377.89 \frac{1}{s}$ $v_A := \omega_{EI} \cdot d_{CG} = -21.357 \frac{m}{s}$

$t_a := t_I, (t_I + .001s) .. 1s$ $\theta_{AI} := \theta_{EI}, \theta_{EI} + .25rad .. 2rad$

$$\Omega_{Fa} := \frac{\theta_{EI}}{t_I} = -449.682 \frac{1}{s}$$

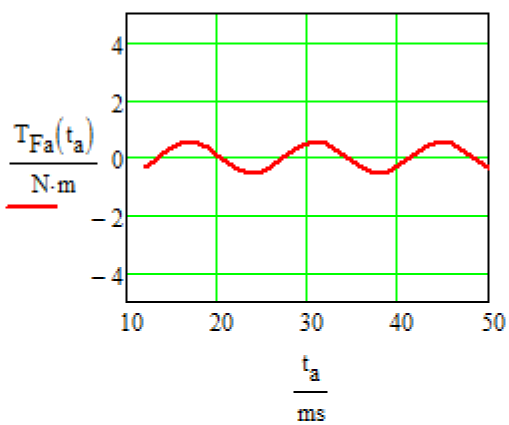
$$\Omega_{Fha} := \frac{19.161}{s}$$

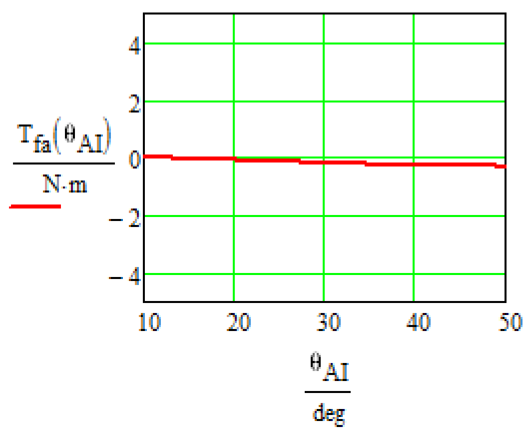
$$T_{Fa}(t_a) := -m_{head} \cdot g \cdot d_{CGz} \cdot \sin(\Omega_{Fa} \cdot t_a) + (m_{head} \cdot g \cdot d_{CGx}) \cdot \cos(\Omega_{Fa} \cdot t_a)$$

$$T_{fa}(\theta_{AI}) := -(m_{head} \cdot g \cdot d_{CGz}) \cdot \sin(\theta_{AI}) + (m_{head} \cdot g \cdot d_{CGx}) \cdot \cos(\theta_{AI})$$

$$T_{Fa}(t_I) = -0.319 J$$

$$T_{fa}(\theta_{EI}) = -0.319 J$$





Sum of Moments After Impulse

$$M_{OCa} = \left(m_{head}\cdot g\cdot d_{CGz}\right)\cdot\sin(\theta) - \left(m_{head}\cdot g\cdot d_{CGx}\right)\cdot\cos(\theta) - k_{necks}\cdot\theta - k_{damp}\cdot\frac{d}{dt}\theta = I_y\cdot\left(\frac{d^2}{dt^2}\theta\right)$$

Complimentary Solution

$$\frac{d^2}{dt^2}\theta_a + \frac{k_{damp}}{I_y}\cdot\frac{d}{dt}\theta_a + \frac{k_{necks}}{I_y}\cdot\theta_a = \frac{\left(m_{head}\cdot g\cdot d_{CGz}\right)\cdot\sin\left(\Omega_{Fa}\cdot t_a\right) - \left(m_{head}\cdot g\cdot d_{CGx}\right)\cdot\cos\left(\Omega_{Fa}\cdot t_a\right)}{I_y}$$

$$\theta_{aI}\left(t_a\right) = c_3\cdot e^{r_1\cdot\left(t_a\right)} + c_4\cdot e^{r_2\cdot\left(t_a\right)}$$

Particular Solution

Guess

$$A_2\cdot\sin\left(\Omega_{Fa}\cdot t\right) + B_2\cdot\cos\left(\Omega_{Fa}\cdot t\right)$$

$$\omega_k(t) = A_2\cdot\Omega_{Fa}\cdot\cos\left(\Omega_{Fa}\cdot t\right) - B_2\cdot\Omega_{Fa}\cdot\sin\left(\Omega_{Fa}\cdot t\right)$$

$$\alpha_k(t) = -A_2\cdot\Omega_{Fa}^{2}\cdot\sin\left(\Omega_{Fa}\cdot t\right) - B_2\cdot\Omega_{Fa}^{2}\cdot\cos\left(\Omega_{Fa}\cdot t\right)$$

$$\frac{d^2}{dt^2}\theta_k + \frac{k_{damp}}{I_y}\cdot\frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y}\cdot\theta_k = -A\cdot\Omega_{Fa}^{2}\cdot\sin\left(\Omega_{Fa}\cdot t\right) - B\cdot\Omega_{Fa}^{2}\cdot\cos\left(\Omega_{Fa}\cdot t\right) + \frac{k_{damp}}{I_y}\cdot\left(A\cdot\Omega_{Fa}\cdot\cos\left(\Omega_{Fa}\cdot t\right) - B\cdot\Omega_{Fa}\cdot\sin\left(\Omega_{Fa}\cdot t\right)\right) + \frac{k_{necks}}{I_y}\cdot\left(A\cdot\sin\left(\Omega_{Fa}\cdot t\right) + B\cdot\cos\left(\Omega_{Fa}\cdot t\right)\right)$$

Rearrange

$$\left(-A_2\cdot\Omega_{Fa}^{2} - \frac{k_{damp}}{I_y}\cdot B_2\cdot\Omega_{Fa} + \frac{k_{necks}}{I_y}\cdot A_2\right)\cdot\sin\left(\Omega_{Fa}\cdot t\right) + \left(-B_2\cdot\Omega_{Fa}^{2} + \frac{k_{damp}}{I_y}\cdot A_2\cdot\Omega_{Fa} + \frac{k_{necks}}{I_y}\cdot B_2\right)\cdot\cos\left(\Omega_{Fa}\cdot t\right) = \frac{\left(m_{head}\cdot g\cdot d_{CGz}\right)\cdot\sin\left(\theta_k\right) - \left(m_{head}\cdot g\cdot d_{CGx}\right)\cdot\cos\left(\theta_k\right)}{I_y}$$

$$-A_2\cdot\Omega_{Fa}^{2} - \frac{k_{damp}}{I_y}\cdot B_2\cdot\Omega_{Fa} + \frac{k_{necks}}{I_y}\cdot A_2 = \left(\frac{m_{head}\cdot g\cdot d_{CGz}}{I_y}\right)$$

$$-B_2 \cdot \Omega_{Fa}^{2} + \frac{k_{damp}}{I_y} \cdot A_2 \cdot \Omega_{Fa} + \frac{k_{necks}}{I_y} \cdot B_2 = \left(\frac{-m_{head} \cdot g \cdot d_{CGx}}{I_y} \right)$$

$$-A_2 \cdot I_y \cdot \Omega_F^{2} - k_{damp} \cdot B_2 \cdot \Omega_F + k_{necks} \cdot A_2 = m_{head} \cdot g \cdot d_{CGz}$$

$$\left(k_{necks} - I_y \cdot \Omega_F^{2}\right) \cdot A_2 - k_{damp} \cdot \Omega_F \cdot B_2 = m_{head} \cdot g \cdot d_{CGz}$$

$$-B_2 \cdot I_y \cdot \Omega_F^{2} + k_{damp} \cdot A_2 \cdot \Omega_F + k_{necks} \cdot B_2 = -m_{head} \cdot g \cdot d_{CGx}$$

$$\left(k_{necks} - I_y \cdot \Omega_F^{2}\right) \cdot B_2 + k_{damp} \cdot \Omega_F \cdot A_2 = -m_{head} \cdot g \cdot d_{CGx}$$

$$B_2 = \frac{-m_{head} \cdot g \cdot d_{CGx} - k_{damp} \cdot \Omega_{Fa} \cdot A_2}{\left(k_{necks} - I_y \cdot \Omega_{Fa}^{2}\right)}$$

$$\left(k_{necks} - I_y \cdot \Omega_{Fa}^{2}\right) \cdot A_2 - k_{damp} \cdot \Omega_{Fa} \cdot \left(\frac{-m_{head} \cdot g \cdot d_{CGx} - k_{damp} \cdot \Omega_{Fa} \cdot A_2}{k_{necks} - I_y \cdot \Omega_{Fa}^{2}} \right) = m_{head} \cdot g \cdot d_{CGz}$$

$$\left(k_{necks} - I_y \cdot \Omega_{Fa}^{2}\right)^{2} \cdot A_2 + k_{damp} \cdot \Omega_{Fa} \cdot m_{head} \cdot g \cdot d_{CGx} + \left(k_{damp} \cdot \Omega_{Fa}\right)^{2} \cdot A_2 = \left(k_{necks} - I_y \cdot \Omega_{Fa}^{2}\right) \cdot \left(m_{head} \cdot g \cdot d_{CGz}\right)$$

$$\left[\left(k_{necks} - I_y \cdot \Omega_{Fa}^{2}\right)^{2} + \left(k_{damp} \cdot \Omega_{Fa}\right)^{2}\right] \cdot A_2 + k_{damp} \cdot \Omega_{Fa} \cdot m_{head} \cdot g \cdot d_{CGx} = \left(k_{necks} - I_y \cdot \Omega_{Fa}^{2}\right) \cdot \left(m_{head} \cdot g \cdot d_{CGz}\right)$$

$$A_2 := \frac{\left(k_{necks} - I_y \cdot \Omega_{Fa}^{2}\right) \cdot \left(m_{head} \cdot g \cdot d_{CGz}\right) - k_{damp} \cdot \Omega_{Fa} \cdot m_{head} \cdot g \cdot d_{CGx}}{\left[\left(k_{necks} - I_y \cdot \Omega_{Fa}^{2}\right)^{2} + \left(k_{damp} \cdot \Omega_{Fa}\right)^{2}\right]} = -4.891 \times 10^{-4}$$

$$B_2 := \frac{-m_{head} \cdot g \cdot d_{CGx} - k_{damp} \cdot \Omega_{Fa} \cdot A_2}{k_{necks} - I_y \cdot \Omega_{Fa}^{2}} = 1.662 \times 10^{-4}$$

$$\theta_{aI}(t_a) = c_3 \cdot e^{r_1 \cdot (t_a)} + c_4 \cdot e^{r_2 \cdot (t_a)} + A_2 \cdot \sin\left[\Omega_F \cdot (t_a)\right] + B_2 \cdot \cos\left[\Omega_F \cdot (t_a)\right]$$

$$\theta_{aI}(t_I) = c_3 \cdot e^{r_1 \cdot (t_I)} + c_4 \cdot e^{r_2 \cdot (t_I)} + A_2 \cdot \sin\left[\Omega_F \cdot (t_I)\right] + B_2 \cdot \cos\left[\Omega_F \cdot (t_I)\right] = \theta_{EI}$$

$$c_3 = \frac{\theta_{EI} - A_2 \cdot \sin\left[\Omega_{Fa} \cdot (t_I)\right] - B_2 \cdot \cos\left[\Omega_{Fa} \cdot (t_I)\right] - c_4 \cdot e^{r_2 \cdot (t_I)}}{e^{r_1 \cdot (t_I)}}$$

$$\alpha_{aI}(t_I) = c_3 \cdot r_1^{2} \cdot e^{r_1 \cdot (t_I)} + c_4 \cdot r_2^{2} \cdot e^{r_2 \cdot (t_I)} - A_2 \cdot \Omega_{Fa}^{2} \cdot \sin\left[\Omega_{Fa} \cdot (t_I)\right] - B_2 \cdot \Omega_{Fa}^{2} \cdot \cos\left[\Omega_{Fa} \cdot (t_I)\right] = \alpha_I(t_I)$$

$$\left[\omega_{EI} - A_2\cdot\Omega_{Fa}\cdot\cos\left[\Omega_{Fa}\cdot(t_I)\right] + B_2\cdot\Omega_{Fa}\cdot\sin\left[\Omega_{Fa}\cdot(t_I)\right] - c_4\cdot r_2\cdot e^{r_2\cdot(t_I)}\right]\cdot r_1 + c_4\cdot r_2^{\,2}\cdot e^{r_2\cdot(t_I)} - A_2\cdot\Omega_{Fa}^{\,2}\cdot\sin\left[\Omega_{Fa}\cdot(t_I)\right] - B_2\cdot\Omega_{Fa}^{\,2}\cdot\cos\left[\Omega_{Fa}\cdot(t_I)\right] = \alpha_I(t_I)$$

$$\omega_{EI}\cdot r_1 - A_2\cdot r_1\cdot\Omega_{Fa}\cdot\cos\left[\Omega_{Fa}\cdot(t_I)\right] + B_2\cdot r_1\cdot\Omega_{Fa}\cdot\sin\left[\Omega_{Fa}\cdot(t_I)\right] - c_4\cdot r_1\cdot r_2\cdot e^{r_2\cdot(t_I)} + c_4\cdot r_2^{\,2}\cdot e^{r_2\cdot(t_I)} - A_2\cdot\Omega_{Fa}^{\,2}\cdot\sin\left[\Omega_{Fa}\cdot(t_I)\right] - B_2\cdot\Omega_{Fa}^{\,2}\cdot\cos\left[\Omega_{Fa}\cdot(t_I)\right] = \alpha_I(t_I)$$

$$c_4\cdot\left[r_2^{\,2}\cdot e^{r_2\cdot(t_I)} - r_1\cdot r_2\cdot e^{r_2\cdot(t_I)}\right] = \alpha_I(t_I) + A_2\cdot\Omega_{Fa}^{\,2}\cdot\sin\left[\Omega_{Fa}\cdot(t_I)\right] + B_2\cdot\Omega_{Fa}^{\,2}\cdot\cos\left[\Omega_{Fa}\cdot(t_I)\right] - \omega_{EI}\cdot r_1 + A_2\cdot r_1\cdot\Omega_{Fa}\cdot\cos\left[\Omega_{Fa}\cdot(t_I)\right] - B_2\cdot r_1\cdot\Omega_{Fa}\cdot\sin\left[\Omega_{Fa}\cdot(t_I)\right]$$

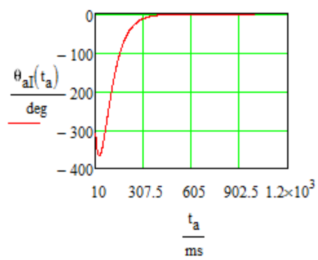
$$c_4 := \left[\frac{\alpha_I(t_I) + A_2\cdot\Omega_{Fa}^{\,2}\cdot\sin\left[\Omega_{Fa}\cdot(t_I)\right] + B_2\cdot\Omega_{Fa}^{\,2}\cdot\cos\left[\Omega_{Fa}\cdot(t_I)\right] - \omega_{EI}\cdot r_1 + A_2\cdot r_1\cdot\Omega_{Fa}\cdot\cos\left[\Omega_{Fa}\cdot(t_I)\right] - B_2\cdot r_1\cdot\Omega_{Fa}\cdot\sin\left[\Omega_{Fa}\cdot(t_I)\right]}{\left[r_2^{\,2}\cdot e^{r_2\cdot(t_I)} - r_1\cdot r_2\cdot e^{r_2\cdot(t_I)}\right]}\right] = 23.342$$

$$c_3 := \frac{\theta_{EI} - A_2\cdot\sin\left[\Omega_{Fa}\cdot(t_I)\right] - B_2\cdot\cos\left[\Omega_{Fa}\cdot(t_I)\right] - c_4\cdot e^{r_2\cdot(t_I)}}{e^{r_1\cdot(t_I)}} = -26.925$$

$$\frac{d^2}{dt^2}\theta_k + \frac{k_{damp}}{I_y}\cdot\frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y}\cdot\theta_k = -A\cdot\Omega_{Fa}^{\,2}\cdot\sin(\Omega_{Fa}\cdot t) - B\cdot\Omega_{Fa}^{\,2}\cdot\cos(\Omega_{Fa}\cdot t) + \frac{k_{damp}}{I_y}\cdot(A\cdot\Omega_{Fa}\cdot\cos(\Omega_{Fa}\cdot t) - B\cdot\Omega_{Fa}\cdot\sin(\Omega_{Fa}\cdot t)) + \frac{k_{necks}}{I_y}\cdot(A\cdot\sin(\Omega_{Fa}\cdot t) + B\cdot\cos(\Omega_{Fa}\cdot t))$$

Complete Solution

$$\theta_{aI}(t_a) := c_3\cdot e^{r_1\cdot(t_a)} + c_4\cdot e^{r_2\cdot(t_a)} + A_2\cdot\sin\left[\Omega_{Fa}\cdot(t_a)\right] + B_2\cdot\cos\left[\Omega_{Fa}\cdot(t_a)\right]$$



$\theta_{raI} := \theta_{aI}(t_I) = -309.178\cdot deg$ $\qquad \theta_{EI} = -309.178\cdot deg$

$\theta_{aI}(.502s) = -0.542\cdot deg$

$\theta_{aI}(17ms) = -334.331\cdot deg$

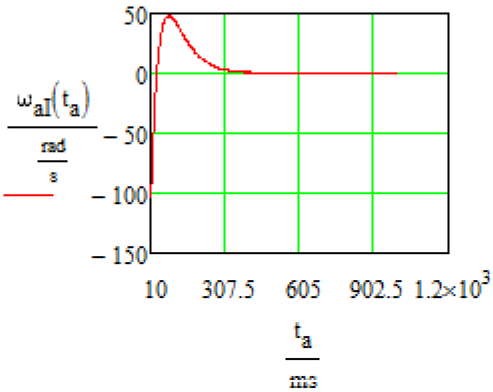
Guess $\qquad t_g := 350ms$

Given

$$\left[c_3\cdot e^{r_1\cdot(t_g)} + c_4\cdot e^{r_2\cdot(t_g)} + A_2\cdot\sin\left[\Omega_{Fa}\cdot(t_g)\right] + B_2\cdot\cos\left[\Omega_{Fa}\cdot(t_g)\right] = 0deg\right]$$

$\text{Find}(t_g) = 0.687\,s$

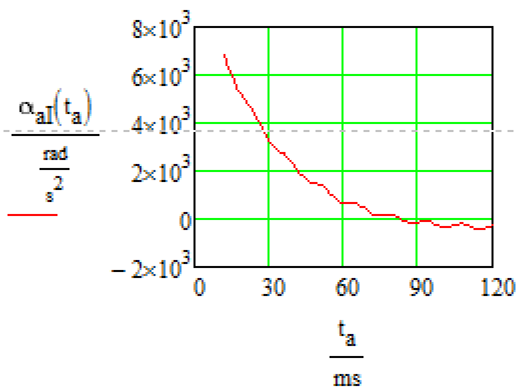
$$\omega_{aI}(t_a) := c_3 \cdot r_1 \cdot e^{r_1 \cdot (t_a)} + c_4 \cdot r_2 \cdot e^{r_2 \cdot (t_a)} + A_2 \cdot \Omega_{Fa} \cdot \cos[\Omega_{Fa} \cdot (t_a)] - B_2 \cdot \Omega_{Fa} \cdot \sin[\Omega_{Fa} \cdot (t_a)]$$



$$\omega_{EI} = -377.89 \cdot \frac{rad}{sec}$$

$$\omega_{raI} := \omega_{aI}(t_I) = -103.721 \frac{1}{s} \cdot rad$$

$$\alpha_{aI}(t_a) := c_3 \cdot r_1{}^2 \cdot e^{r_1 \cdot (t_a)} + c_4 \cdot r_2{}^2 \cdot e^{r_2 \cdot (t_a)} - A_2 \cdot \Omega_{Fa}{}^2 \cdot \sin[\Omega_{Fa} \cdot (t_a)] - B_2 \cdot \Omega_{Fa}{}^2 \cdot \cos[\Omega_{Fa} \cdot (t_a)]$$

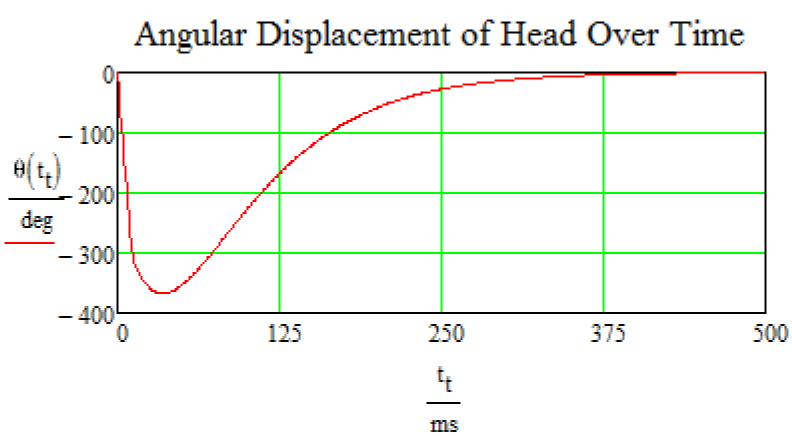


$$\alpha_{EI} = 1.118 \times 10^4 \frac{1}{s^2}$$

$$\alpha_{aI}(t_I) = 6.835 \times 10^3 \frac{1}{s^2}$$

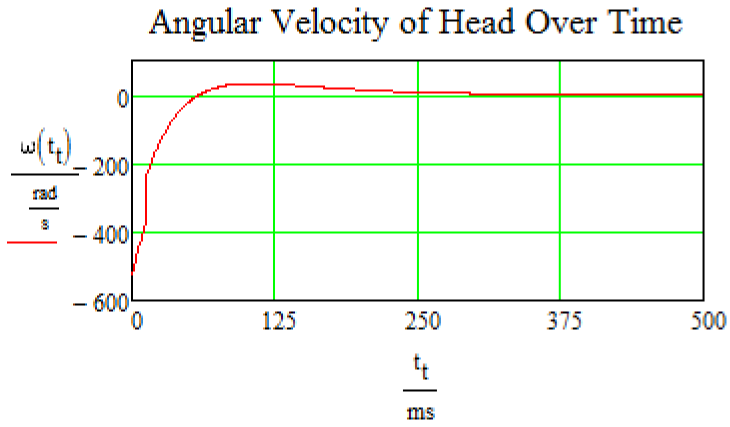
$t_t := 0s, .001s .. .5s$

$$\theta(t_t) := \begin{cases} c_1 \cdot e^{r_1 \cdot t_t} + c_2 \cdot e^{r_2 \cdot t_t} + A_c \cdot \sin(\Omega_F \cdot t_t) + B_c \cdot \cos(\Omega_F \cdot t_t) & \text{if } 0s \le t_t \le t_I \\ c_3 \cdot e^{r_1 \cdot (t_t)} + c_4 \cdot e^{r_2 \cdot (t_t)} + A_2 \cdot \sin[\Omega_{Fa} \cdot (t_t)] + B_2 \cdot \cos[\Omega_{Fa} \cdot (t_t)] & \text{if } t_I < t_t \le .5s \end{cases}$$



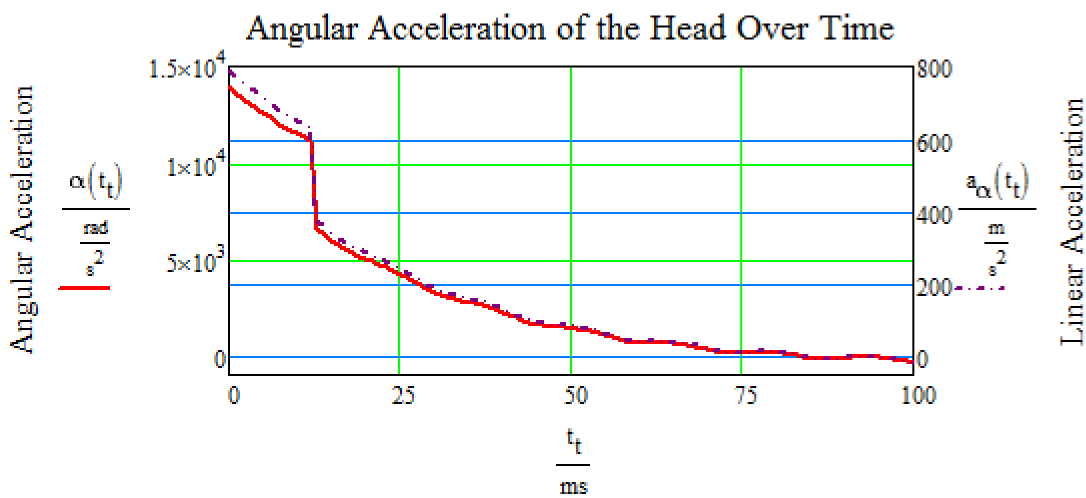
$$\Omega_{Fa} = -449.682 \frac{1}{s}$$

$$\omega(t_t) := \begin{cases} \left(c_1 \cdot r_1 \cdot e^{r_1 \cdot t_t} + c_2 \cdot r_2 \cdot e^{r_2 \cdot t_t} + A_c \cdot \Omega_F \cdot \cos(\Omega_F \cdot t_t) - B_c \cdot \Omega_F \cdot \sin(\Omega_F \cdot t_t)\right) & \text{if } 0s \le t_t \le t_I \\ \left(c_3 \cdot r_1 \cdot e^{r_1 \cdot t_t} + c_2 \cdot r_2 \cdot e^{r_2 \cdot t_t} + A_2 \cdot \Omega_{Fa} \cdot \cos(\Omega_{Fa} \cdot t_t) - B_2 \cdot \Omega_{Fa} \cdot \sin(\Omega_{Fa} \cdot t_t)\right) & \text{if } t_I < t_t \le .5s \end{cases}$$

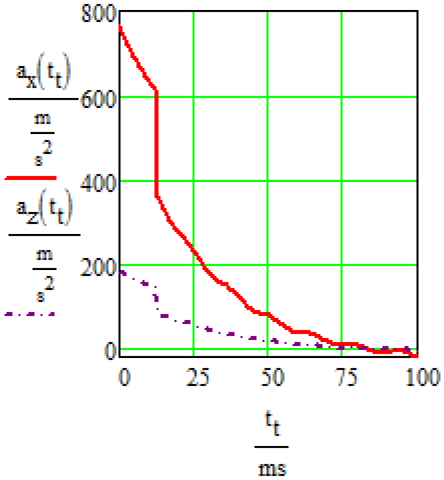


$$\alpha(t_t) := \begin{cases} \left(c_1 \cdot r_1^2 \cdot e^{r_1 \cdot t_t} + c_2 \cdot r_2^2 \cdot e^{r_2 \cdot t_t} - A_c \cdot \Omega_F^2 \cdot \sin(\Omega_F \cdot t_t) - B_c \cdot \Omega_F^2 \cdot \cos(\Omega_F \cdot t_t)\right) & \text{if } 0ms \le t_t \le t_I \\ \left[c_3 \cdot r_1^2 \cdot e^{r_1 \cdot (t_t)} + c_4 \cdot r_2^2 \cdot e^{r_2 \cdot (t_t)} - A_2 \cdot \Omega_{Fa}^2 \cdot \sin\left[\Omega_{Fa} \cdot (t_t)\right] - B_2 \cdot \Omega_{Fa}^2 \cdot \cos\left[\Omega_{Fa} \cdot (t_t)\right]\right] & \text{if } t_I < t_t \le .5s \end{cases}$$

$$a_\alpha(t_t) := \alpha(t_t) \cdot d_{CG}$$



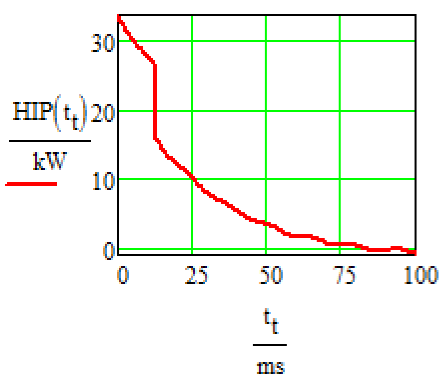
$a_x(t_t) := \alpha(t_t) \cdot d_{CGz}$ $\qquad$ $a_z(t_t) := \alpha(t_t) \cdot d_{CGx}$



$a_{xmax} := a_x(0ms) = 767.45 \frac{m}{s^2}$

$a_{zmax} := a_z(0ms) = 181.397 \frac{m}{s^2}$

$$HIP(t_t) := m_{head} \cdot a_x(t_t) \cdot \int_{0ms}^{80ms} a_x(t_t)\, dt_t + m_{head} \cdot a_z(t_t) \cdot \int_{0ms}^{80ms} a_z(t_t)\, dt_t + I_y \cdot \alpha(t_t) \cdot \int_{0ms}^{80ms} \alpha(t_t)\, dt_t$$



$HIP(0ms) = 33.605 \cdot kW$

C-6: Calculations for Finding Necessary Damping Coefficient of Neck Support

$P := 100\,\mathrm{psi}$ $\qquad k_{oobleck} := .15115\,\frac{m^2 \cdot kg}{s}$ $\qquad k_{damp} = 0.215\,\frac{m^2 \cdot kg}{s}$

$$I_y \cdot \frac{d^2}{dt^2}\theta_k + \left(k_{damp} + k_{oobleck}\right) \cdot \frac{d}{dt}\theta_k + k_{necks} \cdot \theta_k = \frac{\left(P \cdot Area_{Bore} \cdot d_{CGx} + m_{head} \cdot g \cdot d_{CGz}\right) \cdot \sin\left(\Omega_F \cdot t\right) - \left(P \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right) \cdot \cos\left(\Omega_F \cdot t\right)}{I_y}$$

Complimentary Solution (Left side of equation)

$$I_y \cdot \frac{d^2}{dt^2}\theta_k + \left(k_{damp} + k_{oobleck}\right) \cdot \frac{d}{dt}\theta_k + k_{necks} \cdot \theta_k$$

$$\frac{d^2}{dt^2}\theta_k + \frac{\left(k_{damp} + k_{oobleck}\right)}{I_y} \cdot \frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y} \cdot \theta_k = 0 \qquad Area_{Bore} = 11.401\,cm^2$$

$$r_{n1} := \frac{-\frac{\left(k_{damp} + k_{oobleck}\right)}{I_y} + \sqrt{\left(\frac{k_{damp} + k_{oobleck}}{I_y}\right)^2 - 4 \cdot \frac{k_{necks}}{I_y}}}{2} = -6.433\,\frac{1}{s}$$

$$r_1 = -15.853\,\frac{1}{s}$$

$$r_{n2} := \frac{-\frac{\left(k_{damp} + k_{oobleck}\right)}{I_y} - \sqrt{\left[\frac{\left(k_{damp} + k_{oobleck}\right)}{I_y}\right]^2 - 4 \cdot \frac{k_{necks}}{I_y}}}{2} = -66.75\,\frac{1}{s}$$

$$r_2 = -27.087\,\frac{1}{s}$$

$$\theta_{nI}(t) = c_{n1} \cdot e^{r_{n1} \cdot t} + c_{n2} \cdot e^{r_{n2} \cdot t}$$

Particular Solution (right side of equation)

$$\frac{\left(P \cdot Area_{Bore} \cdot d_{CGx} + m_{head} \cdot g \cdot d_{CGz}\right) \cdot \sin\left(\Omega_F \cdot t\right) - \left(P \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right) \cdot \cos\left(\Omega_F \cdot t\right)}{I_y}$$

Guess

$$A \cdot \sin\left(\Omega_F \cdot t\right) + B \cdot \cos\left(\Omega_F \cdot t\right)$$

$$\omega_k(t) = A \cdot \Omega_F \cdot \cos\left(\Omega_F \cdot t\right) - B \cdot \Omega_F \cdot \sin\left(\Omega_F \cdot t\right)$$

$$\alpha_k(t) = -A \cdot \Omega_F^2 \cdot \sin\left(\Omega_F \cdot t\right) - B \cdot \Omega_F^2 \cdot \cos\left(\Omega_F \cdot t\right)$$

$$\frac{d^2}{dt^2}\theta_k + \frac{\left(k_{damp} + k_{oobleck}\right)}{I_y} \cdot \frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y} \cdot \theta_k = -A \cdot \Omega_F^2 \cdot \sin\left(\Omega_F \cdot t\right) - B \cdot \Omega_F^2 \cdot \cos\left(\Omega_F \cdot t\right) + \frac{\left(k_{damp} + k_{oobleck}\right)}{I_y} \cdot \left(A \cdot \Omega_F \cdot \cos\left(\Omega_F \cdot t\right) - B \cdot \Omega_n \cdot \sin\left(\Omega_F \cdot t\right)\right) + \frac{k_{necks}}{I_y} \cdot \left(A \cdot \sin\left(\Omega_F \cdot t\right) + B \cdot \cos\left(\Omega_F \cdot t\right)\right)$$

Rearrange

$$\left[-A \cdot \Omega_F^2 - \frac{\left(k_{damp} + k_{oobleck}\right)}{I_y} \cdot B \cdot \Omega_F + \frac{k_{necks}}{I_y} \cdot A\right] \cdot \sin\left(\Omega_F \cdot t\right) + \left[-B \cdot \Omega_n^2 + \frac{\left(k_{damp} + k_{oobleck}\right)}{I_y} \cdot A \cdot \Omega_F + \frac{k_{necks}}{I_y} \cdot B\right] \cdot \cos\left(\Omega_F \cdot t\right) = \frac{\begin{pmatrix} P \cdot Area_{Bore} \cdot d_{CGx} \cdots \\ + m_{head} \cdot g \cdot d_{CGz} \end{pmatrix} \cdot \sin\left(\theta_k\right) - \begin{pmatrix} P \cdot Area_{Bore} \cdot d_{CGz} \cdots \\ + m_{head} \cdot g \cdot d_{CGx} \end{pmatrix} \cdot \cos\left(\theta_k\right)}{I_y}$$

$$\left[-A \cdot \Omega_F^2 - \frac{\left(k_{damp} + k_{oobleck}\right)}{I_y} \cdot B \cdot \Omega_F + \frac{k_{necks}}{I_y} \cdot A\right] = \frac{\begin{pmatrix} P \cdot Area_{Bore} \cdot d_{CGx} \cdots \\ + m_{head} \cdot g \cdot d_{CGz} \end{pmatrix}}{I_y}$$

$$\left[-B \cdot \Omega_F^2 + \frac{\left(k_{damp} + k_{oobleck}\right)}{I_y} \cdot A \cdot \Omega_F + \frac{k_{necks}}{I_y} \cdot B\right] = \frac{-\begin{pmatrix} P \cdot Area_{Bore} \cdot d_{CGz} \cdots \\ + m_{head} \cdot g \cdot d_{CGx} \end{pmatrix}}{I_y}$$

$$\left(k_{necks} - I_y \cdot \Omega_F^2\right) \cdot B + \left(k_{damp} + k_{oobleck}\right) \cdot \Omega_F \cdot A = -\left(P \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right)$$

$$\left(k_{necks} - I_y \cdot \Omega_F^2\right) \cdot A + \left(k_{damp} + k_{oobleck}\right) \cdot \Omega_F \cdot B = \left(P \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right)$$

$$A_n = \frac{P \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx} - \left(k_{damp} + k_{oobleck}\right) \cdot \Omega_F \cdot B_n}{\left(k_{necks} - I_y \cdot \Omega_F^2\right)}$$

$$B_n = \frac{-\left(P \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right) - \left(k_{damp} + k_{oobleck}\right) \cdot \Omega_F \cdot A_n}{\left(k_{necks} - I_y \cdot \Omega_F^2\right)}$$

$$B_n = \frac{-\left(P \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right) - \left(k_{damp} + k_{oobleck}\right) \cdot \Omega_F \cdot \left[\frac{P \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx} - \left(k_{damp} + k_{oobleck}\right) \cdot \Omega_F \cdot B_n}{\left(k_{necks} - I_y \cdot \Omega_F^2\right)}\right]}{\left(k_{necks} - I_y \cdot \Omega_F^2\right)}$$

$$B_n \cdot \left(k_{necks} - I_y \cdot \Omega_F^2\right) = -\left(P \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right) + \frac{-\Omega_F \cdot \left(k_{damp} + k_{oobleck}\right) \cdot P \cdot Area_{Bore} \cdot d_{CGz}}{k_{necks} - I_y \cdot \Omega_F^2} - \frac{\Omega_F \cdot \left(k_{damp} + k_{oobleck}\right) \cdot m_{head} \cdot g \cdot d_{CGx}}{k_{necks} - I_y \cdot \Omega_F^2} + \frac{\left[\Omega_F \cdot \left(k_{damp} + k_{oobleck}\right)\right]^2}{\left(k_{necks} - I_y \cdot \Omega_F^2\right)} \cdot B_n$$

$$B_n \cdot \left[\left(k_{necks} - I_y \cdot \Omega_F^2\right) - \frac{\left[\Omega_F \cdot \left(k_{damp} + k_{oobleck}\right)\right]^2}{\left(k_{necks} - I_y \cdot \Omega_F^2\right)}\right] = -\left(P \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right) + \frac{-\Omega_F \cdot \left(k_{damp} + k_{oobleck}\right) \cdot P \cdot Area_{Bore} \cdot d_{CGz}}{k_{necks} - I_y \cdot \Omega_F^2} - \frac{\Omega_F \cdot \left(k_{damp} + k_{oobleck}\right) \cdot m_{head} \cdot g \cdot d_{CGx}}{k_{necks} - I_y \cdot \Omega_F^2}$$

$$A_n := \frac{\left[\left(k_{damp} + k_{oobleck}\right) \cdot \Omega_F\right] \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGz} + m_{head} \cdot g \cdot d_{CGx}\right) - \left(k_{necks} - I_y \cdot \Omega_F^2\right) \cdot \left(Pressure \cdot Area_{Bore} \cdot d_{CGx} + m_{head} \cdot g \cdot d_{CGz}\right)}{-\left[\left(k_{necks} - I_y \cdot \Omega_F^2\right)^2 + \left[\left(k_{damp} + k_{oobleck}\right) \cdot \Omega_F\right]^2\right]} = -3.978$$

$$B_n := \frac{\left[\left(Pressure \cdot Area_{Bore} \cdot d_{CGx}\right) + m_{head} \cdot g \cdot d_{CGz}\right] - \left(k_{necks} - I_y \cdot \Omega_F^2\right) \cdot A_n}{-\left[\left(k_{damp} + k_{oobleck}\right) \cdot \Omega_F\right]} = -0.125$$

$$\theta_{nI}(t) = \left(c_{n1} \cdot e^{r_{n1} \cdot t} + c_{n2} \cdot e^{r_{n2} \cdot t}\right) + \left(A_n \cdot \sin\left(\Omega_F \cdot t\right) + B_n \cdot \cos\left(\Omega_F \cdot t\right)\right)$$

$$\theta_{nI}(0ms) = \left(c_{n1} + c_{n2}\right) + \left(B_n \cdot \cos(0)\right) = 0deg \qquad c_{n1} = -\left(B_n \cdot \cos(0)\right) - c_{n2}$$

$$\alpha_{na}(t) = c_{n1} \cdot r_{n1}^2 \cdot e^{r_{n1} \cdot t} + c_{n2} \cdot r_{n2}^2 \cdot e^{r_{n2} \cdot t} - A_n \cdot \Omega_F^2 \cdot \sin\left(\Omega_F \cdot t\right) - B_n \cdot \Omega_F^2 \cdot \cos\left(\Omega_F \cdot t\right)$$

$$\alpha_a(0ms) = c_{n1} \cdot r_{n1}^2 + c_{n2} \cdot r_{n2}^2 - B_n \cdot \Omega_F^2 = \alpha_{hi}$$

$$\left[-\left(B_n \cdot \cos(0)\right) - c_{n2}\right] \cdot r_1^2 + c_{n2} \cdot m_2^2 - B_n \cdot \Omega_F^2 \cdot \cos(0) = \alpha_{hi}$$

$$-r_{n1}^2 \cdot \left(B_n \cdot \cos(0)\right) - r_{n1}^2 \cdot c_{n2} + c_{n2} \cdot r_{n2}^2 = \alpha_{hi} + B_n \cdot \Omega_F^2$$

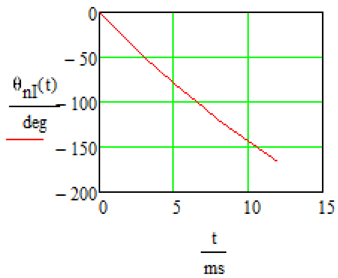
$$c_{n2} := \frac{\alpha_{hi} + B_n \cdot \Omega_F^2 + r_{n1}^2 \cdot \left(B_n\right)}{\left(r_{n2}^2 - r_{n1}^2\right)} = 3.134 \qquad c_2 = 30.598$$

$$c_1 = -31.299$$

$$c_{n1} := -B_n - c_{n2} = -3.009$$

Complete Solution

$$\theta_{nI}(t) := \left(c_{n1}\cdot e^{r_{n1}\cdot t} + c_{n2}\cdot e^{r_{n2}\cdot t}\right) + \left(A_n\cdot \sin(\Omega_F\cdot t) + B_n\cdot \cos(\Omega_F\cdot t)\right)$$



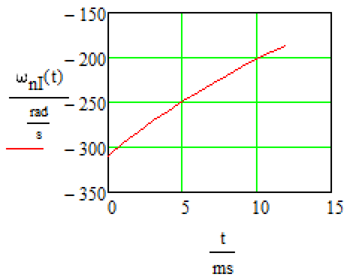
$$\theta_{nI}(0ms) = 0$$

$$\theta_{nEI} := \theta_{nI}(t_I) = -165.997\cdot deg$$

Distance cylinder is in contact with head

$$d_{nEI} := \sqrt{2\cdot d_{CG}^2 - 2\cdot d_{CG}^2\cdot \cos(\theta_{nEI})} = 4.417\cdot in$$

$$\omega_{nI}(t) := c_{n1}\cdot r_{n1}\cdot e^{r_{n1}\cdot t} + c_{n2}\cdot r_{n2}\cdot e^{r_{n2}\cdot t} + A_n\cdot \Omega_F\cdot \cos(\Omega_F\cdot t) - B_n\cdot \Omega_F\cdot \sin(\Omega_F\cdot t)$$

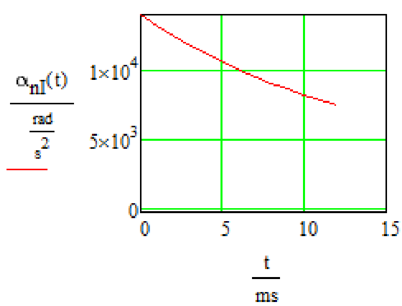


$$\omega_{nI}(0ms) = -309.203\,\frac{1}{s}$$

$$\omega_{nEI} := \omega_{nI}(t_I) = -186.363\cdot \frac{rad}{sec}$$

$$\omega_{hi} = -22.159\,\frac{1}{s}$$

$$\alpha_{nI}(t) := c_{n1}\cdot r_{n1}^2\cdot e^{r_{n1}\cdot t} + c_{n2}\cdot r_{n2}^2\cdot e^{r_{n2}\cdot t} - A_n\cdot \Omega_F^2\cdot \sin(\Omega_F\cdot t) - B_n\cdot \Omega_F^2\cdot \cos(\Omega_F\cdot t)$$



$$\alpha_{nEI} := \alpha_{nI}(t_I) = 7.52\times 10^3\cdot \frac{rad}{s^2}$$

$$\alpha_{nI}(0ms) = 1.395\times 10^4\,\frac{1}{s^2}$$

$$\alpha_{hi} = 1.395\times 10^4\,\frac{1}{s^2}$$

Sum of Moments After Impulse

$$M_{OCan} = (m_{head}\cdot g\cdot d_{CGz})\cdot \sin(\theta) - (m_{head}\cdot g\cdot d_{CGx})\cdot \cos(\theta) - k_{necks}\cdot \theta - (k_{oobleck} + k_{damp})\cdot \frac{d}{dt}\theta = I_y\cdot\left(\frac{d^2}{dt^2}\theta\right)$$

Complimentary Solution

$$\frac{d^2}{dt^2}\theta_a + \frac{(k_{oobleck} + k_{damp})}{I_y}\cdot \frac{d}{dt}\theta_a + \frac{k_{necks}}{I_y}\cdot \theta_a = \frac{(m_{head}\cdot g\cdot d_{CGz})\cdot \sin(\Omega_{Fa}\cdot t_a) - (m_{head}\cdot g\cdot d_{CGx})\cdot \cos(\Omega_{Fa}\cdot t_a)}{I_y}$$

$$\theta_{aI}(t_a) = c_{n3}\cdot e^{r_{n1}\cdot (t_a)} + c_{n4}\cdot e^{r_{n2}\cdot (t_a)}$$

Particular Solution

Guess

$$A_{n2}\cdot\sin(\Omega_{Fa}\cdot t) + B_{n2}\cdot\cos(\Omega_{Fa}\cdot t)$$

$$\omega_k(t) = A_{n2}\cdot\Omega_{Fa}\cdot\cos(\Omega_{Fa}\cdot t) - B_{n2}\cdot\Omega_{Fa}\cdot\sin(\Omega_{Fa}\cdot t)$$

$$\alpha_k(t) = -A_{n2}\cdot\Omega_{Fa}^{2}\cdot\sin(\Omega_{Fa}\cdot t) - B_{n2}\cdot\Omega_{Fa}^{2}\cdot\cos(\Omega_{Fa}\cdot t)$$

$$\frac{d^2}{dt^2}\theta_k + \frac{(k_{oobleck} + k_{damp})}{I_y}\cdot\frac{d}{dt}\theta_k + \frac{k_{necks}}{I_y}\cdot\theta_k = -A_{n2}\cdot\Omega_{Fa}^{2}\cdot\sin(\Omega_{Fa}\cdot t) - B_{n2}\cdot\Omega_{Fa}^{2}\cdot\cos(\Omega_{Fa}\cdot t) + \frac{(k_{oobleck} + k_{damp})}{I_y}\cdot(A_{n2}\cdot\Omega_{Fa}\cdot\cos(\Omega_{Fa}\cdot t) - B_{n2}\cdot\Omega_{Fa}\cdot\sin(\Omega_{Fa}\cdot t)) + \frac{k_{necks}}{I_y}\cdot(A_{n2}\cdot\sin(\Omega_{Fa}\cdot t) + B_{n2}\cdot\cos(\Omega_{Fa}\cdot t))$$

Rearrange

$$\left[-A_{n2}\cdot\Omega_{Fa}^{2} - \frac{(k_{oobleck} + k_{damp})}{I_y}\cdot B_{n2}\cdot\Omega_{Fa} + \frac{k_{necks}}{I_y}\cdot A_2\right]\cdot\sin(\Omega_{Fa}\cdot t) + \left[-B_{n2}\cdot\Omega_{Fa}^{2} + \frac{(k_{oobleck} + k_{damp})}{I_y}\cdot A_{n2}\cdot\Omega_{Fa} + \frac{k_{necks}}{I_y}\cdot B_{n2}\right]\cdot\cos(\Omega_{Fa}\cdot t) = \frac{(m_{head}\cdot g\cdot d_{CGz})\cdot\sin(\theta_k) - (m_{head}\cdot g\cdot d_{CGx})\cdot\cos(\theta_k)}{I_y}$$

$$-A_{n2}\cdot\Omega_{Fa}^{2} - \frac{(k_{oobleck} + k_{damp})}{I_y}\cdot B_{n2}\cdot\Omega_{Fa} + \frac{k_{necks}}{I_y}\cdot A_{n2} = \left(\frac{m_{head}\cdot g\cdot d_{CGz}}{I_y}\right)$$

$$-B_{n2}\cdot\Omega_{Fa}^{2} + \frac{(k_{oobleck} + k_{damp})}{I_y}\cdot A_{n2}\cdot\Omega_{Fa} + \frac{k_{necks}}{I_y}\cdot B_{n2} = \left(\frac{-m_{head}\cdot g\cdot d_{CGx}}{I_y}\right)$$

$$-A_{n2}\cdot I_y\cdot\Omega_F^{2} - (k_{oobleck} + k_{damp})\cdot B_{n2}\cdot\Omega_F + k_{necks}\cdot A_{n2} = m_{head}\cdot g\cdot d_{CGz}$$

$$\left(k_{necks} - I_y\cdot\Omega_F^{2}\right)\cdot A_{n2} - (k_{oobleck} + k_{damp})\cdot\Omega_F\cdot B_{n2} = m_{head}\cdot g\cdot d_{CGz}$$

$$-B_{n2}\cdot I_y\cdot\Omega_F^{2} + (k_{oobleck} + k_{damp})\cdot A_{n2}\cdot\Omega_F + k_{necks}\cdot B_{n2} = -m_{head}\cdot g\cdot d_{CGx}$$

$$\left(k_{necks} - I_y\cdot\Omega_F^{2}\right)\cdot B_{n2} + (k_{oobleck} + k_{damp})\cdot\Omega_F\cdot A_{n2} = -m_{head}\cdot g\cdot d_{CGx}$$

$$B_{n2} = \frac{-m_{head}\cdot g\cdot d_{CGx} - (k_{oobleck} + k_{damp})\cdot\Omega_{Fa}\cdot A_{n2}}{\left(k_{necks} - I_y\cdot\Omega_{Fa}^{2}\right)}$$

$$\left(k_{necks} - I_y\cdot\Omega_{Fa}^{2}\right)\cdot A_{n2} - (k_{oobleck} + k_{damp})\cdot\Omega_{Fa}\cdot\left[\frac{-m_{head}\cdot g\cdot d_{CGx} - (k_{oobleck} + k_{damp})\cdot\Omega_{Fa}\cdot A_{n2}}{k_{necks} - I_y\cdot\Omega_{Fa}^{2}}\right] = m_{head}\cdot g\cdot d_{CGz}$$

$$\left(k_{necks} - I_y\cdot\Omega_{Fa}^{2}\right)^{2}\cdot A_{n2} + (k_{oobleck} + k_{damp})\cdot\Omega_{Fa}\cdot m_{head}\cdot g\cdot d_{CGx} + \left[(k_{oobleck} + k_{damp})\cdot\Omega_{Fa}\right]^{2}\cdot A_{n2} = \left(k_{necks} - I_y\cdot\Omega_{Fa}^{2}\right)\cdot(m_{head}\cdot g\cdot d_{CGz})$$

$$\left[\left(k_{necks} - I_y\cdot\Omega_{Fa}^{2}\right)^{2} + \left[(k_{oobleck} + k_{damp})\cdot\Omega_{Fa}\right]^{2}\right]\cdot A_{n2} + (k_{oobleck} + k_{damp})\cdot\Omega_{Fa}\cdot m_{head}\cdot g\cdot d_{CGx} = \left(k_{necks} - I_y\cdot\Omega_{Fa}^{2}\right)\cdot(m_{head}\cdot g\cdot d_{CGz})$$

$$A_{n2} := \frac{\left(k_{necks} - I_y\cdot\Omega_{Fa}^{2}\right)\cdot(m_{head}\cdot g\cdot d_{CGz}) - (k_{oobleck} + k_{damp})\cdot\Omega_{Fa}\cdot m_{head}\cdot g\cdot d_{CGx}}{\left[\left(k_{necks} - I_y\cdot\Omega_{Fa}^{2}\right)^{2} + \left[(k_{oobleck} + k_{damp})\cdot\Omega_{Fa}\right]^{2}\right]} = -4.73\times10^{-4}$$

$$A_2 = -4.891\times10^{-4}$$

$$B_{n2} := \frac{-m_{head}\cdot g\cdot d_{CGx} - (k_{oobleck} + k_{damp})\cdot\Omega_{Fa}\cdot A_{n2}}{k_{necks} - I_y\cdot\Omega_{Fa}^{2}} = 1.965\times10^{-4}$$

$$B_2 = 1.662\times10^{-4}$$

$$\theta_{naI}(t_a) = c_{n3}\cdot e^{r_{n1}\cdot(t_a)} + c_{n4}\cdot e^{r_{n2}\cdot(t_a)} + A_{n2}\cdot\sin\left[\Omega_F\cdot(t_a)\right] + B_{n2}\cdot\cos\left[\Omega_F\cdot(t_a)\right]$$

$$\theta_{naI}(t_I) = c_{n3}\cdot e^{r_{n1}\cdot(t_I)} + c_{n4}\cdot e^{r_{n2}\cdot(t_I)} + A_{n2}\cdot\sin\left[\Omega_F\cdot(t_I)\right] + B_{n2}\cdot\cos\left[\Omega_F\cdot(t_I)\right] = \theta_{nEI}$$

$$\theta_{naI}(t_I) = c_{n3}\cdot e^{r_{n1}\cdot(t_I)} + c_{n4}\cdot e^{r_{n2}\cdot(t_I)} + A_{n2}\cdot\sin[\Omega_F\cdot(t_I)] + B_{n2}\cdot\cos[\Omega_F\cdot(t_I)] = \theta_{nEI}$$

$$\left[\frac{\alpha_{nEI} + A_{n2}\cdot\Omega_F\cdot\sin[\Omega_F\cdot(t_I)] + B_{n2}\cdot\Omega_F\cdot\cos[\Omega_F\cdot(t_I)] - c_{n4}\cdot r_{n2}^{2}\cdot e^{r_{n2}\cdot(t_I)}}{r_{n1}^{2}}\right] + c_{n4}\cdot e^{r_{n2}\cdot(t_I)} + A_{n2}\cdot\sin[\Omega_F\cdot(t_I)] + B_{n2}\cdot\cos[\Omega_F\cdot(t_I)] = \theta_{nEI}$$

$$c_{n4} := \left[\frac{\alpha_{nI}(t_I) + A_{n2}\cdot\Omega_{Fa}^{2}\cdot\sin[\Omega_{Fa}\cdot(t_I)] + B_{n2}\cdot\Omega_{Fa}^{2}\cdot\cos[\Omega_{Fa}\cdot(t_I)] - \omega_{nEI}\cdot r_{n1} + A_{n2}\cdot r_{n1}\cdot\Omega_{Fa}\cdot\cos[\Omega_{Fa}\cdot(t_I)] - B_{n2}\cdot r_{n1}\cdot\Omega_{Fa}\cdot\sin[\Omega_{Fa}\cdot(t_I)]}{\left[r_{n2}^{2}\cdot e^{r_{n2}\cdot(t_I)} - r_{n1}\cdot r_{n2}\cdot e^{r_{n2}\cdot(t_I)}\right]}\right] = 3.47$$

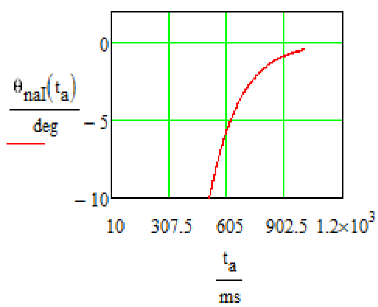
$c_4 = 23.342$

$c_3 = -26.925$

$$c_{n3} := \frac{\theta_{nEI} - A_{n2}\cdot\sin[\Omega_{Fa}\cdot(t_I)] - B_{n2}\cdot\cos[\Omega_{Fa}\cdot(t_I)] - c_{n4}\cdot e^{r_{n2}\cdot(t_I)}}{e^{r_{n1}\cdot(t_I)}} = -4.812$$

Complete Solution

$$\theta_{naI}(t_a) := c_{n3}\cdot e^{r_{n1}\cdot(t_a)} + c_{n4}\cdot e^{r_{n2}\cdot(t_a)} + A_{n2}\cdot\sin[\Omega_{Fa}\cdot(t_a)] + B_{n2}\cdot\cos[\Omega_{Fa}\cdot(t_a)]$$



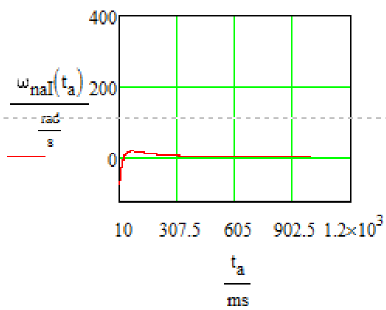
$\theta_{nraI} := \theta_{naI}(t_I) = -165.997\cdot deg$ $\theta_{nEI} = -165.997\cdot deg$

$\theta_{naI}(.502s) = -10.916\cdot deg$

$\theta_{naI}(17ms) = -183.2\cdot deg$

Guess $t_{ng} := 350ms$

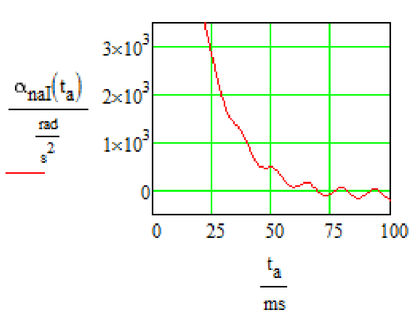
$$\omega_{naI}(t_a) := c_{n3}\cdot r_{n1}\cdot e^{r_{n1}\cdot(t_a)} + c_{n4}\cdot r_{n2}\cdot e^{r_{n2}\cdot(t_a)} + A_{n2}\cdot\Omega_{Fa}\cdot\cos[\Omega_{Fa}\cdot(t_a)] - B_{n2}\cdot\Omega_{Fa}\cdot\sin[\Omega_{Fa}\cdot(t_a)]$$



$\omega_{nEI} = -186.363\cdot\frac{rad}{sec}$

$\omega_{nraI} := \omega_{naI}(t_I) = -75.108\,\frac{1}{s}\cdot rad$

$$\alpha_{naI}(t_a) := c_{n3}\cdot r_{n1}^{2}\cdot e^{r_{n1}\cdot(t_a)} + c_{n4}\cdot r_{n2}^{2}\cdot e^{r_{n2}\cdot(t_a)} - A_{n2}\cdot\Omega_{Fa}^{2}\cdot\sin[\Omega_{Fa}\cdot(t_a)] - B_{n2}\cdot\Omega_{Fa}^{2}\cdot\cos[\Omega_{Fa}\cdot(t_a)]$$



$\alpha_{nEI} = 7.52\times10^{3}\,\frac{1}{s^2}$

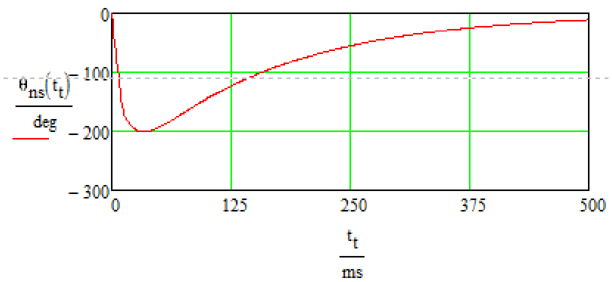
$\alpha_{naI}(t_I) = 6.804\times10^{3}\,\frac{1}{s^2}$

$\alpha_{nconstant} := \alpha_{naI}(t_I) - \alpha_{nEI} = -715.681\,\frac{1}{s^2}$

$t_t := 0s, .001s .. .5s$

$$\theta_{ns}(t_t) := \begin{cases} c_{n1}\cdot e^{r_{n1}\cdot t_t} + c_{n2}\cdot e^{r_{n2}\cdot t_t} + A_n\cdot \sin(\Omega_F\cdot t_t) + B_n\cdot \cos(\Omega_F\cdot t_t) & \text{if } 0s \le t_t \le t_I \\ c_{n3}\cdot e^{r_{n1}\cdot (t_t)} + c_{n4}\cdot e^{r_{n2}\cdot (t_t)} + A_{n2}\cdot \sin[\Omega_{Fa}\cdot (t_t)] + B_{n2}\cdot \cos[\Omega_{Fa}\cdot (t_t)] & \text{if } t_I < t_t \le .5s \end{cases}$$

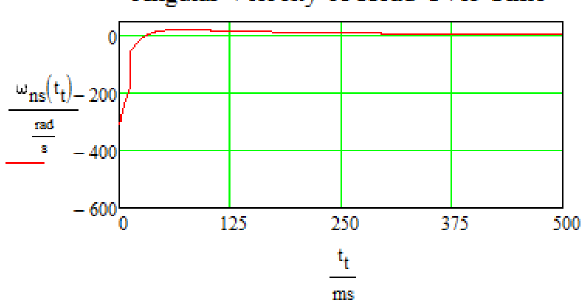
Angular Displacement of Head Over Time



$$\omega_{ns}(t_t) := \begin{cases} \left(c_{n1}\cdot r_{n1}\cdot e^{r_{n1}\cdot t_t} + c_{n2}\cdot r_{n2}\cdot e^{r_{n2}\cdot t_t} + A_n\cdot \Omega_F\cdot \cos(\Omega_F\cdot t_t) - B_n\cdot \Omega_F\cdot \sin(\Omega_F\cdot t_t)\right) & \text{if } 0s \le t_t \le t_I \\ \left(c_{n3}\cdot r_{n1}\cdot e^{r_{n1}\cdot t_t} + c_{n2}\cdot r_{n2}\cdot e^{r_{n2}\cdot t_t} + A_{n2}\cdot \Omega_{Fa}\cdot \cos(\Omega_{Fa}\cdot t_t) - B_{n2}\cdot \Omega_{Fa}\cdot \sin(\Omega_{Fa}\cdot t_t)\right) & \text{if } t_I < t_t \le .5s \end{cases}$$

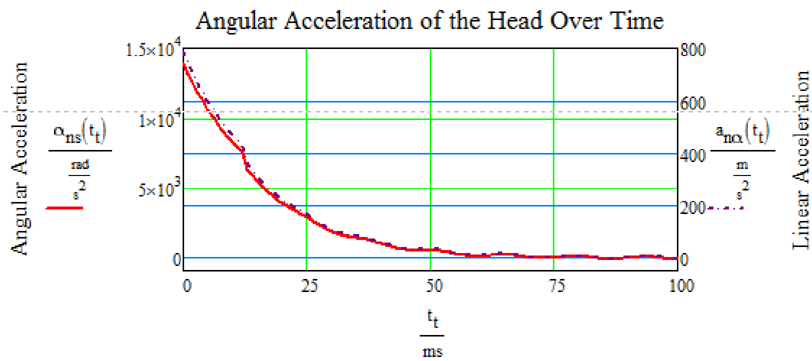
Angular Velocity of Head Over Time

$\omega(t_I) = -377.89\,\frac{1}{s}$

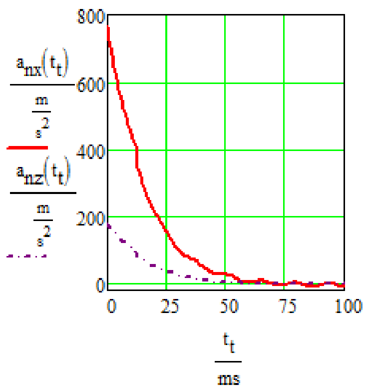


$$\alpha_{ns}(t_t) := \begin{cases} \left(c_{n1} \cdot r_{n1}^{2} \cdot e^{r_{n1} \cdot t_t} + c_{n2} \cdot r_{n2}^{2} \cdot e^{r_{n2} \cdot t_t} - A_n \cdot \Omega_F^{2} \cdot \sin(\Omega_F \cdot t_t) - B_n \cdot \Omega_F^{2} \cdot \cos(\Omega_F \cdot t_t)\right) & \text{if } 0ms \le t_t \le t_I \\ \left[c_{n3} \cdot r_{n1}^{2} \cdot e^{r_{n1} \cdot (t_t)} + c_{n4} \cdot r_{n2}^{2} \cdot e^{r_{n2} \cdot (t_t)} - A_{n2} \cdot \Omega_{Fa}^{2} \cdot \sin\left[\Omega_{Fa} \cdot (t_t)\right] - B_{n2} \cdot \Omega_{Fa}^{2} \cdot \cos\left[\Omega_{Fa} \cdot (t_t)\right]\right] & \text{if } t_I < t_t \le .5s \end{cases}$$

$a_{n\alpha}(t_t) := \alpha_{ns}(t_t) \cdot d_{CG}$ $\qquad \alpha_{ns}(0ms) = 1.395 \times 10^{4} \, \frac{1}{s^2} \cdot rad$ $\qquad a_{n\alpha}(0ms) = 788.597 \, \frac{m}{s^2}$



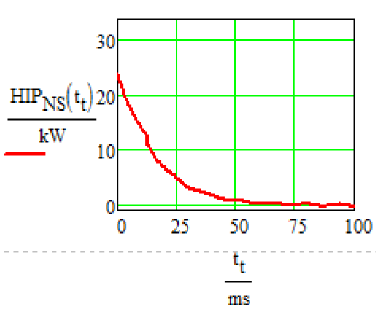
$a_{nx}(t_t) := \alpha_{ns}(t_t) \cdot d_{CGz}$ $\qquad a_{nz}(t_t) := \alpha_{ns}(t_t) \cdot d_{CGx}$



$a_{nxmax} := a_{nx}(0ms) = 767.45 \, \frac{m}{s^2}$

$a_{nzmax} := a_{nz}(0ms) = 181.397 \, \frac{m}{s^2}$

$$HIP_{NS}(t_t) := m_{head} \cdot a_{nx}(t_t) \cdot \int_{0ms}^{60ms} a_{nx}(t_t) \, dt_t + m_{head} \cdot a_{nz}(t_t) \cdot \int_{0ms}^{60ms} a_{nz}(t_t) \, dt_t + I_y \cdot \alpha_{ns}(t_t) \cdot \int_{0ms}^{60ms} \alpha_{ns}(t_t) \, dt_t$$



$HIP_{NS}(0ms) = 23.999 \cdot kW$

$k_{oobleck} = 0.151 \, \frac{m^2 \cdot kg}{s}$

$k_{damp} = 0.215 \, \frac{m^2 \cdot kg}{s}$

C-7: Calculation for Determining Necessary Thickness of Plywood for the Test Rig Base

How thick of plywood to get?

young's modulus of plywood $E_w := 12.4GPa$ }from http://www.tecotested.com/techtips/pdf/tt_plywooddesigncapacities

Length of wood board $L_b := 24in$ width $wd_p := 12in$ Thickness $Th_p := .25in$

$x_b := 0 \cdot in, 1in .. L_b$

Volume of wood board $A_{cw} := wd_p \cdot Th_p = 1.935 \times 10^{-3} m^2$

Assume weight of air cylinder support structure (& air cylinder) are distributed uniformly

Length from end to start of air cylinder structure $L_a := 13in$

Density of plywood $\delta_p := 3.47 \frac{kg}{m^3}$ }from http://www.tecotested.com/techtips/pdf/tt_plywooddesigncapacities

		Amount
mass per length of wood base	$m_{wb} := A_{cw} \cdot \delta_p = 6.716 \times 10^{-3} \frac{kg}{m}$	
Weight per length of wood base	$W_w := m_{wb} \cdot g = 0.066 \cdot \frac{N}{m}$	
Weight of wood base	$W_W := W_w \cdot L_b$	
Perforated framing 304 steel		
Cross section aread	$A_{cF} := [.074in \cdot (1.5in + 1.426in)] = 0.217 \cdot in^2$	
Length of frame base	$L_F := L_b - L_a = 11 \cdot in$	
total mass 1ft frame	$m_{1F} := A_{cF} \cdot \delta_{ss} \cdot 12in = 0.342 kg$	4
total mass of 2ft column frame	$m_{2F} := A_{cF} \cdot \delta_{ss} \cdot 24in = 0.684 kg$	4
total mass 1ft frame	$m_{halfF} := A_{cF} \cdot \delta_{ss} \cdot .6in = 0.017 kg$	6
total mass of support structure (approx.)	$m_F := 4m_{1F} + 4 \cdot m_{2F} + 6 \cdot m_{halfF} = 4.205 kg$	
total mass of support structure with air cylinder	$m_L := m_F + m_{ac} = 6.763 kg$	
weight of support structure with air cylinder	$W_L := m_L \cdot g = 14.911 \cdot lbf$	

weight of load per inch $W_1 := \frac{W_L}{L_F} = 1.356 \cdot \frac{lbf}{in}$

Moment of Inertia of wood board $I_w := \frac{1}{12} \cdot L_b \cdot Th_p^{\,3} = 0.031 \cdot in^4$

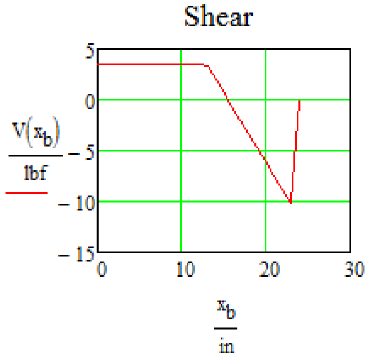
Forces from hands while carrying set-up at ends

$$N_2 := \frac{W_w \cdot \frac{L_b^{\,2}}{2} + W_L \cdot \left(L_a + \frac{L_F}{2}\right)}{L_b} = 11.498 \cdot lbf$$

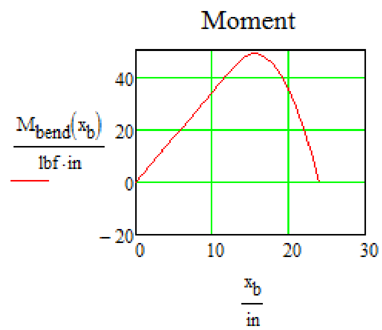
$N_1 := W_w \cdot L_b + W_L - N_2 = 3.422 \cdot lbf$

$S(x, a) := \text{if}(x \geq a, 1, 0)$

Shear $V(x_b) := N_1 \cdot S(x_b, 0in) - W_w \cdot S(x_b, 0in) \cdot (x_b) - W_1 \cdot S(x_b, L_a) \cdot (x_b - L_a) + N_2 \cdot S(x_b, L_b)$



$$M_{bend}(x_b) := N_1 \cdot S(x_b, 0in) \cdot x_b - \frac{W_w}{2} \cdot S(x_b, 0in) \cdot x_b^2 - \frac{W_1}{2} \cdot S(x_b, L_a) \cdot (x_b - L_a)^2 + N_2 \cdot S(x_b, L_b)(x_b - L_b)$$



$$\theta_{slope}(x_b) = \frac{1}{E_w \cdot I_w} \cdot \left[\frac{N_1}{2} \cdot S(x_b, 0in) \cdot x_b^2 - \frac{W_w}{6} \cdot S(x_b, 0in) \cdot x_b^3 - \frac{W_1}{6} \cdot S(x_b, L_a) \cdot (x_b - L_a)^3 + \frac{N_2}{2} \cdot S(x_b, L_b)(x_b - L_b)^2 + C_1 \right]$$

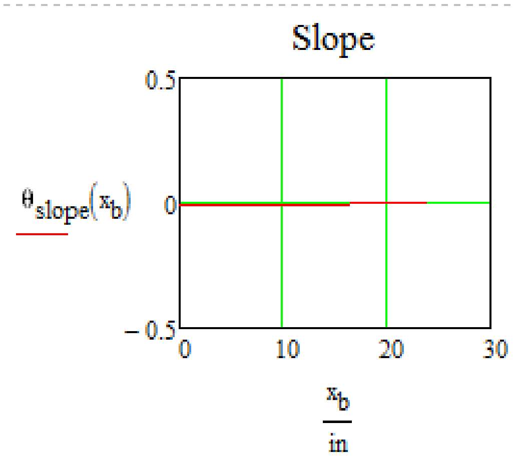
$$D(x_b) = \frac{1}{E_w \cdot I_w} \cdot \left[\frac{N_1}{6} \cdot S(x_b, 0in) \cdot x_b^3 - \frac{W_w}{24} \cdot S(x_b, 0in) \cdot x_b^4 - \frac{W_1}{24} \cdot S(x_b, L_a) \cdot (x_b - L_a)^4 + \frac{N_2}{6} \cdot S(x_b, L_b)(x_b - L_b)^3 + C_1 \cdot x_b + C_2 \right]$$

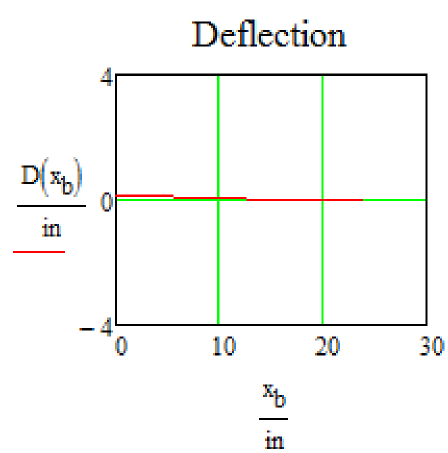
$$C_{w1} := \frac{-N_1}{2} \cdot L_b^2 + \frac{W_w}{6} \cdot L_b^3 + \frac{W_1}{6} \cdot (L_b - L_a)^3 = -683.833 \cdot lbf \cdot in^2$$

$$C_{w2} := \frac{-N_1}{6} \cdot L_b^3 + \frac{W_w}{24} \cdot L_b^4 + \frac{W_1}{24} \cdot (L_b - L_a)^4 - C_{w1} \cdot L_b = 9.361 \times 10^3 \cdot lbf \cdot in^3$$

$$\theta_{slope}(x_b) := \frac{1}{E_w \cdot I_w} \cdot \left[\frac{N_1}{2} \cdot S(x_b, 0in) \cdot x_b^2 - \frac{W_w}{6} \cdot S(x_b, 0in) \cdot x_b^3 - \frac{W_1}{6} \cdot S(x_b, L_a) \cdot (x_b - L_a)^3 + \frac{N_2}{2} \cdot S(x_b, L_b)(x_b - L_b)^2 + C_{w1} \right]$$

$$D(x_b) := \frac{1}{E_w \cdot I_w} \cdot \left[\frac{N_1}{6} \cdot S(x_b, 0in) \cdot x_b^3 - \frac{W_w}{24} \cdot S(x_b, 0in) \cdot x_b^4 - \frac{W_1}{24} \cdot S(x_b, L_a) \cdot (x_b - L_a)^4 + \frac{N_2}{6} \cdot S(x_b, L_b)(x_b - L_b)^3 + C_{w1} \cdot x_b + C_{w2} \right]$$





Appendix D: **Procedures**

D-1: Procedure for Determining Oobleck's Viscosity to Force Relationship

Experimental procedure to find equation relation of viscosity to force for Oobleck:

Materials:

- *2000mL graduated cylinder*
- *Digital scale*
- *Micrometer*
- *Stop watch*
- *Ruler*
- *5 spheres of different masses*
- *Oobleck sample*

Procedure:

1) *Measure the diameter of the graduated cylinder using the ruler and record the result.*
2) *Find the mass of the empty graduated cylinder using the scale and record the result.*
3) *Measure the diameter of one of the spheres using the micrometer and record the result in a table.*
4) *Find the mass of the same sphere using the scale and record the result in a table.*
5) *Find the density of the sphere using the formula:* $\rho = \frac{m}{\frac{3}{16}\pi d^2}$ *where m is the mass found in step 4 and d is the diameter found in step 3. Record the result in a table.*
6) *Repeat steps 3-5 for the remaining spheres.*
7) *Take an Oobleck sample and pour it into the graduated cylinder until the 1600mL mark.*
8) *Measure the height the Oobleck fills the graduated cylinder to and record the result. (This only needs to be done once)*
9) *Calculate the volume of the Oobleck using the formula:* $V = \frac{1}{4}\pi d^2 h$ *where d is the diameter found in step 1 and h is the height found in step 8 and record the result.*
10) *Find the mass of the graduated cylinder with the Oobleck in it using the scale.*
11) *Find the mass of the Oobleck by subtracting the mass of the empty graduated cylinder found in step 2 from the mass found in step 10 and record the result.*
12) *Find the density of the Oobleck by using the following formula:* $\rho = \frac{m}{\frac{3}{16}\pi d^2}$ *where m is the mass found in step 11 and d is the diameter found in step 1. Record your results.*
13) *Use the stopwatch to time how long it takes for the sphere to reach the bottom of the graduated cylinder and record the results in a table.*
14) *Find the viscosity of the Oobleck for each sphere using the following formulas:* $F_d = 6\pi\mu V d$, $F_d = mg - F_b$, *and* $F_b = \frac{4}{3}\pi r^3 \rho_{fluid} g$, *where* F_d *is the drag force,* μ *is the viscosity of the Oobleck, V is the velocity of the sphere, d is the diameter of the sphere, m is the mass of the sphere, g is the acceleration of gravity,* F_b *is the buoyancy force, r is the radius of the sphere, and* ρ_{fluid} *is the density of the Oobleck found in step 12. Record your result in a table.*
15) *Repeat steps 13-15 for each sphere in the same sample of Oobleck.*
16) *Repeat steps 7-15 for each sample of Oobleck.*
17) *Once all the viscosities have been found for each sphere in each sample of Oobleck, plot the viscosities vs. weight of the sphere (mg).*
18) *Fit an equation to the curve to find the shear thickening relationship the Oobleck has with force.*

D-2 : Hot Water Oobleck Cooking Procedure

Oobleck cooking procedure using hot water

1. *Mix one batch 2:1 cornstarch to water in order to have the same concentration*
2. *Divide mixture into zip lock bags*
3. *Fill a large pot with enough water (at the same temperature of that used to make the suspension) to cover the suspension. Use a metal steamer to set the bags on in the pot to make sure none of the plastic touches the side of the pot*
4. *Fill a bowl with cold water and ice*
5. *Turn gas stove on medium (5) and heat water until bubbles are seen sticking to the sides then "stir" the bags*
6. *Reduce heat to low (3) and leave the bags in for 3 minutes*
7. *"stir" and remove 1 bag and place it in the ice water for 2 minutes*
8. *After two minutes "stir" the bags and remove one and place it in the ice water for two minutes*
9. *Repeat steps 7 and 8 for the remaining bags*
10. *Dry off and label bags accordingly after the ice water bath*

**if cooking for longer, "stir" every two minutes*

**The last bag was cooked for 15 minutes to get a more striking difference between the cooked and uncooked oobleck.*

D-3: ANSYS Static Analysis Procedure

1. *Open Ansys*
2. *In the analysis systems drop down on the left hand side of the screen, double click static structural*
3. *Right click the Geometry cell, select import file, then browse. Import the parasolid file of the model.*
4. *Change the materials by selecting Engineering Data Sources, and adding polyethylene and polypropylene.*
5. *Under View, select reset workplace layout.*
6. *Under the Geometry tab in the model window, select each part of the model and in the details window, change each part to the desired material (All to polyethylene).*
7. *Right click on the Model cell, select mesh in the menu, then update. A mesh is created on the part.*
8. *In the sizing section of mesh, change the sizing relevance to fine. The update the mesh.*
9. *Right click static structural, insert, fixed, select the bottom cylinder of the neck, select apply.*
10. *Under static structural, right click, insert, acceleration. Select the impact point on the model and enter an acceleration of 1.938*10^3 m/s^2 as used in prior MathCad calculations.*
11. *Right click solution, insert total deformation, repeat and select equivalent stress (von mises).*
12. *Right click static structural and select solve.*

13. *Repeat steps 6-12 with a force of 416.738 N.*

D-4: ANSYS Dynamic Analysis Procedure

1. *Open Ansys*
2. *In the analysis systems drop down on the left hand side of the screen, double click explicit dynamics.*
3. *Right click the Geometry cell, select import file, then browse. Import the parasolid file of the model.*
4. *Change the materials by selecting Engineering Data Sources, and adding polyethylene.*
5. *Under View, select reset workplace layout.*
6. *Under the Geometry tab in the model window, select each part of the model and in the details window, change each part to the desired material (All to polyethylene).*
7. *Right click on the Model cell, select mesh in the menu, then update. A mesh is created on the part.*
8. *In the sizing section of mesh, change the sizing relevance to fine. The update the mesh.*
9. *Right click explicit dynamics, insert, fixed, select the bottom cylinder of the neck, and select apply.*
10. *Under explicit dynamics, right click, insert, acceleration. Select the impact point on the model and enter a force of 416.738 N as used in prior MathCad calculations.*
11. *Under explicit dynamics and analysis settings, add a time of 0.012 seconds (12 ms as used in previous calculations).*
12. *Right click solution, insert total deformation, repeat and select equivalent stress (von mises) and total acceleration.*
13. *Right click explicit dynamics and select solve.*

Appendix E: **Completed Assessment Forms**

Appendix F: **Miscellaneous**

Standardized Assessment of Concussion (SAC)

ORIENTATION — Score: ____ / 5

What month is it?	0	☐	1	☐
What is the date?	0	☐	1	☐
What day of the week is it?	0	☐	1	☐
What year is it?	0	☐	1	☐
What time of day is it? (*w/in 1 hour*)	0	☐	1	☐

IMMEDIATE MEMORY — Score: ____ / 15

Form A	Form B	Form C	Form D
Elbow	Candle	Baby	Monkey
Apple	Paper	Monkey	Penny
Carpet	Sugar	Perfume	Blanket
Saddle	Sandwich	Sunset	Lemon
Bubble	Wagon	Iron	Insect

	Trial 1	Trail 2	Trail 3
Word 1	0 ☐ 1 ☐	0 ☐ 1 ☐	0 ☐ 1 ☐
Word 2	0 ☐ 1 ☐	0 ☐ 1 ☐	0 ☐ 1 ☐
Word 3	0 ☐ 1 ☐	0 ☐ 1 ☐	0 ☐ 1 ☐
Word 4	0 ☐ 1 ☐	0 ☐ 1 ☐	0 ☐ 1 ☐
Word 5	0 ☐ 1 ☐	0 ☐ 1 ☐	0 ☐ 1 ☐

NEUROLOGIC SCREENING

Loss of Consciousness: *(occurrance, duration)*

Retrograde Amnesia

Antegrade Amnesia

Strength

Sensation

Coordination

CONCENTRATION: *Digits Backwards* — Score: ____ / 5

Form A					
4-9-3	6-2-9	0	☐	1	☐
3-8-1-4	3-2-7-9	0	☐	1	☐
6-2-9-7-1	1-5-2-8-5	0	☐	1	☐
7-1-8-4-6-2	5-3-9-1-4-8	0	☐	1	☐
Form B					
5-2-6	4-1-5	0	☐	1	☐
1-7-9-5	4-9-6-8	0	☐	1	☐
4-8-5-2-7	6-1-8-4-3	0	☐	1	☐
8-3-1-9-6-4	7-2-4-8-6-5	0	☐	1	☐
Form C					
1-4-2	6-5-8	0	☐	1	☐
1-8-3-1	3-4-8-1	0	☐	1	☐
4-9-1-5-3	6-8-2-5-1	0	☐	1	☐
3-7-6-5-1-9	9-2-6-5-1-4	0	☐	1	☐

Months in Reverse Order

Dec_Nov_Oct_Sept_Aug_ Jul_Jun_May_Apr_Mar_Feb_Jan

0 ☐ 1 ☐

DELAYED RECALL — Score: ____ / 5

Word 1	0	☐	1	☐
Word 2	0	☐	1	☐
Word 3	0	☐	1	☐
Word 4	0	☐	1	☐
Word 5	0	☐	1	☐

SCORE TOTALS

Orientation	=	___ / 5
Immediate Memory	=	___ / 15
Concentration	=	___ / 5
Delayed Recall	=	___ / 5

Overall Score

/ 30

Standard Assessment of Concussion