Laboratory: Vibration Measurements

1. OBJECTIVES

This laboratory uses strain gauge to measure the dynamic characteristic and the elastic material properties of a cantilever.

Vibration data will be analyzed to:

- Determine the vibration amplitude, velocity, and acceleration in various units of measure;
- Determine natural frequencies;
- Measure and express damping characteristics as logarithmic decrement and percentage of critical damping;
- Compare measurements with analytical and/or computational models of a cantilever; and
- Determine elastic modulus of a cantilever.

Uncertainty analysis of the results will be performed.

2. BACKGROUND

Health monitoring is the process of studying and assessing the integrity of structures, which is crucial for preventing failure and for achieving reliable designs. Health monitoring can be done by dynamic or static analysis, or a combination of both. In static analysis, deformations or changes in the orientation of structures, due to application of loads, or unexpected damages, are determined via comparisons with reference models. For dynamic analysis, dynamic characteristics of the structures, including natural frequencies, modal shapes, and damping factors, are determined via modal analysis.

In either static or dynamic health monitoring, the utilization of appropriate transducers is required to provide accurate measurement of structural responses in both frequency and time domains. Conventional devices utilized for health monitoring are based on piezoelectric transducers. These transducers are usually large in size, require high actuation power, and have narrow frequency bandwidths, which reduce their accuracy, versatility, and applicability to study smaller structures. The advanced developments of IC microfabrication and microelectromechanical systems (MEMS) have led to the progressive designs of small footprint, low dynamic mass and actuation power MEMS inertial sensors. Due to their high natural frequencies, these MEMS inertial sensors provide wide frequency bandwidths and high measuring accuracies.

2.1 Static Analysis of a Simple Cantilever Beam

2.1.1 Stress, Strain, and Deflection Associated with Bending

A bending moment exists in a structural element when a moment is applied so that the element bends. The bending moment at a section of a structural element is defined as the sum of the moments about that the section of external forces acting to one side of the section. Moments are calculated by multiplying the external vector forces by the vector distance at which they are applied.

Bending occurs locally when a slender object is subjected to an external load applied perpendicular to a longitudinal axis of the object. On a bending beam, compressive and tensile forces develop in the direction of the beam axis under bending loads. The forces induce stresses on the beam. The maximum compressive force occurs on at the lower edge of the beam, and the maximum tensile force occurs at the upper most edge. The equation for determining the bending stress is

$$\sigma = \frac{Mc}{I} \qquad \text{Eq.1}$$

where M is the applied moment, c is the distance from the neutral axis to the outer fiber of the beam, and I is the moment of inertia. The derivation of Eq.1 is shown in Appendix A.

The maximum bending stress in a beam is

$$\sigma_{max} = \frac{Mt}{2I} \qquad \text{Eq.2}$$

where t is the thickness of the beam.

Hooke's law describes the relationship between stress and induced strains for linear elastic materials.

$$\varepsilon = \frac{\sigma}{E} \qquad \text{Eq.3}$$

where E is the elastic modulus of the beam's material.

Deflection is the degree to which a structural element is displaced under a load; it may refer to an angle or a distance. As shown in Figure 1, on the neutral axis of a beam subjected to bending, for a very small angle, slope of the beam $\frac{dw}{dx} = \tan\theta = \theta$. The curvature of a beam is defined as $\kappa = \frac{1}{\rho} = \frac{d\theta}{ds}$; ρ is the radius of the curve. Since $d\theta$ is small, $dx = ds$. Therefore we have

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds} = \frac{d\theta}{dx} = \frac{d}{dx}\left(\frac{dw}{dx}\right) = \frac{d^2w}{dx^2} \qquad \text{Eq.4}$$

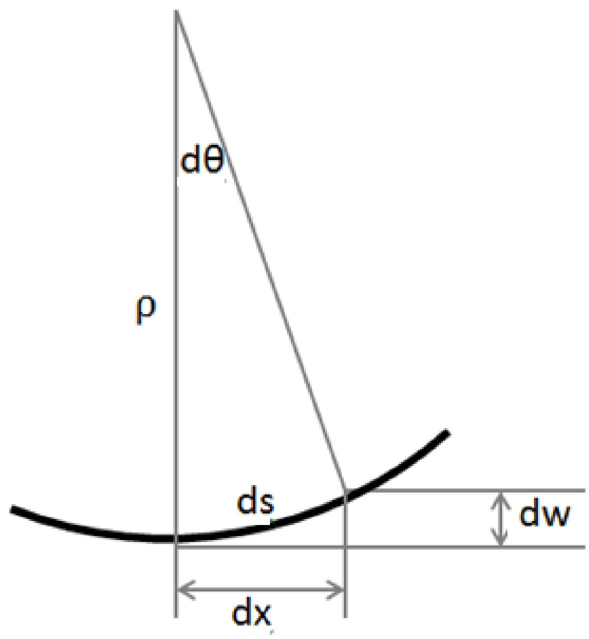


Figure 1 Neutral axis of a beam subjected to bending

Euler—Bernoulli beam theory relates curvature of a bending beam to bending moment and rigidity of the material

$$\mathrm{EI}\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} = \mathrm{M} \qquad \textbf{Eq.4}$$

where E is the elastic modulus of the material, I is the second moment of area. I must be calculated with respect to the centroidal axis perpendicular to the applied loading. w is the deflection in distance, $\frac{\mathrm{dw}}{\mathrm{dx}}$ is the slope of the beam, and $\frac{\mathrm{d}^2\mathrm{w}}{\mathrm{dx}^2}$ equals to the beam curvature κ , or $\frac{1}{\rho}$. The second moment of inertia of rectangle about the centroidal axis perpendicular to the applied loading is expressed as

$$I = \frac{bT^3}{12} \qquad \textbf{Eq.5}$$

where b is the width and T is the height or thickness.

2.1.2 Calculation of Static Characteristics

Macaulay's method (the double integration method) is a technique used in structural analysis to determine the deflection of Euler-Bernoulli beams. Use of Macaulay's technique is very convenient for cases of discontinuous and/or discrete loading. Typically partial uniformly distributed loads and uniformly varying loads over the span and a number of concentrated loads are conveniently handled using this technique.

For general loadings, the bending moment M can be expressed in the form

$$M = M(x) + P_1\langle x - a_1\rangle + P_2\langle x - a_2\rangle + \cdots + P_n\langle x - a_n\rangle \qquad \text{Eq.6}$$

The quantity $\langle x - a_i\rangle$ is a Macaulay bracket, it is defined as

$$\langle x - a_i\rangle = \begin{cases} 0 & \text{if } x < a_i \\ x - a_i & \text{if } x > a_i \end{cases} \qquad \text{Eq.7}$$

When integrating expressions containing Macaulay brackets, we have

$$\int P\langle x - a\rangle \mathrm{dx} = \frac{P\langle x-a\rangle^2}{2} + C_m \qquad \text{Eq.8}$$

Consider a simple cantilever beam fixed at one end and loaded with a force on the free end. The dimensions of the cantilever beam are defined in the figure below.

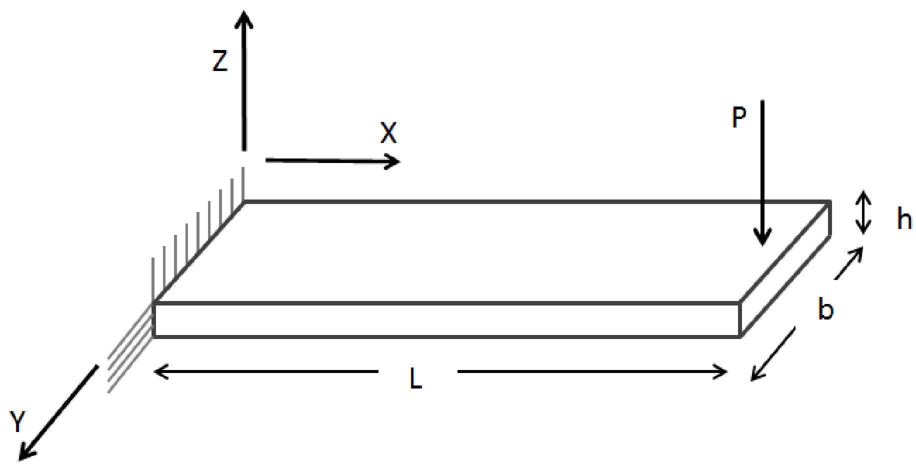


Figure 2 Dimensions of a Simple Cantilever Beam

Figure 3 equations and plots for deflection in terms of distance, deflection in terms of slope, bending moment and shear stress at arbitrary location in the beam on the neutral axis. In the equations, x is the distance from the fixed end of the beam to the point of interest, P is the applied load, L is the length of the beam, E is the elastic modulus, and I is the second moment of inertia.

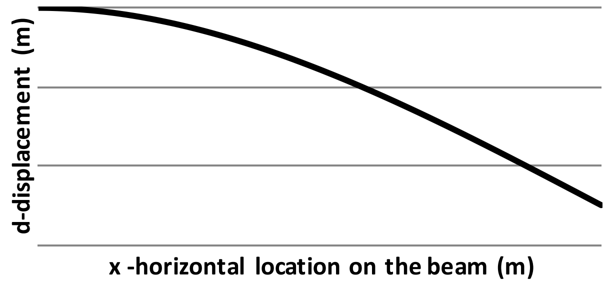
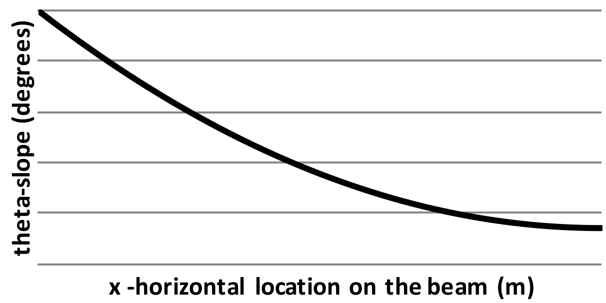
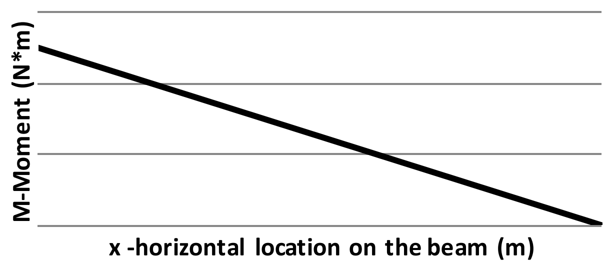
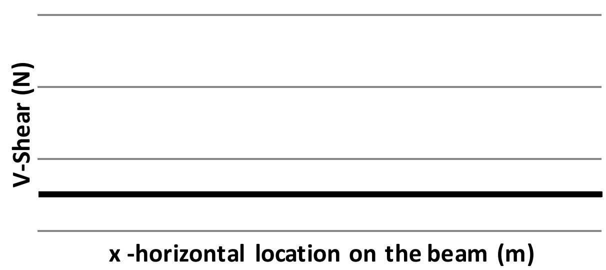
$$w = \frac{Px^2(3L-x)}{6EI}$$ **Eq.9**	
$$\theta = \frac{Px(2L-x)}{2EI}$$ **Eq.10**	
$$M = P(L-x)$$ **Eq.11**	
$$V = P$$ **Eq.12**	

Figure 3 Deflection, Bending Moment and Shear Stress

Recall Eq.2 and Eq.3, and substitute with $M = P(L-x)$, we have the expressions for maximum bending stress and corresponding strain at arbitrary location on beam.

$$\sigma_{max} = \frac{P(L-x)t}{2I} = \frac{6P(L-x)}{bT^2} \quad \textbf{Eq.13}$$

$$\varepsilon = \frac{P(L-x)t}{2EI} = \frac{6P(L-x)}{EbT^2} \quad \textbf{Eq.14}$$

2.2 Dynamic Characteristics of a Cantilever Beam under Free Vibration

Vibration is a mechanical phenomenon whereby oscillations occur about an equilibrium point. Free vibration occurs when a mechanical system is set off with an initial input and then allowed to vibrate freely. The mechanical system will then vibrate at one or more of its "natural frequency" and damp down to zero. Forced vibration is when an alternating force or motion is applied to a mechanical system.

A normal mode of an oscillating system is a pattern of motion in which all parts of the system move sinusoidally with the same frequency and with a fixed phase relation. The motion described by the normal modes is called resonance. The frequencies of the normal modes of a system are known as its natural frequencies or resonant frequencies. Each physical object has a set of normal modes that depend on its structure, materials and boundary conditions.

A mode of vibration is characterized by a modal frequency and a mode shape, and is numbered according to the number of half waves in the vibration. In a system with two or more dimensions, such as the pictured disk, each dimension is given a mode number. Each mode is entirely independent of all other modes. Thus all modes have different frequencies (with lower modes having lower frequencies) and different mode shapes.

2.2.1 Natural Frequencies of a Cantilever Beam under Free Vibration [1]

For an Euler-Bernoulli beam under free vibration, the Euler-Lagrange equation is

$$EI\,\frac{d^4w}{dx^4} + \rho A\frac{d^2w}{dt^2} = 0 \quad \textbf{Eq.15}$$

Since deflection is a function of time and distance, we have

$$w(x,t) = W(x)\cos(\omega t - \alpha) \quad \textbf{Eq.16}$$

This makes eq.15:

[1] Volterra, E. (01/01/1966). "Dynamics of Vibrations". *Journal of applied mechanics (0021-8936)*, 33(4), p.956.

$$\frac{EI}{\rho A}\frac{d^4w}{dx^4} - \omega^2 w = 0 \qquad \text{Eq.17}$$

Solution for displacement is:

$$W(x) = C_1\cos(x\lambda) + C_2\sin(x\lambda) + C_3\cosh(x\lambda) + C_4\sinh(x\lambda) \qquad \text{Eq.18}$$

Where: $\lambda = \left(\frac{\rho A}{EI}\omega^2\right)^{\frac{1}{4}}$

For a cantilever beam, the displacement and slope are zero at the fixed end, and the moment and shear are zero at the free end. Thus the boundary conditions are:

when $x = 0$, $y = 0$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.

when x=L, $\frac{\mathrm{d}^2y}{\mathrm{d}^2x} = 0$, $\frac{\mathrm{d}^3y}{\mathrm{d}^3x} = 0$.

Applying the boundary conditions yields

$$\cos(\lambda L)\cosh(\lambda L) = -1 \qquad \text{Eq.19}$$

The equation for time is

$$T(t) = b_1\sin[(\beta_n^2\sqrt{\frac{EI}{\rho A}})\,t] + b_2\cos[(\beta_n^2\sqrt{\frac{EI}{\rho A}})\,t] \qquad \text{Eq.20}$$

So the exact expression of n^{th}natural frequency in rad/sec is

$$\omega_n = \frac{(\lambda L)^2}{L^2}\sqrt{\frac{EI}{\rho A}} = \frac{\alpha_n^2}{L^2}\sqrt{\frac{EI}{\rho A}} \qquad \text{Eq.21}$$

where E is Young's modulus of elasticity, I is moment of inertia of cross section, L is effective length of beam, and ρ is the density, A is the area of cross section. The dimensionless wave number β = 2π/wavelength. β_n values for cantilever beams are: $\beta_1 L$ = 1.8751=α_1, $\beta_2 L$ = 4.6941= α_2, $\beta_3 L$ = 7.8548=α_3, $\beta_4 L$ = 10.99557=α_4, $\beta_5 L$ = 14.1372=α_5, $\beta_6 L$ = 17.279=α_6.

Therefore, the natural frequency of cantilever beam with a rectangular cross section is

$$\omega_n = \alpha_n^2\sqrt{\frac{EI}{mL^4}} = \alpha_n^2\sqrt{\frac{EI}{\rho AL^4}} = \alpha_n^2\sqrt{\frac{E(\frac{bT^3}{12})}{\rho(bT)L^4}} = \frac{\alpha_n^2}{2\sqrt{3}}\sqrt{\frac{ET^2}{\rho L^4}} \qquad \text{Eq.22}$$

A simple method of approximating the natural frequency of cantilever beams is shown below. The method also estimates equivalent stiffness and equivalent mass of the beam.

Recall the generic expression of natural frequency in rad/sec is $\omega_n = \sqrt{\frac{k}{m}}$. To find the natural frequency of a cantilever beam, the equivalent stiffness and equivalent mass are needed.

As given in section 2.1.2, the deflection w at the tip of a cantilever beam (x=L) is

$$w = \frac{Px^2(3L-x)}{6EI} = \frac{PL^3}{3EI} \qquad \textbf{Eq.23}$$

Using Hook's law, the deflection at the end of the cantilever can be expressed as

$$P = kw \qquad \textbf{Eq.24}$$

where k is the stiffness of the cantilever beam. Combining eq. 17 and eq. 18, k can be given as

$$k = \frac{3EI}{L^3} \qquad \textbf{Eq.25}$$

Therefore, the frequency of a cantilever with a point load m at length x can be given as

$$\omega_n = \sqrt{\frac{3EI}{mx^3}} \qquad \textbf{Eq.26}$$

The same frequency can be provided by a load m_{eq} at the end of beam

$$\omega_n = \sqrt{\frac{3EI}{m_{eq}L^3}} \qquad \textbf{Eq.27}$$

Consider a cantilever beam with constant cross section and uniformly distributed mass of value m per meter along the length. At any time t during vibration, the relationship between generic deflection (measured at an abscissa y from free end), denoted by $w_y(t)$ and the deflection at the free end, denoted by $w_{max}(t)$ can be expressed as:

$$w_y(t) = \left[1 - \frac{3}{2}\left(\frac{y}{L}\right) + \frac{1}{2}\left(\frac{y}{L}\right)^3\right] w_{max}(t) \qquad \textbf{Eq.28}$$

The kinetic energy of the distributed parameter cantilever is expressed as:

$$T = \frac{1}{2}\int_0^l \rho A \left[\frac{\partial w_y(t)}{\partial t}\right]^2 dy = \frac{1}{2}\left[\frac{\partial w_{max}(t)}{\partial t}\right]^2 \int_0^l \rho A \left[1 - \frac{3}{2}\left(\frac{y}{L}\right) + \frac{1}{2}\left(\frac{y}{L}\right)^3\right]^2 dy \qquad \textbf{Eq.29}$$

The lumped load m_{eq} at the end of beam has the kinetic energy:

$$T_c = \frac{1}{2} m_{eq} \left[\frac{\partial w_{max}(t)}{\partial t}\right]^2 \qquad \text{Eq.30}$$

The two kinetic energies of Eq. 29 and Eq.30 need to be equal. The equivalent mass is:

$$m_{eq} = \int_0^L \rho A \left[1 - \frac{3}{2}\left(\frac{y}{L}\right) + \frac{1}{2}\left(\frac{y}{L}\right)^3\right]^2 dy = \frac{33}{140} m \qquad \text{Eq.31}$$

Therefore, the natural frequency in rad/sec is expressed as:

$$\omega_n = 3.5675\sqrt{\frac{EI}{mL^3}} \qquad \text{Eq.32}$$

The error of the estimation is within 2%.

2.2.2 Mode Shapes of a Cantilever Beam under Free Vibration

The mode shapes of a vibrating beam can be determined through solving the relevant equations. The video below shows the vibration mode shapes of a simply supported beam and a cantilever beam.

http://www.youtube.com/watch?v=kun62B7VUg8

2.2.3 Damping Factor of a Cantilever Beam under Free Vibration

The vibrating object dissipates energy through damping, and the oscillation amplitude decays with time as a result. The damping ratio is a dimensionless measure describing how rapidly the oscillations decay during each cycle.

Where the system is completely lossless, the mass would oscillate indefinitely, with constant amplitude. This hypothetical case is called undamped.

If the system contained high losses, for example if the system vibrates in a viscous fluid, the mass could slowly return to its rest position without ever overshooting. This case is called overdamped. Commonly, the mass tends to overshoot its starting position, and then return, overshooting again. With each overshoot, some energy in the system is dissipated, and the oscillations die towards zero. This case is called underdamped. Between the overdamped and underdamped cases, there exists a certain level of damping at which the system will just fail to overshoot and will not make a single oscillation. This case is called critical damping. The key difference between critical damping and overdamping is that, in critical damping, the system returns to equilibrium in the minimum amount of time.

The damping ratio expresses the level of damping in a system relative to critical damping. For a damped harmonic oscillator with mass m, damping coefficient c, and spring constant k, it can be defined as the ratio of the damping coefficient in the system's differential equation to the critical damping coefficient:

$$\zeta = \frac{c}{c_c} \qquad \textbf{Eq.33}$$

where the system's equation of motion is

$$m\frac{d^2 w}{dt^2} + c\frac{dw}{dt} + kw = 0 \qquad \textbf{Eq.34}$$

and the corresponding critical damping coefficient is

$$c_c = 2\sqrt{km} \qquad \textbf{Eq.35}$$

A common method for analyzing the damping of an underdamped oscillation is the logarithmic decrement method, for which the following relationships apply.

$$\ln\left(\frac{y_i}{y_{i+n}}\right) = n\delta \qquad \textbf{Eq.36}$$

$$\varsigma = \frac{\delta}{\sqrt{(2\pi)^2+\delta^2}} \qquad \textbf{Eq.37}$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\varsigma^2}} \qquad \textbf{Eq.38}$$

where y_i is the amplitude of peak i (i is an integer counting each peak), n is the number of cycles being considered, δ is the log decrement, ω_n is the undamped natural frequency, and ω_d is the damped natural frequency. Both frequencies are in radiance per second. Note, it is assumed that object oscillates about zero. If there is an offset in y, the y_i amplitude must be defined relative to that offset.

According to Eq.25 and Eq.33, the equivalent stiffness and equivalent mass are expressed as:

$$k = \frac{3EI}{L^3}$$

$$m_{eq} = \frac{33}{140}\, m$$

The critical damping factor of a cantilever beam is

$$c_c = 1.6818\sqrt{\frac{EI}{L^3}\, m} = 0.2428\sqrt{\frac{Eb^3 t}{L^3}\, m} \qquad \text{Eq.39}$$

2.3 Measurement Methods of Dynamic Characteristics

The dynamic characteristics of a vibrating object, including vibrating frequency and damping factor extracted from strain and acceleration data acquired during the vibration .

2.3.1 Measurement of Vibration Frequency: Fourier Transformation

Fourier series decomposes periodic signals into the sum of an infinite series of simple oscillating functions, namely sines and cosines, or complex exponentials. The technique can be applied to mathematical and physical problems, especially electrical engineering, vibration analysis, acoustics, optics, signal processing, image processing, quantum mechanics, econometrics, etc.

2.3.1.1 Fourier Series

The Fourier series of a periodic function consists of an offset value, an even (cosine) component, and an odd (sine) component. The offset value a_0 of a periodic function f(x) with period T is defined as the average value of the periodic function over a period. The even component relates to the portion of the periodic function behaving as $f(x) = f(-x)$, which is a property of the cosine function. The odd component relates to the portion of the periodic function behaving as $f(x) = -f(-x)$, which is a property of the sine function.

The components a_0 , a_n , and b_n are given by the relationships

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(x)\,\mathrm{dx} \qquad \text{Eq.40}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\cos(nx)\,\mathrm{dx} \qquad \text{Eq.41}$$

$$b_n = \frac{2}{\mathrm{T}} \int_{-\mathrm{T}/2}^{\mathrm{T}/2} f(x) \sin(nx) \mathrm{dx}$$ **Eq.42**

The three components are combined to form the Fourier series:

$$f(x) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(nx) + b_n \sin(nx)]$$ **Eq.43**

The limit of the Fourier series approaches the exact value of the periodic function as the number of terms in the series approaches infinity. The Fourier series become an approximation when the series includes a finite number of terms. More terms in the series expansion, closer the approximation of the original function, as demonstrated in Figure 4 Fourier serious expansion of a periodic sawtooth wave (L=1). The number of terms in the series varies from one, two, to five and 25.

, which contains Fourier series approximations of a saw tooth signal with 1 term, 2 terms, 5 terms and 25 terms.

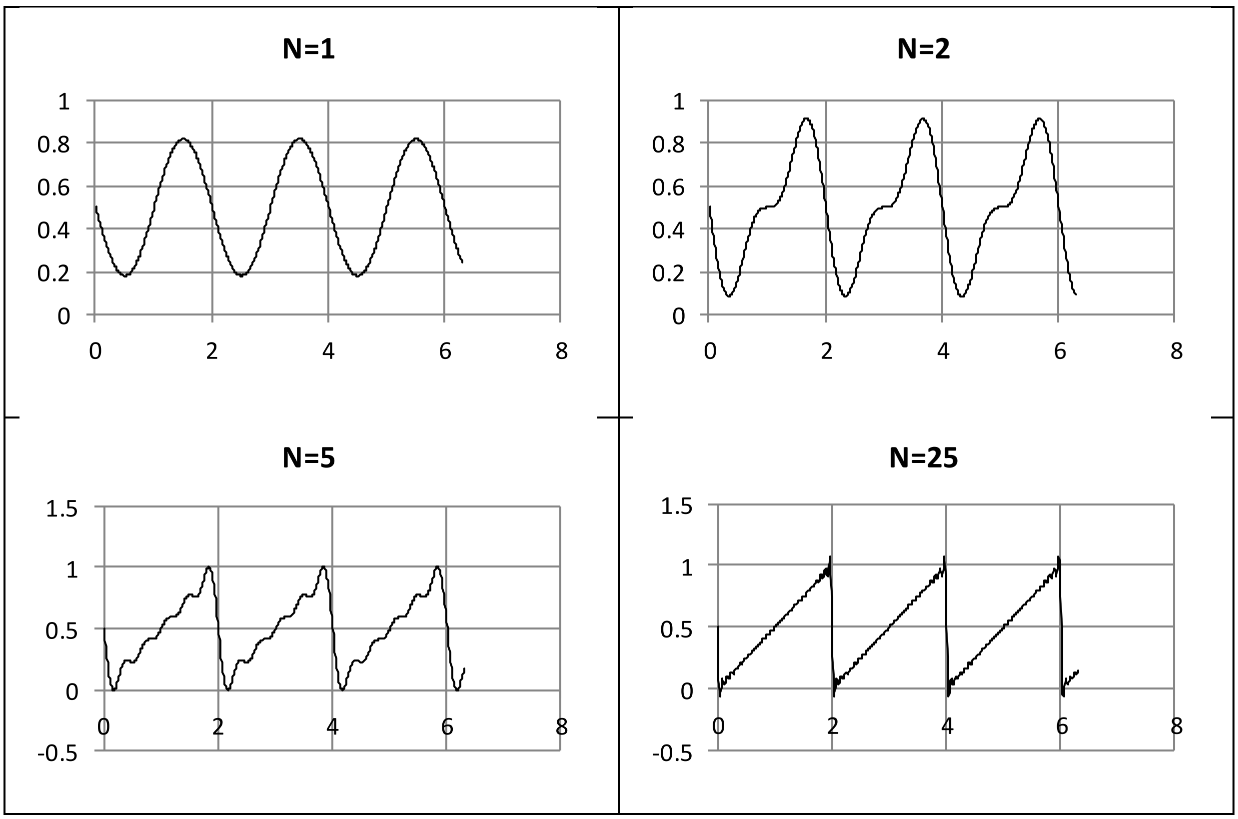


Figure 4 Fourier serious expansion of a periodic sawtooth wave (L=1). The number of terms in the series varies from one, two, to five and 25.

The derivation of the Fourier functions for a periodic sawtooth wave is shown below.

Consider a string of length 2L plucked at the right end and fixed at the left. The functional form of this configuration is

$$f(x) = \frac{x}{2L}$$ **Eq.44**

The components of the Fourier series are given by

$$a_0 = \frac{1}{L}\int_0^{2L} \frac{x}{2L}\,\mathrm{dx} = 1 \qquad \text{Eq.45}$$

$$a_n = \frac{1}{L}\int_0^{2L} \frac{x}{2L}\cos\left(\frac{\mathrm{n\pi x}}{\mathrm{L}}\right)\mathrm{dx} = \frac{[2\mathrm{n\pi}\cos(\mathrm{n\pi})]\sin(\mathrm{n\pi})}{\mathrm{n}^2\pi^2} = 0 \qquad \text{Eq.46}$$

$$b_n = \frac{1}{L}\int_0^{2L} \frac{x}{2L}\sin\left(\frac{n\pi x}{L}\right)\mathrm{dx} = \frac{-2n\pi\cos(2n\pi)+\sin(2n\pi)}{2n^2\pi^2} = -\frac{1}{n\pi} \qquad \text{Eq.47}$$

The Fourier series is therefore given by

$$f(x) = \frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{L}\right) \qquad \text{Eq.48}$$

The example of periodic square wave can be also used to illustrate Fourier approximation.

Consider a square wave of length 2L over the range [0, 2L]. The functional form of the configuration

$$f(x) = 2\left[H\left(\frac{x}{L}\right) - H\left(\frac{x}{L} - 1\right)\right] - 1 \qquad \text{Eq.49}$$

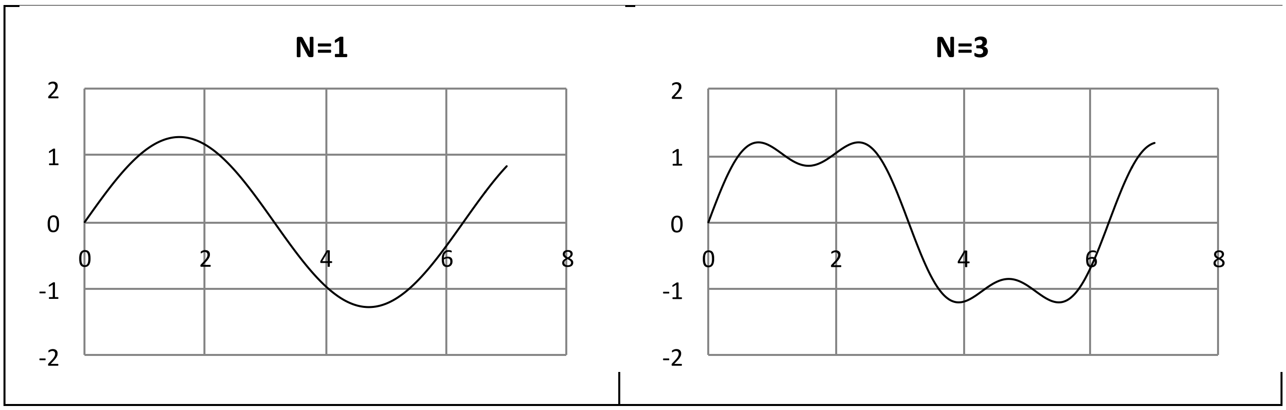
where H(x) is the Heaviside step function. Since $f(x) = f(2L - x)$, the function is odd, so $a_0 = a_n = 0$, and

$$b_n = \frac{1}{L}\int_0^{2L} f(x)\sin\left(\frac{n\pi x}{L}\right)\mathrm{dx} = \frac{4}{n\pi}\sin^2\left(\frac{1}{2}n\pi\right)]$$

$$= \frac{2}{n\pi}[1 - (-1)^n] \qquad \text{Eq.50}$$

The Fourier series is therefore

$$f(x) = \frac{4}{\pi}\sum_{n=1,3,5,\dots}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{L}\right) \qquad \text{Eq.51}$$



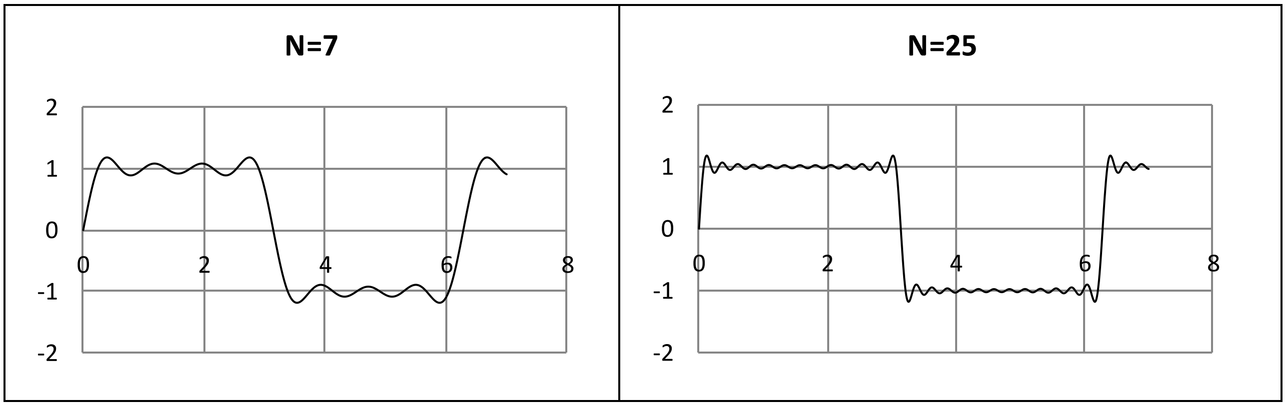


Figure 5 Fourier serious expansion of a periodic square wave (L=1). The number of terms in the series varies from one, three, to seven and 25.

2.3.1.2 Introduction to Fast Fourier Transforms (FFT)

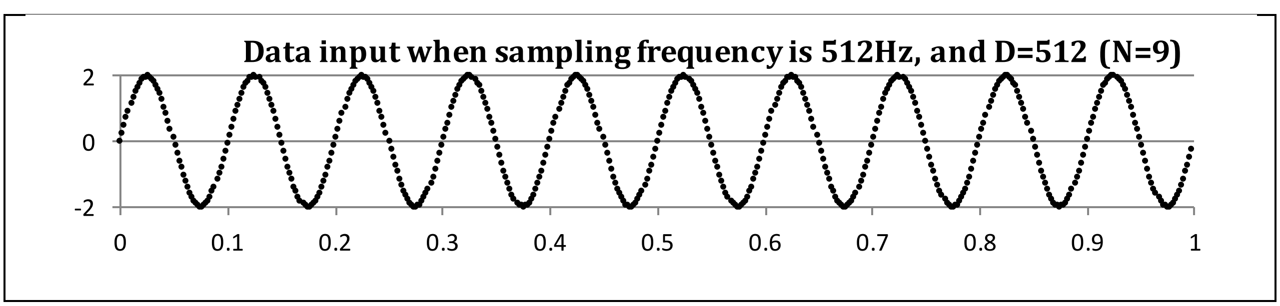
Fast Fourier transformation (FFT) is a technique used to rapidly convert data from time domain to frequency domain. It decomposes a sequence of values into components of different frequencies. The input to a FFT consists of a series of 2^ndata points sampled in time domain at a constant sampling frequency (equally spaced intervals). The output consists of a series of $2^n - 1$ data points in frequency domain showing the contribution of each frequency to the overall signal. The resolution of the FFT is given by

$$Resolution = \frac{f_{sampling}}{D} \qquad \text{Eq.52}$$

Sampling frequency $f_{sampling}$ is determined by dividing the number of data points $D = 2^N$ by the time interval of sampling t:

$$f_{sampling} = \frac{D}{t} \qquad \text{Eq.53}$$

Higher the sampling frequency, higher the accuracy of the FFT. Below are the FFT analysis of the function $f(x) = 2\sin(20\pi x)$, over the range of [0, 1] second. The function has a frequency of 10 Hz. The input data and FFT analysis results are listed in Figure 3.



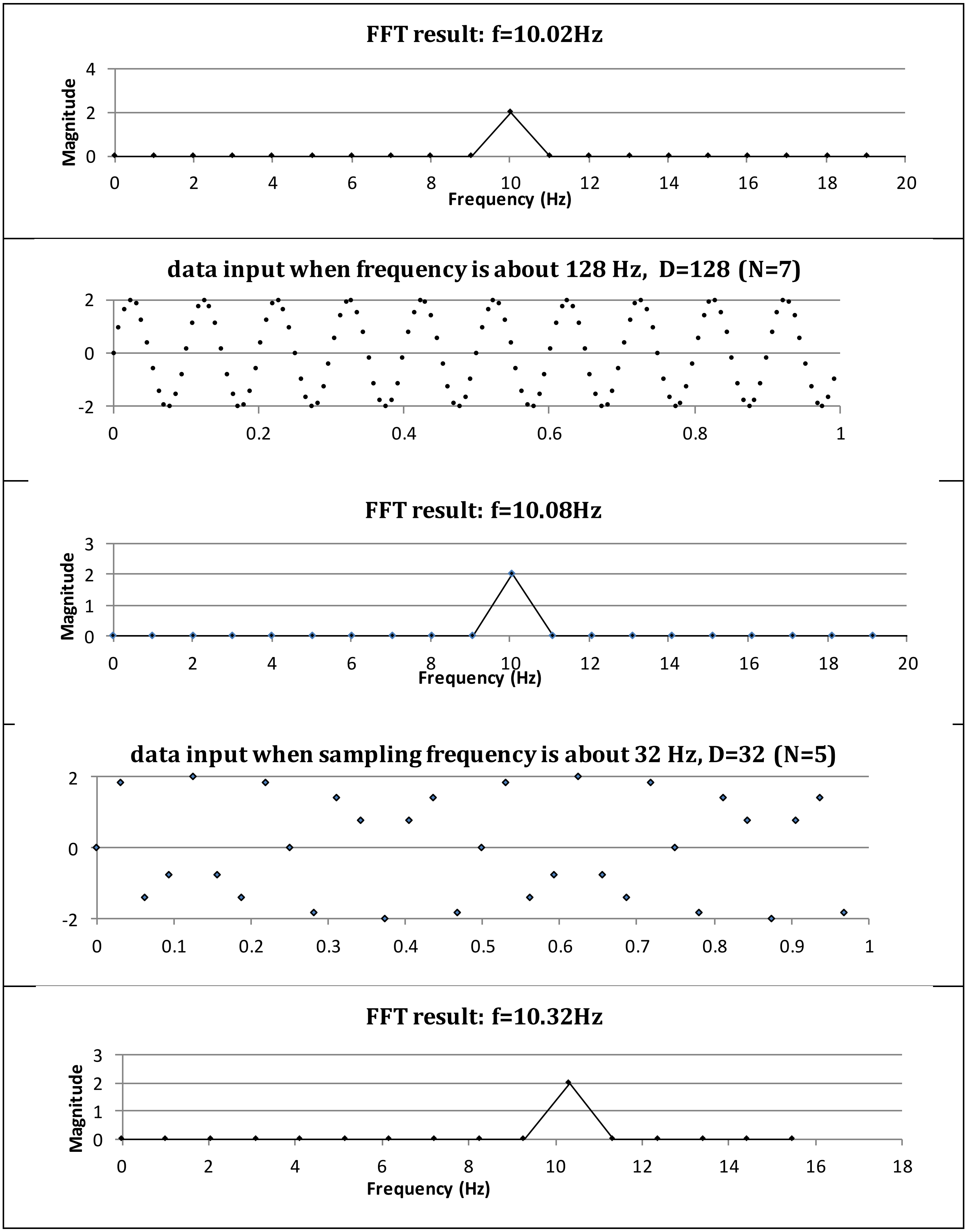


Figure 3 Data input to Fourier analysis and results

The result of FFT includes a real and an imaginary component. The magnitude (or power) and phase of the FFT data is computed by

$$Magnitude = \sqrt{real^2 + imaginary^2} \quad \text{Eq.54}$$

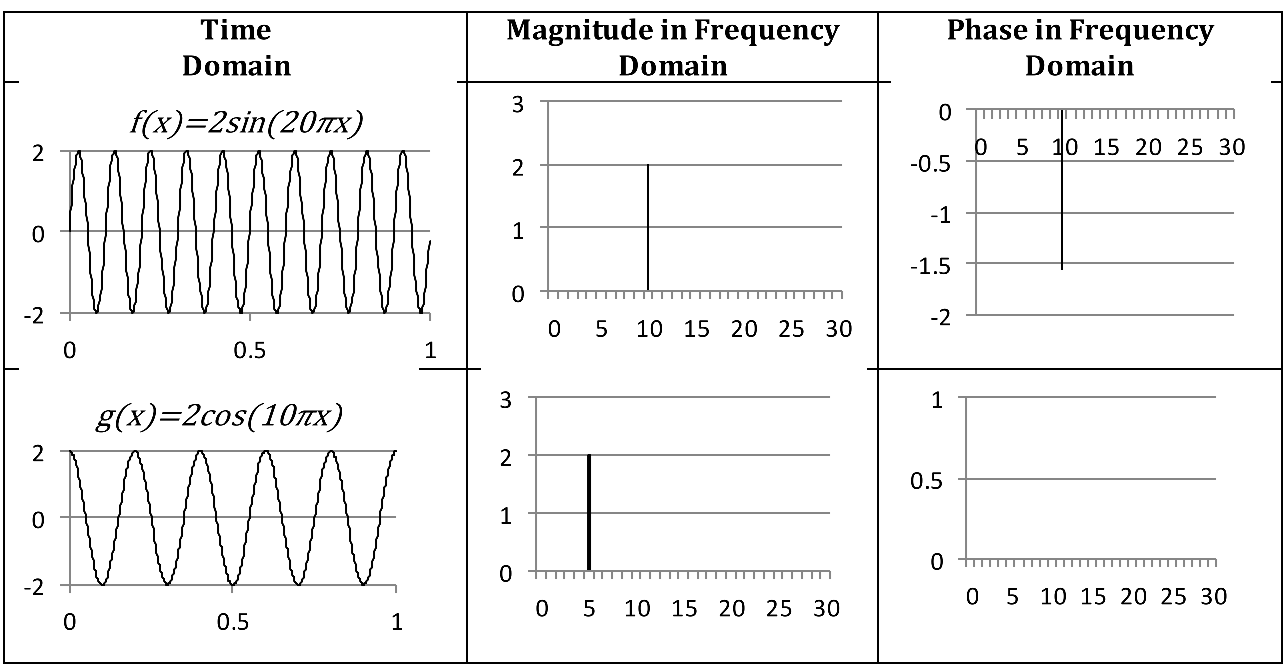
$$Phase = \tan^{-1}\left(\frac{imaginary}{real}\right) \quad \text{Eq.55}$$

For example, at 10Hz, the magnitude of the function $f(x) = 2\sin(20\pi x)$ has magnitude of 2, and a phase of -0.5π or -90^{o}; while the function $g(x) = 2\cos(10\pi x)$ has a magnitude of 2 and a phase of 0 at 5Hz.

2.3.1.4 Properties of Fourier Transforms

The Fourier transform is linear. It possesses the properties of homogeneity and additivity. That is, scaling in one domain corresponds to scaling in another domain, and addition in one domain correspond to addition in another domain.

Figure 4 shows scaling and addition of f(x) and g(x) mentioned in previous paragraph. We can clearly see that scaling the input in time domain results in same scaling in magnitude, but has no effect in phase. And addition of inputs in time domain correspond to a combination of magnitude and phase of the two inputs' frequency domain.



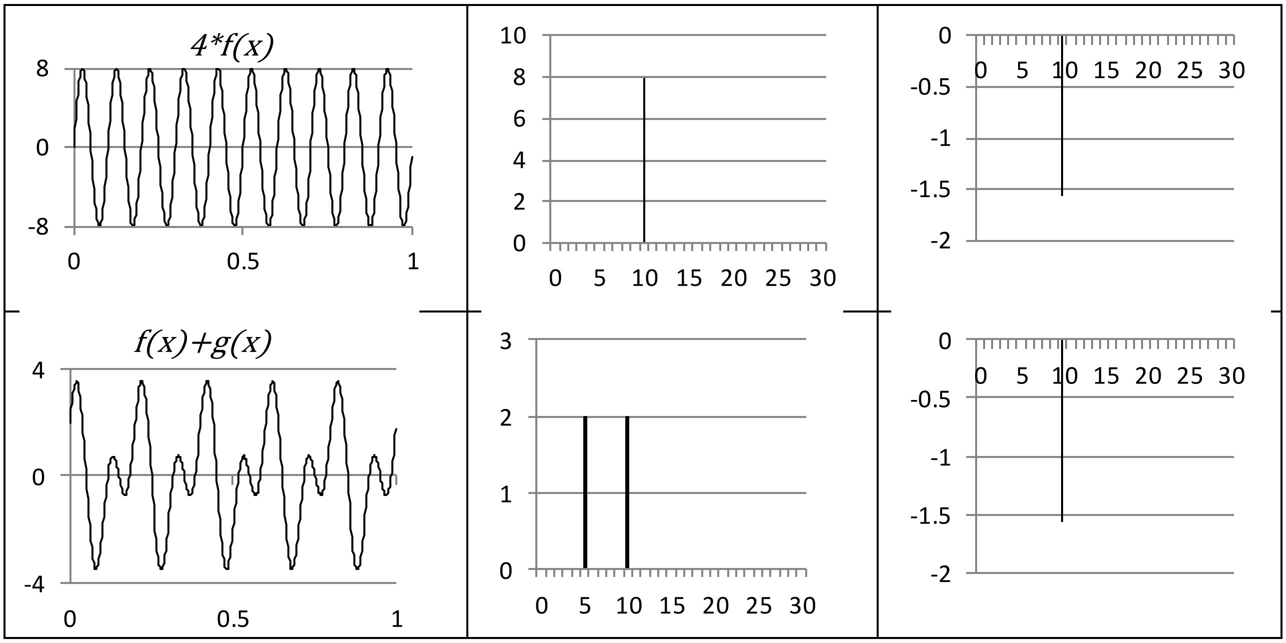


Figure 4 properties of Fourier Transformation

This additivity can be understood in terms of how sinusoids behave. Consider adding two sinusoids with the same frequency but different amplitudes) and phases If the two phases happen to be same, the amplitudes will add when the sinusoids are added. If the two phases happen to be exactly opposite, the amplitudes will subtract when the sinusoids are added. When sinusoids (or spectra) are in polar form, they cannot be added by simply adding the magnitudes and phases.

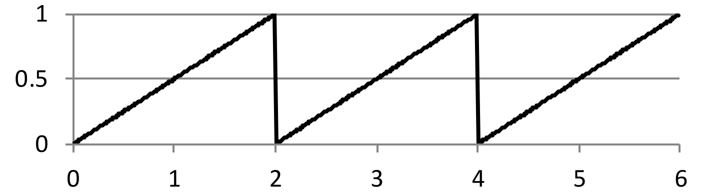
In spite of being linear, the Fourier transform is not shift invariant. In other words, a shift in the time domain does not correspond to a shift in the frequency domain. Instead, a shift in the time domain corresponds to changing the slope of the phase.

2.3.1.5 Examples of Fourier Transforms

Recall the periodic sawtooth function used in section 1, Fourier transforms can be used to find its frequencies. In Figure 5, the first "peak" in positive frequency domain indicates 0.5 Hz as the function's first frequency. Note the offset of the function results in a peak of magnitude at 0 Hz, and the time shift in the function causes the shift in slope of phase. Compare the result after removing the offset and time shift (shown in Figure 6) with the original result.

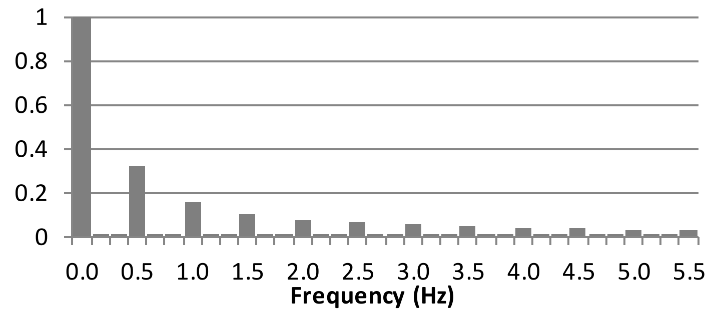
Time Domain:

T=2 sec



Magnitude in Frequency Domain

Frequency: 0.5 Hz



Phase (degrees) in Frequency Domain

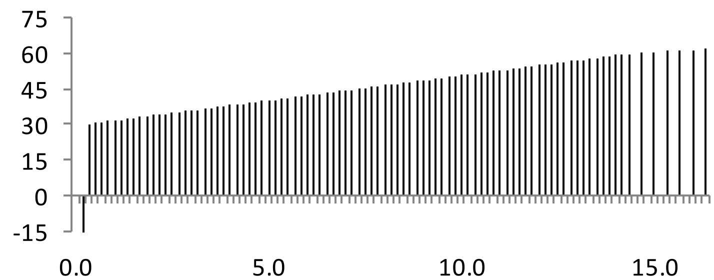
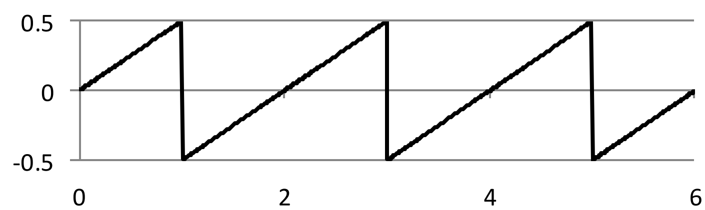


Figure 5 Fourier Transform of Periodic Sawtooth Function

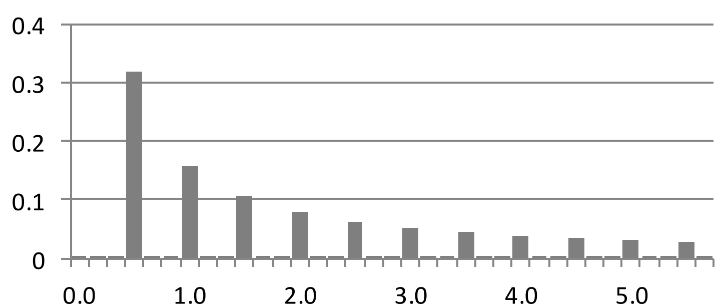
Time Domain:

T=2 sec



Magnitude in Frequency Domain

First Frequency: 0.5 Hz



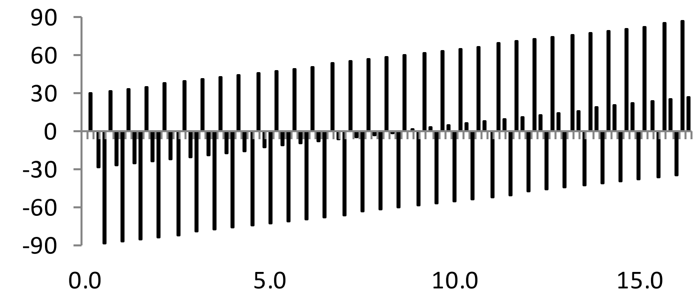
Phase (degrees) in Frequency Domain	

Figure 6 Fourier transformation of periodic sawtooth function without offset and time shift.

Applying Fourier transform to the periodic square wave function used in section 1 yield results in Figure 7. Comparing the results in Figure 8 with Figure 7, we can see that a time shift leads to a shift in phase, but have no impact on magnitude.

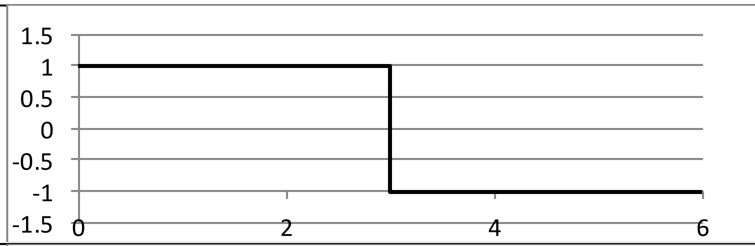
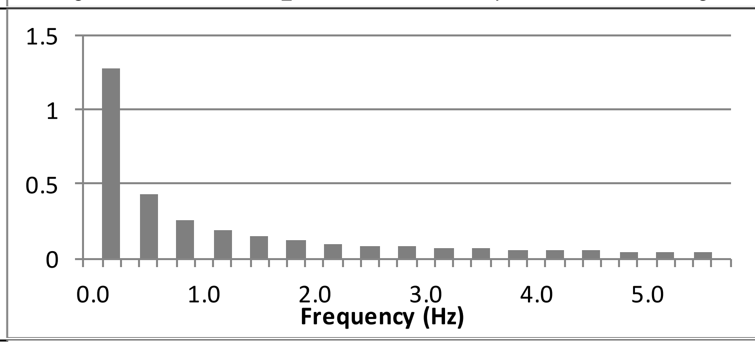
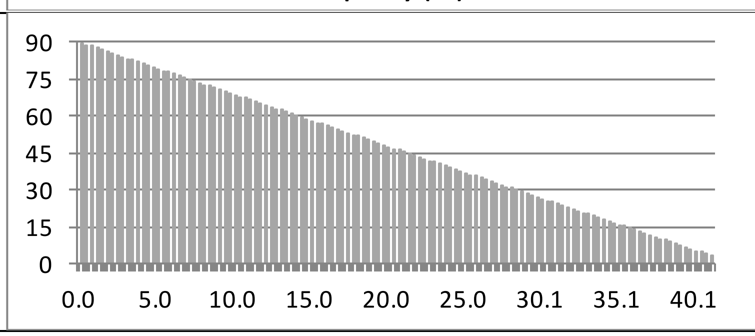
Time Domain: T=6 sec	
Magnitude in Frequency Domain	
Phase (degrees) in Frequency Domain	

Figure 7 Fourier transform result of periodic square wave.

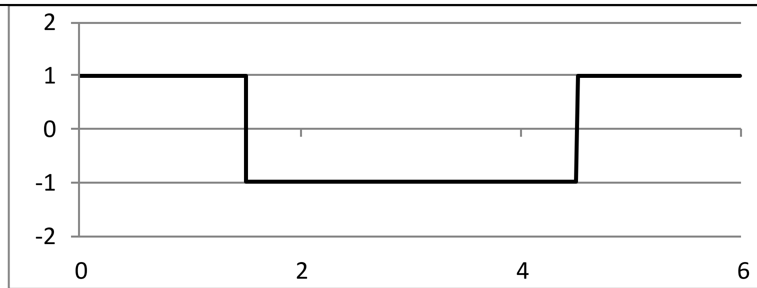
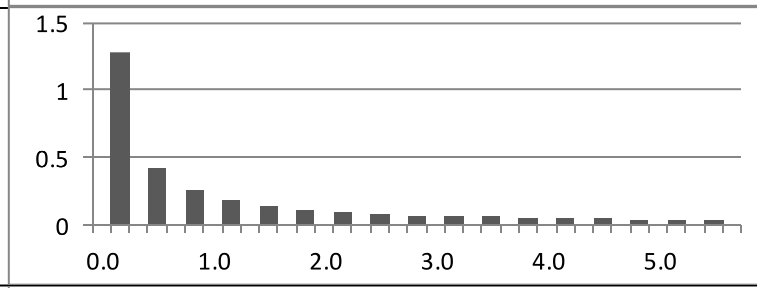
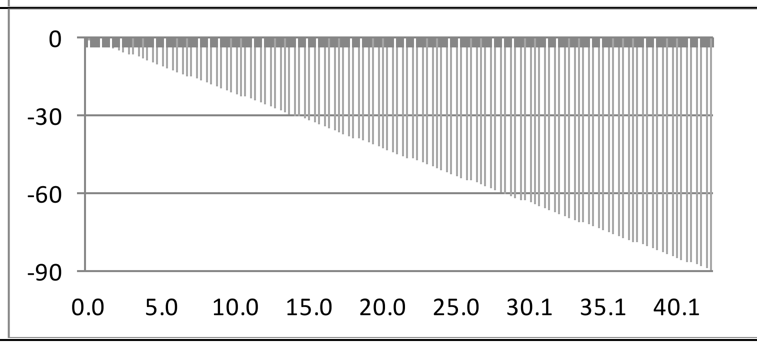
Time Domain: T=6 sec	
Magnitude in Frequency Domain First Frequency: 0.167 Hz	
Phase (degrees) in Frequency Domain	

Figure 8 Fourier transform result of periodic square wave with time shift.

2.3.2 Determining Damping Factor: Logarithmic Decrement

Logarithmic decrement, δ, is used to find the damping ratio of an underdamped system in the time domain. The logarithmic decrement is the natural log of the ratio of the amplitudes of any two successive peaks, as shown in eq.36.

$$\delta = \frac{1}{n} \ln\left(\frac{y_i}{y_{i+n}}\right)$$

where y_i is the amplitude of peak i (i is an integer counting each peak), n is the number of cycles being considered, δ is the log decrement. If there is an offset in y, the y_i amplitude must be defined relative to that offset.

The damping ratio is then found from the logarithmic decrement, as shown in eq.37.

$$\varsigma = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

2.3.3 Determining Vibration Amplitude, Velocity, and Acceleration

Eq.28 shows the relationship between the deflection at the free end of the beam and at any point on the beam. The distance between the free end and the point is denoted by y.

$$w_y(t) = \left[1 - \frac{3}{2}\left(\frac{y}{L}\right) + \frac{1}{2}\left(\frac{y}{L}\right)^3\right] w_{max}(t)$$

Eq. 9 and Eq.14 addressed the derivations of strain and deflection of the beam at a point with distance x from the clamped end.

$$\varepsilon = \frac{P(\mathrm{L-x})T}{2EI} = \frac{6P(\mathrm{L-x})}{EbT^2}$$

$$w = \frac{Px^2(3L-x)}{6EI} = \frac{2Px^2(3L-x)}{EbT^3}$$

Therefore, the expression for the deflection can be updated:

$$w = \frac{\varepsilon x^2(3L-x)}{3T(L-x)} \qquad \textbf{Eq.56}$$

Since it is obvious that $L=x+y$ for any point chosen, we have the expression for the peak altitude in terms of strain, and the location of measured strain, length of the beam, and thickness of the beam:

$$A_{peak} = w_{max} = \frac{\varepsilon}{3T(L-x)} \qquad \textbf{Eq.57}$$

And taking a derivation in regards of time gives the peak velocity of the tip:

$$v_{peak} = \frac{dw_{max}}{dt} = \frac{d\varepsilon/\mathrm{d}t}{3T(L-x)} \qquad \textbf{Eq.58}$$

And a second order derivative of the deflection gives the peak acceleration:

$$a_{peak} = \frac{d^2 w_{max}}{dt^2} = \frac{d^2\varepsilon/\mathrm{d}t^2}{3T(L-x)} \qquad \textbf{Eq.59}$$

The root mean square (abbreviated RMS), is a statistical measure of the magnitude of a varying quantity. It is especially useful when variants are positive and negative, e.g., sinusoids, RMS is used in various fields.

The RMS value of a set of values (or a continuous-time waveform) is the square root of the arithmetic mean (average) of the squares of the original values (or the square of the function that defines the continuous waveform). In the case of a set of n values$\{x_1, x_2, x_3, \ldots x_n\}$, the RMS is given by:

$$x_{RMS} = \sqrt{\frac{1}{n}(x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2} \qquad \textbf{Eq.60}$$

The RMS of a sine wave function$y = k\sin(2\pi f t)$ is given by:

$$y_{RMS} = \frac{k}{\sqrt{2}} \qquad \textbf{Eq.61}$$

The RMS value of the vibration altitude, velocity and acceleration can be calculated by Eq.61 with the peak values provided by Eq.57, Eq.58 and Eq.59.

2.3.4 Determining the Elastic Modulus

Recall the expression of natural frequency in rad/sec in eq.22:

$$\omega_n = \alpha_n^2\sqrt{\frac{EI}{mL^4}} = \alpha_n^2\sqrt{\frac{EI}{\rho AL^4}} = \alpha_n^2\sqrt{\frac{E(\frac{bT^3}{12})}{\rho(bT)L^4}} = \frac{\alpha_n^2}{2\sqrt{3}}\sqrt{\frac{ET^2}{\rho L^4}}$$

Since we have $\alpha_1 = 1.8751$, the first frequency in rad/sec can be expressed as:

$$\omega_n = 1.015\sqrt{\frac{ET^2}{\rho L^4}} \qquad \textbf{Eq.62}$$

Therefore, the elastic modulus can be given by:

$$E = \frac{0.9707\omega_n^2\rho L^4}{T^2} \qquad \textbf{Eq.63}$$

2.4 Basics of Strain Gages

2.4.1 Operating Principle and Application of Strain Gages

Strain-gauge sensor is one of the most commonly used means of load, weight, and force detection. Strain gauges are frequently used in mechanical engineering research and development to measure the stresses generated by machinery, and in Aircraft component testing to structural measure stress of members, linkages, and any other critical component of an airframe.

A strain gauge operates on the principle that the electrical resistance of a wire changes when the length of the wire varies. It is used for measuring deformations in solid bodies. The strain experienced by the sensor is directly proportional to the change in resistance of the gauge used, as shown in Eq 7.When unstressed, usual strain gauge resistances range from 30 Ohms to 3 kOhms.

Eq.64

$$R = \rho \frac{L}{A}$$

An ideal strain gage is small in size and mass, low in cost, easily attached, and highly sensitive to strain but insensitive to ambient or process temperature variations. The ideal strain gauge would undergo change in resistance only because of the deformations of the surface to which the sensor is coupled. However, in real applications, there are many factors which influence detected resistance such as temperature, material properties, the adhesive that bonds the gage to the surface, and the stability of the metal.

The strain sensitivity, which is also known as the gage factor (GF) of the sensor, is given by:

Eq.65

$$F = \frac{dR/R}{\varepsilon_x}$$

where R is the resistance of the gauge without deformation, dR is the change in resistance caused by strain, and ε_x is the strain to be measured. Therefore, the strain can be expressed as:

$$\varepsilon_x = \frac{1}{F}\frac{dR}{R}$$

Eq.66

2.4.2 Materials and Selection of Strain Gauges

Typical materials for strain gages include: constantan (copper-nickel alloy), nichrome v (nickel-chrome alloy), platinum alloys (usually tungsten), isoelastic (nickel-iron alloy), karma-type alloy wires (nickel-chrome alloy), foils, and semiconductor materials. The most popular alloys for strain gages are copper-nickel alloys and nickel-chromium alloys.

Temperature change can affect the internal structure of strain-sensing material, and also can amend properties of the material of the surface the strain gage is attached to. When there is a temperature change while a measurement is being made, the effects can cause large errors in data unless proper precautions are taken.

Each material has unique reaction to temperature change, as illustrated in figure below. Variation in expansion coefficients between the gage and base materials may cause dimensional changes in the sensor element. Therefore, it is a good practice to select strain gauge made of same type of material as the base structure.

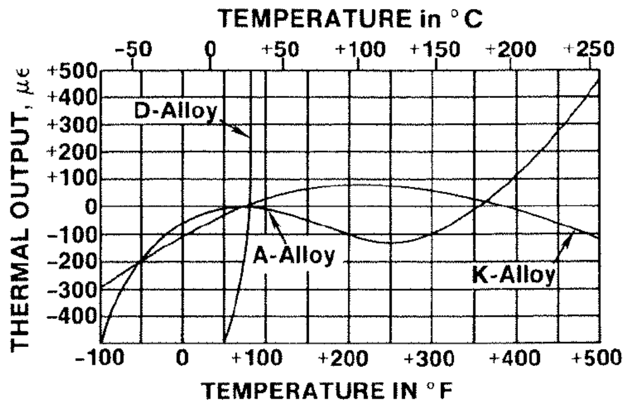


Figure 9 Temperature Effects on Thermal Output of Strain Gauges

Strain gauge's product name contains all critical information needed to select appropriate gauge. The meanings of each part of the name are shown in Figure 10 below. While Figure 11 shows key information of the type of strain gauge selected for this experiment.

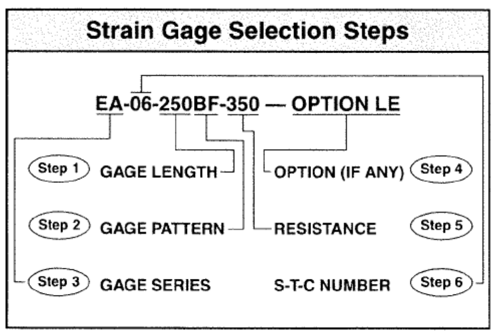
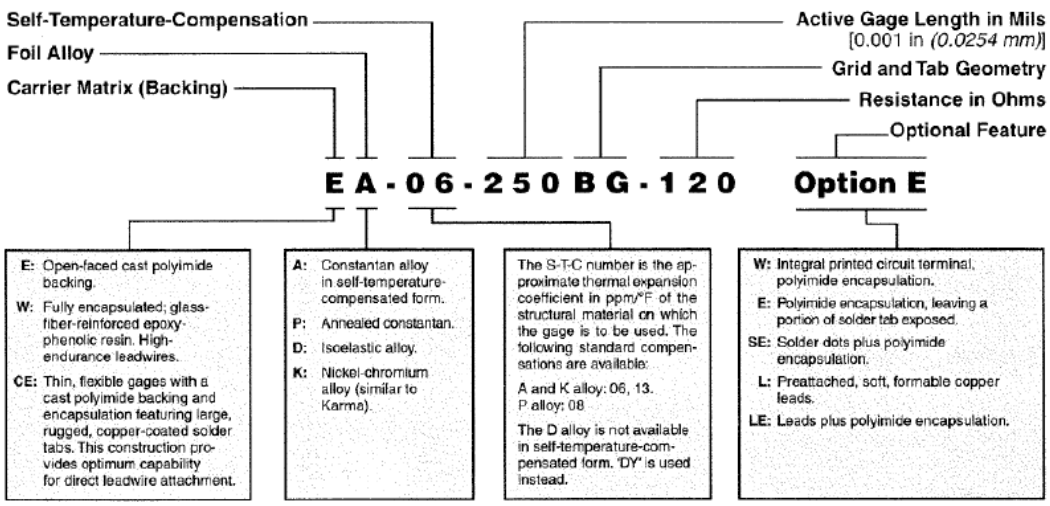


Figure 10 Strain Gage Selection Steps



GAGE SERIES	DESCRIPTION AND PRIMARY APPLICATION	TEMPERATURE RANGE	STRAIN RANGE	FATIGUE LIFE	
				Strain Level in $\mu\varepsilon$	Number of Cycles
EA	General-purpose static and dynamic stress analysis. Wide range of options available.	Normal: −100° to +350°F (−75° to +175°C) Special or Short Term: −320° to +400°F (−195° to +205°C)	±3% for gage lengths under 1/8 in (3.2 mm) ±5% for 1/8 in & over	±1800 ±1500 ±1200	10^5 10^6 10^8

Figure 11 Crucial Information of Strain Gauge Selected

2.5 Basics of Wheatstone bridge

A Wheatstone bridge is an electrical circuit used to measure an unknown electrical resistance (from 1 Ω to 1MΩ) by balancing two legs of a bridge circuit, one leg of which includes the unknown component. A circuit diagram of Wheatstone bridge is shown in figure below, where the battery (symbol "E" serves as an excitation source, and the output is measured by a potentiometer "G").

A "balanced" bridge is one with potential difference between B and D is equal to zero. Balance is sensed by closing switch S2 and measuring output current and voltage – to be near zero. Voltage drop across R2 is equal to voltage drop across R1, since voltage difference between B and D is equal to zero. Therefore,

$$R_x = \frac{R_1\,R_3}{R_2} \qquad \text{Eq.67}$$

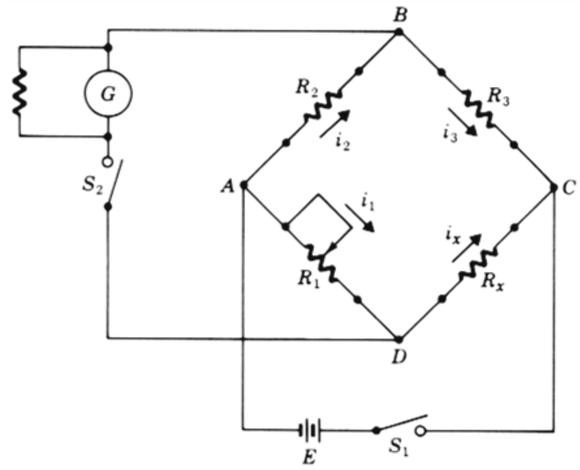


Figure 10 circuit diagram of Wheatstone bridge

When the bridge is unbalanced, equivalent resistance of the circuit is,

$$R = \frac{R_1 * R_4}{R_1 + R_4} + \frac{R_2 * R_3}{R_2 + R_3} \qquad \text{Eq.68}$$

When the circuit is viewed as a circuit divider, the output voltage is,

$$E_g = \left(\frac{E}{R_1 + R_4}\right) * R_1 - \left(\frac{E}{R_2 + R_3}\right) * R_2 = E * \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_4)(R_2 + R_3)} \qquad \text{Eq.69}$$

When the resistance of R_4 changes by a small amount(ΔR_4), the new output voltage is,

$$E_g + \Delta E_g = E * \frac{R_1 R_3 - R_2(R_4 + \Delta R_4)}{(R_1 + R_4 + \Delta R_4)(R_2 + R_3)} = E \left(\frac{1 + \frac{\Delta R_4}{R_4} - \frac{R_3 R_1}{R_4 R_2}}{\left(1 + \frac{R_1}{R_4} + \frac{\Delta R_4}{R_4}\right)\left(1 + \frac{R_3}{R_2}\right)}\right) \qquad \text{Eq.70}$$

If the bridge was originally balanced ($E_g = 0, R_1 = R_2 = R_3 = R_4$), then we have,

$$\Delta E_g = \frac{E * \frac{\Delta R_4}{R_4}}{4 + 2\left(\frac{\Delta R_4}{R_4}\right)} \qquad \text{Eq.71}$$

Since change in resistance is really small($\Delta R_4 \ll 1$), the change in output voltage is,

$$\Delta E_g = \frac{E * \Delta R_4}{4 R_4}$$

or,

$$\Delta E_g = \frac{E * \Delta R}{4R} \qquad \text{Eq.72}$$

3. PROCEDURES

In order to determine the dynamic characteristics and elastic modulus of a vibrating cantilever beam, the procedures of this experiment include research relevant data, initial measurement of the beam, analytical estimations, hardware set-up, signal conditioning, testing with LabVIEW program, taking measurements, and data analysis.

The information acquired from research and part of the measurement process should also be used to produce uncertainty analysis and the contribution of each parameter to total uncertainty.

3.1 Preparations

3.1.1 Research of Relevant Data and Initial Measurements

In order to estimate the natural frequencies of the cantilever beam, the material's elastic modulus and the density need to be found from professional sources. Research online, or use a table in a textbook.

Measure the beam's length and thickness. With a pencil, mark the location to install the strain gauge on the beam at approximately 1 inch from the clamped end.

3.1.2 Undamped Natural Frequency

With the equations provided in Section 2, calculate theoretical undamped natural frequency of the beam. Make sure to use the actual effective length of the beam for calculation. The effective part of the beam is the "free vibrating" part between table and the free end.

3.1.3 Understand the Effect of Gain in the Signal Conditioner

Calculate the amplifier gain required to amplify the output of the Wheatstone bridge so that you get 1 mV/micro-strain.

Recall Gage factor,

$$\mathrm{F} = \frac{\mathrm{dR/R}}{\varepsilon_{\mathrm{x}}}$$

Measured strain can be expressed as

$$\varepsilon_x = \frac{\Delta R}{FR_x} \qquad \text{Eq.73}$$

Recall the expression of change in bridge output voltage caused by a small change in resistance ,

$$\Delta E_g = \frac{E\,\Delta R}{4R}$$

The relationship between measured strain and change in output can be found as,

$$\Delta E_g = \frac{F}{4}\,E\varepsilon_x \qquad \text{Eq.74}$$

To achieve an output signal of 1mV per με, the gain (G) needs to satisfy:

$$\frac{G\Delta E_g}{\varepsilon_x} = \frac{1\times\ 10^{-3}V}{1\times 10^{-6}} = 1\times 10^{3}V$$

Therefore,

$$G\ =\ 1\times 10^{3}V\times\frac{4}{EF} \qquad \text{Eq.75}$$

For this experiment, gage factor (F) is 2.095±0.5%.

3.1.4 Calculate the strain simulated by Shunt Resistors

Calculate the strain simulated by shunt resistors.

The connected shunt resistors are parallel to the gage, the equivalent resistance is:

$$R_{eq} = \frac{R_x R_{cal}}{R_x + R_{cal}} \qquad \text{Eq.76}$$

Therefore,

$$\varepsilon_{eq} = \frac{\Delta R}{F * R_x} = \frac{R_{eq} -\ R_x}{F * R_x} = \frac{R_x}{F(R_x + R_{cal})} \qquad \text{Eq.77}$$

Gage factor is 2.095 $\pm$0.5% for the gauge chosen for this experiment. Resistance without deformation is 120Ω.

3.2 Set-Up

3.2.1 Hardware Set-up

Clamp the beam to the edge of the lab bench. Place a metal plate between the clamp and the beam for noise reduction. Attach the strain gauge to the beam on the marked location.

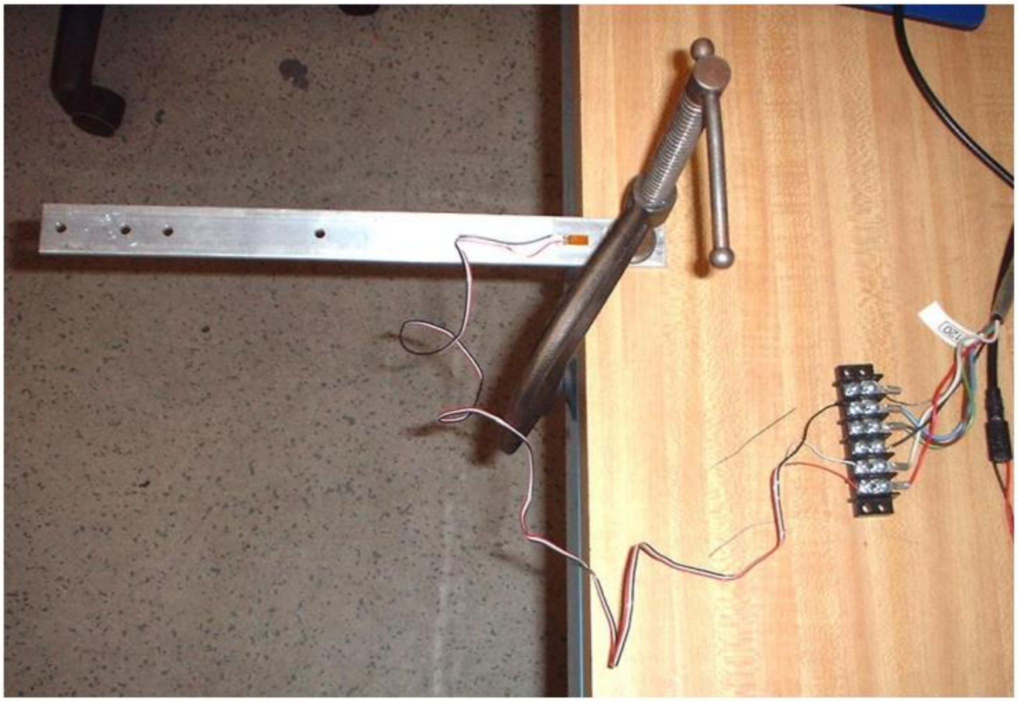


Figure 12 Clamped Cantilever Beam with Strain Gauge Installed

Besides strain gauge and the beam, material needed for attaching the gauge to a surface include: sand paper, degreaser/alcohol, conditioner, neutralizer solutions, cotton balls & swabs, one-side sticky tape , adhesive , low-impedance strain gage wire (about 15 “) , and soldering material. The steps of are explained below.

1) Degreasing: wipe the surface with degreaser or alcohol to remove oil, grease, organic taminants and soluble chemical residues.
2) Surface abrading: sand the surface with sand paper, in order to remove loosely bonded adherents (scalc, rust, paint, coating, oxides, etc.) and develop a surface texture suitable for bonding.
3) Mark layout lines: mark the planned positions to attach strain gauges.
4) Apply neutralizer to the surface, alcohol works as well.

5) Mount on tape: secure strain gauge to the surface with tape, before applying adhesive. When mounting the gauge to the tape, make sure that the side of the gage with soldering terminals should be facing the tape, or "facing up" from the surface.

Carefully remove the strain gauge from its package with tweezers, make sure the strain gauge stay chemically clean. Attach one end of a 4-to-6 inch tape to the surface, carefully attach the strain gage to the tape with tweezers, then pick the gage up by lifting the tape at a shallow angle until the tape comes free with the gage and terminal attached. See figure below for illustration of this step.

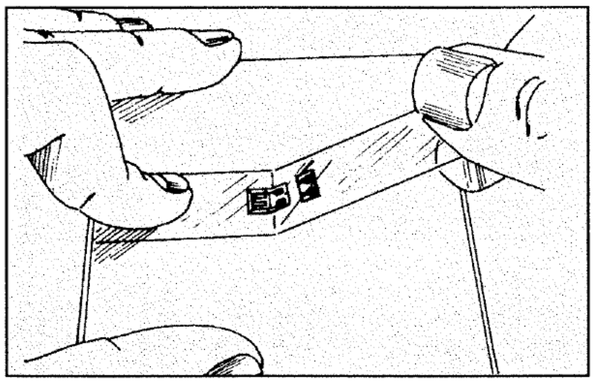


Figure 13 Mount the Strain Gauge on Tape

6) Position the tape: position the gauge/tape assembly so the gauge is over previously marked layout line. Gently apply the assembly onto surface. If the assembly is misaligned, lift the tape again at a shallow angle until the assembly is free from the surface. Reposition.

7) Lift tape: prior to applying adhesive, lift the end of tape opposite the solder tabs at a shallow angle, until the gauge and terminal is free from the surface. Tack the loose end of the tape under and press to the surface, so the gage lies flat with the bonding side exposed.

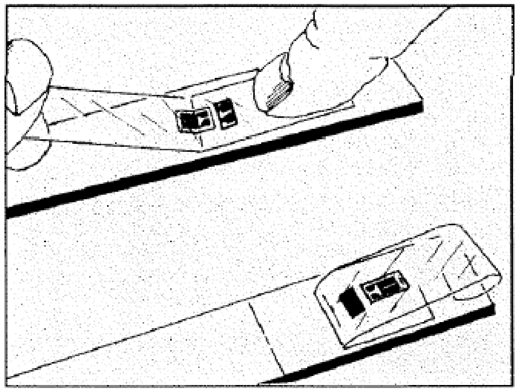


Figure 14 Lift tape

8) Apply adhesive and attach: apply a drop of adhesive to the gage's bonding side, attach the gauge and the surface by pressing on the tape for a minute. Wait two minutes before making a firm wiping stroke over the tape.
9) Remove the tape and clean the terminals with alcohol and a cotton swab.
10) Soldering and stress relief: mask the gage grid area with drafting tape before soldering. After soldering the wires to the terminals, tape the lead-in wires to the surface to prevent the wires from being accidentally pulled from the tabs.
11) Measure the base resistance of the unstrained strain gage after its proper mounting but before complete wiring. Check for surface contamination by measuring the isolation resistance between the gauge grid and the stressed force detector specimen by means of an ohmmeter, if the specimen is conductive. This should be done before connecting the lead wires to the instrumentation.
12) Strain gage should be connected to a Wheatstone bridge with quarter bridge set-up.
13) Connect the signal conditioner properly to provide power to the bridge and amplify the signal. For set-up procedures, refer to Document 2.
14) Connect the inputs from the signal conditioner to the NI DAQ device with a BNC cable, use channel AI0.

3.2.2 Construct the LabVIEW program

Refer to Document 3 for the tutorial to construct a basic VI program for this laboratory.

3.2.3 Verify the Set-up

Before starting the measurements, the strain gauge installations needs to be verified, the following steps should be followed:

a. Run the VI program to monitor the readings.

b. Check for irrelevant induced voltages in the circuit by reading the voltage when the power supply to the bridge is disconnected. Ensure that bridge output voltage readings for each strain-gage channel are practically zero.

c. Connect the excitation power supply to the bridge and verify both the correct voltage level and its stability.

d. Test out the strain gage bond by applying pressure to the gage. The reading should not be affected.

e. Observe corresponding change in the time domain graph as the beam is gently bent.

f. Take a weight provided by the lab and attach it to the beam, record a few seconds of voltage readings after the system stabilizes. Take an average of the stabilized data and calculate the corresponding measured strain. Calculate the theoretical strain at the point of the stain gauge and compare with the measurement result.

3.3 Taking Measurements

Set the sampling rate to over 1kHz. Pluck the beam a few times and record the data with provided program. Note that the program only records last group of data before clicking “stop” button to end the program. The length of the recorded data is the number of samples divided by sampling rate.

4. DATA ANALYSIS AND DISCUSSIONS

Determine the vibration amplitude, velocity, and acceleration in various units of measure; determine natural frequencies; measure and express damping characteristics as logarithmic decrement and percentage of critical damping; determine elastic modulus of a cantilever; compare measurements with analytical and/or computational models.

Conduct uncertainty analysis on the results. Assume 3% of uncertainty in strain measurements. Refer to provided sample uncertainty analysis.

Identify, in order of importance, percentage contribution of all uncertainties to the overall uncertainty in pressure characterizations and Poisson's ratio measurements.

*For optional activities during this laboratory, refer to Document 4.

Attachments

- Sample VI
- Sample Lab Report
- User Manual of Signal Conditioner Used in the Experiment

Document 1: Bending Stress and Strain in Cantilever Beam

Recall, the definition of normal strain is

$$\varepsilon = \Delta L/L \qquad \text{Eq.1}$$

Using the line segments shown in Figure 1, the before and after length can be used to give

$$\varepsilon = \frac{\overline{A'B'} - \overline{AB}}{\overline{AB}} \qquad \text{Eq.2}$$

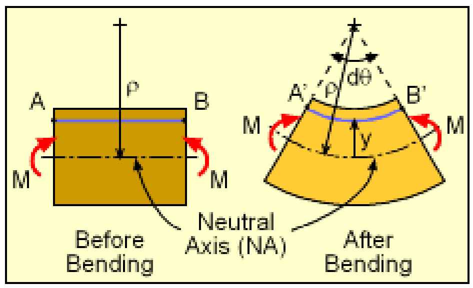


Figure 15 Bending of a Cantilever Beam

The line length on neutral axis remains same after bending. The length becomes shorter above the neutral axis (for positive moment) and longer below. The line AB and A'B' can be described using the radius of curvature, ρ, the differential angle, dθ, and the distance from A'B' to the neutral axis, y. The y coordinate is assumed upward from the neutral axis, where there is no strain.

$$\overline{AB} = \rho d\theta \qquad \text{Eq.3}$$

$$\overline{A'B'} = (\rho - y)d\theta \qquad \text{Eq.4}$$

Therefore we have

$$\varepsilon = \frac{(\rho - y)d\theta - \rho d\theta}{\rho d\theta} = -\frac{y}{\rho} \qquad \text{Eq5}$$

This relationship gives the bending strain at any location as a function of the beam curvature and the distance from the neutral axis.

The strain equation above can be converted to stress by using Hooke's law, $\sigma = E\varepsilon$, giving,

$$\sigma = Ey/\rho \qquad \text{Eq.6}$$

This relationship between radius of curvature and the bending moment can be determined by summing the moment due to the normal stresses on an arbitrary beam cross section and equating it to the applied internal moment. This is the same as applying the moment equilibrium equation about the neutral axis (NA).

$$\sum M_{NA} = 0 \qquad \text{Eq.7}$$

$$\int y(-\mathrm{d}F) = M = -\int y\sigma \mathrm{d}A \qquad \text{Eq.8}$$

Combining Eq.7 and Eq.8 gives

$$\frac{E}{\rho}\int y^2\, \mathrm{d}A = M \qquad \text{Eq.9}$$

Note that the integral is the area moment of inertia, I, or the second moment of the area. Using the area moment of inertia gives

$$\frac{EI}{\rho} = M \qquad \text{Eq.10}$$

Eq. 10 can be used again to eliminate ρ, giving,

$$M = \frac{EI}{-\frac{Ey}{\sigma}} = -\frac{\sigma I}{y} \qquad \text{Eq.11}$$

Rearranging gives,

$$\sigma = -\frac{My}{I} \qquad \text{Eq.12}$$

This equation gives the bending normal stress, and is also commonly called the flexure formula. The y term is the distance from the neutral axis (up is positive). The *I* term is the moment of inertia about the neutral axis.

Document 2: Set-up Procedure for the Signal Conditioner (Tacuna)

a. Connection

Connect the wires as indicated in Figure 3.

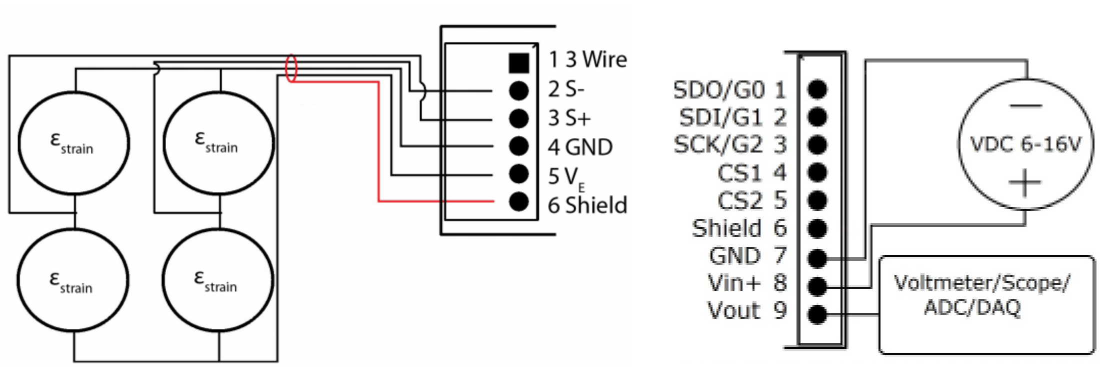


Figure 1 Connections for Tacuna Systems Strain Gauge or Load Cell Amplifier/Conditioner Interface Manual

b. Gain Setting

To get a gain of 220, make sure the switches (location shown in Figure 4) are set as indicated in Table 7.

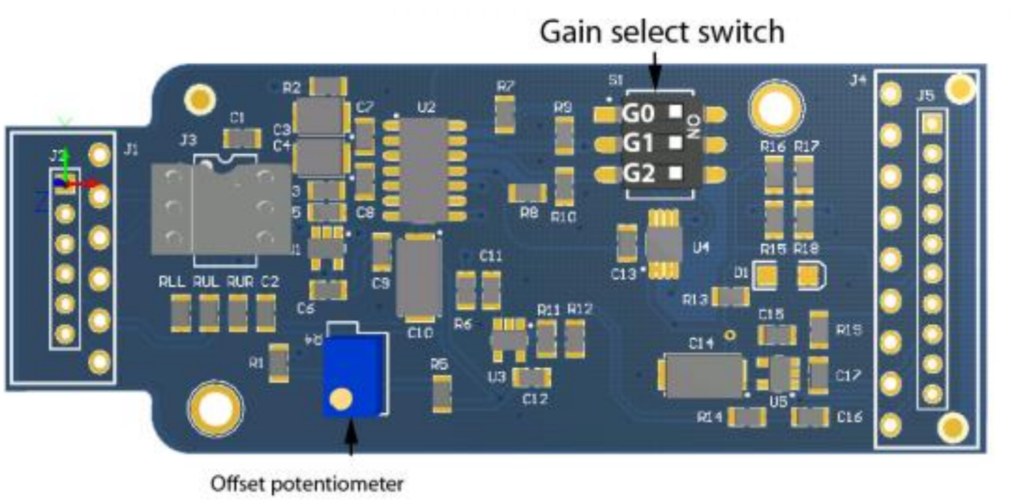


Figure 2 Location of Gain select switch and offset potentiometer

G0	G1	G2
ON	OFF	OFF

Table 1 Switch settings for Tacuna for 220 Gain

c. Bridge Balance

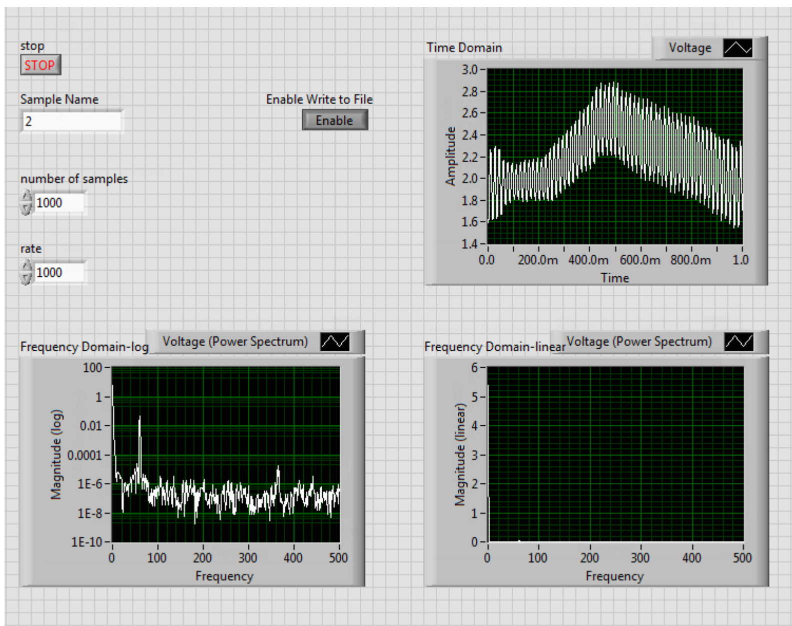
Use the offset potentiometer to adjust the output voltage to 2.5V, which is half of the output range.

It is required to open the enclosure to adjust the gain switches but not the offset potentiometer. The wire connections are located outside of the enclosure.

Document 3: LabVIEW Construction Tutorial

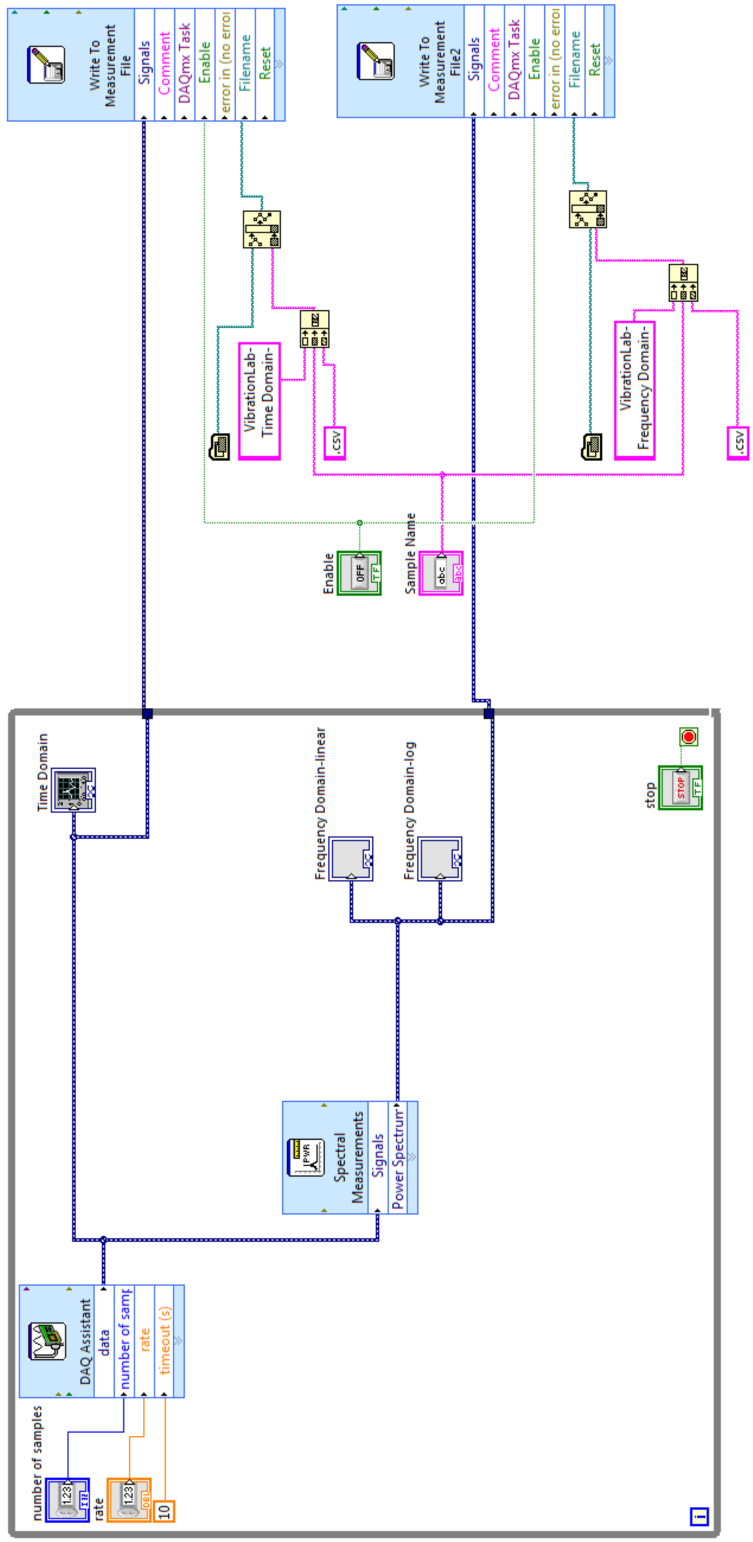
This sample LabVIEW program for the Vibration Laboratory acquires the voltage input from connected NI DAQ device, performs spectral analysis of the input over a specified time period, then saves data in both time domain and frequency domain to separate .csv files in the same folder where the LabVIEW program is saved. Around 1kHz acquisition rate is used for the experiment. This is a basic program to complete the experiment; there are many other ways to write an advanced VI.

The front panel of the program is shown below. The block diagram is shown on page 2. This document walks through the steps of constructing this program.



Before opening LabVIEW program, make sure that the NI DAQ device is probably connected to the desktop and turned on.

On *Tools Palette*, make sure that *Automatic Tools Selection* is enabled (the box/button on top of the palette). This setting automatically selects the appropriate pointer tools from the palette based on the mouse- over object.

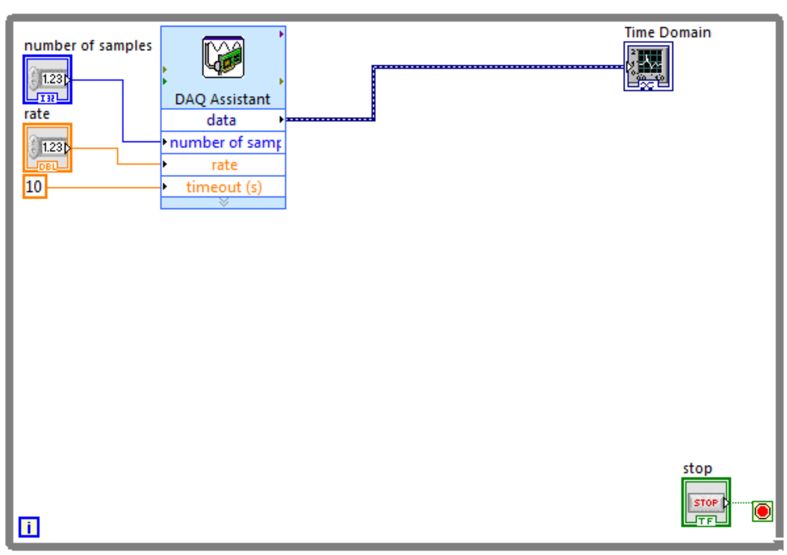


Write To Measurement File
Signals
Comment
DAQmx Task
Enable
error in (no erro
Filename
Reset
Write To Measurement File2
Signals
Comment
DAQmx Task
Enable
error in (no erro
Filename
Reset
VibrationLab-Time Domain-
.csv
VibrationLab-Frequency Domain-
.csv
Enable
OFF
Sample Name
abc
Time Domain
Frequency Domain-linear
Frequency Domain-log
stop
STOP
Spectral Measurements
Signals
Power Spectrum
DAQ Assistant
data
number of samp
rate
timeout (s)
number of samples
rate
10

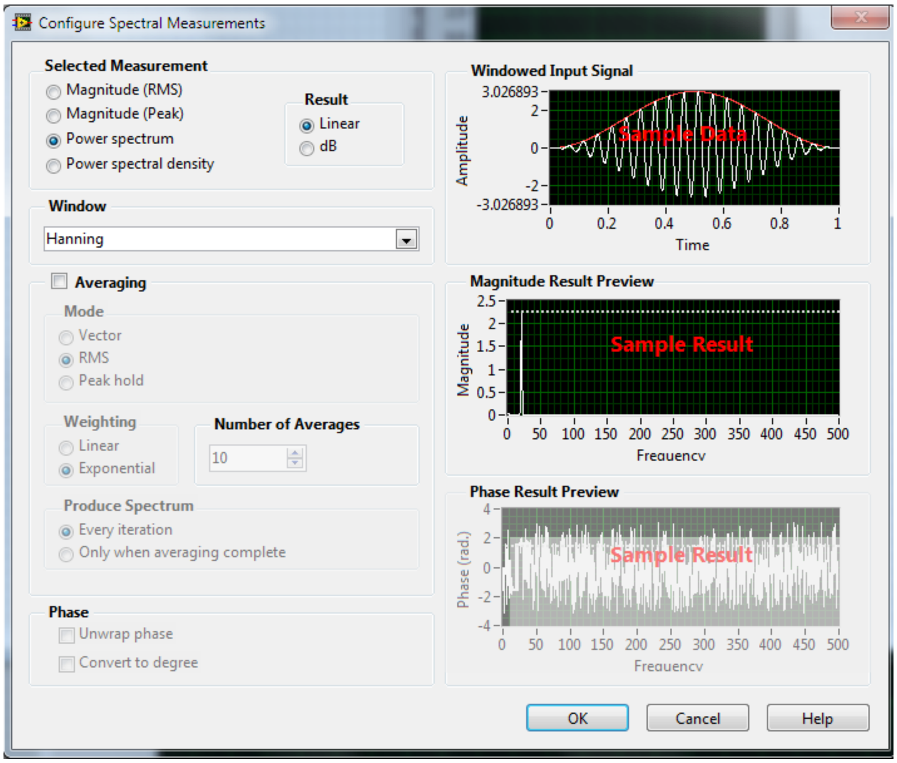
Add a *While Loop* and connect the (already created) *Stop Button* with the *Loop Condition* icon. (*Functions Palette* ➔ *Programming* ➔ *Structures* ➔*While Loop*). The modules can also be accessed by *Search* toolbox in *Function Palette*.

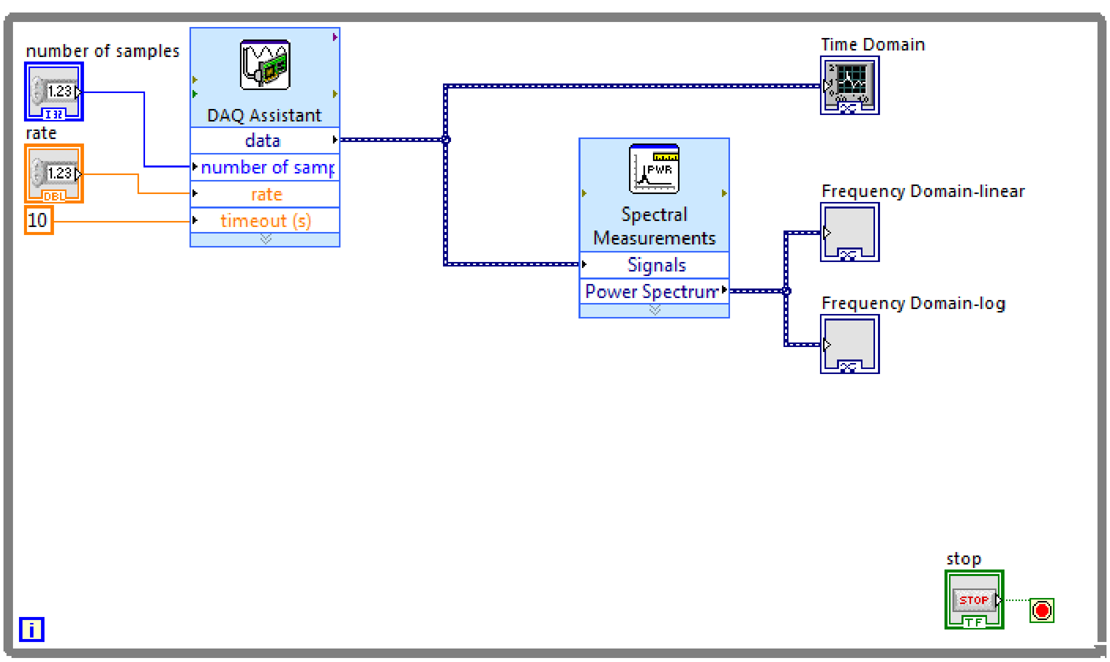
Add a *DAQ Assistant* in the *While Loop* and configure the subVI with the wizard. (*Functions Palette* ➔ *Measurement I/O*➔ *NI DAQ mx* ➔ *DAQ Assistant*). For the measurement type, select *Acquire Signals* ➔ *Analog Input* ➔ *Voltage*. For the physical channel, select the channel of incoming signal. Since channel AI0 of NI 6229 is connected to the input, select this specific channel. Next, configure the channel settings: *N Samples* for acquisition mode. Note that the DAQ box needs to be connected to the computer and turned on before starting of LabVIEW program. Save the work and restart the program if the module fails to initialize.

Drag down the downward arrow on the icon and create *Numeric Controls* for “number of samples” (samples to read) and “rate” (Rate Hz). The calculated timeout is the number of seconds for recorded data. Create a *Graphical Indicator* for data output of the *DAQ Assistant* and change the label of the graph into “Time Domain”.

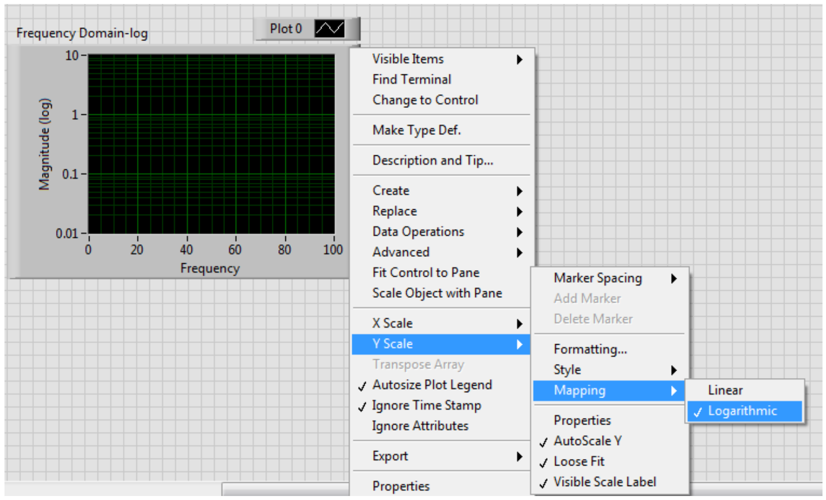


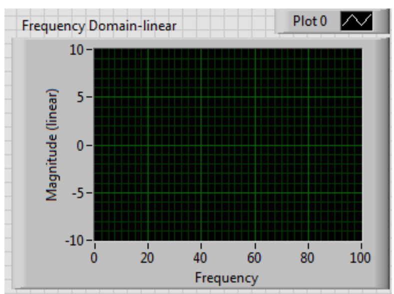
Create a *Spectral Measurement* for the data output of *DAQ Assistant* and create two *Graphical Indicators* for the power spectrum output of the function. In the configuration wizard, select “Power spectrum” as measurement. Change the labels of the *Graphical Indicators* into “Frequency Domain – Linear” and “Frequency Domain – Log”.

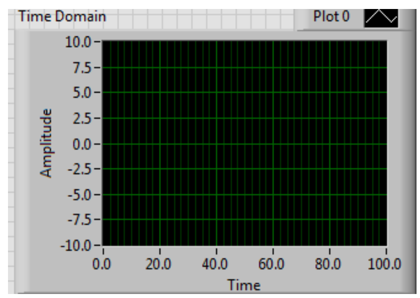




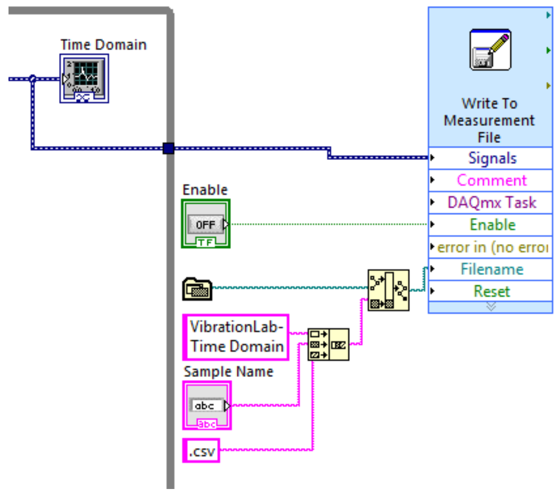
Go to *Front Panel* and configure the three *Waveform Graphs*. Replace the default axis labels with appropriate names (left clicking on the label texts enables editing). Make the mapping of Y axis on the Frequency Domain-Log graph "Logarithmic"; the menu is accessed by right clicking anywhere on the module.



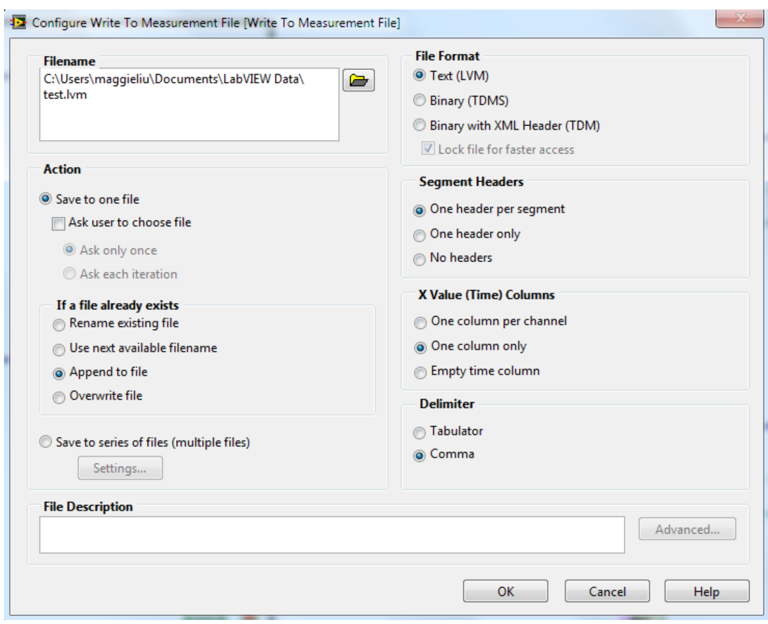




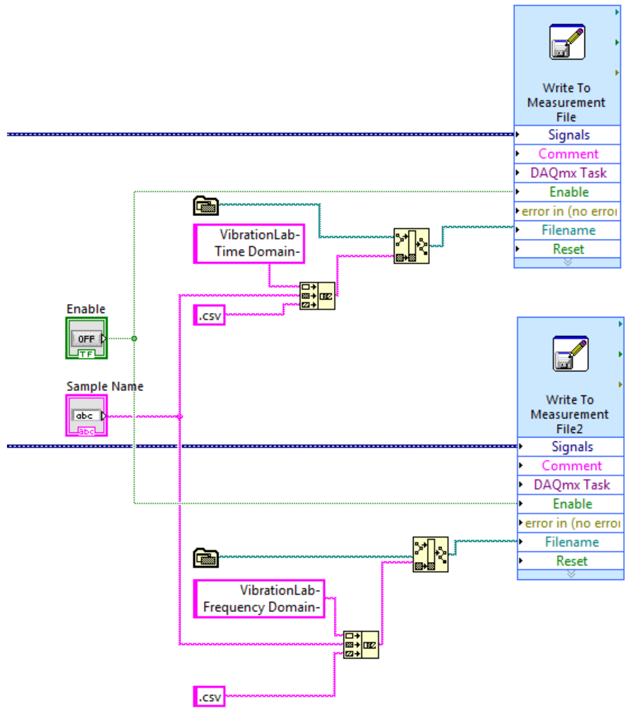
Create a *Write to Measurement File* module outside of the *While Loop*. Drag down the downward arrow to show the input and outputs of the module. Extend the *Dynamic Data* wire for the time domain data out of the *While Loop* and connect it to the signals input. Create a control for the "*Enable*" input, and rename the button "Enable Write to File". The *Filename* can be constructed with *Build Path* function. It builds the file path with an *Application Directory* function, which points to the folder where the VI is saved, and a Concatenated String (Use *Concatenate String* function in *String Palette*) which consists of the lab name, the text "time domain data", the user inputted sample name, and a ".csv" (comma separated values) as file extension, so that the data file can be opened with Microsoft Excel.



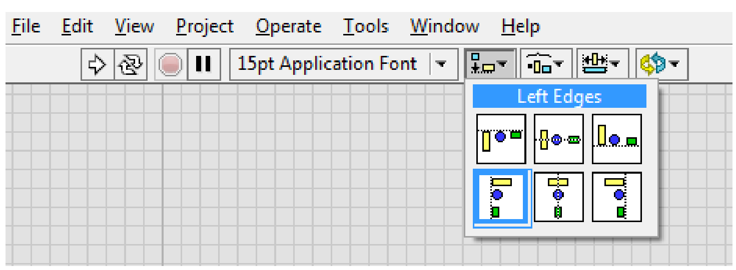
The *Write to Measurement File* should be configured as shown below. The filename in this wizard will be overwritten by the input; it should "save to one file"; the format should be text, with one header only or no headers; there should be only one time column; and the delimiter should be comma.



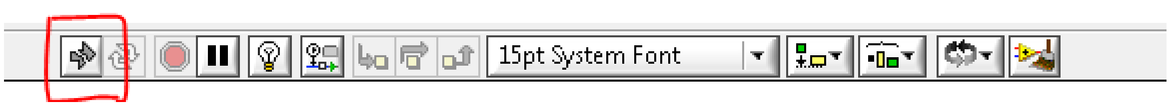
Create a second *Write to Measurement File* module for frequency domain data. The steps are the same as the other *Write to Measurement File*, so one could simple select all elements connected to the previous module and edit the elements later. The two modules should share the "Sample name" and Enable button. The filename of the second module should say frequency domain.

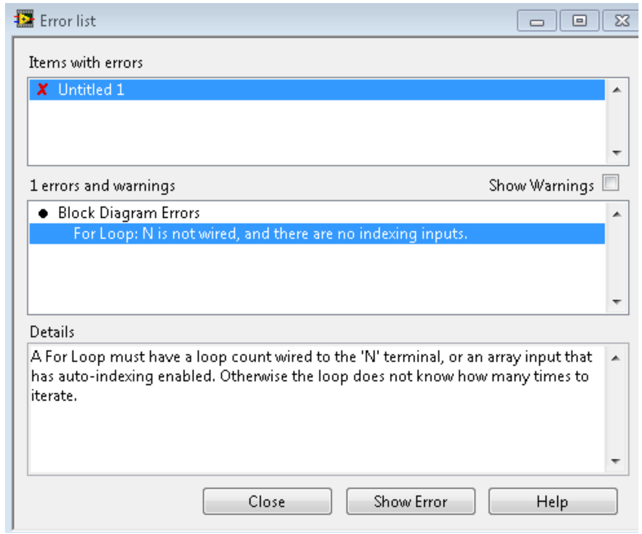


Rearrange the objects for a desirable layout. Drag the icon and drop them at appropriate locations. The objects can be arranged with the tools on the top tool bar, alignment, distribution and resizing tools can be used on selected objects.



Now we have completed constructing the VI. If there is any error in the program, the run button will appear "broken" as shown in the figure below. Click on the button to view the error list, the "details" should explain the error. Debug until all errors are resolved; use other debugging functions on the menu bar if needed.





When the run button appears as a rightward arrow, enter appropriate parameters on the *Front Panel*, connect a BNC cable to AI0 of the DAQ device with two idle clips (this will provide some varied voltage inputs), and test run the program. Use *Edit*➔ *Make current values default* to save the entered parameters as default values. If there is no error interrupting the run, we can check the data file under the specified directory for satisfactory results. Trouble shoots until the program is ready for use.

Now the VI is ready for the Vibration Measurement Laboratory. Can you make it better?

Document 4: Optional Activities

1. Create a shared data file for the class; consolidate measured internal pressure from all the students. What is the average and standard deviation of the measured value? What are some of the possible causes of these variations?
2. Take two data recordings, one with the DAQ Assistants' input voltage range set to -10V to 10V, one with it set to -2V to 2V. Analyze the data and find out the resolution of each recording. Why are they different?
3. Read the user manual for the signal conditioner and change the gain setting. Compare the resolutions of strain readings under different gains.
4. A US nickel weighs 5 grams. Stack nickels on the further end of the beam after the completing the set-up of this experiment. Calculate measured strain when 1 to 10 nickels are on the beam. Compare the results with theoretical values. What are possible causes of deviations? Plot the results. If there is any nonlinearity, try to explain it.
5. Measure the elastic modulus in another way: apply various known weight onto the beam and plot the measured strains with calculated theoretical stresses. Modify the VI for this purpose if interested. Compare this measurement result with the result from vibration measurement and theoretical value.