A Bayesian Analysis of BMI Data of Children from Small Domains: Adjustment for Nonresponse

by

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ABSTRACT

We analyze data on body mass index (BMI) in the third National Health and Nutrition Examination survey, predict finite population BMI stratified by different domains of race, sex and family income, and investigate what adjustment needed for nonresponse mechanism.

We built two types of models to analyze the data. In the ignorable nonresponse models, each model is within the hierarchical Bayesian framework. For Model 1, BMI is only related to age. For Model 2, the linear regression is height on weight, and weight on age. The parameters, nonresponse and the nonsampled BMI values are generated from each model. We mainly use the composition method to obtain samples for Model 1, and Gibbs sampler to generate samples for Model 2.

We also built two nonignorable nonresponse models corresponding to the ignorable nonresponse models. Our nonignorable nonresponse models have one important feature: the response indicators are not related to BMI and neither weight nor height, but we use the same parameters corresponding to the ignorable nonresponse models. We use sample important resampling (SIR) algorithm to generate parameters and nonresponse, nonsample values.

Our results show that the ignorable nonresponse Model 2 (modeling height and weight) is more reliable than Model 1 (modeling BMI), since the predicted finite population mean BMI of Model 1 changes very little with age. The predicted finite population mean of BMI is affected by different domain of race, sex and family income.

Our results also show that the nonignorable nonresponse models infer smaller standard deviation of regression coefficients and population BMI than in the ignorable nonresponse models. It is due to the fact that we are incorporating information from the response indicators, and there are no additional parameters. Therefore, the nonignorable nonresponse models allow wider inference.

ACKNOWLEDGMENTS

I would like to express my gratitude to my advisor, Dr. Balgobin Nandram, whose scientific expertise, understanding, and patience, added considerably to my graduate experience. I appreciate his vast knowledge and endless ideas in many areas, and his assistance in writing this thesis. I am also proud that at the same week of my master's capstone presentation, Balgobin Nandram was appointed as Associate Editor for the prestigious Journal of the American Statistical Association (JASA), Applications and Case Studies, the premier statistical journal in the world.

I am grateful to Dr. Jai W. Choi, National Center of Health Statistics, for his assistance with the data on body mass index from the third National Health and Nutrition Examination Survey (NHANES III).

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A very special thanks goes out to my friends Khriss Toto, Ruijuan Guo and Danny Y. Jin, who encourage, help me along the way whenever I need a hand. Especially, Khriss Toto, who reviewed this paper and gave me tons of advice at times of critical need.

I would also like to thank my parents for the support they provided me through my entire life, especially who stayed in America for half a year only to take care of my son and me. And in particular, I must acknowledge my husband, Shiping, without whose love, understanding and encouragement, I would not have finished this thesis, even the masters' program. I also acknowledge my son, Tiantian, whose smile is the best "medicine" to release my pressure and exhaustion.

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Chapter 1: Introduction

1. 1 Background

Currently there is a great interest in the study of obesity, especially for children. The Body mass index (BMI), defined as the ratio of the body weight (in kilograms) over the squared of height (in square meters), provides a reliable indicator of body fatness for children and adolescents. BMI can be used to screen for weight categories that may lead to health problems.

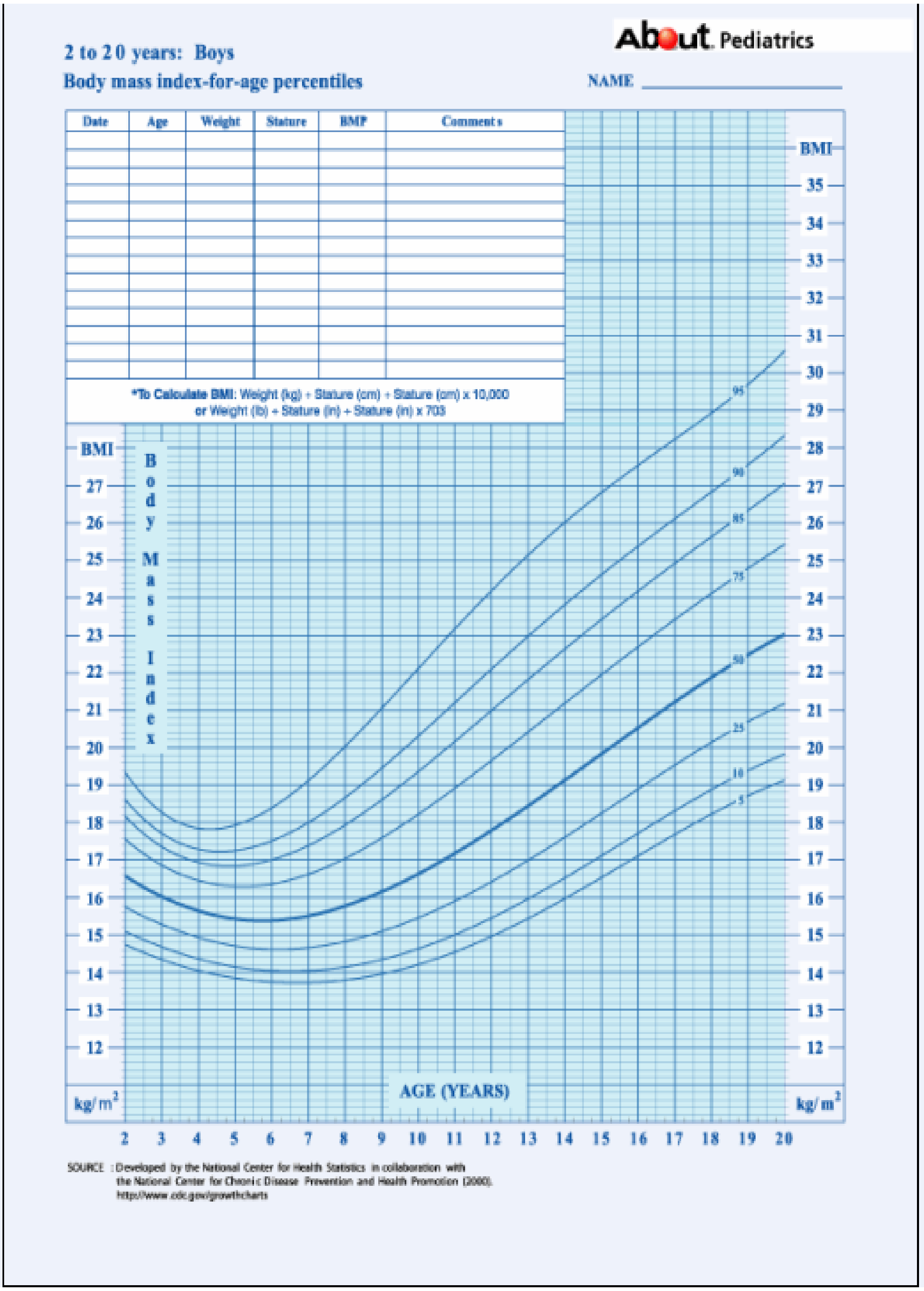
BMI can be used to categorize children as either overweight, at risk of overweight, or underweight. Table 1.1 describes the BMI-for-age weight status categories and the corresponding percentiles (reference [1]).

Table 1.1 BMI-for-age weight status categories

Weight status category	Percentile range
Underweight	Less than the 5th percentile
Healthy weight	5th percentile to less than the 85th percentile
At risk of overweight	85th to less than the 95th percentile
Overweight	Equal to or greater than the 95th percentile

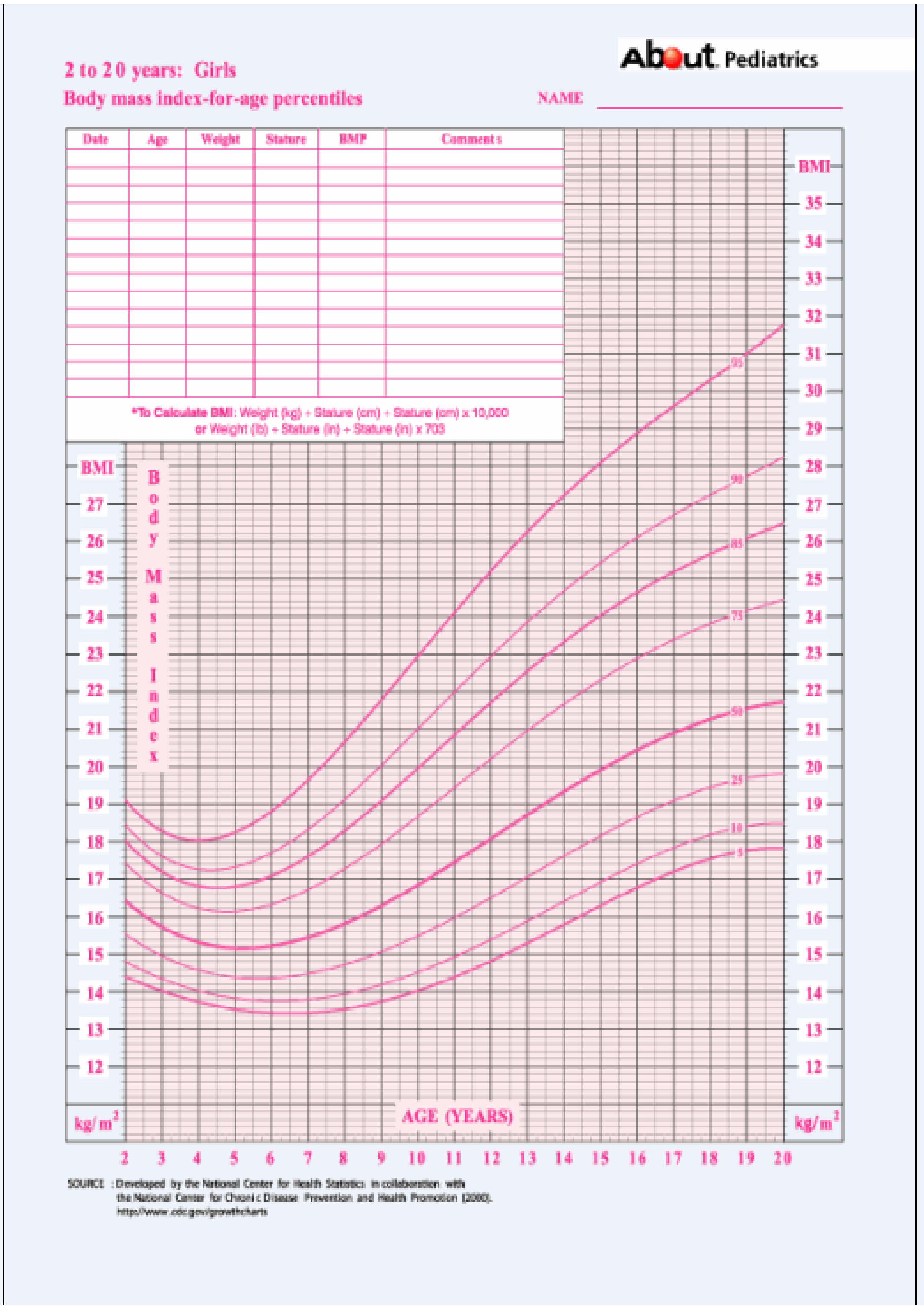
Figure 1.1 and 1.2 show the BMI-for-age percentile of boys and girls aged 2-20 years, respectively (reference [1]).

Figure 1.1 BMI-for-age: boys: 2 to 20 years



Note: see reference [1].

Figure 1.2 BMI-for-age: girls: 2 to 20 years



Note: see reference [1].

As seen in Table 1.1, a child is underweight if his or her BMI-for-age-and-gender is less than the 5th percentile. Studies have shown that an under-nourished child is more likely to become sick. The child may feel weak or tired, and have trouble focusing and concentrating. He or she may have stunted growth or a delay in the onset of puberty. Studies have shown that 17.6 percent of children experience food insecurity, a possible cause of under-nourishment.

On the other hand, several disorders have been linked to overweight in childhood. For instance, a potential increase in type 2 diabetes mellitus is related to the increase in overweight among children (Fagot-Campagna, 2000). Using NHANES III data, Ogden, Flegal, Carroll and Johnson (2002) stated in their study that, “the prevalence of overweight among children in the United States is continuing to increase especially among Mexican-American and non-Hispanic black adolescents.” See Ogden, M., Carroll, L. Curtin, M. McDowell, C, Flegal, K. (2006).

Large nonresponse rates are observed in most complex sample survey, which could impede accurate analysis. Thus, a better data collection mechanisms to minimize nonresponse, like improved data augmentation, is of vital importance. For institutions like the National Center of Health Statistics, which relies on surveys for much information about the condition of the U.S. population, minimization of nonresponse rates is crucial. These may be a very efficient way of collecting data, but to achieve accurate estimates, a more sophisticated method for analyzing the nonignorable nonresponse is necessary.

As mentioned, BMI-for-age percentile of children is useful information to learn about overweight percentile among children. The purpose in this study is to predict the finite population mean BMI for children (2-8 years old), post-stratified by family income, race and gender from the third National Health and Nutrition Examination Survey (NHANES III). But due to significant nonresponse rates observed in this survey without concluding further field work, an investigation of accuracy and the adjustment for nonresponse will be considered to attain a more precise estimation. This study will also attempt to formulate a nonresponse mechanism to address this problem.

1.2 NHANES III Data

The third National Health and Nutrition Examination Survey (NHANES III) is one of the surveys used to assess an aspect of the health of the U.S. population. NHANES III data were collected October 1988 to September 1993 at mobile examination centers (MECs) set up across the U.S.

The collection of these data consists of two parts: the first part is the sample selection and the interview of a sampled household for basic information. The second part is the examination and the interview of those sampled household at the MECs. The health examination at MECs gives information on a physical examination, a set of tests and measurements performed by technicians, and a specimen collection.

In the interview of the first part of the NHANES III survey, a responsible household member is asked about the height and weight of family members. This information might not be reliable due to possible error and hence, may not be useful for prediction. However, since the examination in the second part includes a physical examination where the height and weight of household members were taken, a more reliable source of height and weight is made available.

Nonresponse occurs in both the interview and examination stages. The interview nonresponse arises from sampled persons who either did not participate in the interviews, participated in the interview and introduced to participate in the health examination, missed the examination at home or at the MECs. A partial reason for the nonresponse for young children is that the parents or older mothers were extremely protective and would not allow their children to leave home for a physical examination. In this paper, “nonresponse” refers to a missing BMI value for those sampled “persons” whose age, sex, race and family income information were obtained. We note also that for children (2-8 years old) the observed nonresponse rate is 1331 out of 6878 as approximately 19%.

Because there are significant numbers of height nonresponse, which may contain the important difference between respondents and nonrespondents, there can be serious nonresponse bias in inference. Therefore the main issue we address here is that missing values should not be ignored because respondents and nonrespondents may differ.

1.3 Data description

In this study, we consider 12 domains on race, sex and family income. Race contains two levels as white (W) and nonwhite (N); sex has two levels, male (M) and female (F); family income is evaluated by three levels, low (L), median (M) and high (H). For example, a white boy in low income family will be represented as WML. The other domains are WMM, WMH, WFL, WFM, WFH, NML, NMM, NMH, NFL, NFM, and NFH. It is true that income is important in the studies of BMI (Miech, R., Kumanyika, S., Stettler, N., Link, B., Phelan, J., Chang, W., 2006).

Many researchers have analyzed NHANES III data with nonresponse. See Nandram, B., and Choi, J. W. (2006), Nandram, B., Cox, L. and Choi, J. W. (2005) and Nandram, B. (2006).

In this study, the sample size from 2-8 years old is 6921. There are three types of missing data:

1) missing height, 19% (1331/6921),
2) missing weight, 0.62% (43/6921),
3) missing both height and weight, 0.19% (13/6921).

Since the numbers of type (2) and (3) are not significant, we will assume that all weight values are observed by deleting all data with missing weight. Table 1.2 displays the respondents and nonrespondents sample sizes corresponding to the height over domains

Table1.2 Respondents and nonrespondents sample sizes corresponding to the height over domains

Dom	Response Height	Nonrespone Height	Total
WML	222	66	288
WMM	1066	305	1371
WMH	404	154	558
WFL	269	58	327
WFM	1102	324	1426
WFH	395	143	538
NML	329	57	386
NMM	607	70	677
NMH	133	20	153
NFL	283	45	328
NFM	632	71	703
NFH	105	48	153
Total	5547	1331	6908

Throughout, the four variables, age, height, weight and BMI are represented by a, h, w and b, respectively.

1.4 Preliminary Study

1.4.1 Transformation

We assume that variables are normal. So first of all, we checked the normality and applied a set of possible transformations. We found that an appropriate transformation is logarithm. Therefore log transformation of age, height, weight and BMI is used throughout the paper. We use $\tilde{a}, \tilde{h}$, $\tilde{w}$ and $\tilde{b}$ to indicate log-transformed variables, respectively.

1.4.2 Linear regression

a) **Log-height vs. log-weight.** SAS output gives $R^2_{adjusted}$ equal to 83.48%. The plots of residual vs. predicted log-height and normal-quartile (Figure 1.3) indicate the distribution of residuals is approximately close to normal, even though there are several outliers. This suggests there is strong positive linear relationship between log-height and log-weight. Therefore, it is reasonable to predict missing height by linear regressing weight on height.

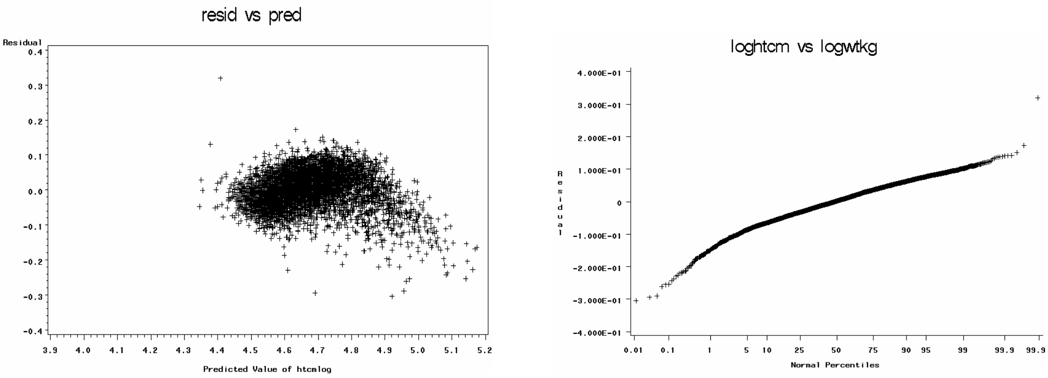


Figure 1.3: Plots of residual vs. predicted values, and normal-quantile plot by regression of log-height vs. log-weight.

b) **Log-weight vs. log-age.** The overall F test is significant and $R^2_{adjusted}$ is 16%, which is not satisfactory. The Normal-Quantile plot (Figure 1.4) looks a little bit skewed to right, even after log transformation. Thus for the regression of log-weight vs. log-age, normalization is doubtful.

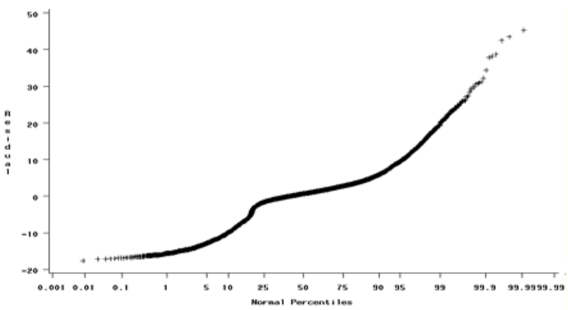


Figure 1.4: Normal-Quartile plot of the residual of log-weight vs. log-age.

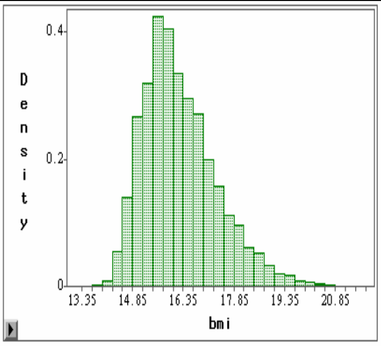
c) **Log-BMI vs. log-age**. The linear regression of log-BMI on log-age is also not satisfied. The overall F test is significant and $R^2_{adjusted}$ is just 12.8%, Thus for the regression of log-BMI on log-age, normalization is doubtful.

1.4.3 Predicting BMI

Since the regression of log-height vs. log-weight turns out to be the best model among the three models checked, we use this model to predict the nonresponse BMI, and use the results to analyze the distribution of BMI of the population. Figure 1.5 shows histograms of the observed BMI and predicted BMI.

Figure 1.5: Histograms of observed BMI and Predicated nonresponse BMI

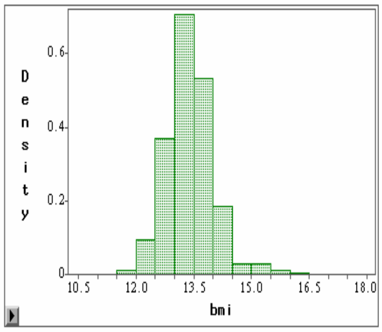
Observed BMI



Moments

N	5547.0000	Sum Wgts	5547.0000
Mean	16.2700	Sum	90249.9077
Std Dev	1.1259	Variance	1.2677
Skewness	0.8917	Kurtosis	1.0467
USS	1475400.36	CSS	7030.8202
CV	6.9203	Std Mean	0.0151

Predicted BMI



Moments

N	1331.0000	Sum Wgts	1331.0000
Mean	13.4120	Sum	17851.4366
Std Dev	0.6568	Variance	0.4314
Skewness	0.8973	Kurtosis	4.1638
USS	239998.122	CSS	573.7880
CV	4.8973	Std Mean	0.0180

1.4.4 Regression of Domains

We also provided linear regression of the 12 domains. Table 1.3 indicates all 12 adjusted R-Square are greater than 80%, so these linear regression models fitted the data well. We notice that there is not much difference among the coefficients in the12 coefficients.

Table 1.3 Estimate log-height in regression model: $\tilde{h}_{ij} = \beta_0 + \beta_1 \tilde{w}_{ij} + e_{ij}$ by domains.

Dom.	MEAN	STD	$R^2_{adjusted}$	β_0	β_1
WML	4.5849	0.1656	0.8449	3.5162	0.3930
WMM	4.5998	0.1701	0.8284	3.5394	0.3862
WMH	4.5817	0.1888	0.8784	3.4553	0.4180
WFL	4.5916	0.1679	0.8074	3.5587	0.3774
WFM	4.5830	0.1747	0.8163	3.5722	0.3746
WFH	4.5681	0.1975	0.8692	3.4431	0.4233
NML	4.6147	0.1697	0.8778	3.4785	0.4095
NMM	4.6549	0.1549	0.8375	3.4789	0.4107
NMH	4.6605	0.1590	0.8430	3.5860	0.3759
NFL	4.6370	0.1615	0.8144	3.6279	0.3599
NFM	4.6510	0.1568	0.8374	3.6007	0.3681
NMH	4.6537	0.1712	0.8355	3.5932	0.3740

Figures 1.6 and 1.7 show the linear regression of log-height vs. log-weight fit the data well, since all residual plots have no pattern and Q_Q plot are close to the normal. Figure 1.8 and 1.9 imply that histograms of predicted BMI are approximately symmetric expect NMH and NFH domains.

Figure 1.6: Scatter plots of Residuals vs. Predicted log-Height in regression model: $\tilde{h}_{ij} = \beta_0 + \beta_1 \tilde{w}_{ij} + e_{ij}$ by domains

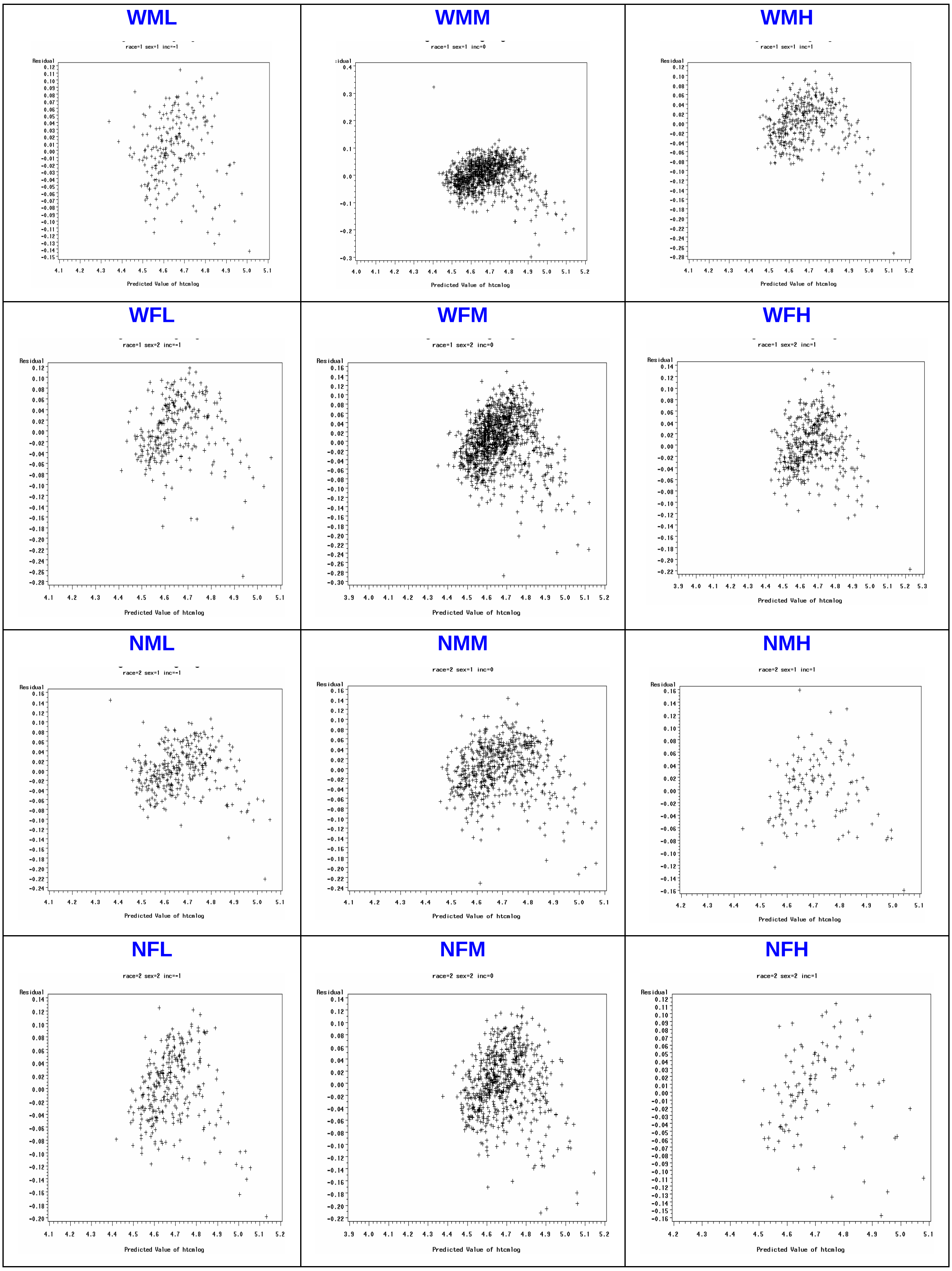
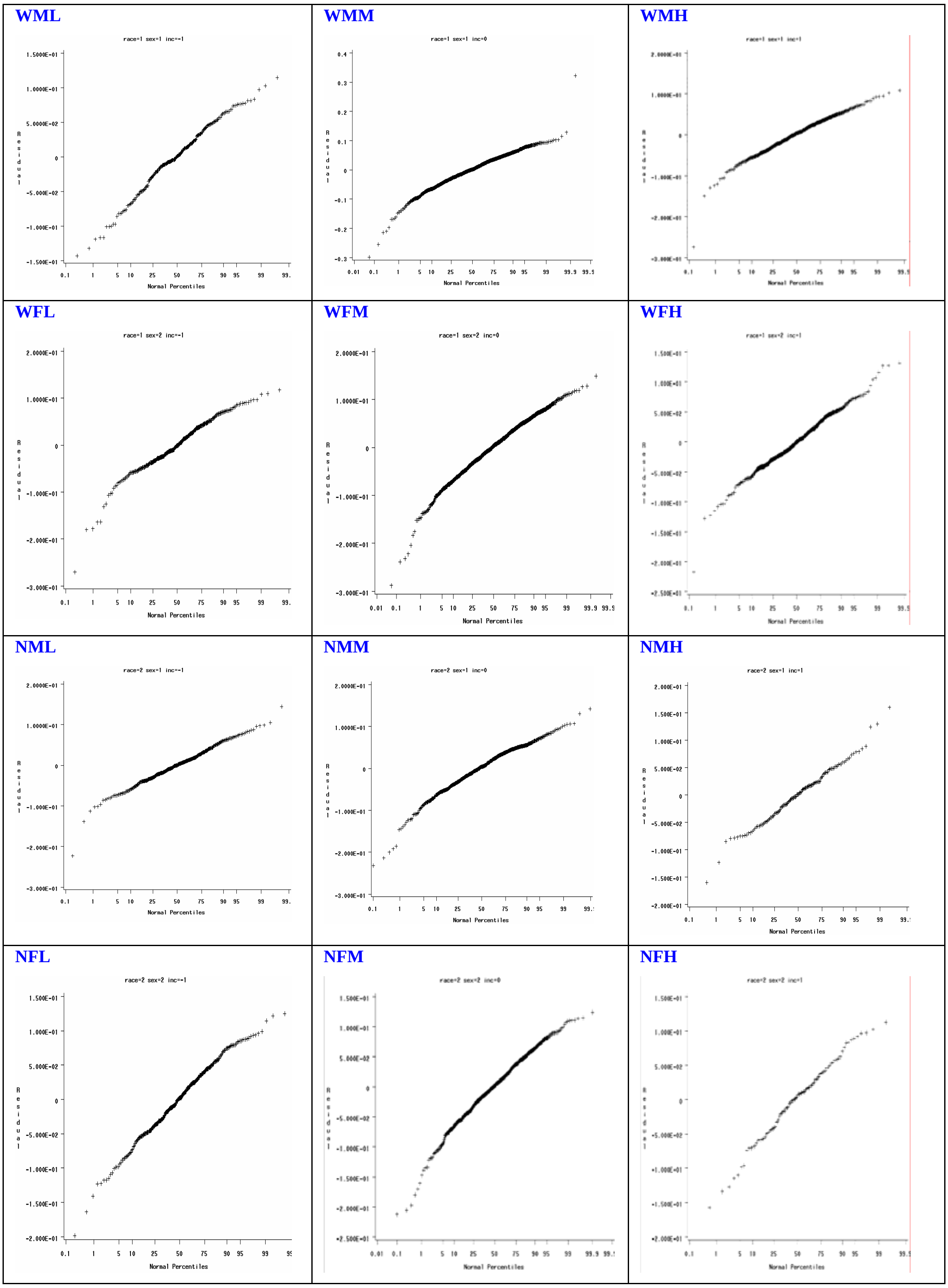


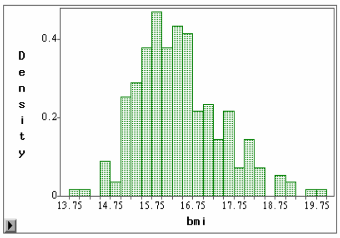
Figure 1.7: Normal-Quantile plots by regression of log-height vs. log-weight by domains.



Note: all 12 N-Q plots are close to straight line.

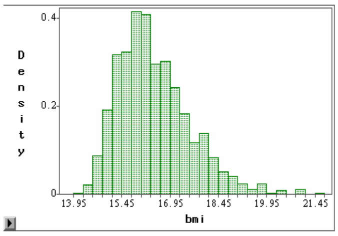
Figure 1.8: Observed BMI by regression of log-height vs. log-weight by domains.

WML



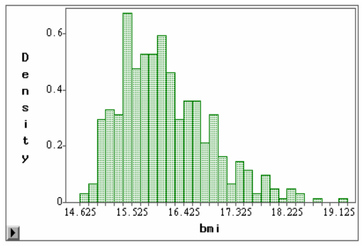
Moments			
N	222.0000	Sum Wgts	222.0000
Mean	16.4154	Sum	3644.2088
Std Dev	1.0290	Variance	1.0588
Skewness	0.6305	Kurtosis	0.4978
USS	60054.9849	CSS	234.0035
CV	6.2685	Std Mean	0.0691

WMM



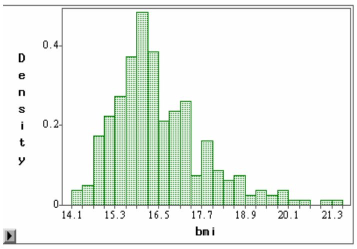
Moments			
N	1066.0000	Sum Wgts	1066.0000
Mean	16.4610	Sum	17547.4754
Std Dev	1.1387	Variance	1.2966
Skewness	0.9589	Kurtosis	1.3081
USS	290230.649	CSS	1380.8437
CV	6.9174	Std Mean	0.0349

WMH



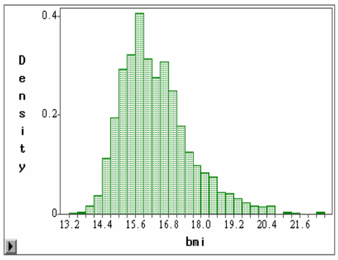
Moments			
N	404.0000	Sum Wgts	404.0000
Mean	16.1179	Sum	6511.6460
Std Dev	0.7823	Variance	0.6120
Skewness	0.8403	Kurtosis	0.6042
USS	105200.944	CSS	246.6526
CV	4.8538	Std Mean	0.0389

WFL



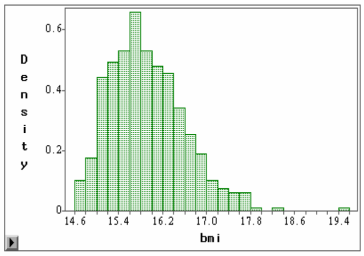
Moments			
N	269.0000	Sum Wgts	269.0000
Mean	16.5124	Sum	4441.8429
Std Dev	1.2682	Variance	1.6083
Skewness	1.0770	Kurtosis	1.3878
USS	73776.6319	CSS	431.0246
CV	7.6802	Std Mean	0.0773

WFM



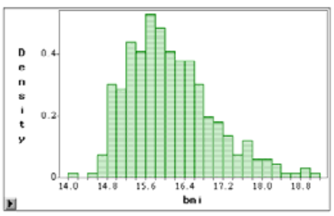
Moments			
N	1102.0000	Sum Wgts	1102.0000
Mean	16.3777	Sum	18048.1903
Std Dev	1.2966	Variance	1.6811
Skewness	0.9899	Kurtosis	1.3831
USS	297438.140	CSS	1850.8691
CV	7.9167	Std Mean	0.0391

WFH



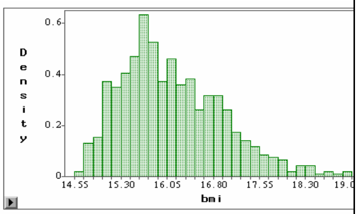
Moments			
N	395.0000	Sum Wgts	395.0000
Mean	15.9229	Sum	6289.5429
Std Dev	0.6917	Variance	0.4784
Skewness	0.8317	Kurtosis	1.4655
USS	100336.215	CSS	188.4926
CV	4.3439	Std Mean	0.0348

NML



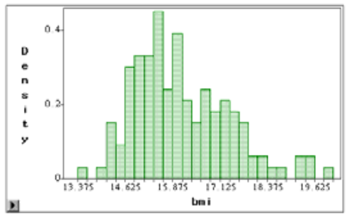
Moments			
N	329.0000	Sum Wgts	329.0000
Mean	16.1062	Sum	5298.9403
Std Dev	0.8816	Variance	0.7773
Skewness	0.7501	Kurtosis	0.4607
USS	85601.0122	CSS	254.9532
CV	5.4739	Std Mean	0.0486

NMM



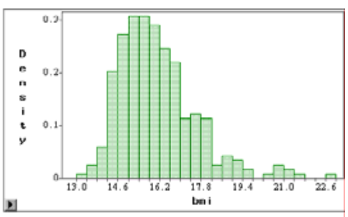
Moments			
N	607.0000	Sum Wgts	607.0000
Mean	16.1369	Sum	9795.0960
Std Dev	0.8332	Variance	0.6943
Skewness	0.6923	Kurtosis	0.2087
USS	158483.164	CSS	420.7163
CV	5.1634	Std Mean	0.0338

NMH



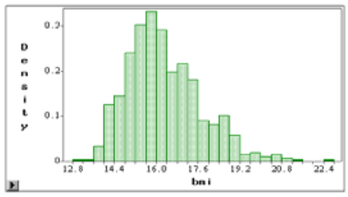
Moments			
N	133.0000	Sum Wgts	133.0000
Mean	16.1122	Sum	2142.9180
Std Dev	1.2710	Variance	1.6155
Skewness	0.7494	Kurtosis	0.3139
USS	34740.2940	CSS	213.2451
CV	7.8886	Std Mean	0.1102

NFL



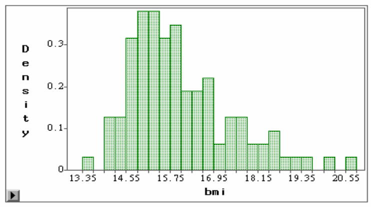
Moments			
N	283.0000	Sum Wgts	283.0000
Mean	16.1348	Sum	4566.1355
Std Dev	1.5171	Variance	2.3015
Skewness	1.2166	Kurtosis	2.1822
USS	74322.5019	CSS	649.0273
CV	9.4025	Std Mean	0.0902

NFH



Moments			
N	632.0000	Sum Wgts	632.0000
Mean	16.2873	Sum	10293.5605
Std Dev	1.4363	Variance	2.0629
Skewness	0.7824	Kurtosis	0.7598
USS	168955.804	CSS	1301.7105
CV	8.8185	Std Mean	0.0571

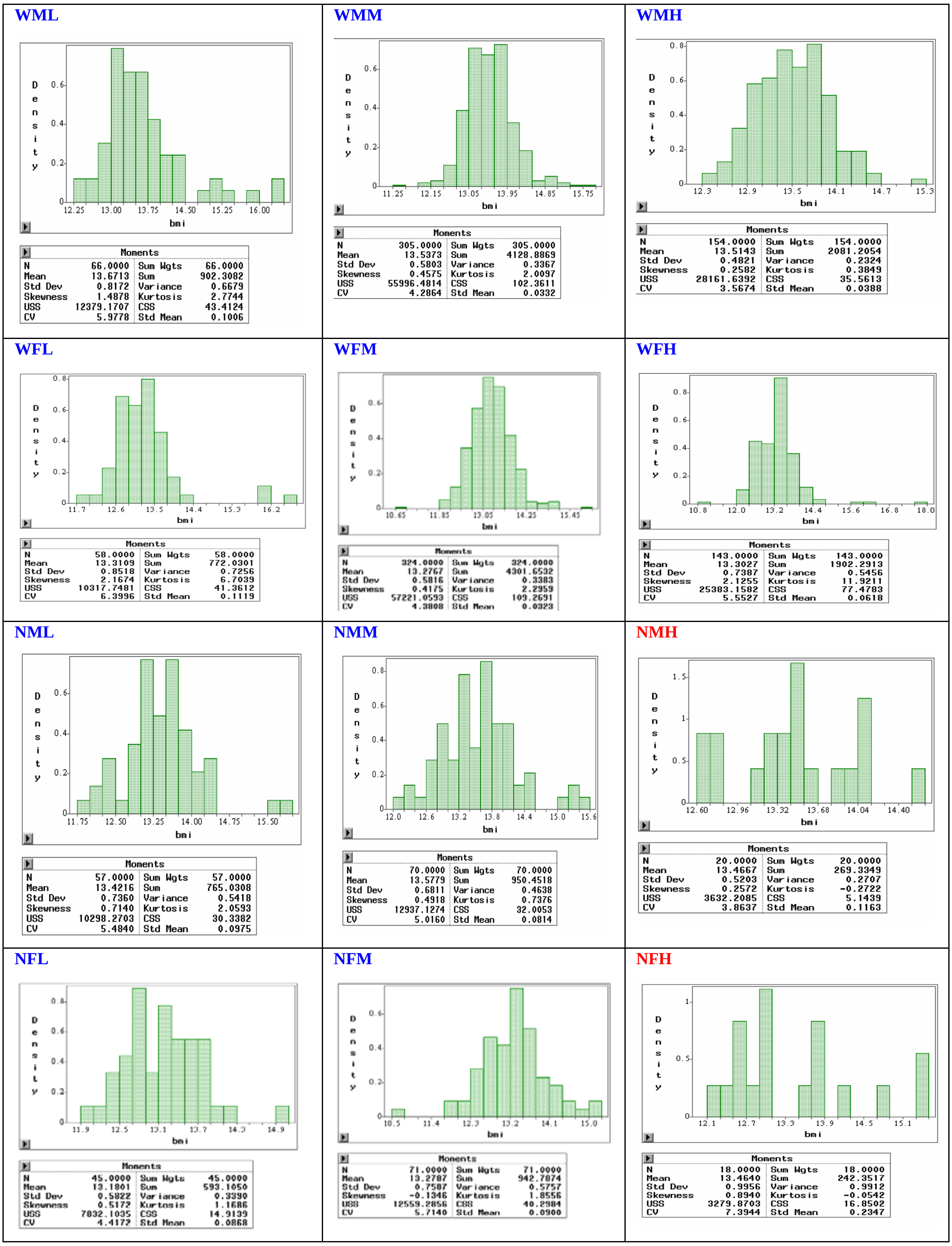
NFH



Moments			
N	105.0000	Sum Wgts	105.0000
Mean	16.0371	Sum	1683.8927
Std Dev	1.3908	Variance	1.9343
Skewness	0.9757	Kurtosis	0.7631
USS	27205.8748	CSS	201.1651
CV	8.6723	Std Mean	0.1357

Note: Most histograms are approximately symmetric expect WFL and NFH domain.

Figure 1.9: Predicted Missing BMI by domains



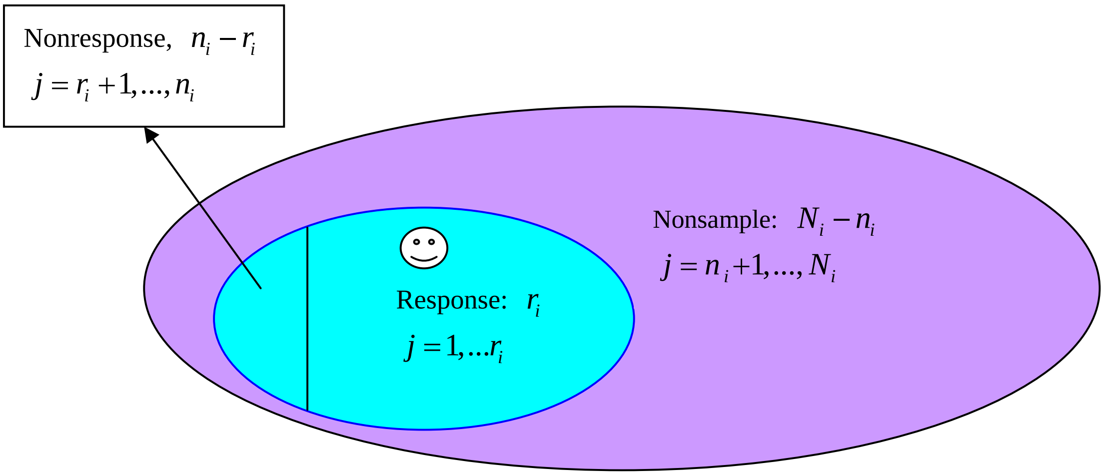
Note: Most histograms of predicted BMI are approximately symmetric expect NMH and NFH domains.

So far, we have provided a background of the BMI and NHANES III data. We also gave a preliminary analysis of the data, which illustrates the basic relationship among BMI and height, weight, age, sex, family income and race. We will use these results to build the hierarchical Bayesian models later.

1.5 Predict finite population mean

Our goal is to predict the finite population mean of BMI for the 12 combinations formed by crossing race, sex and family income. As we know, the finite population mean comes from two big parts, the sample set and nonsample set. Within the sample set, there are observed units and unobserved units. So we use the response sample data to predict nonresponse and nonsample values. Figure 1.10 explains the basic idea of predicting the finite population mean of BMI.

Figure 1.10 The illustration of the relationship of target populations (N_i), sample (n_i) and response (r_i) in the i^{th} domain



Let N_i denote the population on i^{th} domain, and n_i the sample size, so that $N_i - n_i$ is nonsample size. Within sample n_i, let r_i denote the response individuals, and $n_i - r_i$ the nonresponse individuals. Thus the finite population mean on each domain is given by:

$$\bar{y}_i = \frac{r_i \bar{y}_{ir} + (n_i - r_i)\bar{y}_{inr} + n_i \bar{y}_{ins}}{N_i} = (\frac{r_i}{N_i})\bar{y}_{ir} + (\frac{n_i - r_i}{N_i})\bar{y}_{inr} + (\frac{N_i - n_i}{N_i})\bar{y}_{ins},$$

where, for the i^{th} domain, $\bar{y}_r$ is the mean of response, $\bar{y}_{nr}$ is the mean of nonresponse, and $\bar{y}_{ns}$ is the mean of nonsample. Thus, we need to predict $\bar{y}_{nr}$ and $\bar{y}_{ns}$.

We need to know the N_i, the population size by domains. Using census data, we were able to get rough estimates of the population size of each domain. These are presented in Table 1.4.

Table1.4 Estimated population size by domains

Dom, Age	2	3	4	5	6	7	8
1	30290	30290	30290	30290	30290	30290	30290
2	30290	30290	30290	30290	30290	30290	30290
3	30290	30290	30290	30290	30290	30290	30290
4	31139	31139	31139	31139	31139	31139	31139
5	31139	31139	31139	31139	31139	31139	31139
6	31139	31139	31139	31139	31139	31139	31139
7	186585	186585	186585	186585	186585	186585	186585
8	186585	186585	186585	186585	186585	186585	186585
9	186585	186585	186585	186585	186585	186585	186585
10	195548	195548	195548	195548	195548	195548	195548
11	195548	195548	195548	195548	195548	195548	195548
12	195548	195548	195548	195548	195548	195548	195548

1.6 Thesis Overview

We will build two ignorable nonresponse hierarchical Bayesian models to fit the data, then predict finite population mean of BMI by predicting nonresponse and nonsample values. We call them Model 1 (modeling BMI) and Model 2 (modeling height and weight). Each model is within the hierarchical Bayesian framework. For Model 1, BMI is linearly related to age. For Model 2, the linear regression is height on weight, and weight on age. The BMI values of the nonresponse and the nonsampled individuals are predicted from each model. We also use respondent sample data to predict all parameters. As mentioned in this chapter, all variables are taken after log transformation, and also prediction is done on the log scale, the BMI values are retransformed to original scale.

However, both ignorable nonresponse models do not include the data missing mechanism; we assume that response values and nonresponse values have the same distribution. That may be wrong, so that we construct nonignorable nonresponse models in Chapter 3.

In Chapter 3, we build nonignorable nonresponse models for Model 1 (modeling BMI) and Model 2 (modeling height and weight). Here the big issue is that the response indicators are not related to BMI and neither weight nor height, but we use the same parameters of the ignorable nonresponse models described in Chapter 2.

Chapter 4 provides concluding remarks.

Chapter 2: Ignorable nonresponse models

2.1 Model 1 (modeling BMI)

In this chapter, we discuss the two ignorable nonresponse models. We will show how to fit them to the BMI data over the 12 domains.

2.1.1 Model building

Here we consider the regression of BMI on age. Let $\tilde{b}_{ij}$ (on log scale) be the BMI of j^{th} individual in the i^{th} domain, $\tilde{a}_{ij}$ is the corresponding age, $i = 1,...,L, L = 12, j = 1,...,N_i, N_i$ is the population size in the i^{th} domain.

Our ignorable nonresponse model with covariate age is

$$\tilde{b}_{ij} \sim Normal(\beta_0 + \beta_1 \tilde{a}_{ij} + \nu_i, \sigma^2), \quad (2.1)$$

$$\nu_i \overset{ind}{\sim} Normal(0, \frac{1-\rho}{\rho}\sigma^2). \quad (2.2)$$

We use non-informative but proper priors on the hyperparameters,

$$p(\sigma^2) \propto \frac{1}{\sigma^2}, \quad (2.3)$$

$$\rho \sim Uniform(0,1), \quad (2.4)$$

where, ν_i is the ith domain effect. See Battese, G., Harter, R. and Fuller, W. (1988) for a discussion of (2.1) and (2.2).

Letting $\underset{\sim}{\beta} = (\beta_0, \beta_1)'$, $\underset{\sim}{\nu} = (\nu_1,...,\nu_{12})'$ and $\underset{\sim}{a}_{ij} = (1, a_{ij})'$, $\bar{a}_i$ is the average of the age on i^{th} domain. By using Bayes' theorem, the joint posterior density of each parameter is,

$$p(\underset{\sim}{\beta},\underset{\sim}{\nu},\rho,\sigma^2 \mid \underset{\sim}{\tilde{b}})$$

$$\propto \frac{1}{\sigma^2}\prod_i \left\{ \left[\prod_j \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{1}{2\sigma^2}[\tilde{b}_{ij} - (\beta_0 + \beta_1 \tilde{a}_{ij} + \nu_i)]^2\} \right] \frac{1}{\sqrt{2\pi(\frac{1-\rho}{\rho})\sigma^2}} \exp(-\frac{1-\rho}{2\rho\sigma^2}\nu_i^2) \right\}$$

$$\propto (\frac{1-\rho}{\rho})^{l/2} \frac{1}{(\sigma^2)^{\sum r_i + l/2+1}} \prod_{i=1}^{l} \left\{ \exp[-\frac{1}{2\sigma^2}(\sum_{j=1}^{r_i}(\tilde{b}_{ij} - \tilde{a}'_{ij}\underset{\sim}{\beta} - \nu_i) + \frac{\rho}{1-\rho}\nu_i^2)] \right\}. \qquad (2.5)$$

We can show propriety of $p(\underset{\sim}{\beta},\underset{\sim}{\nu},\rho,\sigma^2 \mid \underset{\sim}{\tilde{b}})$ for $p \ge 1$, see Appendix for proof of propriety. In (2.5), $p = 2$.

From the Appendix, we can define the following conditional posterior densities, given $0 \le \rho \le 1$, $\lambda_i = \dfrac{r_i}{r_i + \frac{1-\rho}{\rho}}$ and

$$\hat{\underset{\sim}{\beta}} = \left[\sum_{r=1}^{l}\sum_{j=1}^{r_i}(\underset{\sim}{a}_{ij} - \underset{\sim}{\bar{a}}_i)'(\underset{\sim}{a}_{ij} - \underset{\sim}{\bar{a}}_i) + \sum_{i=1}^{l}\lambda_i(\frac{\rho}{1-\rho})\underset{\sim}{\bar{a}}_i\underset{\sim}{\bar{a}}'_i \right]^{-1} \left[\sum_{r=1}^{l}\sum_{j=1}^{r_i}(b_{ij} - \bar{b}_i)(\underset{\sim}{a}_{ij} - \underset{\sim}{\bar{a}}_i) + \sum_{i=1}^{l}\lambda_i(\frac{\rho}{1-\rho})\bar{b}_i\bar{a}_i \right], \qquad (2.6)$$

$$p(\rho|y) \propto \frac{\prod_{i=1}^{l}\sqrt{\frac{(1-\rho)r_i}{\rho r_i + (1-\rho)}}}{\left[\sum_{r=1}^{l}\sum_{j=1}^{r_i}(y_{ij} - \bar{y}_i - (\underset{\sim}{x}_{ij} - \underset{\sim}{\bar{x}}_i)'\hat{\underset{\sim}{\beta}})^2 + \sum_{i=1}^{l}\frac{(1-\rho)r_i}{\rho r_i + (1-\rho)}\left(\bar{y}_i - \underset{\sim}{\bar{x}}'_i\hat{\underset{\sim}{\beta}}\right)^2 \right]^{\frac{\sum_{i=1}^{l} r_i - p}{2}}}, \qquad (2.7)$$

$$\sigma^{-2} \,|\, \rho, \underset{\sim}{b} \sim Gamma\left\{ \frac{\sum r_i - p}{2}, \sum_{i=1}^{l}\sum_{j=1}^{r_i}\frac{(b_{ij} - \bar{b}_i - (a_{ij} - \bar{a}_i)'\hat{\underset{\sim}{\beta}})^2 + \sum_{i=1}^{l}\lambda_i(\frac{1-\rho}{\rho})(\bar{b}_i - \bar{a}'_i\hat{\underset{\sim}{\beta}})}{2} \right\}, \qquad (2.8)$$

$$\underset{\sim}{\beta} \,|\, \sigma^2, \rho, \underset{\sim}{b} \sim Normal\left\{ \hat{\underset{\sim}{\beta}}, \sigma^2\{\sum_{i=1}^{l}\sum_{j=1}^{r_i}[(a_{ij} - \bar{a}_i)(\tilde{a}_{ij} - \bar{a}_i)' + \lambda_i(\frac{\rho}{1-\rho})\bar{a}_i\bar{a}'_i]\} \right\}, \qquad (2.9)$$

$$\nu_i \,|\, \underset{\sim}{\beta}, \sigma^2, \rho, \underset{\sim}{\tilde{b}} \overset{iid}{\sim} Normal\left\{ \lambda_i(\bar{b}_i - \bar{a}'_i\underset{\sim}{\beta}), \frac{\lambda_i}{\gamma_i}\sigma^2 \right\}. \qquad (2.10)$$

2.1.2 Simulation (composition and grid sampler)

We use independent samples via the composition method to draw parameters $\underset{\sim}{\nu}, \underset{\sim}{\beta}, \sigma^2$, because $p(\underset{\sim}{\nu}, \underset{\sim}{\beta}, \sigma^2, \rho | \underset{\sim}{\tilde{b}}) = p_1(\underset{\sim}{\nu} | \underset{\sim}{\beta}, \sigma^2, \rho, \underset{\sim}{\tilde{b}}) p_2(\underset{\sim}{\beta} | \sigma^2, \rho, \underset{\sim}{\tilde{b}}) p_3(\sigma^2 | \rho, \underset{\sim}{\tilde{b}}) p_4(\rho | \underset{\sim}{\tilde{b}})$.

First we use a grid method to draw a sample ρ from $p_4(\rho | \underset{\sim}{\tilde{b}})$. This is easy to carry out, because $0 \le \rho \le 1$, and the function is easy to compute for such ρ.

Then we use observed data $\underset{\sim}{\tilde{b}}$ and generated samples of ρ to draw σ^2 from (2.7), and with these values of σ^2, ρ, we draw β from (2.8); and with these given values of $\underset{\sim}{\beta}, \sigma^2, \rho$, we draw ν from (2.9). We draw 1000 iterations to get $\underset{\sim}{\nu}^{(h)}, \underset{\sim}{\beta}^{(h)}, \sigma^{2(h)}$, $h = 1, ..., 1000$.

The posterior predictive distribution for a nonresponse and nonsample values, $\tilde{b}_{ij}$ can be written as a mixture, $p(\underset{\sim}{\tilde{b}}_{ns}, \underset{\sim}{\tilde{b}}_{nr} | \underset{\sim}{\tilde{b}}_r) \sim \int\int\int p(\underset{\sim}{\tilde{b}} | \underset{\sim}{\beta}, \underset{\sim}{\nu}, \sigma^2, \underset{\sim}{\tilde{b}}) p(\underset{\sim}{\beta}, \underset{\sim}{\nu}, \sigma^2 | \underset{\sim}{\tilde{b}}) d\underset{\sim}{\beta} d\underset{\sim}{\nu} d\sigma^2$, that is,

$$\underset{\sim}{\tilde{b}}_{ns}, \underset{\sim}{\tilde{b}}_{nr} | \underset{\sim}{\tilde{b}}_r \sim \int\int\int \prod_{i=1}^{L} \prod_{j=r_i+1}^{N_i} \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\left[\frac{\tilde{b}_{ij} - (\beta_0 + \beta_1 \tilde{a}_{ij} + \nu_i)}{2\sigma^2} \right]^2 \right\} p(\underset{\sim}{\beta}, \underset{\sim}{\nu}, \sigma^2 | \underset{\sim}{\tilde{b}}) d\underset{\sim}{\beta} d\underset{\sim}{\nu} d\sigma^2 \qquad (2.11)$$

To draw posterior predictive distribution, we use the result of the previously generated β, ν, σ^2 and then simulate $\tilde{b}_{ij} \sim Normal(\beta_0 + \beta_1 \tilde{a}_{ij} + \nu_i, \sigma^2)$, $i = 1, ..., L, \quad j = r_i + 1, ..., N_i$.

2. 2 Model 2 (modeling height and weight)

Here we describe the regression of log-height vs. log-weight and log-weight vs. log-age.

2.2.1 Model building

The big difference from modeling of BMI is that we directly use height and weight. Note that, $\log(bmi) = \log(\frac{weight}{height^2}) = \log(weight) - 2\log(height)$, so that

$\tilde{b}_{ij} = \tilde{w}_{ij} - 2\tilde{h}_{ij}$, $i = 1,\ldots,L,\ j = r_i + 1,\ldots,N_i$.

The hierarchical model is as below,

$$\tilde{h}_{ij} \sim Normal(\alpha_1 \tilde{w}_{ij} + \nu_{1i}, \sigma_1^2), \qquad i = 1,\ldots,L \quad L = 12, \quad j = 1,\ldots,r_i, \tag{2.12}$$

$$\tilde{w}_{ij} \sim Normal(\gamma_1 \tilde{a}_{ij} + \nu_{2i}, \sigma_2^2), \tag{2.13}$$

$$\underset{\sim}{\nu}_1 \overset{ind}{\sim} Normal(\alpha_0, \frac{1-\rho_1}{\rho_1}\sigma_1^2), \tag{2.14}$$

$$\underset{\sim}{\nu}_2 \overset{ind}{\sim} Normal(\gamma_0, \frac{1-\rho_2}{\rho_2}\sigma_2^2), \tag{2.15}$$

$$\rho_1, \rho_2 \overset{ind}{\sim} Uniform(0,1), \tag{2.16}$$

$$P(\sigma_1^2, \sigma_2^2) \propto \frac{1}{\sigma_1^2}\frac{1}{\sigma_2^2}. \tag{2.17}$$

We write σ_3 and σ_4 as $\sigma_3 = \frac{1-\rho_1}{\rho_1}\sigma_1^2$ and $\sigma_4 = \frac{1-\rho_2}{\rho_2}\sigma_2^2$.

2.2.2 Simulation (Gibbs sampler)

Using Bayes' theorem, the joint posterior density function of all parameters is as follows:

$$p(\alpha_0,\alpha_1,\gamma_0,\gamma_1,\underset{\sim}{v}_1,\underset{\sim}{v}_2,\rho_1,\rho_2,\sigma_1^2,\sigma_2^2 \mid \underset{\sim}{\tilde{h}},\underset{\sim}{\tilde{w}}) \propto$$

$$\frac{1}{\sigma_1^2}\frac{1}{\sigma_2^2}\prod_i\prod_j\left(\frac{1}{\sqrt{2\pi\sigma_1^2}}\exp\{-\frac{1}{2\sigma_1^2}[\tilde{h}_{ij}-(\alpha_0+\alpha_1\tilde{w}_{ij}+v_{1i})]^2\}\right)$$

$$\times\prod_i\prod_j\left(\frac{1}{\sqrt{2\pi\sigma_2^2}}\exp\{-\frac{1}{2\sigma_2^2}[\tilde{w}_{ij}-(\gamma_0+\gamma_1\tilde{a}_{ij}+v_{2i})]\right)$$

$$\times\prod_i\frac{1}{\sqrt{2\pi(\frac{1-\rho_1}{\rho_1})\sigma_1^2}}\exp\left[-\frac{1}{2(\frac{1-\rho_1}{\rho_1})\sigma_1^2}v_{1i}^2\right]$$

$$\times\prod_i\frac{1}{\sqrt{2\pi(\frac{1-\rho_2}{\rho_2})\sigma_2^2}}\exp\left[-\frac{1}{2(\frac{1-\rho_2}{\rho_2})\sigma_2^2}v_{2i}^2\right] \qquad (2.18)$$

To use the Gibbs sampler, we need the conditional posterior density of each parameter given the others. Let $i=1,\ldots,l$, $j=1,\ldots,n_i$,

$$\left(v_{1i}\,\middle|\,\alpha_0,\alpha_1,\gamma_0,\gamma_1,\sigma_1,\sigma_2,\sigma_3,\sigma_4,\underset{\sim}{\tilde{h}},\underset{\sim}{\tilde{w}}\right)\overset{Ind.}{\sim}$$

$$Normal\left\{\frac{\sum_{j=1}^{n_i}(\tilde{h}_{ij}-\alpha_1\tilde{w}_{ij})\sigma_3}{(\sigma_3+\sigma_1/r_i)/r_i}+\alpha_0(1-\frac{\sigma_3\alpha_0}{(\sigma_3+\sigma_1/r_i)}),\frac{1-\frac{\sigma_3}{\sigma_3+\sigma_1/r_i}}{\sigma_3}\right\}, \qquad (2.19)$$

$$\left(\nu_{2i}\left|\alpha_{0,}\alpha_1,\gamma_0,\gamma_1,\sigma_1,\sigma_2,\sigma_3,\sigma_4,\underset{\sim}{\tilde{h}},\underset{\sim}{\tilde{w}}\right.\right)\overset{Ind}{\sim}$$

$$Normal\left\{\frac{\sum_{j=1}^{n_i}(\tilde{w}_{ij}-\gamma_1\tilde{w}_{ij})\sigma_4}{(\sigma_4+\sigma_2/n_i)/r_i}+\gamma_0(1-\frac{\sigma_4\alpha_1}{(\sigma_4+\sigma_2/r_i)}),\frac{1-\frac{\sigma_4}{\sigma_4+\sigma_2/r_i}}{\sigma_4}\right\}, \qquad (2.20)$$

$$\left(\alpha_0\left|\underset{\sim}{\alpha}_1,\underset{\sim}{\nu},\sigma_1,\sigma_2,\sigma_3,\sigma_{4,}\underset{\sim}{\tilde{h}},\underset{\sim}{\tilde{w}}\right.\right)\sim Normal\left\{\left(\sum_{i=1}^{l}\sum_{j=1}^{n_i}\tilde{w}_{ij}\right)^{-1}\sum_{i=1}^{l}\sum_{j=1}^{n_i}(\tilde{h}_{ij}-\nu_{1i}\tilde{w}_{ij}),\frac{\sigma_3}{l}\right\}, \qquad (2.21)$$

$$\left(\alpha_1\left|\underset{\sim}{\gamma},\underset{\sim}{\nu},\sigma_1,\sigma_2,\sigma_3,\sigma_{4,}\underset{\sim}{\tilde{h}},\underset{\sim}{\tilde{w}}\right.\right)\sim Normal\left\{(\sum_{i=1}^{l}\nu_{1i})/L,\quad \sigma_1(\sum_{i=1}^{l}\sum_{j=1}^{r_i}\tilde{w}_{ij})^{-1}\right\}, \qquad (2.22)$$

$$\left(\gamma_0\left|\alpha_0,\alpha_1,\underset{\sim}{\nu}_1,\underset{\sim}{\nu}_2,\sigma_1,\sigma_2,\sigma_3,\sigma_{4,}\underset{\sim}{\tilde{h}},\underset{\sim}{\tilde{w}}\right.\right)\sim Normal\left\{(\sum_{i=1}^{l}\nu_{2i})/l,\sigma_2/l\right\}, \qquad (2.23)$$

$$\left(\gamma_1\left|\alpha_0,\alpha_1,\underset{\sim}{\nu}_1,\underset{\sim}{\nu}_2,\sigma_1,\sigma_2,\sigma_3,\sigma_{4,}\underset{\sim}{\tilde{h}},\underset{\sim}{\tilde{w}}\right.\right)\sim$$

$$Normal\left\{(\sum_{i=1}^{l}\sum_{j=1}^{n_i}\tilde{a}_{ij})^{-1}\sum_{i=1}^{l}\sum_{j=1}^{n_i}(\tilde{w}_{ij}-\nu_{2i}\tilde{a}_{ij}),(\sum_{i=1}^{l}\sum_{j=1}^{n_i}\tilde{a}_{ij})^{-1}\sigma_2\right\}, \qquad (2.24)$$

$$\left(\sigma_1^2\left|\alpha_0,\alpha_1,\gamma_0,\gamma_1,\underset{\sim}{\nu}_1,\underset{\sim}{\nu}_2,\rho_1,\rho_2,\underset{\sim}{\tilde{h}},\underset{\sim}{\tilde{w}}\right.\right)\sim Gamma\left\{(\sum_{i=1}^{l}r_i+l-2)/2,\right.$$

$$\left.\sum_{i=1}^{l}\sum_{j=1}^{r_i}(\tilde{h}_{ij}-\alpha_1\tilde{w}_{ij}-\nu_{1i})^2+\frac{(1-\rho_1)}{2\rho_1}\sum_{i=1}^{l}(\nu_{1i}-\alpha_0)^2\right\}, \qquad (2.25)$$

$$\left(\sigma_2^2\left|\alpha_0,\alpha_1,\gamma_0,\gamma_1,\underset{\sim}{\nu}_1,\underset{\sim}{\nu}_2,\rho_1,\rho_2,\underset{\sim}{\tilde{h}},\underset{\sim}{\tilde{w}}\right.\right)\sim Gamma\left\{(\sum_{i=1}^{l}r_i+l-2)/2,\right.$$

$$\left.\sum_{i=1}^{l}\sum_{j=1}^{r_i}(\tilde{w}_{ij}-\gamma_1\tilde{a}_{ij}-\nu_{2i})^2+\frac{(1-\rho_2)}{2\rho_2}\sum_{i=1}^{l}(\nu_{2i}-\gamma_0)^2\right\}. \qquad (2.26)$$

It is not so easy to draw ρ_1 and ρ_2, since,

$$\left(\rho_1 \middle| \alpha_0, \alpha_1, \gamma_0, \gamma_2, \underset{\sim}{v}_1, \underset{\sim}{v}_2, \underset{\sim}{\tilde{h}}, \underset{\sim}{\tilde{w}}\right) \propto \left(\frac{1-\rho_1}{\rho_1}\right)^{l/2} \exp\left(-\frac{1-\rho_1}{\rho_1}\sum_{i=1}^{L}\frac{v_{1i}^2}{2\sigma_1^2}\right), \tag{2.27}$$

$$\left(\rho_2 \middle| \alpha_0, \alpha_1, \gamma_0, \gamma_2, \underset{\sim}{v}_1, \underset{\sim}{v}_2, \underset{\sim}{\tilde{h}}, \underset{\sim}{\tilde{w}}\right) \propto \left(\frac{1-\rho_2}{\rho_2}\right)^{l/2} \exp\left(-\frac{1-\rho_2}{\rho_2}\sum_{i=1}^{L}\frac{v_{2i}^2}{2\sigma_2^2}\right). \tag{2.28}$$

We use accept-reject-sampling to generate ρ_1 and ρ_2. We first transform ρ_1 to $\delta_1 = \frac{\rho_1}{1-\rho_1}$. Then $\rho_1 = \frac{\delta_1}{1+\delta_1}$, with jacobian, $\frac{d\rho_1}{d\delta_1} = \frac{1}{(1+\delta_1)^2}$, giving

$$\pi(\delta_1 \mid \alpha_0, \alpha_1, \gamma_0, \gamma_2, \underset{\sim}{v}_1, \underset{\sim}{v}_2, \underset{\sim}{\tilde{h}}, \underset{\sim}{\tilde{w}}) \propto \delta_1^{l/2} \exp\left(-\delta_1 \sum_{i=1}^{L}\frac{v_{1i}^2}{2\sigma_1^2}\right)$$

$$= \left(\frac{\delta_1}{1+\delta_1}\right)^2 \delta_1^{(\frac{l-2}{2})-1} \exp\left(-\delta_1 \sum_{i=1}^{L}\frac{v_{1i}^2}{2\sigma_1^2}\right)$$

$$= \left(\frac{\delta_1}{1+\delta_1}\right)^2 \pi_a(\delta_1 \mid \alpha_0, \alpha_1, \gamma_0, \gamma_2, \underset{\sim}{v}_1, \underset{\sim}{v}_2, \underset{\sim}{\tilde{h}}, \underset{\sim}{\tilde{w}}) \tag{2.29}$$

where $\pi_a(\delta_1 \mid \alpha_0, \alpha_1, \gamma_0, \gamma_2, \underset{\sim}{v}_1, \underset{\sim}{v}_2, \underset{\sim}{\tilde{h}}, \underset{\sim}{\tilde{w}}) \sim Gamma\left(\frac{l-2}{2}, \sum_{i=1}^{L}\frac{v_{1i}^2}{2\sigma_1^2}\right)$. (2.30)

We first draw a sample δ_1 from $\delta_1 \sim Gamma\left(\frac{l-2}{2}, \sum_{i=1}^{L}\frac{v_{1i}^2}{2\sigma_1^2}\right)$, and draw $u \sim Uniform(0,1)$. If $u < \delta_1$, take δ_1; otherwise draw another δ_1, and continue until we get $\delta_1 > u$ and use these to get sample ρ_1 using $\rho_1 = \frac{\delta_1}{1+\delta_1}$.

Since conditional posterior density of ρ_2 is similar to ρ_1, we use the same procedure to draw sample ρ_2.

Predicted conditional posterior density of nonresponse height given weight and parameters is,

$$(\tilde{h}_{ij} \left| \alpha_1, \underset{\sim}{\nu}_1, \sigma_1^2, \tilde{w}_{ij}) \sim Normal(\alpha_1 \tilde{w}_{ij} + \nu_{1i}, \sigma_1^2) \quad j = r_i + 1, \ldots, n_i, i = 1, \ldots, L, L = 12 \right. \tag{2.31}$$

$$\tilde{b}_{ij} = \tilde{w}_{ij} - 2\tilde{h}_{ij}, \quad i = 1, \ldots L, \; j = r_i + 1, \ldots, n_i$$

Starting with crude estimates of the mean height, $\overline{h}_i^{(0)}$ and hyperparameters, $\underset{\sim}{\nu}_1^{(0)}, \underset{\sim}{\nu}_2^{(0)}, \alpha_0^{(0)}, \alpha_1^{(0)}, \gamma_0^{(0)}, \gamma_1^{(0)}, \sigma_1^{(0)}, \sigma_2^{(0)}, \rho_1^{(0)}, \rho_2^{(0)}$, we draw $\underset{\sim}{\nu}_1^{(1)}$ from (2.19), then draw $\underset{\sim}{\nu}_2^{(1)}$ from (2.20), then draw $\alpha_0^{(1)}$ from (2.21). Thus, each subvector is updated conditional on the latest values of the other components.

We then draw $\alpha_0^{(1)}, \alpha_1^{(1)}, \gamma_0^{(1)}, \gamma_1^{(1)}, \sigma_1^{(1)}, \sigma_2^{(1)}$ from (2.22), (2.23), (2.24), (2.25) and (2.26) to complete an iteration of the scheme. We repeat the iterations t times, these t iterations simulate a Markov chain which converges to the joint posterior distributions of $\pi\left(\underset{\sim}{\nu}_1^{(t)}, \underset{\sim}{\nu}_2^{(t)}, \alpha_0^{(t)}, \alpha_1^{(t)}, \gamma_0^{(t)}, \gamma_1^{(t)}, \sigma_1^{(t)}, \sigma_2^{(t)}, \rho_1^{(t)}, \rho_2^{(t)}\right)$. In order to diminish the effect of the starting distribution, we draw 11000 iterations, discarding "burn-in" 1000 interactions, and then take every tenth of the remaining 10000 and then make inferences.

The predicted nonresponse BMI values are given by $\tilde{b}_{ij} = \tilde{w}_{ij} - 2\tilde{h}_{ij}$, $i = 1, \ldots L, \; j = r_i + 1, \ldots, n_i$. (We do not need draw nonresponse weight, because we treated weight as known in this study.)

The next step is to generate nonsample values using simulated results.

2.2.3 Predict finite population mean

We use generated the parameters on each iteration $(\underset{\sim}{\nu}_1, \underset{\sim}{\nu}_2, \alpha_0, \alpha_1, \gamma_0, \gamma_1, \sigma_1, \sigma_2, \rho_1, \rho_2)$ and predicted nonresponse $\tilde{h}_{ij}$, $j = r_i + 1, \ldots, n_i$, to draw nonsample values of BMI from their conditional posterior densities,

$$\tilde{w}_{ij} - 2\tilde{h}_{ij} \left| (\underset{\sim}{\nu}_1, \underset{\sim}{\nu}_2, \alpha_0, \alpha_1, \gamma_0, \gamma_1, \sigma_1, \sigma_2) \right. \sim$$

$$Normal\left\{\left[(1-2\alpha_1)(\gamma_1 \tilde{a} + \nu_{2i}) - 2\nu_{1i}\right], 4\sigma_1^2 + \left[\sigma_2^2 (1-2\alpha_1)^2\right]\right\}. \qquad (2.32)$$

The following procedure show how we get the mean and variance of $\tilde{W} - 2\tilde{H}$.

Since, $\tilde{H} \left| \tilde{w} \right. \sim Normal(\alpha_1 \tilde{w} + \nu_1, \sigma_1^2)$ and $\tilde{W} \left| \tilde{a} \right. \sim Normal(\gamma_1 \tilde{a} + \nu_2, \sigma_2^2)$,

thus, $$E\left[w - 2h\right] = E_w E(w - 2h \left| w \right.)$$

$$= E\left[w - 2E(H \left| w \right.)\right]$$

$$= E\left[(1-2\alpha_1) w - 2\nu_1\right]$$

$$= (1-2\alpha_1)(\gamma_1 a + \nu_2) - 2\nu_1, \qquad (2.33)$$

and $$Var(W - 2H) = E_w Var(W - 2H \left| w \right.) + Var_w \left\{ E\left[W - 2H \left| w \right. \right] \left| w \right. \right\}$$

$$= 4E_w (H \left| w \right.) + Var_w (w - 2E(H \left| w \right.))$$

$$= 4\sigma_1^2 + (1-2\alpha_1)^2 \sigma_2^2. \qquad (2.34)$$

We draw $\tilde{b}_{ij} = \tilde{w}_{ij} - 2\tilde{h}_{ij}$, $i = 1, \ldots, L \quad j = n_i + 1, \ldots, N_i$ from (2.32)

Finally, we calculate the finite population mean of BMI as follows,

$$\bar{b}_i = \frac{\sum_{j=1}^{r_i} \tilde{b}_{ij(response)} + \sum_{j=r_i+1}^{n_i} \tilde{b}_{ij(nonresponse)} + \sum_{j=n_i+1}^{N_i} \tilde{b}_{ij(nonsample)}}{N_i} \qquad (2.35)$$

2.3 Data analysis

In table 2.1, under Model 1 (modeling BMI), the estimated coefficient β_1 =0.03161, and the 95% credible interval does not contains zero, which suggests that the mean of BMI is slightly positively associated with age.

Table 2.1 Inference on regression coefficients on ignorable nonresponse Model 1 (modeling BMI)

Coef.	AVG.	STD	95% CI
β_0	2.74167	0.01543	(2.71148, 2.77178)
β_1	0.03161	0.00984	(0.01197, 0.05064)
σ	0.11252	0.00216	(0.10830, 0.11690)
δ	0.00032	0.00025	(0.00001, 0.00094)
ρ	0.00283	0.00222	(0.00013, 0.00825)

In table 2.2, under Model 2 (modeling height and weight), the coefficient of $\alpha_1 = 0.0286$, and the 95% credible interval does not contain zero, which indicates that weight is related with height. Also $\gamma_1 = 0.39964$ implies a positive association of weight and age.

Table 2.2 Inference on regression coefficients on ignorable nonresponse Model 2 (modeling height and weight)

Coef.	AVG.	STD	95% CI
α_0	0.02629	0.01203	(0.00246, 0.05285)
α_1	0.02860	0.00188	(0.02443, 0.03199)
γ_0	2.21641	0.11303	(2.00567, 2.44849)
γ_1	0.39964	0.01084	(0.37930, 0.42168)
σ_1	0.02397	0.00120	(0.02161, 0.02632)
σ_2	0.16002	0.00281	(0.15450, 0.16594)
σ_3	0.00140	0.00083	(0.00046, 0.00383)
σ_4	4.88265	2.00713	(2.28852, 9.67768)

Figure 2.1 show plots of sampled parameters $\alpha_0, \alpha_1, \gamma_1, \gamma_2, \sigma_1, \sigma_2, \sigma_3, \sigma_4$ versus iterations, which shows of Gibbs sampler is stable.

Figure 2.1 Plots of the parameters vs. iteration of Gibbs sampler (cont.)

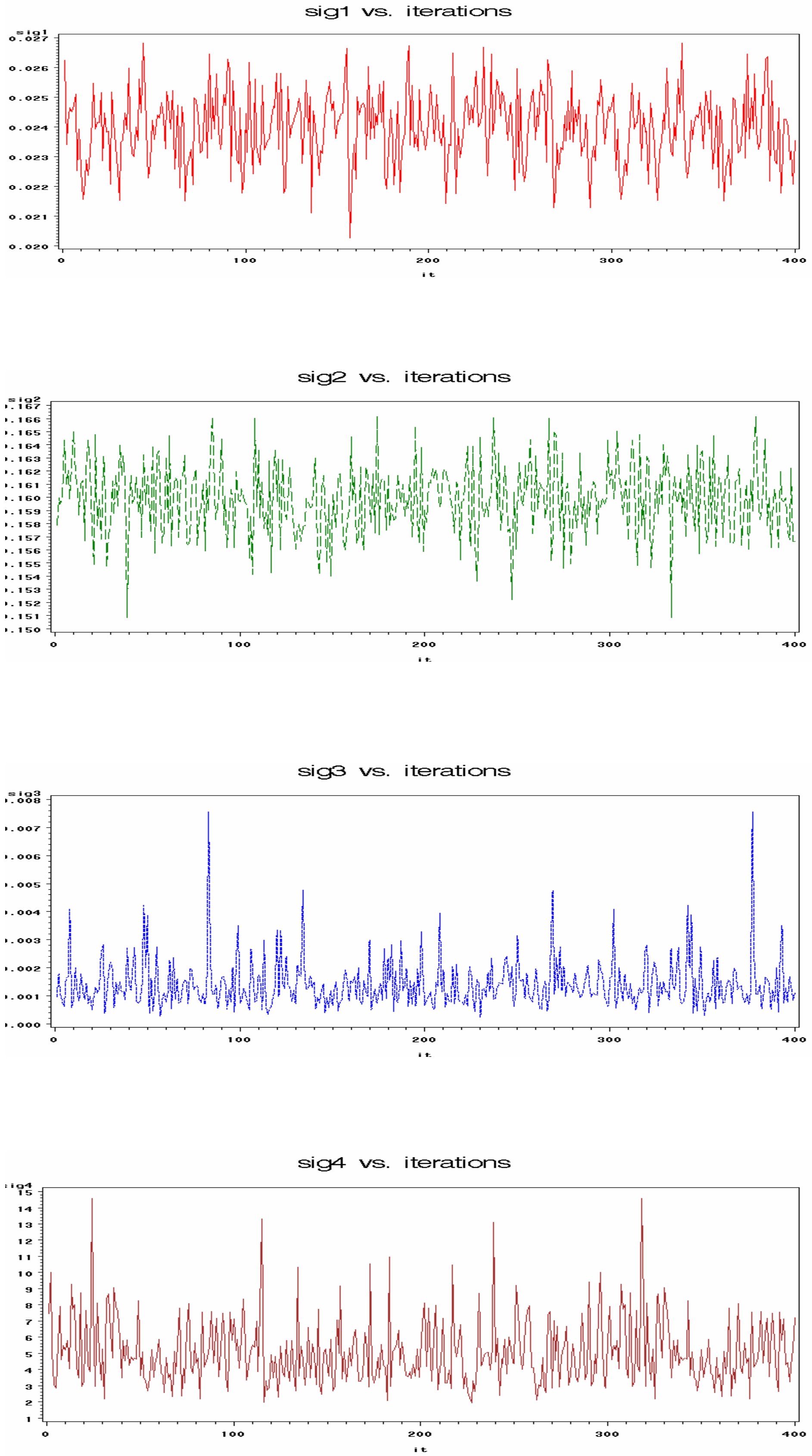
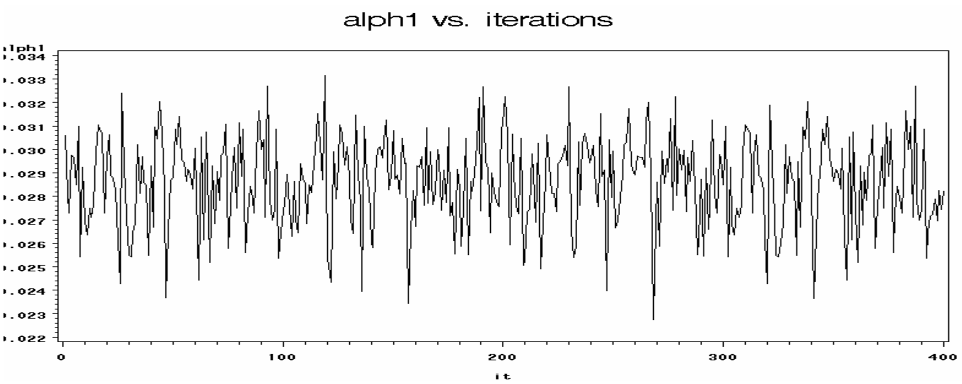
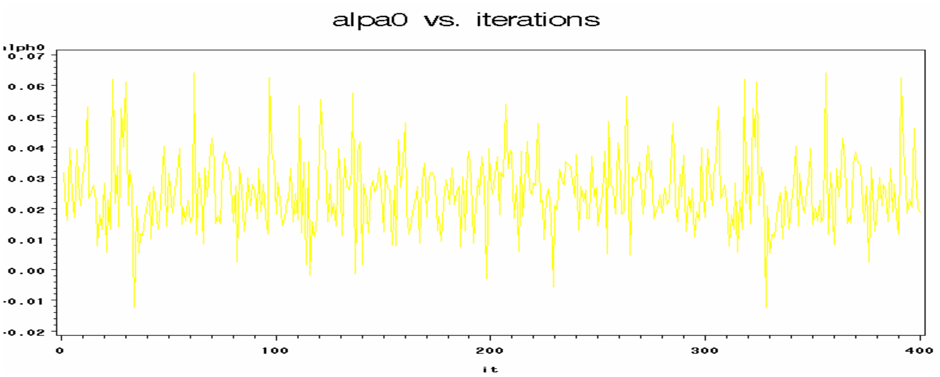
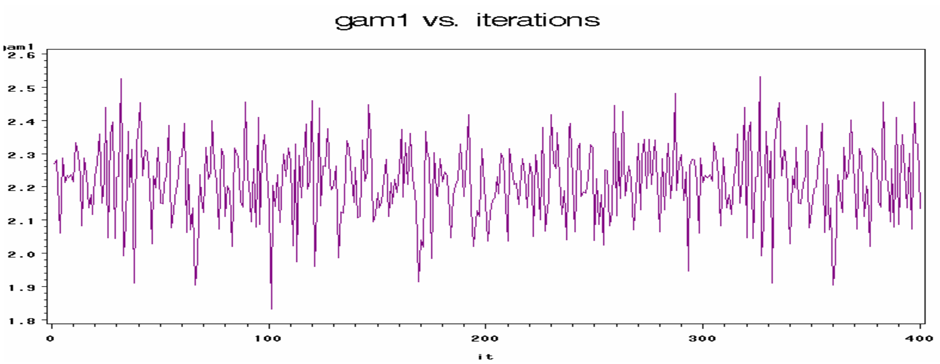
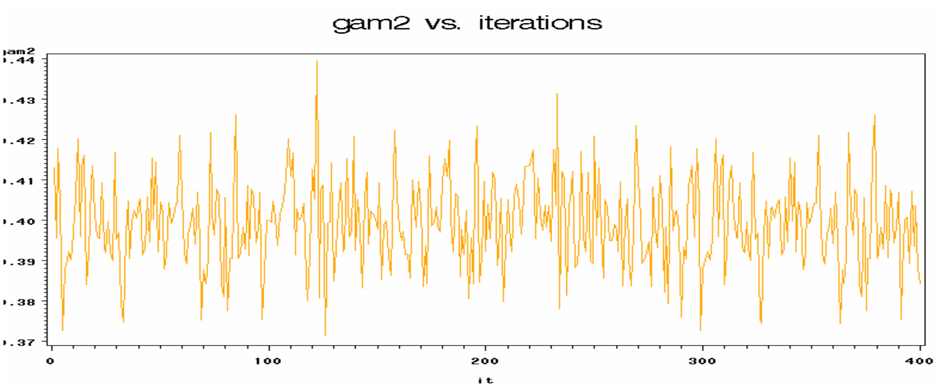


Figure 2.1 Plots of the parameters vs. iteration of Gibbs sampler









We also check the independence of sampled parameters. In Table 2.4, the autocorrelations of parameters are very small except for α_1 and γ_1 ($\alpha_1 = 0.396$ and $\gamma_1 = 0.352$). The procedure might benefit from longer runs and longer gaps.

Table 2.3 Autocorrelation of parameters

Par.		ACF	Str	Par.		ACF	Str	Par.		ACF	Str	Par.		ACF	Str
1	1	0.047	0.032	3	1	-0.037	0.032	5	1	0.317	0.032	7	1	0.000	0.032
1	2	-0.060	0.032	3	2	-0.082	0.032	5	2	0.014	0.032	7	2	-0.060	0.032
1	3	0.023	0.032	3	3	-0.028	0.032	5	3	0.075	0.032	7	3	0.069	0.032
1	4	0.041	0.032	3	4	-0.067	0.032	5	4	0.155	0.032	7	4	-0.130	0.032
1	5	0.116	0.032	3	5	-0.096	0.032	5	5	0.039	0.032	7	5	0.049	0.032
1	6	0.061	0.032	3	6	-0.105	0.032	5	6	-0.020	0.032	7	6	0.125	0.032
1	7	-0.080	0.032	3	7	-0.076	0.032	5	7	-0.060	0.032	7	7	-0.120	0.032
1	8	-0.080	0.032	3	8	0.147	0.032	5	8	-0.080	0.032	7	8	0.070	0.032
1	9	0.000	0.031	3	9	0.001	0.031	5	9	-0.030	0.031	7	9	0.054	0.031
1	10	-0.030	0.031	3	10	0.041	0.031	5	10	0.076	0.031	7	10	-0.120	0.031
1	11	0.019	0.031	3	11	0.095	0.031	5	11	-0.040	0.031	7	11	-0.060	0.031
1	12	-0.090	0.031	3	12	0.041	0.031	5	12	-0.200	0.031	7	12	-0.020	0.031
1	13	0.013	0.031	3	13	0.039	0.031	5	13	-0.050	0.031	7	13	-0.120	0.031
1	14	-0.070	0.031	3	14	-0.068	0.031	5	14	-0.030	0.031	7	14	0.110	0.031
1	15	-0.020	0.031	3	15	-0.065	0.031	5	15	0.004	0.031	7	15	-0.030	0.031
1	16	0.027	0.031	3	16	0.024	0.031	5	16	-0.120	0.031	7	16	-0.180	0.031
1	17	0.045	0.031	3	17	-0.087	0.031	5	17	-0.130	0.031	7	17	0.050	0.031
1	18	0.023	0.031	3	18	0.005	0.031	5	18	-0.030	0.031	7	18	-0.100	0.031
1	19	-0.100	0.031	3	19	0.037	0.031	5	19	0.088	0.031	7	19	-0.030	0.031
1	20	0.016	0.031	3	20	0.022	0.031	5	20	-0.010	0.031	7	20	0.031	0.031
2	1	0.396	0.032	4	1	0.352	0.032	6	1	-0.060	0.032	8	1	-0.040	0.032
2	2	0.125	0.032	4	2	0.075	0.032	6	2	0.043	0.032	8	2	-0.020	0.032
2	3	0.021	0.032	4	3	0.001	0.032	6	3	-0.030	0.032	8	3	0.085	0.032
2	4	0.099	0.032	4	4	-0.044	0.032	6	4	-0.020	0.032	8	4	0.013	0.032
2	5	0.127	0.032	4	5	-0.048	0.032	6	5	-0.170	0.032	8	5	-0.140	0.032
2	6	0.021	0.032	4	6	0.020	0.032	6	6	0.013	0.032	8	6	-0.020	0.032
2	7	-0.080	0.032	4	7	-0.025	0.032	6	7	-0.100	0.032	8	7	0.040	0.032
2	8	-0.060	0.032	4	8	-0.031	0.032	6	8	-0.060	0.032	8	8	0.042	0.032
2	9	-0.050	0.031	4	9	-0.117	0.031	6	9	-0.050	0.031	8	9	0.036	0.031
2	10	0.000	0.031	4	10	-0.129	0.031	6	10	0.087	0.031	8	10	-0.020	0.031
2	11	-0.010	0.031	4	11	-0.177	0.031	6	11	-0.100	0.031	8	11	0.029	0.031
2	12	-0.110	0.031	4	12	-0.154	0.031	6	12	0.069	0.031	8	12	-0.010	0.031
2	13	-0.100	0.031	4	13	-0.024	0.031	6	13	-0.020	0.031	8	13	0.048	0.031
2	14	-0.150	0.031	4	14	0.047	0.031	6	14	0.004	0.031	8	14	-0.020	0.031
2	15	-0.010	0.031	4	15	-0.019	0.031	6	15	-0.090	0.031	8	15	0.081	0.031
2	16	-0.060	0.031	4	16	-0.015	0.031	6	16	0.035	0.031	8	16	-0.040	0.031
2	17	-0.090	0.031	4	17	-0.005	0.031	6	17	0.001	0.031	8	17	0.024	0.031
2	18	-0.070	0.031	4	18	-0.011	0.031	6	18	0.136	0.031	8	18	0.066	0.031
2	19	-0.060	0.031	4	19	0.105	0.031	6	19	0.093	0.031	8	19	-0.070	0.031
2	20	-0.010	0.031	4	20	0.192	0.031	6	20	0.010	0.031	8	20	0.008	0.031

Note: in Table 2.4, Par.1,2,…,8 represent α_0, α_1, γ_0, γ_1, σ_1, σ_2, σ_3 and σ_4, respectively. Table 2.4 shows the posterior mean of BMI have the similar values on all age groups in Model 1(modeling BMI); while posterior mean of BMI on Model 2 (modeling height and weight) increases when age increases. Therefore, we say Model 2 is more reasonable than Model 1, because the mean of BMI is expected to increase with age among children.

Table 2.4: Comparison of posterior means of ignorable nonresponse models (cont.)

Age	Domain	Model 1				Model 2			
		PM	PSD	95%	CI	PM	PSD	95%	CI
2	WML	16.842	0.264	(16.353,	17.366)	10.932	0.328	(10.300	11.604)
	WMM	15.715	0.348	(15.188,	16.581)	10.810	0.172	(10.492	11.186)
	WMH	16.721	0.238	(16.228,	17.184)	9.809	0.233	(9.306	10.244)
	WFL	16.891	0.247	(16.418,	17.389)	11.552	0.333	10.851	12.206
	WFM	15.124	0.486	(14.475,	16.462)	10.688	0.161	10.370	10.990
	WFH	16.637	0.243	(16.152,	17.117)	9.875	0.263	9.300	10.358
	NML	16.737	0.239	(16.244,	17.190)	11.952	0.320	11.323	12.621
	NMM	16.722	0.211	(16.306,	17.128)	12.192	0.241	11.722	12.616
	NMH	16.733	0.282	(16.163,	17.263)	11.648	0.491	10.822	12.814
	NFL	16.713	0.259	(16.170,	17.240)	11.655	0.362	11.030	12.368
	NFM	16.793	0.211	(16.371,	17.215)	12.312	0.263	11.887	12.872
	NMH	16.721	0.291	(16.125,	17.279)	11.419	0.511	10.436	12.672
3	WML	17.058	0.245	(16.601,	17.557)	12.732	0.380	12.032	13.494
	WMM	15.913	0.340	(15.405,	16.778)	12.561	0.188	12.186	12.953
	WMH	16.936	0.216	(16.484,	17.361	11.411	0.260	10.841	11.937
	WFL	17.108	0.230	(16.689,	17.579)	13.450	0.372	12.669	14.134
	WFM	15.312	0.483	(14.689,	16.644)	12.426	0.176	12.089	12.753
	WFH	16.850	0.220	(16.375,	17.264)	11.495	0.294	10.847	12.029
	NML	16.952	0.218	(16.507,	17.388)	13.921	0.358	13.210	14.626
	NMM	16.937	0.188	(16.565,	17.294)	14.200	0.267	13.685	14.676
	NMH	16.948	0.264	(16.377,	17.455)	13.567	0.566	12.614	14.861
	NFL	16.927	0.241	(16.411,	17.424)	13.575	0.410	12.899	14.362
	NFM	17.009	0.187	(16.641,	17.375)	14.341	0.293	13.877	14.980
	NMH	16.936	0.279	(16.357,	17.458)	13.301	0.593	12.132	14.750
4	WML	17.213	0.243	16.753	17.703	14.184	0.427	13.385	15.034
	WMM	16.057	0.339	15.53	16.956	13.991	0.207	13.598	14.419
	WMH	17.089	0.211	16.669	17.494	12.711	0.29	12.092	13.312
	WFL	17.264	0.23	16.842	17.741	14.983	0.408	14.129	15.714
	WFM	15.45	0.484	14.833	16.765	13.831	0.2	13.443	14.201
	WFH	17.003	0.218	16.534	17.415	12.804	0.323	12.132	13.402
	NML	17.106	0.215	16.653	17.548	15.512	0.394	14.717	16.265
	NMM	17.091	0.185	16.722	17.466	15.821	0.297	15.261	16.425
	NMH	17.103	0.262	16.542	17.609	15.119	0.63	14.048	16.521
	NFL	17.082	0.239	16.59	17.566	15.127	0.453	14.376	16.007
	NFM	17.164	0.184	16.802	17.522	15.980	0.324	15.449	16.685
	NMH	17.091	0.282	16.502	17.628	14.822	0.661	13.579	16.416

Table 2.4: Comparison of posterior means of ignorable nonresponse models (cont.)

Age	Domain	Model 1				Model 2			
		PM	PSD	95%	CI	PM	PSD	95%	CI
5	WML	17.335	0.249	16.843	17.829	15.424	0.469	14.570	16.390
	WMM	16.170	0.343	15.645	17.047	15.203	0.231	14.764	15.663
	WMH	17.209	0.216	16.768	17.631	13.822	0.316	13.155	14.467
	WFL	17.384	0.236	16.954	17.890	16.294	0.442	15.411	17.086
	WFM	15.560	0.491	14.925	16.864	15.033	0.224	14.587	15.458
	WFH	17.123	0.223	16.675	17.553	13.923	0.348	13.229	14.573
	NML	17.228	0.222	16.753	17.665	16.870	0.428	16.000	17.739
	NMM	17.212	0.192	16.837	17.608	17.207	0.326	16.572	17.922
	NMH	17.224	0.266	16.646	17.737	16.443	0.687	15.233	17.940
	NFL	17.202	0.245	16.683	17.712	16.451	0.493	15.631	17.489
	NFM	17.285	0.190	16.900	17.650	17.379	0.355	16.760	18.107
	NMH	17.212	0.289	16.605	17.781	16.120	0.723	14.739	17.848
6	WML	17.436	0.257	16.919	17.953	16.519	0.509	15.616	17.566
	WMM	16.263	0.351	15.718	17.165	16.273	0.258	15.770	16.763
	WMH	17.311	0.226	16.861	17.755	14.800	0.342	14.084	15.497
	WFL	17.487	0.248	17.032	18.014	17.450	0.473	16.508	18.302
	WFM	15.647	0.496	15.005	16.966	16.095	0.251	15.623	16.579
	WFH	17.223	0.232	16.749	17.669	14.907	0.373	14.156	15.565
	NML	17.327	0.232	16.833	17.784	18.068	0.461	17.141	19.017
	NMM	17.311	0.204	16.913	17.736	18.429	0.357	17.720	19.239
	NMH	17.323	0.274	16.725	17.836	17.610	0.739	16.262	19.176
	NFL	17.302	0.253	16.787	17.819	17.620	0.530	16.731	18.764
	NFM	17.385	0.201	16.977	17.771	18.613	0.388	17.891	19.391
	NMH	17.311	0.299	16.686	17.914	17.265	0.778	15.746	19.099
7	WML	17.522	0.270	16.957	18.056	17.506	0.549	16.535	18.638
	WMM	16.343	0.360	15.786	17.264	17.246	0.283	16.691	17.745
	WMH	17.396	0.236	16.913	17.872	15.681	0.369	14.934	16.448
	WFL	17.572	0.261	17.087	18.112	18.494	0.503	17.451	19.400
	WFM	15.726	0.502	15.070	17.050	17.056	0.276	16.531	17.599
	WFH	17.308	0.243	16.815	17.767	15.799	0.399	14.999	16.547
	NML	17.412	0.242	16.908	17.899	19.148	0.493	18.175	20.108
	NMM	17.396	0.215	16.990	17.853	19.530	0.387	18.748	20.443
	NMH	17.408	0.283	16.815	17.947	18.662	0.788	17.189	20.302
	NFL	17.387	0.263	16.829	17.888	18.673	0.566	17.714	19.940
	NFM	17.470	0.214	17.046	17.873	19.725	0.418	18.924	20.568
	NMH	17.396	0.310	16.752	18.028	18.297	0.829	16.662	20.235

Table 2.4: Comparison of posterior means of ignorable nonresponse models

Age	Domain	Model 1				Model 2			
		PM	PSD	95%	CI	PM	PSD	95%	CI
8	WML	17.595	0.278	17.028	18.153	18.407	0.584	17.368	19.631
	WMM	16.416	0.366	15.824	17.347	18.138	0.307	17.519	18.703
	WMH	17.471	0.248	16.943	17.978	16.495	0.394	15.693	17.328
	WFL	17.648	0.270	17.170	18.212	19.446	0.533	18.340	20.382
	WFM	15.794	0.509	15.118	17.123	17.931	0.304	17.358	18.519
	WFH	17.382	0.255	16.879	17.849	16.609	0.424	15.721	17.426
	NML	17.486	0.254	16.971	17.996	20.135	0.525	19.110	21.126
	NMM	17.470	0.228	17.045	17.938	20.536	0.419	19.680	21.529
	NMH	17.482	0.293	16.890	18.048	19.625	0.833	18.062	21.328
	NFL	17.460	0.274	16.886	17.975	19.635	0.599	18.616	21.008
	NFM	17.545	0.228	17.096	17.972	20.741	0.449	19.872	21.636
	NMH	17.470	0.320	16.809	18.118	19.239	0.877	17.471	21.265

Only looking at ignorable nonresponse Model 2 (modeling height and weight) on Table 2.4, there are small differences in the posterior means of BMI among most domains, except in domains WMH, WFH and NFM. Domains WMH and WFH contain the smallest posterior means of BMI, which may indicate that "white" and "high family income" children have small BMI. Meanwhile, domain NFM carries the largest posterior means of BMI, which implies that non-white girls, living on middle income families, have the big BMI. All these results lead to the conclusion that the 12 combinations of race, sex and family income influence the finite population BMI.

2.4 Conclusion

From these results, we therefore conclude that Model 2 (modeling height and weight) is more reasonable than model 1 (modeling BMI), because posterior means of BMI for Model 1 do not follow our belief about the data. That is, for children, BMI should increase as age increases. Furthermore, the finite population mean is slightly influenced by different combination of family income, sex and race, especially by race and sex of high level income family.

Chapter 3: Nonignorable nonresponse models

An important distinction between models for nonresponse is the difference between ignorable and nonignorable models.

Let x denote covariates and y the response variable, Little and Rubin (2002, Sec. 1.3) describe three types of missing-data mechanism. These types differ according to whether the probability of response (a) is independent of x and y (b) depends on x but not on y and (c) depends on y and possible x. The missing data are missing completely at random (MCAR) in (a), missing at random (MAR) in (b), and missing not at random (NMAR) in (c). Models for MCAR and MAR missing-data mechanisms are called ignorable (if the parameters of the dependent variable and response are distinct (Rubin 1976)). Models for MNAR missing-data mechanism are called nonignorable. See also Nandram, B. (2006).

It is important that the parameters of the dependent variable and response are distinct (Rubin 1976). We construct nonignorable nonresponse models with response indicators having the same parameters as in the ignorable nonresponse. Our idea is to construct the nonignorable nonresponse models by centering them on their ignorable counterpart.

3.1 Model fitting

We extend the two models in Chapter 2. Essentially, the samples obtained in Chapter 2 are subsampled to get samples from the new posterior densities using the sample importance resampling (SIR) algorithm.

3.1.1 Model 1 (modeling BMI):

Let $\tilde{b}_{ij}$ be the BMI of j^{th} individual in the i^{th} domain, $\tilde{a}_{ij}$ is the corresponding age, $i = 1,..., L,\ j = 1,..., N_i$. Let r_{ij} be the response indicator given by

$$r_{ij} = \begin{cases} 1, \text{ if the j}^{\text{th}} \text{ individual responds} \\ 0, otherwise \end{cases}, \quad i = 1,..., L,\ j = 1,..., n_i \,.$$

We define the nonignorable nonresponse model with covariate age as

$$r_{ij} \sim Bernoulli\left(\frac{e^{\beta_0 + \beta_1 \tilde{a}_{ij} + \nu_i}}{1 + e^{\beta_0 + \beta_1 \tilde{a}_{ij} + \nu_i}}\right), \tag{3.1}$$

$$\tilde{b}_{ij} \sim Normal(\beta_0 + \beta_1 \tilde{a}_{ij} + \nu_i, \sigma^2), \tag{2.1}$$

$$\nu_i \overset{ind}{\sim} Normal(0, \frac{1-\rho}{\rho}\sigma^2). \tag{2.2}$$

We use non-informative priors on the hyperparameters

$$p(\sigma^2) \propto \frac{1}{\sigma^2}, \tag{2.3}$$

$$\rho \sim Uniform(0,1). \tag{2.4}$$

Note that formulations of (2.1)—(2.4) are the same as those in the ignorable nonresponse models of Chapter 2. Here the response indicators r_{ij} are not related to the BMI values. However, the key idea here is to use the same parameters of the ignorable nonresponse models,

$$r_{ij} \middle| \tilde{b}_{ij} \sim Bernouli\left(\frac{e^{\beta_0 + \beta_1 \tilde{a}_{ij} + \nu_i}}{1 + e^{\beta_0 + \beta_1 \tilde{a}_{ij} + \nu_i}}\right).$$

Let $p_{ij} = \frac{e^{\beta_0 + \beta_1 \tilde{a}_{ij} + \nu_i}}{1 + e^{\beta_0 + \beta_1 \tilde{a}_{ij} + \nu_i}}$ denote the propensity scores. Thus,

$$\log(\frac{p_{ij}}{1-p_{ij}}) = \beta_0 + \beta_1 \tilde{a}_{ij} + v_i$$

That is to say, $\tilde{b}_{ij} \sim Normal\left\{\log(\frac{p_{ij}}{1-p_{ij}}), \sigma^2\right\}$; so that the mean of $\tilde{b}_{ij}$ is the logit of the propensity scores.

The joint posterior density is described below,

Let $i = 1,...,l$, $j = 1,...,n_i$, $\Omega = (\underset{\sim}{\beta}, \underset{\sim}{v}, \rho, \sigma^2)$

$$p(\Omega \mid \underset{\sim}{b}) = A(\Omega)\pi(\Omega \mid \underset{\sim}{b}) \text{, where} \tag{3.2}$$

$$A(\Omega) = \prod_i \prod_j \frac{\left[e^{\beta_0 + \beta_1 \tilde{a}_{ij} + v_i}\right]^{r_{ij}}}{1 + e^{\beta_0 + \beta_1 \tilde{a}_{ij} + v_i}}, \tag{3.3}$$

and $\pi(\Omega \mid \underset{\sim}{b}) = \frac{1}{\sigma^2} \prod_i \left[\prod_j \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{1}{2\sigma^2}[\tilde{b}_{ij} - (\beta_0 + \beta_1 \tilde{a}_{ij} + v_i)]^2\} \right]$

$$\times \prod_i \frac{1}{\sqrt{2\pi(\frac{1-\rho}{\rho})\sigma^2}} \exp(-\frac{1-\rho}{2\rho\sigma^2} v_i^2) \; . \tag{3.4}$$

In Chapter 2, we show how to get sample from the posterior density $\pi(\Omega^{(h)}), h = 1,...,M$. We will show how to get a sample from $p(\Omega \mid \underset{\sim}{b})$ using the sampling importance resampling (SIR) algorithm.

3.1.2 Model 2 (modeling height and weight)

Recall the conditional distribution of $\tilde{W}\text{-}2\tilde{H}$ in Chapter 2,

$$\tilde{w}_{ij} - 2\tilde{h}_{ij} \left| (\underset{\sim}{\nu}_1, \underset{\sim}{\nu}_2, \alpha_0, \alpha_1, \gamma_0, \gamma_1, \sigma_1, \sigma_2) \right. \sim$$

$$Normal\left\{ \left[(1-2\alpha_1)(\gamma_1 \tilde{a}_{ij} + \nu_{2i}) - 2\nu_{1i} \right], 4\sigma_1^2 + \left[\sigma_2^2 (1-2\alpha_1)^2 \right] \right\}, \quad (2.34)$$

where, $\tilde{b}_{ij} = \tilde{w}_{ij} - 2\tilde{h}_{ij}$.

We define the nonignorable nonresponse Model 2 (modeling height and weight) as below,

$$r_{ij} \sim Bernouli\left\{ \frac{e^{(1-2\alpha_1)(\gamma_1 \tilde{a} + \nu_{2i}) - 2\nu_{1i}}}{1 + e^{(1-2\alpha_1)(\gamma_1 \tilde{a} + \nu_{2i}) - 2\nu_{1i}}} \right\}, \quad (3.\ 5)$$

$$\tilde{h}_{ij} \left| \tilde{w}_{ij} \right. \sim Normal(\alpha_1 \tilde{w}_{ij} + \nu_{1i}, \sigma_1^2) \qquad i = 1, ..., L, \quad j = 1, ..., N_i, \quad (2.12)$$

$$\tilde{w}_{ij} \left| \tilde{a}_{ij} \right. \sim Normal(\gamma_1 \tilde{a}_{ij} + \nu_{2i}, \sigma_2^2), \quad (2.13)$$

$$\nu_{1i} \overset{iid}{\sim} Normal(\alpha_0, \frac{1-\rho_1}{\rho_1} \sigma_1^2), \quad (2.14)$$

$$\nu_{2i} \overset{iid}{\sim} Normal(\gamma_0, \frac{1-\rho_2}{\rho_2} \sigma_2^2), \quad (2.15)$$

$$\rho_1, \rho_2 \overset{iid}{\sim} Uniform(0,1), \quad (2.16)$$

$$P(\sigma_1^2, \sigma_2^2) \propto \frac{1}{\sigma_1^2} \frac{1}{\sigma_2^2}. \quad (2.17)$$

Here the response indicators r_{ij} are not related to the height and weight. However since $\tilde{b}_{ij} = \tilde{w}_{ij} - 2\tilde{h}_{ij}$, the response indicator density uses the same parameters of ignorable nonresponse density of height and weight. Similarly, in Section 3.1.1, the response indicators r_{ij} of nonignorable nonresponse Model 1 (modeling BMI), are also not related to the BMI. However, we use the same parameters of the corresponding ignorable nonresponse model.

The joint density of nonignorable nonresponse Model 2 given the data is as follows.

Let $i = 1,...,l$, $j = 1,...,n_i$, $\Omega = (\alpha_{0,}\alpha_1, \gamma_0, \gamma_1, \underset{\sim}{\nu}_1, \underset{\sim}{\nu}_2, \rho_1, \rho_2, \sigma_1^{\ 2}, \sigma_2^2)$,

$p(\Omega \mid \underset{\sim}{\tilde{h}}, \underset{\sim}{\tilde{w}}) = A(\Omega)\pi(\Omega \big| \underset{\sim}{\tilde{h}}, \underset{\sim}{\tilde{w}})$, where

$$A(\Omega) = \prod_i \prod_j \frac{e^{\left[(1-2\alpha_1)(\gamma_1 \tilde{a} + \nu_{2i}) - 2\nu_{1i}\right]^{r_{ij}}}}{1 + e^{(1-2\alpha_1)(\gamma_1 \tilde{a} + \nu_{2i}) - 2\nu_{1i}}}, \tag{3.6}$$

$$\pi(\Omega \big| \underset{\sim}{\tilde{h}}, \underset{\sim}{\tilde{w}}) = \frac{1}{\sigma_1^2} \frac{1}{\sigma_2^2} \prod_i \prod_j \left(\frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\{-\frac{1}{2\sigma_1^2}[\tilde{h}_{ij} - (\alpha_0 + \alpha_1 \tilde{w}_{ij} + \nu_{1i})]^2\} \right)$$
$$\times \prod_i \prod_j \left(\frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\{-\frac{1}{2\sigma_2^2}[\tilde{w}_{ij} - (\gamma_0 + \gamma_1 \tilde{a}_{ij} + \nu_{2i})] \right)$$
$$\times \prod_i \frac{1}{\sqrt{2\pi(\frac{1-\rho_1}{\rho_1})\sigma_1^{\ 2}}} \exp\left[-\frac{1}{2(\frac{1-\rho_1}{\rho_1})\sigma_1^2} \nu_{1i}^2 \right]$$
$$\times \prod_i \frac{1}{\sqrt{2\pi(\frac{1-\rho_2}{\rho_2})\sigma_2^{\ 2}}} \exp\left[-\frac{1}{2(\frac{1-\rho_2}{\rho_2})\sigma_2^2} \nu_{2i}^2 \right]. \tag{3.7}$$

Here, we see $\pi(\Omega \big| \underset{\sim}{\tilde{h}}, \underset{\sim}{\tilde{w}})$ is the joint density of the ignorable nonresponse model in Chapter 2, but it has different form from the ignorable nonresponse model, since in the model above, $\pi(\Omega \big| \underset{\sim}{\tilde{h}}, \underset{\sim}{\tilde{w}})$ is constrained by response indicator r_{ij} . Meanwhile, the distributions of α_1 and γ_1 are the factors for r_{ij} . This behavior suggested that nonsample values are affected by the response indicator r_{ij} .

Furthermore, note that $p(\Omega \mid \underset{\sim}{\tilde{h}}, \underset{\sim}{\tilde{w}})$ is very complex, thus again we use the sampling importance resampling (SIR) to get sample from $p(\Omega \mid \underset{\sim}{\tilde{h}}, \underset{\sim}{\tilde{w}})$.

3.2 Sample Importance Resampling (SIR) algorithm

We describe now the basic idea of SIR algorithm. Suppose we have a random sample $\{\underset{\sim}{x}_1,\ldots,\underset{\sim}{x}_n\}$ from $h(\underset{\sim}{x})$, we want to get a random sample from $f(\underset{\sim}{x})$. We can draw the sample $\{\underset{\sim}{x}_1,\ldots,\underset{\sim}{x}_n\}$ by using the sample importance resampling (SIR) algorithm. Let $w_i = \dfrac{f(\underset{\sim}{x}_i)/h(\underset{\sim}{x}_i)}{\sum_{i=1}^{n}\left[f(\underset{\sim}{x}_i)/h(\underset{\sim}{x}_i)\right]}, i=1,\ldots,n$. So we have $\{w_1,\ldots,w_n\}$, we can draw a sample with probability w_i without replacement (Gelman, Stern, Carlin, Rubin, 2002). We draw a subsample of size 10,000 from $\pi(\Omega \mid \underset{\sim}{\tilde{h}}, \underset{\sim}{\tilde{w}})$ in Chapter 2 with importance weights,

$$w^{(h)} = \frac{A(\Omega^{(h)})}{\sum_{h=1}^{M} A(\Omega^{(h)})}, h=1,\ldots,M$$, so we have $\{ (\Omega^{(h)}, w^{(h)}),\ h=1,\ldots,M \}$

For Model 1(modeling BMI),

$$w^{(h)} = \frac{\left\{\prod_i \prod_j \frac{\left[e^{\beta_0+\beta_1\tilde{a}_{ij}+v_i}\right]^{r_{ij}}}{1+e^{\beta_0+\beta_1\tilde{a}_{ij}+v_i}}\right\}^{(h)}}{\sum_{h=1}^{M}\left\{\prod_i \prod_j \frac{\left[e^{\beta_0+\beta_1\tilde{a}_{ij}+v_i}\right]^{r_{ij}}}{1+e^{\beta_0+\beta_1\tilde{a}_{ij}+v_i}}\right\}^{(h)}}, h=1,\ldots,M. \tag{3.8}$$

and for Model 2 (modeling height and weight),

$$w^{(h)} = \frac{\left\{\prod_i \prod_j \frac{e^{\left[(1-2\alpha_1)(\gamma_1\tilde{a}+v_{2i})-2v_{1i}\right]^{r_{ij}}}}{1+e^{(1-2\alpha_1)(\gamma_1\tilde{a}+v_{2i})-2v_{1i}}}\right\}^{(h)}}{\sum_{h=1}^{M}\left\{\prod_i \prod_j \frac{e^{\left[(1-2\alpha_1)(\gamma_1\tilde{a}+v_{2i})-2v_{1i}\right]^{r_{ij}}}}{1+e^{(1-2\alpha_1)(\gamma_1\tilde{a}+v_{2i})-2v_{1i}}}\right\}^{(h)}}, h=1,\ldots,M. \tag{3.9}$$

From Model 1, we draw 10,000 iterations and sample 10% without replacement. For Model 2, we draw 101,000 iterations, discard the first 1000 “burn-in” iterations, and take each tenth to get 10,000. Then we sample 10% without replacement, and perform the following iterations.

(1) From the set $\{(\Omega^{(h)}, w^{(h)}),\ h = 1,\ldots, M\}$, draw a sample of size 1, and keep this one, where the probability of sampling each is proportional to the weight;

(2) Adjust the sampling weights so that they add up to 1;

(3) Then go to step (1), sample a second value using the same procedure, but excluding the already sampled value from the set;

(4) Repeat the procedure 1000 times.

We sample without replacement because it avoids repeating the same values many times.

3.3 Data analysis

In table 3.1, under Model 1, we compared the ignorable and nonignorable nonresponse models. Observe that the values of the regression coefficients are very similar and the standard deviations (Std.) are all smaller in the nonignorable response models than in the ignorable nonresponse models. It makes sense because we are incorporating information from the response indicators, and there are no additional parameters.

Table 3.1 Inference on regression coefficients on nonignorable nonresponse Model 1 (modeling BMI)

Ignorable

Coef.	Mean	Std.	95%	CI
β_1	2.74167	0.01543	2.71148	2.77178
β_2	0.03161	0.00984	0.01197	0.05064
σ	0.11252	0.00216	0.10830	0.11690
δ	0.00032	0.00025	0.00001	0.00094
ρ	0.00283	0.00222	0.00013	0.00825

Nonignorable

Coef.	Mean	Std.	95%	CI
β_1	2.74094	0.01623	2.70852	2.77140
β_2	0.02845	0.00982	0.00982	0.04702
σ	0.11254	0.00218	0.10833	0.11698
δ	0.00064	0.00042	0.00030	0.00166
ρ	0.00569	0.00369	0.00271	0.01453

In Table 3.2, under Model 2, we compared ignorable to nonignorable nonresponse models. Note that the values of the regression coefficients are slightly different but the standard deviations (Std.) are all smaller in the nonignorable response models than in the ignorable nonresponse models. Again this is due to the fact that we are incorporating information from the response indicators, and there are no additional parameters.

Table 3.2 Inference on regression coefficients on nonignorable nonresponse Model 2 (modeling height and weight)

Ignorable

Coef.	Mean	Std.	95%	CI
α_0	0.02629	0.01203	0.00246	0.05285
α_1	0.02860	0.00188	0.02443	0.03199
γ_0	2.21641	0.11303	2.00567	2.44849
γ_1	0.39964	0.01084	0.37930	0.42168
σ_1	0.02397	0.00120	0.02161	0.02632
σ_2	0.16002	0.00281	0.15450	0.16594
σ_3	0.00140	0.00083	0.00046	0.00383
σ_4	4.88265	2.00713	2.28852	9.67768

Nonignorable

Coef.	Mean	Std.	95%	CI
α_0	0.01929	0.00958	0.00117	0.03796
α_1	0.03140	0.00088	0.02962	0.03290
γ_0	2.22751	0.09195	2.03371	2.40178
γ_1	0.39529	0.01044	0.37188	0.41700
σ_1	0.02518	0.00097	0.02330	0.02683
σ_2	0.16042	0.00285	0.15598	0.16874
σ_3	0.00114	0.00062	0.00032	0.00255
σ_4	4.75949	1.84908	2.16977	8.96170

Table 3.3: Comparison of posterior means of nonignorable to ignorable nonresponse Model 1 (modeling BMI) (cont.)

Age	Domain	Model 1:Ignorable				Model 1: Nonignorable			
		PM	PSD	95%	CI	PM	PSD	95%	CI
2	WML	16.842	0.264	(16.353,	17.366)	16.824	0.302	16.241	17.454
	WMM	15.715	0.348	(15.188,	16.581)	15.390	0.193	15.022	15.761
	WMH	16.721	0.238	(16.228,	17.184)	16.604	0.245	16.130	17.084
	WFL	16.891	0.247	(16.418,	17.389)	16.898	0.292	16.326	17.490
	WFM	15.124	0.486	(14.475,	16.462)	14.688	0.194	14.285	15.063
	WFH	16.637	0.243	(16.152,	17.117)	16.492	0.252	15.998	16.993
	NML	16.737	0.239	(16.244,	17.190)	16.682	0.268	16.174	17.204
	NMM	16.722	0.211	(16.306,	17.128)	16.695	0.233	16.224	17.129
	NMH	16.733	0.282	(16.163,	17.263)	16.680	0.345	15.996	17.369
	NFL	16.713	0.259	(16.170,	17.240)	16.669	0.292	16.094	17.242
	NFM	16.793	0.211	(16.371,	17.215)	16.783	0.223	16.335	17.233
	NMH	16.721	0.291	(16.125,	17.279)	16.657	0.354	16.004	17.358
3	WML	17.058	0.245	(16.601,	17.557)	17.019	0.285	16.495	17.623
	WMM	15.913	0.340	(15.405,	16.778)	15.563	0.161	15.235	15.856
	WMH	16.936	0.216	(16.484,	17.361	16.796	0.222	16.380	17.232
	WFL	17.108	0.230	(16.689,	17.579)	17.094	0.278	16.572	17.658
	WFM	15.312	0.483	(14.689,	16.644)	14.851	0.166	14.507	15.173
	WFH	16.850	0.220	(16.375,	17.264)	16.681	0.226	16.238	17.139
	NML	16.952	0.218	(16.507,	17.388)	16.875	0.252	16.402	17.361
	NMM	16.937	0.188	(16.565,	17.294)	16.888	0.206	16.475	17.289
	NMH	16.948	0.264	(16.377,	17.455)	16.873	0.332	16.220	17.536
	NFL	16.927	0.241	(16.411,	17.424)	16.862	0.275	16.330	17.402
	NFM	17.009	0.187	(16.641,	17.375)	16.977	0.200	16.596	17.376
	NMH	16.936	0.279	(16.357,	17.458)	16.850	0.341	16.200	17.534
4	WML	17.213	0.243	16.753	17.703	17.158	0.282	16.641	17.786
	WMM	16.057	0.339	15.530	16.956	15.690	0.149	15.390	15.980
	WMH	17.089	0.211	16.669	17.494	16.933	0.214	16.540	17.351
	WFL	17.264	0.230	16.842	17.741	17.234	0.277	16.721	17.779
	WFM	15.450	0.484	14.833	16.765	14.972	0.158	14.653	15.271
	WFH	17.003	0.218	16.534	17.415	16.818	0.219	16.392	17.268
	NML	17.106	0.215	16.653	17.548	17.014	0.251	16.539	17.504
	NMM	17.091	0.185	16.722	17.466	17.026	0.198	16.625	17.426
	NMH	17.103	0.262	16.542	17.609	17.012	0.329	16.383	17.674
	NFL	17.082	0.239	16.590	17.566	17.000	0.272	16.477	17.530
	NFM	17.164	0.184	16.802	17.522	17.117	0.196	16.733	17.501
	NMH	17.091	0.282	16.502	17.628	16.988	0.339	16.333	17.661

Table 3.3: Comparison of posterior means of nonignorable to ignorable nonresponse Model 1 (modeling BMI) (cont.)

Age	Domain	Model 1:Ignorable				Model 1: Nonignorable			
		PM	PSD	95%	CI	PM	PSD	95%	CI
5	WML	17.335	0.249	16.843	17.829	17.268	0.286	16.742	17.880
	WMM	16.170	0.343	15.645	17.047	15.789	0.149	15.488	16.064
	WMH	17.209	0.216	16.768	17.631	17.040	0.215	16.629	17.461
	WFL	17.384	0.236	16.954	17.890	17.342	0.283	16.832	17.901
	WFM	15.560	0.491	14.925	16.864	15.067	0.158	14.745	15.373
	WFH	17.123	0.223	16.675	17.553	16.925	0.218	16.506	17.352
	NML	17.228	0.222	16.753	17.665	17.122	0.257	16.630	17.633
	NMM	17.212	0.192	16.837	17.608	17.134	0.199	16.743	17.552
	NMH	17.224	0.266	16.646	17.737	17.120	0.333	16.506	17.766
	NFL	17.202	0.245	16.683	17.712	17.108	0.275	16.562	17.657
	NFM	17.285	0.190	16.900	17.650	17.226	0.202	16.812	17.624
	NMH	17.212	0.289	16.605	17.781	17.096	0.344	16.424	17.792
6	WML	17.436	0.257	16.919	17.953	17.358	0.292	16.810	17.989
	WMM	16.263	0.351	15.718	17.165	15.870	0.154	15.557	16.159
	WMH	17.311	0.226	16.861	17.755	17.131	0.224	16.712	17.568
	WFL	17.487	0.248	17.032	18.014	17.434	0.292	16.898	18.011
	WFM	15.647	0.496	15.005	16.966	15.142	0.164	14.807	15.460
	WFH	17.223	0.232	16.749	17.669	17.015	0.224	16.588	17.456
	NML	17.327	0.232	16.833	17.784	17.211	0.265	16.685	17.756
	NMM	17.311	0.204	16.913	17.736	17.224	0.206	16.834	17.657
	NMH	17.323	0.274	16.725	17.836	17.209	0.338	16.565	17.873
	NFL	17.302	0.253	16.787	17.819	17.198	0.281	16.630	17.752
	NFM	17.385	0.201	16.977	17.771	17.316	0.211	16.903	17.734
	NMH	17.311	0.299	16.686	17.914	17.185	0.349	16.494	17.881
7	WML	17.522	0.270	16.957	18.056	17.435	0.300	16.861	18.064
	WMM	16.343	0.360	15.786	17.264	15.941	0.164	15.600	16.268
	WMH	17.396	0.236	16.913	17.872	17.207	0.233	16.786	17.697
	WFL	17.572	0.261	17.087	18.112	17.511	0.302	16.978	18.123
	WFM	15.726	0.502	15.07	17.05	15.211	0.172	14.854	15.540
	WFH	17.308	0.243	16.815	17.767	17.090	0.234	16.641	17.545
	NML	17.412	0.242	16.908	17.899	17.287	0.274	16.756	17.849
	NMM	17.396	0.215	16.99	17.853	17.300	0.216	16.900	17.723
	NMH	17.408	0.283	16.815	17.947	17.285	0.345	16.630	17.974
	NFL	17.387	0.263	16.829	17.888	17.274	0.289	16.700	17.842
	NFM	17.470	0.214	17.046	17.873	17.392	0.222	16.944	17.849
	NMH	17.396	0.310	16.752	18.028	17.261	0.355	16.569	17.970

Table 3.3: Comparison of posterior means of nonignorable to ignorable nonresponse Model 1 (modeling BMI)

Age	Domain	Model 1:Ignorable				Model 1: Nonignorable			
		PM	PSD	95%	CI	PM	PSD	95%	CI
8	WML	17.595	0.278	17.028	18.153	17.501	0.309	16.919	18.166
	WMM	16.416	0.366	15.824	17.347	16.005	0.173	15.655	16.331
	WMH	17.471	0.248	16.943	17.978	17.273	0.240	16.825	17.758
	WFL	17.648	0.270	17.170	18.212	17.579	0.312	17.003	18.195
	WFM	15.794	0.509	15.118	17.123	15.270	0.179	14.905	15.609
	WFH	17.382	0.255	16.879	17.849	17.155	0.241	16.703	17.620
	NML	17.486	0.254	16.971	17.996	17.353	0.285	16.801	17.945
	NMM	17.470	0.228	17.045	17.938	17.366	0.226	16.936	17.809
	NMH	17.482	0.293	16.890	18.048	17.351	0.353	16.699	18.038
	NFL	17.460	0.274	16.886	17.975	17.340	0.299	16.748	17.941
	NFM	17.545	0.228	17.096	17.972	17.459	0.233	16.990	17.933
	NMH	17.470	0.320	16.809	18.118	17.327	0.364	16.619	18.048

Table 3.4: Comparison of posterior means of nonignorable to ignorable nonresponse Model 2 (modeling height and weight) (cont.)

Age	Domain	Model 2: Ignorable				Model 2: Nonignorable			
		PM	PSD	95%	CI	PM	PSD	95%	CI
2	WML	10.932	0.328	(10.300	11.604)	10.826	0.256	10.350	11.302
	WMM	10.810	0.172	(10.492	11.186)	10.816	0.143	10.535	11.044
	WMH	9.809	0.233	(9.306	10.244)	9.737	0.245	9.294	10.119
	WFL	11.552	0.333	10.851	12.206	11.584	0.348	10.827	12.122
	WFM	10.688	0.161	10.370	10.990	10.632	0.152	10.401	10.972
	WFH	9.875	0.263	9.300	10.358	9.874	0.256	9.410	10.258
	NML	11.952	0.320	11.323	12.621	11.962	0.342	11.118	12.582
	NMM	12.192	0.241	11.722	12.616	12.214	0.256	11.727	12.737
	NMH	11.648	0.491	10.822	12.814	11.699	0.509	10.604	12.826
	NFL	11.655	0.362	11.030	12.368	11.612	0.322	11.082	12.192
	NFM	12.312	0.263	11.887	12.872	12.425	0.230	12.031	12.793
	NMH	11.419	0.511	10.436	12.672	11.521	0.513	10.686	12.775
3	WML	12.732	0.380	12.032	13.494	12.580	0.303	12.030	13.133
	WMM	12.561	0.188	12.186	12.953	12.539	0.155	12.271	12.807
	WMH	11.411	0.260	10.841	11.937	11.299	0.282	10.841	11.697
	WFL	13.450	0.372	12.669	14.134	13.457	0.412	12.570	14.135
	WFM	12.426	0.176	12.089	12.753	12.331	0.174	12.063	12.683
	WFH	11.495	0.294	10.847	12.029	11.467	0.292	10.949	11.916
	NML	13.921	0.358	13.210	14.626	13.900	0.387	13.017	14.630
	NMM	14.200	0.267	13.685	14.676	14.192	0.297	13.649	14.867
	NMH	13.567	0.566	12.614	14.861	13.595	0.577	12.357	14.861
	NFL	13.575	0.410	12.899	14.362	13.494	0.354	12.909	14.109
	NFM	14.341	0.293	13.877	14.980	14.438	0.264	13.969	14.903
	NMH	13.301	0.593	12.132	14.750	13.388	0.589	12.415	14.838
4	WML	14.184	0.427	13.385	15.034	13.992	0.333	13.414	14.656
	WMM	13.991	0.207	13.598	14.419	13.941	0.161	13.694	14.215
	WMH	12.711	0.290	12.092	13.312	12.565	0.314	12.081	13.028
	WFL	14.983	0.408	14.129	15.714	14.967	0.463	13.974	15.762
	WFM	13.831	0.200	13.443	14.201	13.703	0.195	13.444	14.135
	WFH	12.804	0.323	12.132	13.402	12.752	0.322	12.173	13.281
	NML	15.512	0.394	14.717	16.265	15.462	0.428	14.588	16.253
	NMM	15.821	0.297	15.261	16.425	15.788	0.332	15.225	16.581
	NMH	15.119	0.630	14.048	16.521	15.124	0.634	13.765	16.442
	NFL	15.127	0.453	14.376	16.007	15.011	0.382	14.386	15.645
	NFM	15.980	0.324	15.449	16.685	16.060	0.295	15.531	16.588
	NMH	14.822	0.661	13.579	16.416	14.893	0.647	13.790	16.472

Table 3.4: Comparison of posterior means of nonignorable to ignorable nonresponse Model 2 (modeling height and weight) (cont.)

Age	Domain	Model 2: Ignorable				Model 2: Nonignorable			
		PM	PSD	95%	CI	PM	PSD	95%	CI
5	WML	15.424	0.469	14.570	16.390	15.195	0.368	14.600	15.916
	WMM	15.203	0.231	14.764	15.663	15.129	0.179	14.876	15.446
	WMH	13.822	0.316	13.155	14.467	13.648	0.349	13.105	14.176
	WFL	16.294	0.442	15.411	17.086	16.257	0.518	15.165	17.143
	WFM	15.033	0.224	14.587	15.458	14.870	0.226	14.539	15.385
	WFH	13.923	0.348	13.229	14.573	13.850	0.346	13.211	14.427
	NML	16.870	0.428	16.000	17.739	16.794	0.469	15.857	17.630
	NMM	17.207	0.326	16.572	17.922	17.148	0.366	16.583	18.037
	NMH	16.443	0.687	15.233	17.940	16.426	0.681	14.964	17.776
	NFL	16.451	0.493	15.631	17.489	16.303	0.405	15.626	16.941
	NFM	17.379	0.355	16.760	18.107	17.444	0.329	16.831	18.042
	NMH	16.120	0.723	14.739	17.848	16.176	0.700	15.010	17.880
6	WML	16.519	0.509	15.616	17.566	16.255	0.399	15.595	17.059
	WMM	16.273	0.258	15.770	16.763	16.181	0.195	15.870	16.552
	WMH	14.800	0.342	14.084	15.497	14.594	0.378	14.034	15.185
	WFL	17.450	0.473	16.508	18.302	17.390	0.564	16.214	18.371
	WFM	16.095	0.251	15.623	16.579	15.908	0.249	15.533	16.462
	WFH	14.907	0.373	14.156	15.565	14.811	0.374	14.131	15.458
	NML	18.068	0.461	17.141	19.017	17.967	0.502	16.902	18.846
	NMM	18.429	0.357	17.720	19.239	18.346	0.398	17.745	19.332
	NMH	17.610	0.739	16.262	19.176	17.574	0.724	16.034	18.988
	NFL	17.620	0.530	16.731	18.764	17.442	0.427	16.746	18.088
	NFM	18.613	0.388	17.891	19.391	18.663	0.358	18.054	19.305
	NMH	17.265	0.778	15.746	19.099	17.306	0.749	16.114	19.120
7	WML	17.506	0.549	16.535	18.638	17.208	0.427	16.474	18.049
	WMM	17.246	0.283	16.691	17.745	17.131	0.210	16.785	17.492
	WMH	15.681	0.369	14.934	16.448	15.451	0.401	14.898	16.093
	WFL	18.494	0.503	17.451	19.400	18.414	0.605	17.171	19.473
	WFM	17.056	0.276	16.531	17.599	16.843	0.280	16.384	17.458
	WFH	15.799	0.399	14.999	16.547	15.682	0.398	14.958	16.394
	NML	19.148	0.493	18.175	20.108	19.024	0.529	17.866	19.966
	NMM	19.530	0.387	18.748	20.443	19.424	0.432	18.706	20.502
	NMH	18.662	0.788	17.189	20.302	18.607	0.759	16.985	20.115
	NFL	18.673	0.566	17.714	19.940	18.468	0.448	17.772	19.156
	NFM	19.725	0.418	18.924	20.568	19.760	0.384	19.092	20.450
	NMH	18.297	0.829	16.662	20.235	18.324	0.792	17.120	20.222

Table 3.4: Comparison of posterior means of nonignorable to ignorable nonresponse Model 2 (modeling height and weight)

Age	Domain	Model 2: Ignorable				Model 2 : Nonignorable			
		PM	PSD	95%	CI	PM	PSD	95%	CI
8	WML	18.407	0.584	17.368	19.631	18.082	0.465	17.282	19.012
	WMM	18.138	0.307	17.519	18.703	18.004	0.224	17.618	18.384
	WMH	16.495	0.394	15.693	17.328	16.240	0.426	15.645	16.939
	WFL	19.446	0.533	18.340	20.382	19.346	0.638	18.027	20.480
	WFM	17.931	0.304	17.358	18.519	17.691	0.296	17.213	18.359
	WFH	16.609	0.424	15.721	17.426	16.475	0.418	15.705	17.206
	NML	20.135	0.525	19.110	21.126	19.989	0.560	18.756	20.981
	NMM	20.536	0.419	19.680	21.529	20.409	0.463	19.568	21.590
	NMH	19.625	0.833	18.062	21.328	19.550	0.798	17.841	21.125
	NFL	19.635	0.599	18.616	21.008	19.405	0.471	18.658	20.148
	NFM	20.741	0.449	19.872	21.636	20.761	0.413	20.008	21.514
	NMH	19.239	0.877	17.471	21.265	19.254	0.831	18.002	21.243

Table 3.3 and 3.4 show the predicted posterior means (PM) of BMI under the nonignorable and ignorable nonresponse models. We see that most values of posterior means (PM) and standard deviations (PSD) in nonignorable nonresponse models are smaller than in ignorable nonresponse models. So we plot the histogram of importance weights under two models to check if the weight values are stable.

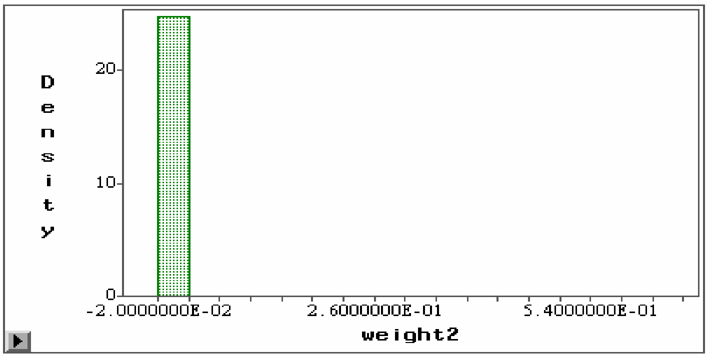
Table 3.5, 3.6 and Figure 3.1, 3.2 display the distributions of importance weights. Under nonignorable nonresponse Model 1 (modeling BMI), there is a large outlier, 0.646, and 0.90% of the samples are greater than 2.464E-05. Under nonignorable nonresponse Model 2 (modeling height and weight), the maximum value is 0.04, and 90% of the samples are greater than 1.423E-06. Since there are more similar weights in Model 2, which is better for generation of samples, we can say the SIR method works better for nonignorable nonresponse Model 2.

Table 3.5 Nonignorable nonresponse Model 1, the top 100 importance weights

0.646288	6.02E-04	1.73E-04	6.83E-05	4.15E-05
7.82E-02	5.75E-04	1.68E-04	6.43E-05	4.04E-05
7.15E-02	5.06E-04	1.46E-04	6.27E-05	3.98E-05
5.54E-02	4.36E-04	1.27E-04	6.25E-05	3.91E-05
5.48E-02	4.34E-04	1.26E-04	6.02E-05	3.67E-05
3.03E-02	3.70E-04	1.18E-04	5.90E-05	3.61E-05
1.37E-02	3.65E-04	1.08E-04	5.87E-05	3.57E-05
8.73E-03	3.52E-04	1.03E-04	5.86E-05	3.47E-05
6.82E-03	3.29E-04	1.01E-04	5.81E-05	3.42E-05
3.34E-03	3.24E-04	9.86E-05	5.69E-05	3.25E-05
2.82E-03	2.91E-04	9.78E-05	5.57E-05	3.23E-05
2.69E-03	2.78E-04	9.01E-05	5.48E-05	3.19E-05
2.64E-03	2.49E-04	9.00E-05	5.30E-05	3.17E-05
2.55E-03	2.37E-04	8.32E-05	4.89E-05	3.10E-05
2.25E-03	2.21E-04	8.19E-05	4.88E-05	3.07E-05
1.61E-03	2.19E-04	8.14E-05	4.83E-05	3.01E-05
1.43E-03	2.01E-04	8.10E-05	4.59E-05	2.96E-05
1.19E-03	1.89E-04	7.46E-05	4.58E-05	2.85E-05
1.04E-03	1.81E-04	7.37E-05	4.41E-05	2.83E-05
7.66E-04	1.81E-04	6.83E-05	4.18E-05	2.61E-05

Sum=0.998462

Figure 3.1 Histogram of important weights under nonignorable nonresponse Model 1



Moments			
N	1001.0000	Sum Wgts	1001.0000
Mean	0.0010	Sum	1.0000
Std Dev	0.0209	Variance	0.0004
Skewness	29.7744	Kurtosis	918.0438
USS	0.4363	CSS	0.4353
CV	2088.4208	Std Mean	0.0007

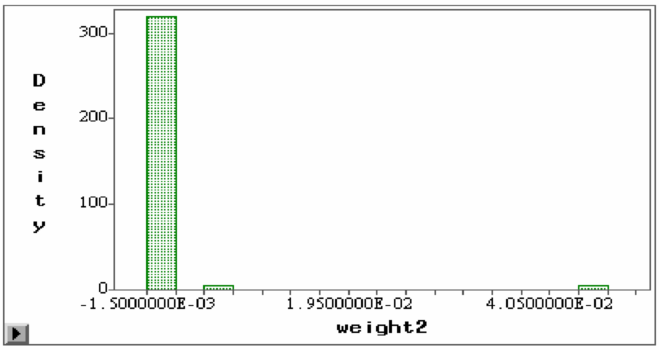
Quantiles				
100%	Max	0.6463	99.0%	0.0028
75%	Q3	2.207E-06	97.5%	0.0004
50%	Med	2.532E-07	95.0%	9.782E-05
25%	Q1	7.444E-08	90.0%	2.464E-05
0%	Min	2.495E-08	10.0%	3.470E-08
	Range	0.6463	5.0%	2.870E-08
	Q3-Q1	2.132E-06	2.5%	2.718E-08
	Mode	2.599E-08	1.0%	2.593E-08

Table 3.6 Nonignorable nonresponse Model 2, the top 100 importance weights

4.82E-02	4.60E-02	4.60E-02	4.82E-02	4.59E-02
4.60E-02	4.59E-02	4.59E-02	4.59E-02	4.59E-02
4.60E-02	4.48E-02	6.95E-03	4.59E-02	4.59E-02
4.59E-02	4.47E-02	6.63E-03	4.55E-02	4.22E-02
4.53E-02	6.63E-03	6.54E-03	6.94E-03	6.57E-03
6.62E-03	6.54E-03	6.54E-03	6.70E-03	6.54E-03
6.57E-03	3.54E-04	3.54E-04	6.57E-03	3.56E-04
6.54E-03	3.53E-04	9.49E-05	6.54E-03	3.55E-04
6.54E-03	3.52E-04	2.65E-05	6.54E-03	3.52E-04
6.54E-03	3.52E-04	2.64E-05	6.54E-03	3.52E-04
3.54E-04	9.50E-05	2.64E-05	2.65E-05	9.51E-05
2.65E-05	2.65E-05	2.63E-05	2.65E-05	2.72E-05
2.64E-05	2.25E-05	2.25E-05	2.65E-05	2.65E-05
2.63E-05	2.17E-05	6.65E-06	2.63E-05	2.64E-05
4.11E-06	6.65E-06	4.66E-06	2.22E-05	2.63E-05
4.07E-06	4.65E-06	4.65E-06	4.59E-06	2.25E-05
3.14E-06	4.11E-06	4.65E-06	2.08E-06	2.22E-05
1.38E-06	4.10E-06	4.11E-06	1.42E-06	1.54E-05
1.38E-06	3.25E-06	2.98E-06	1.38E-06	4.66E-06
1.16E-06	1.38E-06	2.87E-06	1.38E-06	3.25E-06

SUM=9.99932E-01

Figure 3.2 Histogram of important weights under nonignorable nonresponse Model 2



Moments			
N	997.0000	Sum Wgts	997.0000
Mean	0.0010	Sum	1.0000
Std Dev	0.0063	Variance	3.984E-05
Skewness	6.8494	Kurtosis	45.8251
USS	0.0407	CSS	0.0397
CV	629.3069	Std Mean	0.0002

Quantiles				
100%	Max	0.0482	99.0%	0.0459
75%	Q3	2.218E-08	97.5%	0.0066
50%	Med	5.5824E-10	95.0%	9.498E-05
25%	Q1	2.1133E-11	90.0%	1.423E-06
0%	Min	3.6563E-12	10.0%	5.8915E-12
	Range	0.0482	5.0%	4.546E-12
	Q3-Q1	2.216E-08	2.5%	3.829E-12
	Mode	1.9732E-11	1.0%	3.7881E-12

3.4 Conclusion

The results show that sampling importance resampling (SIR) works better on nonignorable nonresponse Model 2 (modeling height and weight). The nonignorable nonresponse models infer smaller standard deviation of regression coefficients and population BMI than in the ignorable nonresponse models. It is due to the fact that we are incorporating information from the response indicator, and there are no additional parameters. Therefore, the nonignorable nonresponse models allow more general inference.

Chapter 4: Conclusion

In this thesis, we present Bayesian ignorable nonresponse models and nonignorable nonresponse models for small areas by studying BMI in NHANES III. We predicted finite population mean of BMI for children (2-8 years old), post-stratified by family income, race and gender, and we investigated what adjustment needs to be made for nonignorable nonresponse.

4.1 Preliminary study

We first analyze the distribution of the data over domains, we found the missing values of height are as big as 19%, but the number of missing weight is not significant. So we only consider missing height, but ignore any missing weight.

Second, we checked the normality assumption of variables, like BMI, age, height and weight. We also found log is the preferred transformation for all variables. Thus we use log transformation throughout the study. Furthermore, we created simple linear regression of log-BMI vs. log-age, log-height vs. log-weight and log-weight vs. log-age. The results show that there are positive associations among these pairs of variables. Then we use these results to build the ignorable nonresponse models in Chapter 2.

To predict the finite population mean of BMI, we need to generate parameters, nonresponse and nonsample BMI values. In this study, we applied several sampler methods, such as simple grid method, composition method, accept-reject sampling method, Gibbs sampler method and sampling importance resampling (SIR) method to get samples based on the distribution of the posterior densities.

4.2 Ignorable nonresponse models

In Chapter 2, we use Bayesian methods to construct two ignorable nonresponse models for BMI. Model 1 is simply modeling BMI vs. age; since BMI is calculated by the ratio of weight over height squared, we construct hierarchical Model 2 by modeling height on weight, and modeling weight on age.

Under Model 1 (modeling BMI), we want to obtain samples for four sets of parameters, $\underset{\sim}{\nu}, \underset{\sim}{\beta}, \sigma^2, \rho$. Based on the complex conditional posterior densities, we use composition method to obtain samples for $\underset{\sim}{\nu}, \underset{\sim}{\beta}, \sigma^2$. But for ρ, since we set it as $0 \le \rho \le 1$, which is bounded and easy to control, we are able to apply simple simulation approach, the composition method with a grid method to get samples of ρ.

Under Model 2 (modeling height and weight), we generate the parameters, $\underset{\sim}{\nu}_1, \underset{\sim}{\nu}_2, \alpha_0, \alpha_1, \gamma_0, \gamma_1, \sigma_1, \sigma_2, \sigma_3, \sigma_4$. Since their posterior densities are complex, we get sample by using Gibbs sampler. All parameters have standard conditional densities except for ρ_1 and ρ_2, which are sampled using accept-reject sampling method.

In addition, we check the independence of the sampled parameters. The results show the autocorrelations of parameters are very small except for α_1 and γ_1 ($\alpha_1 = 0.396$ and $\gamma_1 = 0.352$), which seem to have multilateral autocorrelation. We also check the procedure of the Gibbs sampling, which satisfies our assumption. This is, considered stable.

Finally, we compared the estimates given by two ignorable models. The results displays that Model 2 (modeling height and weight) is more reasonable than Model 1 (modeling BMI). The posterior mean of BMI of Model 2 are affected by domains stratified by race, sex and family income.

However, in Chapter 2, both ignorable nonresponse models do not include the data missing mechanism. We assume that response values and nonresponse values have the same distribution, which may be wrong, so we construct nonignorable nonresponse models in Chapter 3.

4.3 Nonignorable nonresponse models

In Chapter 3, we built nonignorable nonresponse models for Model 1(modeling BMI) and Model 2 (modeling height and weight). Here the important feature is that the response indicators are not related to BMI, weight nor height, but we use the same parameters of the ignorable nonresponse function of BMI on age, height on weight, and weight on age.

As in the ignorable nonresponse models, to predict finite population mean of BMI, we need to simulate the parameters, the nonresponse and nonsample values. Under both nonignorable nonresponse models, the joint densities are very complex, so we use the sampling importance resampling (SIR) algorithm to get sample.

We constructed table by setting up Bayesian ignorable and nonignorable nonresponse models, and compared the average, standard deviation and 95% credible interval. We found some differences between the two models. In Table 3.1 and 3.2, inference on regression coefficients in nonignorable nonresponse models are slightly different than in ignorable nonresponse models (they should be the same, since we use the same parameters), while the standard deviations (Std.) are all smaller in nonignorable response models than in ignorable nonresponse models. It due to the situation that we are incorporating information from the response indicators, and there are no additional parameters.

Table 4.1 Inference on regression coefficients on nonignorable and ignorable nonresponse Model 1 (modeling BMI)

Ignorable

ef.	Mean	Std.	95%	CI
β_1	2.74167	0.01543	2.71148	2.77178
β_2	0.03161	0.00984	0.01197	0.05064
σ	0.11252	0.00216	0.10830	0.11690
δ	0.00032	0.00025	0.00001	0.00094
ρ	0.00283	0.00222	0.00013	0.00825

Nonignorable

Coef.	Mean	Std.	95%	CI
β_1	2.74094	0.01623	2.70852	2.77140
β_2	0.02845	0.00982	0.00982	0.04702
σ	0.11254	0.00218	0.10833	0.11698
δ	0.00064	0.00042	0.00030	0.00166
ρ	0.00569	0.00369	0.00271	0.01453

Table 4.2 Inference of regression coefficients on nonignorable and ignorable nonresponse Model 2 (modeling height and weight)

Ignorable

Coef.	Mean	Std.	95%	CI
α_0	0.02629	0.01203	0.00246	0.05285
α_1	0.02860	0.00188	0.02443	0.03199
γ_0	2.21641	0.11303	2.00567	2.44849
γ_1	0.39964	0.01084	0.37930	0.42168
σ_1	0.02397	0.00120	0.02161	0.02632
σ_2	0.16002	0.00281	0.15450	0.16594
σ_3	0.00140	0.00083	0.00046	0.00383
σ_4	4.88265	2.00713	2.28852	9.67768

Nonignorable

Coef.	Mean	Std.	95%	CI
α_0	0.01929	0.00958	0.00117	0.03796
α_1	0.03140	0.00088	0.02962	0.03290
γ_0	2.22751	0.09195	2.03371	2.40178
γ_1	0.39529	0.01044	0.37188	0.41700
σ_1	0.02518	0.00097	0.02330	0.02683
σ_2	0.16042	0.00285	0.15598	0.16874
σ_3	0.00114	0.00062	0.00032	0.00255
σ_4	4.75949	1.84908	2.16977	8.96170

We have reproduced Table 3.1, 3.2 and 3.4 as Table 4.1, 4.2 and 4.4 to finally convey the right impression.

Table 4.4: Comparison of posterior means of nonignorable to ignorable nonresponse Model 2 (modeling height and weight) (cont.)

Age	Domain	Model 2: Ignorable				Model 2: Nonignorable			
		PM	PSD	95%	CI	PM	PSD	95%	CI
2	WML	10.932	0.328	(10.300	11.604)	10.826	0.256	10.350	11.302
	WMM	10.810	0.172	(10.492	11.186)	10.816	0.143	10.535	11.044
	WMH	9.809	0.233	(9.306	10.244)	9.737	0.245	9.294	10.119
	WFL	11.552	0.333	10.851	12.206	11.584	0.348	10.827	12.122
	WFM	10.688	0.161	10.370	10.990	10.632	0.152	10.401	10.972
	WFH	9.875	0.263	9.300	10.358	9.874	0.256	9.410	10.258
	NML	11.952	0.320	11.323	12.621	11.962	0.342	11.118	12.582
	NMM	12.192	0.241	11.722	12.616	12.214	0.256	11.727	12.737
	NMH	11.648	0.491	10.822	12.814	11.699	0.509	10.604	12.826
	NFL	11.655	0.362	11.030	12.368	11.612	0.322	11.082	12.192
	NFM	12.312	0.263	11.887	12.872	12.425	0.230	12.031	12.793
	NMH	11.419	0.511	10.436	12.672	11.521	0.513	10.686	12.775
3	WML	12.732	0.380	12.032	13.494	12.580	0.303	12.030	13.133
	WMM	12.561	0.188	12.186	12.953	12.539	0.155	12.271	12.807
	WMH	11.411	0.260	10.841	11.937	11.299	0.282	10.841	11.697
	WFL	13.450	0.372	12.669	14.134	13.457	0.412	12.570	14.135
	WFM	12.426	0.176	12.089	12.753	12.331	0.174	12.063	12.683
	WFH	11.495	0.294	10.847	12.029	11.467	0.292	10.949	11.916
	NML	13.921	0.358	13.210	14.626	13.900	0.387	13.017	14.630
	NMM	14.200	0.267	13.685	14.676	14.192	0.297	13.649	14.867
	NMH	13.567	0.566	12.614	14.861	13.595	0.577	12.357	14.861
	NFL	13.575	0.410	12.899	14.362	13.494	0.354	12.909	14.109
	NFM	14.341	0.293	13.877	14.980	14.438	0.264	13.969	14.903
	NMH	13.301	0.593	12.132	14.750	13.388	0.589	12.415	14.838
4	WML	14.184	0.427	13.385	15.034	13.992	0.333	13.414	14.656
	WMM	13.991	0.207	13.598	14.419	13.941	0.161	13.694	14.215
	WMH	12.711	0.290	12.092	13.312	12.565	0.314	12.081	13.028
	WFL	14.983	0.408	14.129	15.714	14.967	0.463	13.974	15.762
	WFM	13.831	0.200	13.443	14.201	13.703	0.195	13.444	14.135
	WFH	12.804	0.323	12.132	13.402	12.752	0.322	12.173	13.281
	NML	15.512	0.394	14.717	16.265	15.462	0.428	14.588	16.253
	NMM	15.821	0.297	15.261	16.425	15.788	0.332	15.225	16.581
	NMH	15.119	0.630	14.048	16.521	15.124	0.634	13.765	16.442
	NFL	15.127	0.453	14.376	16.007	15.011	0.382	14.386	15.645
	NFM	15.980	0.324	15.449	16.685	16.060	0.295	15.531	16.588
	NMH	14.822	0.661	13.579	16.416	14.893	0.647	13.790	16.472

Table 4.4: Comparison of posterior means of nonignorable to ignorable nonresponse Model 2 (modeling height and weight) (cont.)

Age	Domain	Model 2: Ignorable				Model 2: Nonignorable			
		PM	PSD	95%	CI	PM	PSD	95%	CI
5	WML	15.424	0.469	14.570	16.390	15.195	0.368	14.600	15.916
	WMM	15.203	0.231	14.764	15.663	15.129	0.179	14.876	15.446
	WMH	13.822	0.316	13.155	14.467	13.648	0.349	13.105	14.176
	WFL	16.294	0.442	15.411	17.086	16.257	0.518	15.165	17.143
	WFM	15.033	0.224	14.587	15.458	14.870	0.226	14.539	15.385
	WFH	13.923	0.348	13.229	14.573	13.850	0.346	13.211	14.427
	NML	16.870	0.428	16.000	17.739	16.794	0.469	15.857	17.630
	NMM	17.207	0.326	16.572	17.922	17.148	0.366	16.583	18.037
	NMH	16.443	0.687	15.233	17.940	16.426	0.681	14.964	17.776
	NFL	16.451	0.493	15.631	17.489	16.303	0.405	15.626	16.941
	NFM	17.379	0.355	16.760	18.107	17.444	0.329	16.831	18.042
	NMH	16.120	0.723	14.739	17.848	16.176	0.700	15.010	17.880
6	WML	16.519	0.509	15.616	17.566	16.255	0.399	15.595	17.059
	WMM	16.273	0.258	15.770	16.763	16.181	0.195	15.870	16.552
	WMH	14.800	0.342	14.084	15.497	14.594	0.378	14.034	15.185
	WFL	17.450	0.473	16.508	18.302	17.390	0.564	16.214	18.371
	WFM	16.095	0.251	15.623	16.579	15.908	0.249	15.533	16.462
	WFH	14.907	0.373	14.156	15.565	14.811	0.374	14.131	15.458
	NML	18.068	0.461	17.141	19.017	17.967	0.502	16.902	18.846
	NMM	18.429	0.357	17.720	19.239	18.346	0.398	17.745	19.332
	NMH	17.610	0.739	16.262	19.176	17.574	0.724	16.034	18.988
	NFL	17.620	0.530	16.731	18.764	17.442	0.427	16.746	18.088
	NFM	18.613	0.388	17.891	19.391	18.663	0.358	18.054	19.305
	NMH	17.265	0.778	15.746	19.099	17.306	0.749	16.114	19.120
7	WML	17.506	0.549	16.535	18.638	17.208	0.427	16.474	18.049
	WMM	17.246	0.283	16.691	17.745	17.131	0.210	16.785	17.492
	WMH	15.681	0.369	14.934	16.448	15.451	0.401	14.898	16.093
	WFL	18.494	0.503	17.451	19.400	18.414	0.605	17.171	19.473
	WFM	17.056	0.276	16.531	17.599	16.843	0.280	16.384	17.458
	WFH	15.799	0.399	14.999	16.547	15.682	0.398	14.958	16.394
	NML	19.148	0.493	18.175	20.108	19.024	0.529	17.866	19.966
	NMM	19.530	0.387	18.748	20.443	19.424	0.432	18.706	20.502
	NMH	18.662	0.788	17.189	20.302	18.607	0.759	16.985	20.115
	NFL	18.673	0.566	17.714	19.940	18.468	0.448	17.772	19.156
	NFM	19.725	0.418	18.924	20.568	19.760	0.384	19.092	20.450
	NMH	18.297	0.829	16.662	20.235	18.324	0.792	17.120	20.222

Table 4.4: Comparison of posterior means of nonignorable to ignorable nonresponse Model 2 (modeling height and weight)

Age	Domain	Model 2: Ignorable				Model 2 : Nonignorable			
		PM	PSD	95%	CI	PM	PSD	95%	CI
8	WML	18.407	0.584	17.368	19.631	18.082	0.465	17.282	19.012
	WMM	18.138	0.307	17.519	18.703	18.004	0.224	17.618	18.384
	WMH	16.495	0.394	15.693	17.328	16.240	0.426	15.645	16.939
	WFL	19.446	0.533	18.340	20.382	19.346	0.638	18.027	20.480
	WFM	17.931	0.304	17.358	18.519	17.691	0.296	17.213	18.359
	WFH	16.609	0.424	15.721	17.426	16.475	0.418	15.705	17.206
	NML	20.135	0.525	19.110	21.126	19.989	0.560	18.756	20.981
	NMM	20.536	0.419	19.680	21.529	20.409	0.463	19.568	21.590
	NMH	19.625	0.833	18.062	21.328	19.550	0.798	17.841	21.125
	NFL	19.635	0.599	18.616	21.008	19.405	0.471	18.658	20.148
	NFM	20.741	0.449	19.872	21.636	20.761	0.413	20.008	21.514
	NMH	19.239	0.877	17.471	21.265	19.254	0.831	18.002	21.243

Table 4.4 displays the predicted posterior means (PM) of BMI under nonignorable and ignorable nonresponse models. Note that most PM and PSD values of nonignorable nonresponse models are smaller than corresponding ignorable nonresponse models. They should be similar, because we use the same parameters.

4.4 Recommendation

In the future, we need to compare the models defined in Chapter 3 with those in Nandram and Choi (2005) (Nandram, B., Cox, L. and Choi, J. W., 2005) and Nandram and Choi (2006), particularly the inclusion of survey weights. We will also need to do some diagnostic to assure how well the models fit, and which is the best model.

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Appendix: Propriety of $p(\underset{\sim}{v},\underset{\sim}{\beta},\sigma^2,\rho\big|\underset{\sim}{y})$

The joint posterior density of $\underset{\sim}{v},\underset{\sim}{\beta},\sigma^2,\rho$ is,

$$p(\underset{\sim}{v},\underset{\sim}{\beta},\sigma^2,\rho\big|\underset{\sim}{y}) = p_1(\underset{\sim}{v}\big|\underset{\sim}{\beta},\sigma^2,\rho,\underset{\sim}{y})\,p_2(\underset{\sim}{\beta}\big|\sigma^2,\rho,\underset{\sim}{y})\,p_3(\sigma^2\big|\rho,\underset{\sim}{y})\,p_4(\rho\big|\underset{\sim}{y})\,, \tag{A.1}$$

where, $\underset{\sim}{\beta}$ is p x 1 vector, p=2 in Model 1,

$$p(\underset{\sim}{\beta},\underset{\sim}{v},\rho,\sigma^2 \mid \underset{\sim}{y})$$

$$\propto \frac{1}{\sigma^2}\prod_{i=1}\left\{\left[\prod_j \frac{1}{\sqrt{2\pi\sigma^2}}\exp\{-\frac{1}{2\sigma^2}[y_{ij}-\underset{\sim}{x}'_{ij}\underset{\sim}{\beta}-v_i)]^2\}\right]\frac{1}{\sqrt{2\pi(\frac{1-\rho}{\rho})\sigma^2}}\exp(-\frac{1-\rho}{2\rho\sigma^2}v_i^2)\right\}$$

$$\propto (\frac{1-\rho}{\rho})^{l/2}\frac{1}{(\sigma^2)^{\sum r_i+l/2+1}}\prod_{i=1}^{l}\left\{\exp[-\frac{1}{2\sigma^2}(\sum_{j=1}^{r_i}(y_{ij}-\underset{\sim}{x}'_{ij}\underset{\sim}{\beta}-v_i)+\frac{\rho}{1-\rho}v_i^2)]\right\}. \tag{A.2}$$

We show that $p(\underset{\sim}{\beta},\underset{\sim}{v},\rho,\sigma^2 \mid \underset{\sim}{y})$ is proper.

Consider $\sum_{j=1}^{r_i}(y_{ij}-\underset{\sim}{x}'_{ij}\underset{\sim}{\beta}-v_i)+\frac{\rho}{1-\rho}v_i^2)$

$$=(r_i+\frac{\rho}{1-\rho})\left[v_i-\frac{r_i}{r_i+\frac{1-\rho}{\rho}}(\underset{\sim}{\bar{y}}-\underset{\sim}{x}'_{ij})\right]^2$$

$$+\left[\frac{r_i(\frac{1-\rho}{\rho})}{r_i+\frac{1-\rho}{\rho}}\left(\bar{y}_i-\underset{\sim}{\bar{x}}'_i\underset{\sim}{\beta}\right)^2+\sum_{j=1}^{r_i}(y_{ij}-\bar{y}_i-(\underset{\sim}{x}_{ij}-\underset{\sim}{\bar{x}}_i)'\underset{\sim}{\beta})^2\right], \tag{A.3}$$

letting $\lambda_i=\frac{r_i}{r_i+\frac{1-\rho}{\rho}}$, thus,

$$p(\underset{\sim}{\beta},\underset{\sim}{v},\rho,\sigma^2 \mid \underset{\sim}{y}) \propto (\frac{1-\rho}{\rho})^{l/2}\frac{1}{(\sigma^2)^{\sum r_i+l/2+1}}$$

$$\times\prod_{i=1}^{l}\exp\left\{-\frac{1}{2\sigma^2}\left[\frac{r_i}{\lambda_i}(v_i-\lambda_i(\bar{y}_i-\bar{x}'\underset{\sim}{\beta}_i)^2+\sum_{i=1}^{l}\lambda_i(\frac{\rho}{1-\rho})\left(\bar{y}_i-\underset{\sim}{\bar{x}}_i'\underset{\sim}{\beta}\right)^2]+\sum_{j=1}^{r_i}(y_{ij}-\bar{y}_i-(\underset{\sim}{x}_{ij}-\underset{\sim}{\bar{x}}_i)'\underset{\sim}{\beta})^2\right]\right\},$$

(A.4)

and $v_i\left|\underset{\sim}{\beta},\sigma^2,\rho,\underset{\sim}{y}\right.\overset{ind}{\sim} Normal\left\{\lambda_i(\bar{y}_i-\bar{x}_i'\underset{\sim}{\beta}),\frac{\lambda_i}{\gamma_i}\sigma^2\right\}.$ (A.5)

Then integrating out v_i, we have

$$p(\underset{\sim}{\beta},\sigma^2,\rho\left|y\right.)\propto(\frac{1-\rho}{\rho})^{l/2}\frac{1}{(\sigma^2)^{\sum r_i+l/2+1}}\prod_{i=1}^{l}\sqrt{\frac{\lambda_i}{r_i}}$$

$$\times\exp(-\frac{1}{2\sigma^2}[\sum_{r=1}^{l}\sum_{j=1}^{r_i}(y_{ij}-\bar{y}_i-(\underset{\sim}{x}_{ij}-\underset{\sim}{\bar{x}}_i)'\underset{\sim}{\hat{\beta}})^2+\sum_{i=1}^{l}\lambda_i(\frac{\rho}{1-\rho})\left(\bar{y}_i-\underset{\sim}{\bar{x}}_i'\underset{\sim}{\hat{\beta}}\right)^2]$$

$$\times\exp\{-\frac{1}{2\sigma^2}(\underset{\sim}{\beta}-\underset{\sim}{\hat{\beta}})[\sum_{r=1}^{l}\sum_{j=1}^{r_i}(\underset{\sim}{x}_{ij}-\underset{\sim}{\bar{x}}_i)'(\underset{\sim}{x}_{ij}-\underset{\sim}{\bar{x}}_i)+\sum_{i=1}^{l}\lambda_i(\frac{\rho}{1-\rho})\underset{\sim}{\bar{x}}_i\underset{\sim}{\bar{x}}_i']^{-1}(\underset{\sim}{\beta}-\underset{\sim}{\hat{\beta}}),$$

(A.6)

where, $\rho=\frac{\delta^2}{\delta^2+\sigma^2}$,

$$\underset{\sim}{\hat{\beta}}=[\sum_{r=1}^{l}\sum_{j=1}^{r_i}(\underset{\sim}{x}_{ij}-\underset{\sim}{\bar{x}}_i)'(\underset{\sim}{x}_{ij}-\underset{\sim}{\bar{x}}_i)+\sum_{i=1}^{l}\lambda_i(\frac{\rho}{1-\rho})\underset{\sim}{\bar{x}}_i\underset{\sim}{\bar{x}}_i']^{-1}\times\left[\sum_{r=1}^{l}\sum_{j=1}^{r_i}(y_{ij}-\bar{y}_i)(\underset{\sim}{x}_{ij}-\underset{\sim}{\bar{x}}_i)+\sum_{i=1}^{l}\lambda_i(\frac{\rho}{1-\rho})\bar{y}_i\underset{\sim}{\bar{x}}_i\right],$$

(A.7)

thus, $\underset{\sim}{\beta}\left|\sigma^2,\rho,\underset{\sim}{y}\right.\sim Normal\left\{\underset{\sim}{\hat{\beta}},\sigma^2\{\sum_{i=1}^{l}\sum_{j=1}^{r_i}[(\underset{\sim}{x}_{ij}-\underset{\sim}{\bar{x}}_i)(\underset{\sim}{x}_{ij}-\underset{\sim}{\bar{x}}_{\boldsymbol{i}})'+\lambda_i(\frac{\rho}{1-\rho})\underset{\sim}{x}_{ij}\underset{\sim}{\bar{x}}_{\boldsymbol{i}}']\}\right\}.$ (A.8)

Integrating out $\underset{\sim}{\beta}$, we have,

$$p(\sigma^2,\rho\left|y\right.)\propto(\frac{1-\rho}{\rho})^{l/2}\frac{1}{(\sigma^2)^{\sum r_i+l/2+1}}\prod_{i=1}^{l}\sqrt{\frac{\lambda_i}{r_i}}$$

$$\times \exp\left\{-\frac{1}{2\sigma^2}\left[\sum_{r=1}^{l}\sum_{j=1}^{r_i}\left(y_{ij}-\bar{y}_i-(\underset{\sim}{x}_{ij}-\underset{\sim}{\bar{x}}_i)\underset{\sim}{\hat{\beta}}\right)^2+\sum_{i=1}^{l}\lambda_i(\frac{\rho}{1-\rho})(\bar{y}_i-\underset{\sim}{\bar{x}}_i'\underset{\sim}{\hat{\beta}})^2\right]\right\}, \text{(A.9)}$$

thus,

$$\sigma^{-2}\left|\rho,\underset{\sim}{b}\right. \sim Gamma\left\{\frac{\sum r_i-p}{2},\sum_{i=1}^{l}\sum_{j=1}^{r_i}\frac{(y_{ij}-\bar{y}_i-(\underset{\sim}{x}_{ij}-\underset{\sim}{\bar{x}}_i)'\underset{\sim}{\hat{\beta}})^2+\sum_{j=1}^{r_i}\lambda_i(\frac{1-\rho}{\rho})(\bar{y}_i-\underset{\sim}{\bar{x}}_i'\underset{\sim}{\hat{\beta}})^2}{2}\right\}.$$

(A.10)

And integrating out σ^2, we have,

$$p(\rho|y) \propto \frac{\prod_{i=1}^{l}\sqrt{\frac{(1-\rho)r_i}{\rho r_i+(1-\rho)}}}{\left[\sum_{r=1}^{l}\sum_{j=1}^{r_i}(y_{ij}-\bar{y}_i-(\underset{\sim}{x}_{ij}-\underset{\sim}{\bar{x}}_i)'\underset{\sim}{\hat{\beta}})^2+\sum_{i=1}^{l}\frac{(1-\rho)r_i}{\rho r_i+(1-\rho)}\left(\bar{y}_i-\underset{\sim}{\bar{x}}_i'\underset{\sim}{\hat{\beta}}\right)^2\right]^{\frac{\sum_{i=1}^{l}r_i-p}{2}}}, \quad \text{(A.11)}$$

$0 \le p \le 1$, note that $\int_0^1 p(\rho|y)d\rho < \infty$.

Thus, the joint posterior density $p(\underset{\sim}{\beta},\underset{\sim}{v},\rho,\sigma^2 \mid \underset{\sim}{y})$ is proper.