

**EXPLORATIONS OF TRADING STRATEGIES FOR LEVERAGED
EXCHANGE-TRADED FUNDS**

A Directed Research Project

Submitted to the Faculty of the

WORCESTER POLYTECHNIC INSTITUTE

in partial fulfillment of the requirements for the

Professional Degree of Master of Science

in

Financial Mathematics

by

Barry J. Posterro

November 2009

Approved:

Professor Marcel Blais, Advisor

Professor Bogdan Vernescu, Head of Department

Abstract

This paper describes our work in exploring trading strategies for the leveraged exchange-traded funds, Direxion Daily Financial Bull 3X (FAS) and Direxion Daily Financial Bear 3X (FAZ) over the first three quarters of 2009. Using minute-by-minute stock data we are able to verify the accuracy of these ETFs in regards to their target of the Russell 1000 Financial Index (RIFIN). We are then able to quantify the returns and risks involved with trading strategies that seek to exploit the ETFs objectives, specifically momentum trades, tracking-error discrepancy trades, and a combination of the two strategies we term “discount-and-up.” Bootstrap simulation techniques are employed to measure values at risk and conditional tail expectations over 30 day time horizons for each strategy. Lastly, we demonstrate the dangers of traditional buy-and-hold investing with regards to leveraged ETFs.

Acknowledgements

This work could not have been completed without the support of my wife, Kristen, whose understanding and support throughout the process of finishing this project and during the entire pursuit of this degree made this achievement possible. Also, thanks to Professor Marcel Blais whose guidance and enthusiasm for this work served to make this an enjoyable exercise.

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1. Introduction

An exchange-traded fund (ETF) is an investment vehicle that behaves like a mutual fund, but is traded like a stock. Like a mutual fund, ETFs hold a basket of assets of which investors can buy shares. Similarities to mutual funds end at this point. Unlike mutual funds, these shares are traded on open exchanges during normal trading hours and even off-hours, marked to market throughout the day, not just following the close of trading. As such ETFs can be subject to a bid-ask spread. Many ETFs also hold financial derivatives, a practice not available to mutual funds. Lastly, as ETFs are exchanged between traders, one can close his position in an ETF at any time without incurring early redemption fees.

Typically an ETF is created to track a market sector, usually a market sector index. QQQQ (Powershares QQQ Trust) tracks the NASDAQ 100 Index. Similarly SPY (SPDR S&P 500) tracks the returns of the S&P500. Recently, two new types of ETFs have gained favor with investors. These are daily leveraged and/or daily inverse ETFs. These ETFs promise to return a multiple of the daily return of a particular index. This paper looks at trading strategies involving two such leveraged ETFs, Direxion Daily Financial Bull 3X (FAS) and Direxion Daily Financial Bear 3X (FAZ), which aim to replicate triple and negative triple the daily return of the Russell 1000 Financial Index (RIFIN), respectively.

Given that their daily objectives are so clearly defined, we looked at trading strategies that seek to exploit the investment objective of FAS and FAZ. Developing a strategy we term “discount-and-up” we use the recently documented late day volatility of these ETFs and others to place buy orders in the last half hour of trading in the event

either FAS or FAZ is trading up for the day and is currently below its objective of triple or negative triple the daily return of the RIFIN. We then exit this position shortly before the market closes. Over the first three quarters of 2009, we demonstrate that this strategy would have earned a total return of 132.5%. Having been on the markets for less than a year, FAS and FAZ trading strategy results are being presented here for the first time, quantifying the conjectures of industry traders as to the proper way to exploit the behavior of these ETFs.

Furthermore, we show that requiring the ETF to be trading both up for the day and below its target before buying it is statistically superior to both momentum trading, a practice advocated often in industry of buying an ETF late in the day if it is positive for the day thus far, and tracking-error trading, buying the ETF simply because it is trading for less than its implied assets under management. Of the three strategies, discount-and-up offered the highest return and least amount of risk by several measures.

Lastly, this paper demonstrates the dangers of using leveraged ETFs as part of a buy-and-hold strategy. With examples, both contrived for demonstration and pulled from actual market data, we show that a buy-and-hold position in either FAS or FAZ can lead to unpredictable results, even when the market's overall return is in line with the investor's projections.

For further work, we suggest those interested continue to test the discount-and-up and other trading strategies in markets of varying volatility. We show here that the strategy performed better during the more volatile markets of the first five months of 2009 than it did in the second five months of 2009 when economic stability began to retake hold of the RIFIN and the markets in general.

2. Background

2.1 Exchange-Traded Funds

Exchange-Traded Funds (ETFs) are investment vehicles that are most akin to mutual funds in their structure and behavior. An ETF will hold a set of assets that investors can purchase shares of on the open market. Unlike mutual funds which post their values at the end of each trading day and all buy and sell orders during the day are executed at that price, ETFs trade on open exchanges during normal trading hours with price fluctuations throughout the day. As such, ETFs are also subject to a bid-ask spread and the liquidity issues of typical securities.

While mutual funds usually have broad objectives, such as investing in high growth stocks or large cap securities, ETFs are more often designed to track a particular index. For example, one of the first ETFs QQQQ (Powershares QQQ Trust) tracks the NASDAQ 100 Index. Similarly SPY (SPDR S&P 500) tracks the returns of the S&P500. As indices are not traded on exchanges, previously the only way investors could take positions on these composites was to buy more sophisticated investment vehicles, such as swaps and futures. ETFs have grown in popularity because they allow an investor to directly “buy stock” in the direction of an index using the same tools he/she already uses to buy any other stock. As ETFs do not charge early redemption fees, like most mutual funds, they also allow an investor to hold their position for a much shorter period of time than one typically would with a regular mutual fund.

In 2006, ETF offerings expanded with the introduction of leveraged ETFs and leveraged inverse ETFs. These ETFs promise to return a multiple of the return of the

index being tracked, usually 2 or 3 times the index's return or -2 or -3 times the index's return. Examples of these include DTO (Powershares Dbl Crude Oil) which returns twice the inverse of the daily movement in the price of a barrel of crude oil. TYH (Direxion Daily Tech Bull 3X) replicates three times the daily return of the Russell 1000 Technology Index. Investors strongly believing oil will drop on a given day can take a double short position by buying DTO and investors believing technology stocks will have a positive day can take a triple long position by purchasing TYH.

As of the end of January 2009, there are over 100 leveraged and inverse ETFs being traded with assets under management of about \$22 billion. Daily volumes of individual ETFs typically exceed the volumes of common stocks. For example, the ETFs studied herein, FAS (Direxion Daily Financial Bull 3X) and FAZ (Direxion Daily Financial Bear 3X), typically averaged volumes of 250 million shares per day prior to their recent reverse splits. Their high volume is one reason they are the focus of this study, as liquidity and bid-ask spreads were assumed to be irrelevant to their dynamics.

2.2 Literature Search

Because leveraged ETF's are relatively new investment vehicles, a literature search did not reveal any mathematical-based papers or quantitative reports that discussed trading strategies or investment returns. *The Dynamics of Leveraged and Inverse Exchange-Traded Funds* published by Barclays Global Investors in May 2009 does discuss the dangers of buy-and-hold strategies in regards to these vehicles and concludes that they are not suitable for that type of long-term, inactively managed investment style. Our paper goes further and expressly shows that, in the case of

FAS/FAZ, the buy-and-hold strategy is vastly inferior to any of the three simple trading strategies we offer as alternatives.

Barclays also discusses the observed volatility of leveraged ETFs as the market approaches its daily close¹. This phenomenon had also been noted by the general media and on investment message boards across financial web-sites. In December 2008, the Wall Street Journal published an article in which they discuss the impact of leveraged ETFs on late day market volatility². In that report, they give examples of days in which the majority of the market's swing for the day took place in the last hour or half-hour of trading. Pointing to high volume spikes in ETFs as the market enters its last hour of trading, they theorize that the need to rebalance ETFs adds momentum to whichever direction the market had already been trending and that some investors play this momentum as a trading strategy. This paper shows that while such a trading strategy does yield high returns, it offers higher variance than does the strategy explored herein which offers nearly identical returns with much decreased standard deviation, value at risk and conditional tail expectation.

3. FAS/FAZ Reliability in Tracking RIFIN

3.1 RIFIN

The Russell 1000 Financial Services Index (RIFIN) is published by Russell Investments. It is an index that tracks the performance of the financial services companies that are part of the more general Russell 1000 Index. The RIFIN is maintained regularly and totally recreated annually to ensure that it includes only the most relevant financial services-oriented companies. New emerging companies are added and any companies that have significantly shifted their economic orientation away from the financial services sector are removed.

As of August 31, 2009, the top 10 holdings in the RIFIN were⁵:

JP Morgan, Bank of America, Wells Fargo, Goldman Sachs, Citigroup, US Bancorp, Morgan Stanley, Bank of New York Mellon, American Express, and Visa Inc.

3.2 FAS

Direxion Daily Financial Bull 3X Shares (FAS) is an ETF offered by Direxion Shares. The goal of this ETF is to return triple the daily return of RIFIN. Therefore, if the RIFIN were to rise 10% in one day, Direxion would attempt to increase the market price of FAS by 30% over that same day. Likewise, a 10% drop in the RIFIN in one day would materialize as a 30% fall in FAS for that same day. Because this is a daily ETF, Direxion makes no attempt with FAS to mimic any returns longer than the horizon of the

market close of the previous trading day to the market close of the succeeding trading day.

To achieve this objective, FAS is composed of approximately 1/3 securities that compose the RIFIN index and 2/3 derivatives of the RIFIN, specifically swaps. As of September 20, 2009, FAS' balance sheet was approximately \$3B, with \$2B in Russell 1000 Financial Index Swaps. Table 1 shows FAS' top holdings as of that day⁴.

Account Ticker	Stock Ticker	Security Description	Shares	Price	Market Value
FAS		RUSSELL 1000 FINANCIAL INDEX SWAP	1,704,800	795.69	1,356,492,312
FAS		RUSSELL 1000 FINANCIAL INDEX SWAP	380,000	795.69	302,362,200
FAS		RUSSELL 1000 FINANCIAL INDEX SWAP	347,000	795.69	276,104,430
FAS		RUSSELL 1000 FINANCIAL INDEX SWAP	195,000	795.69	155,159,550
FAS		AIM S/T INVEST. TRUST TREASURY	138,132,590	1	138,132,591
FAS	JPM	Jpmorgan Chase & Co.	1,675,415	44.95	75,309,904
FAS		AIM S/T INVEST. TRUST TREASURY	70,727,063	1	70,727,063
FAS	BAC	Bank Of America Corp	3,855,862	17.63	67,978,847
FAS	WFC	Wells Fargo & Company	2,119,729	28.49	60,391,079
FAS	GS	GOLDMAN SACHS GROUP INC	224,369	183.18	41,099,913
FAS	C	CITIGROUP INC.	5,056,847	4.26	21,542,168
FAS	USB	Us Bancorp	845,736	22.76	19,248,951
FAS	MS	MORGAN STANLEY	605,782	31.38	19,009,439
FAS	BK	Bank Of Ny Mellon Corp	532,799	30.3	16,143,810
FAS	AXP	American Express Co	452,229	34.77	15,724,002
FAS	V	VISA INC-CLASS A SHARES	200,262	73.79	14,777,333
FAS	TRV	Travelers Cos Inc/The	260,921	47.37	12,359,828
FAS	STT	State Street Corp	220,074	54.39	11,969,825
FAS	PRU	Prudential Financial, Inc.	205,324	52.69	10,818,522
FAS	MET	Metlife Inc	257,186	39.32	10,112,554

Table 1: Top Holdings of FAS as of September 20, 2009

3.3 FAZ

Direxion Daily Financial Bear 3X Shares (FAZ) is another ETF offered by Direxion Shares. Opposite of FAS, the goal of this ETF is to return triple the inverse of the daily return of RIFIN. Therefore, if the RIFIN were to rise 10% in one day, Direxion would attempt to decrease the market price of FAZ by 30% over that same day.

Likewise, a 10% drop in the RIFIN in one day would materialize as a 30% increase in FAZ for that same day.

To achieve this objective, FAZ is composed primarily of short positions in financial swaps of the RIFIN. As of September 20, 2009, FAZ's balance sheet was short approximately \$3.5B of Russell 1000 Financial Index Swaps and \$1.5B long in Treasury Notes. Table 2 shows FAZ's complete list of holdings as of that day⁴.

Account Ticker	Stock Ticker	Security Description	Shares	Price	Market Value
FAZ		RUSSELL 1000 FINANCIAL INDEX SWAP	(2,396,600)	795.69	(1,906,950,654)
FAZ		RUSSELL 1000 FINANCIAL INDEX SWAP	(1,519,000)	795.69	(1,208,653,110)
FAZ		RUSSELL 1000 FINANCIAL INDEX SWAP	(145,000)	795.69	(115,375,050)
FAZ		RUSSELL 1000 FINANCIAL INDEX SWAP	(350,000)	795.69	(278,491,500)
FAZ		AIM S/T INVEST. TRUST TREASURY	621,398,094	1	621,398,094
FAZ		AIM S/T INVEST. TRUST TREASURY	362,859,795	1	362,859,795
FAZ		AIM S/T INVEST. TRUST TREASURY	34,210,543	1	34,210,543
FAZ		AIM S/T INVEST. TRUST TREASURY	120,510,000	1	120,510,000
FAZ		AIM S/T INVEST. TRUST TREASURY	408,594,291	1	408,594,291

Table 2: Complete Holdings of FAZ as of September 20, 2009

3.4 FAS/FAZ History

Both FAS and FAZ have inception dates of November 6, 2008. Because one cannot directly invest in an index, these ETFs offer purchasers a way to add a general index to their portfolios and invest according to their beliefs of the future direction of the financial services sector. In this case, the investor is doing so with three times the volatility of the RIFIN itself.

On July 9, 2009, with the share price of each ETF under \$10, Direxion instituted a 1-for-5 reverse split for FAS and a 1-for-10 reverse split for FAZ. Prior to this adjustment, the average daily volume for FAS was approximately 250 million shares and for FAZ was approximately 300 million shares. These volume levels made FAS/FAZ

more ideal for this research than other triple leveraged ETFs with lower volumes because liquidity assumptions did not need to be given consideration.

3.5 Correlations of FAS/FAZ with RIFIN

The first avenue of investigation in studying and developing a trading strategy for these ETFs was to confirm that they do in fact consistently meet their investment objective of returning three times the daily return (directly or inversely) of the RIFIN index. Correlation coefficients were calculated over the 92 trading days of test data for both FAS with RIFIN and FAZ with RIFIN. Correlations were calculated for each of the 390 trading minutes of the market day. That is, a correlation was calculated between FAS and RIFIN for the 9:30AM trading minute by looking across the 92 trading days. Another correlation was calculated for 9:31AM, and onwards until 3:59PM. Samples of these results are shown in Table 4. The complete results are represented graphically in Chart 1 and Chart 2.

Table 3 below summarizes the results of the correlation exercises.

		Time
Best FAS/RIFIN Correl	0.99602	3:55 PM
Worst FAS/RIFIN Correl	0.95293	9:30 AM
FAS/RIFIN Correl	0.99495	3:59 PM
FAS/RIFIN Correl	0.99425	3:30 PM
Avg FAS/RIFIN Correl	0.98950	
Best FAZ/RIFIN Correl	-0.99642	3:44 PM
Worst FAZ/RIFIN Correl	-0.95640	9:30 AM
FAZ/RIFIN Correl	-0.99538	3:59 PM
FAZ/RIFIN Correl	-0.99529	3:30 PM
Avg FAZ/RIFIN Correl	-0.99012	

Table 3: Correlation Statistics (1/2/2009 – 5/14/2009)

Firstly, we see that both FAS and FAZ show their worst correlation with the RIFIN at 9:30AM. This is expected and logical for two reasons. Firstly, the financial stocks that compose the RIFIN experience both after-market and pre-market trading at large volumes since the previous close. At open, the RIFIN instantly takes these new values into account in calculating its own value and instantly posts a market open gain or loss from the previous close. Secondly, the trading of both FAS and FAZ post-market and pre-market can draw it out of line. When FAS and FAZ are traded off-market hours, they behave like any typical security, following auction rules. As such, their values are, for that period of time, being determined by bid/ask rules, not being marked to market by Direxion. Because the 9:30AM correlations are still very high, we have evidence that the high volumes of off-hour trading does help to keep them from being drawn too far off their targets. However, when the market opens, they will still need to be brought back into line by Direxion and that shift is recorded when the first FAS or FAZ trades are posted on-hours.

As expected, the best correlations for FAS and FAZ with the RIFIN are both close to the end of the trading day. The best correlation of FAS with the RIFIN is seen at 3:55PM. Likely Direxion does not have enough time after 3:55PM to account for and execute trades in response to last minute moves by RIFIN components. For FAZ, the time of best correlation is 3:44PM. We know that FAZ is composed entirely of RIFIN derivatives. As such, last minute moves by the RIFIN will necessitate actions by Direxion to bring FAZ into line with its daily objective. More moves will be required for FAZ than for FAS because FAS is 1/3 comprised of the actual stocks of the RIFIN. While the best correlations are not at 3:59PM, it is important to note that the 3:59PM

correlations for both FAS and FAZ only differ from their best correlations by 0.00107 and 0.00104 respectively.

Time	FAS/RIFIN Correlation	FAZ/RIFIN Correlation	Time	FAS/RIFIN Correlation	FAZ/RIFIN Correlation	Time	FAS/RIFIN Correlation	FAZ/RIFIN Correlation
9:30:00 AM	0.95293	-0.95640	12:00:00 PM	0.99093	-0.99161	3:30:00 PM	0.99425	-0.99529
9:31:00 AM	0.96661	-0.96866	12:01:00 PM	0.99133	-0.99181	3:31:00 PM	0.99456	-0.99549
9:32:00 AM	0.97160	-0.97207	12:02:00 PM	0.99190	-0.99214	3:32:00 PM	0.99432	-0.99522
9:33:00 AM	0.97437	-0.97406	12:03:00 PM	0.99110	-0.99168	3:33:00 PM	0.99477	-0.99552
9:34:00 AM	0.97669	-0.97704	12:04:00 PM	0.99220	-0.99237	3:34:00 PM	0.99406	-0.99495
9:35:00 AM	0.97420	-0.97456	12:05:00 PM	0.99168	-0.99226	3:35:00 PM	0.99460	-0.99531
9:36:00 AM	0.97289	-0.97169	12:06:00 PM	0.99147	-0.99271	3:36:00 PM	0.99514	-0.99564
9:37:00 AM	0.97165	-0.97203	12:07:00 PM	0.99217	-0.99256	3:37:00 PM	0.99510	-0.99558
9:38:00 AM	0.97389	-0.97456	12:08:00 PM	0.99182	-0.99236	3:38:00 PM	0.99539	-0.99617
9:39:00 AM	0.97003	-0.96906	12:09:00 PM	0.99174	-0.99212	3:39:00 PM	0.99556	-0.99597
9:40:00 AM	0.97515	-0.97615	12:10:00 PM	0.99211	-0.99274	3:40:00 PM	0.99526	-0.99584
9:41:00 AM	0.97616	-0.97656	12:11:00 PM	0.99154	-0.99240	3:41:00 PM	0.99503	-0.99567
9:42:00 AM	0.97289	-0.97444	12:12:00 PM	0.99194	-0.99249	3:42:00 PM	0.99507	-0.99586
9:43:00 AM	0.97042	-0.97160	12:13:00 PM	0.99094	-0.99180	3:43:00 PM	0.99545	-0.99595
9:44:00 AM	0.97086	-0.97190	12:14:00 PM	0.99129	-0.99221	3:44:00 PM	0.99536	-0.99642
9:45:00 AM	0.97135	-0.97273	12:15:00 PM	0.99147	-0.99178	3:45:00 PM	0.99533	-0.99571
9:46:00 AM	0.97265	-0.97201	12:16:00 PM	0.99139	-0.99229	3:46:00 PM	0.99501	-0.99596
9:47:00 AM	0.97123	-0.97204	12:17:00 PM	0.99168	-0.99247	3:47:00 PM	0.99538	-0.99595
9:48:00 AM	0.97043	-0.97090	12:18:00 PM	0.99072	-0.99198	3:48:00 PM	0.99571	-0.99615
9:49:00 AM	0.97122	-0.97088	12:19:00 PM	0.99133	-0.99239	3:49:00 PM	0.99505	-0.99591
9:50:00 AM	0.97224	-0.97437	12:20:00 PM	0.99211	-0.99269	3:50:00 PM	0.99520	-0.99602
9:51:00 AM	0.97416	-0.97595	12:21:00 PM	0.99160	-0.99238	3:51:00 PM	0.99540	-0.99615
9:52:00 AM	0.97342	-0.97416	12:22:00 PM	0.99190	-0.99249	3:52:00 PM	0.99568	-0.99634
9:53:00 AM	0.97500	-0.97582	12:23:00 PM	0.99098	-0.99148	3:53:00 PM	0.99539	-0.99589
9:54:00 AM	0.97380	-0.97346	12:24:00 PM	0.99124	-0.99186	3:54:00 PM	0.99563	-0.99623
9:55:00 AM	0.97434	-0.97437	12:25:00 PM	0.99178	-0.99218	3:55:00 PM	0.99602	-0.99641
9:56:00 AM	0.97307	-0.97288	12:26:00 PM	0.99200	-0.99238	3:56:00 PM	0.99577	-0.99636
9:57:00 AM	0.97248	-0.97231	12:27:00 PM	0.99191	-0.99209	3:57:00 PM	0.99563	-0.99612
9:58:00 AM	0.97434	-0.97513	12:28:00 PM	0.99193	-0.99256	3:58:00 PM	0.99591	-0.99608
9:59:00 AM	0.97179	-0.97242	12:29:00 PM	0.99118	-0.99194	3:59:00 PM	0.99495	-0.99538
10:00:00 AM	0.97302	-0.97430	12:30:00 PM	0.99174	-0.99208			

Table 4: Samples of Correlation Coefficients of FAS/FAZ with RIFIN

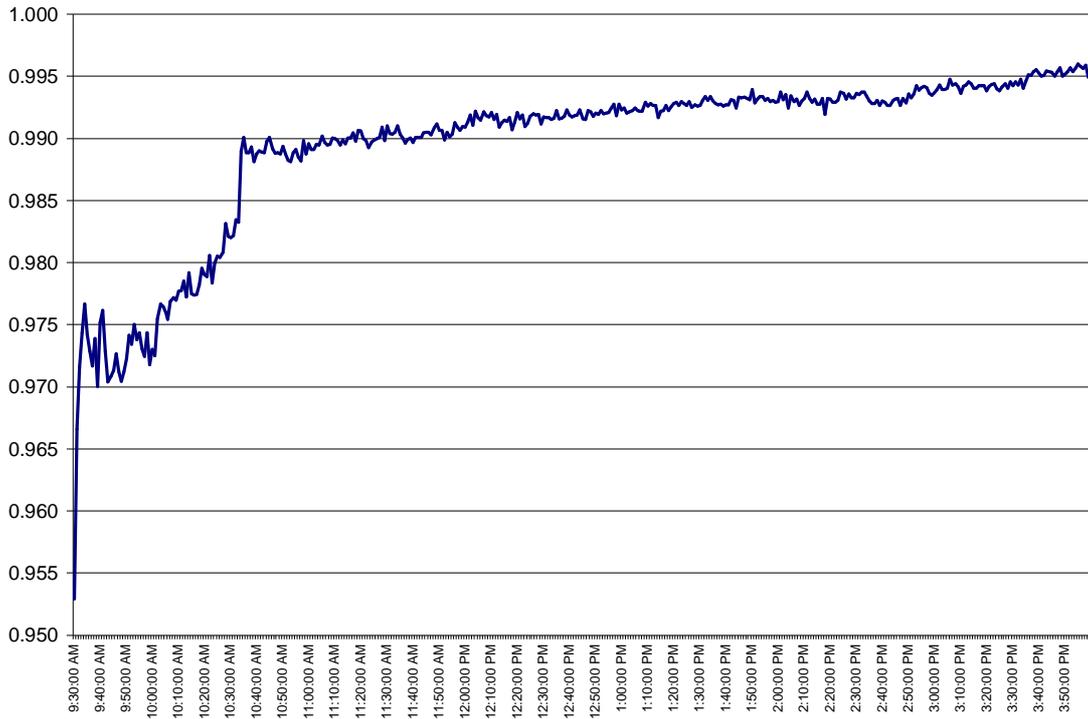


Chart 1: Minute by Minute Correlations between FAS and RIFIN (1/2/09 – 5/14/09)

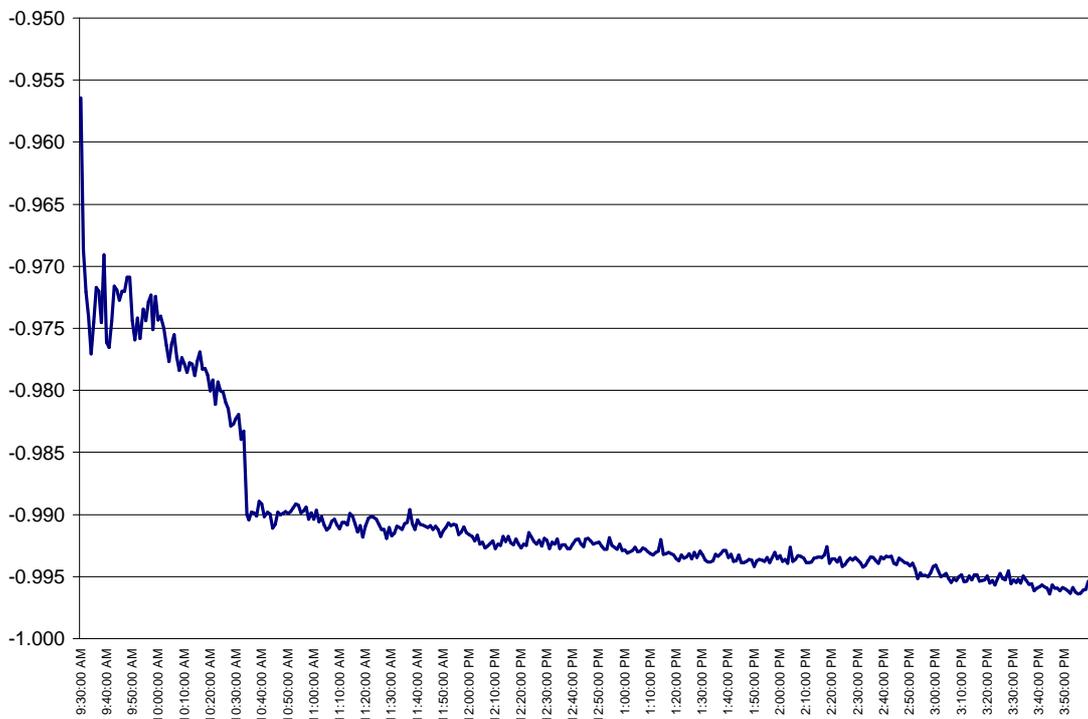


Chart 2: Minute by Minute Correlations between FAZ and RIFIN (1/2/09 – 5/14/09)

3.6 Regression of FAS/FAZ with RIFIN

Continuing to explore whether these ETFs do in fact consistently meet their investment objective of returning three times the daily return (directly or inversely) of the RIFIN index, the returns of FAS/FAZ were regressed against the returns of the RIFIN. Straight line regression coefficients were calculated over the 92 trading days of test data for both FAS with RIFIN and FAZ with RIFIN. Least squares equations were calculated for each of the 390 trading minutes of the market day. That is, a linear equation:

$$\text{ETF} = m \cdot \text{RIFIN} + b$$

was calculated between FAS and RIFIN for the 9:30AM trading minute by looking across the 92 trading days. Another equation was calculated for 9:31AM, and onwards until 3:59PM. Samples of the slopes of these equations are shown in Table 5. The complete results are represented graphically in Chart 3 and Chart 4.

As was seen with the correlation coefficients, both FAS and FAZ show their worst regressed slope with the RIFIN at 9:30AM. This is again expected and logical for the same reasons the correlation between FAS/FAZ and the RIFIN is worst at the opening bell. The off-hours market trading of FAS/FAZ and the components of the RIFIN draw the ETFs and the RIFIN out of line such that at market open, FAS/FAZ are far off from their targets. Like the correlations, though, the regression coefficient is brought back in line with the target of three within minutes and is held within a close neighborhood for the remainder of the day.

Time	FAS/RIFIN Regression Slope	FAZ/RIFIN Regression Slope	Time	FAS/RIFIN Regression Slope	FAZ/RIFIN Regression Slope	Time	FAS/RIFIN Regression Slope	FAZ/RIFIN Regression Slope
9:30:00 AM	4.64701	-4.70262	12:00:00 PM	2.91920	-2.91126	3:30:00 PM	2.92577	-2.92487
9:31:00 AM	3.43552	-3.47932	12:01:00 PM	2.92315	-2.91574	3:31:00 PM	2.92148	-2.93109
9:32:00 AM	3.10879	-3.13460	12:02:00 PM	2.93322	-2.92105	3:32:00 PM	2.92138	-2.92576
9:33:00 AM	2.95824	-2.98668	12:03:00 PM	2.93768	-2.92334	3:33:00 PM	2.94049	-2.93640
9:34:00 AM	2.92024	-2.95767	12:04:00 PM	2.93847	-2.92878	3:34:00 PM	2.94674	-2.94491
9:35:00 AM	2.89718	-2.92399	12:05:00 PM	2.93100	-2.92580	3:35:00 PM	2.94482	-2.93983
9:36:00 AM	2.86886	-2.89760	12:06:00 PM	2.93755	-2.92346	3:36:00 PM	2.95737	-2.95949
9:37:00 AM	2.87750	-2.90636	12:07:00 PM	2.92424	-2.91288	3:37:00 PM	2.94509	-2.94336
9:38:00 AM	2.85547	-2.87574	12:08:00 PM	2.91099	-2.90369	3:38:00 PM	2.93406	-2.93576
9:39:00 AM	2.83308	-2.85147	12:09:00 PM	2.92711	-2.91874	3:39:00 PM	2.94164	-2.94412
9:40:00 AM	2.85041	-2.86967	12:10:00 PM	2.92190	-2.91081	3:40:00 PM	2.92219	-2.91967
9:41:00 AM	2.83710	-2.85832	12:11:00 PM	2.91514	-2.90403	3:41:00 PM	2.91571	-2.92126
9:42:00 AM	2.83349	-2.84052	12:12:00 PM	2.92061	-2.90555	3:42:00 PM	2.91366	-2.91721
9:43:00 AM	2.82085	-2.84634	12:13:00 PM	2.92314	-2.90668	3:43:00 PM	2.91755	-2.92402
9:44:00 AM	2.86547	-2.87548	12:14:00 PM	2.93271	-2.91178	3:44:00 PM	2.92604	-2.92969
9:45:00 AM	2.85451	-2.86997	12:15:00 PM	2.92457	-2.91722	3:45:00 PM	2.92943	-2.93125
9:46:00 AM	2.87983	-2.88740	12:16:00 PM	2.92932	-2.92190	3:46:00 PM	2.92420	-2.92583
9:47:00 AM	2.87063	-2.88434	12:17:00 PM	2.92493	-2.91259	3:47:00 PM	2.92739	-2.92707
9:48:00 AM	2.83937	-2.84473	12:18:00 PM	2.92262	-2.90781	3:48:00 PM	2.91100	-2.91646
9:49:00 AM	2.87417	-2.87312	12:19:00 PM	2.92787	-2.91126	3:49:00 PM	2.91639	-2.92217
9:50:00 AM	2.85848	-2.87761	12:20:00 PM	2.92666	-2.91388	3:50:00 PM	2.91723	-2.92586
9:51:00 AM	2.85614	-2.86137	12:21:00 PM	2.92207	-2.90679	3:51:00 PM	2.92143	-2.93373
9:52:00 AM	2.85242	-2.86377	12:22:00 PM	2.92550	-2.90883	3:52:00 PM	2.91576	-2.92448
9:53:00 AM	2.83676	-2.85953	12:23:00 PM	2.91970	-2.90813	3:53:00 PM	2.90637	-2.91989
9:54:00 AM	2.83658	-2.85156	12:24:00 PM	2.91871	-2.90836	3:54:00 PM	2.89484	-2.91525
9:55:00 AM	2.84668	-2.85709	12:25:00 PM	2.93919	-2.92568	3:55:00 PM	2.88828	-2.90893
9:56:00 AM	2.84848	-2.85648	12:26:00 PM	2.92755	-2.91050	3:56:00 PM	2.87325	-2.89788
9:57:00 AM	2.84340	-2.86969	12:27:00 PM	2.92060	-2.90470	3:57:00 PM	2.85787	-2.89379
9:58:00 AM	2.85377	-2.87567	12:28:00 PM	2.93518	-2.91993	3:58:00 PM	2.84938	-2.88128
9:59:00 AM	2.85491	-2.86677	12:29:00 PM	2.92477	-2.91185	3:59:00 PM	2.79493	-2.83731
10:00:00 AM	2.88756	-2.89316	12:30:00 PM	2.93238	-2.91261			

Table 5: Samples of Regressed Y-Intercepts of FAS/FAZ with RIFIN

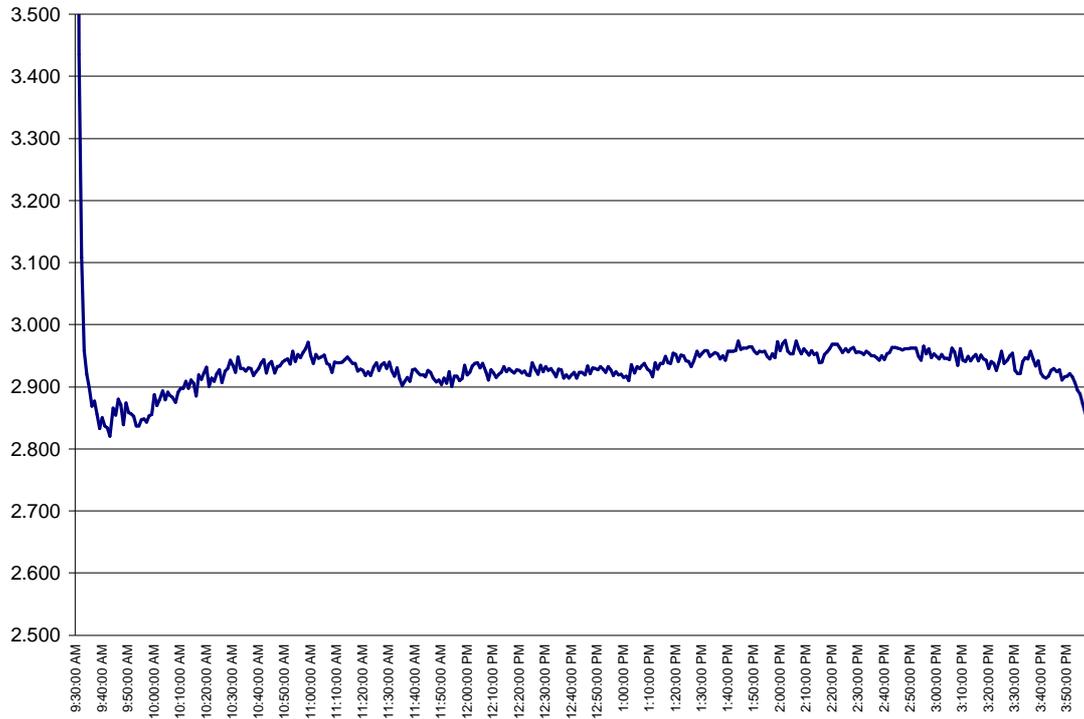


Chart 3: Minute by Minute Slopes between FAS and RIFIN (1/2/09 – 5/14/09)



Chart 4: Minute by Minute Slopes between FAZ and RIFIN (1/2/09 – 5/14/09)

4. Discount-And-Up Trading Strategy

4.1 Definition

The trading strategy that was the focus of our study was based on the idea of exploiting the excellent tracking history of FAS/FAZ with the RIFIN and the observed late day volatility that had been observed in the markets, which observers have theorized has been due recently to the increased presence of ETFs². Under our strategy, a buy order is placed for FAS or FAZ at 3:30PM if at that time the ETF is both up for the day and also trading at a discount. This position is then closed at the end of the day, that is, 30 minutes later. We term this strategy “discount-and-up.”

We define discount to mean that the ETF is trading below the level at which the RIFIN’s return at 3:30PM would imply its price should be. For example, if at 3:30PM, the RIFIN is up 5% for the day, but FAS is only up 14% for the day, a buy order would be placed for FAS. Our position in FAS would then be closed at 3:59PM, exiting us completely from the market. If, at 3:30PM, the ETF which is positive for the day is not trading at below its expected level, then no trade is made that day. For example, if the RIFIN is down 4% at 3:30PM, but FAZ is trading at 12.5% up for the day, no trade is placed. Obviously only FAS or FAZ, not both, is ever expected to be up at any point during the day.

By purchasing the positive ETF at a discount, we hope that the price will continue to rise as it attempts to catch up to its objective. Also, in rebalancing, the ETF will acquire assets related to the RIFIN that may help push the RIFIN further in the direction it is already moving, compounding the gains in the already positively trading ETF. An

ETF that is simply positive for the day, but not at a discount, will not experience this compounding and is more likely to decrease from its current level as it is trading at a premium. Being at a premium, our investment would not gain as greatly from a rise in the RIFIN, as that rise would not require the ETF to move unless the rise is so great that it pushes past the level the at which the ETF is already positioned.

Likewise, buying an ETF simply based on it being traded below its target is not sufficient to expect it to rise. If the ETF is discounted, but not trading up for the day, it may be that the ETF is being purposely held out of balance by its custodians who anticipate that the RIFIN will continue to move in its current direction and the ETF will end up in the proper spot with minimal rebalancing from its current position. For Direxion, the potential of the RIFIN to move in line to the ETF has financial rewards over shifting assets and incurring the transaction costs associated with the sale and purchase of the RIFIN swaps and RIFIN component securities, especially in that rebalancing early is no guarantee that another rebalance with additional costs will not be necessary nearer the close.

4.2 Results on Initial Data (Jan 2, 2009 – May 14, 2009)

The discount-and-up strategy was first tested against our original set of minute-by-minute data for the period of January 2, 2009 through May 14, 2009, which consists of 92 trading days. Chart 5 shows the cumulative returns for the discount-and-up strategy. Notice that there are six paths graphed. Each path represents a different purchase time. While the decision to invest or not was made at 3:30PM, we recognize that placing the

buy order may be naturally delayed a few minutes. Therefore, we present different results for the possible delays.

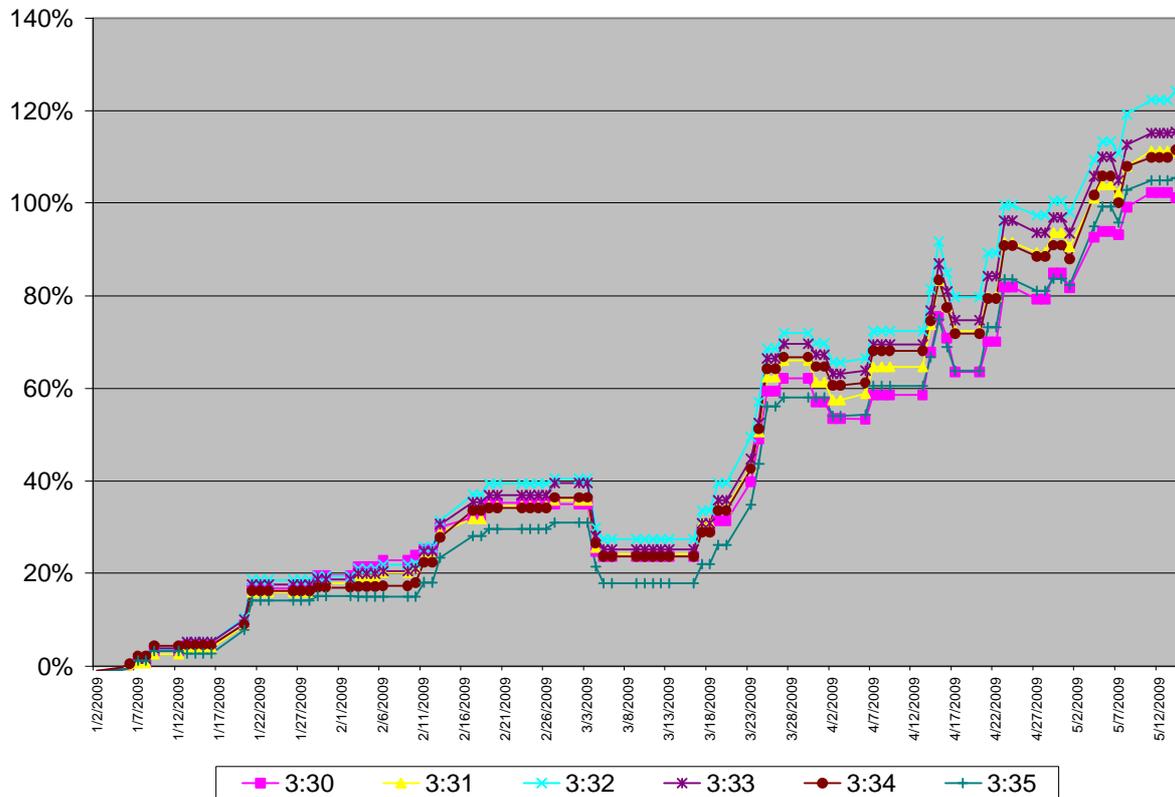


Chart 5: Cumulative returns for discount-and-up trading strategy – 1/2/09 – 5/14/09

Following the discount-and-up strategy, we see that orders perfectly placed at 3:30PM, followed by a close of that position at 3:59PM would have generated a 101.2% return over the given time period.

The following graph, Chart 6, shows the returns over the same time period had we only chosen to apply the strategy to FAS, and had ignored FAZ.

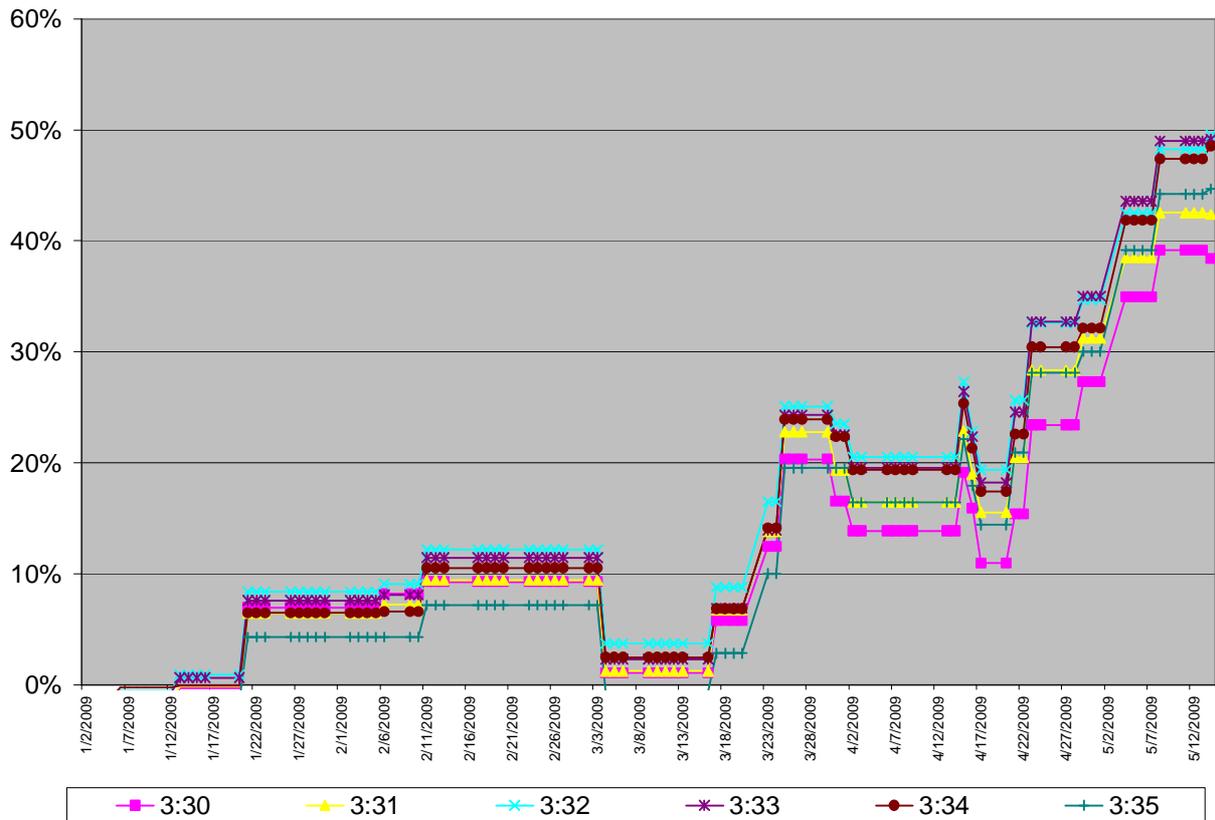


Chart 6: Cumulative returns for discount-and-up trading strategy: FAS Only

The following graph, Chart 7, this time shows the returns over the same time period had we only chosen to apply the strategy to FAZ, and had ignored FAS.

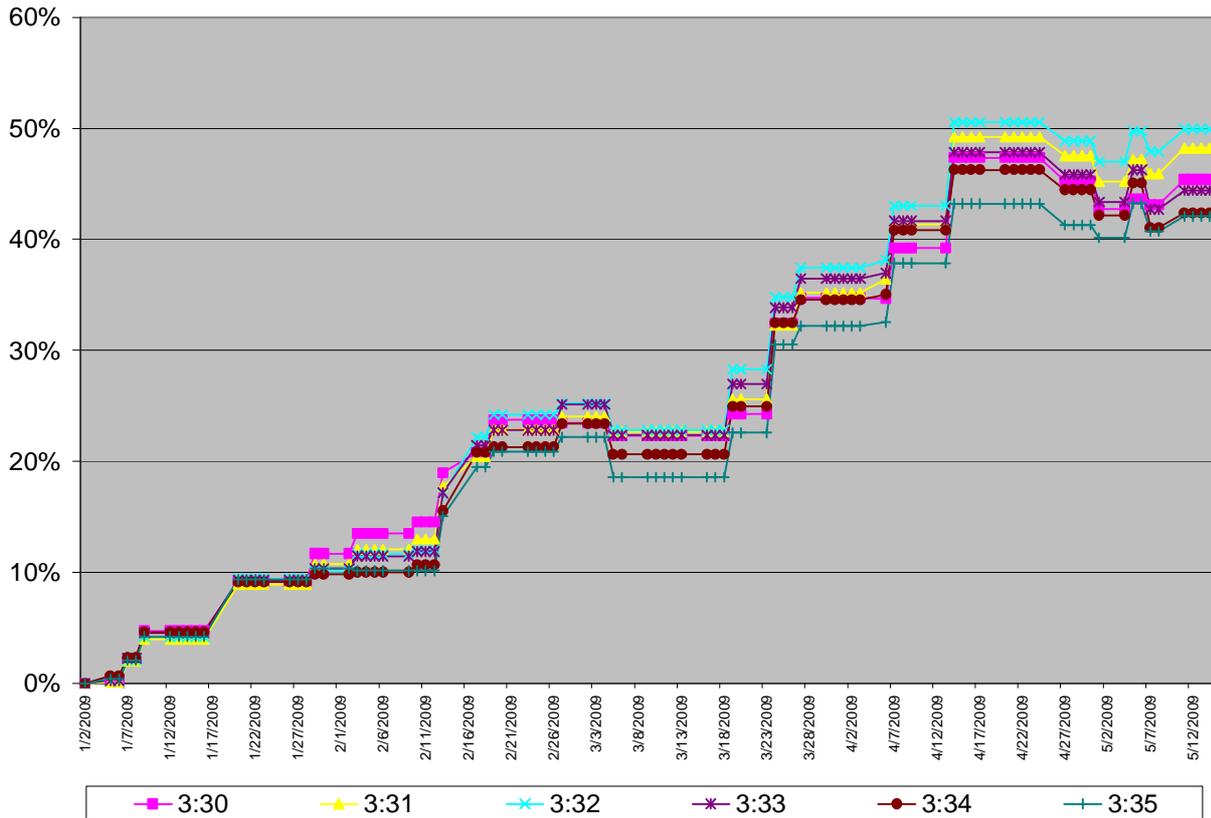


Chart 7: Cumulative returns for discount-and-up trading strategy: FAZ Only

We present the two previous graphs to demonstrate that the discount-and-up strategy performed equally well on both FAS and FAZ and that one ETF did not subsidize the other ETF in the combined results presented initially.

The table below summarizes the results of the discount-and-up strategy as applied to both ETFs and as applied to each ETF individually.

Discount-and-Up Strategy						
Cumulative Returns: Jan 2, 2009 - May 14, 2009						
Purchase Times						
	3:30PM	3:31PM	3:32PM	3:33PM	3:34PM	3:35PM
ETFs						
FAS	38.4%	42.4%	49.5%	49.2%	48.5%	44.7%
FAZ	45.4%	48.2%	49.9%	44.4%	42.4%	42.1%
Both	101.2%	111.0%	124.2%	115.4%	111.5%	105.5%

Table 6: Discount-and-Up Cumulative Returns Varying Purchase Times

While these results lend evidence to the idea that a slight delay in being able to purchase the ETF does not destroy the return on the strategy, we also explored the idea that execution delays may make it prudent to sell off our holdings slightly earlier than 3:59PM to ensure the closure of the position. The next table shows the results over the same time period had the purchases been made at 3:30PM and sold off between 3:55PM and 3:59PM of the same day and the results had the trades not been placed in time, but made between 9:30AM and 9:35AM of the next trading day. As with purchasing the ETF, a less than perfect timing of the sell transaction does not destroy the investment return. However, failure to place one's order before the close of the day does have severe consequences to performance. Again, we present the results for having applied discount-and-up to both ETFs and to FAS and FAZ individually.

Discount-and-Up Strategy
Cumulative Returns: Jan 2, 2009 - May 14, 2009

Time of Sale	FAS	FAZ	Both
3:55PM	40.2%	43.3%	100.9%
3:56PM	43.8%	52.9%	119.8%
3:57PM	43.5%	56.5%	124.6%
3:58PM	50.8%	54.5%	133.1%
3:59PM	38.4%	45.4%	101.2%
9:30AM	-42.9%	58.2%	-9.7%
9:31AM	-43.9%	71.1%	-4.0%
9:32AM	-43.5%	66.5%	-6.0%
9:33AM	-39.5%	72.0%	4.0%
9:34AM	-40.0%	80.0%	8.0%
9:35AM	-38.2%	85.9%	14.9%

Table 7: Discount-and-Up Cumulative Returns Varying Sell-Off Times

Here, we see that failing to close one's position by the end of the day turned a 101.2% gain into as much as a 10% loss. Unlike the changes in purchases times, though, we see that FAS and FAZ are not equally affected. FAZ actually seems to benefit from having to hold the position overnight. Recalling that FAZ is the inverse ETF, it is likely benefiting from the fact that on down market days, people who did not close their positions by the end of the day may leave them open to sell in the morning as they are exiting the market in general. Also, investors may choose to exit the market following the down day.

4.3 Simulation Results on Initial Data (Jan 2, 2009 – May 14, 2009)

With the promising results based on the original data, we then ran simulations to gather more descriptive statistics. Using the bootstrap with replacement technique, we generated 1,000 30-day trading periods from the original data using the discount-and-up strategy. Bootstrap with replacement is a sampling method³. In the data set, there are 92 trading days and hence 92 results for the discount-and-up strategy. These results include several returns of 0%, as the conditions to place a buy order are not always met. From the set of 92 returns, we randomly choose 30, one at a time, replacing them after selection. This methodology is called bootstrap with replacement.

We repeat this procedure 1,000 times, hence creating 1,000 simulated 30-day trading periods. From these 1,000 results, we calculate the most frequently investigated statistical measures of investment return and risk. The two less familiar measures in the following table are Value at Risk (VaR) and Conditional Tail Expectation (CTE).

VaR is a percentile measurement. VaR(95) is the result that is beaten 95% of the time. In the table, VaR(95) is 1.99%. Therefore, in 95% of the 1,000 simulated trading horizons, the discount-and-up strategy generated a total return of 1.99% or better.

CTE is an offshoot of VaR. Of the scenarios that fall below the VaR(95) level, the CTE(95) is the average of those results. In the table below, CTE(95) is -2.97%. Therefore, when the trading strategy result is less than 1.99%, it is on average -2.97%. CTE is often interpreted as “how bad do I expect things to go when things go bad?” A CTE close to the VaR usually implies more stability in worst case scenarios. However, a

CTE significantly less than the VaR can mean that one's investment strategy is much more precarious and exposed to significant downside risk.

Simulated 30-Day Trading Period Results	
	Original Data
Avg 30-Day Return	27.45%
Variance of Returns	2.77%
Std Dev of Returns	16.64%
Median Return	26.47%
VaR (95)	1.99%
VaR (90)	7.46%
CTE (95)	-2.97%
CTE (90)	0.81%

Table 8: Statistical Results for Discount-and-Up

With an average return of 27.45% in 30-trading days, the simulated results are inline with the actual results we saw over the original 92-day data set. Compounding 27.45% three times, we get a 107% return, which closely match the 101.2% return showcased earlier. More promising than the return, though, are the value at risk and conditional tail expectation results. With a positive VaR at both the 95th and 90th levels, it implies that there is less than a 5% chance that the strategy will lead to a loss over the 30-day trading period. Also, the CTE(95) tells us that when returns are in the lower 5th percentile, we should expect a loss of only 3% of the investment. The CTE(90) of 0.81% shows that we should expect to be at about the break-even level if the 30-day trading period turns out to be in the lower 10th percentile (that is one month per year.)

The results above assume perfect trade execution. That is, buy orders are placed (if necessary) at 3:30PM and sell orders are placed at 3:59PM. We show earlier in the paper that delays in placing purchase orders do not have significant impact on the results of the strategy. We ran simulation exercises for each of the 6 purchase times (3:30PM to

3:35PM) and those results confirmed that indeed, delays in buying the ETFs do not noticeably impact results. We do not show those results here, but we do demonstrate again the importance of closing the positions before the end of the day.

Simulated 30-Day Trading Period Results							
Data from Jan 2, 2009 - May 14, 2009							
Sell Times	Avg	Varaince	Std Dev	VaR(95)	VaR(90)	CTE 95	CTE 90
3:55 PM	26.99%	2.20%	14.85%	4.55%	8.25%	0.07%	3.19%
3:56 PM	30.94%	2.77%	16.63%	5.32%	10.57%	0.66%	4.40%
3:57 PM	31.93%	3.03%	17.40%	4.88%	10.51%	-0.08%	3.95%
3:58 PM	33.82%	3.43%	18.52%	6.03%	10.68%	0.42%	4.52%
3:59 PM	27.45%	2.77%	16.64%	1.99%	7.46%	-2.97%	0.81%
9:30 AM	1.46%	11.00%	33.17%	-41.87%	-35.84%	-46.33%	-42.34%
9:31 AM	3.62%	11.82%	34.39%	-40.70%	-34.27%	-46.24%	-41.88%
9:32 AM	3.28%	12.36%	35.16%	-41.81%	-35.04%	-47.64%	-43.06%
9:33 AM	6.83%	13.23%	36.38%	-39.95%	-33.26%	-45.96%	-41.31%
9:34 AM	7.91%	12.75%	35.71%	-38.31%	-31.66%	-44.39%	-39.68%
9:35 AM	9.58%	12.33%	35.12%	-36.25%	-28.98%	-43.08%	-37.82%

Table 9: Statistical Results for Discount-and-Up with Varying Sell-Off Times

This table shows that all statistical measures of success of the discount-and-up strategy decay precipitously if the position is held overnight. The average return drops to as little as 1.5%; variances quadruple; standard deviations double; and VaR's and CTE's become so unacceptably high that the use of the strategy would be deemed imprudent.

4.4 Comparison to Popular Strategies

Earlier we mentioned that we believe the discount-and-up strategy is superior to the popularly mentioned momentum play of simply buying into an ETF in the last hour of trading if it is trading up for the day. We also believed that discount-and-up would perform better than simply buying an ETF if it is at a discount. This strategy is often mentioned on ETF message boards, but not documented with actual results. In the graph below, we show the statistical comparison between the three strategies, discount-and-up, buy if up, and buy if discounted, running each strategy through the 1,000 30-day simulations.

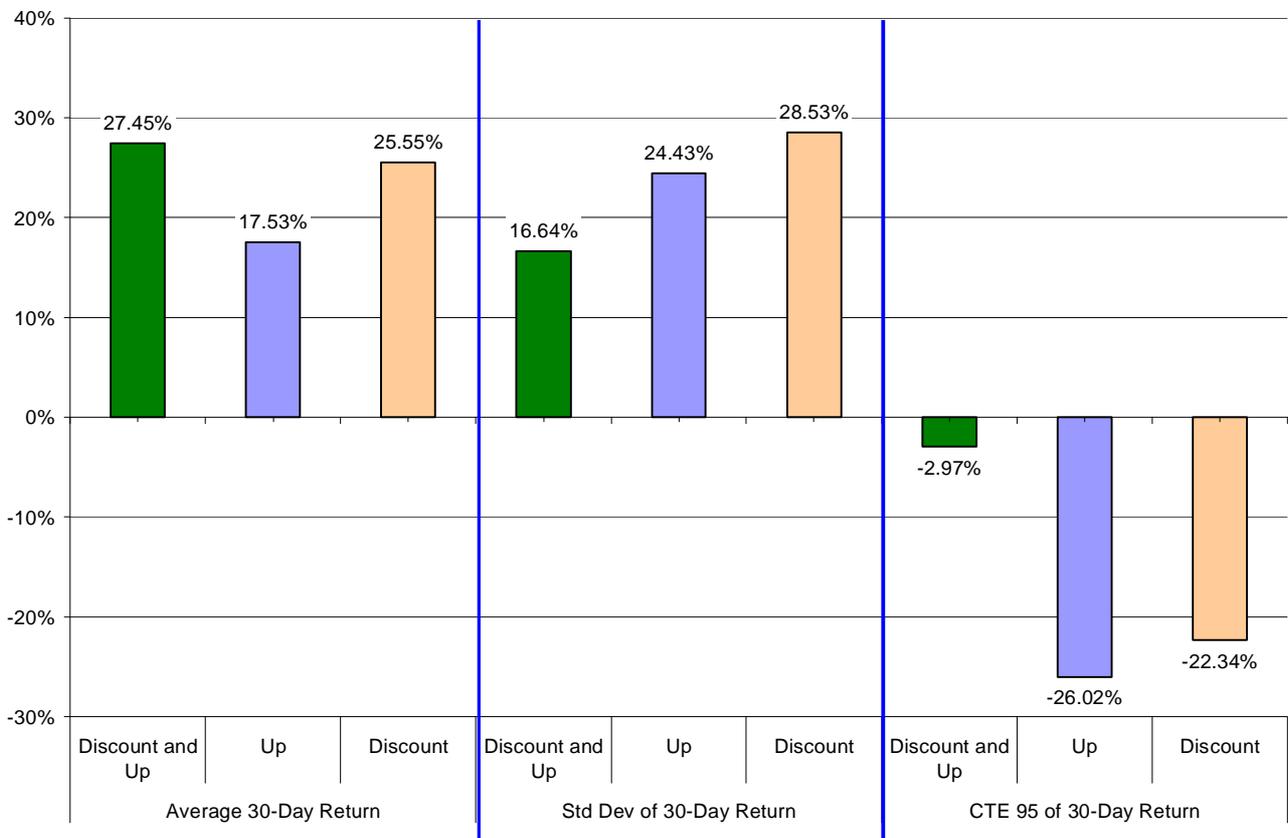


Chart 8: Comparison of Three ETF Trading Strategies

Here, we see that the discount-and-up strategy outperforms the two simpler alternatives in all three categories of risk and reward. While the pure discount strategy is close enough to the discount-and-up strategy in terms of average return (27.45% vs 25.55%), discount-and-up outperforms pure discount in both standard deviation, being about half as volatile, and in CTE(95), having only about 10% of the expected loss of the pure discount strategy in the case of a lower 5th percentile performance. Surprisingly, the strategy most often advocated in industry, playing the ETF momentum², performs worst

in all three categories. It offered the lowest returns, higher volatility, and the highest expected loss in the case of unfavorable results.

4.5 Results on Expanded Data (Jan 2, 2009 – Oct 14, 2009)

When the data became available, the discount-and-up strategy was retested against the actual ETF behavior documented through October 15, 2009. Below, we show the graph of the cumulative returns of discount-and-up for the whole year through Oct 15. The following graph, Chart I, assumes perfect buy and sell executions at 3:30PM and 3:59PM, respectively.

We see that while the strategy continued to do well, the growth slowed significantly shortly after the end of the original data set. The vertical line in the graph marks the date of May 14, which is the last day of the original data set. Year to date, on October 15, the strategy returned 132.5%. Therefore, the 101.2% gain as of May 14 was followed by only a 31% gain thereafter.

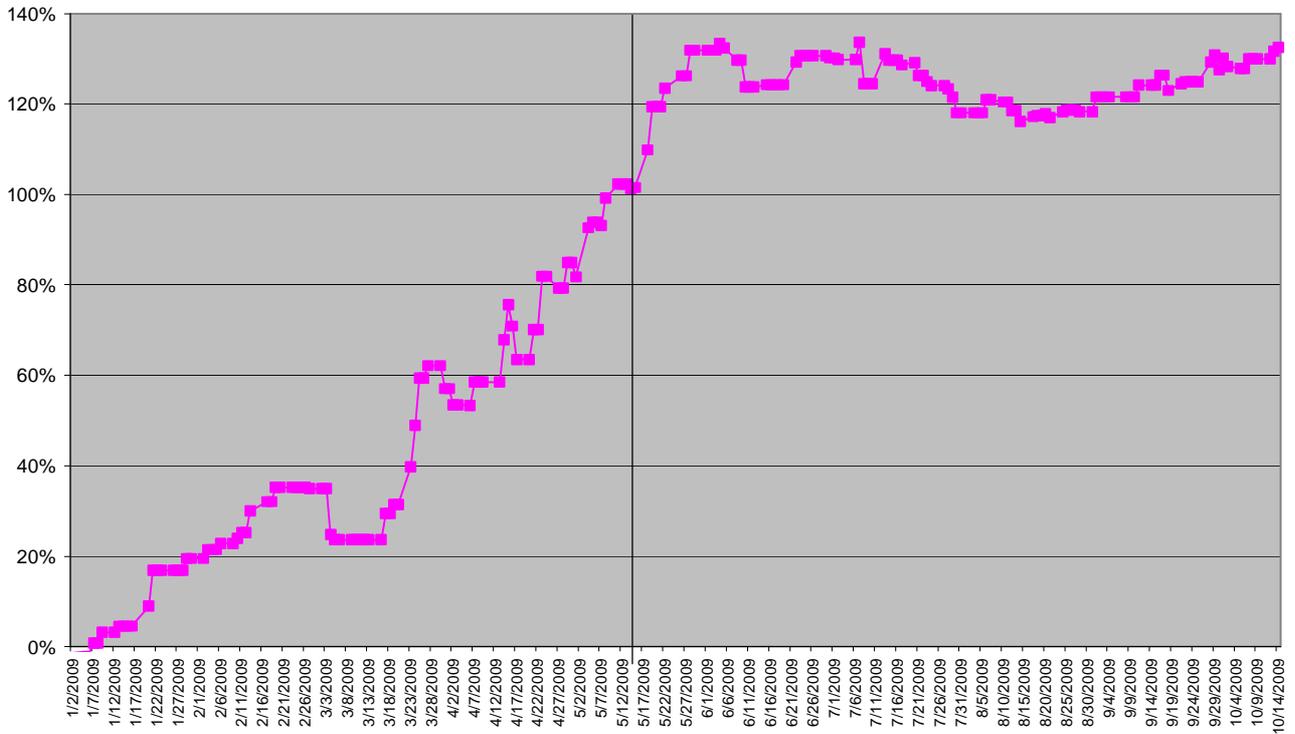


Chart 9: Discount-and-up returns Jan 2, 2009 – Oct 14, 2009

Again, we present the results of the strategy had it been applied to FAS only and had it been applied to FAZ only. As with the original data set, we see that there is no significant difference in behavior between the two ETFs. One ETF is not responsible for either lifting or dragging down the strategy as a whole.

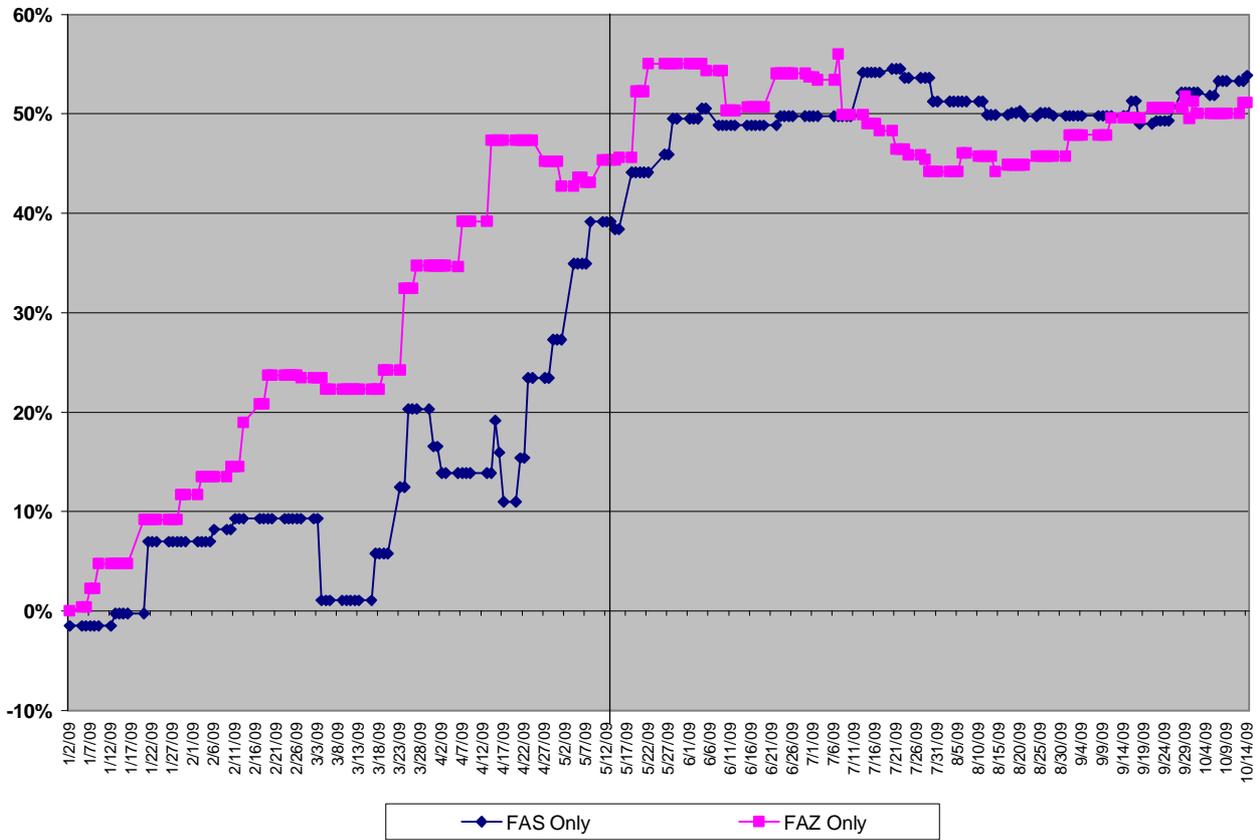


Chart 10: Discount-and-up returns Jan 2, 2009 – Oct 14, 2009: Applied to FAS and FAZ Only

4.6 Simulation Results on Expanded Data (Jan 2, 2009 – Oct 14, 2009)

While the results were not as promising on the expanded set of data compared to the original data, the period from May 15 to Oct 14 does only represent one realization of the ETF behavior. Having a larger set of data, we generated a new set of 1,000 30-day scenarios and recalculated the relevant statistics. Again, bootstrap with replacement was used to generate the returns for the discount-and-up strategy.

Below we compare the results of the original and expanded data sets.

Simulated 30-Day Trading Period Results		
	Original Data	Expanded Data
Avg 30-Day Return	27.45%	13.59%
Variance of Returns	2.77%	1.20%
Std Dev of Returns	16.64%	10.95%
Median Return	26.47%	12.72%
VaR (95)	1.99%	-3.20%
VaR (90)	7.46%	0.15%
CTE (95)	-2.97%	-6.84%
CTE (90)	0.81%	-4.14%

Table 10: Comparison of Results Between Two Data Sets

Between the two data sets, we see a 50% drop in expected return, slightly improved volatility, but slightly degenerated VaR and CTE values. The more complete set of data tells us that discount-and-up may not perform as well as previously thought in terms of expected 30-day return, but the return is more stable. However, the potential for poor results is greater than the original data set revealed. Var(95) and VaR(90) are now both negative and the CTE's are in turn lower than first calculated.

4.7 Comparison of RIFIN Between Data Set Time Periods

To try to understand what drove the change in results, we compared the RIFIN's behavior between Jan 2, 2009 – May 14, 2009 to its behavior during May 15, 2009 – Oct 14, 2009. Simply graphing the empirical cumulative distribution functions of the RIFIN from the two time periods, we see that the earlier time period was much more volatile than the later.

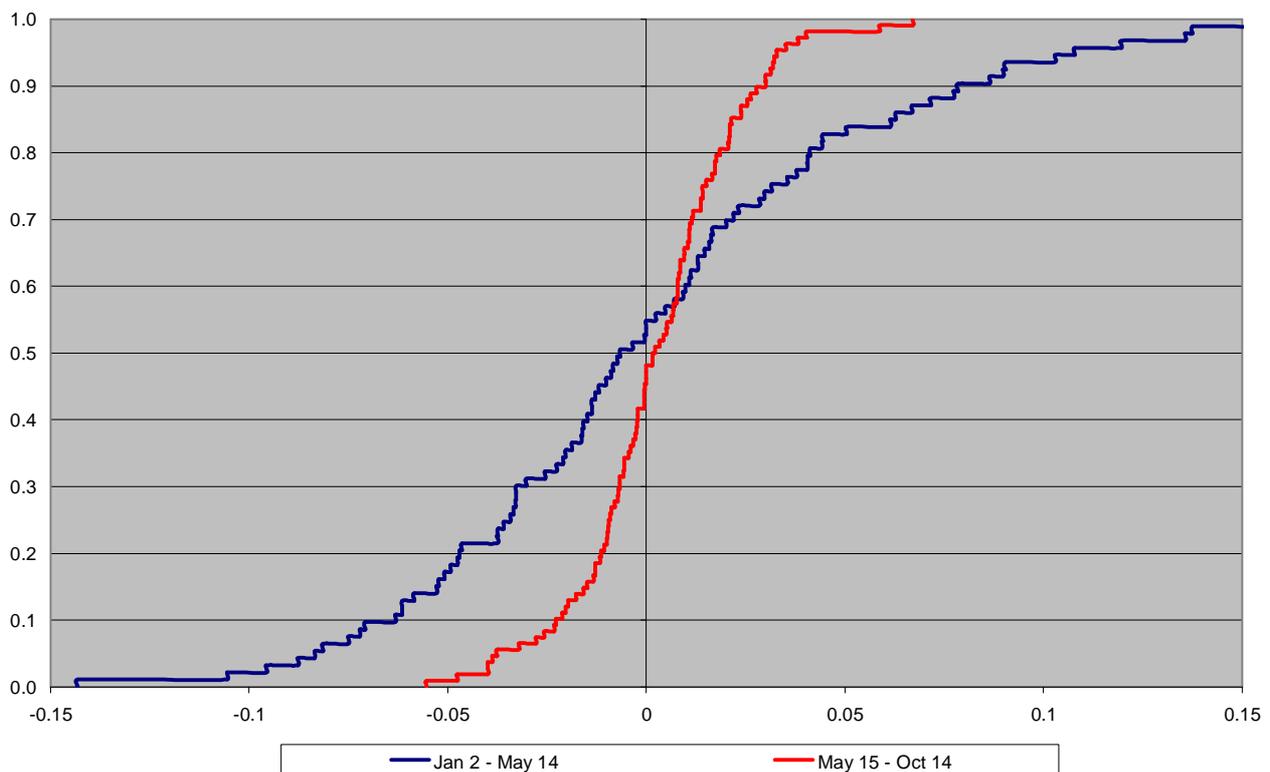


Chart 11: RIFIN - Cumulative Distribution Functions - Daily Returns

While these plots hint at normality, we did turn to Minitab to more definitively check the results of the daily RIFIN results in each of the two time periods. Results of those runs are seen in Chart L and Chart M.

The mean daily RIFIN return for the original time period was calculated at 0.001085 with a 95% confidence interval of (-0.0106, 0.01279), so we cannot say with any statistical certainty that its mean is not zero. The standard deviation of the RIFIN over this time period is 0.0568.

The mean daily RIFIN return for the later time period was calculated at 0.002754 with a 95% confidence interval of (-0.012, 0.0067), so we again cannot say with enough certainty that its mean is not zero either. The standard deviation of the RIFIN over this period is 0.02068.

The lower standard deviation and the observed spread of returns in the graphs above tell us that the time period Jan 2 – May 14 was more volatile for the RIFIN than the time period May 15 – Oct 14. This conclusion is also supported by the economic climate adjustments that began to occur in Q3 2009 as the economy began to exit the recession and overall stock market stability took hold.

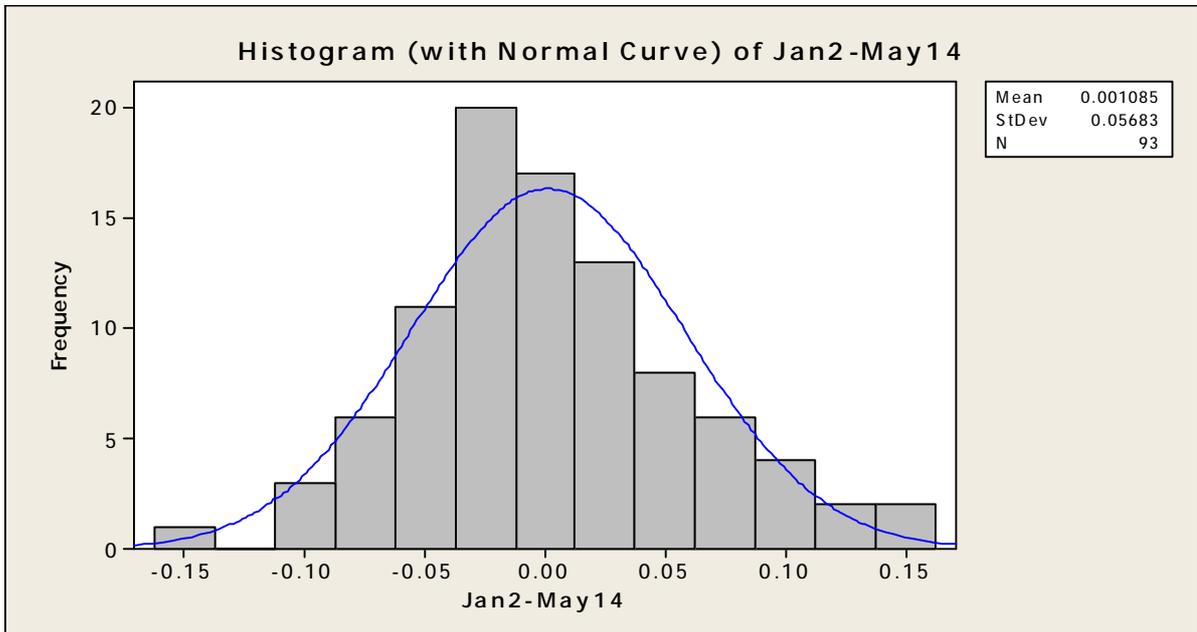


Chart 12: Normality fit of RIFIN daily returns from Jan 2 – May 14

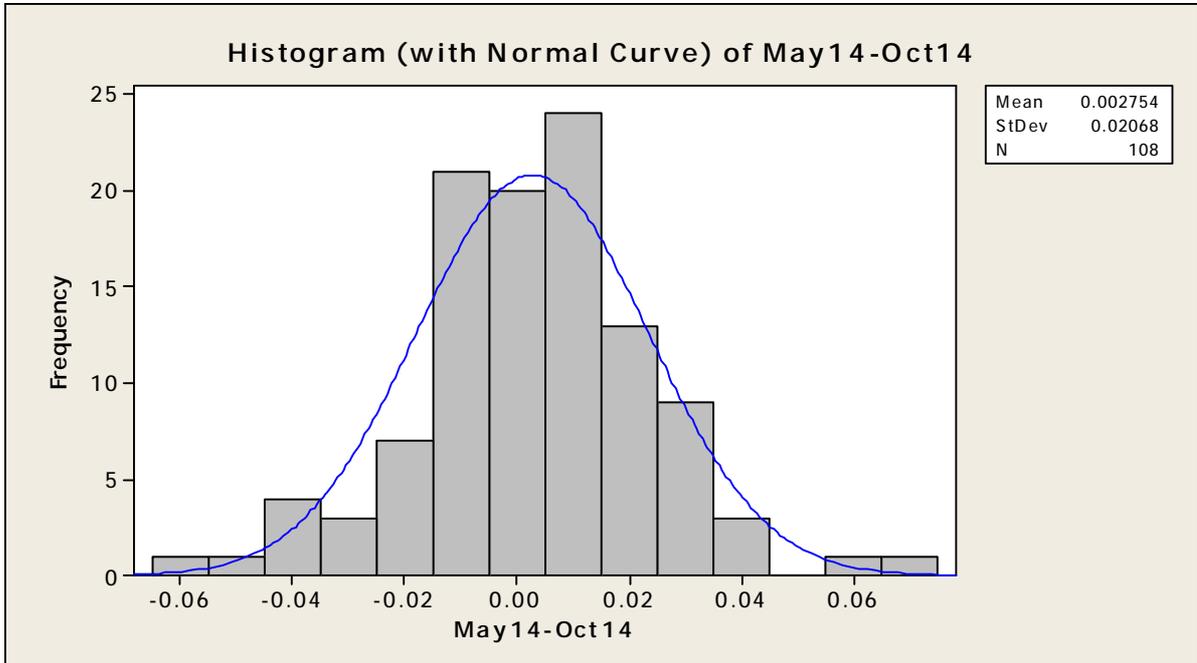


Chart 13: Normality fit of RIFIN daily returns from May 14 – Oct 14

The following box plot further shows that the spread of RIFIN returns was much tighter in the second time period than in the original time period.

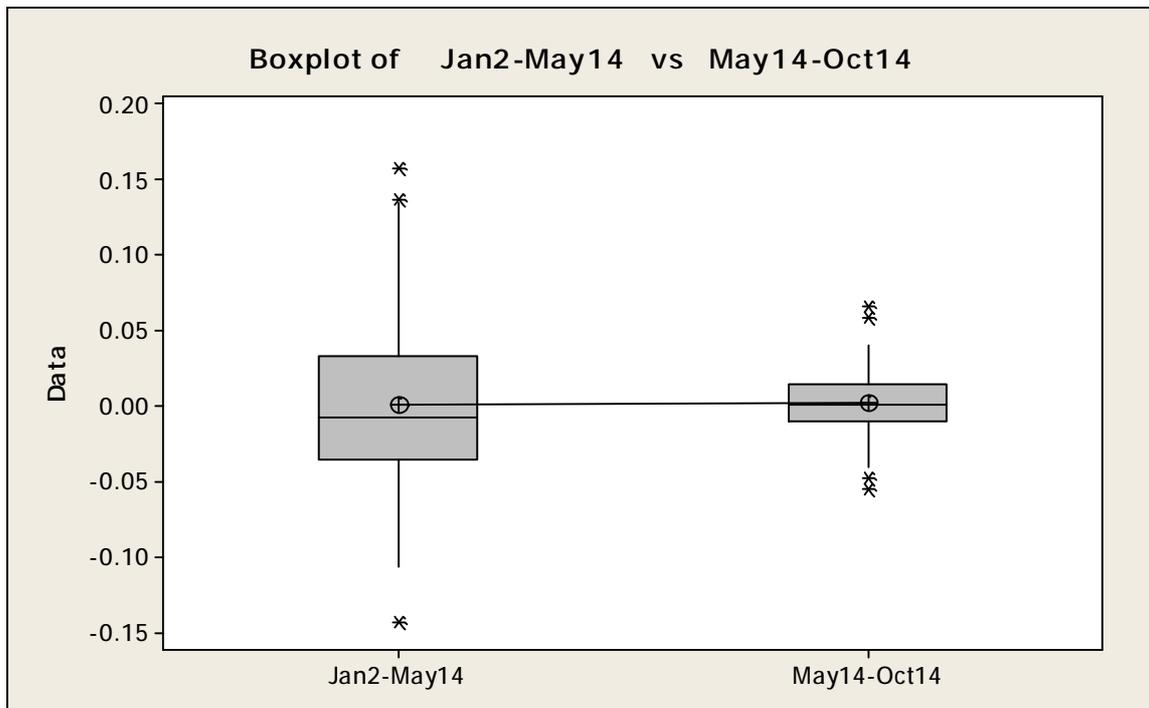


Chart 14: Box Plots of RIFIN returns over separate time periods

Furthermore, the two-sample t-test of the null hypothesis that the difference between means is not zero returned a P-Value of 0.789, which does not allow us to reject that hypothesis. We again infer that the means of the RIFIN returns between the two time periods should be assumed to be equal and assumed to be zero.

Minitab Results for Comparison of mu's

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Difference = mu (Jan2-May14) - mu (May14-Oct14)
Estimate for difference: -0.001669
95% CI for difference: (-0.013993, 0.010654)
T-Test of difference = 0 (vs not =): T-Value = -0.27 P-Value = 0.789
```

5. Issues of Buy-and-hold Strategy with Leveraged ETFs

5.1 Year to Date Returns

The discount-and-up strategy involves minimal transaction costs, being that at most two trades would need to be placed on a given day. The question arises, though, if the effort of using discount-and-up is worthwhile compared to a pure buy-and-hold strategy. For example, from Jan 2, 2009 to Oct. 15, 2009, the RIFIN rose 26%. With FAS being triple leveraged, shouldn't a buy-and-hold investor expect a 78% return for this time period? The answer is no.

Investors must remember that FAS and FAZ are leveraged *daily* returns. Accordingly, FAS and FAZ are only obligated to replicate triple the change in RIFIN from the opening bell to the closing bell, not for any longer period of time. The following graph, Chart O, shows the YTD returns of FAS and FAZ in 2009. It also plots the YTD return of RIFIN and the YTD return of 3*RIFIN. Note that 3*RIFIN is three times the YTD return of the RIFIN. This measure is different than FAS which is the compounded return of three times the daily returns. We see that despite a 26% rise in RIFIN, both FAS and FAZ are down over the same time period. Subsequently, we see that despite the naturally intuitive assumption, FAS and FAZ year to date returns are not reflective inverses of each other.

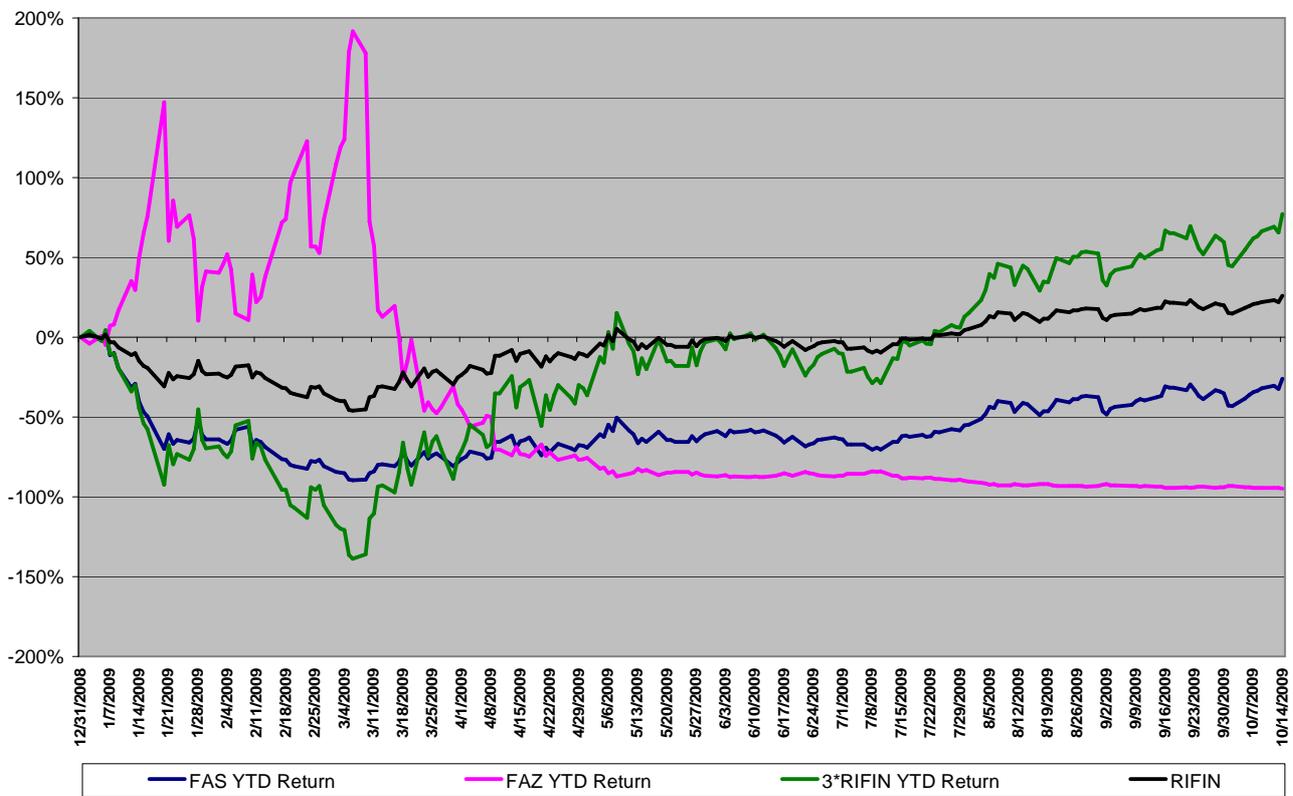


Chart 15: YTD Returns

Below are the year to date returns as of Oct. 15, 2009.

Buy-and-hold returns - Oct 15, 2009	
RIFIN	25.7%
3*RIFIN YTD Return	77.0%
FAS YTD Return	-26.3%
FAZ YTD Return	-95.1%

Table 11: Cumulative Returns for Buy-and-Hold

5.2 Daily Leveraged Returns vs. Leveraged Total Returns

The difference between being leveraged and being daily leveraged can be seen in the following simple two-day example. For this example, we start with RIFIN, FAS, and FAZ all set at 100.

RIFIN day1: Up 10%

$$\begin{aligned} \text{RIFIN} &= 110 \\ \text{FAS} &= 130 \\ \text{FAZ} &= 70 \end{aligned}$$

RIFIN day 2: Down 10%

$$\begin{aligned} \text{RIFIN} &= 99 \\ \text{FAS} &= 91 \\ \text{FAZ} &= 91 \end{aligned}$$

Here, we see that a net change of 1% in the RIFIN created 9% swings in both FAS and FAZ, not 3% swings. If the leveraging were based on the total returns, FAS would be at 97 and FAZ would be at 103.

The issue is in the basic algebra to calculate each type of return. Below are the general formulas for the leveraged daily return (LDR) and leveraged total return (LTR). Here r is the daily return and x is the leveraging rate.

$$\text{LDR} = \prod_{i=1}^n (1 + xr_i) - 1 \quad \neq \quad x \cdot \left(\prod_{i=1}^n (1 + r_i) - 1 \right) = \text{LTR}$$

For the simple case of $n = 2$, we see that:

$$\text{LDR} = (1+xr_1) (1+xr_2) - 1 = xr_1 + xr_1 + x^2r_1r_2$$

$$\text{LTR} = x * ((1+r_1) (1+r_2) - 1) = xr_1 + xr_1 + xr_1r_2$$

Firstly, we see that the two results are only equal in the case that $x = 1$, that is, without the presence of leveraging. The difference between the returns of daily leveraging and total leveraging after two days is shown below.

$$\text{LDR} - \text{LTR} = x^2r_1r_2 - xr_1r_2$$

$$\text{LDR} - \text{LTR} = x(x-1)r_1r_2$$

For a triple leveraged ETF, like FAS which has $x=3$, we see that the $x(x-1)$ term generates a difference between results of a factor of 6. We see this manifest in the numerical example above where FAS ends the second day at 91, 6 below a total leveraged result of 97. However, for a triple inverse ETF, like FAZ, the $x(x-1)$ term generates a difference between results of a factor of 12. We see this phenomenon demonstrated in the numerical example above where FAZ ends the second day at 91, 12 below what would have been a total leveraged result of 103.

The importance of these dynamics should be noted by the investor. Simply believing that the RIFIN will drop over some time period is not sufficient justification for taking a position in FAZ. In the simple example above, the belief that the RIFIN would fall (even in the short run) was correct, but an investment in FAZ still lost 9%.

We turn now to the more general case of n days, and look at the difference between the leveraged daily returns and the leveraged total return.

For general n ,

$$\text{LDR} = (1+xr_1) (1+xr_2) (1+xr_3) \dots(1+xr_n) - 1$$

$$\text{LDR} = \sum_{i=1}^n xr_i + \sum_{i \neq j}^n x^2 r_i r_j + \sum_{i \neq j \neq k}^n x^3 r_i r_j r_k + \dots + x^n \prod_{i=1}^n r_i$$

$$\text{LDR} = x \sum_{i=1}^n r_i + x^2 \sum_{i \neq j}^n r_i r_j + x^3 \sum_{i \neq j \neq k}^n r_i r_j r_k + \dots + x^n \prod_{i=1}^n r_i$$

and,

$$\text{LTR} = x * ((1+r_1) (1+r_2) (1+r_3) \dots (1+r_n) - 1)$$

$$\text{LTR} = x \sum_{i=1}^n r_i + x \sum_{i \neq j}^n r_i r_j + x \sum_{i \neq j \neq k}^n r_i r_j r_k + \dots + x \prod_{i=1}^n r_i$$

Now, looking at LDR – LTR, the difference between leveraging daily returns and leveraging the cumulative returns,

$$\begin{aligned}
\text{LDR} - \text{LTR} &= \left(x \sum_{i=1}^n r_i - x \sum_{i=1}^n r_i \right) + \left(x^2 \sum_{i \neq j}^n r_i r_j - x \sum_{i \neq j}^n r_i r_j \right) + \\
&\quad \left(x^3 \sum_{i \neq j \neq k}^n r_i r_j r_k - x \sum_{i \neq j \neq k}^n r_i r_j r_k \right) + \dots + \left(x^n \prod_{i=1}^n r_i - x \prod_{i=1}^n r_i \right) \\
&= x(x-1) \sum_{i \neq j}^n r_i r_j + x(x^2-1) \sum_{i \neq j \neq k}^n r_i r_j r_k + \dots + x(x^{n-1}-1) \prod_{i=1}^n r_i
\end{aligned}$$

Recall that r is the daily return of the RIFIN. Earlier data showed that the RIFIN's average daily return is about 0.002. More important than the actual number is the order of that number. Here, r is of the order $O(10^{-3})$. Therefore, as early as the second term of the difference above, the order becomes $O(10^{-9})$, five orders of ten beyond that of the common financial limit of significance of $O(10^{-4})$, called basis points.

The first term containing $x(x-1)$, is also a sum of terms of $O(10^{-6})$. However, with triple leveraged funds, the $x(x-1)$ is approximately 10^1 and for $n = 100$, the summation naturally pushes itself up one or two orders, pushing the difference up to between $O(10^{-3})$ and $O(10^{-2})$. The leveraging factors and similar size n do not move the remaining terms into significance for consideration.

Therefore, we the difference between leveraged daily returns and leveraged total returns, as n gets large, is approximately:

$$\text{LDR} - \text{LTR} \approx x(x-1) \sum_{i \neq j}^n r_i r_j + O(10^{-9})$$

For FAS,

$$\text{LDR}_{\text{FAS}} - \text{LTR}_{\text{FAS}} \approx 6 \sum_{i \neq j}^n r_i r_j$$

And for FAZ,

$$\text{LDR}_{\text{FAZ}} - \text{LTR}_{\text{FAZ}} \approx -12 \sum_{i \neq j}^n r_i r_j$$

In general, we see that daily leveraging will lead to dramatically different returns compared to longer-term leveraging. In some cases, this difference benefits the daily leveraged investor and in some cases the daily leveraged investor is worse off. If one invests in FAS and the RIFIN has a positive daily gain for some time period, then the summation of all the mixed products of daily returns is guaranteed to be positive and will be multiplied by a factor of 6 to create a much larger gain for having leveraged daily than having leveraged the end cumulative result. Likewise, over that same time period, an investor in FAZ would see a factor of negative 12 applied to the summation of the mixed products of daily returns and will be much more worse off from having leveraged daily.

For markets more volatile than all positive returns, though, the sums of the mixed products of daily returns are not guaranteed to be either positive or negative. Examples containing mixtures of positive and negative daily RIFIN returns can be constructed that show the ETF investor did better by being exposed to leveraging daily and, as we showed

earlier, examples can be constructed that show he would have been better off to invest in a vehicle that leveraged the cumulative return.

For an actual example of the former, we turn back to our data. From Jan 2, 2009 to Jan 20, 2009, the RIFIN fell 31% from 654 to 452. Over that same time period, FAZ grew 147%, from \$36 to \$88. Here being invested in a buy-and-hold of FAZ did an extra 5,300 basis points better than investing in a way that triples the cumulative return, which would have returned 93%. Likewise, over that same time period, FAS fell from \$25 to about \$8, a drop of 70%. While a loss, this return is still 2,000 basis points better than had the investor triple leveraged the cumulative return of the RIFIN.

The inability to track the RIFIN for longer than one day not only makes these ETFs inappropriate investments for the individual buy-and-hold investor, it also makes these ETFs a poor choice for institutional investors seeking to hedge a position in the financial sector. Hedge positions are typically established with the goal of protecting one investment with a second investment meant to track and guard against the downside risk of the original investment. As such, FAZ would only be useful as a daily hedge against drops in the RIFIN, not as a long-term hedge against deterioration in the financial services sector as a whole.

6. Conclusions and Suggestions for Future Study

While leveraged ETFs are not proper investments for the buy-and-hold long term investor or as long term hedges to a related position, we see evidence that more sophisticated investors can use the rules of the ETF to profit, especially in periods of economic uncertainty and high market volatility. In times of market uneasiness, investors could have turned to the hard rules of leveraged ETFs to find investment opportunities. Our investigation into the behavior of the FAS and FAZ ETFs through the first three quarters of 2009 revealed that relatively simple trading strategies that attempt to exploit the ETFs' replication goals would have lead to tremendous returns with relatively low risk. While investors have anecdotally reported using the idea of buying ETFs based on momentum, or buying ETFs based on their discrepancy from their published targets, we show that combining the two into the discount-and-up strategy not only produced the best actual results year to date, but also offered the statistically least amount of risk. The risk, though, was magnified to unacceptable levels if a strategy failed to close its position before the day's close.

Future work could involve continuing to test the discount-and-up strategy, especially against periods of different RIFIN volatility. We believe that the higher the volatility in RIFIN, the greater the return of the strategy. For instance, we saw that the discount-and-up strategy worked exceptionally well during the period of Jan 2, 2009 – May 14, 2009, a period of severe market decline, but high volatility. While the discount-and-up returns continued to be very good from May 15, 2009 – Oct 14, 2009, they were

less impressive. Volatility in the market diminished and the opportunities to exploit the ETFs objectives were hampered.

7. References and Data

References - Literature

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Data

The stock price data used in this project was obtained through the Bloomberg Professional Service, including the software and terminal hardware. Special thanks to the help desk at Bloomberg for excellent service in helping me obtain the minute-by-minute historical data necessary to make this work possible.