

Worcester Polytechnic Institute

# Analysis of Fund Mapping Techniques for Variable Annuities

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Grant Fredricks  
Erin Ingalls  
Angela McAlister

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**Jon Abraham, Advisor**

**Guillaume Briere-Giroux, Liaison**

## **Abstract**

Our project tested the accuracy of projected mutual fund returns (via fund mappings) relative to the actual performance of the mutual fund. To achieve this, we performed Monte-Carlo simulations using geometric Brownian motion. Utilizing Excel's VBA, we generated many scenarios to draw conclusions about fund mapping reliability. We modified parameters within the simulation in order to isolate factors that affected the fund mapping's accuracy, and analyzed the specific effects of these variables to provide our sponsor, Towers Perrin, with recommendations.

## Executive Summary

Variable annuities operate like mutual funds, except variable annuities have specific benefits that provide the policyholder with an insurance feature. These benefits include guaranteed minimum withdrawal benefits (GMWB), guaranteed minimum income benefits (GMIB), guaranteed minimum accumulation benefits (GMAB), and guaranteed minimum death benefits (GMDB). The benefits provide the policyholder with some protection against unfavorable performance of their assets, which are held in a separate account within the variable annuity. In order to protect themselves against these unfavorable outcomes, insurance companies use hedging programs. Hedging programs are used with the goal of building a portfolio that is expected to increase in value in the event that the separate account (the one providing insurance) decreases in value.

In order to build these hedging portfolios that advance in a declining market, managers use derivative instruments that protect them against losses caused by realized volatility, interest rate changes, and implied volatility costs. The policyholder's separate account is invested into mutual funds that typically use an active management strategy; however, these funds do not always have a clear benchmark. When the benchmark is unclear, insurance companies will need to assign the mutual fund to hedgeable indices that they will use for hedging transactions. When companies do not know the exact benchmark, they typically regress the mutual fund returns against the historical returns of indices that are assumed to explain the return of certain asset classes (e.g. the S&P 500 or the Russell 2000, which we abbreviate as SPY and RUS, respectively). The process of assigning the mutual fund to hedgeable indices is known as fund mapping. Fund mappings are used to project the expected mutual fund performance given movement in the referenced indices.

This process inherently has a type of risk known as basis risk. Basis risk arises between an exposure and a hedge mechanism that is not perfectly correlated with the exposure, where the degree of the risk is based on the correlation between the exposure and the hedge (Banks, 2004). This means that there is always a risk from inaccurate fund mappings or active management. For an insurance company, it is impossible to directly trade derivatives with the separate account mutual funds as the underlying asset. Consequently, insurance companies use hedgeable indices to define proxy hedge instruments (Briere-Giroux, 2009). When the proxy hedge instrument does not use the correct hedgeable indices, it could result in losses because of basis risk. The reason for this could be due to an incorrect estimation of the fund mappings. Another possible source of basis risk could be that the insurance company found the correct fund mappings given the historical performance, but the fund managers change their strategies from one period to the next. The final source of basis risk is that the

insurance company found the correct fund mapping to use going forward as a benchmark, but the manager is trying to beat the benchmark. Therefore, there is some deviation between the benchmark and the performance the manager is expecting, which would give rise to basis risk.

Our project studied the last source of basis risk. To do this, we tested the accuracy of projected mutual fund returns (using fund mappings) relative to the actual performance of a mutual fund (called MFI). MFI was created by us to follow the returns of SPY with some additional random error, which represents the manager alpha. Manager alpha is the amount of randomness of the MFI, the risk resulting from a manager trying to outperform the benchmark. To achieve this, we performed Monte-Carlo simulations using geometric Brownian motion. With Excel's Visual Basic for Applications (VBA), we were able to generate and analyze a large number of stock simulations and regressions in order to help us draw conclusions about the fund mapping reliability, and how manager alpha affects the accuracy of the fund mappings. Furthermore, we modified certain parameters within the simulation, such as manager alpha, in order to isolate specific factors that may have affected the fund mapping's accuracy.

By analyzing the results, we were able to make conclusions about the outcomes of our research. We discovered that basis risk arising from active management is insignificant using our model. In general, the returns of the proxy MFI using the fund mappings were very close to the true MFI returns. Additionally, the two-index fund mapping was almost exactly the same as the fund mapping using just the SPY, showing that adding other indices to a fund mapping does not adversely affect the outcome, even if those indices are not accurate. However, fund mappings based entirely on inappropriate indices generated some error. We also found that regressing over a larger period of time created regression coefficients that were closer to the proper fund mappings. However, this did not have a significant impact on the actual accuracy of the fund mappings when looking ten years into the future. That means that we found no difference between the accuracy of the fund mappings using a 1-year regression and a 10-year regression, even though the coefficients for longer regressions were more accurate.

After observing a single case and its results, we created more cases with different parameters in order to see how they affected the outcome. Throughout 15 different cases, we changed variables such as the manager alpha of the MFI, proportion of randomness from SPY returns for the MFI, correlation between the SPY and RUS, and volatility of the SPY and RUS. There are several factors that we looked at to determine the differences among the different cases, including the regressions' coefficients of determination ( $R^2$ ), realized manager alpha, realized correlation, and the ending difference between MFI and the fund mappings. First of all, we found that increasing and decreasing the correlation and volatility had the largest impact on the fund mapping accuracy, especially when using improper indices.

As the difference between the SPY and RUS volatilities increased, the accuracy of the RUS-only fund mapping decreased. Multicollinearity did not occur, even though the SPY and RUS are highly correlated. This may be because using the SPY as a fund mapping was too perfect for the MFI. All other findings were ordinary and expected; for example, there is slightly more error in the fund mappings when the MFI has a higher manager alpha. Our results allowed us to provide our sponsor, Towers Perrin, with recommendations on how to account for basis risk and its effects.

Although we did not find surprising results in general, future research may provide greater insight into basis risk and the effects of active management. If our model were improved to be more applicable to the real world, more surprising or interesting results may be found. Some ways to improve the model may be to implement stochastic parameters, add more indices and have a more diversified portfolio, or change the expected returns for the indices. For example, it is questionable to assume that indices correlated in the past will continue to be correlated in the future, so future research could create scenarios wherein the correlation or volatility changes over time. It may also be interesting to attempt to study other sources of basis risk. For example, to study the effects of managers changing investment strategies from one period to the next, researchers may have to do a more qualitative study on human nature and how it affects management and investment styles. We hope that by continuing research on the effects and sources of basis risk, insurance companies and asset managers will eventually be able to improve their investment strategy and performance, and reduce the risk associated with variable annuities and mutual funds.

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## 1. Introduction

Variable annuities operate like mutual funds; however, variable annuities have specific benefits that provide the policyholder with an insurance feature. These benefits include guaranteed minimum withdrawal benefits (GMWB), guaranteed minimum income benefits (GMIB), guaranteed minimum accumulation benefits (GMAB), and guaranteed minimum death benefits (GMDB). The benefits provide the policyholder with some protection against unfavorable performance of their assets, which are held in a separate account within the variable annuity. In order to protect themselves against these unfavorable outcomes, insurance companies use hedging programs. Hedging programs are used with the goal of building a portfolio that is expected to increase in value in the event that the separate account (the one providing insurance) decreases in value. Shorting the market is one possible option for hedging this risk.

In order to build these hedging portfolios that advance in a declining market, managers use derivative instruments which protect them against losses caused by realized volatility, interest rate changes, and implied volatility costs. The policyholder's separate account is invested in mutual funds that typically use an active management strategy; however, these funds do not always have a clear benchmark. When the benchmark is unclear, insurance companies will need to assign the mutual fund to hedgeable indices that they will use for their hedging transactions. When companies do not know the exact benchmark, they typically regress the mutual fund returns against the historical returns of indices that are assumed to explain the return of certain asset classes (e.g. the S&P 500 or the Russell 2000). The process of assigning the mutual fund to hedgeable indices is known as fund mapping. Fund mappings are used to project the expected mutual fund performance given the movement in the reference indices.

Our project tested the accuracy of the projected mutual fund returns (using fund mappings) relative to the actual performance of a mutual fund (called MFI). MFI was created to follow the returns of the S&P 500 with some random error, which represents the manager alpha. To achieve this, we performed Monte-Carlo simulations using geometric Brownian motion. With Excel's Visual Basic for Applications (VBA), we were able to generate and analyze a large number of scenarios to help us draw conclusions about the fund mapping reliability. Furthermore, we modified certain parameters within the simulation, such as volatility and correlation, in order to isolate specific factors that may have affected the fund mapping's accuracy. This allowed us to analyze the specific effects of these variables and provide our sponsor, Towers Perrin, with recommendations.

## 2. Background

In this background section, we present information about various subjects in order to help the reader fully understand the process and goal of our project. We will begin with a very broad description of what a variable annuity consists of and a general description of how mutual funds work. We will then move on to discuss what hedging is and techniques used in hedge funds, which is followed by a description on active management and basis risk. Lastly, we introduce various mathematical tools and equations used to provide us with a hedging technique that can be analyzed and discussed.

### 2.1. Variable Annuities

Variable annuities are the most rapidly growing financial product today; nominal sales more than doubled in the United States between 1996 and 2004 (Poterba, 2006). By definition, an annuity is a contract issued by an insurance company which usually has two phases, the accumulation phase and the payout phase (Wyss, 2001). An individual purchases a variable annuity contract by making a single purchase payment or a series of such payments over a set period of time (Variable Annuities: What you should know, 2009).

During the accumulation phase the annuity operates much like a mutual fund where one can choose a variety of investment options, which consist of stocks, bonds, money market instruments, or a combination of the three. Over time, the value of the annuity will increase or decrease depending on the investment combination and how well the chosen funds are performing in the market (Variable Annuities: What you should know, 2009). Furthermore, the contract value is developed using fixed or variable investments that are tax deferrable as long as the earnings stay within the annuity (Wyss, 2001). The fixed investment is an option given by the issuer, where the insured can allocate a portion of his or her purchase payment into a fixed account, which accumulates using a fixed interest rate (Variable Annuities: What you should know, 2009). This fixed account is typically known as a guaranteed minimum accumulation benefit (GMAB). More benefit options for variable annuities will be discussed later. The insured is typically allowed to transfer funds from one investment to another without tax charges, but other charges may be set depending on the limitations arranged by the insurer. However, if the insured withdraws money from the account, particularly in the early stages of the accumulation phase, the insurer will issue a surrender charge. Additionally, if withdrawals are made before the policyholder is 59 ½ years of age there will be a 10% federal tax penalty (Variable Annuities: What you should know, 2009).

When the insured is prepared to enter the payout phase, there are a number of options available. The three most common payout plans are as follows:

- **Lump-sum:** The recipient can choose to withdraw all the funds in a lump-sum payment, where taxes are charged on the total accumulated earnings. This will result in a large tax liability, making this option less desirable.
- **Systematic Payments:** This option allows a fixed amount to be paid out to the recipient at specific time intervals until it exhausts all accumulated earnings. This option spreads out the tax liability but must be monitored to make sure payments do not end prior to death of the recipient.
- **Annuitization:** This option makes a steady stream of payments, where the insurance company agrees to provide the recipient with an income for a specified period. Changes to these plans are limited and tax liabilities are spread out. A portion of each payment is treated as a return of principal (Wyss, 2001).

There are many different annuitization options: guaranteed income until the death of the recipient, at which time payments are stopped; guaranteed payments for a specified amount of time and in case of the recipient death, the beneficiary receives payments until funds are exhausted; and lifetime payments to two designated people wherein if one dies the other will receive payments until his or her death (Wyss, 2001). The variable annuity also has a guaranteed minimum income benefit (GMIB) option, which guarantees a minimum level of the annuity income payments even if the account doesn't hold enough money at the time of the payment (possibly due to investment losses) (Variable Annuities: What you should know, 2009). In order to protect against decreasing account earnings, there is a guaranteed minimum withdrawal benefit (GMWB) option. This benefit enables the policyholder to be able to withdraw a percentage of the initial investment every year until the entire investment has been withdrawn (Investopedia, 2009).

The payout period differentiates the two types of accounts, fixed and variable. When dealing with the fixed account, the policyholder will receive fixed dollar payments depending on earnings accrued during the accumulation phase. However, when dealing with the variable account, the income payments depend on the market value of the portfolio created during the accumulation phase. As the value of the portfolio rises or declines, the amount of the variable income payment could increase or decrease. During rising markets, this may be an opportunity for the income to keep pace with inflation during the retirement years (Wyss, 2001).

There are many reasons why individuals are attracted to variable annuities, one being the desire to accumulate wealth while not being taxed on the earnings until the policyholder makes withdrawals.

Another appealing reason is the insurance component of the variable annuity. The guaranteed minimum death benefit (GMDB) option states that if the policyholder dies before they retire, their beneficiary will receive either all the earnings accumulated thus far or some guaranteed minimum, whichever is greater (Variable Annuities: What you should know, 2009). There is also a 'step up' option that locks in any increase in market value of the variable account for the beneficiaries. Moreover, the benefit is paid directly to the beneficiary without any costs or time delays (Wyss, 2001).

An additional reason why variable annuities may be attractive is because of the convenience factor. Variable annuities offer a rebalancing option that restores the policyholder's assets to their initial state and original target allocations (Wyss, 2001). Furthermore, there is also an option of turning the account into a life annuity at the end of the accumulation phase (mentioned previously as annuitization), but does not force withdrawal when the policyholder reaches 70 ½ years of age (Poterba, 2006). Some insurance companies have also offered features such as 'bonus credits'. When making a purchasing payment, the contract promises to add a bonus credit to your account determined by a specified percentage (usually between 1% and 5%) of the purchasing payment. However, when the contract offers bonus credits it usually means there is an offsetting downside to the annuity, such as a higher surrender charge, a longer surrender period and/or a higher annual mortality or risk charge (Variable Annuities: What you should know, 2009).

The variable account accumulates earnings, as stated previously, by investing in stocks, bonds and/or money market instruments. When creating this account, the insurance company and the client develop a contract that acts as a mutual fund. They are able to decide which combination of investments to choose, which are generally more risky than those investments found in the fixed account. Furthermore, after the client chooses the contract which best suits their preferences, the insurance company sees their contract and hedges against the risk that may arise due to the chosen funds and benefit options. We will discuss the hedging procedure and risk in subsequent sections; however, mutual funds represent a large portion of the variable account and are extremely important for successful accounts.

## **2.2. Mutual Funds**

A mutual fund is a diversified portfolio funded by various investors, with assets such as stocks, bonds, short-term money market instruments or other securities (Mutual Funds, 2007). The investors purchase shares of the mutual fund, which are redeemable, meaning that they can sell the shares back to the fund at any time. When purchasing and selling funds, investors use four main sources:

professional financial advisers, employer sponsored retirement plans, fund companies, and fund supermarkets. When individuals are just beginning to invest, most depend on financial advisers to make intelligent decisions for them in order to earn a good return on their investments. In 1990, financial advisers were responsible for 72% of households who had assets in mutual funds (Investment Company Fact Book, 2009). Many investors study the performances of mutual funds in the past, study the progress of the funds, and may turn to a stockbroker for an informed decision (Wolfinger, 2005). Generally, funds focus on specific characteristics of stocks, such as buying stocks for dividends or long-term growth, while others focus on industries or the geographic location (Wolfinger, 2005). Other investment options will be discussed in further detail later.

Today, direct contribution (DC) retirement funds have become more popular. The percentage of mutual fund holders using DC retirement funds has risen from 8% in 1990 to 26% in 2008. Figure 1 shows the change in composition of ownership within the four main sources of long-term investment options from 1990 to 2008 (Investment Company Fact Book, 2009).

### Composition of Ownership within Four Main Investment Options

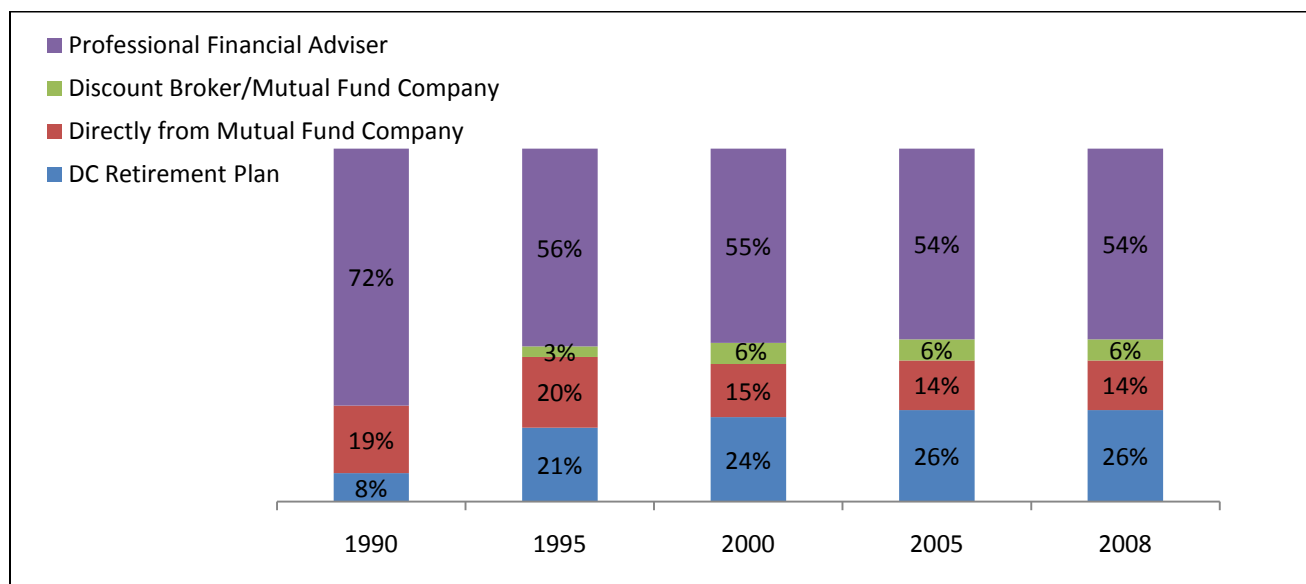


Figure 1: Change in Composition of Ownership within Four Main Investment Options

Mutual funds are becoming a very important part of the investment world, and in 2008, data shows that 92 million individuals or 45 percent of all U.S. households had investments in mutual funds.

Figure 2 shows the percentage of households owning mutual funds between 1980 and 2008. In May of 2008, 76% of households who owned mutual funds indicated that their main reason for the investment was for retirement purposes. Full statistics of all households saving money show that 95%

were saving for retirement, 52% held mutual funds to reduce taxable income and 45% were saving for emergencies (investors can be saving for more than one purpose) (Investment Company Fact Book, 2009).

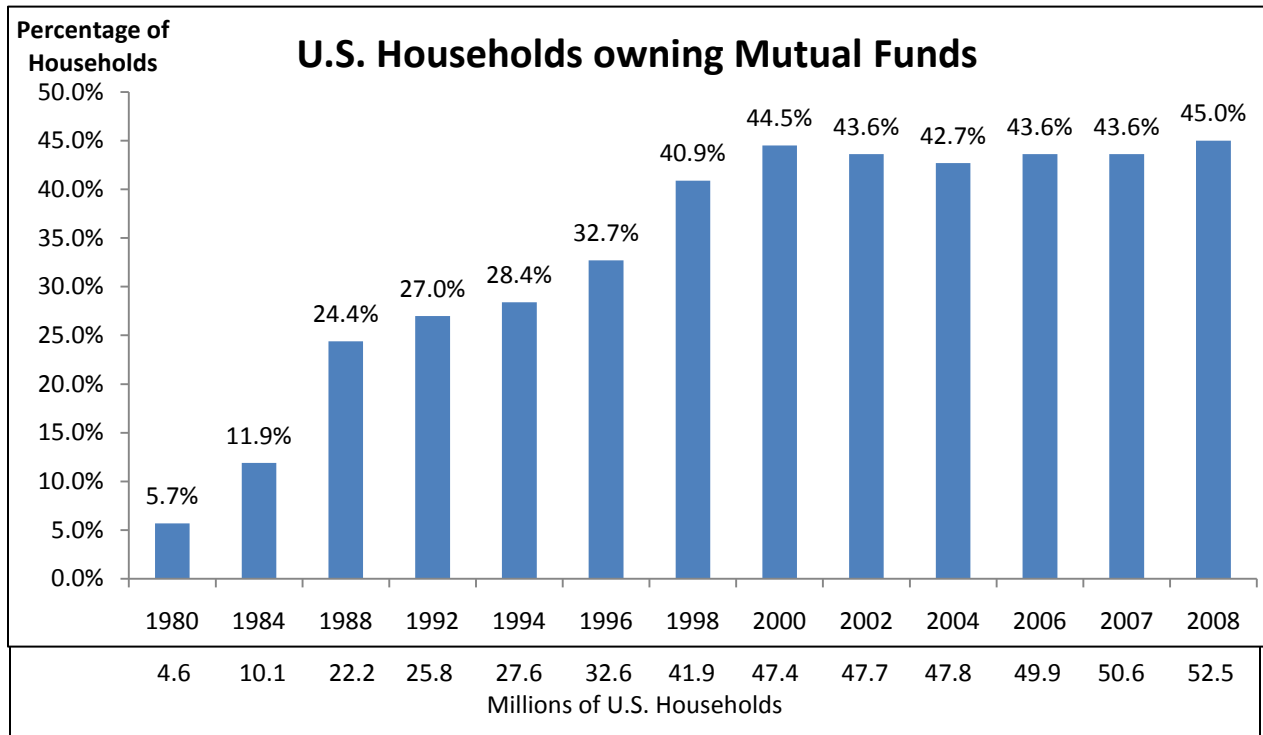


Figure 2: U.S. Households owning Mutual Funds

There are two different types of retirement plans: inside-employer sponsored and outside-employer sponsored plans. The inside-employer sponsored retirement plans are the main reason why so many individuals have been introduced to mutual funds, through their 401(k) and other retirement opportunities brought to them by their employer. In 2008, it was found that 59% of households owning mutual funds purchased their first mutual fund through their employer (Investment Company Fact Book, 2009). Since retirement plans are becoming more popular in the workplace, this is the first exposure most have to mutual funds in general; 47% of individuals who made their first mutual fund purchase prior to 1990 made the purchase through an outside-employer sponsored plan. A person who uses an outside-employer plan purchases through either a fund company directly, a discount broker/mutual fund supermarket, or a professional financial adviser (Investment Company Fact Book, 2009). After the year 2000, only 32% of households made their first mutual fund purchase through an outside-employer sponsored plan.

Professional financial advisers continue to be an extremely important aspect of the mutual fund world, advising 54% of households owning mutual funds in 2008 (Figure 1). Financial advisers offer

investment services and planning services. Investment services include regular portfolio reviews and recommendations, and recommendations for the employee sponsored retirement plans. Planning services include discussing financial goals, planning to achieve specific goals (such as college and retirement), and managing assets in retirement. The recipients also have access to specialists regarding areas such as tax planning (Investment Company Fact Book, 2009).

There are various ways for individuals to invest; however, mutual funds hold the majority of investments. Table 1 shows the diversity in individual retirement account (IRA) holdings. Variable annuities are the fourth largest investment option available, providing 23% of households with individual retirement accounts. Individual bonds and U.S. saving bonds hold a much lower percentage than previous years because the purchasing of these bonds have declined dramatically since 2005 (Treasury Direct, 2008).

**How Households Invest their IRA's During 2008**  
(percentage, multiple responses included)

<b>Mutual Funds (total)</b>	<b>73</b>
Stock MFs	61
Bond MFs	37
Hybrid MFs	22
Money Market MFs	33
<b>Individual Stocks</b>	<b>42</b>
<b>Annuities (total)</b>	<b>34</b>
Variable Annuity	23
Fixed Annuity	21
<b>Bank Savings Accounts</b>	<b>28</b>
<b>Individual Bonds</b>	<b>13</b>
<b>U.S. Savings Bonds</b>	<b>10</b>
<b>ETFs</b>	<b>8</b>
<b>Other</b>	<b>7</b>

Table 1: How Households Invest their IRA's During 2008

Source: (Investment Company Fact Book, 2009)



There are various types of mutual funds that are offered to investors, including:

**Corporate Bonds:** Bonds issued by corporations, which pay higher rates due to their riskiness (Corporate Bonds, 2009).

**Growth Funds:** Growth of capital is the main objective of this type of fund, which is composed mostly of common stocks. These funds can be either conservative (investing in large cap) or aggressive (investing in small cap).

**Sector Funds:** Invest in companies in a specific geographic area or industry.

**Income Funds:** Provide investors with a high yield by investing in stocks and bonds that make dividend payments to shareholders.

**Balanced Funds:** Have a conservative investment policy invested in common stock, preferred stock and bonds.

**US Government Securities Funds:** Invest in securities offered by the U.S. government, such as Treasury bills, notes, and bonds.

**Money Market Funds:** These investors have a high return and high liquidity, which are high-yield short-term debt securities.

**International Funds:** Invest in common stocks of foreign countries.

(Finance, 2005)

Mutual funds have many benefits and conveniences. When investing in a mutual fund, investment decisions are made by full-time investment managers, where they can invest in full or fractional shares. These shares can be used as collateral for a loan, which is very beneficial to the shareholders. The shareholders are also offered a conversion privilege, which allows the investor to change from one fund to another (within the same family) if the investor thinks it will provide capital gains (Finance, 2005).

When investing in mutual funds, the shares are always open to the public and continuously redeemable shares are sold by the investor. The investor buys shares at the ask price and sells the

shares at the bid price, where the ask price cannot be larger than the bid price by 8.5% under National Association of Securities Dealers law (Finance, 2005).

After the client has chosen their mutual fund contract, the next step is to send the contract to the insurance company. When the insurance company receives this contract, they try to hedge against the funds based on the riskiness of the investments they chose. Insurance companies use many hedging techniques in order to provide them with confidence about the risk that may arise. Furthermore, there are also hedge funds that are used by many investors. Insurance companies hedging for variable annuities do not use hedge funds due to their risky nature; however, typical techniques for hedging can be seen in hedge funds.

### **2.3. Hedging Techniques and Hedge Funds**

When working with any investment there are always risks and rewards involved. Generally, the lower the risk of an investment, the lower the potential reward. Managers are constantly trying to develop a trade-off between the two. Anytime an investor is defraying risk by investing in an offsetting investment, they are hedging their risk (Chriss, 1996). For example, a manager with a call option is always exposed to the risk that their stock price would rise above their strike price (price in which the security can be bought, at expiration) (Strike Price, 2009). At expiration of the stock, the manager will need to have cash equal to the difference between the market price of the stock and the strike price, assuming the market price exceeds the strike price. In order to hedge against this risk, the manager would invest in a separate security which, at expiration, would provide the cash difference needed for the previous investment (Chriss, 1996). This is one example of a hedging technique.

Black-Scholes is a method used for hedging techniques, and if the weights on each investment are kept in balance, then the value of the portfolio and the option are always equal (Chriss, 1996). When hedging techniques are used for variable annuities, the manager uses this method in order to provide the necessary benefits the insured had chosen (described above: GMAB, GMDB, GBIB, GMWB). An example of hedging techniques can be seen in hedge funds.

As mentioned above, variable annuities are usually split between two accounts. The variable account is invested into mutual funds that are actively managed. In order to minimize risk, the manager needs to create an account that will increase as the market decreases, also known as shorting the market. Hedge funds work the same way as traditional mutual funds do, where various investors pool together funds in order to create a diversified portfolio. However, a hedge fund manager's main concern is reducing the risk within the portfolio. Due to this, the managers are much more flexible

when choosing their investments and are given investment tools and techniques that managers of traditional mutual funds do not implement (Wolfinger, 2005).

Since the 1990's, hedge funds have soared in popularity. There are an estimated 6000 hedge funds active today managing \$600 billion in assets, whereas only 68 hedge funds were active in 1984 (Lhabitant, 2004). There are many different definitions of a hedge fund, but the most important characteristic is that they are not open to the public. François-Serge Lhabitant, the author of *Hedge Funds: Quantitative Insights*, has provided a starting definition: "Hedge funds are privately organized, loosely regulated and professionally managed pools of capital not widely available to the public." Due to the private nature of a hedge fund, regulators do not feel it is a traditional investment vehicle (like mutual funds) and therefore there is no reason to regulate it or require specific disclosures (Lhabitant, 2004). Table 2 shows some typical differences between traditional mutual funds and hedge funds that were described in James Hedges book, *Hedges on Hedge Funds: How to Successfully Analyze and Select and Investment* (2004).

### Traditional Mutual Funds vs. Hedge Funds

	Traditional	Hedge Funds
<b>Performance Objectives</b>	Relative Returns	Absolute Returns
<b>Investment Vehicles</b>	Stocks, Bonds, Cash	All Asset Classes
<b>Regulation Structure</b>	Regulated	Largely Unregulated
<b>Performance Drivers</b>	Asset Class and Market Correlation	Fund Manager Skill
<b>Fees</b>	Management Fee Only	Management Fee, Performance Incentive Fee

Table 2: Traditional Mutual Funds versus Hedge Funds

(Hedges, 2004)

Hedge funds are able to use many different investment tools, such as derivative products, short selling, or using leverage, which are not used in traditional mutual funds. Using leverage means that they can borrow money if they believe it would enhance their returns without causing the size of the investment to increase (Wolfinger, 2005). Due to the fact that hedge funds can play both directions of the market, they can profit in both bull (rising price) and bear (falling price) markets, which is what 'absolute return' means (Lhabitant, 2004). Managers of these funds must be very intelligent and skilled professionals. Therefore, performance fees apply and usually amount to five times the fees of traditional mutual funds (Lhabitant, 2004). Since hedge funds are so flexible, the managers also rely on many different styles of management and investment strategies in order to utilize opportunity.

## Management Strategies and Characteristics

Category	Sub strategy	Risk	Volatility	Focus
<b>Global Macro (tactical)</b>	Global Macro Managed futures Emerging Mkts	High	Very High	Opportunistic Exploitation of Macro trends One way speculation on future movements
<b>Event Driven</b>	Merger Arbitrage Distressed Securities Special Situations	Medium	Low	Exploit unrealized value Transaction and time driven
<b>Equity Hedges</b>	Long/Short Short Selling	Medium to high	Medium to high	Combine L/S equity and bonds to reduce market exposure and isolate performance while managing risk
<b>Relative Value/Mkt Neutral</b>	Convertible Fixed Income Equity Mkt Neutral	Low	Low	Exploit pricing inefficiencies Neutralize L/S positions Minimize directional effect
<b>Funds of Funds</b>	Strategy Specific Sector Specific	Low	Low to moderate	Stock diversification Long-term stabilization

Table 3: Management Strategies and Characteristics

(Hedges, 2004)

Global Macro, or tactical trading investment style, focuses on the direction of the market prices representing currencies, commodities, equities and/or bonds (Lhabitant, 2004). Global Macros are the purest form of top-down investing techniques and base their actions on changes within fundamental economic, political and market factors (Lhabitant, 2004). Global Macro managers' portfolios are very large, but very concentrated on specific investment themes. These managers rely heavily on derivatives and leverage, along with goals of high returns and more liberal attitudes towards risk than other styles of investing (Hedges, 2004). Commodity trading advisers and managed futures managers are included in this category, and are usually split into two groups: systematic traders and discretionary traders. Systematic traders believe that the future movements of the market can be explained by historical data and spend much of their time using computer-generated systems to reduce volatility. Discretionary traders proceed with decisions based on fundamentals and technical market analysis, along with their past experiences and trading skills (Lhabitant, 2004).

The event-driven style focuses on price fluctuations and imbalances during a specific life cycle of a company, such as during mergers and acquisitions, bankruptcy, corporate reconstruction or spin-offs

(Lhabitant, 2004). This style can be broken down into four sub strategies: distressed securities, risk (merger) arbitrage, special situations and sector funds (Hedges, 2004). Distressed security funds focus on the debt or equity of companies that are expected to be in financial or operational distress, which are long-term with long redemption periods (Lhabitant, 2004). The managers of these funds are often referred to as vulture investors and become actively involved with the reconstruction of the company (Hedges, 2004). Risk arbitrage funds focus on companies that are involved in a merger or acquisition, where they buy stocks of the acquired company. This tends to be a riskier investment because if the take-over is broken the investor could lose a large amount; therefore, the managers prefer not to get involved with companies who have hostile takeovers (Hedges, 2004). Special situations fall under multiple themes, including distressed securities and risk arbitrage. The managers focus on underlying problems within the companies, such as emerging market debt, depressed stocks, unannounced mergers/acquisitions or reorganization that would drop the stock prices temporarily (Hedges, 2004). Lastly, sector funds focus on specific sectors of the economy, such as technology companies, financial institutions, healthcare and biotech companies, etc. (Hedges, 2004).

Equity hedges, much like the name indicates, are when managers invest in equity and make long investments with short sales in order to reduce, but not eliminate, market exposure (Lhabitant, 2004). These funds tend to have more long gross exposure than short gross exposure, which implies that the funds have a significant correlation with traditional mutual funds and experience large downturns when the market experiences large downturns (Lhabitant, 2004). Equity hedges can also be classified into several sub strategic categories: global, regional, sectoral, emerging and dedicated short bias, which proves them to be very opportunistic (Lhabitant, 2004).

Relative value/market neutral funds seek to effectively neutralize market influences, where their profit will come from the difference in price between two securities (Hedges, 2004). The strategies used by these funds include convertible arbitrage, fixed-income arbitrage, index arbitrage, closed-end fund arbitrage, and equity market neutral (Hedges, 2004). Convertible arbitrage exploits pricing abnormalities between bonds that are convertible and their equity (Lhabitant, 2004). The fixed-income arbitrage exploits pricing inefficiencies within and across global fixed income markets. These funds are known to be very profitable, but unpredictable (Hedges, 2004). Index arbitrage exploits the mispricing of indices and index derivative securities (Lhabitant, 2004). However, since computerized trading has increased in popularity, the mispricing of indices has decreased and therefore this method does not provide much profit (Hedges, 2004). Closed-end fund arbitrage buys a basket of stocks or securities and replicates a closed-end mutual fund. The key interest for these managers is to find mutual funds that

are selling securities at substantially different prices from their net asset value (Hedges, 2004). Lastly, equity market neutral strategies exploit pricing inefficiencies between related equity and equity derivative securities using tools designed to completely eliminate nearly all market risk (Hedges, 2004).

Funds of Funds are defined as a breed of mutual funds where managers invest in traditional mutual funds and hedge funds (Wolfinger, 2005). This method gives the ability to use all strategies that each manager prefers instead of only using a single strategy. The investments in hedge funds will reduce the potential for poor performance and the diversification of the portfolio will yield long-term returns (Lhabitant, 2004).

Today, equity hedging funds are the dominant strategy and are a direct result of the bear market in the early 2000s. Long-only managers were forced to drop their traditional mutual funds and open hedge funds in order to short sell short and obtain performance fees (Lhabitant, 2004).

In order for insurance companies to gain profits for clients, and also for their own benefit, they must be very active in their management styles. One component of active management is the manager alpha, which represents the risk that is uncorrelated with the market, with a goal of decreasing the deviation between the fund mapping and the indices used (Litterman, 2005).

## **2.4. Active Management**

There are two main strategies for managing a portfolio, active management and passive management. Active management involves constantly buying and selling a specific mix of securities in hopes of out-performing the market (Curtis, 2001). Passive management is the opposite of active management, wherein the manager of the mutual fund tries to mirror already existing market indices (Lim, 2005). Jack Bogle, founder of the Vanguard Group, describes the difference between active and passive management as follows: "Active fund managers seek to find those needles in the haystack that lead to outsized results; passively managed index funds simply buy the whole haystack" (Lim, 2005). According to the tracking service of Morningstar, of 16,000 mutual funds in existence an estimated 95% of them are actively managed (Lim, 2005).

One type of active management is qualitative management. A qualitative manager will choose stocks based on an analysis of their income statements and balance sheets, financial ratios, phone interviews with company personnel, research reports, and the manager's intuition and experience (Chincarini & Kim, 2006). Another strategy is quantitative management (also known as QEPM, quantitative equity portfolio management), which is not based on intuition like qualitative management. These managers base decisions on mathematical and statistical analyses. Quantitative managers look at

the stock fundamentals from the income statement and balance sheet, technical data, macroeconomic data, survey data, analyst's recommendations or any other data stored in a database (Chincarini & Kim, 2006). With the technological advancements of the last decade, quantitative management is growing in popularity, since computers can process information faster and the internet offers more information which is available at all times. However, even with all of the information available, qualitative management is also growing in popularity.

In Grinold and Kahn's *Active Portfolio Management* (2000), they developed a fundamental law of active management. The variables in the formula consist of the following:

**Breadth Coefficient (BC):** The number of independent forecasts taken per year.

**Information Coefficient (IC):** The measure of skill correlating each forecast and the actual outcome.

**Information Ratio (IR):** Measure of the manager's opportunity.

With these variables, the measure of the manager's opportunity (IR) can be computed as:

$$IR = IC \cdot \sqrt{BC}$$

This formula can be used for giving insight into active management, but is not an investment tool (Grinold & Kahn, 2000). In short, this law indicates that low-turnover strategies (lower BC) have to be very accurate (higher IC) in order to produce the same information ratio (IR) as high-turnover strategies (Fundamental Law of Active Management, 2007).

There are a few ways an insurance company could improve their risk-adjusted returns, such as by separating their interest rate, market, and active risk. Furthermore, an insurance company could manage their market risk more effectively and decrease the exposure their liabilities may have to interest rate fluctuations. Some also believe that taking more active risk could be beneficial to their risk-adjusted returns (Active Alpha and Beyond, 2009).

## 2.5. Tracking Error or Active Risk

Tracking error or active risk represents how well a portfolio tracks the fund mapping. When using hedging procedures, managers use Monte-Carlo simulation to develop many possible scenarios in order to test how their mutual fund may react to specific market changes. The simulated mutual fund represents how they believe the market will move and is known as a fund mapping. The fund mapping is used as a tool to analyze how well their mutual fund is performing. The size of the tracking risk

depends generally on the size of the active portfolio, not on the scope of the fund mapping (Hull, 2006). In order for the manager to minimize the tracking error, they need to study the stocks' expected returns, volatilities, and the correlation between the stocks (Chincarini & Kim, 2006). After taking all of these factors into consideration, the manager will decide the correct weights of each stock in order to minimize the error and maximize the return. The determined weights are very important in minimizing the tracking error and there are many methods for determining the best weights.

The first method is known as the Ad Hoc method, which is when the manager chooses a specified number of stocks with the largest market capitalization as the fund mapping (Chincarini & Kim, 2006). However, there are some downfalls to this method. The first problem is that this fund mapping would be very sensitive depending on its size. The second problem is that there is no attempt to choose a fund mapping that minimizes tracking error, because the manager only chooses the stocks based on their market capitalization. Lastly, the fund mapping has to be very large in order to represent the market correctly. For example, if the manager chooses the 50 stocks that have the largest market capitalization, it only represents 50% of the S&P 500, and represents a smaller percentage of certain larger indices (Chincarini & Kim, 2006). Although simple, this method is not the best method in determining the fund mapping.

Another method in determining the fund mapping is stratification. This was originally used by statisticians in order to gather characteristics about a certain population when they could not afford to gather information on all the members. Therefore, this method uses a representative sample of the population (Chincarini & Kim, 2006). The strategy is to select portions of each type of stock in order to represent the universe of stocks. The risk is minimized by the diversification of the portfolio; however, it does not give the manager confidence to judge what kind or how much risk they are dealing with (Chincarini & Kim, 2006).

The best method that managers use is the minimization-of-tracking-error approach. There are two methods used in this approach. One method is to minimize the tracking error, while the other is to maximize the expected excess returns without going over the maximum tracking error determined by the manager (Chincarini & Kim, 2006). Most managers define the tracking error as the standard deviation of the difference between the portfolio's returns and the fund mapping's returns. The following equation is used to minimize the tracking error:

$$TE = SD(r_p - r_B) = \sqrt{Var(r_p - r_B)}$$



where  $r_p$  = portfolio return,  $r_B$  = fund mapping return,  $SD$  represents standard deviation,  $Var$  represents variance (Chincarini & Kim, 2006).

The manager's goal is to get the tracking error to equal zero; however, in reality this is very difficult due to transaction costs, reinvestment of dividends and methods of sampling the fund mapping (Chincarini & Kim, 2006). What is most important is keeping the tracking error stable. Furthermore, the tracking error is usually annualized; the manager can use the same equation given above except to annualize it they would need to multiply the result by the square root of the number of periods per year they are analyzing. For example, if a person were dealing with months he or she could multiply the result by  $\sqrt{12}$  to annualize it (Chincarini & Kim, 2006).

When tracking how well the portfolio is doing relative to the chosen fund mapping, there is always a risk that arises between the hedging instruments and the movement of the liability. This is known as basis risk. This is the topic of the next section.

## 2.6. Basis Risk

When an insurance company performs hedging for liabilities, there will be a mismatch between the performance of the hedging instruments and the movement in the liability. A large contributor for that difference is called basis risk. Basis risk is a risk that arises between an exposure and a hedge mechanism that is not perfectly correlated with the exposure, where the degree of the risk is based on the correlation between the exposure and the hedge (Banks, 2004). Basis risk typically takes place when an imperfect hedging product is used as a proxy hedge for the exposure because the correct hedge is expensive or very hard to find (Horcher, 2005). For an insurance company, it is impossible to directly trade derivatives with the separate account mutual funds as the underlying. As a consequence insurance companies use hedgeable indices to define proxy hedge instruments (Briere-Giroux, 2009).

When the proxy hedge instrument does not use the correct hedgeable indices, it could result in losses because of basis risk. The reason for this could be due to an incorrect estimation of the fund mappings. Another possible source of basis risk could be that the insurance company found the correct fund mappings given the historical performance, but the fund managers change their strategies from one period to the next. The final source of basis risk is that the insurance company found the correct fund mapping to use going forward as a benchmark, but the manager is trying to beat the benchmark. Therefore, there is some deviation between the benchmark and the performance the manager is expecting, which would give rise to basis risk.

In the end, the risk is present when the predicted fund performance does not match the actual fund performance. This means that the change in value of the hedge instruments does not properly offset the change in the value of the liabilities. The following equation could give some insight into the interpretation of basis risk:

**Basis = Spot price of asset to be hedged - Futures price of contract used**

The spot price is the price that the manager expects the asset to be, and the future price is the market determined price of the asset at a certain date in the future. The basis is zero when the spot price of assets to be hedged and the future price of contract used are equal. The basis increases when the spot price increases more than the futures contract, which is referred to as a strengthening of the basis. The weakening of the basis happens when the future price is more than the spot price (Hull, 2006).

When basis risk is being measured, there is a hedging tool already set in place, as mentioned above. The hedging procedures are used to reduce this risk as well, by using mathematical tools in order to predict how the hedging tool may react to specific conditions. To predict this, we created a model for equity returns that attempted to simulate various market conditions.

## 2.7. Model for Equity Returns

Our model for equity returns can be thought of as a Monte-Carlo method. The Monte-Carlo method consists of observing a large number of possible random outcomes in order to predict what the true result or outcome will be. We employed Monte-Carlo methods in our model for equity returns by creating many scenarios (or 'worlds') of how markets may move in the future. Specifically, our Monte-Carlo simulation occurs in an artificial environment in order to generate the random process of market prices and rates (Crouhy, Galai, & Mark, 2001). Each simulation is a scenario that represents a possible 'world' which produces future stock prices and is used for risk management (Dash, 2004). When generating many scenarios, the distribution of the stock prices is expected to converge towards the true distribution (Crouhy, Galai, & Mark, 2001).

The Monte-Carlo simulation using Brownian motion involves two main steps. The first step involves finding the risk factors and estimating parameters such as volatility and correlation using the historical data of the stock prices and returns (Crouhy, Galai, & Mark, 2001). The second step is to use geometric Brownian motion to construct price paths, which are created using a normal random number generator.

### 2.7.1. Black-Scholes Model for Estimating Volatility

In order to model a single stock or index, a manager must know some historical information about the stock or index. Specifically, he or she must be able to estimate the drift rate and volatility. The drift rate is the expected return of a stock in a given time period. If a stock price increases 6% per year on average, then the drift rate is 6%. Volatility is a measure of uncertainty about the returns of a stock or index (Hull, 2006). Volatility is more difficult than drift rate to estimate, since it depends on the amount of variance over all times, as opposed to just the beginning and end of a period of time.

Given a series of stock prices from  $S_0$  to  $S_n$ , the Black-Scholes method for estimating volatility from historical data is

$$\sigma = \frac{\sqrt{\frac{1}{n-1} \sum_{i=1}^n ((u_i - \bar{u})^2)}}{\sqrt{\tau}} = \frac{SD(u_i)}{\sqrt{\tau}}$$

where  $u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$ ,  $\bar{u} = \frac{\sum_{i=1}^n u_i}{n}$ ,  $n + 1 =$  number of observations, and  $\tau =$  length of time between  $S_i$  and  $S_{i+1}$  in years (Hull, 2006). In words, the estimate of the volatility is the standard deviation of all log returns divided by the square root of the amount of time between each observation (in years). Once the annual volatility and drift rate are estimated, one can begin to simulate the stock price in the future.

### 2.7.2. Geometric Brownian Motion

Geometric Brownian motion is a method used to simulate stock prices over time. This model is known to be a very powerful and flexible one, but it can only be used for portfolios of a limited size due to the amount of computer resources it requires (Crouhy, Galai, & Mark, 2001). Given a drift rate (expected return)  $\mu$ , annual volatility  $\sigma$ , initial stock price  $S_t$ , and a standard normal random number  $\varepsilon$  ( $\varepsilon \sim N(0,1)$ ), the stock price at time  $t + \Delta t$  is given by the equation

$$S_{t+\Delta t} = S_t \exp\left(\left(\mu - \frac{\sigma^2}{2}\right) \Delta t + \sigma \varepsilon \sqrt{\Delta t}\right).$$

This is the same as saying that  $S_{t+\Delta t}$  is lognormal, or that

$$\ln(S_{t+\Delta t}) \sim N\left(\ln(S_t) + \left(\mu - \frac{\sigma^2}{2}\right) t, \sigma^2 t\right)$$

(Hull, 2006).

This process can be used over and over in order to simulate a stock price at various times, beginning at  $S_t$  and continuing for each  $\Delta t$  interval. At each time, the previous value of  $S_{t+\Delta t}$  becomes the new  $S_t$ , a new  $\varepsilon$  is found, and a new  $S_{t+\Delta t}$  is calculated.

### 2.7.3. Stock Correlation and Cholesky Decomposition

Generating a simulation for one index is straightforward, because during the simulation there is no other factor which it may affect or be affected by. However, when generating two or more correlated indices, the effect that one index has on the others must be considered. Although each index moves randomly during each iteration of geometric Brownian motion depending upon the value of  $\varepsilon$ , if the indices are correlated, the values of  $\varepsilon$  used for each index should be correlated.

The first step in determining how to correlate indices using geometric Brownian motion is to estimate their correlation in the past. Although it may be questionable to assume that indices correlated in the past will continue to be correlated in the future, estimating correlation in the past can still guide the modeling of correlation in the future. Given two indices,  $X_1$  and  $X_2$ , the correlation between the indices is given by

$$\text{Corr}(X_1, X_2) = \rho = \frac{\text{Cov}(X_1, X_2)}{\sqrt{\text{Var}(X_1) * \text{Var}(X_2)}}$$

where  $\text{Cov}(X_1, X_2) = E[X_1 X_2] - E[X_1] * E[X_2]$  and  $\text{Var}(X_n) = E[(X_n)^2] - (E[X_n])^2$ .

Once the correlation is found, it can be used to model correlated random numbers. In our case, we want to create a correlation matrix that will be used in the geometric Brownian motion formula to generate correlated indices. The size of the correlation matrix depends upon the number of correlated indices being modeled; if there are  $n$  indices, then the correlation matrix will be  $n \times n$ . For the two-index case, the correlation matrix  $C$  is easy to create:

$$C = \begin{bmatrix} \sigma_1 & 0 \\ \sigma_2 \rho & \sigma_2 \sqrt{1 - \rho^2} \end{bmatrix}$$

where  $\sigma_1$  and  $\sigma_2$  are the estimated volatilities of the indices, and  $\rho$  is the desired correlation between the two (Haugh, 2004). This matrix  $C$  can be written as two horizontal vectors:

$$C = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

Where  $\mathbf{V}_1 = (\sigma_1, 0)$  and  $\mathbf{V}_2 = (\sigma_2 \rho, \sigma_2 \sqrt{1 - \rho^2})$ . When modeling two indices, this is all that is needed.

However, when three or more correlated indices are to be generated, finding the correlation matrix is more difficult. Given  $n$  indices  $X_1 \dots X_n$ , we must first create a covariance matrix  $\Sigma$ :

$$\Sigma = \begin{bmatrix} \text{Cov}(X_1, X_1) & \text{Cov}(X_1, X_2) & \dots & \text{Cov}(X_1, X_n) \\ \text{Cov}(X_2, X_1) & \text{Cov}(X_2, X_2) & \dots & \text{Cov}(X_2, X_n) \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}(X_n, X_1) & \text{Cov}(X_n, X_2) & \dots & \text{Cov}(X_n, X_n) \end{bmatrix}$$

Note that  $Cov(X_i, X_i) = Var(X_i)$  and that  $Cov(X_i, X_j) = Cov(X_j, X_i)$ . To find the correlation matrix  $C$  based on this covariance matrix  $\Sigma$ , we must find the Cholesky Decomposition of  $\Sigma$  (Haugh, 2004).

The Cholesky Decomposition of a matrix  $\Sigma$  is defined as  $C$  such that  $\Sigma = C^T C$ . In other words, the correlation matrix  $C$  multiplied by its transpose  $C^T$  should equal the covariance matrix  $\Sigma$ . The process of finding  $C$  such that  $\Sigma = C^T C$  is very difficult for large matrices, and often requires the use of a computational program, such as MATLAB or VBA (Haugh, 2004). In general, the form of  $C$  will be

$$C = \begin{bmatrix} a_{1,1} & 0 & 0 & \dots & 0 \\ a_{2,1} & a_{2,2} & 0 & \dots & 0 \\ a_{3,1} & a_{3,2} & a_{3,3} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & a_{n,3} & \dots & a_{n,n} \end{bmatrix}$$

where each  $a_{i,j}$  is a positive real number. Like the 2x2 case,  $C$  can be written as a series of horizontal vectors:

$$C = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \vdots \\ \mathbf{V}_n \end{bmatrix}$$

with each  $\mathbf{V}_i$  representing a row in  $C$ . Note that the 2x2 form of  $C$  can also be found using this method of Cholesky decomposition, but it is much easier to just use the general form of the 2x2 correlation matrix.

#### 2.7.4. Generating Correlated Brownian Motions

Once the correlation matrix is found, geometric Brownian motion can be used to generate correlated random indices. The process is similar to the process above, except that instead of having one standard normal random number  $\varepsilon$  ( $\varepsilon \sim N(0,1)$ ), there must be  $n$  independent standard normal random numbers  $\varepsilon_1 \dots \varepsilon_n$ , where  $n$  is the number of correlated indices. Each  $\varepsilon_i$  must be generated independently from all of the others. Once all  $\varepsilon_i$  are generated, put them in a vector

$$\mathbf{Z} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

Finally, once the random vector is created, use it in the formula for geometric Brownian motion to find the next value of each index. The price of index  $i$  at time  $t$  is denoted  $S_{i,t}$ , and the price of each index at time  $t + \Delta t$  is

$$S_{1,t+\Delta t} = S_{1,t} \exp\left(\left(\mu_1 - \frac{\sigma_1^2}{2}\right)\Delta t + \mathbf{V}_1 \cdot \mathbf{Z}\sqrt{\Delta t}\right)$$

$$\begin{aligned}
S_{2,t+\Delta t} &= S_{2,t} \exp\left(\left(\mu_2 - \frac{\sigma_2^2}{2}\right)\Delta t + \mathbf{V}_2 \cdot \mathbf{Z}\sqrt{\Delta t}\right) \\
&\vdots \\
S_{n,t+\Delta t} &= S_{n,t} \exp\left(\left(\mu_n - \frac{\sigma_n^2}{2}\right)\Delta t + \mathbf{V}_n \cdot \mathbf{Z}\sqrt{\Delta t}\right)
\end{aligned}$$

where  $\mu_1 \dots \mu_n$  are the drift rates (expected returns) of each index,  $\sigma_1 \dots \sigma_n$  are the estimated volatilities of each index,  $\mathbf{V}_1 \dots \mathbf{V}_n$  are the rows of the correlation matrix, and  $\mathbf{Z}$  is the vector composed of independent standard normal random variables. Note that  $\mathbf{V}_i \cdot \mathbf{Z}$  is the dot product of  $\mathbf{V}_i$  and  $\mathbf{Z}$  (Haugh, 2004). Once one iteration of geometric Brownian motion is complete, it can be repeated over and over with a new standard normal random variable vector  $\mathbf{Z}$ .

The two-index case that is used throughout our simulations is a bit simpler than the general case above for  $n$  correlated indices. When geometric Brownian motion is used to generate two correlated indices, the price at time  $t + \Delta t$  is

$$\begin{aligned}
S_{1,t+\Delta t} &= S_{1,t} \exp\left(\left(\mu_1 - \frac{\sigma_1^2}{2}\right)\Delta t + \sigma_1 \varepsilon_1 \sqrt{\Delta t}\right) \\
S_{2,t+\Delta t} &= S_{2,t} \exp\left(\left(\mu_2 - \frac{\sigma_2^2}{2}\right)\Delta t + \left(\sigma_2 \rho \varepsilon_1 + \sigma_2 \varepsilon_2 \sqrt{1 - \rho^2}\right)\sqrt{\Delta t}\right)
\end{aligned}$$

where  $\mu_1$  and  $\mu_2$  are the drift rates (expected returns) of each index,  $\sigma_1$  and  $\sigma_2$  are the estimated volatilities of each index,  $\rho$  is the correlation between the two indices, and  $\varepsilon_1$  and  $\varepsilon_2$  are two independent standard normal random numbers. Note that these formulas are the same as the ones above, except that  $\mathbf{V}_1$  and  $\mathbf{V}_2$  have been replaced with their exact values from the 2x2 correlation matrix, and  $\mathbf{Z}$  replaced with  $\varepsilon_1$  and  $\varepsilon_2$ . This model can be repeated as many times as desired, creating a random walk of correlated indices into the future. Using geometric Brownian motion to generate a large number of random scenarios of stock prices is a type of Monte-Carlo simulation.

## 2.8. Regression Analysis

While conducting the Monte-Carlo Simulation, it was previously explained that historical data were used to provide the managers with parameters in order to execute the Brownian motion. After the Monte-Carlo simulation is complete, the mutual fund being used as a benchmark is regressed against the market indices chosen in order to see which market index best explains the movement of the mutual fund. The regression analysis is extremely important in determining the relationship between two or more variables. Linear regressions find the relationship between a dependent variable

and an independent variable, whereas multiple regressions have a dependent variable and two or more independent variables (Multiple Regression, 2008). When finding the relationship between the variables, it does not necessarily mean that one variable causes the other, but there is some significant involvement between the variables (Linear Regression, 1997-1998). In other words, regressions can determine correlation but not causation.

The numerical measurement of the relation between the two variables is known as the correlation coefficient (or the slope of the regression line), denoted as beta ( $\beta$ ) (Linear Regression, 1997-1998). When  $\beta$  equals one, it shows that the variables are perfectly correlated and will move along the same path. When  $\beta$  is negative it means that the variables move in opposite directions (Levinson, 2005). When saying that the variables move in opposite directions, it means that when one variable increases, the other is expected to decrease. Furthermore, when  $\beta$  equals zero, it means that there is no relationship between the two variables. In most cases involving stock regressions, the independent variable represents the market and the dependent variable represents the stock to be benchmarked. When the  $\beta$  coefficient is 1.5, this shows that the stock is expected to rise or fall 1.5% when the market rises or falls 1% (Levinson, 2005). A high positive  $\beta$  means that the share is riskier and when beta is positive but less than one it represents a conservative investment (Levinson, 2005).

For a linear regression, the equation will only involve two variables:

$$Y = \alpha + \beta * X$$

where Y represents the dependent variable,  $\alpha$  is the intercept,  $\beta$  is the correlation coefficient and X is the independent variable (Linear Regression, 1997-1998). When we have a multiple regression, the equation will be:

$$Y = \alpha + \beta_1 * X_1 + \beta_2 * X_2 + \dots + \beta_n * X_n.$$

In this equation, the X's represent the different independent variables with the corresponding correlation coefficients (Multiple Regression, 2008).

## 3. Methodology

This chapter discusses the procedures we used in order to achieve our goals and objectives. The first section provides a brief outline of the steps we performed, including subsections discussing each step in further detail. The subsections explain how we used historical data to develop parameters needed to carry out a Monte-Carlo Simulation and how we applied regression analysis to analyze benchmark accuracy. The second section discusses the spreadsheet we constructed using Excel's Visual Basic for Applications (VBA).

### 3.1. Outline of the Simulation Process

First, we performed ten-year simulations of the RUS (iShares Russell 2000 Index) and SPY (S&P depository receipts or SPDR) using geometric Brownian motion based on their historical data. In addition, for each simulation of the SPY we created MFI (our simulated mutual fund, called Mutual Funds Incorporated), which had the same expected daily returns as the SPY, except with a small random error every day that represented the manager alpha. Next, we performed regressions comparing the returns of the MFI with the returns of the SPY and RUS, which gave us fund mappings for the MFI. Following this, we performed additional ten-year simulations of the SPY, RUS, and MFI that began where each of the initial simulations ended. We also computed what the value of the MFI would have been given the fund mapping determined in the regression. By comparing the differences between how the MFI actually performed and how its fund mappings performed, we were able to determine the accuracy of regression analysis for creating fund mappings. For a diagram explaining our simulations, see Figure 3.

This process can be summarized in four steps:

- 1) Analyzing actual historical data for the SPY and RUS.
- 2) Simulating the initial ten years (the 'Future-Past').
- 3) Regressing the MFI returns against the SPY and RUS returns.
- 4) Simulating ten more years, beginning where the first simulations ended, as well as computing the value of the MFI based on the fund mappings. This final simulation is done many times, to get a large distribution of outcomes (the 'Future-Future').



## Past Data (Daily)

9/4/2009-5/26/2000



Using info from past data to predict the stock prices for the next 10 years and create a mutual fund which follows the SPY. 50 Scenarios (See Input Tab)

## Future-Past (Daily/Monthly)

Scenario 1

Scenario 50



Calculate Regressions 1-10 years:  
**MFI**: Represents our mutual fund created in FP  
**MF12VAR**: Represents 2 Variable Regression  
**MF1SPY**: Represents 1 Variable Regression MFI vs. SPY  
**MF1RUS**: Represents 1 Variable Regression MFI vs. RUS

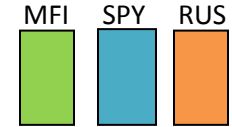
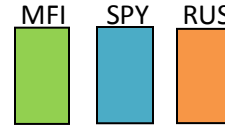
## Future-Present 2020

Predict the next 10 years for MFI, SPY, and RUS using data from the previous 10 years (Future-Past). Then use regressions to create MF12VAR, MF1SPY and MF1RUS. Compare these to original indices.

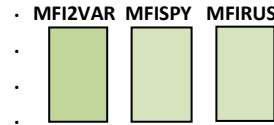
## Future-Futures (Daily/Monthly)

Scenario 1

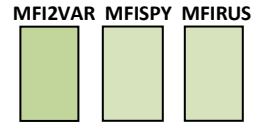
Scenario 50



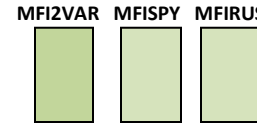
Scenario 1.1 (10-yr Reg)



Scenario 50.1 (10-yr Reg)



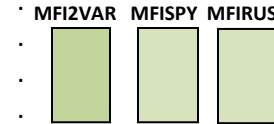
Scenario 1.1 (1-yr Reg)



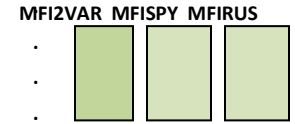
Scenario 50.1 (1-yr Reg)



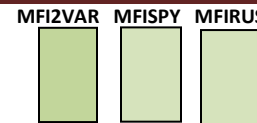
Scenario 1.50 (10-yr Reg)



Scenario 50.50 (10-yr Reg)



Scenario 1.50 (1-yr Reg)



Scenario 50.50 (1-yr Reg)

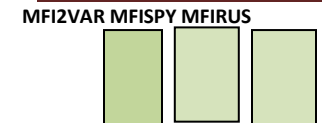


Figure 3: Diagram

After completing this process for one scenario, we can continue to repeat steps 2-4 in order to generate a large distribution of possible outcomes. For our purposes, we performed 50 simulations of the Future-Past, with 50 Future-Future simulations for each Future-Past.

### **3.1.1. Past Data**

We began our research by analyzing historical data of the SPY and RUS. We obtained daily values of the SPY and RUS dating back to May 26, 2000 from Yahoo Finance (SPY - S&P Dep Receipts, 2009). Both the SPY and RUS indexes represent total returns (including dividends). Using the Black-Scholes method for estimating volatility, we estimated the volatility of the SPY and RUS from May 26, 2000 to September 4, 2009. In addition to estimating the volatility of the SPY and RUS, we determined the correlation between their daily returns. Based on the SPY and RUS's volatility and correlation, we were able to create a covariance matrix and find the Cholesky decomposition of this matrix. The Cholesky decomposed matrix was used during the simulation to model the correlation between the SPY and RUS, based on their past correlation and volatility.

### **3.1.2. The Future-Past**

The first part of our simulation was called the Future-Past (FP). First, we simulated the SPY and RUS using geometric Brownian motion for ten years. Then, we generated a MFI that followed the SPY simulation with some amount of random error to represent manager alpha. In other words, we created a MFI that had the SPY as its perfect fund mapping. This task was done many times in order to generate a large number of FPs. For each simulation, the values of the SPY, RUS, and MFI were outputted at many points in time so that we could perform regressions on the data.

### **3.1.3. Regressions**

If we were an insurance company and wanted to find the fund mappings for our MFI, we would perform regressions comparing the returns of our MFI with various stock indices. Therefore, by performing regressions after creating simulations, we are assuming the role of an insurance company in many different market scenarios, attempting to find the proper fund mapping for our MFI. For the purposes of our study, we assume that our company decides to perform regressions using only the SPY and RUS.

For each FP, we had three types of regressions. The first regression compared the returns of the MFI to the returns of both the SPY and RUS, a multivariate regression. The second and third regressions each compared the returns of the MFI to the returns of the SPY and RUS separately. In other words, one regression compared only the MFI and SPY, and one regression compared only the MFI and RUS.

In addition, we wanted to know if the number of years that were regressed in the past affected the accuracy of the regression. So, for every  $n$  years (1, 2, 3...) up to the number of years in the FP, we performed regressions on the last  $n$  years of the FP. Each of these periods had the three types of regressions discussed earlier. In this way, for each FP, the number of regressions performed was equal to three times the number of years in the FP.

These regressions served as different fund mappings for the MFI. If we were an insurance company and didn't know that the MFI was following the SPY exactly, these fund mappings would be our best guesses as to the factors affecting our MFI. However, since we know the exact fund mapping of the MFI and also have the fund mappings resulting from regressions, we can perform further stock simulations to test the accuracy of the estimated fund mapping versus the real fund mapping.

#### **3.1.4. The Future-Future**

The final step in our methodology was to perform more stock market simulations, continuing from the end of each FP. This was called the Future-Future (FF). We wanted to know if, as an insurance company, the regression fund mappings would be accurate. Therefore, in order to create a large number of different 'worlds', we created 50 FFs for each FP in order to have a wide range of possible outcomes. In other words, every FP has its own large set of unique FFs.

For each FF, we again simulated the SPY and RUS using geometric Brownian motion for a ten year period, and generated a MFI in the same way as we did in the FP. Then, for each regression, we computed what the value of the mutual fund would have been if the regression had been used to compute it. This fund mapping of the MFI depended upon the SPY, RUS, or both, depending on the regression being used to compute it. Finally, we compared the difference between the true mutual fund simulation and the fund mapping generated from the regression. This showed us the accuracy of the various fund mappings.

By generating 50 FFs for each FP, and then creating 50 FPs, there were many distinct FFs in aggregate. Precisely, the total number of comparisons we could make between the MFI fund mapping and its true value was the number of FPs times the number of FFs in each FP. In effect, we had a large sample size for a given set of parameters. By modifying the parameters and running the entire simulation again, and then comparing results, we could determine what effects the parameters had on the accuracy of the regression fund mappings.

## 3.2. Creating the Simulation

Using VBA in Excel, we created a Macro that would complete the entire simulation based on specific inputs. All necessary random numbers are generated in the Macro, as well as all necessary computations and simulations. The runtime of the Macro depends upon the speed of the computer being used to run it. During our simulations, it typically took about 50 minutes to run 50 scenarios for one case. The Macro outputs the results of the case in Excel, including the values of the indices and MFI at all points in time. It also performs simple analysis of certain factors, such as realized SPY/RUS correlation, MFI tracking error, accuracy of the regressions, differences between the real MFI and the fund mappings, and graphs displaying pertinent data.

### 3.2.1. Inputs

The VBA Macro performed the entire simulation based upon inputs contained in the 'Inputs' tab in the Excel spreadsheet. The inputs that may be changed are highlighted in blue font and light blue background in Excel. Some inputs may only be text strings and must be selected from a drop-down list. The inputs can be grouped into five categories, for each part of the simulation.

#### 3.2.1.1. *The Future-Past*

In the FP, the user may specify how many FPs to run and how many years each FP will be. Both of these must be whole numbers. The number of FPs is also known as the number of scenarios, since each FP will contain an entire scenario including FP, regressions, and FPs. For our simulations, we used 50 FPs of ten years each.

The user may also specify how often the index and MFI values are calculated in the FP, as well as how often they are displayed. Both options may be either daily or monthly. However, if the values are calculated monthly, then they must be displayed monthly, since you can't display daily data if it doesn't exist. Finally, the user may specify whether to print the values of the index, or the returns during the FP. No matter which is printed, the simulation is still based on geometric Brownian motion using logarithmic returns. For our simulations, we calculated and displayed daily in the FP, and printed the index values.

In the Excel spreadsheet, another component is listed alongside the other FP inputs. The component,  $\Delta t$ , is the proportion of a year between each period in the FP. It is automatically updated based on how often the values in the FP are calculated, and does not need to be changed.

### ***3.2.1.2. SPY and RUS***

The SPY and RUS both have similar inputs. Each of them has a volatility  $\sigma$ , and a drift rate  $\mu$ . These components define the geometric Brownian motion used to generate the SPY and RUS in both the FP and FF. In addition, the SPY has listed under it a correlation parameter. This is the desired correlation between the SPY and RUS throughout the simulation. Finally, a 2x2 Cholesky decomposed matrix is created based on the volatility and correlation of the SPY and RUS. This matrix is used to properly simulate the correlation between the two indices. Since the Excel spreadsheet does not update calculations automatically, it is necessary to manually update calculations if the volatility or correlation is changed, so that the correct Cholesky matrix is used during the simulation.

We chose the volatility and correlation of the SPY and RUS based on past data: the SPY volatility was found to be 0.22345; the RUS volatility was found to be 0.26677; and the correlation between the SPY and RUS was found to be 0.89379. The Cholesky matrix resulting from our volatility and correlation was  $\begin{bmatrix} 0.22345 & 0 \\ 0.23844 & 0.11965 \end{bmatrix}$ . In addition, we chose the SPY's drift rate to be 0.08 and the RUS's drift rate to be 0.09.

Finally, the last parameters effecting the SPY and RUS are their initial stock prices,  $S(0)$ . The values are based on the actual SPY and RUS closing values as of September 4, 2009, and thus it is not recommended that they be changed.

### ***3.2.1.3. Mutual Fund***

Since the MFI is made to follow the SPY with slight random error, the only factors affecting the MFI are the degrees of random error. There are two inputs that affect the randomness of the MFI. The first,  $\alpha$ , is a measure of the degree of randomness associated only with the MFI. Generally,  $\alpha$  is the coefficient of an additional standard normal random number that is added to the SPY's return each year to create the MFI. The second parameter affecting the randomness of the MFI modifies  $\alpha$ . Known as the 'Proportion of SPY' on the 'Inputs' tab in Excel, it modifies  $\alpha$  depending on how much the SPY changes in a given period. Specifically, a given SPY return is multiplied by the 'Proportion of SPY' and added to  $\alpha$  to create the total random error of the MFI. Intuitively, the 'Proportion of SPY' component makes sense because the more volatile the SPY is, the more volatile the MFI should be as well. Alpha and the 'Proportion of SPY' were two of the major factors we analyzed, but for the base case we set  $\alpha$  as 0.020 and 'Proportion of SPY' as 2.50.

The third component listed under the MFI is the initial value of the MFI,  $S(0)$ . Since we are creating the MFI to follow the SPY, the initial value should not be changed, as it should be exactly the same as the SPY's initial value.

#### ***3.2.1.4. Regressions***

The regressions on the FP can be modified by three parameters. The first is the maximum number of years to regress back in the FP. For example, if a user only wanted to regress up to five years back, but the FP was 10 years long, he or she could set the maximum number of years to regress equal to 5. The number must be a whole number and must be less than or equal to the number of years in the FP. If it is set larger than the number of years in the FP, the Macro will automatically set it to the number of years in the FP. We always used the maximum number of years to regress set equal to 10, which was also the number of years in the FP.

The second regression parameter is a choice to regress every single day in the FP, or only once a month. If the FP was only outputted monthly, then the regressions were forced to be done monthly. For all of our simulations, we used daily regressions.

Lastly, we allowed the option to regress on the index returns or index values. Both types of regressions could be done regardless of what was printed in the FP, since index returns could be calculated based on index values, or vice versa. Generally, it is not useful to regress on the index values, since then the regression would be entirely dependent upon the starting value of the indices, and would be inaccurate for estimating returns. Therefore, for all of our regressions, we regressed the returns of the indices and MFI, not the values.

#### ***3.2.1.5. The Future-Future***

The variables in the FF parallel those in the FP. A user may choose the number of years in each FF, how many FFs to create for each FP, how often to calculate the values in the FF, how often to display the values in the FF, and whether to print index values or index returns. Once again,  $\Delta t$  is automatically calculated based on how often the values in the FF are calculated. If the values are only calculated monthly, they can only be displayed monthly. Note that all parameters in the FF are completely independent of those in the FP. No matter what was chosen in the FP, a user may choose something completely different for the FF.

For all of our simulations, we used the same FF parameters. We did 50 FFs for each FP, and the number of years in each FF was 10. The values were calculated daily, but only displayed monthly. This

was done to save space in the Excel spreadsheet. Lastly, we opted to have the Macro print out the values of the indices and MFI, rather than the returns.

### 3.2.2. Scenario Output

This sheet contains the data for the entire simulation. Every time the Macro is run, a new ‘Scenario Output’ sheet is created, and any old ones are backed up by renaming them. This sheet is what is seen when the Macro is running. The top-left cell is automatically selected, and displays what the Macro is currently doing, until it is complete. The rest of the sheet is divided into three major sections.

The first section has every FP. Each FP is printed vertically and has three columns. Each column corresponds with the values or returns of the MFI, SPY, or RUS. Each period is one row, and the leftmost column of the sheet displays which period number it is.

The second section of the ‘Scenario Output’ sheet contains information on all of the regressions that were done. Below each FP, there is a list of regressions corresponding with that FP. The three types of regressions are labeled MFI2Var (SPY and RUS), MFISPY, and MFIRUS. They are grouped into sets, based on how many years were regressed. The number of years regressed is listed in the leftmost column of the sheet. Each regression contains a formula for the MFI based on that regression, followed by the coefficients used in the formula, followed by an array of numbers. The array of numbers contains data about the regression. For the 2-variable regressions, the data in the array corresponds to the following table:

#### 2-Variable Regression Data Output

Coefficient for RUS	Coefficient for SPY	Constant term
Standard error of the RUS term	Standard error of the SPY term	Standard error of the constant
Coefficient of Determination ( $R^2$ )	Standard error of the MFI estimate	
F-Statistic	Degrees of Freedom	
Sum of Squares	Residual Sum of Squares	

Table 4: 2-Variable Regression Data Output

For the 1-variable regressions, the data in the array corresponds with the following table:

### 1-Variable Regression Data Output

Coefficient of the independent variable	Constant term
Standard error of the independent variable	Standard error of the constant
Coefficient of Determination ( $R^2$ )	Standard error of the MFI estimate
F-Statistic	Degrees of Freedom
Sum of Squares	Residual Sum of Squares

Table 5: 1-Variable Regression Data Output

Finally, after the regressions follow the FFs. For every FP, all of its FFs are contained below its regressions. They are labeled in the form ‘Scenario XX.YY’, where XX is the number of the FP and YY is the number of the FF. So, ‘Scenario 2.3’ is the 3<sup>rd</sup> FF of the 2<sup>nd</sup> FP. Each FF begins the same as the FP, with a simulation using geometric Brownian motion for a certain number of years. Following this simulation, there are lists of values of what the MFI would be if it had followed the fund mapping of a certain regression. This is done for every number of regression years. Since we regressed from one to ten years in our simulations, we have ten lists following each FF of what the MFI would be if it had followed a certain regression fund mapping. Then, after all of the fund mapping values of the MFI are completed, the next FF is calculated below that, and the process repeats until all FFs for that FP are done.

### 3.2.3. Output

This sheet is printed after the simulation is done. The first part reprints the values that were used as inputs, to ensure that they remained the same throughout the entire simulation. It is formatted the same way as the ‘Input’ tab. It also lists the start time and end time of the Macro, so that a user may see how long it took to run the entire simulation. To the right of this, some simple analysis is conducted on the scenario data.

The top-right contains a brief overview of what happened during each FP. First, the average yearly return of the SPY, RUS, and MFI is calculated. Next, the realized  $\alpha$  is calculated below the average returns. Also known as the tracking error, it is the standard deviation of the difference between the MFI and the SPY. Below that, the realized correlation between the SPY and RUS is calculated. After all of this, the average and standard deviation of all of these different calculations are printed on the right



side. These calculations are mostly for error checking, to make sure that the correlation, MFI random error, and expected returns were implemented correctly.

Finally, the lower-right portion contains simple analysis of the regression data. It finds the averages of several key parameters of the regression for each number of years regressed. Specifically, the average of the coefficients, standard error, and  $R^2$  for each type of regression is calculated for each of the regression years. The last column on the right contains averages among all regression years for the various parameters.

### 3.2.4. Graph Data

After the Macro generates the entire simulation, it prints certain data into this sheet that will be displayed in graphs. In a way, it is an extension of the 'Output' tab. It is divided into two main sections: the top portion has data concerning regression coefficients, and the lower portion contains the differences between the real MFI and benchmark MFI in the FF.

For the regressions, The Macro looks separately at each coefficient. Each 1-variable regression has one coefficient, and the 2-variable regressions have 1 SPY coefficient and 1 RUS coefficient. For each of these four coefficients, the Macro does the same thing. Looking at each year separately, it analyzes the minimum, first quartile, median, mean, third quartile, and maximum of a coefficient. This is done for all years of regressions. For example, in our case, since we regressed up to 10 years, we have 10 different sets of regressions to look at for each of the four types of coefficients.

Below the regression analysis are the percentage differences between the benchmarks and the real values of the MFI. For each of the 30 regressions we performed (three types of regressions times ten years), the Macro looked at the percentage difference between the real MFI and the value based on that benchmark every month. The Macro finds the minimum, first quartile, median, mean, third quartile, and maximum difference every month for each type of regression.

### 3.2.5. Graphs

The last things that are output by the Macro are the graphs. There are two types of graphs. The first type of graph shows a scatter plot of the regression coefficients for the various regressions, and the second type of graph is a line chart displaying the difference between the real MFI and the MFI fund mapping from the regression.

There are four different graphs showing coefficients. These correspond with the SPY coefficient from the 2-variable regression, RUS coefficient from the 2-variable regression, the SPY coefficient from the 1-variable SPY regression, and the RUS coefficient from the 1-variable RUS coefficient. Each graph

shows the maximum, third quartile, median, first quartile, and minimum coefficients of the 50 scenarios, for every number of years regressed. These values are pulled from the 'Graph Data' tab, thus the charts are a graphical representation of the numbers displayed there. For an example of this type of graph, see Figure 4.

The Macro creates 30 of the second type of graph. Each one of these shows the monthly maximum, third quartile, median, first quartile, and minimum difference between the real MFI and one of the proxy MFI fund mappings found via regression. Since there were thirty different regressions (three types of regressions times ten years), there is a different graph for each one. As before, the data for these graphs is pulled from the 'Graph Data' sheet. See Figure 5 for an example of this type of graph.

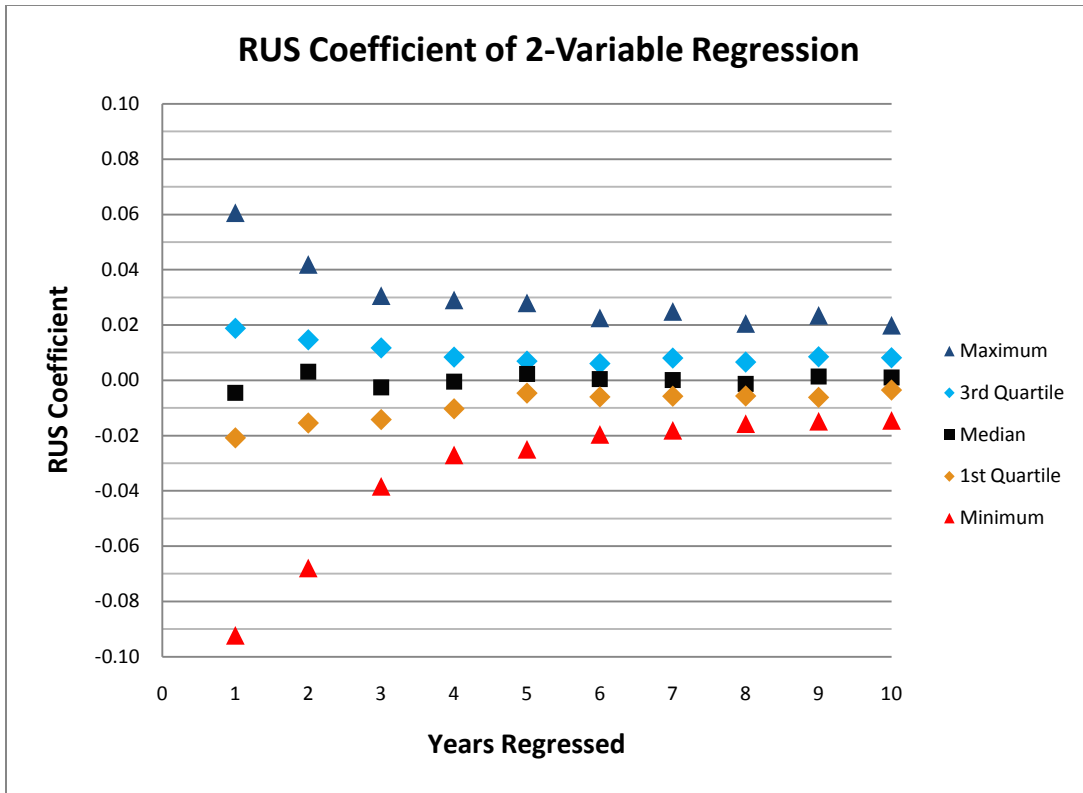


Figure 4: Example of the first type of chart generated by the Macro

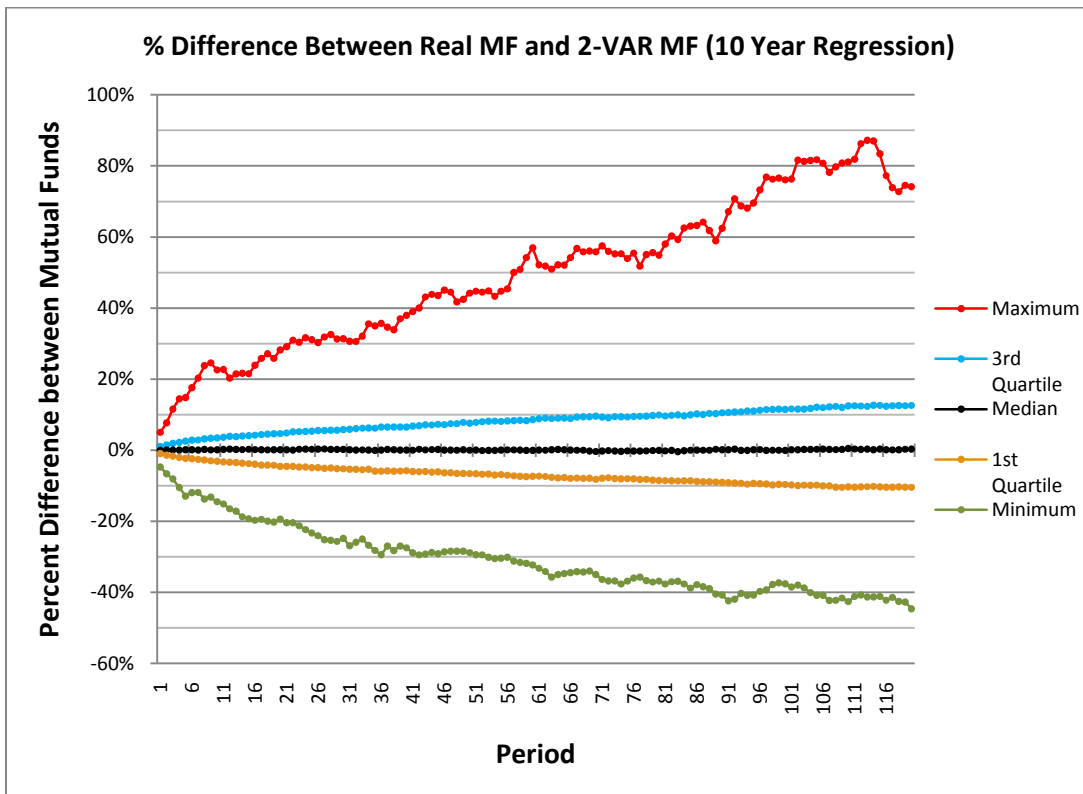


Figure 5: Example of the second type of chart generated by the Macro

## 4. Analysis and Discussion

In order to analyze the effects of different inputs on the accuracy of our fund mappings, we created 15 different cases and changed only one or two variables in each. A summary of the values of each variable in each case can be found in Table 6. The variables that were changed were manager alpha, proportion of randomness from SPY returns, correlation between the SPY and RUS, SPY volatility, and RUS volatility. There are several statistics that we looked at to determine the differences among the different cases, which include coefficient of determination ( $R^2$ ), realized alpha, realized correlation, and the difference between MFI and each of the other simulated MFIs (MFI2Var, MFISPY, and MFIRUS) after ten years.

	Alpha	Proportion	Correlation	SPY Volatility	RUS Volatility
Base Case	0.020	2.5	0.89379	0.22345	0.26677
Case A	<b>0.015</b>	2.5	0.89379	0.22345	0.26677
Case B	<b>0.025</b>	2.5	0.89379	0.22345	0.26677
Case C	0.020	<b>2.0</b>	0.89379	0.22345	0.26677
Case D	0.020	<b>3.0</b>	0.89379	0.22345	0.26677
Case E	<b>0.015</b>	<b>3.0</b>	0.89379	0.22345	0.26677
Case F	<b>0.025</b>	<b>3.0</b>	0.89379	0.22345	0.26677
Case G	<b>0.015</b>	<b>2.0</b>	0.89379	0.22345	0.26677
Case H	<b>0.025</b>	<b>2.0</b>	0.89379	0.22345	0.26677
Case I	0.020	2.5	0.89379	<b>0.17000</b>	0.26677
Case J	0.020	2.5	0.89379	<b>0.27000</b>	0.26677
Case K	0.020	2.5	0.89379	0.22345	<b>0.21000</b>
Case L	0.020	2.5	0.89379	0.22345	<b>0.31000</b>
Case M	0.020	2.5	<b>0.80000</b>	0.22345	0.26677
Case N	0.020	2.5	<b>0.95000</b>	0.22345	0.26677

Table 6: Defining Cases

### 4.1. Regression Coefficients

When comparing between different cases, we only looked at the mean of the regression coefficients for the ten year regressions. As evidenced in Table 7, the coefficients for the two-variable regression and the single variable SPY regression vary very little among the different cases. However,

the regression coefficients for the single-variable regressions using RUS returns change quite a bit when volatility and correlations are changed (Cases I-N). Because the regression coefficients of the 2-Var SPY are very close to one and the coefficients of the 2-Var RUS are very close to zero, all factors analyzed were very similar when comparing the values for the two-variable regression with the values for the single-variable regression using SPY returns, since the fund mappings were so similar.

	2-Var SPY	2-Var RUS	1-Var SPY	1-Var RUS
Base Case	1.0001	-0.0001	1.0024	0.7495
Case A	0.9998	-0.0008	0.9996	0.7484
Case B	1.0016	0.0004	1.0022	0.7504
Case C	1.0044	-0.0014	1.0007	0.7489
Case D	1.0041	-0.0037	1.0020	0.7468
Case E	0.9981	0.0017	0.9985	0.7492
Case F	1.0018	0.0009	1.0001	0.7463
Case G	1.0009	-0.0011	0.9987	0.7461
Case H	0.9980	0.0008	1.0000	0.7511
Case I	0.9961	0.0027	0.9999	0.5688
Case J	1.0011	-0.0029	0.9988	0.9070
Case K	1.0023	-0.0016	1.0016	0.9529
Case L	0.9986	0.0007	1.0012	0.6444
Case M	0.9975	-0.0009	0.9973	0.6669
Case N	1.0004	0.0019	1.0027	0.9793

Table 7: Regression Coefficients

## 4.2. Values of R<sup>2</sup>

The coefficient of determination (R<sup>2</sup>), a statistical measure of how much of the mutual fund's movement is explained by the underlying fund mapping, is almost exactly the same for the regressions on the single variable SPY return and the two-variable regression in all 15 cases. A summary of the values of R<sup>2</sup> for the 2-Var, SPY, and RUS regressions for all 15 cases can be found in Table 8.

	2-Var	SPY	RUS
Base Case	0.9485	0.9485	0.7580
Case A	0.9560	0.9559	0.7635
Case B	0.9391	0.9391	0.7532
Case C	0.9600	0.9600	0.7661
Case D	0.9345	0.9345	0.7457
Case E	0.9429	0.9428	0.7539
Case F	0.9245	0.9244	0.7398
Case G	0.9670	0.9670	0.7712
Case H	0.9518	0.9517	0.7606
Case I	0.9363	0.9362	0.7461
Case J	0.9538	0.9357	0.7630
Case K	0.9481	0.9480	0.7567
Case L	0.9473	0.9473	0.7567
Case M	0.9475	0.9474	0.6073
Case N	0.9483	0.9483	0.8557

Table 8: Average Values of  $R^2$

As evidenced in the Table 8, the values of  $R^2$  for the 2-Var and SPY regressions are all within a reasonably small range. The lowest value of  $R^2$  is from Case F with a value of 0.9245, where we raised alpha from 0.020 to 0.025 and we raised the proportion of randomness from the SPY returns from 2.5 to 3.0. This is to be expected because this combination of alpha and proportion allows for the most randomness among all of the cases. The highest value is from Case G with a value of 0.9670, where we lowered alpha from 0.020 to 0.015 and lowered the proportion from 2.5 to 2.0. As with the lowest value, this is to be expected because this combination of alpha and proportion allows for the least amount of randomness, so MFI follows the SPY more closely and the regression is more accurate. In general, the values of  $R^2$  for the 2-Var and SPY regressions all hover around 0.95, meaning that about 95% of the momentum of MFI can be explained by SPY alone or by the SPY and RUS combined. When there is a difference between the value of  $R^2$  for the 2-Var and RUS regressions, it is a difference of only about 0.0001, which may be a rounding error. Because there is such a high correlation between the SPY

and MFI returns, adding the RUS returns in does not help explain the movement of MFI. This means that the SPY already explains the movement well enough such that adding the RUS has no additional positive effect. Effectively, the two fund mappings are virtually identical.

Because the RUS is not the proper benchmark for MFI, the values of  $R^2$  are lower when looking at the RUS fund mapping. Generally, the RUS accounted for about 75% of the movement of the SPY, since the values of  $R^2$  were generally around 0.75. This value is much lower than the  $R^2$  of 0.95 in the other types of regressions because the RUS is not the proper benchmark. In addition, the values of  $R^2$  for the RUS regressions have a much larger range than the values for the other types of regressions. However, this is mostly because of two outliers, Case M and Case N. These are the cases where the correlation between SPY and RUS returns is changed. Changing the correlation has the largest impact on the RUS regression because changing the correlation between RUS and SPY changes the correlation between MFI and RUS returns but not between MFI and SPY returns. Aside from those two outliers, the highest and lowest values of  $R^2$  for the RUS regressions correspond directly with the highest and lowest values of  $R^2$  for the 2-Var and SPY regressions.

### **4.3. Realized Alpha and Correlation**

Realized alpha, or tracking error, is the standard deviation of the difference between MFI and SPY. It is the average of the observed values of manager alpha throughout all 50 FP scenarios, and takes into account both the manager alpha and the 'Proportion of SPY' parameters. The Standard Deviation of Tracking Error in Table 9 is the standard deviation of the realized alphas of all 50 FP scenarios. Average realized correlation was calculated mostly in order to ensure that our calculations were being done correctly as a form of error checking, and the standard deviation across all 50 FP scenarios is also shown in Table 9.

The values of the realized alpha are all within a relatively close range. They tended to be between 5.00% and 5.50%, except in exceptional cases. The lowest value comes from Case G with a value of 4.13%, because in Case G, we lowered the manager alpha from 0.020 to 0.015 and lowered the proportion from 2.5 to 2.0, which limited the amount of randomness. This, in turn, lowered the difference between the MFI and SPY returns. The highest value of realized alpha came from Case F with a value of 6.41%. This also is not surprising because in Case F we raised manager alpha from 0.020 to 0.025 and raised the proportion from 2.5 to 3.0, which allowed more randomness than any other case. These results are not extraordinary, and serve more as a confirmation that the implementation of the

randomness associated with MFI was done correctly. Finally, we noticed no significant differences between any of the standard deviations of the tracking error.

	Average Tracking Error	Standard Deviation of Tracking Error	Average Realized Correlation	Standard Deviation of Realized Correlation
Base Case	5.24%	0.12%	89.42%	0.37%
Case A	4.81%	0.10%	89.44%	0.42%
Case B	5.73%	0.10%	89.42%	0.38%
Case C	4.58%	0.08%	89.46%	0.28%
Case D	5.92%	0.14%	89.38%	0.39%
Case E	5.53%	0.13%	89.39%	0.41%
Case F	6.41%	0.16%	89.34%	0.42%
Case G	4.13%	0.09%	89.30%	0.41%
Case H	5.07%	0.08%	89.42%	0.36%
Case I	4.43%	0.09%	89.26%	0.36%
Case J	5.93%	0.14%	89.42%	0.34%
Case K	5.25%	0.09%	89.39%	0.35%
Case L	5.26%	0.09%	89.36%	0.34%
Case M	5.27%	0.12%	80.15%	0.70%
Case N	5.24%	0.11%	95.00%	0.19%

Table 9: Realized Alpha and Correlation

All values for realized correlation centered on the calculated historical value for correlation (89.38%) that was input into the Macro. The only exceptions are Cases M and N in which we intentionally changed the correlation. In these two cases, the realized correlations of 80.15% and 95.00% are very close to our inputted values of 80.00% and 95.00%. This is evidence that our model was working correctly. Finally, we observed that the standard deviation of the correlation was similar for all cases except M and N, in which we lowered and raised the correlation, respectively. Lowering the correlation caused a higher standard deviation of correlation, and raising the correlation caused a lower standard deviation. This is because as the correlation gets further from 100%, there is more potential for error above the correlation, causing higher standard deviations.



#### 4.4. Ending Difference between MFI and MFI2Var

Table 10 shows the 25<sup>th</sup> percentile, 75<sup>th</sup> percentile, median, and interquartile range (75<sup>th</sup> percentile minus 25<sup>th</sup> percentile) of the ending difference between the MFI and MFI2Var returns at the end of the ten-year FF.

	25 <sup>th</sup> Percentile	Median	75 <sup>th</sup> Percentile	Interval Length
Base Case	-10.43%	0.30%	12.59%	23.02%
Case A	-10.92%	-1.02%	9.88%	20.80%
Case B	-7.89%	0.24%	8.94%	16.83%
Case C	-9.10%	0.49%	10.97%	20.07%
Case D	-12.40%	-0.14%	13.47%	25.87%
Case E	-11.76%	-0.15%	12.09%	23.85%
Case F	-12.51%	0.18%	14.25%	26.76%
Case G	-8.64%	0.02%	10.12%	18.76%
Case H	-9.55%	0.04%	12.17%	21.72%
Case I	-10.16%	-0.67%	9.36%	19.52%
Case J	-12.03%	-1.17%	10.05%	22.08%
Case K	-9.64%	1.70%	13.85%	23.49%
Case L	-8.86%	-0.52%	8.09%	16.95%
Case M	-12.15%	-0.54%	11.83%	23.98%
Case N	-11.30%	-0.73%	11.56%	22.86%

Table 10: Difference between MFI and MFI2Var

In general, there are not any unexpected results. The median is around 0% for all cases, meaning that on average there is no difference between the MFI and the 2-Var fund mapping after ten years. In other words, the fund mapping is accurate on average. This is corroborated by the values of  $R^2$  that we found for the 2-Var regression, which were all around 0.95 and signaled that the regression was accurate. The 25<sup>th</sup> and 75<sup>th</sup> percentiles are generally around -10% and 10% respectively, meaning that about 50% of the time, the fund mapping will be off by less than 10%. The other 50% of the time, there would be even more error. This shows the effect of basis risk; there may be certain situations when things simply go awry due to unexpected problems, when in other situations there is a positive outcome, even with the same inputs.

## 4.5. Ending Difference between MFI and MFISPY

Table 11 shows the 25<sup>th</sup> percentile, 75<sup>th</sup> percentile, median, and interquartile range (75<sup>th</sup> percentile minus 25<sup>th</sup> percentile) of the ending difference between the MFI and MFISPY returns at the end of the ten-year FF. We noticed that the interval lengths in Table 11 are very similar to those in Table 10, above. This is because of the close relationship between the linear regression using SPY returns and the multiple regression using both SPY and RUS returns. This is similar to how the values of  $R^2$  for the MFISPY regression compare with the MFI2-Var regression; they are almost exactly the same. This caused the fund mappings themselves to be almost the same between the MFISPY and MFI2-Var after ten years.

	25 <sup>th</sup> Percentile	Median	75 <sup>th</sup> Percentile	Interval Length
Base Case	-10.49%	0.53%	12.53%	23.02%
Case A	-10.97%	-1.12%	9.86%	20.83%
Case B	-8.00%	0.12%	8.87%	16.87%
Case C	-9.04%	0.54%	10.91%	19.95%
Case D	-12.55%	-0.13%	13.42%	25.97%
Case E	-11.85%	-0.24%	11.96%	23.81%
Case F	-12.62%	0.20%	14.16%	26.78%
Case G	-8.50%	0.18%	9.92%	18.42%
Case H	-9.38%	0.29%	11.97%	21.35%
Case I	-10.12%	-0.69%	9.46%	19.58%
Case J	-12.08%	-1.29%	10.13%	22.21%
Case K	-9.41%	1.68%	14.02%	23.43%
Case L	-8.62%	-0.55%	8.20%	16.82%
Case M	-12.03%	-0.32%	11.82%	23.85%
Case N	-11.16%	-0.58%	11.64%	22.80%

Table 11: Difference between MFI and MFISPY

## 4.6. Ending Difference between MFI and MFIRUS

Table 12 shows the 25<sup>th</sup> percentile, 75<sup>th</sup> percentile, median, and interquartile range (75<sup>th</sup> percentile minus 25<sup>th</sup> percentile) of the ending difference between the MFI and MFIRUS returns at the end of the ten-year FF. The intervals in Table 12 are much larger than those in the two previous sections. This is because although the SPY and RUS returns are highly correlated, the RUS does not

explain the movement of MFI as thoroughly as SPY does. As a result, there is much greater variation in the ending values after ten years. We also found that the variation was greatest when the volatility of the SPY was lowered (Case J) and least when the volatility of SPY was raised (Case I). So, by changing the volatility of the SPY, the MFIRUS was affected. This is because the MFI follows the SPY, so by changing the parameters for the SPY, the MFI is also changed. Finally, we noticed that the medians of the ending RUS values are skewed downward, normally around -11.00%. However, this skew seems to depend on the volatilities and correlation of the indices, since the skew was different in Cases I-N. In Case K, there was even an upward trend, suggesting that the MFIRUS fund mapping was too low rather than too high.

	25 <sup>th</sup> Percentile	Median	75 <sup>th</sup> Percentile	Interval Length
Base Case	-31.38%	-13.45%	11.35%	42.73%
Case A	-30.73%	-11.74%	12.64%	43.37%
Case B	-29.09%	-10.69%	11.20%	40.29%
Case C	-31.51%	-12.29%	12.08%	43.59%
Case D	-30.87%	-11.11%	14.36%	45.23%
Case E	-29.10%	-10.22%	13.70%	42.80%
Case F	-29.87%	-11.02	14.14%	44.01%
Case G	-28.71%	-10.00%	13.32%	42.03%
Case H	-31.45%	-11.48%	13.11%	44.56%
Case I	-36.23%	-23.80%	-7.25%	28.98%
Case J	-26.94%	-0.89%	33.64%	60.58%
Case K	-16.71%	6.72%	33.27%	49.98%
Case L	-35.03%	-20.78%	-1.61%	33.42%
Case M	-37.38%	-14.83%	16.06%	53.44%
Case N	-23.21%	-7.08%	11.28%	34.49%

Table 12: Difference between MFI and MFIRUS

It is plain to see that the biggest variances from the base case occur in the cases where volatility or correlation are changed, Case I through Case N. This has led us to believe that it is the ratio of the two volatilities that makes the most difference in determining the ending value. Overall, this fund mapping shows that using improper indices results in fund mappings that have significant error, on average, after ten years. This suggests that insurance companies and investors should be careful to choose proper indices for their fund mappings, as a poor choice of indices could lead to very inaccurate

fund mappings. One important statistic to pay attention to is  $R^2$ . If  $R^2$  is close to one, the fund mapping can be expected to be very accurate, but if it is further from one, the fund mapping can be expected to be more inaccurate.

## 5. Conclusions and Recommendations

Within this section we will discuss the conclusions and recommendation of our research. We will begin with an analysis of our results and interpretations of their meaning. Following this, we will then discuss ways to improve our model and other suggestions for future research. Finally, we will conclude with a summary of our research.

### 5.1. Conclusions

After analyzing the results of many different cases and situations, we have come up with some conclusions about the research. These conclusions revolve around several different themes, such as correlation, multicollinearity, volatility, and regressions over different periods of time.

#### 5.1.1. Correlation

One interesting observation that we made during the simulations was the effect of correlation on the fund mapping accuracy. When the correlation between the SPY and RUS was raised or lowered, there was no effect on the 1-variable SPY regression or the 2-variable regression. However, the accuracy of the 1-variable RUS regression was directly impacted by the correlation. Precisely, when the correlation was lowered, the 1-variable RUS coefficient became further away from 1, the  $R^2$  was lower, and the fund mapping was less accurate. On the other hand, when the correlation was raised, the coefficient was closer to 1 (and thus more accurate), the  $R^2$  was higher, and the fund mapping was more accurate. Therefore, the correlation between indices affects the accuracy of fund mappings when using improper indices. However, when the chosen indices sufficiently explained the movements of the mutual fund, and thus a proper benchmark was found, the correlation had little effect on the outcome of fund mapping accuracy. Additionally, indices having high correlations may be falsely estimated as being the actual benchmarks, even if though the real benchmark is a different highly correlated index. This error is known as multicollinearity.

#### 5.1.2. Multicollinearity

One issue that could have potentially affected the results was multicollinearity. Multicollinearity occurs when two independent variables in a multiple regression are highly correlated. Although we anticipated multicollinearity to be a source of error, we did not find this to be the case. Even with highly correlated indices (as in Case N with correlation set to 0.95), multicollinearity did not seem to occur. Even though the RUS and SPY were highly correlated, the 2-variable regression still depended mostly on the SPY and was unaffected by the RUS. If multicollinearity had occurred, we would have seen a large

mismatch between the MFISPY and the MFI2-Var, due to the high correlation between the SPY and RUS affecting the fund mappings. Since this did not occur, multicollinearity was not an issue when using this model. However, it may be important to recognize multicollinearity in future research when using different models or indices, and to further study the effect of multicollinearity on fund mappings.

### 5.1.3. Volatility

From Cases I and J, in which the SPY volatility was changed, the biggest effect was actually the accuracy of the RUS regressions. Intuitively, this may not make sense because changing the SPY volatility should not change the RUS's accuracy. However, the reason is because the accuracy of the RUS depends entirely on the difference between the volatility of the SPY and RUS. Putting everything together from Cases I, J, K, L, and the base case, the effect of volatilities on the accuracy of the RUS regression is in Table 13.

	Case I	Case L	Base Case	Case J	Case K
SPY Volatility	0.17000	0.22345	0.22345	0.27000	0.22345
RUS Volatility	0.26677	0.31000	0.26677	0.26677	0.21000
(SPY vol)/(RUS vol)	<b>0.63725</b>	<b>0.72081</b>	<b>0.83761</b>	<b>1.01211</b>	<b>1.06405</b>
Median RUS regression error	<b>-23.78%</b>	<b>-20.81%</b>	<b>-13.43%</b>	<b>-0.89%</b>	<b>6.70%</b>

Table 13: Effect of volatility on improper fund mappings

If we compare the ratio of SPY volatility and RUS volatility to the accuracy of the RUS fund mapping, we can see that the average error of the RUS is directly related to the ratio of the SPY volatility to the RUS volatility. Notice that as the ratio of SPY volatility to RUS volatility increases, the median error of the RUS-only regression after 10 years increases. When the two volatilities are almost equal, the error is about zero (Case J). Therefore, we can conclude that the ratio of the index volatilities directly impacts the accuracy of benchmarks that use improper indices. Therefore, investors should be mindful of the volatilities of indices that they choose to use for fund mappings. If the volatilities vary greatly, there may be errors due to certain indices being more or less volatile than the true benchmark, resulting in errors due to this difference.

### 5.1.4. Regression Periods

One goal of the project was to see if there was any change when regressing a different number of years into the past. Throughout all cases, the number of years regressed seemed to have very little effect on any of the data that we analyzed, including  $R^2$  and fund mapping accuracy. The only difference occurred in the regression coefficients. Generally, the more years regressed, the closer the coefficients

were to what the true fund mapping should be, and the lower the interquartile range (error) of the coefficients was. However, this did not have any significant effect on any other data. We believe that this is because we have used only static volatilities, correlations, and other parameters that do not change at all throughout our entire simulation process. Had we had time to improve our model so that volatility and correlation were stochastic, it would most likely have resulted in visibly different results among the different regression periods analyzed. It may have also made a difference if we had regressed monthly (or some other quantity of time) as opposed to daily. With daily regressions, there is a large amount of data to utilize. Therefore, even with only a one year regression, the regression will be very accurate. However, in the real world, historical data will always be present to use for regressions, so it may be more realistic to simply use all data that is available.

#### **5.1.5. How Our Results Relate to Active Management and Basis Risk**

Our research does not take into account all aspects of basis risk. Since we know the exact benchmark, we do not have to worry about basis risk arising from choosing an incorrect proxy hedge or fund mapping. We also eliminated the possibility that the manager changed his strategy because we controlled all of the variables. Therefore, we only were analyzing the risk arising from the manager trying to outperform the benchmark. We found that, using our static model, this type of basis risk would be nothing on average. However, because the average is just an average and does not always occur, there may be significant error in some cases. This is the basis risk. Sometimes the mutual may outperform its fund mapping, or sometimes it may underperform its fund mapping, due to the random error of a manager trying to outperform the benchmark. Although we have not found a way to eliminate basis risk which would be impossible, we can suggest some ways to minimize it. One way to minimize basis risk would be to ensure that the indices chosen for the fund mapping are representative of the mutual fund. As our data had shown, using improper indices results in a large error and volatility of possible outcomes. Therefore, insurance companies and investors should pay attention to statistical measures such as  $R^2$  in order to obtain the most accurate fund mapping. In addition, the volatilities of the indices may have some effect on the mean accuracy of the fund mappings. Asset managers should ensure that they choose indices with similar volatilities to the mutual fund itself in order to minimize this type of error. These suggestions are only a couple of ways to minimize one type of basis risk, and there may be many more. Future research may be able to find ways in order to further minimize basis risk.

## 5.2. Improvements and Future Research

Our conclusions make it clear that many improvements can be made in order to further study the effects of basis risk and active management on fund mappings. We hypothesize that one of the greatest limitations of our research is the model itself. Therefore, we propose a variety of ideas and techniques that may be implemented by future researchers in order to make the model more versatile. With a better model, more conclusions may be able to be made about basis risk and the accuracy of fund mappings.

### 5.2.1. Creating Stochastic Parameters

Throughout our simulations, we used a constant mean, volatility, and correlation in the geometric Brownian motion. This is not very realistic. In reality, the drift rate, volatility, and correlation of indices are constantly changing. Assuming that they remain constant for years is a large assumption and makes our model less applicable to the real world. Future research might look into creating stochastic volatilities and correlation when modeling indices, and how they affect the accuracy of benchmarks and manager tracking error.

For example, it is questionable to assume that indices correlated in the past will continue to be correlated in the future. If the correlation is changing, it will be most similar to the historical correlation early on, and then begin to drift as time passes. Therefore, a person creating stochastic correlation might want to start with a correlation similar to the historical correlation, and then allow the correlation to decrease or increase depending on various market factors. For example, correlation between indices tends to be highest when there are sharp gains or losses in the market. Consequently, the correlation may be made higher when the absolute returns are very high for a long period of time, and be made lower when the returns are closer to zero.

In addition, market volatility changes over time. This is evidenced by the fact that the market moves more severely over some periods than it does over others. It would be interesting to study the effects of having one volatility in the FP, and then a different volatility (or correlation) in the FF. Or, the volatility estimate could change based on the observed volatility in the past year during the simulation, as if a company kept updating their simulation with the most recent data.

By implementing these stochastic volatilities and correlations, our model for equity returns may become more applicable to the real world. Although we cannot anticipate all factors that may affect the market in the future, improving the model would at least account for certain behaviors. Therefore,



refining the stochastic processes is one of the most fundamental tasks that future researchers should implement.

### 5.2.2. More Scenarios

While working with Excel 2003, we were able to generate 50 different scenarios for a 10 year period. However, we believe that using a larger program and more scenarios may further confirm the conclusions of our research. Using a larger spreadsheet, such as Excel 2007, would increase available space in order to run more scenarios and ensure more confidence with our results. Another method that could potentially provide the simulation with more scenarios would be to use variance reduction techniques.

One example of a variance reduction technique is using antithetic random numbers. For example, if we have a random sample of  $n=2$ , using a random number  $U_1$ , we can define  $U_2=1-U_1$ . In our case we had 50 scenarios (with 50 random numbers), and if we chose to use the antithetic random number technique, we could have produced twice as many random numbers and therefore twice as many scenarios (McLeish, 2005). Another variance reduction technique is Importance Sampling. When using this approach, the simulations are performed on a conditional distribution rather than the original distribution, and may be desirable if the strike price is significantly above or below the market price (Cvitanic & Zapatero, 2004).

### 5.2.3. Adding more indices

After performing our calculations, we found that the SPY and RUS were highly correlated and using these two indices to create fund mappings gave us results we had originally hypothesized. After further analysis, we believe that using more than two indices would have provided us with more diversified and realistic results. In order for our team to use more indices, we would have had to decrease the number of scenario outputs due to limited space available in Excel 2003. We also would have had to apply Cholesky decomposition when performing the geometric Brownian motion, which would have been very computationally expensive.

Generally, portfolios are more diversified and the weights within the portfolio do not normally only follow two indices. Therefore, if we were to use more indices, then the benchmark would be more practical and the weighting options would have developed into a more real world situation. Furthermore, introducing more indices could have given our team a chance to experiment with how volatile our fund mapping could be. This would include introducing securities that were either very risky or very conservative, giving us the chance to see how the benchmark would be affected by this change.

Moreover, this could result in assorted correlation and experimenting with these combinations could give more insight to what techniques are efficient. International indices could also be of interest, since they tend to be very uncorrelated to U.S. indices and have different market factors effecting them.

#### **5.2.4. Choice of Means for SPY and RUS**

Calculating the historical mean returns for the SPY and RUS resulted in -1.60% and 3.59% annual drift rates, respectively. Most of the time, annual returns are not this low, but due to the market decline in recent years the annual returns are sometimes below zero. If we were to use these means in our simulation it would have produced an inaccurate representation of long-term market tendencies; therefore, we decided to use means that were more appropriate for the purpose of our study. Since the Russell 2000 is a broader index with generally more risk, we chose the mean to equal 9.00% and the S&P 500 to equal 8.00%. We believe that choosing annual returns that accurately represented historical data would have provided us with more precise results. Another recommendation would be to research how the annual returns have changed in previous market declines and make valid assumptions based on historical data. A third possible option is to have the mean return constantly changing to replicate a more real world situation. In reality, the mean of the stock return is the drift rate and keeping it constant is an unreliable assumption when developing the simulation. By implementing these ideas, future research could provide a more secure and dependable fund mapping.

### **5.3. Final Conclusions**

Although we identified sources of basis risk and parameters that may affect basis risk, we did not conclude that there are problems with the hedging procedures currently in use. In general, the regressions created fund mappings that were accurate, even when using some improper indices. If we were to use stochastic parameters, we believe this would have yielded more realistic results. Furthermore, creating a more diversified portfolio which would include various indices would also provide our project with more realistic results. More research should be done using the recommendations we have made in this section in order to further validate the fund mapping procedure. We hope that by continuing research on the effects and sources of basis risk, insurance companies and asset managers will eventually be able to improve their investment strategy and performance, and reduce the risk associated with variable annuities and mutual funds.

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## Appendix A: Sponsor Information

Towers, Perrin, Foster, and Crosby (TPF&C) was incorporated in 1934. Their predecessor, Henry W. Brown Co. can be traced back to 1817. Today, Towers Perrin is a global firm with 41 offices open outside the United States, 34 offices open within the United States and 16 alliances located around the world. Towers Perrin serves clients that represent 75% of the world's 500 largest companies and 75% of the companies located on the list of U.S. Fortune 1000 companies. They define themselves as a firm who helps organizations to improve performance through effective people, risk and financial management. Their Mission Statement is as follows:

*“To make significant contributions to improving our clients' business performance — through our unique combination of talent, expertise and commitment — thereby creating value for our stakeholders”.*

Towers Perrin's services include: Actuarial consulting, Enterprise risk & capital management, Executive compensation, Financial modeling software solutions, Health & welfare consulting, Human resources function effectiveness, Insurance consulting, Mergers & acquisitions, Reinsurance, Research & surveys, Retirement risk, Risk financing and transfer, Total rewards effectiveness, and Talent management.

## Appendix B: Acronyms

**BC:** Breadth Coefficient

**DC:** Direct Contribution

**FP:** Future-Past

**FF:** Future-Future

**GMAB:** Guaranteed Minimum Accumulation Benefit

**GMDB:** Guaranteed Minimum Death Benefit

**GMIB:** Guaranteed Minimum Income Benefit

**GMWB:** Guaranteed Minimum Withdrawal Benefit

**IC:** Information Coefficient

**IR:** Information Ratio

**IRA:** Individual Retirement Account

**MFI:** Mutual Fund Incorporated, our created mutual fund

**MFI2Var:** Predicted mutual fund using two-variable regression

**MFISPY:** Predicted mutual fund regressed against SPY returns

**MFIRUS:** Predicted mutual fund regressed against RUS returns

**RUS:** Russell 2000 index

**SPY:** S&P 500 yield index

**TPF&C:** Towers, Perrin, Foster, and Crosby

**VA:** Variable Annuity

## Appendix C: Definitions

**Active Management:** constantly buying and selling a specific mix of securities in hopes of outperforming the market (Curtis, 2001). Managers rely on analytical research, forecasting, and personal experience (Active Management, 2009).

**Basis Risk:** the risk that arises between an exposure and a hedge mechanism that is imperfectly correlated with the exposure, where the degree of the risk is based on the correlation between the exposure and the hedge (Banks, 2004).

**Benchmark:** used to evaluate the performance of a security, mutual fund or investment (Benchmark, 2009).

**Black-Scholes Method for Estimating Volatility:** estimates the volatility of a stock or index based on historical returns.

**Cholesky Decomposition:** the process of calculating the correlation matrix for a group of correlated stock indices, based on the covariance matrix of those indices. It can be used in geometric Brownian motion to create correlated indices.

**Correlation:** a statistical measure of how two or more securities are moving in relation to the others (Correlation, 2009).

**Correlation Coefficient:** a measure determining how much the movement of two or more variables is associated (Correlation Coefficient, 2009).

**Covariance:** measures the strength of the correlation between two or more investments (Wolfram MathWorld, 2009).

**Fund Mapping:** the process of assigning mutual funds to hedgeable indices. Fund mappings are used to project the expected mutual fund performance given the movement in the reference indices.

**Geometric Brownian Motion:** the model we used to generate stock indices in the future.

**Hedge Funds:** portfolio built with a goal of shorting the market.

**Mutual Fund:** a diversified portfolio funded by various investors, with assets such as stocks, bonds, short-term money market instruments or other securities (Mutual Funds, 2007).



**Multicollinearity:** a statistical anomaly that occurs when two or more variables in a multiple regression are highly correlated.

**Regression:** a measure that is used to determine the strength of a relationship between one dependent variable and one or more independent variables (Regression, 2009).

**Shorting the Market:** a portfolio is shorting the market when the value of the portfolio is increasing while the market is declining.

**Tracking Error:** the difference between the price behavior of a portfolio and a benchmark (Tracking Error, 2009).

**Variable Annuity:** an insurance contract that goes through an accumulation phase and then a payout phase.

## Appendix D: VBA Code

```
Sub SPMonteCarlo()  
,  
,  
' SPMonteCarlo Macro  
,  
  
'Start time  
Dim starttime As Date  
  
starttime = Now  
Worksheets("Output").Range("B3").Value = starttime  
  
.....  
'Start Variable Declaration'  
.....  
  
'Declare Variables  
Dim scenarios As Long  
Dim projYears As Long  
Dim projDays As Long  
Dim timeinterval As Double  
Dim a11 As Double  
Dim a21 As Double  
Dim a22 As Double  
Dim randnorm As Double  
Dim FPcalculate As Boolean  
Dim FPdailyDisplay As Boolean  
Dim FPprintType As Boolean  
Dim periodsPerYear As Integer  
  
'Declare S&P Variables  
Dim sigma As Double  
Dim mu As Double  
Dim startvalue As Double  
Dim correlation As Double  
Dim newvalue As Double  
Dim oldvalue As Double  
Dim lnreturn As Double  
Dim FPfinal As Double  
  
'Declare Mutual Fund Variables  
Dim alpha As Double  
Dim SPYalpha As Double  
Dim mfstartvalue As Double  
Dim mfnewvalue As Double  
Dim mfoldvalue As Double  
Dim mflnreturn As Double  
Dim FPMfFinal As Double  
  
'Declare Russell 2000 Variables  
Dim russigma As Double  
Dim rusmu As Double  
Dim russtartvalue As Double  
Dim rusrandnorm As Double  
Dim rusnewvalue As Double  
Dim rusoldvalue As Double  
Dim ruslnreturn As Double  
Dim FPrusFinal As Double  
  
'Declare Regression Variables  
Dim scenarioCounter As Long  
Dim regressionOffset As Long  
Dim currentRegression As Long  
Dim maxRegression As Long  
Dim twolntercept As Double
```

```

Dim SP2VarCoeff As Double
Dim RUS2VarCoeff As Double
Dim SPintercept As Double
Dim SP1VarCoeff As Double
Dim RUSintercept As Double
Dim RUS1VarCoeff As Double
Dim regressDaily As Boolean
Dim regressLN As Boolean
Dim yValues() As Double
Dim xValues() As Double
Dim x1values() As Double
Dim x2values() As Double

```

```
'Declare Future Future Variables
```

```

Dim FFtimeinterval As Double
Dim FFmfiB As Double
Dim FFmfiBnewvalue As Double
Dim FFmfiBoldvalue As Double
Dim FFmfiBlnreturn As Double
Dim FFmfiC As Double
Dim FFmfiClnreturn As Double
Dim FFmfiCnewvalue As Double
Dim FFmfiColdvalue As Double
Dim FFmfiD As Double
Dim FFmfiDlnreturn As Double
Dim FFmfiDnewvalue As Double
Dim FFmfiDoldvalue As Double
Dim FFprojYears As Long
Dim FFprojDays As Long
Dim FFscenarios As Long
Dim FFrowOutput As Long
Dim FFheaderOutput As Long
Dim FFcalculate As Boolean
Dim FFdailyDisplay As Boolean
Dim FFprintType As Boolean
Dim FFperiodsPerYear As Integer

```

```
'Set Starting Values for Variables
```

```

scenarios = Worksheets("Input").Range("B4").Value
scenarioCounter = 0
sigma = Worksheets("Input").Range("B8").Value
mu = Worksheets("Input").Range("C8").Value
correlation = Worksheets("Input").Range("E8").Value
projYears = Worksheets("Input").Range("C4").Value
projDays = projYears * 252
startvalue = Worksheets("Input").Range("D8").Value
timeinterval = Worksheets("Input").Range("G4").Value
alpha = Worksheets("Input").Range("B12").Value
SPYalpha = Worksheets("Input").Range("C12").Value
mfstartvalue = Worksheets("Input").Range("D12").Value
russtartvalue = Worksheets("Input").Range("D16").Value
rusmu = Worksheets("Input").Range("C16").Value
russigma = Worksheets("Input").Range("B16").Value
a11 = Worksheets("Input").Range("B19").Value
a21 = Worksheets("Input").Range("B20").Value
a22 = Worksheets("Input").Range("C20").Value
maxRegression = WorksheetFunction.Min(projYears, Worksheets("Input").Range("B24").Value)
Worksheets("Input").Range("B24").Value = maxRegression
currentRegression = maxRegression
FFtimeinterval = Worksheets("Input").Range("G28").Value
FFprojYears = Worksheets("Input").Range("C28").Value
FFprojDays = FFprojYears * 252
FFscenarios = Worksheets("Input").Range("B28").Value

```

```
'Sets Future Past Calculate variable
```

```
If Worksheets("Input").Range("D4").Value = "Daily" Then
```

```

'Daily calculation of returns means True
FPcalculate = True
Else
If Worksheets("Input").Range("D4").Value = "Monthly" Then
'Monthly calculation of returns means False
FPcalculate = False
Else
'When Neither 'Daily' or 'Monthly' is entered, assume 'Daily'
Worksheets("Input").Range("D4").Value = "Daily"
FPcalculate = True
End If
End If

'Sets Future Past Display variable
If Worksheets("Input").Range("E4").Value = "Daily" Then
'Daily display of data means True
FPdailyDisplay = True
periodsPerYear = 252
Else
If Worksheets("Input").Range("E4").Value = "Monthly" Then
'Monthly display of data means False
FPdailyDisplay = False
periodsPerYear = 12
Else
'When Neither 'Daily' or 'Monthly' is entered, assume 'Daily'
Worksheets("Input").Range("E4").Value = "Daily"
FPdailyDisplay = True
periodsPerYear = 252
End If
End If

'Test
If FPcalculate = False Then
If FPdailyDisplay = True Then
'Scenarios can't be run with monthly calculations but outputting values daily
'So, switch to monthly output.
Worksheets("Input").Range("E4").Value = "Monthly"
FPdailyDisplay = False
periodsPerYear = 12
End If
End If

'If Future Past Display is false (data is shown monthly), then projDays is actually projMonths
If FPdailyDisplay = False Then
projDays = projYears * 12
End If

'Sets Future Past Print variable
If Worksheets("Input").Range("F4").Value = "LN Return" Then
'LN Return output means True
FPprintType = True
Else
If Worksheets("Input").Range("F4").Value = "Values" Then
'Values output means False
FPprintType = False
Else
'When Neither 'LN Return' or 'Values' is entered, assume 'LN Return'
Worksheets("Input").Range("F4").Value = "LN Return"
FPprintType = True
End If
End If

'Sets Regression Calculating Variable
If Worksheets("Input").Range("C24").Value = "Daily" Then
'Regressing daily means True
regressDaily = True

```

```

Else
If Worksheets("Input").Range("C24").Value = "Monthly" Then
'Regressing monthly means False
regressDaily = False
Else
'When Neither 'Daily' or 'Monthly' is entered, assume 'Daily'
Worksheets("Input").Range("C24").Value = "Daily"
regressDaily = True
End If
End If

'Test
If FpdailyDisplay = False Then
If regressDaily = True Then
'Regressions can't be done daily if there is only monthly data for the scenarios
'So, switch to monthly regression.
Worksheets("Input").Range("C24").Value = "Monthly"
regressDaily = False
End If
End If

'Sets whether to regress the ln returns or index values
If Worksheets("Input").Range("D24").Value = "LN Return" Then
'LN Regression means true
regressLN = True
Else
If Worksheets("Input").Range("D24").Value = "Values" Then
'Values Regression means false
regressLN = False
Else
'When Neither 'LN Return' or 'Values' is entered, assume 'LN Return'
Worksheets("Input").Range("D24").Value = "LN Return"
regressLN = True
End If
End If

'Sets Future Future Calculate variable
If Worksheets("Input").Range("D28").Value = "Daily" Then
'Daily calculation of returns means True
FFcalculate = True
Else
If Worksheets("Input").Range("D28").Value = "Monthly" Then
'Monthly calculation of returns means False
FFcalculate = False
Else
'When Neither 'Daily' or 'Monthly' is entered, assume 'Daily'
Worksheets("Input").Range("D28").Value = "Daily"
FFcalculate = True
End If
End If

'Sets Future Future Display variable
If Worksheets("Input").Range("E28").Value = "Daily" Then
'Daily display of data means True
FFdailyDisplay = True
FFperiodsPerYear = 252
Else
If Worksheets("Input").Range("E28").Value = "Monthly" Then
'Monthly display of data means False
FFdailyDisplay = False
FFperiodsPerYear = 12
Else
'When Neither 'Daily' or 'Monthly' is entered, assume 'Daily'
Worksheets("Input").Range("E28").Value = "Daily"
FFdailyDisplay = True
FFperiodsPerYear = 252

```

```

End If
End If

'Test
If FFcalculate = False Then
  If FFdailyDisplay = True Then
    'Scenarios can't be run with monthly calculations but outputting values daily
    'So, switch to monthly output.
    Worksheets("Input").Range("E28").Value = "Monthly"
    FFdailyDisplay = False
    FFperiodsPerYear = 12
  End If
End If

'If Future Future Display is false (data is shown monthly), then FFprojDays is actually FFprojMonths
If FFdailyDisplay = False Then
  FFprojDays = FFprojYears * 12
End If

'Sets Future Future Print variable
If Worksheets("Input").Range("F28").Value = "LN Return" Then
  'LN Return output means True
  FFprintType = True
Else
  If Worksheets("Input").Range("F28").Value = "Values" Then
    'Values output means False
    FFprintType = False
  Else
    'When Neither 'LN Return' or 'Values' is entered, assume 'LN Return'
    Worksheets("Input").Range("F28").Value = "LN Return"
    FFprintType = True
  End If
End If

'End Variable Declaration'
'Backup old "Scenario Output" Worksheet
Dim oldoutputs As Integer
Dim sheet As Integer
sheet = 1
Do While sheet < Worksheets.Count + Charts.Count + 1
  If Sheets(sheet).Name = "Scenario Output" Then
    oldoutputs = 1
    sheet = 1
    Do While sheet < Worksheets.Count + Charts.Count + 1
      If Sheets(sheet).Name = "Scenario Output Old" & oldoutputs Then
        oldoutputs = oldoutputs + 1
        sheet = 0
      End If
      sheet = sheet + 1
    Loop
    Worksheets("Scenario Output").Name = "Scenario Output Old" & oldoutputs
  End If
  sheet = sheet + 1
Loop

'New "Scenario Outputs" Worksheet
Sheets.Add Before:=Worksheets("Output")
ActiveSheet.Name = "Scenario Output"
ActiveWindow.Zoom = 85

'Formatting New Worksheet'

```

.....

```
'Status Update
Worksheets("Scenario Output").Range("A1").Font.Size = 10
Worksheets("Scenario Output").Range("A1").Value = "Initializing..."

'Column Height
Worksheets("Scenario Output").Columns.ColumnWidth = 8.57
Worksheets("Scenario Output").Columns(1).ColumnWidth = 13.51

'Number Formatting
Worksheets("Scenario Output").Columns.NumberFormat = "0.0000"
If FFprintType = True Then
    For i = 1 To projDays + 2
        'If printing LN Returns, use percentage
        Worksheets("Scenario Output").Rows(i).NumberFormat = "0.00%"
    Next i
Else
    For i = 1 To projDays + 2
        'If printing values, use number
        Worksheets("Scenario Output").Rows(i).NumberFormat = "0.00"
    Next i
End If
If FFprintType = False Then
    'If printing values, use number
    For i = 1 To FFscenarios * (maxRegression + 1)
        For j = 1 To FFprojDays
            Worksheets("Scenario Output").Rows(7 + j + 25 * maxRegression + projDays + (i - 1) * (FFprojDays + 2)).NumberFormat = "0.00"
        Next j
    Next i
Else
    'If printing LN Returns, use value
    For i = 1 To FFscenarios * (maxRegression + 1)
        For j = 1 To FFprojDays
            Worksheets("Scenario Output").Rows(7 + j + 25 * maxRegression + projDays + (i - 1) * (FFprojDays + 2)).NumberFormat = "0.00%"
        Next j
    Next i
End If
Worksheets("Scenario Output").Columns(1).NumberFormat = "0"

'Status Update
Worksheets("Scenario Output").Range("A1").Value = "Formatting..."

'Boldng Stuff
Worksheets("Scenario Output").Columns(1).Font.FontStyle = "Bold"
Worksheets("Scenario Output").Rows("1:2").Font.FontStyle = "Bold"
Worksheets("Scenario Output").Range("A1").Font.FontStyle = "Unbold"
For i = 1 To FFscenarios * (maxRegression + 1)
    Worksheets("Scenario Output").Rows(6 + 25 * maxRegression + projDays + (i - 1) * (FFprojDays + 2)).Font.FontStyle = "Bold"
    Worksheets("Scenario Output").Rows(7 + 25 * maxRegression + projDays + (i - 1) * (FFprojDays + 2)).Font.FontStyle = "Bold"
Next i

'Enlarging Fonts
Worksheets("Scenario Output").Rows("1:2").Font.Size = 14
Worksheets("Scenario Output").Range("A1").Font.Size = 10
For i = 1 To FFscenarios * (maxRegression + 1)
    Worksheets("Scenario Output").Rows(6 + 25 * maxRegression + projDays + (i - 1) * (FFprojDays + 2)).Font.Size = 14
    Worksheets("Scenario Output").Rows(7 + 25 * maxRegression + projDays + (i - 1) * (FFprojDays + 2)).Font.Size = 14
Next i

'Borders
With Worksheets("Scenario Output").Rows(2).Borders(xlEdgeBottom)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlThin
```

```

End With
With Worksheets("Scenario Output").Rows(3).Borders(xlEdgeTop)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlThin
End With
With Worksheets("Scenario Output").Columns(1).Borders(xlEdgeRight)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlThin
End With
With Worksheets("Scenario Output").Columns(2).Borders(xlEdgeLeft)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlThin
End With
For i = 1 To FFscenarios * (maxRegression + 1)
    With Worksheets("Scenario Output").Rows(5 + 25 * maxRegression + projDays + (i - 1) * (FFprojDays + 2)).Borders(xlEdgeBottom)
        .LineStyle = xlContinuous
        .ColorIndex = xlAutomatic
        .TintAndShade = 0
        .Weight = xlThick
    End With
    With Worksheets("Scenario Output").Rows(6 + 25 * maxRegression + projDays + (i - 1) * (FFprojDays + 2)).Borders(xlEdgeTop)
        .LineStyle = xlContinuous
        .ColorIndex = xlAutomatic
        .TintAndShade = 0
        .Weight = xlThick
    End With
    With Worksheets("Scenario Output").Rows(7 + 25 * maxRegression + projDays + (i - 1) * (FFprojDays + 2)).Borders(xlEdgeBottom)
        .LineStyle = xlContinuous
        .ColorIndex = xlAutomatic
        .TintAndShade = 0
        .Weight = xlThin
    End With
    With Worksheets("Scenario Output").Rows(8 + 25 * maxRegression + projDays + (i - 1) * (FFprojDays + 2)).Borders(xlEdgeTop)
        .LineStyle = xlContinuous
        .ColorIndex = xlAutomatic
        .TintAndShade = 0
        .Weight = xlThin
    End With
Next i
With Worksheets("Scenario Output").Rows(5 + 25 * maxRegression + projDays + FFscenarios * (maxRegression + 1) * (FFprojDays + 2)).Borders(xlEdgeBottom)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlThick
End With
With Worksheets("Scenario Output").Rows(6 + 25 * maxRegression + projDays + FFscenarios * (maxRegression + 1) * (FFprojDays + 2)).Borders(xlEdgeTop)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlThick
End With
With Worksheets("Scenario Output").Rows(5 + 25 * maxRegression + projDays + FFscenarios * maxRegression * (FFprojDays + 2)).Borders(xlEdgeBottom)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlThick
End With

```



```

With Worksheets("Scenario Output").Rows(6 + 25 * maxRegression + projDays + FFscenarios * maxRegression * (FFprojDays +
2)).Borders(xlEdgeTop)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlThick
End With

'Display Period Numbers
If FPdailyDisplay = False Then
    If FPcalculate = True Then
        If FPprintType = True Then
            'For Monthly periods with daily LN Return calculations
            Worksheets("Scenario Output").Range("A2").Value = "Months (LN Returns, calculated Daily)"
        Else
            'For Monthly periods with monthly Value calculations
            Worksheets("Scenario Output").Range("A2").Value = "Months (Index Values, calculated Daily)"
        End If
    Else
        If FPprintType = True Then
            'For Monthly periods with monthly LN Return calculations
            Worksheets("Scenario Output").Range("A2").Value = "Months (LN Returns, calculated Monthly)"
        Else
            'For Monthly periods with monthly Value calculations
            Worksheets("Scenario Output").Range("A2").Value = "Months (Index Values, calculated Monthly)"
        End If
    End If
End If
Else
    If FPprintType = True Then
        'For Daily periods with daily LN Return calculations
        Worksheets("Scenario Output").Range("A2").Value = "Days (LN Returns, calculated Daily)"
    Else
        'For Daily periods with daily Value calculations
        Worksheets("Scenario Output").Range("A2").Value = "Days (Index Values, calculated Daily)"
    End If
End If
For i = 1 To projDays
    Worksheets("Scenario Output").Cells(i + 2, 1).Value = i
Next i

'Print Regression Years on Left Side
If regressLN = True Then
    If regressDaily = True Then
        Worksheets("Scenario Output").Cells(projDays + 4, 1).Value = "Regressions (LN Returns, calculated Daily)"
    Else
        Worksheets("Scenario Output").Cells(projDays + 4, 1).Value = "Regressions (LN Returns, calculated Monthly)"
    End If
Else
    If regressDaily = True Then
        Worksheets("Scenario Output").Cells(projDays + 4, 1).Value = "Regressions (Index Values, calculated Daily)"
    Else
        Worksheets("Scenario Output").Cells(projDays + 4, 1).Value = "Regressions (Index Values, calculated Monthly)"
    End If
End If
Worksheets("Scenario Output").Cells(projDays + 4, 1).Font.Size = 14
Do While currentRegression > 0
    Worksheets("Scenario Output").Cells((maxRegression - currentRegression) * 25 + projDays + 5, 1).Value = currentRegression & " YEAR"
    currentRegression = currentRegression - 1
Loop
currentRegression = maxRegression

.....

'Formatting Stuff Ends Here'
.....

.....

```

```

'Scenario Stuff Starts Here'
.....

'Start Scenario Loop
Do While scenarioCounter < scenarios

'Randomize
Randomize

'Status Update
Worksheets("Scenario Output").Range("A1").Value = "Calculating Scenario " & scenarioCounter + 1 & "..."

'Output Scenario Heading
Worksheets("Scenario Output").Cells(1, scenarioCounter * 4 + 2).Value = "Scenario " & scenarioCounter + 1
Worksheets("Scenario Output").Cells(2, scenarioCounter * 4 + 2).Value = "MFI"
Worksheets("Scenario Output").Cells(2, scenarioCounter * 4 + 3).Value = "SPY"
Worksheets("Scenario Output").Cells(2, scenarioCounter * 4 + 4).Value = "RUS"

'Reset Variables
startvalue = Worksheets("Input").Range("D8").Value
mfstartvalue = Worksheets("Input").Range("D12").Value
russtartvalue = Worksheets("Input").Range("D16").Value
newvalue = startvalue
rusnewvalue = russtartvalue
mfnewvalue = mfstartvalue

.....

'Future Past Stuff Starts Here'
.....

'Start Daily Projection Loop for Future Past
For currentday = 0 To projDays - 1

'Previous New Values become Old Values
oldvalue = newvalue
rusoldvalue = rusnewvalue
mfoldvalue = mfnewvalue

'This top part of the If statement is used when the indexes are calculated at the same times they are outputted
If (FPdailyDisplay = True And FPcalculate = True) Or (FPdailyDisplay = False And FPcalculate = False) Then

'Reset Random Numbers
randnorm = Application.WorksheetFunction.NormSInv(Rnd())
rusrandnorm = Application.WorksheetFunction.NormSInv(Rnd())
mfrandnorm = Application.WorksheetFunction.NormSInv(Rnd())

'Calculate New Values
newvalue = newvalue * Exp((mu - variance ^ 2 / 2) * timeinterval + timeinterval ^ 0.5 * a11 * randnorm)
rusnewvalue = rusnewvalue * Exp((rusmu - rusvariance ^ 2 / 2) * timeinterval + timeinterval ^ 0.5 * (a21 * randnorm + a22 * rusrandnorm))

'Calculate In of Returns
Inreturn = Log(newvalue / oldvalue)
rusInreturn = Log(rusnewvalue / rusoldvalue)
mflnreturn = Inreturn + (alpha + Abs(SPYalpha * Inreturn)) / (periodsPerYear ^ 0.5) * mfrandnorm

'Calculate New value of Mutual Fund
mfnewvalue = Exp(mflnreturn) * mfoldvalue
Else

'This portion is used when the indexes are calculated daily, but values are outputted monthly
For i = 1 To 21
'Reset Random Numbers
randnorm = Application.WorksheetFunction.NormSInv(Rnd())
rusrandnorm = Application.WorksheetFunction.NormSInv(Rnd())
mfrandnorm = Application.WorksheetFunction.NormSInv(Rnd())

```

```

'Calculate New Values
newvalue = newvalue * Exp((mu - variance ^ 2 / 2) * timeinterval + timeinterval ^ 0.5 * a11 * randnorm)
rusnewvalue = rusnewvalue * Exp((rusmu - rusvariance ^ 2 / 2) * timeinterval + timeinterval ^ 0.5 * (a21 * randnorm + a22 *
rusrandnorm))
mfnewvalue = mfnewvalue * Exp((mu - variance ^ 2 / 2) * timeinterval + timeinterval ^ 0.5 * a11 * randnorm + (alpha + Abs(SPYalpha *
((mu - variance ^ 2 / 2) * timeinterval + timeinterval ^ 0.5 * a11 * randnorm))) / (252 ^ 0.5) * mfrandnorm)
Next i

'Calculate ln of Returns
lnreturn = Log(newvalue / oldvalue)
ruslnreturn = Log(rusnewvalue / rusoldvalue)
mflnreturn = Log(mfnewvalue / mfoldvalue)
End If

If FPprintType = True Then
'Output ln of Returns
Worksheets("Scenario Output").Cells(currentday + 3, scenarioCounter * 4 + 2).Value = mflnreturn
Worksheets("Scenario Output").Cells(currentday + 3, scenarioCounter * 4 + 3).Value = lnreturn
Worksheets("Scenario Output").Cells(currentday + 3, scenarioCounter * 4 + 4).Value = ruslnreturn
Else
'Output Index Values
Worksheets("Scenario Output").Cells(currentday + 3, scenarioCounter * 4 + 2).Value = mfnewvalue
Worksheets("Scenario Output").Cells(currentday + 3, scenarioCounter * 4 + 3).Value = newvalue
Worksheets("Scenario Output").Cells(currentday + 3, scenarioCounter * 4 + 4).Value = rusnewvalue
End If

Next currentday

'Preserve Final Values
FPfinal = newvalue
FPrusFinal = rusnewvalue
FPmfFinal = mfnewvalue

'Print Borders
With Worksheets("Scenario Output").Columns(scenarioCounter * 4 + 5).Borders(xlEdgeRight)
.LineStyle = xlContinuous
.ColorIndex = xlAutomatic
.TintAndShade = 0
.Weight = xlThin
End With
With Worksheets("Scenario Output").Columns(scenarioCounter * 4 + 2).Borders(xlEdgeLeft)
.LineStyle = xlContinuous
.ColorIndex = xlAutomatic
.TintAndShade = 0
.Weight = xlThin
End With

'Color Columns
With Worksheets("Scenario Output").Range(Cells(1, scenarioCounter * 4 + 2), Cells(projDays + 2, scenarioCounter * 4 + 2)).Interior
.Pattern = xlSolid
.PatternColorIndex = xlAutomatic
.Color = 5296274
.TintAndShade = 0
.PatternTintAndShade = 0
End With
With Worksheets("Scenario Output").Range(Cells(1, scenarioCounter * 4 + 3), Cells(projDays + 2, scenarioCounter * 4 + 3)).Interior
.Pattern = xlSolid
.PatternColorIndex = xlAutomatic
.Color = 14136213
.TintAndShade = 0
.PatternTintAndShade = 0
End With
With Worksheets("Scenario Output").Range(Cells(1, scenarioCounter * 4 + 4), Cells(projDays + 2, scenarioCounter * 4 + 4)).Interior
.Pattern = xlSolid
.PatternColorIndex = xlAutomatic
.Color = 49407

```

```

        .TintAndShade = 0
        .PatternTintAndShade = 0
    End With

'Merge Top Row
With Worksheets("Scenario Output").Range(Cells(1, scenarioCounter * 4 + 2), Cells(1, scenarioCounter * 4 + 4))
    .Merge
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .Interior.Color = 8421631
End With

.....

'Future Past Stuff Ends Here'
.....

.....

'Regression Stuff Starts Here'
.....

'Status Update
Worksheets("Scenario Output").Range("A1").Value = "Regressing Scenario " & scenarioCounter + 1 & "..."
currentRegression = maxRegression

Do While currentRegression > 0

    regressionOffset = (maxRegression - currentRegression) * 25 + projDays

    .....

'POPULATING ARRAYS WITH DATA'
.....

'This top part of the If statement is used when the regression is performed on all outputted values
If (FPdailyDisplay = True And regressDaily = True) Or (FPdailyDisplay = False And regressDaily = False) Then

    ReDim yValues(1 To projDays - (maxRegression - currentRegression) * periodsPerYear) As Double
    ReDim x1values(1 To projDays - (maxRegression - currentRegression) * periodsPerYear) As Double
    ReDim x2values(1 To projDays - (maxRegression - currentRegression) * periodsPerYear) As Double
    ReDim xValues(1 To 2, 1 To projDays - (maxRegression - currentRegression) * periodsPerYear) As Double

    'Checks if the scenarios are outputted in the same way (ln or values) that the regression will be done
    If (FPprintType = True And regressLN = True) Or (FPprintType = False And regressLN = False) Then

        'Populate the Arrays with the correct values
        For i = 1 To projDays - (projYears - currentRegression) * periodsPerYear
            yValues(i) = Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + i + 2, scenarioCounter * 4 +
2).Value
            x1values(i) = Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + i + 2, scenarioCounter * 4 +
3).Value
            x2values(i) = Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + i + 2, scenarioCounter * 4 +
4).Value
            xValues(1, i) = x1values(i)
            xValues(2, i) = x2values(i)
        Next i

    Else
        'This is used if the scenarios are outputted as values, but the regression will be done on ln
        If regressLN = True And FPprintType = False Then

            'Populate the Arrays with the correct values
            If currentRegression = projYears Then
                yValues(1) = Log(Worksheets("Scenario Output").Cells(3, scenarioCounter * 4 + 2).Value / mfstartvalue)
                x1values(1) = Log(Worksheets("Scenario Output").Cells(3, scenarioCounter * 4 + 3).Value / startvalue)
                x2values(1) = Log(Worksheets("Scenario Output").Cells(3, scenarioCounter * 4 + 4).Value / russtartvalue)
            Else

```

```

yValues(1) = Log(Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + 3, scenarioCounter * 4 + 2).Value / Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + 2, scenarioCounter * 4 + 2).Value)
x1values(1) = Log(Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + 3, scenarioCounter * 4 + 3).Value / Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + 2, scenarioCounter * 4 + 3).Value)
x2values(1) = Log(Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + 3, scenarioCounter * 4 + 4).Value / Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + 2, scenarioCounter * 4 + 4).Value)
End If

```

```

xValues(1, 1) = x1values(1)
xValues(2, 1) = x2values(1)

```

```

For i = 2 To projDays - (projYears - currentRegression) * periodsPerYear
yValues(i) = Log(Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + i + 2, scenarioCounter * 4 + 2).Value / Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + i + 1, scenarioCounter * 4 + 2).Value)
x1values(i) = Log(Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + i + 2, scenarioCounter * 4 + 3).Value / Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + i + 1, scenarioCounter * 4 + 3).Value)
x2values(i) = Log(Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + i + 2, scenarioCounter * 4 + 4).Value / Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + i + 1, scenarioCounter * 4 + 4).Value)
xValues(1, i) = x1values(i)
xValues(2, i) = x2values(i)
Next i

```

Else

'This is used if the scenarios are outputted as ln, but the regression will be done on values

'Populate the Arrays with the correct values

If currentRegression = projYears Then

```

yValues(1) = mfstartvalue * Exp(Worksheets("Scenario Output").Cells(3, scenarioCounter * 4 + 2).Value)
x1values(1) = startvalue * Exp(Worksheets("Scenario Output").Cells(3, scenarioCounter * 4 + 3).Value)
x2values(1) = russtartvalue * Exp(Worksheets("Scenario Output").Cells(3, scenarioCounter * 4 + 4).Value)

```

Else

```

yValues(1) = mfstartvalue * Exp(Application.WorksheetFunction.Sum(Worksheets("Scenario Output").Range(Cells(3, scenarioCounter * 4 + 2), Cells((projYears - currentRegression) * periodsPerYear + 3, scenarioCounter * 4 + 2)).Value))
x1values(1) = startvalue * Exp(Application.WorksheetFunction.Sum(Worksheets("Scenario Output").Range(Cells(3, scenarioCounter * 4 + 3), Cells((projYears - currentRegression) * periodsPerYear + 3, scenarioCounter * 4 + 3)).Value))
x2values(1) = russtartvalue * Exp(Application.WorksheetFunction.Sum(Worksheets("Scenario Output").Range(Cells(3, scenarioCounter * 4 + 4), Cells((projYears - currentRegression) * periodsPerYear + 3, scenarioCounter * 4 + 4)).Value))
End If

```

```

xValues(1, 1) = x1values(1)
xValues(2, 1) = x2values(1)

```

```

For i = 2 To projDays - (projYears - currentRegression) * periodsPerYear
yValues(i) = yValues(i - 1) * Exp(Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + 2 + i, scenarioCounter * 4 + 2).Value)
x1values(i) = x1values(i - 1) * Exp(Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + 2 + i, scenarioCounter * 4 + 3).Value)
x2values(i) = x2values(i - 1) * Exp(Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + 2 + i, scenarioCounter * 4 + 4).Value)
xValues(1, i) = x1values(i)
xValues(2, i) = x2values(i)
Next i

```

End If

End If

'This portion is used when the indexes are outputted daily, but regression is done monthly

Else

```

ReDim yValues(1 To (projDays - (maxRegression - currentRegression) * periodsPerYear) / 21) As Double
ReDim x1values(1 To (projDays - (maxRegression - currentRegression) * periodsPerYear) / 21) As Double
ReDim x2values(1 To (projDays - (maxRegression - currentRegression) * periodsPerYear) / 21) As Double
ReDim xValues(1 To 2, 1 To (projDays - (maxRegression - currentRegression) * periodsPerYear) / 21) As Double

```

'Checks if the scenarios are outputted in the same way (ln or values) that the regression will be done

```

If (FPprintType = True And regressLN = True) Or (FPprintType = False And regressLN = False) Then

    'Populate the Arrays with the correct values (i*21 because 21 trading days per month)
    For i = 1 To (projDays - (projYears - currentRegression) * periodsPerYear) / 21
        yValues(i) = Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + i * 21 + 2, scenarioCounter * 4
+ 2).Value
        x1values(i) = Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + i * 21 + 2, scenarioCounter * 4
+ 3).Value
        x2values(i) = Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + i * 21 + 2, scenarioCounter * 4
+ 4).Value
        xValues(1, i) = x1values(i)
        xValues(2, i) = x2values(i)
    Next i

Else
    'This is used if the scenarios are outputted as values, but the regression will be done on ln
    If regressLN = True And FPprintType = False Then

        'Populate the Arrays with the correct values
        If currentRegression = projYears Then
            yValues(1) = Log(Worksheets("Scenario Output").Cells(23, scenarioCounter * 4 + 2).Value / mfstartvalue)
            x1values(1) = Log(Worksheets("Scenario Output").Cells(23, scenarioCounter * 4 + 3).Value / startvalue)
            x2values(1) = Log(Worksheets("Scenario Output").Cells(23, scenarioCounter * 4 + 4).Value / russtartvalue)
        Else
            yValues(1) = Log(Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + 23, scenarioCounter * 4
+ 2).Value / Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + 2, scenarioCounter * 4 + 2).Value)
            x1values(1) = Log(Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + 23, scenarioCounter * 4
+ 3).Value / Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + 2, scenarioCounter * 4 + 3).Value)
            x2values(1) = Log(Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + 23, scenarioCounter * 4
+ 4).Value / Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + 2, scenarioCounter * 4 + 4).Value)
        End If

        xValues(1, 1) = x1values(1)
        xValues(2, 1) = x2values(1)

        For i = 2 To (projDays - (projYears - currentRegression) * periodsPerYear) / 21
            yValues(i) = Log(Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + i * 21 + 2,
scenarioCounter * 4 + 2).Value / Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + (i - 1) * 21 + 2,
scenarioCounter * 4 + 2).Value)
            x1values(i) = Log(Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + i * 21 + 2,
scenarioCounter * 4 + 3).Value / Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + (i - 1) * 21 + 2,
scenarioCounter * 4 + 3).Value)
            x2values(i) = Log(Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + i * 21 + 2,
scenarioCounter * 4 + 4).Value / Worksheets("Scenario Output").Cells((projYears - currentRegression) * periodsPerYear + (i - 1) * 21 + 2,
scenarioCounter * 4 + 4).Value)
            xValues(1, i) = x1values(i)
            xValues(2, i) = x2values(i)
        Next i

    Else
        'This is used if the scenarios are outputted as ln, but the regression will be done on values

        'Populate the Arrays with the correct values
        If currentRegression = projYears Then
            yValues(1) = mfstartvalue * Exp(Application.WorksheetFunction.Sum(Worksheets("Scenario Output").Range(Cells(3,
scenarioCounter * 4 + 2), Cells(23, scenarioCounter * 4 + 2)).Value))
            x1values(1) = startvalue * Exp(Application.WorksheetFunction.Sum(Worksheets("Scenario Output").Range(Cells(3,
scenarioCounter * 4 + 3), Cells(23, scenarioCounter * 4 + 3)).Value))
            x2values(1) = russtartvalue * Exp(Application.WorksheetFunction.Sum(Worksheets("Scenario Output").Range(Cells(3,
scenarioCounter * 4 + 4), Cells(23, scenarioCounter * 4 + 4)).Value))
        Else
            yValues(1) = mfstartvalue * Exp(Application.WorksheetFunction.Sum(Worksheets("Scenario Output").Range(Cells(3,
scenarioCounter * 4 + 2), Cells((projYears - currentRegression) * periodsPerYear + 23, scenarioCounter * 4 + 2)).Value))

            x1values(1) = startvalue * Exp(Application.WorksheetFunction.Sum(Worksheets("Scenario Output").Range(Cells(3,
scenarioCounter * 4 + 3), Cells((projYears - currentRegression) * periodsPerYear + 23, scenarioCounter * 4 + 3)).Value))

```

```

        x2values(1) = russtartvalue * Exp(Application.WorksheetFunction.Sum(Worksheets("Scenario Output").Range(Cells(3,
scenarioCounter * 4 + 4), Cells((projYears - currentRegression) * periodsPerYear + 23, scenarioCounter * 4 + 4)).Value))
    End If

    xValues(1, 1) = x1values(1)
    xValues(2, 1) = x2values(1)

    For i = 2 To (projDays - (projYears - currentRegression) * periodsPerYear) / 21
        yValues(i) = yValues(i - 1) * Exp(Application.WorksheetFunction.Sum(Worksheets("Scenario Output").Range(Cells((projYears -
currentRegression) * periodsPerYear + 3 + (i - 1) * 21, scenarioCounter * 4 + 2), Cells((projYears - currentRegression) * periodsPerYear + 2 + i *
21, scenarioCounter * 4 + 2)).Value))
        x1values(i) = x1values(i - 1) * Exp(Application.WorksheetFunction.Sum(Worksheets("Scenario Output").Range(Cells((projYears -
currentRegression) * periodsPerYear + 3 + (i - 1) * 21, scenarioCounter * 4 + 3), Cells((projYears - currentRegression) * periodsPerYear + 2 + i *
21, scenarioCounter * 4 + 3)).Value))
        x2values(i) = x2values(i - 1) * Exp(Application.WorksheetFunction.Sum(Worksheets("Scenario Output").Range(Cells((projYears -
currentRegression) * periodsPerYear + 3 + (i - 1) * 21, scenarioCounter * 4 + 4), Cells((projYears - currentRegression) * periodsPerYear + 2 + i *
21, scenarioCounter * 4 + 4)).Value))
        xValues(1, i) = x1values(i)
        xValues(2, i) = x2values(i)
    Next i

    End If

End If

End If

End If

'.....
'POPULATING ARRAYS WITH DATA IS DONE. NOW, REGRESS THE ARRAYS'
'.....

'2 Variable Regression
Worksheets("Scenario Output").Range(Cells(regressionOffset + 9, scenarioCounter * 4 + 2), Cells(regressionOffset + 13, scenarioCounter *
4 + 4)).Value = WorksheetFunction.LinEst(yValues, xValues, True, True)

'Formatting for 2 Variable Regression
Worksheets("Scenario Output").Cells(regressionOffset + 6, scenarioCounter * 4 + 2).Value = "2-VAR"
Worksheets("Scenario Output").Cells(regressionOffset + 7, scenarioCounter * 4 + 2).Value = "SPY"
Worksheets("Scenario Output").Cells(regressionOffset + 8, scenarioCounter * 4 + 2).Value = "RUS"

twoIntercept = Worksheets("Scenario Output").Cells(regressionOffset + 9, scenarioCounter * 4 + 4).Value
SP2VarCoeff = Worksheets("Scenario Output").Cells(regressionOffset + 9, scenarioCounter * 4 + 3).Value
RUS2VarCoeff = Worksheets("Scenario Output").Cells(regressionOffset + 9, scenarioCounter * 4 + 2).Value

Worksheets("Scenario Output").Cells(regressionOffset + 6, scenarioCounter * 4 + 3).Value = twoIntercept
Worksheets("Scenario Output").Cells(regressionOffset + 7, scenarioCounter * 4 + 3).Value = SP2VarCoeff
Worksheets("Scenario Output").Cells(regressionOffset + 8, scenarioCounter * 4 + 3).Value = RUS2VarCoeff

Worksheets("Scenario Output").Range(Cells(regressionOffset + 6, scenarioCounter * 4 + 2), Cells(regressionOffset + 8, scenarioCounter * 4
+ 3)).Font.FontStyle = "Bold"
Worksheets("Scenario Output").Range(Cells(regressionOffset + 11, scenarioCounter * 4 + 4), Cells(regressionOffset + 13, scenarioCounter *
4 + 4)).Value = ""
Worksheets("Scenario Output").Cells(regressionOffset + 12, scenarioCounter * 4 + 2).NumberFormat = "0.00"
Worksheets("Scenario Output").Cells(regressionOffset + 12, scenarioCounter * 4 + 3).NumberFormat = "0"
If regressLN = False Then
    Worksheets("Scenario Output").Cells(regressionOffset + 13, scenarioCounter * 4 + 2).NumberFormat = "0"
    Worksheets("Scenario Output").Cells(regressionOffset + 13, scenarioCounter * 4 + 3).NumberFormat = "0"
End If

With Worksheets("Scenario Output").Rows(regressionOffset + 13).Borders(xlEdgeBottom)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlThin
End With
With Worksheets("Scenario Output").Rows(regressionOffset + 14).Borders(xlEdgeTop)

```

```

.LineStyle = xlContinuous
.ColorIndex = xlAutomatic
.TintAndShade = 0
.Weight = xlThin
End With

'Output Formula for 2 Variable Regression
Worksheets("Scenario Output").Cells(regressionOffset + 5, scenarioCounter * 4 + 2).Value = "MFIB= " & Round(twoIntercept, 4) & " + " &
Round(SP2VarCoeff, 4) & "*SPY + " & Round(RUS2VarCoeff, 4) & "*RUS"
Worksheets("Scenario Output").Cells(regressionOffset + 5, scenarioCounter * 4 + 2).Font.FontStyle = "Bold"

'SPY Regression
Worksheets("Scenario Output").Range(Cells(regressionOffset + 17, scenarioCounter * 4 + 2), Cells(regressionOffset + 21, scenarioCounter *
4 + 3)).Value = WorksheetFunction.LinEst(yValues, x1values, True, True)

'Formatting for SPY Regression
Worksheets("Scenario Output").Cells(regressionOffset + 15, scenarioCounter * 4 + 2).Value = "1-VAR"
Worksheets("Scenario Output").Cells(regressionOffset + 16, scenarioCounter * 4 + 2).Value = "SPY"

SPintercept = Worksheets("Scenario Output").Cells(regressionOffset + 17, scenarioCounter * 4 + 3).Value
SP1VarCoeff = Worksheets("Scenario Output").Cells(regressionOffset + 17, scenarioCounter * 4 + 2).Value

Worksheets("Scenario Output").Cells(regressionOffset + 15, scenarioCounter * 4 + 3).Value = SPintercept
Worksheets("Scenario Output").Cells(regressionOffset + 16, scenarioCounter * 4 + 3).Value = SP1VarCoeff

Worksheets("Scenario Output").Range(Cells(regressionOffset + 15, scenarioCounter * 4 + 2), Cells(regressionOffset + 16, scenarioCounter *
4 + 3)).Font.FontStyle = "Bold"
Worksheets("Scenario Output").Cells(regressionOffset + 20, scenarioCounter * 4 + 2).NumberFormat = "0.00"
Worksheets("Scenario Output").Cells(regressionOffset + 20, scenarioCounter * 4 + 3).NumberFormat = "0"
If regressLN = False Then
    Worksheets("Scenario Output").Cells(regressionOffset + 21, scenarioCounter * 4 + 2).NumberFormat = "0"
    Worksheets("Scenario Output").Cells(regressionOffset + 21, scenarioCounter * 4 + 3).NumberFormat = "0"
End If

With Worksheets("Scenario Output").Rows(regressionOffset + 21).Borders(xlEdgeBottom)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlThin
End With
With Worksheets("Scenario Output").Rows(regressionOffset + 22).Borders(xlEdgeTop)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlThin
End With

'Output Formula for SPY Regression
Worksheets("Scenario Output").Cells(regressionOffset + 14, scenarioCounter * 4 + 2).Value = "MFIC = " & Round(SPintercept, 4) & " + " &
Round(SP1VarCoeff, 4) & "*SPY"
Worksheets("Scenario Output").Cells(regressionOffset + 14, scenarioCounter * 4 + 2).Font.FontStyle = "Bold"

'RUS Regression
Worksheets("Scenario Output").Range(Cells(regressionOffset + 25, scenarioCounter * 4 + 2), Cells(regressionOffset + 29, scenarioCounter *
4 + 3)).Value = WorksheetFunction.LinEst(yValues, x2values, True, True)

'Formatting for RUS Regression
Worksheets("Scenario Output").Cells(regressionOffset + 23, scenarioCounter * 4 + 2).Value = "1-VAR"
Worksheets("Scenario Output").Cells(regressionOffset + 24, scenarioCounter * 4 + 2).Value = "RUS"

RUSintercept = Worksheets("Scenario Output").Cells(regressionOffset + 25, scenarioCounter * 4 + 3).Value
RUS1VarCoeff = Worksheets("Scenario Output").Cells(regressionOffset + 25, scenarioCounter * 4 + 2).Value

Worksheets("Scenario Output").Cells(regressionOffset + 23, scenarioCounter * 4 + 3).Value = RUSintercept
Worksheets("Scenario Output").Cells(regressionOffset + 24, scenarioCounter * 4 + 3).Value = RUS1VarCoeff

```



```

Worksheets("Scenario Output").Range(Cells(regressionOffset + 23, scenarioCounter * 4 + 2), Cells(regressionOffset + 24, scenarioCounter *
4 + 3)).Font.FontStyle = "Bold"
Worksheets("Scenario Output").Cells(regressionOffset + 28, scenarioCounter * 4 + 2).NumberFormat = "0.00"
Worksheets("Scenario Output").Cells(regressionOffset + 28, scenarioCounter * 4 + 3).NumberFormat = "0"
If regressLN = False Then
    Worksheets("Scenario Output").Cells(regressionOffset + 29, scenarioCounter * 4 + 2).NumberFormat = "0"
    Worksheets("Scenario Output").Cells(regressionOffset + 29, scenarioCounter * 4 + 3).NumberFormat = "0"
End If

With Worksheets("Scenario Output").Rows(regressionOffset + 29).Borders(xlEdgeBottom)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlMedium
End With
With Worksheets("Scenario Output").Rows(regressionOffset + 30).Borders(xlEdgeTop)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlMedium
End With

'Output Formula for RUS Regression
Worksheets("Scenario Output").Cells(regressionOffset + 22, scenarioCounter * 4 + 2).Value = "MFRUS = " & Round(RUSintercept, 4) & " + "
& Round(RUS1VarCoeff, 4) & "*RUS"
Worksheets("Scenario Output").Cells(regressionOffset + 22, scenarioCounter * 4 + 2).Font.FontStyle = "Bold"

'Set Variable for Next Regression
currentRegression = currentRegression - 1

Loop

'Fill with Color
Worksheets("Scenario Output").Range(Cells(projDays + 4, scenarioCounter * 4 + 2), Cells(maxRegression * 25 + projDays + 4, scenarioCounter
* 4 + 5)).Interior.Color = 8421631

.....
'Regression Stuff Ends Here'
.....

.....
'Future Future Stuff Starts Here'
.....

'Display Period Numbers
If scenarioCounter = 0 Then
    '(Only do it once)
    FFheaderOutput = 25 * maxRegression + projDays + (FFprojDays + 2) * FFcurrScenario * (maxRegression + 1)
    If FFdailyDisplay = False Then
        If FFcalculate = True Then
            If FFprintType = True Then
                'For Monthly periods with daily LN Return calculations
                Worksheets("Scenario Output").Cells(7 + FFheaderOutput, 1).Value = "Months (LN Returns, calculated Daily)"
            Else
                'For Monthly periods with monthly Value calculations
                Worksheets("Scenario Output").Cells(7 + FFheaderOutput, 1).Value = "Months (Index Values, calculated Daily)"
            End If
        End If
    Else
        If FFprintType = True Then
            'For Monthly periods with monthly LN Return calculations
            Worksheets("Scenario Output").Cells(7 + FFheaderOutput, 1).Value = "Months (LN Returns, calculated Monthly)"
        Else
            'For Monthly periods with monthly Value calculations
            Worksheets("Scenario Output").Cells(7 + FFheaderOutput, 1).Value = "Months (Index Values, calculated Monthly)"
        End If
    End If
End If

```

```

Else
  If FFprintType = True Then
    'For Daily periods with daily LN Return calculations
    Worksheets("Scenario Output").Cells(7 + FFheaderOutput, 1).Value = "Days (LN Returns, calculated Daily)"
  Else
    'For Daily periods with daily Value calculations
    Worksheets("Scenario Output").Cells(7 + FFheaderOutput, 1).Value = "Days (Index Values, calculated Daily)"
  End If
End If
For i = 0 To FFprojDays - 1
  Worksheets("Scenario Output").Cells(8 + i + FFheaderOutput, 1).Value = i + 1
Next i
End If

'Display Period Numbers for Regression MF's
If scenarioCounter = 0 Then
  '(Only do it once)
  For i = 1 To maxRegression
    FFheaderOutput = 25 * maxRegression + projDays + (FFprojDays + 2) + (FFprojDays + 2) * FFcurrScenario * (maxRegression + 1) +
(maxRegression - i) * (FFprojDays + 2)
    If FFdailyDisplay = False Then
      If FFcalculate = True Then
        If FFprintType = True Then
          'For Monthly periods with daily LN Return calculations
          Worksheets("Scenario Output").Cells(7 + FFheaderOutput, 1).Value = "Months (LN Returns, calculated Daily)"
        Else
          'For Monthly periods with monthly Value calculations
          Worksheets("Scenario Output").Cells(7 + FFheaderOutput, 1).Value = "Months (Index Values, calculated Daily)"
        End If
      Else
        If FFprintType = True Then
          'For Monthly periods with monthly LN Return calculations
          Worksheets("Scenario Output").Cells(7 + FFheaderOutput, 1).Value = "Months (LN Returns, calculated Monthly)"
        Else
          'For Monthly periods with monthly Value calculations
          Worksheets("Scenario Output").Cells(7 + FFheaderOutput, 1).Value = "Months (Index Values, calculated Monthly)"
        End If
      End If
    End If
  Next i
End If
For j = 1 To FFprojDays
  Worksheets("Scenario Output").Cells(7 + j + FFheaderOutput, 1).Value = j
Next j
End If

'Start Scenario Worksheet Loop
For FFcurrScenario = 0 To FFscenarios - 1

  'Randomize
  Randomize

  'Status Update
  Worksheets("Scenario Output").Range("A1").Value = "Calculating Scenario " & scenarioCounter + 1 & ". " & FFcurrScenario + 1 & "... "

  'Output Scenario Heading
  FFheaderOutput = 25 * maxRegression + projDays + (FFprojDays + 2) * FFcurrScenario * (maxRegression + 1)
  Worksheets("Scenario Output").Cells(6 + FFheaderOutput, scenarioCounter * 4 + 2).Value = "Scenario " & scenarioCounter + 1 & ". " &
FFcurrScenario + 1

```

```

Worksheets("Scenario Output").Cells(7 + FFheaderOutput, scenarioCounter * 4 + 2).Value = "MFI"
Worksheets("Scenario Output").Cells(7 + FFheaderOutput, scenarioCounter * 4 + 3).Value = "SPY"
Worksheets("Scenario Output").Cells(7 + FFheaderOutput, scenarioCounter * 4 + 4).Value = "RUS"

'Reset Variables
newvalue = FPfinal
rusnewvalue = FPrusFinal
mfnewvalue = FPmfFinal
FFmfiBnewvalue = FPmfFinal
FFmfiCnewvalue = FPmfFinal
FFmfiDnewvalue = FPmfFinal

'Color Columns
FFheaderOutput = 25 * maxRegression + projDays + (FFprojDays + 2) * FFcurrScenario * (maxRegression + 1)
With Worksheets("Scenario Output").Range(Cells(6 + FFheaderOutput, scenarioCounter * 4 + 2), Cells(7 + FFheaderOutput + FFprojDays,
scenarioCounter * 4 + 2)).Interior
    .Pattern = xlSolid
    .PatternColorIndex = xlAutomatic
    .Color = 5296274
    .TintAndShade = 0
    .PatternTintAndShade = 0
End With
With Worksheets("Scenario Output").Range(Cells(6 + FFheaderOutput, scenarioCounter * 4 + 3), Cells(7 + FFheaderOutput + FFprojDays,
scenarioCounter * 4 + 3)).Interior
    .Pattern = xlSolid
    .PatternColorIndex = xlAutomatic
    .Color = 14136213
    .TintAndShade = 0
    .PatternTintAndShade = 0
End With
With Worksheets("Scenario Output").Range(Cells(6 + FFheaderOutput, scenarioCounter * 4 + 4), Cells(7 + FFheaderOutput + FFprojDays,
scenarioCounter * 4 + 4)).Interior
    .Pattern = xlSolid
    .PatternColorIndex = xlAutomatic
    .Color = 49407
    .TintAndShade = 0
    .PatternTintAndShade = 0
End With

'Merge Top Row
With Worksheets("Scenario Output").Range(Cells(6 + FFheaderOutput, scenarioCounter * 4 + 2), Cells(6 + FFheaderOutput,
scenarioCounter * 4 + 5))
    .Merge
    .HorizontalAlignment = xlCenter
    .VerticalAlignment = xlBottom
    .Interior.Color = 8421631
End With

For i = 1 To maxRegression

'Output Scenario Heading
FFheaderOutput = 25 * maxRegression + projDays + (FFprojDays + 2) + (FFprojDays + 2) * FFcurrScenario * (maxRegression + 1) +
(maxRegression - i) * (FFprojDays + 2)
Worksheets("Scenario Output").Cells(6 + FFheaderOutput, scenarioCounter * 4 + 2).Value = "Scenario " & scenarioCounter + 1 & ". " &
FFcurrScenario + 1 & " (" & i & "-Year Regression)"
Worksheets("Scenario Output").Cells(7 + FFheaderOutput, scenarioCounter * 4 + 2).Value = "MFI 2Var"
Worksheets("Scenario Output").Cells(7 + FFheaderOutput, scenarioCounter * 4 + 3).Value = "MFI SPY"
Worksheets("Scenario Output").Cells(7 + FFheaderOutput, scenarioCounter * 4 + 4).Value = "MFI RUS"

'Color Columns
With Worksheets("Scenario Output").Range(Cells(6 + FFheaderOutput, scenarioCounter * 4 + 2), Cells(7 + FFheaderOutput + FFprojDays,
scenarioCounter * 4 + 2)).Interior
    .Pattern = xlSolid
    .PatternColorIndex = xlAutomatic
    .Color = 5296274
    .TintAndShade = 0

```

```

        .PatternTintAndShade = 0
    End With
    With Worksheets("Scenario Output").Range(Cells(6 + FFheaderOutput, scenarioCounter * 4 + 3), Cells(7 + FFheaderOutput + FFprojDays,
scenarioCounter * 4 + 3)).Interior
        .Pattern = xlSolid
        .PatternColorIndex = xlAutomatic
        .Color = 8575688
        .TintAndShade = 0
        .PatternTintAndShade = 0
    End With
    With Worksheets("Scenario Output").Range(Cells(6 + FFheaderOutput, scenarioCounter * 4 + 4), Cells(7 + FFheaderOutput + FFprojDays,
scenarioCounter * 4 + 4)).Interior
        .Pattern = xlSolid
        .PatternColorIndex = xlAutomatic
        .Color = 11855054
        .TintAndShade = 0
        .PatternTintAndShade = 0
    End With

'Merge Top Row
    With Worksheets("Scenario Output").Range(Cells(6 + FFheaderOutput, scenarioCounter * 4 + 2), Cells(6 + FFheaderOutput,
scenarioCounter * 4 + 5))
        .Merge
        .HorizontalAlignment = xlCenter
        .VerticalAlignment = xlBottom
        .Interior.Color = 8421631
    End With
Next i

'Start Projection Loop
For FFcurrentPeriod = 0 To FFprojDays - 1

    'Previous New Values become Old Values
    oldvalue = newvalue
    rusoldvalue = rusnewvalue
    mfoldvalue = mfnewvalue

    'This top part of the If statement is used when the indexes are calculated at the same times they are outputted
    If (FFdailyDisplay = True And FFcalculate = True) Or (FFdailyDisplay = False And FFcalculate = False) Then

        'Reset Random Numbers
        randnorm = Application.WorksheetFunction.NormSInv(Rnd())
        rusrandnorm = Application.WorksheetFunction.NormSInv(Rnd())
        mfrandnorm = Application.WorksheetFunction.NormSInv(Rnd())

        'Calculate New Values
        newvalue = newvalue * Exp((mu - variance ^ 2 / 2) * FFtimeinterval + FFtimeinterval ^ 0.5 * a11 * randnorm)
        rusnewvalue = rusnewvalue * Exp((rusmu - rusvariance ^ 2 / 2) * FFtimeinterval + FFtimeinterval ^ 0.5 * (a21 * randnorm + a22 *
rusrandnorm))

        'Calculate ln of Returns
        lnreturn = Log(newvalue / oldvalue)
        ruslnreturn = Log(rusnewvalue / rusoldvalue)
        mflnreturn = lnreturn + (alpha + Abs(SPYalpha * lnreturn)) / (FFperiodsPerYear ^ 0.5) * mfrandnorm

        'Calculate New Value of Mutual fund
        mfnewvalue = mfoldvalue * Exp(mflnreturn)

    Else
        'This portion is used when the indexes are calculated daily, but values are outputted monthly
        For i = 1 To 21
            'Reset Random Numbers
            randnorm = Application.WorksheetFunction.NormSInv(Rnd())
            rusrandnorm = Application.WorksheetFunction.NormSInv(Rnd())
            mfrandnorm = Application.WorksheetFunction.NormSInv(Rnd())

```

```

'Calculate New Values
newvalue = newvalue * Exp((mu - variance ^ 2 / 2) * FFtimeinterval + FFtimeinterval ^ 0.5 * a11 * randnorm)
rusnewvalue = rusnewvalue * Exp((rusmu - rusvariance ^ 2 / 2) * FFtimeinterval + FFtimeinterval ^ 0.5 * (a21 * randnorm + a22 *
rusrandnorm))
mfnewvalue = mfnewvalue * Exp((mu - variance ^ 2 / 2) * FFtimeinterval + FFtimeinterval ^ 0.5 * a11 * randnorm + (alpha +
Abs(SPYalpha * ((mu - variance ^ 2 / 2) * FFtimeinterval + FFtimeinterval ^ 0.5 * a11 * randnorm))) / (252 ^ 0.5) * mfrandnorm)
Next i

'Calculate Ln of Returns
Inreturn = Log(newvalue / oldvalue)
ruslnreturn = Log(rusnewvalue / rusoldvalue)
mflnreturn = Log(mfnewvalue / mfoldvalue)

End If

'Output Values
FFrowOutput = 8 + 25 * maxRegression + projDays + FFcurrentPeriod + FFcurrScenario * (FFprojDays + 2) * (maxRegression + 1)
If FFprintType = True Then
'Output Ln of Returns
Worksheets("Scenario Output").Cells(FFrowOutput, scenarioCounter * 4 + 2).Value = mflnreturn
Worksheets("Scenario Output").Cells(FFrowOutput, scenarioCounter * 4 + 3).Value = Inreturn
Worksheets("Scenario Output").Cells(FFrowOutput, scenarioCounter * 4 + 4).Value = ruslnreturn
Else
'Output Index Values
Worksheets("Scenario Output").Cells(FFrowOutput, scenarioCounter * 4 + 2).Value = mfnewvalue
Worksheets("Scenario Output").Cells(FFrowOutput, scenarioCounter * 4 + 3).Value = newvalue
Worksheets("Scenario Output").Cells(FFrowOutput, scenarioCounter * 4 + 4).Value = rusnewvalue
End If

'Set Starting Values for Variables
currentRegression = maxRegression

FFmfiBoldvalue = FFmfiBnewvalue
FFmfiColdvalue = FFmfiCnewvalue
FFmfiDoldvalue = FFmfiDnewvalue

Do While currentRegression > 0

'Reset Variables
regressionOffset = (maxRegression - currentRegression) * 25 + projDays
twoIntercept = Worksheets("Scenario Output").Cells(regressionOffset + 6, scenarioCounter * 4 + 3).Value
SP2VarCoeff = Worksheets("Scenario Output").Cells(regressionOffset + 7, scenarioCounter * 4 + 3).Value
RUS2VarCoeff = Worksheets("Scenario Output").Cells(regressionOffset + 8, scenarioCounter * 4 + 3).Value
SPintercept = Worksheets("Scenario Output").Cells(regressionOffset + 15, scenarioCounter * 4 + 3).Value
SP1VarCoeff = Worksheets("Scenario Output").Cells(regressionOffset + 16, scenarioCounter * 4 + 3).Value
RUSintercept = Worksheets("Scenario Output").Cells(regressionOffset + 23, scenarioCounter * 4 + 3).Value
RUS1VarCoeff = Worksheets("Scenario Output").Cells(regressionOffset + 24, scenarioCounter * 4 + 3).Value

'Calculate New value of Mutual Funds
mfnewvalue = Exp(mflnreturn) * mfoldvalue

'If regressions were done on LN...
If regressLN = True Then
FFmfiBlreturn = twoIntercept + SP2VarCoeff * Inreturn + RUS2VarCoeff * ruslnreturn
FFmfiClreturn = SPintercept + SP1VarCoeff * Inreturn
FFmfiDlreturn = RUSintercept + RUS1VarCoeff * ruslnreturn
FFmfiBnewvalue = Exp(FFmfiBlreturn) * FFmfiBoldvalue
FFmfiCnewvalue = Exp(FFmfiClreturn) * FFmfiColdvalue
FFmfiDnewvalue = Exp(FFmfiDlreturn) * FFmfiDoldvalue

'Or if regressions were done on values...
Else
FFmfiBnewvalue = twoIntercept + SP2VarCoeff * newvalue + RUS2VarCoeff * rusnewvalue
FFmfiCnewvalue = SPintercept + SP1VarCoeff * newvalue
FFmfiDnewvalue = RUSintercept + RUS1VarCoeff * rusnewvalue
FFmfiBlreturn = Log(FFmfiBnewvalue / FFmfiBoldvalue)

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```

        FFmfiClnreturn = Log(FFmfiCnewvalue / FFmfiColdvalue)
        FFmfiDlnreturn = Log(FFmfiDnewvalue / FFmfiDoldvalue)
    End If

    'Output Values
    FFrowOutput = 8 + 25 * maxRegression + projDays + (FFprojDays + 2) + FFcurrentPeriod + (maxRegression - currentRegression) *
(FFprojDays + 2) + (FFprojDays + 2) * FFcurrScenario * (maxRegression + 1)
    If FFprintType = True Then
        'Output In of Returns
        Worksheets("Scenario Output").Cells(FFrowOutput, scenarioCounter * 4 + 2).Value = FFmfiBlnreturn
        Worksheets("Scenario Output").Cells(FFrowOutput, scenarioCounter * 4 + 3).Value = FFmfiClnreturn
        Worksheets("Scenario Output").Cells(FFrowOutput, scenarioCounter * 4 + 4).Value = FFmfiDlnreturn

    Else
        'Output Index Values
        Worksheets("Scenario Output").Cells(FFrowOutput, scenarioCounter * 4 + 2).Value = FFmfiBnewvalue
        Worksheets("Scenario Output").Cells(FFrowOutput, scenarioCounter * 4 + 3).Value = FFmfiCnewvalue
        Worksheets("Scenario Output").Cells(FFrowOutput, scenarioCounter * 4 + 4).Value = FFmfiDnewvalue
    End If

    currentRegression = currentRegression - 1
Loop

Next FFcurrentPeriod

Next FFcurrScenario

.....
'Future Future Stuff Ends Here'
.....

'Advance Counter Variable
scenarioCounter = scenarioCounter + 1

Loop

.....
'Scenario Stuff Ends Here'
.....

.....
'Formatting Stuff Starts Here'
.....

'Status Update
Worksheets("Scenario Output").Range("A1").Value = "Finalizing..."

'Put black bar between Future Past and Regressions
Worksheets("Scenario Output").Rows(projDays + 3).RowHeight = 9.75
With Worksheets("Scenario Output").Rows(projDays + 3).Interior
    .Pattern = xlSolid
    .PatternColorIndex = xlAutomatic
    .Color = 0
    .TintAndShade = 0
    .PatternTintAndShade = 0
End With
With Worksheets("Scenario Output").Rows(projDays + 3).Borders(xlEdgeBottom)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlThin
End With
With Worksheets("Scenario Output").Rows(projDays + 3).Borders(xlEdgeTop)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0

```

```

        .Weight = xlThin
    End With

'Put black bar between Regressions and Future Future
Worksheets("Scenario Output").Rows(25 * maxRegression + projDays + 5).RowHeight = 9.75
With Worksheets("Scenario Output").Rows(25 * maxRegression + projDays + 5).Interior
    .Pattern = xlSolid
    .PatternColorIndex = xlAutomatic
    .Color = 0
    .TintAndShade = 0
    .PatternTintAndShade = 0
End With
With Worksheets("Scenario Output").Rows(25 * maxRegression + projDays + 5).Borders(xlEdgeBottom)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlThin
End With
With Worksheets("Scenario Output").Rows(25 * maxRegression + projDays + 5).Borders(xlEdgeTop)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlThin
End With

'Formatting Stuff Ends Here'

'Graph Stuff Starts Here'

'Clear Old Worksheet
Worksheets("Graph Data").Cells.Clear

'Formatting Stuff Starts Here'

'Number Formatting
Worksheets("Graph Data").Columns.NumberFormat = "0.00%"
Worksheets("Graph Data").Rows(1).NumberFormat = "0"
Worksheets("Graph Data").Rows(38).NumberFormat = "0"
For i = 2 To 37
    Worksheets("Graph Data").Rows(i).NumberFormat = "0.0000"
Next i

'Bolding Stuff
Worksheets("Graph Data").Columns(1).Font.FontStyle = "Bold"
Worksheets("Graph Data").Rows(1).Font.FontStyle = "Bold"
Worksheets("Graph Data").Rows(38).Font.FontStyle = "Bold"

'Enlarging Fonts
Worksheets("Graph Data").Columns(1).Font.Size = 14

'Borders
With Worksheets("Graph Data").Rows(1).Borders(xlEdgeBottom)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic
    .TintAndShade = 0
    .Weight = xlThin
End With
With Worksheets("Graph Data").Rows(2).Borders(xlEdgeTop)
    .LineStyle = xlContinuous
    .ColorIndex = xlAutomatic

```

```

        .TintAndShade = 0
        .Weight = xlThin
    End With
    With Worksheets("Graph Data").Rows(38).Borders(xlEdgeBottom)
        .LineStyle = xlContinuous
        .ColorIndex = xlAutomatic
        .TintAndShade = 0
        .Weight = xlThin
    End With
    With Worksheets("Graph Data").Rows(39).Borders(xlEdgeTop)
        .LineStyle = xlContinuous
        .ColorIndex = xlAutomatic
        .TintAndShade = 0
        .Weight = xlThin
    End With
    With Worksheets("Graph Data").Columns(1).Borders(xlEdgeRight)
        .LineStyle = xlContinuous
        .ColorIndex = xlAutomatic
        .TintAndShade = 0
        .Weight = xlThin
    End With
    With Worksheets("Graph Data").Columns(2).Borders(xlEdgeLeft)
        .LineStyle = xlContinuous
        .ColorIndex = xlAutomatic
        .TintAndShade = 0
        .Weight = xlThin
    End With
    With Worksheets("Graph Data").Columns(2).Borders(xlEdgeRight)
        .LineStyle = xlContinuous
        .ColorIndex = xlAutomatic
        .TintAndShade = 0
        .Weight = xlThin
    End With
    With Worksheets("Graph Data").Columns(3).Borders(xlEdgeLeft)
        .LineStyle = xlContinuous
        .ColorIndex = xlAutomatic
        .TintAndShade = 0
        .Weight = xlThin
    End With

    'Black bars
    For i = 0 To 3
        Worksheets("Graph Data").Rows(10 + 9 * i).RowHeight = 7
        With Worksheets("Graph Data").Rows(10 + 9 * i).Interior
            .Pattern = xlSolid
            .PatternColorIndex = xlAutomatic
            .Color = 0
            .TintAndShade = 0
            .PatternTintAndShade = 0
        End With
        With Worksheets("Graph Data").Rows(10 + 9 * i).Borders(xlEdgeBottom)
            .LineStyle = xlContinuous
            .ColorIndex = xlAutomatic
            .TintAndShade = 0
            .Weight = xlThin
        End With
        With Worksheets("Graph Data").Rows(10 + 9 * i).Borders(xlEdgeTop)
            .LineStyle = xlContinuous
            .ColorIndex = xlAutomatic
            .TintAndShade = 0
            .Weight = xlThin
        End With
    Next i
    For i = 1 To maxRegression * 3
        Worksheets("Graph Data").Rows(38 + 7 * i).RowHeight = 7
        With Worksheets("Graph Data").Rows(38 + 7 * i).Interior

```



```

        .Pattern = xlSolid
        .PatternColorIndex = xlAutomatic
        .Color = 0
        .TintAndShade = 0
        .PatternTintAndShade = 0
    End With
    With Worksheets("Graph Data").Rows(38 + 7 * i).Borders(xlEdgeBottom)
        .LineStyle = xlContinuous
        .ColorIndex = xlAutomatic
        .TintAndShade = 0
        .Weight = xlThin
    End With
    With Worksheets("Graph Data").Rows(38 + 7 * i).Borders(xlEdgeTop)
        .LineStyle = xlContinuous
        .ColorIndex = xlAutomatic
        .TintAndShade = 0
        .Weight = xlThin
    End With
Next i

'-----
'Formatting Stuff Ends Here'
'-----

'-----
'Filling the Regression Graphs Starts Here'
'-----

'Print Left Side
Worksheets("Graph Data").Range("A2").Value = "2-VAR SPY"
Worksheets("Graph Data").Range("A11").Value = "2-VAR RUS"
Worksheets("Graph Data").Range("A20").Value = "1-VAR SPY"
Worksheets("Graph Data").Range("A29").Value = "1-VAR RUS"
For i = 0 To 3
    Worksheets("Graph Data").Cells(9 * i + 2, 2) = "Min"
    Worksheets("Graph Data").Cells(9 * i + 3, 2) = "Min valid"
    Worksheets("Graph Data").Cells(9 * i + 4, 2) = "1st Quartile"
    Worksheets("Graph Data").Cells(9 * i + 5, 2) = "Median"
    Worksheets("Graph Data").Cells(9 * i + 6, 2) = "Mean"
    Worksheets("Graph Data").Cells(9 * i + 7, 2) = "3rd Quartile"
    Worksheets("Graph Data").Cells(9 * i + 8, 2) = "Max valid"
    Worksheets("Graph Data").Cells(9 * i + 9, 2) = "Max"
Next i

'Print Regression Years on Top
Worksheets("Graph Data").Range("B1").Value = "Regression Years"
For i = 1 To maxRegression
    Worksheets("Graph Data").Cells(1, i + 2) = i
Next i

'Fill the chart
Dim dataArray() As Double
ReDim dataArray(1 To scenarios) As Double

For i = 1 To maxRegression
    regressionOffset = (maxRegression - i) * 25 + projDays

    '2-VAR SPY
    'Fill an Array with the 2-VAR SPY Coefficients
    For j = 1 To scenarios
        dataArray(j) = Worksheets("Scenario Output").Cells(regressionOffset + 7, (j - 1) * 4 + 3).Value
    Next j
    'Evaluate max, min, etc. of the 2-VAR SPY coefficients
    Worksheets("Graph Data").Cells(2, i + 2).Value = Application.WorksheetFunction.Min(dataArray())
    Worksheets("Graph Data").Cells(4, i + 2).Value = Application.WorksheetFunction.Quartile(dataArray(), 1)
    Worksheets("Graph Data").Cells(5, i + 2).Value = Application.WorksheetFunction.Median(dataArray())

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Worksheets("Graph Data").Cells(6, i + 2).Value = Application.WorksheetFunction.Average(dataArray())
Worksheets("Graph Data").Cells(7, i + 2).Value = Application.WorksheetFunction.Quartile(dataArray(), 3)
Worksheets("Graph Data").Cells(9, i + 2).Value = Application.WorksheetFunction.Max(dataArray())
Worksheets("Graph Data").Cells(3, i + 2).Value = Application.WorksheetFunction.Max(Worksheets("Graph Data").Cells(2, i + 2).Value,
Worksheets("Graph Data").Cells(4, i + 2).Value - 1.5 * (Worksheets("Graph Data").Cells(7, i + 2).Value - Worksheets("Graph Data").Cells(4, i +
2).Value))
Worksheets("Graph Data").Cells(8, i + 2).Value = Application.WorksheetFunction.Min(Worksheets("Graph Data").Cells(9, i + 2).Value,
Worksheets("Graph Data").Cells(7, i + 2).Value + 1.5 * (Worksheets("Graph Data").Cells(7, i + 2).Value - Worksheets("Graph Data").Cells(4, i +
2).Value))

'2-VAR RUS
'Fill an Array with the 2-VAR RUS Coefficients
For j = 1 To scenarios
    dataArray(j) = Worksheets("Scenario Output").Cells(regressionOffset + 8, (j - 1) * 4 + 3).Value
Next j
'Evaluate max, min, etc. of the 2-VAR RUS coefficients
Worksheets("Graph Data").Cells(11, i + 2).Value = Application.WorksheetFunction.Min(dataArray())
Worksheets("Graph Data").Cells(13, i + 2).Value = Application.WorksheetFunction.Quartile(dataArray(), 1)
Worksheets("Graph Data").Cells(14, i + 2).Value = Application.WorksheetFunction.Median(dataArray())
Worksheets("Graph Data").Cells(15, i + 2).Value = Application.WorksheetFunction.Average(dataArray())
Worksheets("Graph Data").Cells(16, i + 2).Value = Application.WorksheetFunction.Quartile(dataArray(), 3)
Worksheets("Graph Data").Cells(18, i + 2).Value = Application.WorksheetFunction.Max(dataArray())
Worksheets("Graph Data").Cells(12, i + 2).Value = Application.WorksheetFunction.Max(Worksheets("Graph Data").Cells(11, i + 2).Value,
Worksheets("Graph Data").Cells(13, i + 2).Value - 1.5 * (Worksheets("Graph Data").Cells(16, i + 2).Value - Worksheets("Graph Data").Cells(13, i
+ 2).Value))
Worksheets("Graph Data").Cells(17, i + 2).Value = Application.WorksheetFunction.Min(Worksheets("Graph Data").Cells(18, i + 2).Value,
Worksheets("Graph Data").Cells(16, i + 2).Value + 1.5 * (Worksheets("Graph Data").Cells(16, i + 2).Value - Worksheets("Graph Data").Cells(13, i
+ 2).Value))

'1-VAR SPY
'Fill an Array with the 1-VAR SPY Coefficients
For j = 1 To scenarios
    dataArray(j) = Worksheets("Scenario Output").Cells(regressionOffset + 16, (j - 1) * 4 + 3).Value
Next j
'Evaluate max, min, etc. of the 1-VAR SPY coefficients
Worksheets("Graph Data").Cells(20, i + 2).Value = Application.WorksheetFunction.Min(dataArray())
Worksheets("Graph Data").Cells(22, i + 2).Value = Application.WorksheetFunction.Quartile(dataArray(), 1)
Worksheets("Graph Data").Cells(23, i + 2).Value = Application.WorksheetFunction.Median(dataArray())
Worksheets("Graph Data").Cells(24, i + 2).Value = Application.WorksheetFunction.Average(dataArray())
Worksheets("Graph Data").Cells(25, i + 2).Value = Application.WorksheetFunction.Quartile(dataArray(), 3)
Worksheets("Graph Data").Cells(27, i + 2).Value = Application.WorksheetFunction.Max(dataArray())
Worksheets("Graph Data").Cells(21, i + 2).Value = Application.WorksheetFunction.Max(Worksheets("Graph Data").Cells(20, i + 2).Value,
Worksheets("Graph Data").Cells(22, i + 2).Value - 1.5 * (Worksheets("Graph Data").Cells(25, i + 2).Value - Worksheets("Graph Data").Cells(22, i
+ 2).Value))
Worksheets("Graph Data").Cells(26, i + 2).Value = Application.WorksheetFunction.Min(Worksheets("Graph Data").Cells(27, i + 2).Value,
Worksheets("Graph Data").Cells(25, i + 2).Value + 1.5 * (Worksheets("Graph Data").Cells(25, i + 2).Value - Worksheets("Graph Data").Cells(22, i
+ 2).Value))

'1-VAR RUS
'Fill an Array with the 1-VAR RUS Coefficients
For j = 1 To scenarios
    dataArray(j) = Worksheets("Scenario Output").Cells(regressionOffset + 24, (j - 1) * 4 + 3).Value
Next j
'Evaluate max, min, etc. of the 1-VAR RUS coefficients
Worksheets("Graph Data").Cells(29, i + 2).Value = Application.WorksheetFunction.Min(dataArray())
Worksheets("Graph Data").Cells(31, i + 2).Value = Application.WorksheetFunction.Quartile(dataArray(), 1)
Worksheets("Graph Data").Cells(32, i + 2).Value = Application.WorksheetFunction.Median(dataArray())
Worksheets("Graph Data").Cells(33, i + 2).Value = Application.WorksheetFunction.Average(dataArray())
Worksheets("Graph Data").Cells(34, i + 2).Value = Application.WorksheetFunction.Quartile(dataArray(), 3)
Worksheets("Graph Data").Cells(36, i + 2).Value = Application.WorksheetFunction.Max(dataArray())
Worksheets("Graph Data").Cells(30, i + 2).Value = Application.WorksheetFunction.Max(Worksheets("Graph Data").Cells(29, i + 2).Value,
Worksheets("Graph Data").Cells(31, i + 2).Value - 1.5 * (Worksheets("Graph Data").Cells(34, i + 2).Value - Worksheets("Graph Data").Cells(31, i
+ 2).Value))
Worksheets("Graph Data").Cells(35, i + 2).Value = Application.WorksheetFunction.Min(Worksheets("Graph Data").Cells(36, i + 2).Value,
Worksheets("Graph Data").Cells(34, i + 2).Value + 1.5 * (Worksheets("Graph Data").Cells(34, i + 2).Value - Worksheets("Graph Data").Cells(31, i
+ 2).Value))

```

```

Next i
.....
'Filling the Regression Graphs Ends Here'
.....

.....
'Filling the Future Future Graphs Starts Here'
.....

'Print Left Side
For i = 1 To maxRegression
  Worksheets("Graph Data").Cells(i * 21 + 18, 1).Value = "MFI 2-Var " & i & "-Yr. REG"
  Worksheets("Graph Data").Cells(i * 21 + 25, 1).Value = "MFI SPY " & i & "-Yr. REG"
  Worksheets("Graph Data").Cells(i * 21 + 32, 1).Value = "MFI RUS " & i & "-Yr. REG"
Next i
For i = 1 To maxRegression * 3
  Worksheets("Graph Data").Cells(7 * i + 32, 2) = "Min"
  Worksheets("Graph Data").Cells(7 * i + 33, 2) = "1st Quartile"
  Worksheets("Graph Data").Cells(7 * i + 34, 2) = "Median"
  Worksheets("Graph Data").Cells(7 * i + 35, 2) = "Mean"
  Worksheets("Graph Data").Cells(7 * i + 36, 2) = "3rd Quartile"
  Worksheets("Graph Data").Cells(7 * i + 37, 2) = "Max"
Next i

'Print Periods on Top
If FFdailyDisplay = False Then
  Worksheets("Graph Data").Range("B38") = "Month"
Else
  Worksheets("Graph Data").Range("B38") = "Day"
End If
For i = 1 To FFprojDays
  Worksheets("Graph Data").Cells(38, i + 2) = i
Next i

'Fill the chart
Dim FFarray() As Double
Dim offset As Long
Dim offset2 As Long
ReDim FFarray(1 To (FFscenarios * scenarios)) As Double

'For each period in the future future...

'MFIB percentage differences
'Fill an Array with the MFIB percentage differences

'For each number of regression years...
For regression = 1 To maxRegression
  For Period = 1 To FFprojDays
    'MFI 2-Var Stuff
    For FPscenario = 1 To scenarios
      For FFscenario = 0 To FFscenarios - 1
        offset = projDays + 25 * maxRegression + 7 + Period + FFscenario * (FFprojDays + 2) + FFscenario * maxRegression * (FFprojDays + 2) +
          (FFprojDays + 2) + (regression - 1) * (FFprojDays + 2)
        offset2 = projDays + 25 * maxRegression + 7 + Period + FFscenario * (FFprojDays + 2) + FFscenario * maxRegression * (FFprojDays + 2)
        FFarray(scenarios * FFscenario + FPscenario) = Worksheets("Scenario Output").Cells(offset, (FPscenario - 1) * 4 + 2).Value /
          Worksheets("Scenario Output").Cells(offset2, (FPscenario - 1) * 4 + 2).Value - 1
      Next FFscenario
    Next FPscenario

    'Evaluate max, min, etc. of the MFI 2-Var percentage differences
    Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 18, Period + 2).Value =
      Application.WorksheetFunction.Min(FFarray())
    Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 19, Period + 2).Value =
      Application.WorksheetFunction.Quartile(FFarray(), 1)
  
```

```

Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 20, Period + 2).Value =
Application.WorksheetFunction.Median(FFarray())
Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 21, Period + 2).Value =
Application.WorksheetFunction.Average(FFarray())
Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 22, Period + 2).Value =
Application.WorksheetFunction.Quartile(FFarray(), 3)
Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 23, Period + 2).Value =
Application.WorksheetFunction.Max(FFarray())

'MFI SPY Stuff
For FPscenario = 1 To scenarios
  For FFscenario = 0 To FFscenarios - 1
    offset = projDays + 25 * maxRegression + 7 + Period + FFscenario * (FFprojDays + 2) + FFscenario * maxRegression * (FFprojDays + 2) +
(FfprojDays + 2) + (regression - 1) * (FFprojDays + 2)
    offset2 = projDays + 25 * maxRegression + 7 + Period + FFscenario * (FFprojDays + 2) + FFscenario * maxRegression * (FFprojDays + 2)
    FFarray(scenarios * FFscenario + FPscenario) = Worksheets("Scenario Output").Cells(offset, (FPscenario - 1) * 4 + 3).Value /
Worksheets("Scenario Output").Cells(offset2, (FPscenario - 1) * 4 + 2).Value - 1
  Next FFscenario
Next FPscenario

'Evaluate max, min, etc. of the MFI SPY percentage differences
Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 25, Period + 2).Value =
Application.WorksheetFunction.Min(FFarray())
Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 26, Period + 2).Value =
Application.WorksheetFunction.Quartile(FFarray(), 1)
Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 27, Period + 2).Value =
Application.WorksheetFunction.Median(FFarray())
Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 28, Period + 2).Value =
Application.WorksheetFunction.Average(FFarray())
Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 29, Period + 2).Value =
Application.WorksheetFunction.Quartile(FFarray(), 3)
Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 30, Period + 2).Value =
Application.WorksheetFunction.Max(FFarray())

'MFI RUS Stuff
For FPscenario = 1 To scenarios
  For FFscenario = 0 To FFscenarios - 1
    offset = projDays + 25 * maxRegression + 7 + Period + FFscenario * (FFprojDays + 2) + FFscenario * maxRegression * (FFprojDays + 2) +
(FfprojDays + 2) + (regression - 1) * (FFprojDays + 2)
    offset2 = projDays + 25 * maxRegression + 7 + Period + FFscenario * (FFprojDays + 2) + FFscenario * maxRegression * (FFprojDays + 2)
    FFarray(scenarios * FFscenario + FPscenario) = Worksheets("Scenario Output").Cells(offset, (FPscenario - 1) * 4 + 4).Value /
Worksheets("Scenario Output").Cells(offset2, (FPscenario - 1) * 4 + 2).Value - 1
  Next FFscenario
Next FPscenario

'Evaluate max, min, etc. of the MFI SPY percentage differences
Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 32, Period + 2).Value =
Application.WorksheetFunction.Min(FFarray())
Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 33, Period + 2).Value =
Application.WorksheetFunction.Quartile(FFarray(), 1)
Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 34, Period + 2).Value =
Application.WorksheetFunction.Median(FFarray())
Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 35, Period + 2).Value =
Application.WorksheetFunction.Average(FFarray())
Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 36, Period + 2).Value =
Application.WorksheetFunction.Quartile(FFarray(), 3)
Worksheets("Graph Data").Cells((maxRegression - regression + 1) * 21 + 37, Period + 2).Value =
Application.WorksheetFunction.Max(FFarray())

Next Period
Next regression

.....
'Filling the Future Future Graphs Ends Here'
.....

```

.....

'Graph Stuff Ends Here'

.....

'Data Output

Run (OutputTab(scenarios, projYears, projDays, FPcalculate, FPdailyDisplay, FPprintType, sigma, mu, startvalue, correlation, alpha, SPYalpha, mfstartvalue, russigma, rusmu, russtartvalue, a11, a21, a22, maxRegression, regressDaily, regressLN, FFscenarios, FFprojYears, FFprojDays, FFcalculate, FFdailyDisplay, FFprintType))

'Status Update

Worksheets("Scenario Output").Range("A1").Value = ""

'End Time

Dim endtime As Date

endtime = Now

Worksheets("Output").Range("C3").Value = endtime

End Sub

---

Private Function OutputTab(scenarios As Long, projYears As Long, projDays As Long, FPcalculate As Boolean, FPdailyDisplay As Boolean, FPprintType As Boolean, sigma As Double, mu As Double, startvalue As Double, correlation As Double, alpha As Double, SPYalpha As Double, mfstartvalue As Double, russigma As Double, rusmu As Double, russtartvalue As Double, a11 As Double, a21 As Double, a22 As Double, maxRegression As Long, regressDaily As Boolean, regressLN As Boolean, FFscenarios As Long, FFprojYears As Long, FFprojDays As Long, FFcalculate As Boolean, FFdailyDisplay As Boolean, FFprintType As Boolean)

Worksheets("Output").Range("I1:I65536").Value = ""

Worksheets("Output").Range("B7:F7").Value = ""

Worksheets("Output").Range("B11:E11").Value = ""

Worksheets("Output").Range("B15:D15").Value = ""

Worksheets("Output").Range("B19:D19").Value = ""

Worksheets("Output").Range("B22:C23").Value = ""

Worksheets("Output").Range("B27:D27").Value = ""

Worksheets("Output").Range("B31:F31").Value = ""

'Reprint the Input tab in the Output tab

Worksheets("Output").Range("B7").Value = scenarios

Worksheets("Output").Range("C7").Value = projYears

If FPcalculate = True Then

    Worksheets("Output").Range("D7").Value = "Daily"

Else

    Worksheets("Output").Range("D7").Value = "Monthly"

End If

If FPdailyDisplay = True Then

    Worksheets("Output").Range("E7").Value = "Daily"

Else

    Worksheets("Output").Range("E7").Value = "Monthly"

End If

If FPprintType = True Then

    Worksheets("Output").Range("F7").Value = "LN Return"

Else

    Worksheets("Output").Range("F7").Value = "Values"

End If

Worksheets("Output").Range("B11").Value = sigma

Worksheets("Output").Range("C11").Value = mu

Worksheets("Output").Range("D11").Value = startvalue

Worksheets("Output").Range("E11").Value = correlation

Worksheets("Output").Range("B15").Value = alpha

Worksheets("Output").Range("C15").Value = SPYalpha

Worksheets("Output").Range("D15").Value = mfstartvalue

Worksheets("Output").Range("B19").Value = russigma

Worksheets("Output").Range("C19").Value = rusmu

Worksheets("Output").Range("D19").Value = russtartvalue

```

Worksheets("Output").Range("B22").Value = a11
Worksheets("Output").Range("B23").Value = a21
Worksheets("Output").Range("C22").Value = 0
Worksheets("Output").Range("C23").Value = a22

Worksheets("Output").Range("B27").Value = maxRegression
If regressDaily = True Then
    Worksheets("Output").Range("C27").Value = "Daily"
Else
    Worksheets("Output").Range("C27").Value = "Monthly"
End If
If regressLN = True Then
    Worksheets("Output").Range("D27").Value = "LN Return"
Else
    Worksheets("Output").Range("D27").Value = "Values"
End If

Worksheets("Output").Range("B31").Value = FFscenarios
Worksheets("Output").Range("C31").Value = FFprojYears
If FFcalculate = True Then
    Worksheets("Output").Range("D31").Value = "Daily"
Else
    Worksheets("Output").Range("D31").Value = "Monthly"
End If
If FFdailyDisplay = True Then
    Worksheets("Output").Range("E31").Value = "Daily"
Else
    Worksheets("Output").Range("E31").Value = "Monthly"
End If
If FFprintType = True Then
    Worksheets("Output").Range("F31").Value = "LN Return"
Else
    Worksheets("Output").Range("F31").Value = "Values"
End If

'Analyze Data
Dim holdingArray() As Double
Dim holdingArray2() As Double
'If FP is outputted as Values
If FPprintType = False Then

    For i = 1 To scenarios
        Worksheets("Output").Cells(1, 8 + i).Value = "Scenario " & i
        Worksheets("Output").Cells(2, 8 + i).Value = (Worksheets("Scenario Output").Cells(projDays + 2, i * 4 - 1).Value / startvalue) ^ (1 /
projYears) - 1
        Worksheets("Output").Cells(3, 8 + i).Value = (Worksheets("Scenario Output").Cells(projDays + 2, i * 4 - 2).Value / mfstartvalue) ^ (1 /
projYears) - 1
        Worksheets("Output").Cells(4, 8 + i).Value = (Worksheets("Scenario Output").Cells(projDays + 2, i * 4).Value / russtartvalue) ^ (1 /
projYears) - 1
        'Tracking Error (Realized alpha)
        ReDim holdingArray(1 To projDays) As Double
        holdingArray(1) = (Worksheets("Scenario Output").Cells(3, i * 4 - 1).Value / startvalue - 1) - (Worksheets("Scenario Output").Cells(3, i * 4 -
2).Value / mfstartvalue - 1)
        For j = 2 To projDays
            holdingArray(j) = (Worksheets("Scenario Output").Cells(2 + j, i * 4 - 1).Value / Worksheets("Scenario Output").Cells(1 + j, i * 4 - 1).Value -
1) - (Worksheets("Scenario Output").Cells(2 + j, i * 4 - 2).Value / Worksheets("Scenario Output").Cells(1 + j, i * 4 - 2).Value - 1)
        Next j
        Worksheets("Output").Cells(5, 8 + i).Value = Application.WorksheetFunction.stdev(holdingArray()) * Sqr(projDays / projYears)
        'Realized Correlation
        ReDim holdingArray2(1 To projDays) As Double
        holdingArray(1) = Worksheets("Scenario Output").Cells(3, i * 4 - 1).Value / startvalue - 1
        holdingArray2(1) = Worksheets("Scenario Output").Cells(3, i * 4).Value / russtartvalue - 1
        For j = 2 To projDays
            holdingArray(j) = Worksheets("Scenario Output").Cells(2 + j, i * 4 - 1).Value / Worksheets("Scenario Output").Cells(1 + j, i * 4 - 1).Value - 1
            holdingArray2(j) = Worksheets("Scenario Output").Cells(2 + j, i * 4).Value / Worksheets("Scenario Output").Cells(1 + j, i * 4).Value - 1
        Next j
    Next i

```

```

Worksheets("Output").Cells(6, 8 + i).Value = Application.WorksheetFunction.Correl(holdingArray(), holdingArray2())
Next i

'If FP is outputted as LN
Else

For i = 1 To scenarios
Worksheets("Output").Cells(1, 8 + i).Value = "Scenario " & i
'This formula is wrong Worksheets("Output").Cells(2, 8 + i).Value = Application.WorksheetFunction.Sum(Worksheets("Scenario
Output").Range(Cells(3, i * 4 - 1), Cells(projDays + 2, i * 4 - 1)).Value ^ (1 / projYears) - 1
'This formula is wrong Worksheets("Output").Cells(3, 8 + i).Value = Application.WorksheetFunction.Sum(Worksheets("Scenario
Output").Range(Cells(3, i * 4 - 2), Cells(projDays + 2, i * 4 - 2)).Value ^ (1 / projYears) - 1
'This formula is wrong Worksheets("Output").Cells(4, 8 + i).Value = Application.WorksheetFunction.Sum(Worksheets("Scenario
Output").Range(Cells(3, i * 4), Cells(projDays + 2, i * 4)).Value ^ (1 / projYears) - 1
'Tracking Error (Realized alpha)
ReDim holdingArray(1 To projDays) As Double
For j = 1 To projDays
holdingArray(j) = Worksheets("Scenario Output").Cells(2 + j, i * 4 - 1).Value - Worksheets("Scenario Output").Cells(2 + j, i * 4 - 2).Value
Next j
Worksheets("Output").Cells(5, 8 + i).Value = Application.WorksheetFunction.stdev(holdingArray()) * Sqr(projDays / projYears)
'Realized Correlation
ReDim holdingArray2(1 To projDays) As Double
For j = 1 To projDays
holdingArray(j) = Worksheets("Scenario Output").Cells(2 + j, i * 4 - 1).Value
holdingArray2(j) = Worksheets("Scenario Output").Cells(2 + j, i * 4).Value
Next j
Worksheets("Output").Cells(6, 8 + i).Value = Application.WorksheetFunction.Correl(holdingArray(), holdingArray2())
Next i

End If

'Average/Standard Deviation
Worksheets("Output").Cells(1, 10 + scenarios).Value = "Average"
Worksheets("Output").Cells(1, 11 + scenarios).Value = "StDev"
ReDim holdingArray(1 To scenarios)
For i = 1 To 5
For j = 1 To scenarios
holdingArray(j) = Worksheets("Output").Cells(i + 1, 8 + j).Value
Next j
Worksheets("Output").Cells(i + 1, 10 + scenarios).Value = Application.WorksheetFunction.Average(holdingArray())
Worksheets("Output").Cells(i + 1, 11 + scenarios).Value = Application.WorksheetFunction.stdev(holdingArray())
Next i

'Analyze Regressions
For i = 1 To maxRegression
Worksheets("Output").Cells(8, 8 + i).Value = (maxRegression - i + 1) & "-year Regression"

'2-Var Intercept Avg.
ReDim holdingArray(1 To scenarios) As Double
For j = 1 To scenarios
holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 9 + (i - 1) * 25, j * 4).Value
Next j
Worksheets("Output").Cells(9, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

'2-Var Intercept Avg. Error
For j = 1 To scenarios
holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 10 + (i - 1) * 25, j * 4).Value
Next j
Worksheets("Output").Cells(10, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

'2-Var SPY Coef. Avg.
For j = 1 To scenarios
holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 9 + (i - 1) * 25, j * 4 - 1).Value
Next j
Worksheets("Output").Cells(11, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

```

```

'2-Var SPY Coef. Avg. Error
For j = 1 To scenarios
    holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 10 + (i - 1) * 25, j * 4 - 1).Value
Next j
Worksheets("Output").Cells(12, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

'2-Var RUS Coef. Avg.
For j = 1 To scenarios
    holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 9 + (i - 1) * 25, j * 4 - 2).Value
Next j
Worksheets("Output").Cells(13, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

'2-Var RUS Coef. Avg. Error
For j = 1 To scenarios
    holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 10 + (i - 1) * 25, j * 4 - 2).Value
Next j
Worksheets("Output").Cells(14, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

'2-Var R Squared
For j = 1 To scenarios
    holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 11 + (i - 1) * 25, j * 4 - 2).Value
Next j
Worksheets("Output").Cells(15, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

'1-Var SPY Intercept Avg.
For j = 1 To scenarios
    holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 17 + (i - 1) * 25, j * 4 - 1).Value
Next j
Worksheets("Output").Cells(16, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

'1-Var SPY Intercept Avg. Error
For j = 1 To scenarios
    holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 18 + (i - 1) * 25, j * 4 - 1).Value
Next j
Worksheets("Output").Cells(17, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

'1-Var SPY Coef. Avg.
For j = 1 To scenarios
    holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 17 + (i - 1) * 25, j * 4 - 2).Value
Next j
Worksheets("Output").Cells(18, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

'1-Var SPY Coef. Avg. Error
For j = 1 To scenarios
    holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 18 + (i - 1) * 25, j * 4 - 2).Value
Next j
Worksheets("Output").Cells(19, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

'1-Var SPY R Squared
For j = 1 To scenarios
    holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 19 + (i - 1) * 25, j * 4 - 2).Value
Next j
Worksheets("Output").Cells(20, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

'1-Var RUS Intercept Avg.
For j = 1 To scenarios
    holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 25 + (i - 1) * 25, j * 4 - 1).Value
Next j
Worksheets("Output").Cells(21, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

'1-Var RUS Intercept Avg. Error
For j = 1 To scenarios
    holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 26 + (i - 1) * 25, j * 4 - 1).Value
Next j
Worksheets("Output").Cells(22, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

```



```

'1-Var RUS Coef. Avg.
For j = 1 To scenarios
    holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 25 + (i - 1) * 25, j * 4 - 2).Value
Next j
Worksheets("Output").Cells(23, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

'1-Var RUS Coef. Avg. Error
For j = 1 To scenarios
    holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 26 + (i - 1) * 25, j * 4 - 2).Value
Next j
Worksheets("Output").Cells(24, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

'1-Var RUS R Squared
For j = 1 To scenarios
    holdingArray(j) = Worksheets("Scenario Output").Cells(projDays + 27 + (i - 1) * 25, j * 4 - 2).Value
Next j
Worksheets("Output").Cells(25, 8 + i).Value = Application.WorksheetFunction.Average(holdingArray())

Next i

'Average/Standard Deviation
Worksheets("Output").Cells(8, 10 + maxRegression).Value = "Average"
Worksheets("Output").Cells(8, 11 + maxRegression).Value = "StDev"
ReDim holdingArray(1 To maxRegression)
For i = 8 To 24
    For j = 1 To maxRegression
        holdingArray(j) = Worksheets("Output").Cells(i + 1, 8 + j).Value
    Next j
    Worksheets("Output").Cells(i + 1, 10 + maxRegression).Value = Application.WorksheetFunction.Average(holdingArray())
    Worksheets("Output").Cells(i + 1, 11 + maxRegression).Value = Application.WorksheetFunction.stdev(holdingArray())
Next i

End Function

```