

# Simulating Tracking Error in Variable Annuities



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12/17/2010

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# Outline



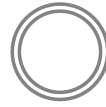
Project Overview

2010 Monte-Carlo Simulator

Analysis and Graphics

Conclusions and Recommendations

# Project Overview

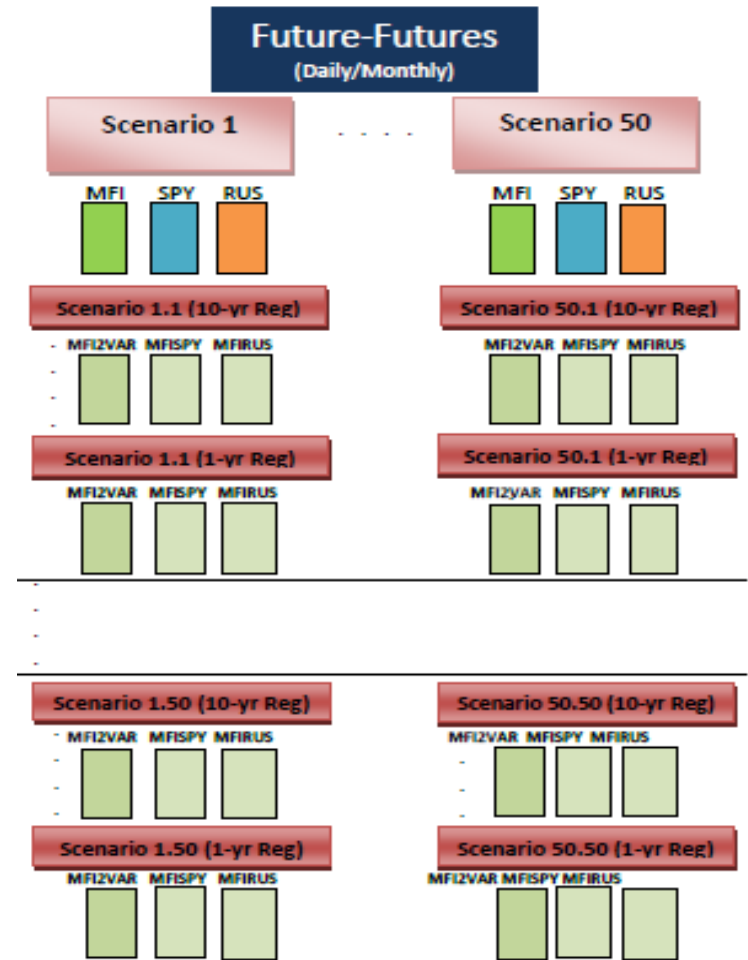
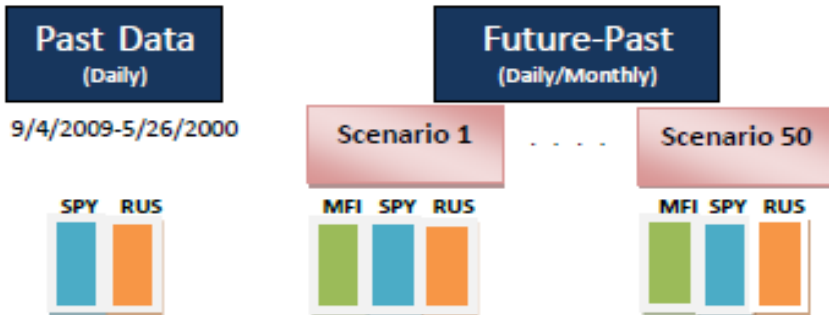


- Mutual Funds as part of variable annuities
- Tested impact of different techniques on tracking error



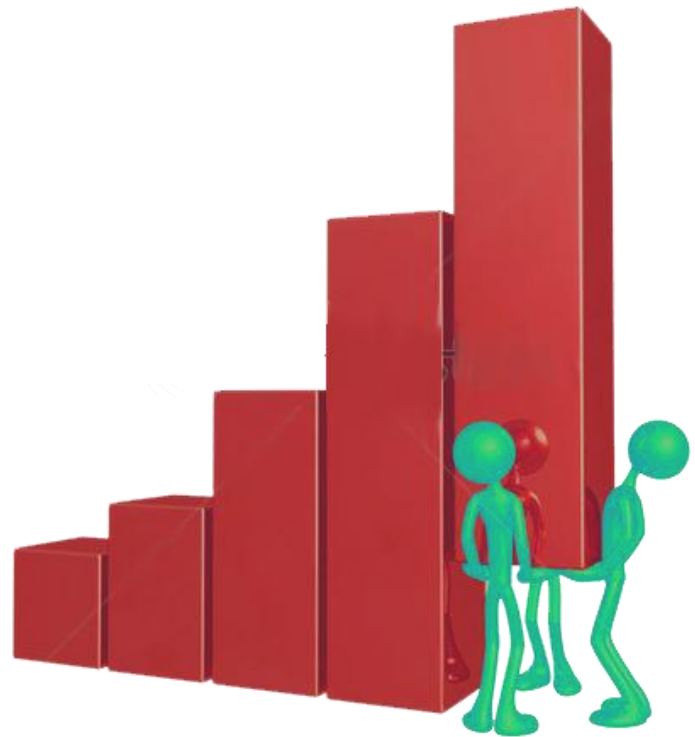
# Overview of Monte-Carlo Simulator

- '09 Monte-Carlo Simulator
  - S&P500 (SPY) and Russell 2000 (RUS)
  - Simulated Mutual Fund (MFI)
  - Future-Past & Future-Future

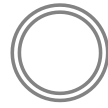


# 2010 Monte-Carlo Simulator

- Model specifications
  - Perfect World
  - Regression
  - Distribution of Alpha
  - Regime Switching



# Perfect World



- **Tracking Error**
  - Comparison of MFI to SPY in future past
- **Follows SPY directly**
  - With the integration of a manager's alpha



# Regression



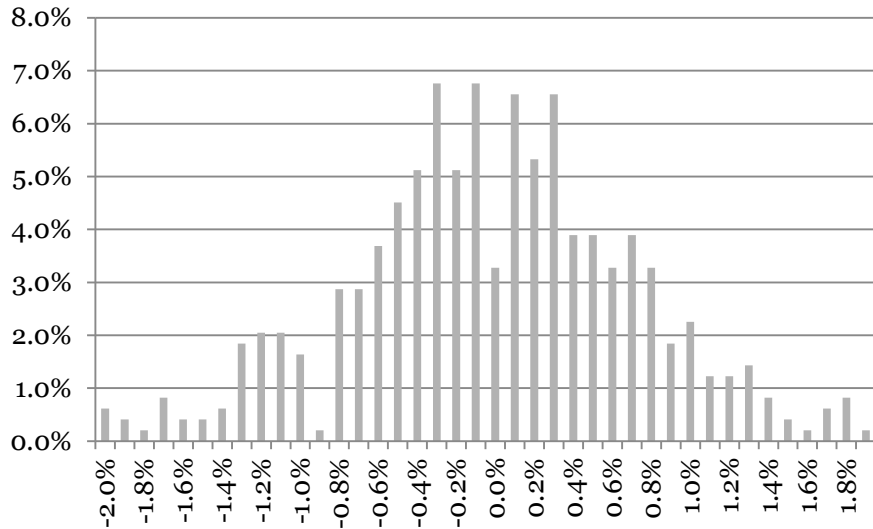
- After the Future Past is Run, the Three Regressions are Built
  - SPY
  - RUS
  - SPY & RUS (2-Var)
- Tracking Error
  - Comparison of regression to MFI in future-future
- Change in correlation



# ETF vs. Mutual Funds

- Exchange Traded Funds (ETFs)

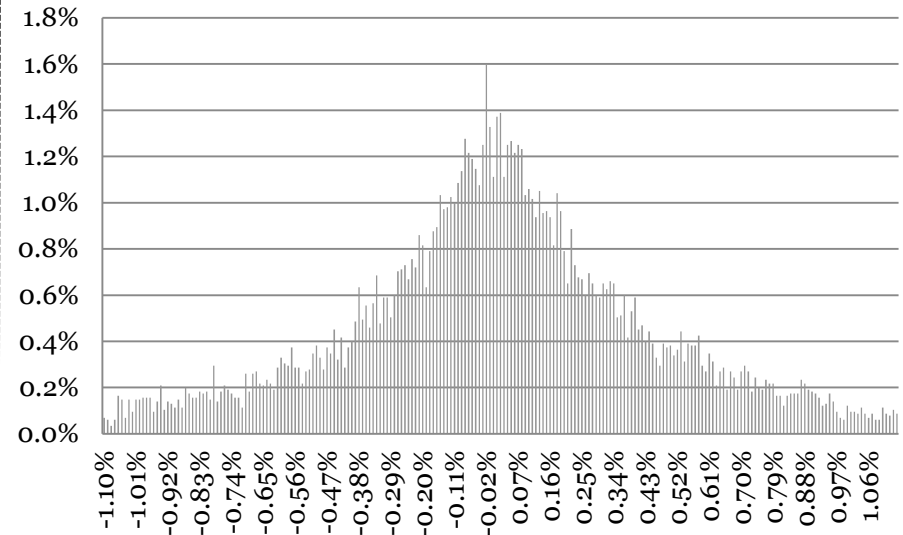
## Daily Manager's Alpha



Expected Value -0.013%  
Standard Deviation 0.73%

- Mutual Funds

## Daily Manager's Alpha



Expected Value 0.014%  
Standard Deviation 0.43%

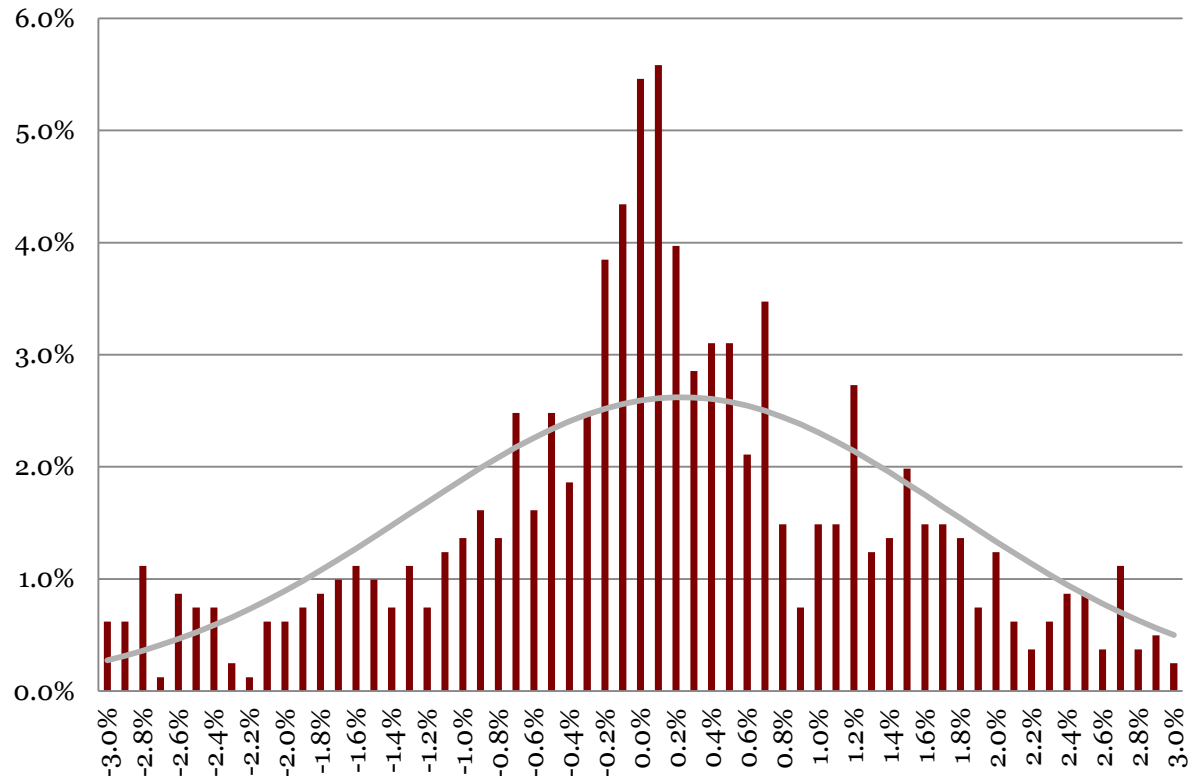


# Distribution of Alpha

	Skewness	Kurtosis
Normal Dist.	0	3
Daily ETF Alpha	-0.09	3.05
Daily MF Alpha	-0.03	3.17
Monthly MF Alpha	0.01	3.21

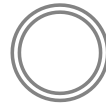


## Manager's Alpha Using Monthly Mutual Fund Returns



	Outliers Removed	Including Outliers
Expected Value	0.18%	0.15%
Standard Deviation	1.52%	2.25%

# Regime Switching

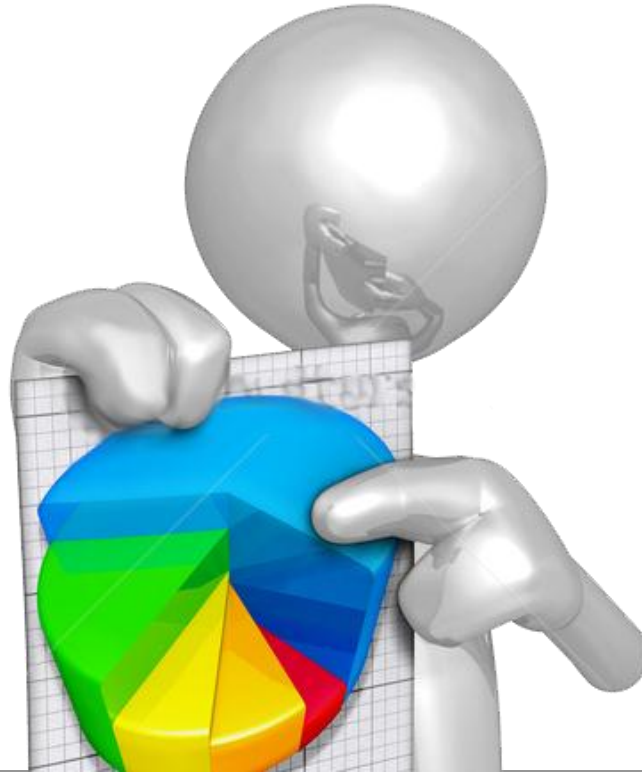


- Mary Hardy's two-regime model
- Monthly Parameter Estimation
  - Used Historical Data for SPY and RUS
- Forcing Probability Matrix

	<b>SPY Monthly Parameters</b>	<b>RUS Monthly Parameters</b>
$p_{1,2}$	5.24%	5.24%
$p_{2,1}$	10.75%	10.75%
$\mu_1$	1.31%	1.22%
$\sigma_1$	2.82%	4.56%
$\mu_2$	-1.17%	-2.05%
$\sigma_2$	7.19%	10.72%



# Results



# Waterfall Graph Order 1

Perfect World

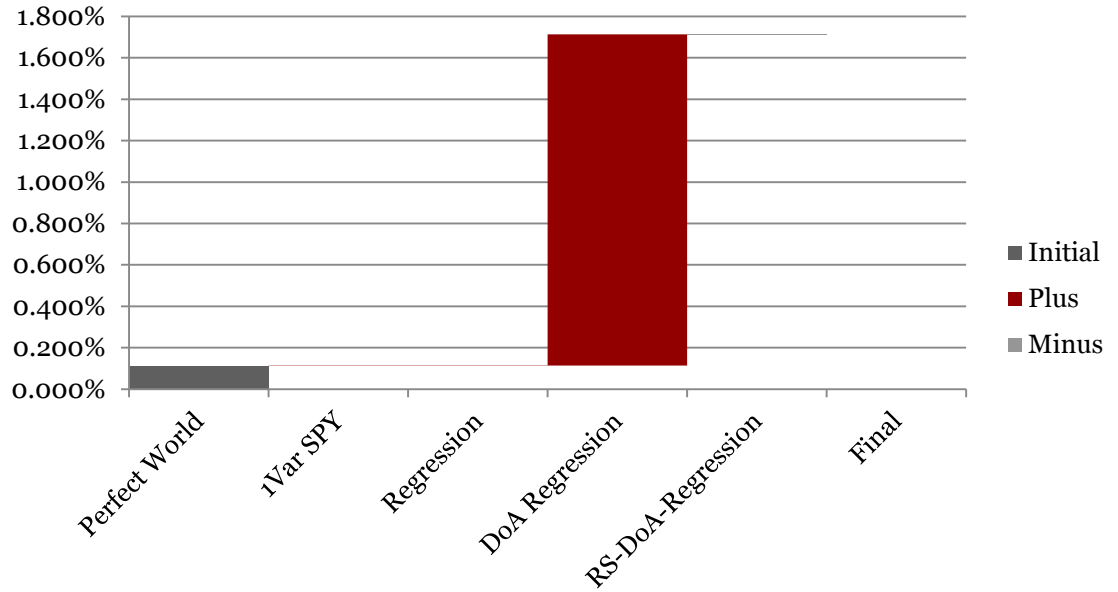
Regression

Distribution of Alpha

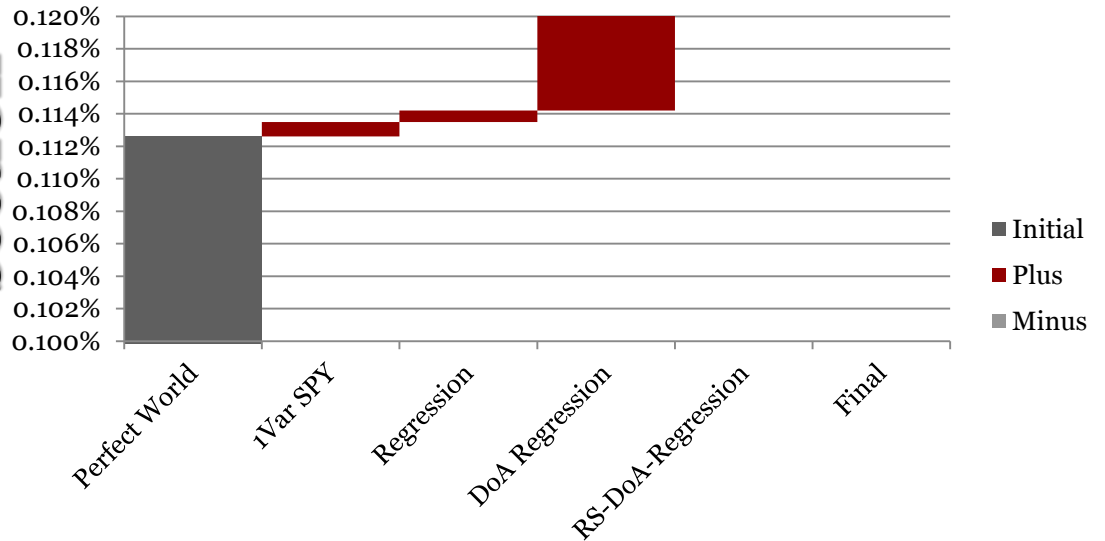
Regime Switching



Full Graph



Magnified section



# Waterfall Graph Order 2

Perfect World

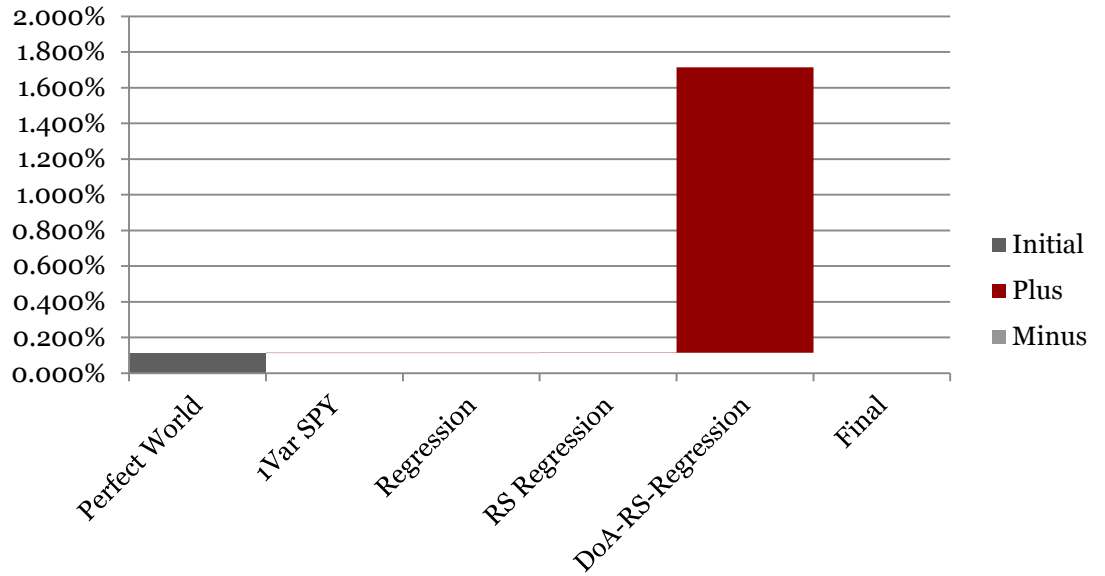
Regression

Regime Switching

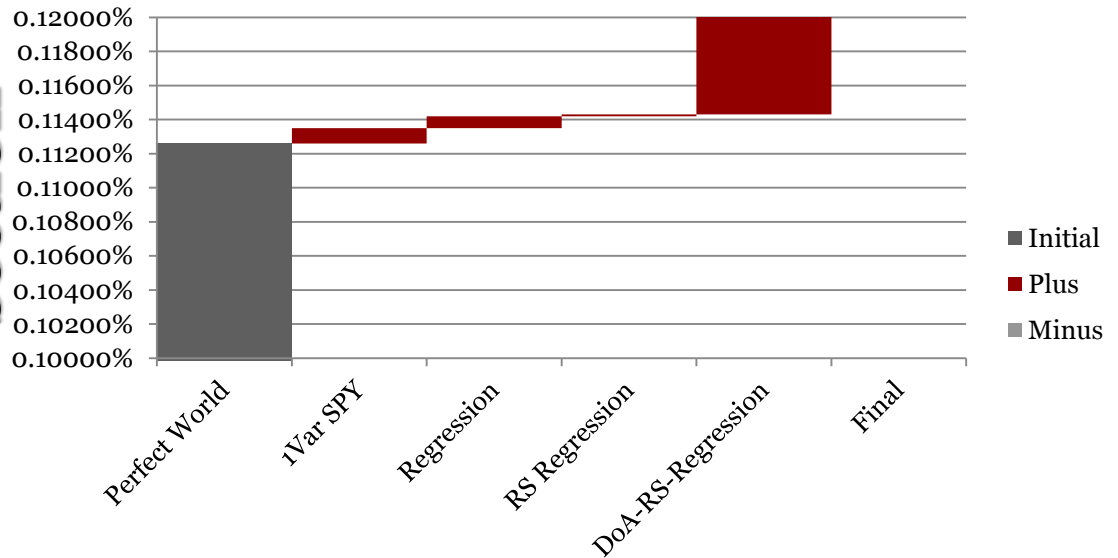
Distribution of Alpha



Full Graph



Magnified section



# Waterfall Graph Order 3

Perfect World

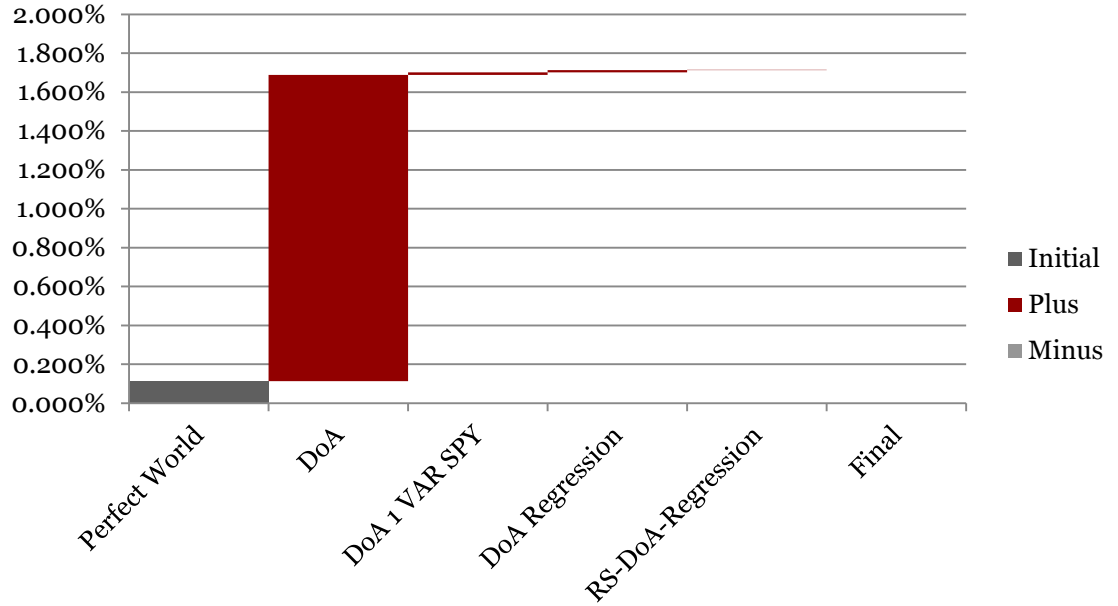
Distribution of Alpha

Regression

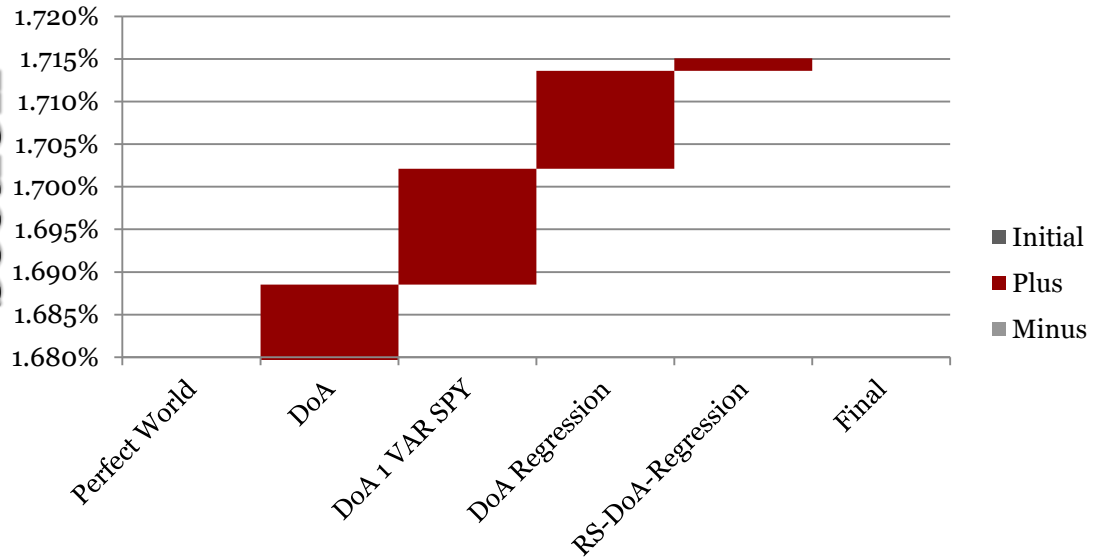
Regime Switching



Full Graph



Magnified section



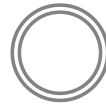
# Recommendations



Our Suggestions for the Future Project Teams



# Suggestions for Next Year



- Find a better method to develop the distribution of Alpha
- Re-Evaluate the tool between FP and FF





# Thank You



**JON ABRAHAM, ADVISOR  
GUILLAUME BRIERE-GIROUX, LIAISON  
GRANT FREDRICKS  
AND  
TOWERS WATSON**



# Questions?



## Sources

Hull, J. C. (2006). *Options, Futures, and Other Derivatives*. Upper Saddle River: Pearson Education, Inc.

Hardy, M. R. (2001). A Regime-Switching Model of Long-Term Stock Returns. *North American Actuarial Journal*/2001/april/naaj0104\_4.pdf, 41-53

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