Interactions of Per-Occurrence and Aggregate Deductibles

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Abstract

Our research investigates the relationship between per-occurrence and aggregate deductibles, with a focus on devising a simplified methodology for estimating loss elimination percentages, particularly pertaining to Table M. Per-occurrence deductibles require the insured to pay for a portion of each loss event. Aggregate deductibles accumulate losses over a defined period, upon reaching a certain threshold the deductible amount is subtracted from the aggregate losses and the insurance coverage begins. Therefore, an aggregate deductible represents the maximum deductible amount that a policyholder must pay for claims over the specified period.

Table M is a widely used tool for determining the extent of loss mitigation through imposing per-occurrence and aggregate limits. Table M is built using entry level ratios and associated aggregate losses categorized by policy size or limit. Typically, this table is generated by modeling aggregate loss distributions using parameterized functions, which offers flexibility for simulations. Actuaries establish Table M by fitting claims data to similarly sized risk groups, ensuring accuracy in estimating loss elimination. However, limitations may arise when data is insufficient for creating credible groups, emphasizing the importance of analyzing entry level ratios to mitigate potential errors. Our primary goal is to propose an improved methodology for Table M, simplifying calculations, therefore facilitating informed decision-making for insurers. We utilized Excel to simulate various distributions while applying different per-occurrence and aggregate deductibles, allowing for comprehensive analysis of their effects on loss reduction percentages. Through these simulations, we derived enhanced methodologies for calculating and presenting Table M charges in a more comprehensive manner.

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Executive Summary

Our study focuses on the complex interplay between per-occurrence and aggregate deductibles, with the goal of deriving a refined methodology for calculating and presenting Table M in a more comprehensive manner. Per-occurrence deductibles require policyholders to cover a portion of each loss event, while aggregate deductibles accumulate losses over a defined period, triggering deductible subtraction once a threshold is reached. Table M is a critical tool for assessing loss mitigation by setting per-occurrence and aggregate limits. It's constructed based on entry level ratios and corresponding aggregate losses, providing insights into risk based on policy size or limit. While actuaries typically construct Table M using parameterized functions to model aggregate loss distributions, limitations can arise from insufficient data for creating credible risk groups, emphasizing the importance of analyzing entry level ratios to mitigate potential inaccuracies.

Our primary objective is to propose a refined methodology for calculating and presenting Table M, simplifying calculations and presentation to support informed decision-making among insurers. Using Excel, we explored techniques for pricing policies that incorporate both occurrence and aggregate deductibles through simulations involving various distributions and different deductible structures. We comprehensively analyzed the impact of these deductible combinations on loss reduction percentages. These simulations allowed us to derive refined methodologies for computing and presenting Table M charges in a more digestible format.

1.0 Introduction

Our study delves into the intricate relationship between per-occurrence and aggregate deductibles, aiming to streamline the process of estimating loss elimination percentages, particularly concerning Table M. Per-occurrence deductibles necessitate policyholders to cover a fraction of each loss event, while aggregate deductibles accumulate losses over a specified period, triggering deductible subtraction once a threshold is met. This denotes the maximum deductible amount a policyholder must bear within the defined period.

Table M serves as a pivotal tool in gauging loss mitigation through the imposition of per-occurrence and aggregate limits. Constructed upon entry level ratios and corresponding aggregate losses, it's crucial for assessing risk based on policy size or limit. Typically, actuaries craft Table M by modeling aggregate loss distributions using parameterized functions, ensuring flexibility for simulations. However, limitations may arise from insufficient data for creating credible risk groups, underscoring the necessity of analyzing entry level ratios to mitigate potential inaccuracies.

Our primary goal is to propose a refined methodology for calculating and presenting Table M, simplifying calculations to facilitate informed decision-making among insurers. Using Excel, we surveyed techniques for pricing policies incorporating both occurrence and aggregate deductibles. To do this we simulated various distributions while applying diverse per-occurrence and aggregate deductibles, enabling a comprehensive analysis of their impact on loss reduction percentages. Through these simulations, we derived refined methodologies for computing and presenting Table M charges in a more comprehensive format.

2.0 Background

2.1 Insurance

Actuaries do not typically use continuous claims distributions in their purest form, where claims are unrestricted. Insurance companies almost always set limitations on claims under a given policy, most commonly an occurrence deductible. Deductibles are a policy adjustment often used to decrease the policy claim count by disregarding small claims that fall below the deductible threshold. A per-occurrence deductible denotes a fixed amount that the insured party must pay for each individual incident or claim covered by their insurance policy before the insurer starts contributing. From the perspective of the insurer, X represents the unlimited claim-size random variable. Incorporating a per-occurrence deductible d, we have payments per loss, Y such that:

$$Y = \begin{cases} 0, & 0 \leq X < d \\ X - d, & d < X \end{cases}$$

Any claims less than or equal to d are paid by the insured, thus the insurer pays \$0. For any claims greater than d, the maximum amount the insured pays is d, while the insurer pays the remainder of the claim, $X - d$. Aggregate loss is the sum of these claims incurred by the insurance company for the entire policy. These losses help the insurer to assess the risk of and set premiums for the policyholder and others of similar classification. There are also aggregate deductibles which accumulate losses over the policy period and set a limit to the total deductible a policyholder would be required to cover on claims throughout the entire policy term. Once this threshold is met, the insurance coverage begins, and the insured is not obligated to cover any additional losses. We provide an example below to illustrate the fundamental concepts using a simple discrete model, which incorporates a policy per-occurrence and aggregate deductibles.

Example 2.1.1: Interaction of Per-Occurrence and Aggregate Deductibles

Assume that $n = 0$, 1 or 2 and there are 3 potential claims sizes: {100, 200, 300}. The associated probability for N and X are provided in the following table.

Claim Count N	
# of Claims	$f_n(n)$
0	0.60
1	0.30
2	0.10

E[N] = 0.50, Var[N] = 0.45

Claim Severity X	
Size x	$f_x(x)$
100	0.40
200	0.50
300	0.10

E[X] = 170, Var[X] = 4,100

Thus, we have 7 distinct total loss amounts: {0, 100, 200, 300, 400, 500, 600}. Provided that:

	n = 0	n = 1	n = 2
Amount	0	100	100 + 100, 100 + 200, 100 + 300
		200	200 + 100, 200 + 200, 200 + 300
		300	300 + 100, 300 + 200, 300 + 300
Pr{S = s | N = n}	1.00	0.40	0.16, 0.20, 0.04
		0.50	0.20, 0.25, 0.05
		0.10	0.04, 0.05, 0.01

Now we will impose a per-occurrence deductible of $150, so for each individual claim the most the insured will pay is $150, and we have that the Pr{X > 150} = 0.60. Thus, we now have 6 distinct total loss amounts: {0, 100, 150, 200, 250, 300}.

	n = 0	n = 1	n = 2
Amount	0	100 150	100 + 100, 100 + 150 150 + 100, 150 + 150
Pr{S = s | N = n}	1.00	0.40 0.60	0.16, 0.24 0.24, 0.36

Now assume there is an aggregate deductible of $250 in addition to the $150 per-occurrence limit, so the insured will pay a maximum of $150 for each claim and the total amount the insured will pay for the entire policy is capped at $250. Thus, we now only have 4 distinct total loss amounts: {0, 100, 150, 200, 250}.

	n = 0	n = 1	n = 2
Amount	0	100 150	100 + 100, 100 + 150 150 + 100
Pr{S = s | N = n}	1.00	0.40 0.60	0.40, 0.60 0.60

2.2 Loss Models

Cumulative distribution functions, or CDFs, are defined for a random variable X such that $F_X(x)$ = Probability (X ≤ *x*). Characteristics of the CDF include:

1. $0 \leq F_X(x) \leq 1$
2. $F_X(x)$ is non-decreasing, so $F_X(n) \leq F_X(n+1)$
3. $F_X(x)$ is right continuous
4. $\lim_{x \to -\infty} F_X(x) = 0 \qquad \lim_{x \to \infty} F_X(x) = 1$

In addition, there is also a survival function $S_X(x)$, which is the complement of $F_X(x)$.

$$S_X(x) = 1 - F_X(x)$$

So $S_X(x) + F_X(x) = 1$, and the survival function has the following characteristics,

1. $0 \leq S_X(x) \leq 1$
2. $S_X(x)$is non-increasing, so $S_X(n) \geq S_X(n+1)$
3. $S_X(x)$ is right continuous
4. $\lim_{x \to -\infty} S_X(x) = 1 \qquad \lim_{x \to \infty} S_X(x) = 0$

f(x) is the derivative of F(x) so we have that,

$$f(x) = \frac{d}{dx}F(x) = \frac{d}{dx}\big(1 - S(x)\big) = -S'(x)$$

This can be used to find the probability that x exists between 2 numbers [a, b],

$$\int_a^b f(x) = F(b) - F(a) = S(a) - S(b) = Prob(x \in [a, b])$$

Example 2.2.1: Finding E[X] and Var[X]

x	p(x)
0	0.5
1	0.25
2	0.12
3	0.08
4	0.05

E(X) = (0 x 0.5) + (1 x 0.25) + (2 x 0.12) + (3 x 0.08) + (4 x 0.05)

= 0.93

x	x-μ
0	= 0 – 0.93 = -0.93
1	= 1 - 0.93 = 0.07
2	= 2 - 0.93 = 1.07
3	= 3 - 0.93 = 2.07
4	= 4 - 0.93 = 3.07

$\text{Var}[X] = 0.5 \cdot (-0.93)^2 + 0.25 \cdot (0.07)^2 + 0.12 \cdot (1.07)^2 + 0.08 \cdot (2.07)^2 + 0.05 \cdot (3.07)^2$

= 1.3851

Standard deviation = $\sqrt{Variance} = \sqrt{1.3851} = 1.1769$

Mean excess loss measures the average loss severity for amounts exceeding a certain value, in actuarial math this threshold is the per-occurrence deductible. It is the expectation of

how much over the deductible a claim will be. To find the limited excess loss for a random variable X, use the random variable Y given,

$$Y^P = X - d \text{ given X > d}$$

This excludes losses below the deductible, d, and therefore can be used to find the average of the checks/payments the insurer pays. Mean excess loss can be calculated such that,

$$E(Y^P) = E[\, X - d \,|X > d\,] = \; e_x(d)$$

$$e_x(d) = \frac{\int_d^{\infty}(x-d)f(x)dx}{1-F(d)} = \frac{E(X) - E(X \wedge d)}{1-F(d)} = average\ check\ size$$

Another method of imposing a per-occurrence deductibles is through left censoring and shifting which includes any loss or accident that the insurer did not pay. To find this consider the random variable Y such that,

$$Y^L = \begin{cases} 0\ if\ X \leq d \\ X - d\ if\ X > d \end{cases}$$

To find the expected value we have that,

$$E(Y^L) = E((X-d)_+) = \; e_x(d)(1 - F_X(d))$$

Y^P calculates payments per payment, while Y^L calculates payment per loss including payments of size zero. The main difference being that Y^P calculates loss based on individual payments made by the insurer for each claim. Y^L, on the other hand, utilizes an approach based on total payments made by the insurer for each risk or claim occurrence, including payments of size zero.

Example 2.2.2: Methods of Imposing Per-Occurrence Deductibles

Possible Losses (*X*)	Prob.	$X - 750$	$E((X-750)_+)$
100	0.4	0	0
500	0.2	0	0
1,000	0.2	250	250
2,500	0.1	1,750	1,750
10,000	0.1	9,250	9,250

The table above shows how to calculate Y^L, or payment per loss including payments of size zero, while applying a deductible, *d*, of 750. Now to find $E[\, X - 750 \,|X > 750\,]$ only include losses above 750, which changes the probability of each loss. There is now only 40% of the initial loss remaining, so the losses now have probability reflecting this:

Possible Losses (*X*)	$E[\, X - 750 \,|X > 750\,]$	Prob.
1,000	250	$= \frac{0.2}{0.4} = 0.5$
2,500	1,750	$= \frac{0.1}{0.4} = 0.25$
10,000	9,250	$= \frac{0.1}{0.4} = 0.25$

With the given information, we can now calculate the expected loss for each method.

E(X)	$\boldsymbol{Y^L = E((X-750)_+)}$	$\boldsymbol{Y^P = E[\, X - 750 \,|X > 750\,]}$
1,590	1,150	2,875

We can also calculate the expected loss of $Y^L$and Y^P by dividing total expected losses by the amount of probability remaining of the given loss for each method, resulting in:

$$E((X-750)_+) = \frac{1{,}150}{1.0} = 1{,}150$$

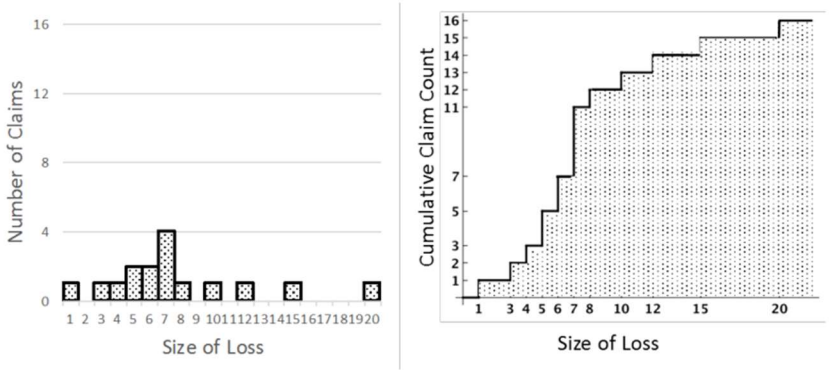
$$E[\,X-750\,|X>750\,] = \frac{1{,}150}{0.4} = 2{,}875$$

2.3 Aggregate Excess Loss Cost Estimation

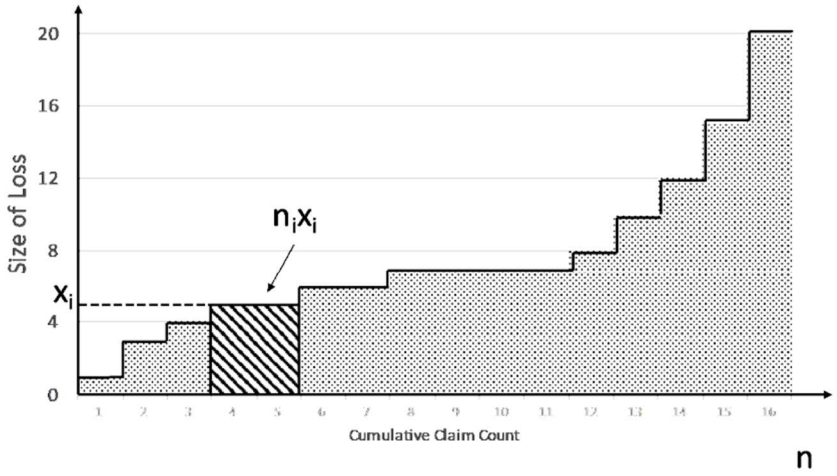
Calculating per-occurrence excess and aggregate excess loss can be complex, therefore it is beneficial to create visual interpretations, to illustrate how much each party will contribute toward the total cost, prior to moving to the intricate calculations. Yoong-Sin Lee describes these graphs in his paper *The Mathematics of Excess of Loss Coverages and Retrospective Rating — A Graphical Approach* earning them the name "Lee diagrams." The key characteristic of these Lee diagrams is that the size, or severity, is on the vertical axis, while the cumulative claim count, or its percent of the loss distribution, is on the horizontal axis. These Lee diagrams provide an illustration of abstract relations and can also make it easier to follow the mathematical procedures instead of diving straight into the complex algebraic concepts.

Example 2.3.1: Lee Diagrams

We will first look at a distribution of a set of losses incrementally and cumulatively, with the number of losses on the vertical axis and the size of the loss on the horizontal axis.



Now we will flip the axes and represent those same losses using a cumulative frequency curve constructed in the method described by Yoong-Sin Lee, where the vertical axis now represents loss size, and the horizontal axis now represents cumulative number of losses.



Consider the shadowed region corresponding to the interval (x_i, n_i) with the area $x_i n_i$. This region can be interpreted as the amount of loss being size x_i and the area below the cumulative frequency curve is the total loss. The cumulative number of losses can be found using,

$c_i = n_1 + \ldots + n_i$ where $i \leq k$. Thus, we can find the total amount of loss by summing all of the vertical strips:

Total loss = $x_1 n_1 + \ldots + x_k n_k$

This provides us with a new way of visualizing the cumulative frequency curve, by arranging losses in ascending order from left to right with each loss taking up 1 unit horizontally. Consider the situation where losses are represented by the randomly distributed variable X with cumulative density function, or cdf,

$$F(x) = Prob(X \leq x)$$

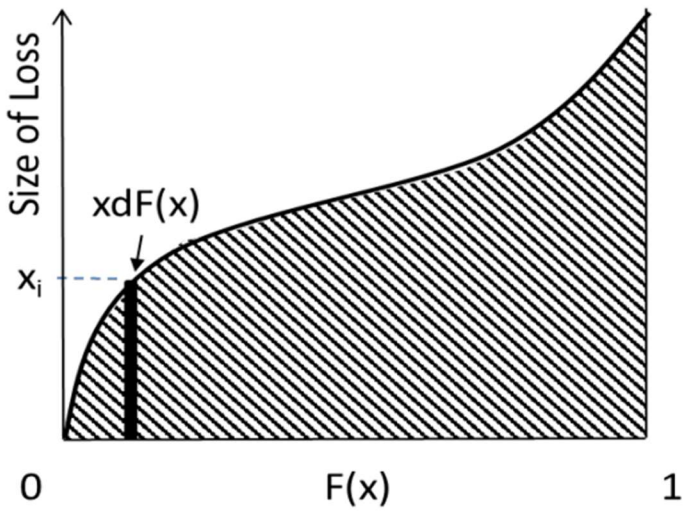
Graphing the continuous cdf, we can sum up each vertical strip with area

$$(x)(d)(F(x))$$

to obtain the expected value of X. E[X] is represented by the shaded region, or the area under the curve. This region is composed of each loss and the strip $(x)(d)(F(x))$ which can be interpreted as the contribution from losses of size between x and $(x) + (d \times x)$.

We can also adapt this diagram to illustrate both limits and deductibles, and how they impact our losses. Consider a policy with a policy occurrence limit of L, as depicted in Figure 2.3.3(a). A loss of size S_1 is below the limit L, so this loss is paid in full. On the other hand, a loss of size S_2 is above L, thus only the amount of loss up to size L is paid by the insurer. Again, to find the expected loss we can sum up the vertical strips. Once

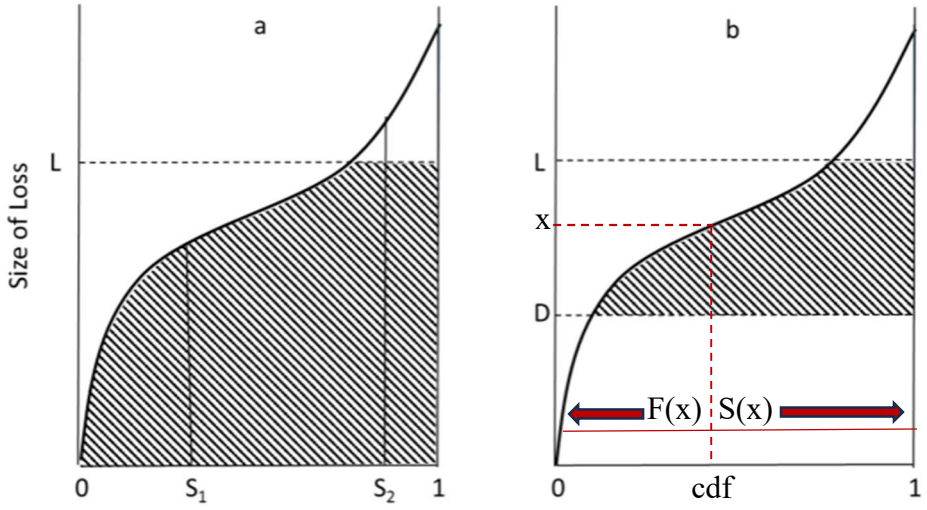
Figure 2.3.2: CDF Curve and Expectation



we reach the losses that are greater than the limit L, each loss now only contributes L to E[X]. This is depicted by the shaded area in Figure 2.3.3(a).

Now consider a policy with a policy occurrence limit L and a deductible D, as depicted in Figure 2.3.3(b). Each loss will now only pay the portion of the loss that is greater than D and less than L. Therefore, we can find the expected losses by subtracting D from all vertical strips and then summing those losses until we reach those that are greater than L. Losses that are greater than L will contribute L-D to E[X]. This is depicted by the shaded area in Figure 2.3.3(b).

Figure 2.3.3: Lee Diagram with Per-Occurrence and Aggregate Deductibles



This can also be approached using the survival function S(x) defined as,

$$S(x) = 1 - F(x)$$

We can also find E[X] using S(x) such that,

$$E[X] = \int_0^\infty S(x)dx$$

Previously we were finding the expected losses by grouping the losses vertically, or by the size of the loss. Using this method, we are grouping the losses horizontally, in other words we are grouping the loss amounts by layer. This method can often be more convenient than grouping the losses by size because you can easily consider any losses within a region by

adjusting the boundaries in the integral. For example, in Figure 2.3.3(b), we could calculate E[X] by limiting the integral to losses between D and L such that,

$$E[X] = \int_D^L S(x)dx \quad [1]$$

Using the previous method to calculate this yields the expression,

$$E[X] = \int_D^L xdF(x) + L \cdot S(L) - D \cdot S(D) \quad [2]$$

Although the form using the layer method (formula [1]) is simpler, it is difficult to integrate S(x), which leads to the form using the size method (formula [2]) being more commonly used in literature.

This leads us to Table M, which has been constructed using the layer method by establishing a distribution of entry level ratios and their associated aggregate losses sorted by size of policy or limit. Entry level ratios are used as a starting point for Table M because they are based on historical data and often incorporate important metrics such as loss, expense, or combined ratios. Table M is used to determine the percentage of loss eliminated by imposing a per-occurrence limit and an aggregate limit. In most cases, it is possible to model these aggregate loss distributions using parameterized functions. This allows for the parameters to be easily modified to fit specific scenarios, thus providing a versatile tool for simulations. This will typically be used when there are not significant similarities to policies, which would typically be used to establish an aggregate loss density function. The actuary must, therefore, fit the claims data to a collection of similarly sized risks using their general forms as a guideline to select the underlying frequency and severity distributions.

Once these are established, the insurer can generate Table M for the given aggregate loss distribution, using the layer method described above. Actuaries use a blend of empirical data and models; however, it is important to note that to establish credible groups there may not be

enough data and thus the insurer may not be able to split the data into small enough layers to accurately generate Table M values. To avoid errors caused by this it is best to look at entry level ratios, or loss as a ratio to its expected loss.

Example 2.3.4: Calculating Table M Charge

Given the following loss ratios for a set of 5 identical risks:

Risk	Loss Ratio	Losses
1	40%	=40%×$1,000,000 = $400,000
2	40%	=40%×$1,000,000 = $400,000
3	80%	=80%×$1,000,000 = $800,000
4	100%	=100%×$1,000,000 = $1,000,000
5	140%	=140%×$1,000,000 = $1,400,000

To calculate E(Loss) take the average of the losses,

E(loss) = ($400,000 + $400,000 + $800,000 + $1,000,000 + $1,400,000) / 5

=$800,000

To calculate loss ratio, find the average of the given loss ratios,

Loss Ratio = (40% + 40% + 80% + 100% + 140%) / 5

= 80%

Assume the sample loss ratio of 80% equals the expected loss ratio. Construct Table M showing the insurance charges for entry ratios from 0.00 to 2.00. We therefore have 9 entry ratios: 0.00, 0.25, 0.50, 0.75, 1.00, 1.25. 1.50, 1.75, and 2.00. To calculate our possible aggregate limits, multiply these entry ratios by the expected losses.

We know E(loss)=$800,000, so we have the following possible aggregate limits:

$0.00 \times \$800,000 = \$0$

$0.25 \times \$800,000 = \$200,000$

$0.50 \times \$800,000 = \$400,000$

$0.75 \times \$800,000 = \$600,000$

$1.00 \times \$800,000 = \$800,000$

$1.25 \times \$800,000 = \$1,000,000$

$1.50 \times \$800,000 = \$1,200,000$

$1.75 \times \$800,000 = \$1,400,000$

$2.00 \times \$800,000 = \$1,600,000$

Now, we must count the number of risks that fall within and above the possible losses, using the 5 losses provided in the original table: 400,000, 400,000, 800,000, 1,000,000, and 1,400,000.

Entry ratio	Possible Aggregate Limits	Number of Risks	Number of Risks Above
0.00	-	0	5
0.25	$200,000	0	5
0.50	$400,000	2	3
0.75	$600,000	2	3
1.00	$800,000	1	2
1.25	$1,000,000	1	1
1.50	$1,200,000	0	1
1.75	$1,400,000	1	0
2.00	$1,600,000	0	0

Now calculate the percent of losses that fall above the possible aggregate loss.

r = 0.00	5/5 = 100%
r = 0.25	5/5 = 100%
r = 0.50	3/5 = 60%
r = 0 .75	3/5 = 60%
r = 1.00	2/5 = 40%
r = 1.25	1/5 = 20%
r = 1.50	1/5 = 20%
r = 1.75	0/5 = 0%
r = 2.00	0/5 = 0%

The next step is to calculate the total dollars above the aggregate limit, by summing all losses that fall above it, subtracting the aggregate limit from the each, so we have that:

Aggregate Limits	$ Above the Aggregate Limit
-	= $400,000 + $400,00 + $800,000 + $1,000,000 + $1,400,000 - $0 = $4,000,000
$200,000	= ($400,000 - $200,000) + ($400,000 - $200,000) + ($800,000 - $200,000) + ($1,000,000 - $200,000) + ($1,400,000 - $200,000) = $3,000,000
$400,000	= ($800,000 - $400,000) + ($1,000,000- $400,000) + ($1,400,000 - $400,000) = $2,000,000
$600,000	= ($800,000 - $600,000) + ($1,000,000- $600,000) + ($1,400,000 - $600,000) = $1,400,000
$800,000	= ($1,000,000 - $800,000) + ($1,400,000 - $800,000) = $800,000
$1,000,000	= $1,400,000 - $1,000,000 = $400,000
$1,200,000	= $1,400,000 - $1,200,000 = $200,000
$1,400,000	= $0
$1,600,000	= $0

Now using everything calculated above we can generate Table M starting at the bottom and moving up using the formula.

Table M Charge $= \ next\ Table\ M\ charge\ + (\Delta entry\ ratio) \times \%\ above\ agg\ limit$

It is important to note that if there are no risks above the aggregate limit, then the Table M charge is 0.00. Working from the bottom up, we have,

Entry Ratio	% Above the Aggregate Limit	Table M Charge
2.00	0.00	0.00
1.75	0.00	$= 0.00 + (2.00 - 1.75) \times 0.00 = 0.00$
1.50	0.20	$= 0.00 + (1.75 - 1.50) \times 0.20 = 0.05$
1.25	0.20	$= 0.05 + (1.50 - 1.25) \times 0.20 = 0.10$
1.00	0.40	$= 0.10 + (1.25 - 1.00) \times 0.40 = 0.20$
0.75	0.60	$= 0.20 + (1.00 - 0.75) \times 0.60 = 0.35$
0.50	0.60	$= 0.35 + (0.75 - 0.5) \times 0.60 = 0.50$
0.25	1.00	$= 0.50 + (0.50 - 0.25) \times 1.0 = 0.75$
0.00	1.00	$= 0.75 + (0.25 - 0) \times 1.0 = 1.0$

An alternative method to find the Table M charge is by finding the ratio of dollars above the aggregate limit to the total losses. Provided that the total loss is $4,000,000, we can generate these Table M values.

$ Above Aggregate Limit	Table M Charge
$4,000,000	$= \frac{\$4,000,000}{\$4,000,000} = 1.00$
$3,000,000	$= \frac{\$3,000,000}{\$4,000,000} = 0.75$
$2,000,000	$= \frac{\$2,000,000}{\$4,000,000} = 0.50$
$1,400,000	$= \frac{\$1,400,000}{\$4,000,000} = 0.35$
$800,000	$= \frac{\$800,000}{\$4,000,000} = 0.20$
$400,000	$= \frac{\$400,000}{\$4,000,000} = 0.10$
$200,000	$= \frac{\$200,000}{\$4,000,000} = 0.50$
$0	$= \frac{\$0}{\$4,000,000} = 0.00$
$0	$= \frac{\$0}{\$4,000,000} = 0.00$

Putting it all together we have that:

Entry Ratio	**Aggregate Limit**	**# of Risks**	**# of Risks Above**	**% of Risks Above**	**$ Above Aggregate Limit**	**Table M Charge**
0.00	-	0	5	100%	$4,000,000	1.00
0.25	$200,000	0	5	100%	$3,000,000	0.75
0.50	$400,000	2	3	60%	$2,000,000	0.50
0.75	$600,000	2	3	60%	$1,400,000	0.35
1.00	$800,000	1	2	40%	$800,000	0.20
1.25	$1,000,000	1	1	20%	$400,000	0.10
1.50	$1,200,000	0	1	20%	$200,000	0.05
1.75	$1,400,000	1	0	0%	$0	0.00
2.00	$1,600,000	0	0	0%	$0	0.00

In some cases, there can be overlap between excess of the per-occurrence limit and the aggregate limit. It is customary to apply the per-occurrence limit first, so any excess loss would be considered excess of the deductible and is therefore included in the per-occurrence excess charge. If the actuary computes Table M charges without restricting the aggregate losses for the impact of the deductible, then it is possible that any losses greater than the deductible could inflate the Table M charge. This would result in an overlap between the Table M charge and the per-occurrence excess charge, leading to inaccurate Table M charges. Thus, the insurer must limit the aggregate losses for the impact of the deductible before generating Table M.

The insurer can use the methods of calculating aggregate excess loss to compute the limited excess charges, because the shape of the distribution of unlimited losses varies from that of limited losses which leads to a distinct type of distribution. Consequently, this limited distribution can be analyzed using the same Table M methods explained above. Deductibles can cover a wide range, requiring the insurer to generate a distinct Table M_D for the specific range. To do this the insurer must limit each individual loss prior to aggregating the losses of each policy. Loss distributions tend to be positively skewed, comprised of several small losses and few large losses which are responsible for a vast majority of the severity distribution's variance. Therefore, after the insurer applies the per-occurrence limit, there is a decrease in the variance of the severity distribution, which limits the probability of unusually large limited aggregate losses each year. Thus, the general rule is when the deductible is lower there is less variance in both the severity distribution and the resulting limited aggregate loss. Therefore, for entry ratios greater than 1.00, lower deductibles usually lead to lower charges.

Example 2.3.5: Calculating Modified Table M Charge

Assume that all risks have equal standard premiums.

Company A:

Number of Risks	Unlimited Loss Ratio
1	10%
4	30%
2	40%
4	50%
1	60%
3	70%
1	80%
2	100%
2	120%

Company B:

Number of Risks	Limited Loss Ratio
1	10%
4	30%
4	40%
3	50%
2	60%
1	70%
2	80%
2	90%
1	100%

Calculate the expected loss ratio for both limited and unlimited.

Unlimited

$$E(Unlimited\ LR) = \frac{\sum(Unlimited\ LR\ \times\ \#\ of\ Risks)}{Total\ Risks}$$

$$= \frac{(10\% \times 1) + (30\% \times 4) + (40\% \times 2) + (50\% \times 4) + (60\% \times 1) + (70\% \times 3) + (80\% \times 1) + (100\% \times 2) + (120\% \times 2)}{1 + 4 + 2 + 4 + 1 + 3 + 1 + 2 + 2}$$

$$= \frac{12}{20} = 0.6$$

Limited

$$E(Limited\ LR) = \frac{\sum(Limited\ LR\ \times\ \#\ of\ Risks)}{Total\ Risks}$$

$$= \frac{(10\% \times 1) + (30\% \times 4) + (40\% \times 4) + (50\% \times 3) + (60\% \times 2) + (70\% \times 1) + (80\% \times 2) + (90\% \times 2) + (100\% \times 1)}{1 + 4 + 4 + 3 + 2 + 1 + 2 + 2 + 1}$$

$$= \frac{10.7}{20} = 0.535$$

Now, we can calculate the loss elimination ratio to find what percentage of the loss ratio is lost due to limiting.

$$LER = \frac{(0.6 - 0.535)}{0.6} = 10.83\%$$

Using loss ratios from 0% to 120% in 10% increments, there is enough information to generate Table M_D. To find the entry ratios, use the unlimited Table M expected loss ratio.

$$Entry\ Ratio = \frac{Loss\ Ratio}{E(Unlimited\ LR)} = \frac{Loss\ Ratio}{0.6}$$

Loss Ratio	Entry Ratio	# of Risks Above	% of Risks Above
0%	0%	20	100%
10%	17%	19	95%
20%	33%	19	95%
30%	50%	15	75%
40%	67%	13	65%
50%	83%	9	45%
60%	100%	8	40%
70%	117%	5	25%
80%	133%	4	20%
90%	150%	4	20%
100%	167%	2	10%
110%	183%	2	10%
120%	200%	0	0%

Now calculate the Table M_D charge using the same formula as ***Example 1.3.4***.

Table M_D Charge $= \ next\ Table\ M\ charge\ + (\Delta entry\ ratio) \times \%\ above\ agg\ limit$

Starting from the bottom up, we have that,

Entry Ratio	% of Risks Above	Table M_D Charge
200%	0%	0
183%	10%	$= 0 + (200\% - 183\%) \times 10\% = 0.0167$
167%	10%	$= 0.0167 + (183\% - 167\%) \times 10\% = 0.0333$
150%	20%	$= 0.0333 + (167\% - 150\%) \times 20\% = 0.0667$
133%	20%	$= 0.0667 + (150\% - 133\%) \times 20\% = 0.1000$
117%	25%	$= 0.1000 + (133\% - 117\%) \times 25\% = 0.1417$
100%	40%	$= 0.1417 + (117\% - 100\%) \times 40\% = 0.2083$
83%	45%	$= 0.2083 + (100\% - 83\%) \times 45\% = 0.2833$
67%	65%	$= 0.2833 + (83\% - 67\%) \times 65\% = 0.3917$
50%	75%	$= 0.3917 + (67\% - 50\%) \times 75\% = 0.5167$
33%	95%	$= 0.5167 + (50\% - 33\%) \times 95\% = 0.6750$
17%	95%	$= 0.6750 + (33\% - 17\%) \times 95\% = 0.8333$
0%	100%	1.0000

Putting it all together we have:

Loss Ratio	**Entry Ratio**	**# of Risks Above**	**% of Risks Above**	**Table M_D Charge**
0%	0%	20	100%	1.000
10%	17%	19	95%	0.8333
20%	33%	19	95%	0.6750
30%	50%	15	75%	0.5167
40%	67%	13	65%	0.3917
50%	83%	9	45%	0.2833
60%	100%	8	40%	0.2083
70%	117%	5	25%	0.1417
80%	133%	4	20%	0.1000
90%	150%	4	20%	0.0667
100%	167%	2	10%	0.0333
110%	183%	2	10%	0.0167
120%	200%	0	0%	0.0000

3.0 Methodology

To achieve the goal of our project, we developed random numbers that follow chosen frequency and severity distributions with chosen constant expected values and created VBA code to run the simulation using the distributions. We followed these steps to accomplish these tasks:

1. Examine how different frequency and severity distributions interact with each other
2. Investigate how an aggregate deductible impacts the insurer's payout
3. Effectively communicate the results in comprehensive tables

3.1 Matching Moments and Percentiles

To minimize difference in our results due to varying distributions, we chose to match their moments or percentiles to create distributions of the same mean or spread. The method used depended on the distributions involved, as some distributions are difficult to match to each other.

First, we defined constants for our distributions – the severity distributions would have a mean of 10,000 and the frequency distributions a mean of 85. With these values in mind, we calculated the parameters for our distributions. We started with the exponential and poisson distributions first, since they each only have one parameter, which is equal to the mean. They served as the baseline distributions for matching moments and finding parameters.

The next frequency distribution we wanted to use was uniform, which has an expected value of:

$$E[X] = \frac{b + a}{2}$$

Thus, we chose 60 for a and 110 for b since we only wanted claim frequencies in this range, which resulted in a mean of 85. From there, we calculated the variance of the uniform

distribution so we could use it to simulate another distribution. The variance of a uniform distribution on the interval [a, b] is equal to:

$$Var(X) = \frac{(b-a)^2}{12}$$

After plugging in our parameters, the variance came out to $\frac{625}{3}$. We used these values to find parameters (r, p) for the negative binomial distribution, setting up these equations:

$$E[X] = \frac{r(1-p)}{p} = 85 \qquad Var(X) = \frac{r(1-p)}{p^2} = \frac{625}{3}$$

By combining and algebraically manipulating these equations, we found $p = 0.408$ and $r = 59$. We repeated this process to generate another severity distribution as well, for a total of two severity and three frequency distributions.

3.2 Inverse Transform

To prepare for the simulation, we created our own data to simulate insurance policies and their claims. We used the inverse transform method to convert uniformly distributed random numbers to random numbers following a target distribution.

The first step is producing uniformly distributed random numbers in excel using the rand() function. The rand() function updates automatically in the spreadsheet, so we copied the values instead, using the same set of uniformly distributed numbers on (0,1) for all distributions. The random numbers represent the probability that the random

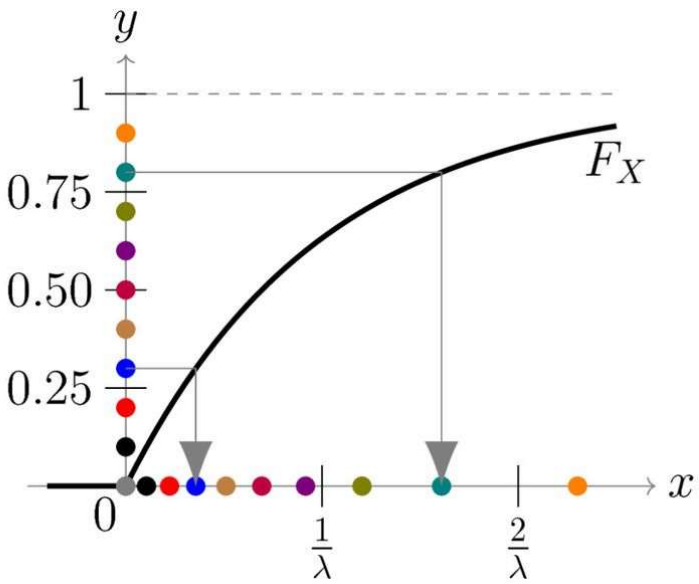


Figure 3.2.1: Inverse Transform Method Exponential Distribution Example Graph

number X, or the value of the CDF F(x), is less than or equal to x – shown in figure 3.2.1 by the dots on the y-axis.

Next, we need the inverse CDF of our target distribution. The first distribution we aimed to generate was exponential, which has CDF:

$$F(x) = 1 - e^{\frac{-x}{\theta}},$$

where θ represents the distribution's mean, F(x) is the random number generated, and x is the value we're solving for – represented by the corresponding dots on the x-axis in figure 3.2.1. We plugged in 10,000 for θ and rearranged the equation to solve for x, which resulted in:

$$x = -10{,}000 * \ln(1 - X),$$

where X represents the value of the CDF at x.

In excel, we created a column to evaluate this equation with the value of the previously generated random numbers as the input X, and it produced exponentially distributed claim amounts. While we were able to invert the exponential CDF algebraically and input into excel, not all distributions are as simple to rearrange.

Another technique for simulating a distribution involves creating the distribution using the CDF and parameters in excel, beginning by initializing a column of 'x' values which are integers greater than or equal to 0. In the next column, the CDF is entered as the formula, complete with parameters, with the x-values as the input. We now have the cumulative probabilities from [0,1] of each x.

In the next column, the previously generated column of random numbers is pasted. In the final column, XLOOKUP is used to search for an F(x) that matches the random number, and then outputs the corresponding x-value, representing the claim severity or frequency.

3.3 Simulation

The expected values of the severity and frequency distributions were $10,000 and 85 claims, respectively, for an aggregate expected value of $850,000. We generated 100,000 possible claims severities and 1000 frequency values, each representing the number of claims made on a given policy.

To begin the simulation, the severity distribution is copied to column A on the CurrentVBARun tab in the simulation workbook, and the frequency distribution to column B. The contents of the 4 tables in R19:AD73 should be cleared before beginning. We then created a table with different occurrence and aggregate deductibles as a ratio of their expected value. They served as inputs for the simulation, which took in the occurrence deductible first and ran the simulation for each of the aggregate deductibles. For example, the first occurrence deductible was 2000, and the first aggregate deductible was 170,000. The simulation runs and spits out the outputs, then continues with the other aggregate deductibles 340,000, 510,000, 680,000, and so on until all possible combinations with the 2000 are exhausted. The simulation then moves on to the next occurrence deductible and continues the same process.

	Expected	Expected
	10,000	850,000
Ratio	Occ Deductible	Agg Deductible
0.0	-	-
0.2	2,000	170,000
0.4	4,000	340,000
0.6	6,000	510,000
0.8	8,000	680,000
1.0	10,000	850,000
1.2	12,000	1,020,000
1.4	14,000	1,190,000
1.6	16,000	1,360,000
1.8	18,000	1,530,000
2.0	20,000	1,700,000

Table 3.3.1: Occurrence and Aggregate Deductibles

After taking in the first occurrence and aggregate deductible, the simulation reads in the first claim number from the frequency distribution. For example, if the first number was 98, the

simulation would iterate through each of the first 98 claims in the severity column. From there, the simulation outputs how much the insured paid, how much the insurer paid because the claim was over the occurrence deductible, how much the insurer paid because the aggregate deductible was reached, and the total ground up claims.

The simulation repeats this for each of the 1000 policies with the same occurrence and aggregate deductible combination, each time tracking where the previous policy had left off in the severity distribution and continuing from the next cell. After completing the run, the simulation outputs 4 values into specified tables. The values are the average insured payment, the average insurer over occurrence payment, the average insurer over aggregate payment, and the average ground up claims, all as a ratio to the aggregate expected value. After the values are put into the corresponding cells in each of the 4 tables, the simulation runs again with the next combination of occurrence and aggregate deductibles. This process is repeated until all combinations are exhausted, producing 4 tables with 121 cells each for a single run.

4.0 Results & Analysis

4.1 Generating Claim Distributions

The procedure for creating the severity and frequency distributions varied based on the distribution and its limitations, as we used our judgment to create a balance between consistent methodology and meaningful results. Initially, we intended to match both the first and second moments to minimize variability in the results due to differences in the mean and variance of the distributions. The exponential severity distribution was generated first, as it was the most straightforward. As indicated in section 3.1, the mean of the severity distribution was chosen as 10,000, which we used to find the variance.

The variance of a distribution X can be expressed in terms of expected value:

$$Var(X) = E[X^2] - E[X]^2.$$

Given the expected value for the exponential distribution:

$$E[X^k] = \theta^k \cdot k! \;\; if\ k\ is\ a\ positive\ integer,$$

we combined these two formulas to find the variance of our exponential distribution as:

$$Var(X) = [\theta^2 \cdot 2!] - [\theta]^2 = 100{,}000{,}000.$$

Thus, we attempted to algebraically solve for parameters θ, α for the pareto distribution using the given formulas:

$$E[X] = \frac{\theta}{\alpha - 1} = 10{,}000; \quad Var(X) = \frac{2\theta^2}{(\alpha-1)(\alpha-2)} - \frac{\theta}{\alpha-1} = 100{,}000{,}000$$

After some algebraic manipulation, the final result was:

$$(\alpha - 1) = (\alpha - 2),$$

which cannot be true, meaning we were unable to match both the expected value and variance. Ultimately, we chose to match the expected value for the distributions so the aggregate expected value, $E[S]$, would remain constant as it is used in calculating the values in the tables.

Below are two tables displaying the distributions we generated for severity and frequency, their probability density functions, and parameters. All six combinations of frequency and severity distributions were used to run the simulation: each run producing its own set of 4 tables.

Table 4.1.1: Claim Severity Distributions

Distribution	PDF	Parameters
Exponential	$f(x) = \frac{e^{-x/\theta}}{\theta}$	$\theta = 10{,}000$
Pareto	$f(x) = \frac{\alpha\theta^{\alpha}}{(x+\theta)^{\alpha+1}}$	$\alpha = 10, \theta = 90{,}000$

Table 4.1.2: Generated Claim Frequency Distributions

Distribution	PDF	Parameters
Poisson	$f(x) = \frac{\lambda^k e^{-\lambda}}{k!}$	$\lambda = 85$
Negative Binomial	$f(x) = \binom{x+r-1}{r-1} p^r (1-p)^k$	$r = 59, p = 0.408$
Uniform	$f(x) = \frac{1}{b-a}$	$a = 60, b = 110$

4.2 Sample Output

We first ran the simulation with exponential severity and poisson frequency. The results are in the following tables:

Insured under occ		Aggregate Limit										
	Ratio to E[X]	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
Occurrence Deductible	0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.2	0.0000	0.1809	0.1813	0.1813	0.1813	0.1813	0.1813	0.1813	0.1813	0.1813	0.1813
	0.4	0.0000	0.2000	0.3293	0.3293	0.3293	0.3293	0.3293	0.3293	0.3293	0.3293	0.3293
	0.6	0.0000	0.2000	0.3983	0.4507	0.4507	0.4507	0.4507	0.4507	0.4507	0.4507	0.4507
	0.8	0.0000	0.2000	0.4000	0.5458	0.5501	0.5501	0.5501	0.5501	0.5501	0.5501	0.5501
	1.0	0.0000	0.2000	0.4000	0.5878	0.6315	0.6315	0.6315	0.6315	0.6315	0.6315	0.6315
	1.2	0.0000	0.2000	0.4000	0.5977	0.6965	0.6984	0.6984	0.6984	0.6984	0.6984	0.6984
	1.4	0.0000	0.2000	0.4000	0.5994	0.7397	0.7533	0.7533	0.7533	0.7533	0.7533	0.7533
	1.6	0.0000	0.2000	0.4000	0.5998	0.7645	0.7981	0.7982	0.7982	0.7982	0.7982	0.7982
	1.8	0.0000	0.2000	0.4000	0.5999	0.7779	0.8339	0.8348	0.8348	0.8348	0.8348	0.8348
	2.0	0.0000	0.2000	0.4000	0.5999	0.7853	0.8613	0.8648	0.8648	0.8648	0.8648	0.8648

Insurer over occ		Aggregate Limit										
	Ratio to E[X]	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
Occurrence Deductible	0.0	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
	0.2	0.8184	0.8184	0.8184	0.8184	0.8184	0.8184	0.8184	0.8184	0.8184	0.8184	0.8184
	0.4	0.6704	0.6704	0.6704	0.6704	0.6704	0.6704	0.6704	0.6704	0.6704	0.6704	0.6704
	0.6	0.5489	0.5489	0.5489	0.5489	0.5489	0.5489	0.5489	0.5489	0.5489	0.5489	0.5489
	0.8	0.4496	0.4496	0.4496	0.4496	0.4496	0.4496	0.4496	0.4496	0.4496	0.4496	0.4496
	1.0	0.3681	0.3681	0.3681	0.3681	0.3681	0.3681	0.3681	0.3681	0.3681	0.3681	0.3681
	1.2	0.3013	0.3013	0.3013	0.3013	0.3013	0.3013	0.3013	0.3013	0.3013	0.3013	0.3013
	1.4	0.2464	0.2464	0.2464	0.2464	0.2464	0.2464	0.2464	0.2464	0.2464	0.2464	0.2464
	1.6	0.2015	0.2015	0.2015	0.2015	0.2015	0.2015	0.2015	0.2015	0.2015	0.2015	0.2015
	1.8	0.1648	0.1648	0.1648	0.1648	0.1648	0.1648	0.1648	0.1648	0.1648	0.1648	0.1648
	2.0	0.1349	0.1349	0.1349	0.1349	0.1349	0.1349	0.1349	0.1349	0.1349	0.1349	0.1349

Insurer over agg		Aggregate Limit										
	Ratio to E[X]	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
Occurrence Deductible	0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.2	0.1813	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.4	0.3293	0.1293	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.6	0.4507	0.2507	0.0525	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.8	0.5501	0.3501	0.1501	0.0043	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1.0	0.6315	0.4315	0.2315	0.0437	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1.2	0.6984	0.4984	0.2984	0.1007	0.0019	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1.4	0.7533	0.5533	0.3533	0.1539	0.0136	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1.6	0.7982	0.5982	0.3982	0.1984	0.0337	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	1.8	0.8348	0.6348	0.4348	0.2350	0.0570	0.0009	0.0000	0.0000	0.0000	0.0000	0.0000
	2.0	0.8648	0.6648	0.4648	0.2648	0.0795	0.0034	0.0000	0.0000	0.0000	0.0000	0.0000

Ground up		Aggregate Limit										
	Ratio to E[X]	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
Occurrence Deductible	0.0	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
	0.2	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
	0.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
	0.6	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
	0.8	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
	1.0	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
	1.2	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
	1.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
	1.6	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
	1.8	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
	2.0	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997

Figure 4.2.1: Output tables for exponential severity, poisson frequency.

The values in each table are ratios of the payment specified to the expected value, which is constant at $850,000. While the table is set up to produce ratios, it can easily be converted to dollars so the payment amounts can be analyzed and presented directly:

Insured under occ		Aggregate Limit										
	Ratio to E[X]	0.0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2.0
Occurrence Deductible	0.0	-	-	-	-	-	-	-	-	-	-	-
	0.2	-	153,755.09	154,074.07	154,074.07	154,074.07	154,074.07	154,074.07	154,074.07	154,074.07	154,074.07	154,074.07
	0.4	-	170,000.00	279,882.81	279,883.05	279,883.05	279,883.05	279,883.05	279,883.05	279,883.05	279,883.05	279,883.05
	0.6	-	170,000.00	338,537.62	383,123.47	383,123.47	383,123.47	383,123.47	383,123.47	383,123.47	383,123.47	383,123.47
	0.8	-	170,000.00	339,998.07	463,947.42	467,576.72	467,576.72	467,576.72	467,576.72	467,576.72	467,576.72	467,576.72
	1.0	-	170,000.00	340,000.00	499,670.10	536,792.80	536,797.55	536,797.55	536,797.55	536,797.55	536,797.55	536,797.55
	1.2	-	170,000.00	340,000.00	508,029.64	592,004.21	593,620.33	593,620.33	593,620.33	593,620.33	593,620.33	593,620.33
	1.4	-	170,000.00	340,000.00	509,527.00	628,728.17	640,318.45	640,318.45	640,318.45	640,318.45	640,318.45	640,318.45
	1.6	-	170,000.00	340,000.00	509,810.64	649,824.39	678,414.81	678,473.14	678,473.14	678,473.14	678,473.14	678,473.14
	1.8	-	170,000.00	340,000.00	509,891.49	661,186.17	708,847.80	709,611.05	709,611.05	709,611.05	709,611.05	709,611.05
	2.0	-	170,000.00	340,000.00	509,931.89	667,473.51	732,124.53	735,044.29	735,044.29	735,044.29	735,044.29	735,044.29

Insurer over occ		Aggregate Limit										
	Ratio to E[X]	0.0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2.0
Occurrence Deductible	0.0	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46
	0.2	695,644.39	695,644.39	695,644.39	695,644.39	695,644.39	695,644.39	695,644.39	695,644.39	695,644.39	695,644.39	695,644.39
	0.4	569,835.41	569,835.41	569,835.41	569,835.41	569,835.41	569,835.41	569,835.41	569,835.41	569,835.41	569,835.41	569,835.41
	0.6	466,594.99	466,594.99	466,594.99	466,594.99	466,594.99	466,594.99	466,594.99	466,594.99	466,594.99	466,594.99	466,594.99
	0.8	382,141.74	382,141.74	382,141.74	382,141.74	382,141.74	382,141.74	382,141.74	382,141.74	382,141.74	382,141.74	382,141.74
	1.0	312,920.91	312,920.91	312,920.91	312,920.91	312,920.91	312,920.91	312,920.91	312,920.91	312,920.91	312,920.91	312,920.91
	1.2	256,098.12	256,098.12	256,098.12	256,098.12	256,098.12	256,098.12	256,098.12	256,098.12	256,098.12	256,098.12	256,098.12
	1.4	209,400.01	209,400.01	209,400.01	209,400.01	209,400.01	209,400.01	209,400.01	209,400.01	209,400.01	209,400.01	209,400.01
	1.6	171,245.32	171,245.32	171,245.32	171,245.32	171,245.32	171,245.32	171,245.32	171,245.32	171,245.32	171,245.32	171,245.32
	1.8	140,107.41	140,107.41	140,107.41	140,107.41	140,107.41	140,107.41	140,107.41	140,107.41	140,107.41	140,107.41	140,107.41
	2.0	114,674.17	114,674.17	114,674.17	114,674.17	114,674.17	114,674.17	114,674.17	114,674.17	114,674.17	114,674.17	114,674.17

Insurer over agg		Aggregate Limit										
	Ratio to E[X]	0.0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2.0
Occurrence Deductible	0.0	-	-	-	-	-	-	-	-	-	-	-
	0.2	154,074.07	318.99	-	-	-	-	-	-	-	-	-
	0.4	279,883.05	109,883.05	0.23	-	-	-	-	-	-	-	-
	0.6	383,123.47	213,123.47	44,585.85	-	-	-	-	-	-	-	-
	0.8	467,576.72	297,576.72	127,578.65	3,629.30	-	-	-	-	-	-	-
	1.0	536,797.55	366,797.55	196,797.55	37,127.45	4.75	-	-	-	-	-	-
	1.2	593,620.33	423,620.33	253,620.33	85,590.69	1,616.12	-	-	-	-	-	-
	1.4	640,318.45	470,318.45	300,318.45	130,791.44	11,590.28	-	-	-	-	-	-
	1.6	678,473.14	508,473.14	338,473.14	168,662.50	28,648.75	58.33	-	-	-	-	-
	1.8	709,611.05	539,611.05	369,611.05	199,719.55	48,424.88	763.25	-	-	-	-	-
	2.0	735,044.29	565,044.29	395,044.29	225,112.40	67,570.78	2,919.76	-	-	-	-	-

Ground up		Aggregate Limit										
	Ratio to E[X]	0.0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2.0
Occurrence Deductible	0.0	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46
	0.2	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46
	0.4	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46
	0.6	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46
	0.8	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46
	1.0	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46
	1.2	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46
	1.4	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46
	1.6	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46
	1.8	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46
	2.0	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46	849,718.46

Figure 1.2.2: Output tables represented in dollars.

When the ratios are converted to dollar amounts, it provides a clearer picture of exactly how much the insurer and insured will pay, along with the total claim amount. Say a policy has occurrence deductible $8000 and aggregate limit $510,000, then we can look at the cell in each table corresponding to a 0.8 occurrence deductible ratio and a 0.6 aggregate ratio to determine the payment allocations. In this example, the ground up claims would be $849,718, just below the expected value of $850,000. The insured would pay $463,947 total, with the insurer paying the remainder, $382,142 due to the occurrence deductible and $3629 due to the aggregate limit being reached. While the tables can be useful & more easily understood in a dollar format, the original ratio values are more important since the expected value was chosen arbitrarily.

4.3 Output Table Features

Now, in a new situation, say the severity distribution is still exponential, while the claim frequency is uniformly distributed. We will now explore and learn how to interpret the output tables produced in this simulation run.

4.3.1 Insured Payment

Insured under occ		Aggregate Limit										
	Ratio to E[X]	0.0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2.0
Occurrence Deductible	0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.2	0.0000	0.1747	0.1806	0.1806	0.1806	0.1806	0.1806	0.1806	0.1806	0.1806	0.1806
	0.4	0.0000	0.2000	0.3257	0.3281	0.3281	0.3281	0.3281	0.3281	0.3281	0.3281	0.3281
	0.6	0.0000	0.2000	0.3847	0.4490	0.4492	0.4492	0.4492	0.4492	0.4492	0.4492	0.4492
	0.8	0.0000	0.2000	0.3986	0.5264	0.5482	0.5482	0.5482	0.5482	0.5482	0.5482	0.5482
	1.0	0.0000	0.2000	0.3999	0.5625	0.6266	0.6293	0.6293	0.6293	0.6293	0.6293	0.6293
	1.2	0.0000	0.2000	0.4000	0.5810	0.6788	0.6959	0.6960	0.6960	0.6960	0.6960	0.6960
	1.4	0.0000	0.2000	0.4000	0.5903	0.7111	0.7494	0.7507	0.7507	0.7507	0.7507	0.7507
	1.6	0.0000	0.2000	0.4000	0.5948	0.7317	0.7899	0.7955	0.7955	0.7955	0.7955	0.7955
	1.8	0.0000	0.2000	0.4000	0.5969	0.7455	0.8195	0.8318	0.8320	0.8320	0.8320	0.8320
	2.0	0.0000	0.2000	0.4000	0.5980	0.7551	0.8410	0.8610	0.8618	0.8618	0.8618	0.8618

Table 1.3.1: Policyholder payments for exponential severity, uniform frequency

The first table represents the proportion of the expected total claims that the policyholder is expected to pay. For example, when both the occurrence deductible and aggregate limit ratios are equal to 1, the policyholder will pay 62.93% of the expected total losses.

If we examine the table column by column, seeing how the payment ratio changes when the aggregate ratio remains constant and the occurrence ratio increases, we can see that the value approaches an asymptote when the limit ratio is 0.2, 0.4, and 0.6. The asymptotic value is equal to the aggregate ratio. Since the aggregate ratio is relatively low, when the occurrence ratio increases, the aggregate limit is reached quicker and the payments from the insured stop immediately. This is not seen for the relatively high aggregate ratios as the aggregate limit is much greater and even when the occurrence ratio is high, there will still be claims that are greater than the expected value in which the responsibility is the insurers.

Row by row, the opposite is true. As the aggregate ratio increases while holding the occurrence ratio constant, the payment ratio approaches an asymptote. The asymptote, however, is not equal to the value of the ratio. It seems to be close to the ratio value when it is relatively low, however this changes as the occurrence ratio increases.

4.3.2 Insurer Payment

Insurer over occ		Aggregate Limit										
	Ratio to E[X]	0.0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2.0
Occurrence Deductible	0.0	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964
	0.2	0.8157	0.8157	0.8157	0.8157	0.8157	0.8157	0.8157	0.8157	0.8157	0.8157	0.8157
	0.4	0.6682	0.6682	0.6682	0.6682	0.6682	0.6682	0.6682	0.6682	0.6682	0.6682	0.6682
	0.6	0.5472	0.5472	0.5472	0.5472	0.5472	0.5472	0.5472	0.5472	0.5472	0.5472	0.5472
	0.8	0.4482	0.4482	0.4482	0.4482	0.4482	0.4482	0.4482	0.4482	0.4482	0.4482	0.4482
	1.0	0.3670	0.3670	0.3670	0.3670	0.3670	0.3670	0.3670	0.3670	0.3670	0.3670	0.3670
	1.2	0.3004	0.3004	0.3004	0.3004	0.3004	0.3004	0.3004	0.3004	0.3004	0.3004	0.3004
	1.4	0.2456	0.2456	0.2456	0.2456	0.2456	0.2456	0.2456	0.2456	0.2456	0.2456	0.2456
	1.6	0.2009	0.2009	0.2009	0.2009	0.2009	0.2009	0.2009	0.2009	0.2009	0.2009	0.2009
	1.8	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644
	2.0	0.1345	0.1345	0.1345	0.1345	0.1345	0.1345	0.1345	0.1345	0.1345	0.1345	0.1345

Table 4.3.2: Insurer occurrence payments for exponential severity, uniform frequency

The table for the insurers' payment due to the loss being greater than the occurrence deductible is a little different in that it is constant across each row. Since this

table is focused on payments as a result of the occurrence deductible, it doesn't change as the aggregate limit varies.

Insurer over agg		Aggregate Limit										
	Ratio to E[X]	0.0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2.0
Occurrence Deductible	0.0	0	0	0	0	0	0	0	0	0	0	0
	0.2	0.1806	0.0060	0	0	0	0	0	0	0	0	0
	0.4	0.3281	0.1281	0.0024	0	0	0	0	0	0	0	0
	0.6	0.4492	0.2492	0.0645	0.0002	0	0	0	0	0	0	0
	0.8	0.5482	0.3482	0.1496	0.0218	0	0	0	0	0	0	0
	1.0	0.6293	0.4293	0.2295	0.0669	0.0027	0	0	0	0	0	0
	1.2	0.6960	0.4960	0.2960	0.1150	0.0172	0	0	0	0	0	0
	1.4	0.7507	0.5507	0.3507	0.1605	0.0397	0.0014	0	0	0	0	0
	1.6	0.7955	0.5955	0.3955	0.2007	0.0638	0.0056	0	0	0	0	0
	1.8	0.8320	0.6320	0.4320	0.2351	0.0865	0.0125	0.0002	0	0	0	0
	2.0	0.8618	0.6618	0.4618	0.2638	0.1067	0.0208	0.0008	0	0	0	0

Table 4.3.3: Insurer aggregate payments for exponential severity, uniform frequency

The table for the insurers' payment due to the losses reaching the aggregate limit accounts for the difference in payment allocation. The values indicate the extra charge that should be applied to the policy for the aggregate limit.

When the aggregate limit is low, as mentioned in the previous section, the insured's payment reaches an asymptote, however the payment allocation's difference is accounted for in this table – it increases down a column because the aggregate limit has a greater impact, meaning the claim severities were relatively high. Across the rows, the value decreases because the aggregate limit is greater and not reached as easily with the given policies.

4.3.3 Total Claim Amount

Ground up		Aggregate Limit										
	Ratio to E[X]	0.0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2.0
Occurrence Deductible	0.0	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964
	0.2	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964
	0.4	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964
	0.6	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964
	0.8	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964
	1.0	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964
	1.2	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964
	1.4	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964
	1.6	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964
	1.8	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964
	2.0	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964	0.9964

Table 4.3.4: Total ground up losses for exponential severity, uniform frequency

The table for ground up losses represents the ratio of the total accumulated losses to the expected value, which in this case is 99.64%. The values in this table are a sum of

the corresponding values in the three preceding tables. The value remains constant throughout the table as the claims used in the simulation are the same throughout the iterations. The only inputs changing are the aggregate limits and occurrence deductibles, which determine the allocation of the payments.

5.0 Recommendations

Our improved methodology for calculating and presenting Table M in a more comprehensive manner allocated the payments between the insured and insurer based on the applied deductibles. We used VBA to write this code, since we required a coding language that could manage substantial amounts of data. Future works could further develop this by upgrading the VBA code to solve the aggregate charge analytically when the given per-occurrence and aggregate deductibles are entered. By automating this process insurers can gain efficiency in handling large datasets. This could also improve accuracy in estimating deductible allocations.

For this project, our data was entirely simulated in Excel. In the future it could be beneficial to explore enhanced integration with data sources and insurance companies. By pulling relevant data such as historical data, claim information, or risk profiles the methodology can provide more realistic and tailored deductible allocations for individual policy holders. This proactive approach could lead to insurers being able to adjust the methodology to suit their specific needs, thus enhancing its usability and overall effectiveness.

By integrating these recommendations into the methodology, insurers could enhance the efficiency and precision of deductible allocations. This would also enhance the overall effectiveness of risk management practices.

6.0 Bibliography

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