Automated Design of Planar Linkages: Slider-Crank Analysis

A report submitted to the Faculty of the WORCESTER POLYTECHNIC INSTITUTE in partial fulfillment of the requirements for the Degree of Bachelor of Science.

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Abstract

The Planar Mechanism Kinematic Simulator (PMKS) is an important software tool for the kinematic analysis of planar linkages. The tool is also capable of carrying out force analysis of linkages with revolute joints. In order to enhance the capability and carry out force analysis of slider-crank linkages, this major qualifying project was involved in developing static and dynamic equations of four-bar and six-bar slider-crank linkages that could be implemented within PMKS. Equations were sourced from standard text-books. These equations were evaluated and implemented in MATLAB and tested on multiple example problems. These examples linkages were also recreated in Working Model 2-D software to compare results.

Acknowledgements

During my rich path to study on four and six bar slider-crank, great minds help me to reach my goals. I would like to thank the following people from Worcester Polytechnic Institute (WPI):

- Professor Pradeep Radhakrishnan, Assistant Teaching Professor, ME, for advising and strengthen my project from its beginning to its end
- Professor David Brown, Professor Emeritus, CS, for his continuous feedback and guidance
- Professor Robert Norton for his book, Kinematics and Dynamics of Machinery, 1st Edition, 2009, that proved very useful for this project
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Table of Contents

Contents

Chapter 1: Introduction

The Planar Mechanism Kinematics Simulator (PMKS: https://designengrlab.github.io/PMKS/) is a browser-based tool that can be used to analyze the kinematics of links and joints and the forces at joints in planar linkages. The tool was originally developed by Professor Matthew I. Campbell of Mechanical Engineering Department at Oregon State University, as a kinematic simulator that can generate position, velocity and acceleration of links and joints in single-degree of freedom planar linkages. The tool can animate these linkages as well as generate all the position, velocity and acceleration data into a comma separated value (CSV) file. The tool was used as a teaching tool at WPI in ME 3310 Kinematics of Mechanisms in 2015, 2016 and 2017 in the courses taught by Prof. Pradeep Radhakrishnan due to this association with the development and usage of the software. The primary usage of the tool was to help students design planar linkages, generate kinematics and compare those values obtained using manual and implementations in MATLAB.

Automated Design Planar Linkages (Andrews et al, 2018) added several capabilities to PMKS such as the ability to carry out static force analysis and dynamic force analysis of linkages with revolute joints. Through these force analyses, the joint forces and the input torque at the motor can be computed. These two features meant that the tool must allow users to specify forces, link dimensions, geometry and material and thus those features was incorporated into the tool. In addition, the tool can also compute stress analysis of links and joints and determine the instantaneous center of rotation of linkages with revolute and prismatic joints. These additions allowed the tool to be used in senior level courses such as ME/RBE 4322 Modeling and Analysis of Mechatronic Systems and ME 4320 Advanced Engineering Design. In those courses, students can compare their solutions to linkage dynamics problems using PMKS.

While PMKS is a valuable tool in education, there are a lot of features that have to be incorporated so that its design and analysis capabilities can enhance outcomes in a number of Mechanical Engineering courses. The features that are majorly lacking in PMKS compared to similar tools are: (i) the ability to add a force onto a pin, (ii) the static, dynamics and stress analysis of a slider crank mechanism, (iii) force analysis of a parallel four-bar linkage and (iv) the ability to incorporate different link geometry. In order to enhance the capabilities of PMKS, this MQP proposes the following

1. Understand the capabilities of PMKS with respect to slider-crank mechanisms

2. Identify the equations related to static, dynamic and stress analysis of links in a slider-crank mechanism
3. Implement the equations in MATLAB and verify solutions for various test cases with similar software

Once the equations have been thoroughly tested as part of this MQP, they can be coded into the PMKS software. The analysis of slider-crank was selected in this MQP since this is one of the most commonly used mechanisms on campus and will be a valuable addition to PMKS.

This MQP report is organized as follows. Chapter 2 will discuss some of the background related to slider-crank linkages. This will be followed by Chapter 3 where the methodology adopted to solve the linkage will be discussed. Chapter 4 will list out all the equations and the test cases will be discussed in Chapter 5. Chapter 6 will present concluding remarks and recommendations for future work.

Chapter 2: Background

In this chapter, all the necessary background related to slider-crank mechanisms will be presented.

2. 1. Slider-Crank Mechanism

The slider crank mechanism has been introduced in to PMKS to mimic the movement of diesel and petrol engines used in the industry. Its idea originates from Scotch Yoke mechanism which converts the linear motion of a mechanism to rotational motion and vice versa (Hastürk, 2016). A sample Scotch mechanism is shown in **Figure 1**.

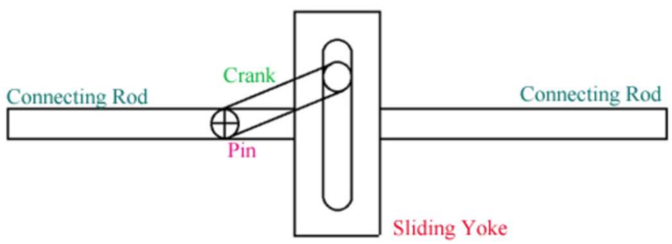


Figure 1: Scotch Yoke mechanism

Source: Scotch Yoke Mechanism Working Principle. (2017). Retrieved from https://techminy.com/scotch-yoke-mechanism/

There are various examples of four-bar slider crank applications in real life. One of the common examples of a four-bar slider crank is the pump jack as shown in **Figure 2**.

Figure 2: A pump jack in an oil field

Source: Hanania, J., Stenhouse, K., & Donev, J. (2015). Pump jack. Retrieved from https://energyeducation.ca/encyclopedia/Pump_jack

The device is composed of long heavy beam which is moved by an external power source causing the end of the beam to rise and fall. As the beam rises and falls, a series of sucker rods, which acts like the slider piston dips in and out of the well increasing the pressure inside.

2.2. Four-bar slider-crank mechanism

A sketch of a slider-crank mechanism is shown in Figure 3. The sketch shown in Figure 3 was created using SolidWorks.

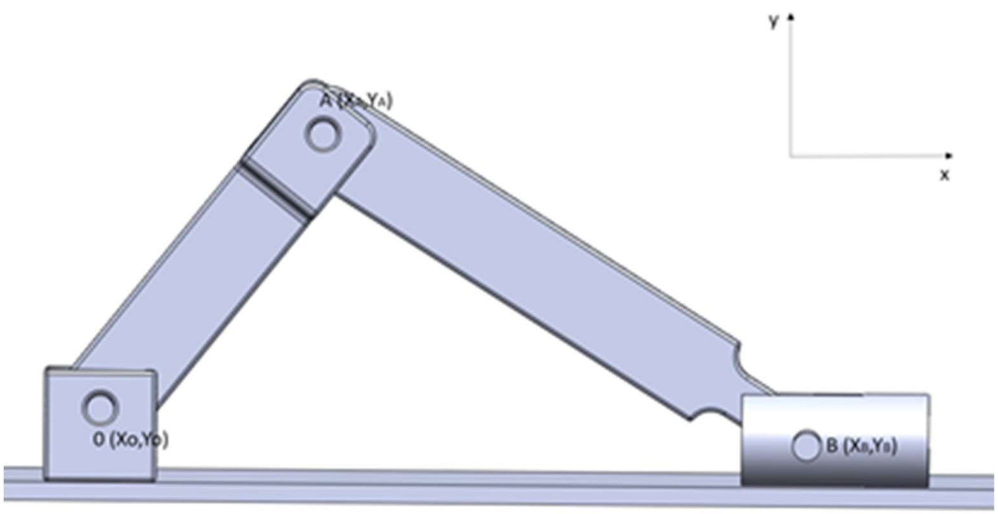


Figure 3: A four-bar slider crank mechanism in SolidWorks

The flat surface is the ground. It is considered as one link. The input link is the link connecting revolute joints O and A and the input is attached to the joint O. The coupler link is the link formed by connecting revolute joints A and B, and the slider is the piston with a joint at B. The slider has another joint, which is the prismatic joint between the sliding block and the surface against which it is sliding. The input can also be the slider instead of the revolute joint at O. An internal combustion engine is an example of a slider-crank mechanism where the piston (slider) is the input.

Such mechanism has wide range of criteria, but one of the beauty we noticed is that it produces a high torque with a small size of size of piston cylinder and spent more time on the top than on other part during the movement. This situation increases the engine efficiency. Additionally, its piston motion is a pure sine wave which occurs overtime and give a constant rotation speed.

2. 3. Six-bar slider crank mechanism

Various examples of six-bar slider crank applications can be seen in mechanical or manufacturing engines. One example of 6-bar slider crank is the double dwell six-bar linkage shown in Figure 4 below.

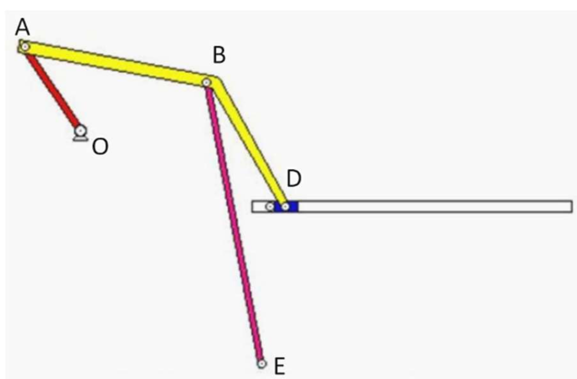


Figure 4: Double Dwell six bar linkage on S. Wang software .

Source: Norton R. L (2003). Design of Machinery, 3rd Edition. Retrieved from: https://slideplayer.com/slide/3449547/

It is composed of two binary links (shown in red) OA and BE, one ternary link (shown in yellow) ABD and a sliding member at its extremity. Three motion functions follow each other and then repeat: a dwell, a rise and a return function (Norton, 2003) during the movement of this linkage.

In this chapter, a brief overview of slider-crank mechanisms is presented. In the next chapter, the project methodology will be presented.

Chapter 3: Methodology

In this chapter, the steps undertaken to fulfill the requirements of this project will be laid out.

3.1. Analysis for Slider Crank

3.1.1. Force Analysis of Four-Bar and Six-Bar Slider Crank mechanisms

Currently, PMKS is able to compute the kinematics of slider-crank mechanisms. However, force analysis cannot be done. Equations for forces analysis on slider-crank linkages will be sourced from Kinematics and Dynamics of Machinery from R. L. Norton, 2009, on forces produced within a slider-crank and verify using MATLAB before implementing into PMKS. Afterward, 4-bar and 6-bar slider-cranks will be created in Working Model. All software implementations will be verified to be matching. To start the work on the four and six-bar slider crank mechanism, we found necessary to begin on the statics and dynamics analysis of a four-bar slider crank mechanism, then finish with the statics and dynamics analysis of a six-bar slider crank mechanism. Two different cases were attempted: one with known input force at the slider and unknown torque at the crank, and the other with unknown input force at slider piston and known torque at the slider.

3.1.2. Determination of Joints Forces in Linkages using the Principle of Virtual Work

The principle of virtual work is an attempt to characterize unequivocally an equilibrium configuration of a mechanical system by observing its reaction to a small kinematical perturbation (Epstein, 1970). Virtual work can be applied on all types of linkages and an input force need to be applied on the slider piston. The benefit of using this principle over the Norton's Design of Machinery book equations analysis is the consideration of a virtual small displacement where the mechanism seems to move creating an energy. It will be used to determine the value of the force when applying known torque on the 4 and 6-bar slider crank mechanism.

In this chapter a brief overview of the roadmap of this project is presented. In the next chapter, we will discuss about the equations statics and dynamics equations to be implemented.

Chapter 4: Slider Crank Forces Analysis

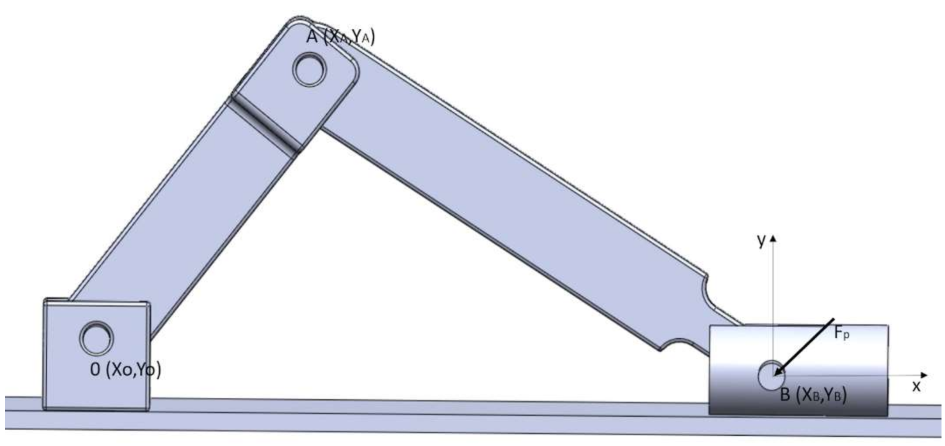
In this chapter, details on the force analysis of four-bar and six-bar slider crank mechanisms are discussed. We do not include the weight of each link and the friction applied on joints neither on statics nor dynamics analysis, but if they have to be considered, the set of equations on this chapter will be different. Additionally, we did not take in consideration slippering case. It would change the whole state of equations in the slider crank portion and the values we could get on reactions.

4.1. Possibilities of Forces Analysis on the Four Bar and Six Bar Slider Crank Slider Crank

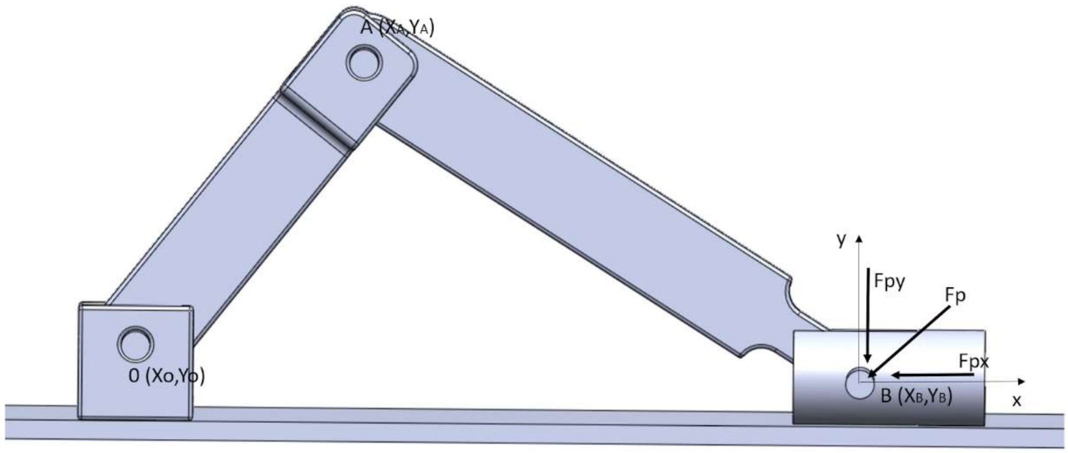
There are multiple possibilities for determining the force analysis in the slider crank four-bar and six-bar mechanisms.

4.1.1. First Possibility: Applying a Constant Input Force on the Piston

The application of a constant input force, Fp, at the piston of the slider mimics the force experienced by a piston in an internal combustion engine. Shown in **Figure 5** are two figures of a four-bar slider-crank mechanism with an applied force at the slider.



5.a



5.b

Figure 5: four-bar slider-crank mechanism with input force applied at the slider piston

In Figure 5.a, the simple representation of the input force applied, F_P, on the mechanism on point B is shown while in Figure 5.b, the decomposition on x and y axis of that same force is presented. Typically, the applied force can remain at the same angle or can vary as the linkage passes through various positions. In the case of slider-crank linkages, the applied force at the slider being at the same angle would be an accurate representation of forces in similar systems.

The coordinate system orientation will be on the way shown above; however, the origin can be the location where the force is applied. In this scenario, the solver would be used to determine all the joint forces and the torque at the crank.

4.1.2. Second Possibility: Applying a Constant Input Torque on the Input Link

The second possibility is the application of a constant input torque at the crank and then the solver can be used to determine all the joint forces and the force at the piston.

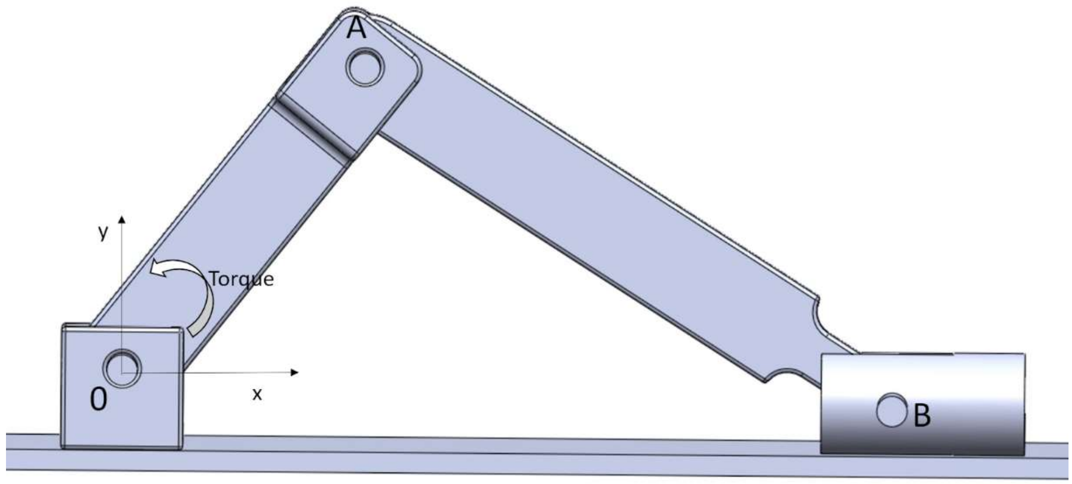
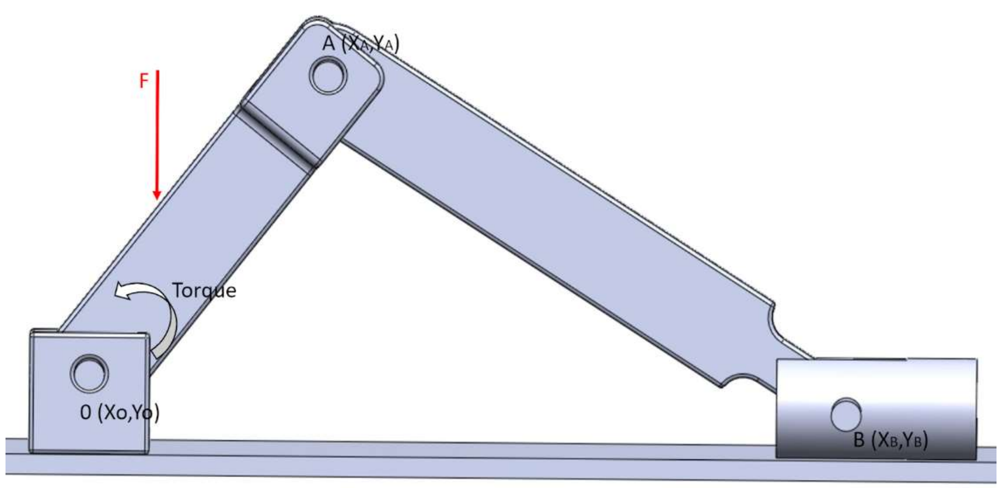


Figure 6: Four-bar slider-crank mechanism with constant input torque applied at the input link

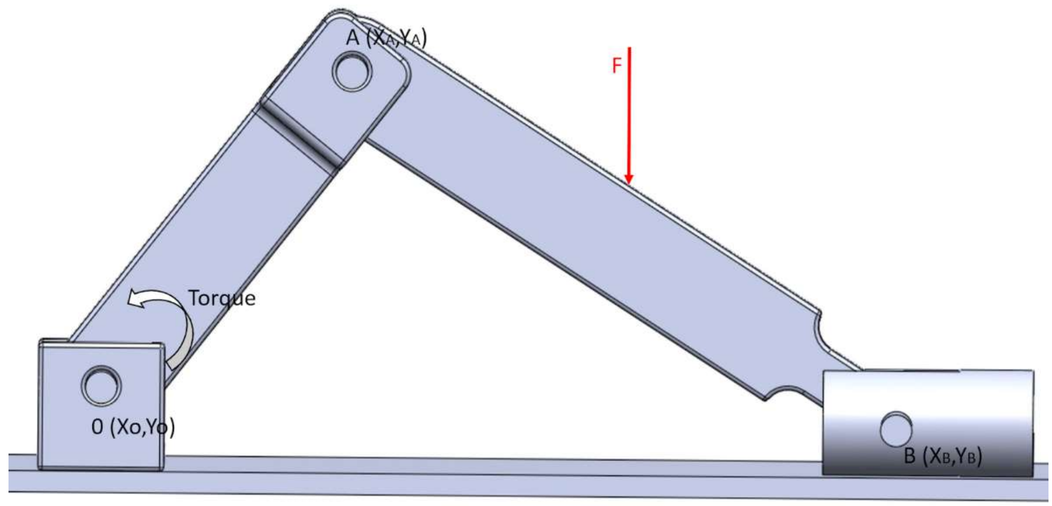
In Figure 6, the direction of the torque is counterclockwise, and it does not produce any angular acceleration because of the constant angular velocity. The axis will stay fixed at the lower extremity of the joint where the torque is applied.

4.1.3. Third Possibility: The Applied Force is not at the Slider Piston

The application of the constant input force at another location other than the piston is also a case to be considered.



7.a



7.b

Figure 7: 4-bar slider crank mechanism with constant input torque applied at the input link

As shown in **Figure 7**, the input force, F, can be applied either on the input link, 7.a, or on the coupler link, 7.b.

4. 2. Analysis Equations for Four-Bar Slider-Crank

The general process in static and dynamic analysis involves the drawing of free body diagrams, which will be followed by deriving equations and subsequently solving them.

4.2.1. Free Body Diagrams

Consider the following four-bar slider-crank mechanism shown below,

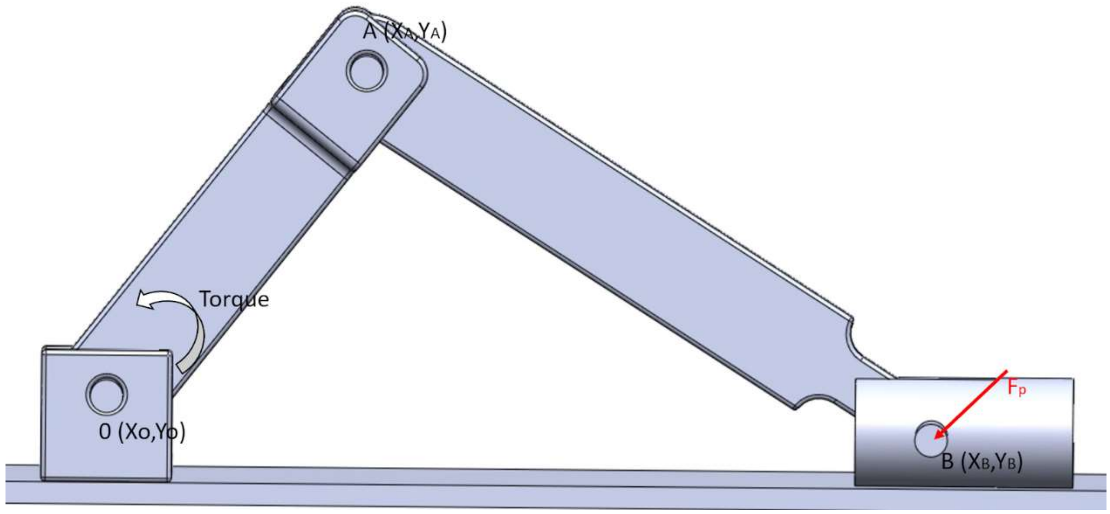


Figure 8: SolidWorks outline of a 4-bar slider crank mechanism with a known input force Fp

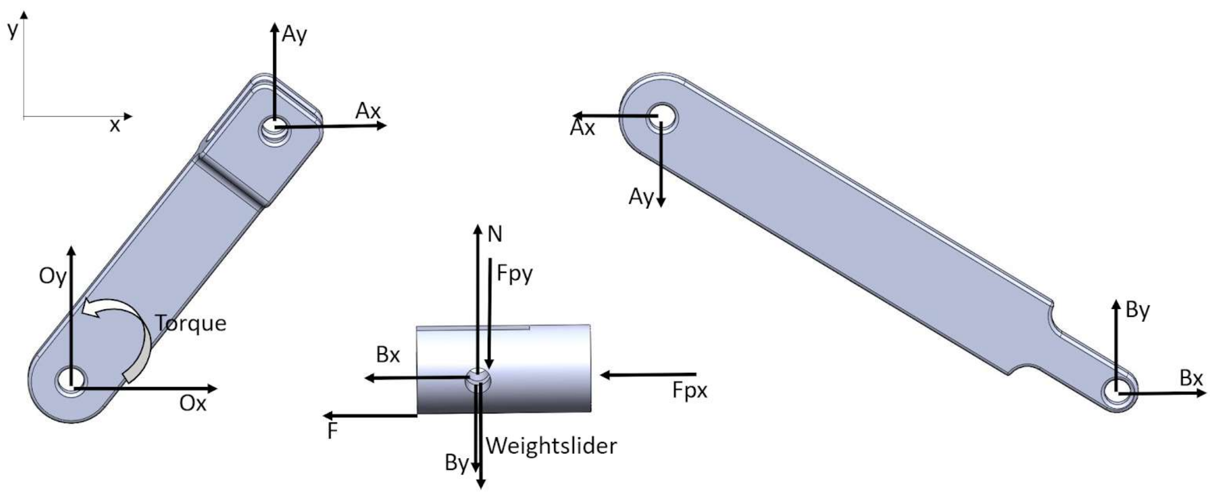
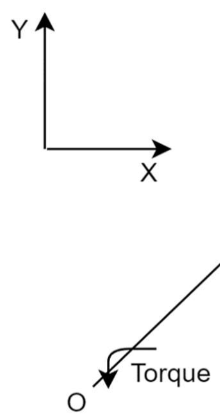


Figure 9: Free body diagram of four-bar slider crank

Figures 8 and **Figure 9** present successively the SolidWorks outline and the free body diagram decomposition of the four- bar slider-crank mechanism. Although the properties of the links such as mass, volume and length are going to vary from mechanism to mechanism, the decomposition and the resolution of linear equations are going to be the same. .

4.2.2. Obtention of Static Equations

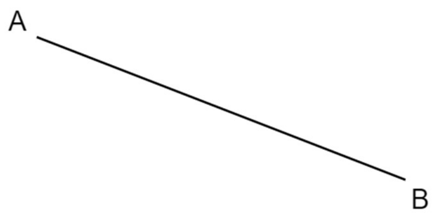
Once the free body diagrams have been drawn, the next step would be to derive the necessary equations. For a four-bar slider-crank mechanism, there are eight equations. Three equations for the first link and the second link each. The slider link will only have two equations.



The statics equations for the input link are:

$$\sum \vec{F} = \vec{0} \quad \rightarrow \quad \vec{F}_O + \vec{F}_A = \vec{0}$$

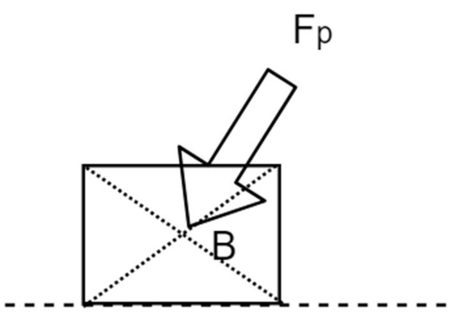
$$\sum M_O = 0 \rightarrow \text{Torque} + [\ \vec{r}_{AOx} \quad \vec{r}_{AOY} \quad 0] \times [A_X \quad A_Y \quad 0\,] = 0$$



The statics equations for the follower link are:

$\sum \vec{F} = \vec{0} \quad \rightarrow \quad \vec{F}_A + \vec{F}_B = \vec{0}$

$\sum M_B = \vec{0} \;\rightarrow\; [\; \vec{r}_{BAx} \quad \vec{r}_{BAy} \quad 0] \times [B_X \;\; B_Y \;\; 0\,] = 0$



The statics equation for the slider is

$\sum \vec{F} = \vec{0} \quad \rightarrow \vec{F}_B + \vec{F}_p + \vec{F}_{riction} + \vec{weight\_slider} + \vec{N} = \vec{0}$

Figure 10: Static equilibrium equations of a four-bar slider-crank mechanism with an applied force at the slider

In the above equations, $F_O = O_x \vec{i} + O_y \vec{j}$ is the vector force applied at joint O. O_x and O_y are its x and y components. $\vec{F}_A = A_x \vec{i} + A_y \vec{j}$ is the vector force applied at joint A. A_x and A_y are its x and y components. $\vec{F}_B = B_x \vec{i} + B_y \vec{j}$ is the vector force applied on joint B. B_x and B_y are its x and y components.

$r_{AOx} = (X_A - X_o)$, $r_{AOY} = (Y_A - Y_o)$, $r_{BAx} = (X_B - X_A)$, $r_{BAY} = (Y_B - Y_A)$ are the x and y coordinates of the position of input and follower links, OA and AB respectively.

$\vec{F}_p = F_{px} \vec{i} + F_{py} \vec{j}$ is the input vector force applied on the slider crank piston. $\vec{F}_{riction}$ is the vector friction force applied on the slider and it is opposite to its movement. $\vec{F}_{riction}$ is directed along the x axis. $\vec{weight\_slider}$ is the vector weight force of the slider. It applies at its center of gravity and it is directed on the y axis. $\vec{N}$ is the normal vector force applied by the horizontal surface on the slider and directed on the y axis. It is linked to the friction force by the formula, $F_{riction} = \mu * N$. If the weight of each link was to consider, those equations set would be different

4.2.3. Solving Equations

The above equations are decomposed into scalar form and solved simultaneously using the A*X=B format depending on the input. Here A is the 8x8 input coefficient matrix, B is the 8x1 know matrix and X is the 8x1 matrix of unknown values. We have two cases.

Case 1: Fp is known and the torque at the crank is unknown quantity that needs to be determined as shown in **Figure 11**.

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_{AOY} & -r_{AOX} & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -r_{BAY} & r_{BAX} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} O_x \\ O_y \\ A_x \\ A_y \\ B_x \\ B_y \\ N \\ \text{Torque} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Fp_x \\ \text{weight\_slider} - Fp_y \end{bmatrix}$$

Figure 11: Linear matrix equation decomposition for a four-bar slider-crank mechanism: statics analysis with Fp known and the torque is unknown

The entries of the third and sixth rows are the ones for the moment equations applied on links. r_{AOx}, r_{AOY}, r_{BAX}, r_{BAY}, μ, weight_slider, Fp_y, Fp_x are known values. The unknowns values to find are the reaction forces coordinates at each joint: Ox, Oy, Ax, Ay, Bx, By, on the mechanism as well as the normal force N applied by the plan on the slider and the Torque on the piston on point O.

Case 2: Fp is unknown that needs to be determined in **Figure 12** and the torque at the crank is known quantity

In this case, the equations from the case 1 are going to be valid here. It is just that the equations have to be rearranged and solved.

Where A_2 is the 8x8 input coefficient matrix, B_2 is the 8x1 know matrix and X_2 is the 8x1 matrix to be determined. For the above 4-bar slider crank mechanism

$$
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & r_{AOy} & -r_{AOx} & 0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -r_{BAy} & r_{BAx} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & \mu & 0 & \mu
\end{bmatrix}
\cdot
\begin{bmatrix}
O_x \\ O_y \\ A_x \\ A_y \\ B_x \\ B_y \\ Fp_x \\ Fp_y
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ \text{Torque} \\ 0 \\ 0 \\ 0 \\ 0 \\ \mu\text{*weight\_slider}
\end{bmatrix}
$$

Figure 12: Linear matrix equation decomposition of matrix for a 4-bar slider crank mechanism: statics analysis with Fp unknown and the torque is known

r_{AOx}, r_{AOY}, r_{BAx}, r_{BAY}, **μ**, weight_slider, Torque are known values. The unknowns values to find are the reaction forces coordinates at each joint: Ox, Oy, Ax, Ay, Bx, By, on the mechanism as well as the normal force N applied by the plan on the slider and the Fp_y, Fp_x on the piston on point B.

4. 3. Dynamics Analysis Equations for Slider-crank

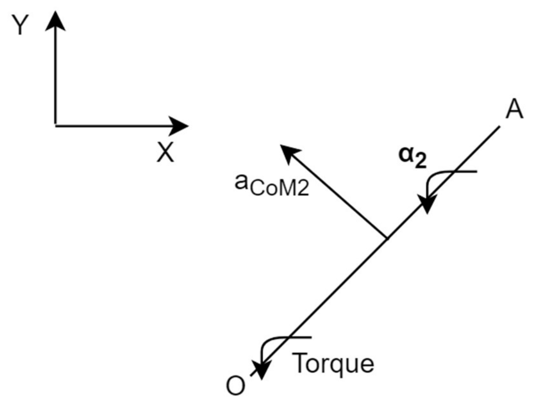
The overall process for dynamic analysis is similar to static analysis. However, angular velocities and angular accelerations of links, acceleration at the mass centers of each link, the mass and the mass moment of inertia of each link needs to be determined. Currently, PMKS can methods to calculate all the aforementioned quantities. During the process, the joints are assumed to be frictionless.

4. 3.1. Free body Diagram of the above Four-Bar Slider Mechanism

The dynamics free body diagram is very similar to the static free body diagrams. The difference will be that the center of mass and the accelerations at mass centers are displayed as shown in Figure 14. Also, the angular accelerations can also be displayed for each link For the slider, the same forces decomposition is displayed like in statics; however, its translational acceleration vector is added and displayed this time.

4.3.2. Obtention of Equations on each Link

The equations are based on Newton's second law and are listed

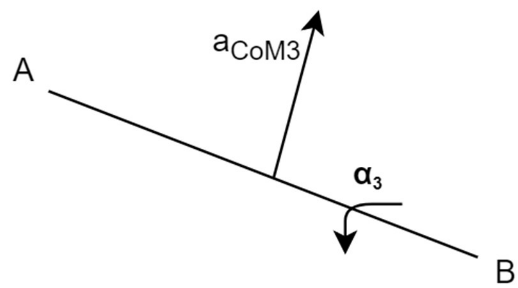


below.

The dynamics equations for the input link are

$$\sum \vec{F} = m_{2*} \vec{a}_{CoM2} \rightarrow \vec{F}o + \vec{F}_A = m_{2*} \vec{a}_{CoM2}$$

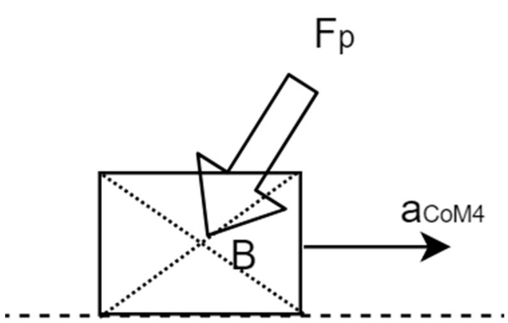
$$\sum M_o = I_{2*}\boldsymbol{\alpha}_2 \rightarrow \text{Torque} + [\ \vec{r}_{AOX}\ \ \vec{r}_{AOY}\ \ 0] \times [A_X\ \ A_Y\ \ 0\,] = I_{2*}\boldsymbol{\alpha}_2$$



The dynamics equations for the follower link are:

$\sum \vec{F} = m_{3*} \vec{a}_{CoM3} \rightarrow \vec{F}_A + \vec{F}_B = m_{3*} \vec{a}_{CoM3}$

$\sum M_B = I_{3*}\boldsymbol{\alpha}_3 \rightarrow [\vec{r}_{BAX} \quad \vec{r}_{BAY} \quad 0] \times [B_X \quad B_Y \quad 0] = I_{3*}\boldsymbol{\alpha}_3$



$\sum \vec{F} = m_{slider\_crank} * \vec{a}_{CoM4} \rightarrow \vec{F}_B + \vec{F}_p + \vec{F}_{riction} + \overrightarrow{weight\_slider} + \vec{N} = m_{slider\_crank} * \vec{a}_{CoM4}$

Figure 13: Free body diagram of a 4-bar slider crank mechanism and equations on each link: dynamics analysis

4.3.3. Solving Equations

The decomposition shown in the statics case for the force vectors $\vec{F}o$, $\vec{F}A$, $\vec{F}B$, $\vec{F}p$ applies in the dynamics case. $\vec{Friction}$ is the vector friction force applied on the slider and it is opposite to its movement. $\vec{Friction}$ is directed alongth x axis. $\vec{weight\_slider}$ is the vector weight force of the slider. It applies at its center of gravity and it is directed along the y axis. $\vec{N}$ is the normal vector force applied by the horizontal surface on the slider and directed along the x axis. It is linked to the friction force by the formula Friction = μ*N. $r_{AOX} = (X_A-X_o)$, $r_{AOY} = (Y_A-Y_o)$, $r_{BAX} = (X_B-X_A)$, $r_{BAY} = (Y_B-Y_A)$ are the x and y coordinates of the position of input and follower links, OA and AB respectively. m_2, I_2, a_{CoM2} ,α_2, are respectively the mass, moment of inertia, linear acceleration at its center of mass and angular acceleration of the input link on the 4-bar slider crank mechanism while m_3, I_3, a_{CoM3} ,α_3 are respectively the mass, moment of inertia, linear acceleration at its center of mass and angular acceleration of the follower link. $m_{slider,}$ and a_{CoM4} are the mass and the acceleration at the center of mass of the slider.

$$\vec{F}o + \vec{F}_A = m_{2*} \vec{a}_{CoM2} \qquad (1)$$

$$Torque + [\ \vec{r}_{AOX} \quad \vec{r}_{AOY} \quad 0] \times [A_X \ \ A_Y \ \ 0\,] = I_{2*}\alpha_2 \qquad (2)$$

$$\vec{F}_A + \vec{F}_B = m_{3*} \vec{a}_{CoM3} \qquad (3)$$

$$[\ \vec{r}_{BAX} \quad \vec{r}_{BAY} \quad 0] \times [B_X \ \ B_Y \ \ 0\,] = I_{3*}\alpha_3 \qquad (4)$$

$$F_B + \vec{F}p + \vec{F}riction + \vec{weight\_slider} + \vec{N} = m_{slider*} \vec{a}_{CoM4} \qquad (5)$$

By solving those equations using linear equations calculations, we got A*X = B;
This time the intensities forces at joints and the torque would be different from the ones in statics analysis.

Case 1: Fp is known and the torque at the crank is unknown quantity that needs to be determined as shown in **Figure 14**.

$$
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & r_{AOY} & -r_{AOX} & 0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -r_{BAY} & r_{BAX} & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & -\mu & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
*
\begin{bmatrix}
O_x \\ O_y \\ A_x \\ A_y \\ B_x \\ B_y \\ N \\ \text{Torque}
\end{bmatrix}
=
\begin{bmatrix}
m_2 * a_{CoM2x} \\
m_2 * a_{CoM2y} \\
I_2 * \alpha_2 \\
m_3 * a_{CoM3x} \\
m_3 * a_{CoM3y} \\
I_3 * \alpha_3 \\
Fp_x + m_{slider} * a_{CoM4x} \\
\text{weight\_slider} + Fp_y + m_{slider} * a_{CoM4y}
\end{bmatrix}
$$

Figure 14: Linear matrix equation decomposition of matrix for a four-bar slider-crank mechanism: dynamics analysis with Fp known and the torque is unknown

Where A_2 is the same 8x8 input coefficient matrix as in statics analysis, B_3 is the 8x1 know matrix and X_3 is the 8x1 matrix to be determined. r_{AOX}, r_{AOY}, r_{BAX}, r_{BAY}, μ, weight_slider, Fp_y, Fp_x, I_2, I_3, α_2, α_3 are known values. The unknown values to be found are the reaction forces coordinates at each joint: Ox, Oy, Ax, Ay, Bx, By, on the mechanism as well as the normal force N applied by the plan on the slider and the Torque on the piston on point O.

Case 2: Fp is unknown that needs to be determined in **Figure 15** and the torque at the crank is known quantity

Here, the same set of equations can be used as in case 1. It is just that the knowns and unknowns change. r_{AOX}, r_{AOY}, r_{BAX}, r_{BAY}, μ, weight_slider, Torque are known values. The unknowns values to find are the reaction forces coordinates at each joint: Ox, Oy, Ax, Ay, Bx, By, on the mechanism as well as the normal force N applied by the plan on the slider and the Fp_y, Fp_x on the piston on point B.

By solving all those equations we got the linear matrix equation we got $A_2 * X_4 = B_4$ where A_2 is the same 8x8 input coefficient matrix as in statics analysis, B_4 is the 8x1 know matrix and X_4 is the 8x1 matrix to be determined.

$$
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & r_{AOy} & -r_{AOx} & 0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -r_{BAy} & r_{BAx} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & \mu & 0 & \mu
\end{bmatrix}
\begin{bmatrix}
O_x \\ O_y \\ A_x \\ A_y \\ B_x \\ B_y \\ Fp_x \\ Fp_y
\end{bmatrix}
=
\begin{bmatrix}
m_2 * a_{CoM2x} \\
m_2 * a_{CoM2y} \\
I_2 * \alpha_2 - \text{Torque} \\
m_3 * a_{CoM3x} \\
m_3 * a_{CoM3y} \\
I_3 * \alpha_3 \\
0 \\
m_{slider} * a_{CoM3x} + \mu * (m_{slider} * a_{CoM3y} + \text{weight\_slider})
\end{bmatrix}
$$

Figure 15: Linear matrix equation decomposition for a four-bar slider-crank mechanism: dynamics analysis with Fp unknown and the torque is known

r_{AOx}, r_{AOY}, r_{BAX}, r_{BAY}, **μ**, weight_slider, Torque are known values. The unknowns values to find are the reaction forces coordinates at each joint: Ox, Oy, Ax, Ay, Bx, By, on the mechanism as well as the normal force N applied by the plan on the slider and the Fp_y, Fp_x on the piston on point B.

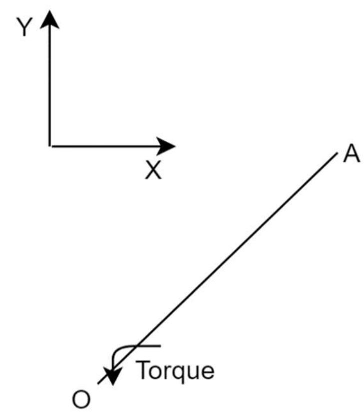
4.4. Generic Case of the Four-Bar Slider-Crank

The generic case of the 4-bar slider crank is when we have the slider on an oblique direction to its axis of reference, the horizontal x-axis.

4.4.1. Statics

4.4.1.1. Free Body Diagram of the Four Bar Slider Mechanism

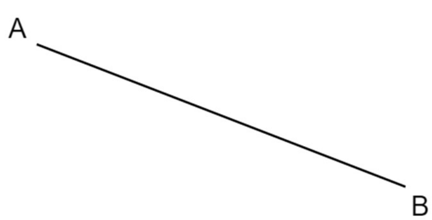
The same process of generating equations as on section **4.2** applies here. However, the angle formed by the slider and the horizontal axis, x, need to be considered as it varies. The joints still assumed to be frictionless.



The statics equations for the input link are:

$\sum \vec{F} = \vec{0} \ \rightarrow \ \vec{F}_O + \vec{F}_A = \vec{0}$

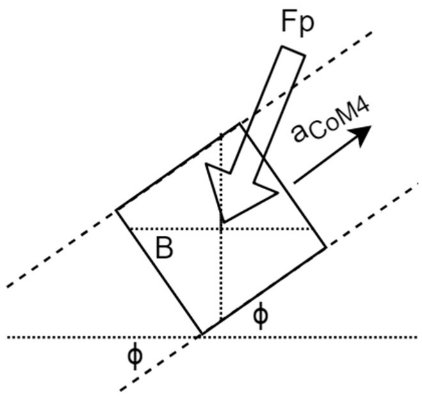
$\sum M_o = 0 \ \rightarrow$ Torque $+ [\ \vec{r}_{AOx} \quad \vec{r}_{AOY} \quad 0] \times [A_X \quad A_Y \quad 0\,] = 0$



The statics equations for the follower link are:

$\sum \vec{F} = \vec{0} \ \rightarrow \ \vec{F}_A + \vec{F}_B = \vec{0}$

$\sum M_B = 0 \ \rightarrow \ [\ \vec{r}_{BAx} \quad \vec{r}_{BAY} \quad 0] \times [B_X \quad B_Y \quad 0\,] = 0$



The statics equation for the slider is

$$\sum \vec{F} = \vec{0} \rightarrow \vec{F}_B + \vec{F}_p + \vec{F}_{riction} + \overrightarrow{weight\_slider} + \vec{N} = \vec{0}$$

Figure 16: Free body diagram of a four-bar slider crank mechanism and equations on each link: statics analysis generic situation

4.4.1.2. Solving Equations

The same decomposition in statics from section **4.2** of forces vectors $\vec{F}o$, $\vec{F}A$, $\vec{F}B$, $\vec{F}p$ applies here. $\vec{F}riction = Friction_x \vec{i} + Friction_y \vec{j}$ is the vector friction force applied on the slider and it is opposite to its movement. $\overrightarrow{weight\_slider}$ is the vector weight force of the slider. It applies at its center of gravity and it is directed on the y axis. $\vec{N} = N_x \vec{i} + N_y \vec{j} = -N * \sin(\phi) \vec{i} + N * \cos(\phi) \vec{j}$ is the normal vector force applied by the horizontal surface on the slider. It is linked to the friction force by the formula Friction = μ*N. $r_{AOX} = (X_A - X_O)$, $r_{AOY} = (Y_A - Y_O)$, $r_{BAX} = (X_B - X_A)$, $r_{BAY} = (Y_B - Y_A)$ are the x and y coordinates of the position of input and follower links, OA and AB respectively. m_{slider} is the mass of the slider.

$\vec{F}_O + \vec{F}_A = \vec{0}$ (1)

Torque + $[\ \vec{r}_{AOX}\ \ \vec{r}_{AOY}\ \ 0] \times [A_X\ \ A_Y\ \ 0\,] = 0$ (2)

$\vec{F}_A + \vec{F}_B = 0$ (3)

$[\ \vec{r}_{BAX}\ \ \vec{r}_{BAY}\ \ 0] \times [B_X\ \ B_Y\ \ 0\,] = 0$ (4)

$\vec{F}_B + \vec{F}_p + \vec{F}_{riction} + \overrightarrow{weight\_slider} + \vec{N} = \vec{0}$ (5)

By replacing the equations of $\vec{F}$riction and $\vec{N}$, and using linear equations calculations, we got the matrix equation: A*X = B;

Case 1: Fp is known and the torque at the crank is unknown quantity that needs to be determined as shown in **Figure 17.**

$$
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & r_{AOY} & -r_{AOX} & 0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -r_{BAY} & r_{BAX} & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & -(\sin\phi+\mu \mathrm{co}\phi) & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & (\cos\phi-\mu\sin\phi) & 0
\end{bmatrix}
*
\begin{bmatrix}
O_x \\ O_y \\ A_x \\ A_y \\ B_x \\ B_y \\ N \\ \text{Torque}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Fp_x \\ \text{weight\_slider} - Fp_y
\end{bmatrix}
$$

Figure 17: Linear matrix equation decomposition of matrix for the general four-bar slider crank mechanism: statics analysis with Fp known and the torque unknown in generic situation

A_3 is the same 8x8 input coefficient matrix as in statics analysis, B_1 is the 8x1 know matrix and X_1 is the 8x1 matrix to be determined. r_{AOX}, r_{AOY}, r_{BAX}, r_{BAY}, μ, m_{slider}, weight_slider, Fp_y, Fp_x are known values while the forces: O_x , O_y, A_x, A_y, B_x, B_y, N and Torque are unknowns.

Case 2: Fp is unknown that needs to be determined in **Figure 18** and the torque at the crank is known quantity

The equations on section 4.4.1.2 still applied here. However, there are changes on unknowns and knowns. Knows are: r_{AOX}, r_{AOY}, r_{BAX}, r_{BAY} , μ, weight_slider, Torque are known values. The unknowns values to find are the reaction forces coordinates at each joint: Ox, Oy, Ax, Ay, Bx, By, on the mechanism as well as the normal force N applied by the plan on the slider and Fp_y, Fp_x on the piston on point B.

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_{AOy} & -r_{AOx} & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -r_{BAy} & r_{BAx} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tan\phi & 0 & -\tan\phi \end{bmatrix} * \begin{bmatrix} O_x \\ O_y \\ A_x \\ A_y \\ B_x \\ B_y \\ Fp_x \\ Fp_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \text{Torque} \\ 0 \\ 0 \\ 0 \\ 0 \\ \text{weight\_slider} \end{bmatrix}$$

Figure 18: Linear matrix equation decomposition of matrix for the general four-bar slider crank mechanism: statics analysis with Fp unknown and the torque known in generic situation

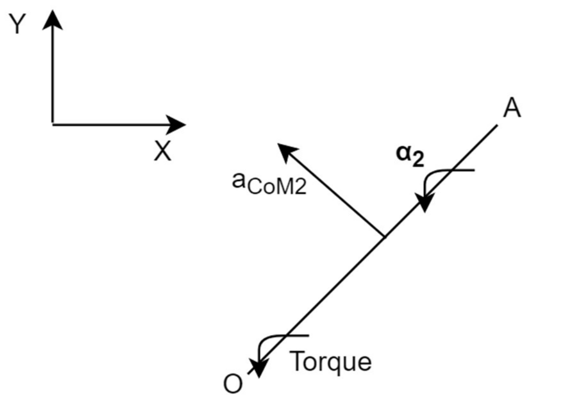
By solving the linear matrix equation, A*X = B. We got A_4 which is the 8x8 input coefficient matrix as in statics analysis, B_4 is the 7x1 know matrix and X_4 is the 8x1 matrix to be determined.

4.4.2. Dynamics

4.4.2.1. Free Body Diagram of the Four-Bar Slider-Crank Mechanism

The same process of generating equations with slider inclined to the horizontal axis applied in the dynamics for the input and follower link. However, the angle formed by the slider and the horizontal axis, x, need to be considered as it varies in the dynamics situation too. The joints are still assumed to be frictionless.

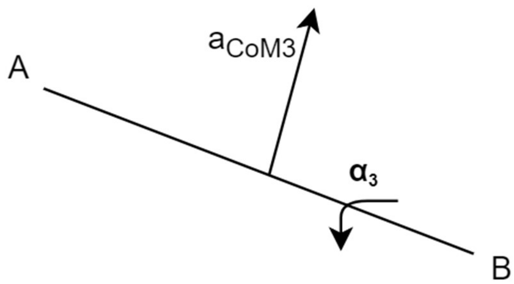
4.4.2.2. Obtention of equations on each link



The dynamics equations for the input link are

$\sum \vec{F} = m_{2*} \vec{a}_{CoM2} \rightarrow \vec{F}o + \vec{F}_A = m_{2*} \vec{a}_{CoM2}$

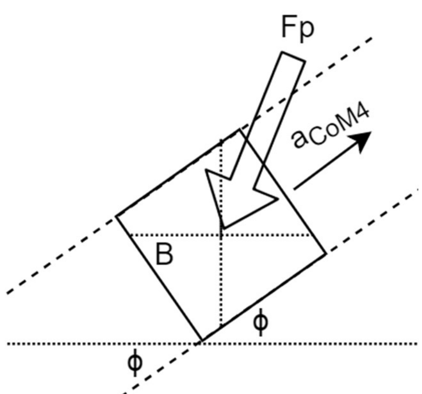
$\sum M_o = I_{2*}\boldsymbol{\alpha}_2 \rightarrow$ Torque + [$\vec{r}_{AOX}$ $\vec{r}_{AOY}$ 0] x [A_X A_Y 0] = $I_{2*}\boldsymbol{\alpha}_2$



The dynamics equations for the follower link are:

$\sum \vec{F} = m_{3*}\ \vec{a}_{CoM3} \rightarrow \vec{F}_A + \vec{F}_B = m_{3*}\ \vec{a}_{CoM3}$

$\sum M_B = I_{3*}\boldsymbol{\alpha}_3 \rightarrow [\ \vec{r}_{BAX}\quad \vec{r}_{BAY}\quad 0]\ x\ [B_X\ \ B_Y\ \ 0\ ] = I_{3*}\boldsymbol{\alpha}_3$



The dynamics equation for the slider is

$\sum \vec{F} = m_{slider\_crank} *\ \vec{a}_{CoM4} \rightarrow \vec{F}_B + \vec{F}_p + \vec{F}_{riction} + \overrightarrow{weight\_slider} + \vec{N} = m_{slider\_crank} *\ \vec{a}_{CoM4}$

Figure 19: Free body diagram of a four-bar slider-crank mechanism and equations on each link: dynamics analysis

4.4.2.3. Solving Equations

The decomposition as in section 4.4.1.2 of forces vectors $\vec{F}o$, $\vec{F}A$, $\vec{F}B$, $\vec{F}p$ applies here. Friction force decomposition, weight_slider and normal force are the same as in 4.4.1.2. r_{AOx}, r_{AOY}, r_{BAX}, r_{BAY} are the x and y coordinates of the position of input and follower links, OA and AB respectively. m_{slider} is the mass of the slider.

$$\vec{F}o + \vec{F}_A = m_{2*} \vec{a}_{CoM2}$$

$$Torque + [\vec{r}_{AOx} \quad \vec{r}_{AOY} \quad 0] \times [A_X \quad A_Y \quad 0] = I_{2*}\alpha_2$$

$$\vec{F}_A + \vec{F}_B = m_{3*} \vec{a}_{CoM3}$$

$$[\vec{r}_{BAX} \quad \vec{r}_{BAY} \quad 0] \times [B_X \quad B_Y \quad 0] = I_{3*}\alpha_3$$

$$\vec{F}_B + \vec{F}p + \vec{F}riction + \overrightarrow{weight\_slider} + \vec{N} = m_{slider\_crank} * \vec{a}_{CoM4}$$

Case 1: Fp is known and the torque at the crank is unknown quantity that needs to be determined as shown in **Figure 19**

Using the relationship $F_{riction} = \boldsymbol{\mu}*N \rightarrow Friction_x \vec{i} + Friction_y \vec{j} = \boldsymbol{\mu}* (N_x \vec{i} + N_y \vec{j})$,

on the x axis: $\sum F_x = (\boldsymbol{\mu} + 1)N_x + B_x - Fp_x$ (a)

on the y axis: $\sum F_y = (\boldsymbol{\mu} + 1)N_y - weight\_slider - B_y + Fp_y$ (b)

By replacing N coordinates in both equations,

we got $\sum F_x = -(\boldsymbol{\mu} + 1)N\sin(\phi) + B_x - Fp_x$

$\sum F_y = (\boldsymbol{\mu} + 1)N\cos(\phi)$ - weight_slider - $B_y + Fp_y$

Applying dynamics equations $\sum F_x = m_{slider*}\, a_{CoM4X}$ and $\sum F_y = m_{slider*}\, a_{CoM4Y}$ on the previous equations,

By solving the linear previous equations, we get the linear matrix equations A*X = B. We got A_5 is the 8x8 input coefficient matrix, B_5 is the 8x1 know matrix and X_1 is the 8x1 matrix to be determined.

$$
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & r_{AOY} & -r_{AOX} & 0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -r_{BAY} & r_{BAX} & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & -(\boldsymbol{\mu}+1)\sin\boldsymbol{\phi} & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & (\boldsymbol{\mu}+1)\cos\boldsymbol{\phi} & 0
\end{bmatrix}
*
\begin{bmatrix}
O_x \\ O_y \\ A_x \\ A_y \\ B_x \\ B_y \\ N \\ \text{Torque}
\end{bmatrix}
=
\begin{bmatrix}
m_2 * a_{CoM2x} \\
m_2 * a_{CoM2y} \\
I_2 * \boldsymbol{\alpha}_2 \\
m_3 * a_{CoM3x} \\
m_3 * a_{CoM3y} \\
I_3 * \boldsymbol{\alpha}_3 \\
Fp_x + m_{slider} * a_{CoM4x} \\
\text{weight\_slider} + Fp_y + m_{slider} * a_{CoM4y}
\end{bmatrix}
$$

Figure 20: Linear matrix equation decomposition of matrix for the general four-bar slider crank mechanism: dynamics analysis with Fp known and the torque unknown in generic situation

r_{AOx}, r_{AOY}, r_{BAX}, r_{BAY}, $\boldsymbol{\mu}$, weight_slider, Torque are known values. The unknowns values to find are the reaction forces coordinates at each joint: Ox, Oy, Ax, Ay, Bx, By, on the mechanism as well as the normal force N applied by the plan on the slider and Fp_y, Fp_x on the piston on point B.

Case 2: Fp is unknown that needs to be determined in **Figure 21** and the torque at the crank is known quantity

Here, the above equation in section 4.4.2.3 need to be arranged. Changes happen on known and unknown values. r_{AOx}, r_{AOY}, r_{BAX}, r_{BAY}, $\boldsymbol{\mu}$, weight_slider, Torque are known values. The unknowns values to find are the reaction forces coordinates at each joint: Ox, Oy, Ax, Ay, Bx, By, on the mechanism as well as the normal force N applied by the plan on the slider and Fp_y, Fp_x on the piston on point B.

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_{AOy} & -r_{AOx} & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -r_{BAy} & r_{BAx} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tan\phi & 0 & -\tan\phi \end{bmatrix} * \begin{bmatrix} O_x \\ O_y \\ A_x \\ A_y \\ B_x \\ B_y \\ Fp_x \\ Fp_y \end{bmatrix} = \begin{bmatrix} m_2 * a_{CoM2x} \\ m_2 * a_{CoM2y} \\ I_2 * \alpha_2 - \text{Torque} \\ m_3 * a_{CoM3x} \\ m_3 * a_{CoM3y} \\ I_3 * \alpha_3 \\ 0 \\ m_{slider} * a_{CoM3x} + \mu * (m_{slider} * a_{CoM3y} + \text{weight\_slider}) \end{bmatrix}$$

Figure 20: Linear matrix equation decomposition of matrix for the general 4-bar slider crank mechanism: statics analysis with Fp unknown and the torque known in generic situation

r_{AOX}, r_{AOY}, r_{BAX}, r_{BAY}, μ, weight_slider, Torque are known values. The unknowns values to find are the reaction forces coordinates at each joint: Ox, Oy, Ax, Ay, Bx, By, on the mechanism as well as the normal force N applied by the plan on the slider and Fp_y, Fp_x on the piston on point B

4. 5. Analysis Equations for Six-Bar Slider-Crank

The analysis process of equations on the six-bar slider crank is the same as in the four-bar slider-crank. However, instead of seven equations and eight unknowns to find, it would be eleven equations to developed and 12 unknowns to find.

This chapter presented all the equations required for statics and dynamics analysis of four-bar slider-crank mechanism. The equations can be scaled for six-bar slider crank mechanisms as well. In the next chapter, the equations are implemented in MATLAB and Working Model results of different tests cases discussed.

Chapter 5: Implementation of Equations

In this chapter, we are discussing the various implementation of the equations developed previously from test cases in MATLAB and Working Model software.

5.1. MATLAB Implementation and Test Case

To make our MATLAB code work, we made some assumptions. The first assumption was to consider the previous distances coordinates of link applied at the middle of joints instead of center of gravity. This means

$$R_{12x} = (x_1 - x_2)/2;$$

$$R_{12y} = (y_1 - y_2)/2;$$

$$R_{23x} = x_1 - x_2;$$

$$R_{23y} = y_1 - y_2;$$

$$R_{32x} = -R_{23x};$$

$$R_{32y} = -R_{23y};$$

$$R_{34x} = R_{23x} - x_3;$$

$$R_{34y} = R_{23y} - y_3;$$

$$R_{43x} = -R_{34x};$$

$$R_{43y} = -R_{34y};$$

The following are the inputs considered:

$$w_{AO} = 1 \text{ rad/s};$$

$$x_O = 0 \text{ m};$$

$$y_O = 0 \text{ m};$$

$$x_A = 0.016 \text{ m};$$

$$y_A = 0.012 \text{ m};$$

$$x_B = 0.049 \text{ m};$$

$$y_B = 0 \text{ m};$$

$$\boldsymbol{\mu} = 0.3;$$

$\boldsymbol{\theta}$ = 38.2924□ ;

Fp_x = -0.087580 N;

Fp_y = -0.133469 N;

linkDensity = 1 g/cm3;

pinDensity = 1 g/cm3;

pinDiameter = 0 m;

MassOfInputLink = 0.000080 kg;

MassOfFollowerLink = 0.000140 kg;

mass_slider = 0.000043 kg;

w_{AO} is the input angular velocity of the input link. Fpx and Fpy are the coordinates of the input forces applying on the slider. Theta is the angle formed by the input link and the ground link. We found the following outputs data of forces and torque applied on the 4-bar slider mechanism in **Figure 21** below

MATLAB outputs	
F_{12X}	0.087569 N
F_{12Y}	-0.03141 N
F_{32X}	-0.04639 N
F_{32Y}	0.080954 N
F_{43X}	0.087577 N
F_{43Y}	-0.03357 N
T_{12}	0.001557 Nm

Figure 21: MATLAB forces outputs

We did not get the normal force F_{14Y} applied by the ground link on the slider. The data are all in the range of 10E-2.

5.2. Implementation in Working Model Software

A simulation of the four-bar slider-crank mechanism was implemented in Working Model 2-D software as shown in **Figure 22**.

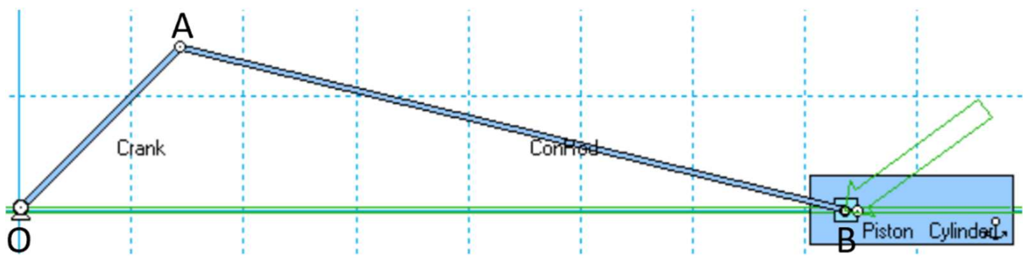


Figure 22: Working Model 4-bar slider crank mechanism

Here are the output forces we got

Working Model outputs	
O_x	-0.46992 N
O_y	-0.13924 N
A_x	0.46992 N
A_y	0.13924 N
B_x	0.46992 N
B_y	0.13924 N
Torque	0.005117 Nm

Figure 23: Working Model outputs forces at joints

O_x, O_y, A_x, A_y, B_x, B_y represent the forces coordinates at joints O, A and B respectively.

5.3. Comparison between MATLAB and Working Model Outputs

On both Working Model and MATLAB, we used the same mass of the slider, 0.000043 kg; we used the same mass at the input and follower link, 0.00008 kg and 0.00014 kg. We used the same input angular velocity on Working Model and MATLAB, 1 rad/s.
Here is the comparison we came up with.

Forces Coordinates	Working Model Results	MATLAB Results
O_x	-0.46992 N	0.087569 N
O_y	-0.13924 N	-0.03141 N
A_x	0.46992 N	-0.04639 N
A_y	0.13924 N	0.080954 N
B_x	0.46992 N	- 0.087577 N
B_y	0.13924 N	- 0.03357 N
Torque	0.005117 Nm	0.001557 Nm

Table 1: Comparison table between Working Model and MATLAB outputs forces at joints

The sign convention of forces coordinates are not even the same on MATLAB and Working Model. We did not get the chance to establish a MATLAB code with a known input torque and compare with the same input torque value at the motor on a four-bar slider crank. We suggest the next team to take a look at it, because we estimated probably data of force could be as close as possible. In the meantime, we realized that Working Model is a commercial software that we do not actually know the mathematics calculations process behind while MATLAB is a commercial tool where we actually wrote the code in order for its to produce such results. So, we are not very surprised of the comparison shown on **Table 1**.

Chapter 6: Conclusion and Recommendations

In this major qualifying project, the equations for forces analysis of slider-crank linkages that can be implemented within PMKS were investigated. Equations for four-bar and six-bar slider-crank mechanisms were sourced from textbooks. The equations were then implemented in MATLAB and different slider-crank mechanisms were tested. Those examples slider-crank mechanisms were also tested in Working Model 2-D software. Upon comparison of the results, there are discrepancies in the joint forces and torque between the two implementations.

In terms of future work, the goal would be to identify the reasons behind the differences in results between the MATLAB and Working Model results. SolidWorks can be used as a third software to see the difference around the results of forces found. Additionally. Another approach could be to study the slippering case on the slider crank and include the weight and friction forces on linkages and joints to see how results vary from what we have found so far.

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Appendix

Appendix A: MATLAB Code of Statics and Dynamics Analysis of the Four and Six Slider Crank

The code has for inputs the different coordinates of joints links, mass of each link, the input angular velocity of the coupler, link depth, link width, link density, pin diameter, mass of the slider, coefficient of friction, input force applied on the slider and angle of inclination of the coupler link. It starts by asking the user to choose if he is looking to do a four or six bar slider-crank calculations. From there the user starts enter the known values in order to generate reactions intensities at each joint of the link and the corresponding torque applied on the coupler.

```
% wBA = input ('Enter wBA = '); %constant angular velocity in z direction
% x1 = input ('Enter x1 = ');
% y1 = input ('Enter y1 = ');
% x2 = input ('Enter x2 = ');
% y2 = input ('Enter y2 = ');
% x3 = input ('Enter x3 = ');
% y3 = input ('Enter y3 = ');

wBA = 1;
x1 = 0;
y1 = 0;
x2 = 0.016;
y2 = 0.012;
x3 = 0.049;
y3 = 0;

ABx = x2 - x1;
mABx = ABx/2;
ABy = y2 - y1;
mABy = ABy/2;
```

```
BCx = x3 - x2;
mBCx = BCx/2;
BCy = y3 - y2;
mBCy = BCy/2;

% depth = input('depth:   '); % 0.000080 kg
% width = input('width:   ');
% linkDensity = input('density of link:  ');
% pinDensity = input('density of pin:  ');
% pinDiameter = input('pin Diameter:   ');

depth = 0.004;%m
width1 = 0.020000;
width2 = 0.035000;
linkDensity = 1;
pinDensity = 1;
pinDiameter = 0;

% friction_coefficient = input('Enter friction_coefficient = '); %coefficient of friction
% mass_slider = input('Enter mass_slider = '); %mass_slider
% theta = input('Enter theta = '); %angle between the ground link and the second link
% phi = input('Enter phi = '); %angle of inclination of the slider
% Fpx = input ('Enter Fpx = '); %force on the x direction
% Fpy = input ('Enter Fpy = '); %force on the y direction
friction_coefficient = 0.3;
mass_slider = 0.000043;
theta = 38.2924; %angle between the ground link and the second link
phi = 0; %angle of inclination of the slider
Fpx = -0.087580;
Fpy = -0.133469; %force on the x direction
```

```
length1 = sqrt(ABx^2 + ABy^2);
length2 = sqrt(BCx^2 + BCy^2);
rectVolume1 = length1 * width * depth;
rectMass1 = rectVolume1 * linkDensity;
rectVolume2 = length2 * width * depth;
rectMass2 = rectVolume2 * linkDensity;
bigSemiCircVolume = (width/2)^2*pi*depth/2;
bigSemiCircMass = bigSemiCircVolume * linkDensity;
pinCircleVolume = pi*(pinDiameter/2)^2 * depth;
pinCircleMass = pinCircleVolume * pinDensity;

% MassOfLink1 = rectMass1 + 2*bigSemiCircMass - 2*pinCircleMass;
% MassOfLink2 = rectMass2 + 2*bigSemiCircMass - 2*pinCircleMass;
MassOfLink1 = 0.000080;
MassOfLink2 = 0.000140;


%%Rectangle MMoI
RectMMoI1 = 1/12*MassOfLink1*(depth^2 + width1^2);
RectMMoI2 = 1/12*MassOfLink2*(depth^2 + width2^2);

%%BigSemiCylinder Mass Moment of Inertia
semiCylinderMMoI = bigSemiCircMass* power(width/2,2)/2;
MovedSemiCylinMMoI1 = semiCylinderMMoI + bigSemiCircMass*power(length1/2,2);
MovedSemiCylinMMoI2 = semiCylinderMMoI + bigSemiCircMass*power(length2/2,2);

%%PinCircles Mass Moment of Inertia
PinCircMMoI = 1/2*pinCircleMass * power(pinDiameter/2,2);
MovedPinCircMMoI1= PinCircMMoI + pinCircleMass* power(((length1/2)),2);
MovedPinCircMMoI2= PinCircMMoI + pinCircleMass* power(((length2/2)),2);

%%Total Mass Moment of Inertia for a Binary Link
```

```
BinaryLinkMMoI1 = RectMMoI1 + (2*MovedSemiCylinMMoI1) - (2*MovedPinCircMMoI1);
BinaryLinkMMoI2 = RectMMoI2 + (2*MovedSemiCylinMMoI2) - (2*MovedPinCircMMoI2);

%V(B/A) + V(B/C) = V_slider
c1 = [0 0 wBA];
c2 = [ABx ABy 0];
c3 = cross(c1, c2);
c4 = [0 0 1];
c5 = [BCx BCy 0];
c6 = cross(c4, c5);
wCB = - c3(1,2,1)/c6(1,2,1);
disp("wCB:   " + wCB);

%AC =                  AB/A     +     AC/B
%   = AB/At      +     AB/An     +     AC/Bt    +    AC/Bn
%   = ABA x AB +  w1 x w1 x AB   + alphaCB x BC + w2 x w2 x BC
alphaBA = 0;%AB/A is zero due to constant angular velocity
a1 = [0 0 alphaBA];
a2 = [ABx ABy 0];
a3 = cross(a1,a2);
a4 = [0 0 wBA];
a5 = [ABx ABy 0];
a6 = cross(a4, a5);
a7 = cross(a4, a6);
a8 = [0 0 1];
a9 = [BCx BCy 0];
a10 = cross(a8,a9);
a11 = [0 0 wCB];
a12 = [BCx BCy 0];
a13 = cross(a11, a12);
a14 = cross(a11, a13);
```

```
% alphaCB = -(a3(1,2,1) + a7(1,2,1) + a14(1,2,1))/ a10(1,2,1);
% disp("alphaCB:   " + alphaCB);
alphaCB = -0.348660;
a15 = [0 0 alphaCB];
a16 = [BCx BCy 0];
a17 = cross(a15,a16);
% a_slider = a3(1,1,1) + a7(1,1,1) + a17(1,1,1) + a14(1,1,1);
% aCoM3x = a_slider;
% disp("a_slider:   " + a_slider);
% aCoM3y = 0;
a_slider =-0.081331;
aCoM3x = -0.081331;
aCoM3y = 0;

%aCoM1
a18 = [0 0 alphaBA];
a19 = [mABx mABy 0];
a20 = cross(a18, a19);
a21 = [0 0 wBA];
a22 = [mABx mABy 0];
a23 = cross(a21, a22);
a24 = cross(a21, a23);
% aCoM1x = a20(1,1,1) + a24(1,1,1);
% disp("aCoM1x:   " + aCoM1x);
% aCoM1y =  a20(1,2,1) + a24(1,2,1);
% disp("aCoM1y:   " + aCoM1y);
aCoM1x =-0.008018;
aCoM1y = -0.005976;

%aCoM2
 a25 = [0 0 alphaCB];
```

```
 a26 = [mBCx mBCy 0];
 a27 = cross(a25, a26);
 a28 = [0 0 wCB];
 a29 = [mBCx mBCy 0];
 a30 = cross(a28, a29);
 a31 = cross(a28, a30);
 a32 = [0 0 alphaBA];
 a33 = [ABx ABy 0];
 a34 = cross(a32, a33);
 a35 = [0 0 wBA];
 a36 = [ABx ABy 0];
 a37 = cross(a35, a36);
 a38 = cross(a35, a37);
%  aCoM2x = a27(1,1,1) + a31(1,1,1) + a34(1,1,1) + a38(1,1,1);
%  disp("aCoM2x:   " + aCoM2x);
%  aCoM2y = a27(1,2,1) + a31(1,2,1) + a34(1,2,1) + a38(1,2,1);
% disp("aCoM2y:   " + aCoM2y);
 aCoM2x =  -0.048703;
 aCoM2y = -0.005994;


R12x= (x1-x2)/2;
R12y= (y1-y2)/2;
R23x= x1-x2;
R23y= y1-y2;
R32x= -R23x;
R32y= -R23y;
R34x= R23x - x3;
R34y= R23y -y3;
R43x= -R34x;
R43y= -R34y;
```

```
% F12x  F12y  F32x  F32y  F43x  F43y  F14y  T12
A = [1    0    1    0       0    0    0    0;
     0    1    0    1       0    0    0    0;
    -R12y R12x -R32y R32x   0    0    0    1;
     0    0   -1    0       1    0    0    0;
     0    0    0   -1       0    1    0    0;
     0    0    R23y -R23x  -R43y R43x 0    0;
     0    0    0    0      -1    0    Θ    0;
     0    0    0    0       0   -1    1    0];

B = [MassOfLink1*aCoM1x;
    MassOfLink1*aCoM1y;
    RectMMoI1*alphaBA ;
    MassOfLink2*aCoM2x ;
    MassOfLink2*aCoM2y;
    RectMMoI2*alphaCB;
    Fpx + mass_slider*aCoM3x;
    (weight_slider + Fpy + mass_slider*aCoM3y)];

x = A^-1 * B;

disp("Force Ax:  " + x(1));
disp("Force Ay:  " + x(2));
disp("Force Bx:  " + x(3));
disp("Force By:  " + x(4));
disp("Force Cx:  " + x(5));
disp("Force Cy:  " + x(6));
disp("Force N:  " + x(7));
disp("Force M:  " + x(8));
```