



Handicapped Pool Chair Lift

A Major Qualifying Project Report

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Abstract

This project was created to improve existing technology and create a design for a new chair lift to assist the disabled in entering and exiting a pool. Our goal was achieved through in-depth research into the problems within technology already on the market, and creating a proposal that improved on the portability weakness most designs possess. Initial concepts were analyzed to determine the lift mechanism and counterweight system for the device. Once a design was chosen, we completed a full CAD model of the device and all of its subassemblies to show and test basic functionality. After the model was complete, we then bought all the components of the device and began manufacturing, using the tools available in Washburn Shops. Finally, construction progress and a list of future recommendations was presented to our advisor for further implementation and utilization.

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1. Introduction

In 2011, the Americans with Disabilities Act (ADA) was put into law, requiring that many public and private aquatic facilities become more accessible to handicapped persons. The most notable and expensive change was the requirement to install a pool chair lift to help persons enter and exit a pool. The overarching problem with existing technology is that the quality of products that exist does not correlate to the prices offered. Many existing devices not only cost a significant amount of money to buy at first, but also do not allow for serviceability, threatening to void warranties if a user even undoes a single screw. In addition, products have a variety of shortcomings, including but not limited to weight, stability, and reliability. Our team's goal is to create a working prototype that is affordable to all pools while still allowing companies to make a profit. This will be done by utilizing low cost and repeatable manufacturing methods that will still lead to a sturdy and effective prototype. Having access to an economical yet safe and effective device is of paramount interest to both potential users with disabilities as well as pool owners and staff.

2. Background

The Americans with Disabilities Act (ADA) was revised in 2011 to incorporate the 2010 Standards for Accessible Design to require that pools have a pool lift capable of moving a disabled person into and out of the pool. The pools covered under these requirements are of a very wide range, including but not limited to hotels and motels, health clubs, recreation centers, universities, public country clubs, and other businesses that have swimming pools, wading pools, and spas. In addition to requiring the pool lift be accessible to those that require it, the ADA includes guidelines describing the location and usage of any pool lift device. These guidelines are essential to designing a successful pool lift device.

2.1 ADA Guidelines

The ADA guidelines explicitly define the area in which a pool lift can be used. The pool lift must be used where the depth of the pool does not exceed 48 inches (ADA 1009.2.1). The exceptions to this regulation include if the depth of the pool is greater than 48 inches at all areas or if there are multiple pool lift devices installed in fixed locations only one needs to meet the requirement. In the loading position, the centerline of the seat must be a minimum of 16 inches from the edge of the pool (ADA 1009.2.2). In addition, there must be a minimum of 36 by 48 square inches clear deck space for the occupant to be able to board the chair (ADA 1009.2.3). The height of the chair must be a minimum of 16 to a maximum of 19 inches from the ground when held above the deck for the user to seat themselves on the device (ADA 1009.2.4). These constraints all serve to allow adequate room for the user to board the pool lift with ease. Diagrams for several of these positioning constraints are featured in Figure 1 for reference.

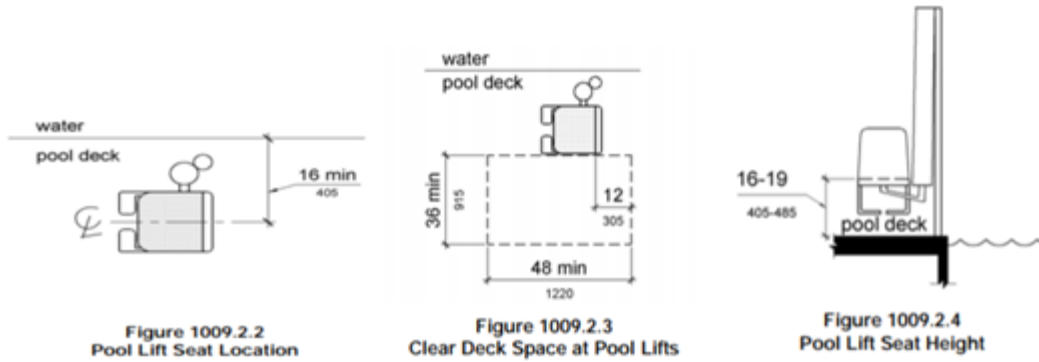


Figure 1: ADA Pool Lift Figures Defining Loading Position

The mechanical design of the pool lift is also controlled by the ADA guidelines. The seat is required to be a minimum of 16 inches wide (ADA 1009.2.5). A foot rest must be included with the seat, and if armrests are employed they must be able to fold back to allow unobstructed boarding and unboarding of the chair (ADA 1009.2.6). Both of these requirements serve to provide a comfortable and stable position for the user while using the lift. The pool lift must also have provisions that allow operation at both the pool deck and water levels by the user unassisted by pool staff (ADA 1009.2.7). This requirement is typically accomplished with a portable waterproof remote the user can hold while in the chair of the pool lift and is crucial to ensure the user is not stranded in the pool in the absence of assistance. The pool lift must also be designed such that the top of the chair is submerged 18 inches from the top of the seat as seen in Figure 2 (ADA 1009.2.8). This allows for the user to easily re-seat themselves while still in the water. The lift must be capable to of lifting a load of 300 pounds minimum (ADA 1009.2.9). This weight threshold is able to cover the majority of the population without imposing more costly design constraints on an already expensive product.

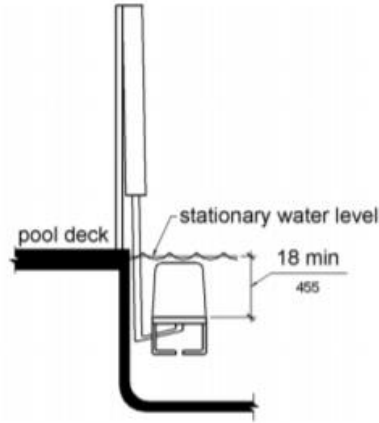


Figure 1009.2.8
Pool Lift Submerged Depth

Figure 2: ADA Figures Defining Submerged Position

2.2 Current Pool Lift Designs

From a design standpoint, in looking to create a new and improved device specifically for this project, it was crucial to look into the existing devices and mechanisms that are already available. As the ADA requirements were enacted about 7 years ago, the market for pool chair lifts is now quite extensive with lifts varying in size, price, capability and style. Most pool lift devices can be split into two different categories as seen in Figure 3: those that are fixed in place (left) and those that are mobile (right).

Fixed designs are characterized by a mast that is attached to the deck of the pool via an anchor point. The anchor is typically installed into the pool deck by drilling into the concrete and adding a sleeve for the mast to attach to, then filling in the extra surrounding area with new concrete.



Figure 3: Fixed (Left) vs. Mobile (Right) Pool Lift Designs

Mobile designs mount the mast to a base with wheels so the whole device may be transferred around the pool or stored conveniently. The base also contains a large counterweight to prevent tipping when the device is in use. Each of these design options have their own pros and cons.

The fixed pool lift design offers the advantage of being structurally secure assuming the anchor is properly installed. The support reactions needed to prevent the device and occupant from tipping over are all transferred through the anchoring point to the pool deck. By counteracting the weight of the occupant this way, the design does not require a significant counterweight to balance the moment caused by the occupant. As a result, fixed designs tend to be much lighter in weight. While the location where a user can be lifted into and out of the pool is limited by the location where the anchor is installed, the mast can be detached from the anchor and stored when necessary.

The mobile pool lift design is able to be used at any location around the pool that complies with the ADA guidelines. This also allows for easy storage when the device is not in use. However, the device requires a large counterweight to prevent tipping during operation. In some cases the weight is so heavy that the device cannot be moved by a single person.

For the pool that is located on Worcester Polytechnic Institute (WPI) facilities --a semi-private, in-ground pool-- the faculty previously purchased a Spectrum brand portable pool chair lift. Other brands include Aqua Creek, Hoyer and PAL (Portable Aquatic Lift). The devices all share similarities; however, some aspects of the design vary depending on price. Some of the cheaper portable lifts have flexible seats that are a mesh material in the shape of a seat as shown in Figure 4. Other chairs have a more stable seat that potentially includes movable armrests, a footrest and a safety strap that fastens across the lap of the user. This particular style of seat design is shown in Figure 5.



Figure 4: A Pool Lift Featuring a Flexible Mesh Chair



Figure 5: A Mobile Pool Lift Featuring a Hard Plastic Chair and Armrests

2.3 Safety Guidelines and Practices

Beyond the guidelines provided in the ADA, there is not much information specific to the design of pool lift devices. In order to develop a device that both accomplishes the required task and is also safe for the user and operator, the team looked to best practices and standards to guide our design. We wanted to come up with a design that improved on existing technology while also not over engineering our design to create extraneous costs.

2.3.1 Human Factors

Any pool lift subjects the occupant to accelerations during operation when moving the person from the pool deck into the water. Large accelerations can cause negative health impacts on the human body, such as feeling pain/pressure or even losing consciousness. These consequences are typical of a high acceleration situation such as space shuttle taking off, and unlikely to be experienced by the occupant of any pool lift device since the motion is considerably slower and has a much lower acceleration. It is important to consider the threshold for voluntary tolerance to acceleration so the occupant is comfortable when using the device. An acceleration can be experienced voluntarily for a certain duration depending on the magnitude and direction, as shown in Figure 6. For example, a person can experience an acceleration of 16G in the +z direction for approximately 0.02 minutes (1.2 seconds) on a voluntary basis. This means the person will not feel uncomfortable or uneasy during this time frame. These factors also depend on the individual under the acceleration, and people can even train themselves to withstand larger magnitudes of acceleration. Figure 6 shows the average value, however, the user of a pool lift device likely has a lower tolerance to acceleration if they are handicapped. These

factors will need to be considered when selecting an appropriate acceleration for the pool lift device to operate.

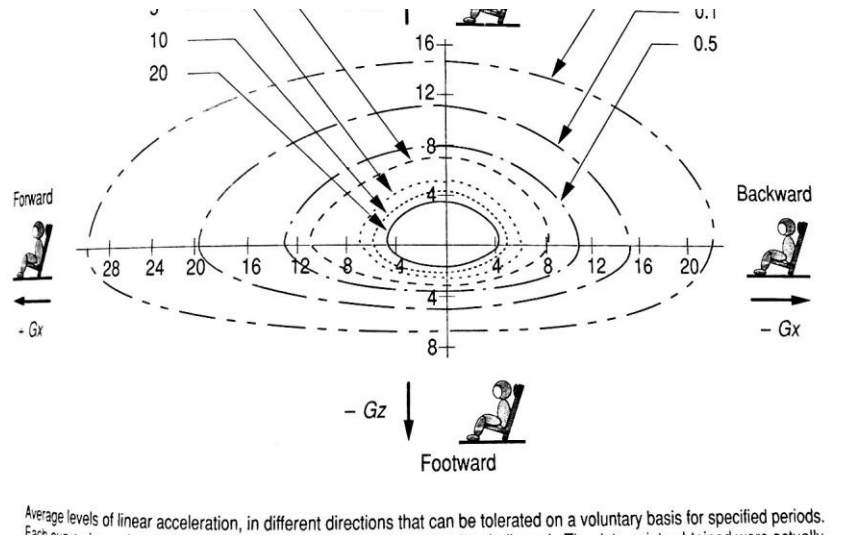


Figure 6: Average Tolerable Acceleration

2.3.2 OSHA Regulations

In addition to understanding the human tolerance of acceleration, other safety compliances that must be considered are the Occupational Safety and Health Administration’s (OSHA) guidelines for pinch points and tipping. OSHA defines a pinch point as “any point other than the point of operation at which it is possible for a part of the body to be caught between the moving parts of a press or auxiliary equipment” (definition 1910.211(d)(44)). Our pool chair lift is likely to have instances of pinch points that must be considered when manufacturing. The biggest concern is the user’s feet hanging over the side of the chair, as the potential for a pinch point exists when the lift is positioned over the side. If the lift is moved to a place that is not far enough over the pool, there exists the possibility that feet could get pinched between the mobile chair and the stationary floor. To overcome this, it will be important to set clear guidelines on where the chair must be positioned with respect to the edge of the pool.

While OSHA does not have direct guidance for pool chair lifts, similar information can be taken from the guidelines for use of heavy machinery such as forklifts. OSHA sets out clear guidelines with regards to avoiding tipping of forklifts, and the same general guideline can be used for chair lifts. As shown in Figure 7, as the center of gravity moves further away from the true center of the machine, the capacity of the machine goes down to avoid tipping over. In order for our team to be able to operate a lift with a 300 pound person, we will have to have a significant counterweight to ensure that the center of gravity does not stray too far from the center of the machine.



Figure 7: OSHA Forklift Tipping Demonstration

2.3.3 ASTM Standards

Another standard we must follow will be the American Society for Testing and Materials (ASTM). As the team builds a device that will be used in a corrosive environment, we must be aware of the materials used in building such a device. ASTM designation number ASTM G78 - 15 defines testing materials for corrosion in chlorinated aqueous environments. It outlines

different testing methods for different materials, the most important being stainless steel and plastics. It emphasizes that “In-service performance data provide the most reliable determination of whether a material would be satisfactory for a particular end use” (ASTM G78 - 15). While we are unable to have huge amounts of in-service data, we should obtain materials ahead of time and test early prototypes submerged in the water to ensure that corrosive effects do not get in the way of our prototyping efforts.

2.4 Functional Requirements

As a result of our research into existing chairs and their characteristics, our team has developed a set of functional requirements. A “functional requirement” specifies the functions that a system or component must perform, typically describing what is needed by the system user as well as requested properties of inputs and outputs. We feel this list best suits the necessary requirements that our design must fulfill to satisfy our goal of producing a reasonable pool chair lift

1. Device must be ADA Standard Compliant.
2. Device shall be designed for a public, in-ground pool.
3. Device shall be able to be tested at the pool in the Recreation Center at WPI.
4. Device shall not tip into pool upon use.
5. Device shall not tip occupant into pool during use.
6. Device shall have a restraint system for use by the occupant during operation which is easily removed by occupant once chair has been submerged.
7. Device shall comply with ASTM Standards for corrosion.
8. Device shall stay stationary during operation.

9. Device shall be OSHA compliant.
10. Device shall move lift mechanism and seated occupant with the following motion:
 - Loading shall occur while the seat of the chair is located fully above the pool deck
 - While in motion, device shall move at a comfortable rate for the occupant.
 - From the loading the position, the device shall position the occupant over the water and then lower the occupant into the water to a depth compliant with ADA standards.
 - The device shall be capable of operating the reverse sequence to assist the user in exiting the pool.
11. Device shall be movable by one person.
12. Device shall be designed for ease of assembly and require only a basic tool kit.
13. Device must be able to be constructed by MQP Team and Washburn Lab Technicians.
14. Device shall not exert accelerations larger than what a human is comfortable experiencing.
15. The time required to position the device at the side of the pool and prepare for operation shall not exceed 15 minutes.
16. The time to move the occupant from the loading position into the pool shall not exceed 3 minutes.

3. Design Concepts

Following initial investigation into existing technology, the team came up with several preliminary design ideas that served as the foundation for the design matrix used in selecting the final design to build and test. These design ideas were sorted into two different categories: the designs for the lifting mechanism for the device, and the designs for the counterweight of the device.

It was decided that design ideas for both semi-permanently installed and mobile style of device would be entertained until we knew more, since we were initially uncertain of what the WPI facilities would allow us to change within the pool area in the Recreation Center. Therefore, the team came up with three designs for potential lifting mechanisms and three designs for potential counterweight mechanisms. Each design within the lifting category is capable of being paired with any of the selections for the counterweight category and vice versa, allowing us to have full range in choice of design.

3.1 Lifting Mechanisms

Detailed within this subsection are the specifics and early sketches of each of our initial ideas for the lifting mechanism. These designs were crucial to the overall progress of the project, as we needed to find the best mechanism to lift the user in and out of the pool and to be able to pivot to allow the user to enter and exit the chairlift at a safe distance in accordance with ADA Standards.

3.1.1 Pulley Mechanism

The first of the three proposed lifting mechanism designs was a pulley system. Our process began with finding the simplest way to lift and lower any type of object, as we were not sure what style of seat or carrying sling would be part of the device. Our first thought went to a pulley system; a simple and affordable way to lift and lower an object. This idea included two pulleys with some variant of rope, chain or belt connecting the pulleys raising the user in and out of the water, shown in Figure 8 below. This pulley system would be mounted on a rotating mast which would satisfy the design requirements needed for the lifting mechanism in accordance with our functional requirements.

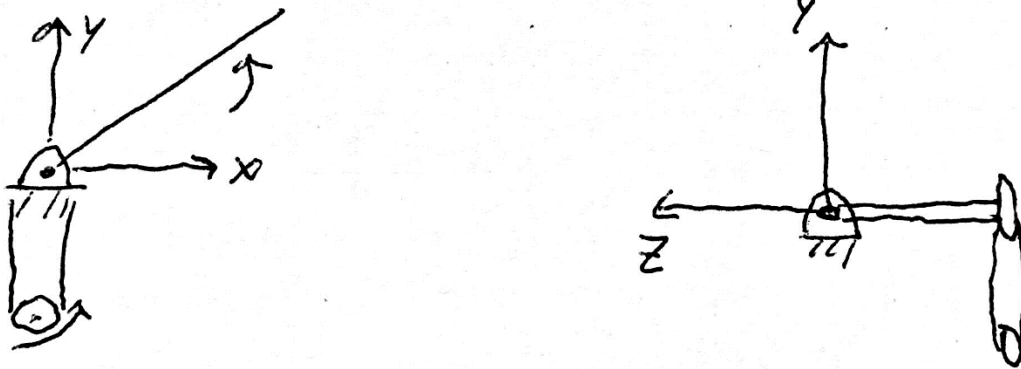


Figure 8: A Rough Sketch of the Proposed Pulley System

3.1.2 Three Bar Inverted Slider Mechanism

The next of the design concepts we developed was the three bar inverted slider. Based on research into technology that already exists on the market, we noticed that another common choice for this type of device was the three bar slider. The design is rather simple (seen in Figure 9) consisting of two pivot points on the same horizontal axis that have the capability of rotating about the pivot point in a radial motion. There is a slider firmly attached to the end of one rod that slides along the axis of the second bar located at the second set pivot point. To achieve this

motion, there is the possibility of using a cranking system or using a linear actuator. More in-depth details were not discussed at this preliminary stage, however a rough sketch was formed and key features were touched upon.

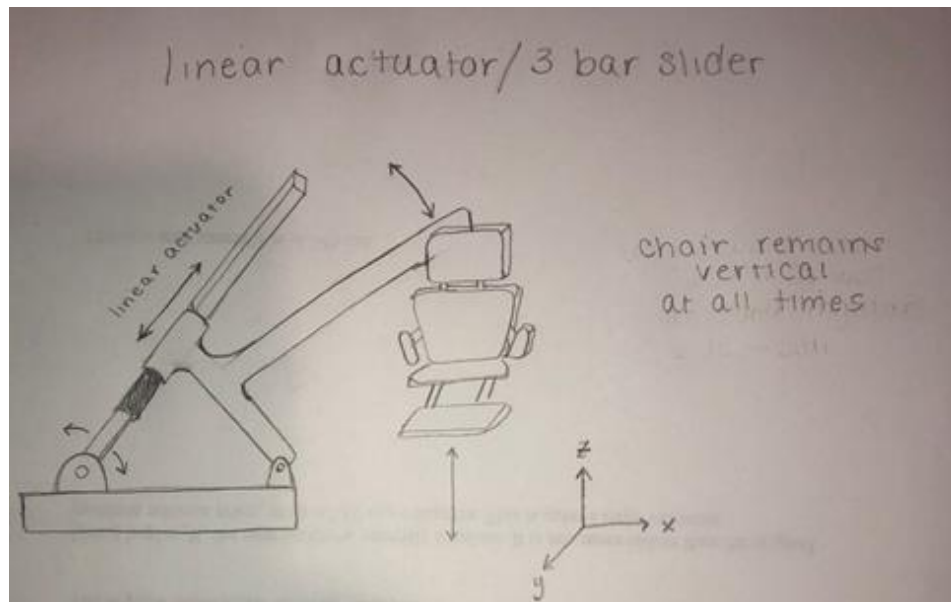


Figure 9: A Rough Sketch of the Preliminary Three Bar Inverted Slider Concept

3.1.3 Four Bar Mechanism

The final design concept we had for the lifting mechanism of the chair was a simple four bar mechanism as shown in Figure 10. This style of assembly consists of a vertical mast that has two parallel bars attached at their ends. These two bars are separated by a specified amount of space depending on design requirements, and at their other ends are connected to a second vertical bar. For our project, the chair would be attached to the second vertical bar and some sort

of motor, crank or actuator would drive the chair up into the raised position and lower it into the water.

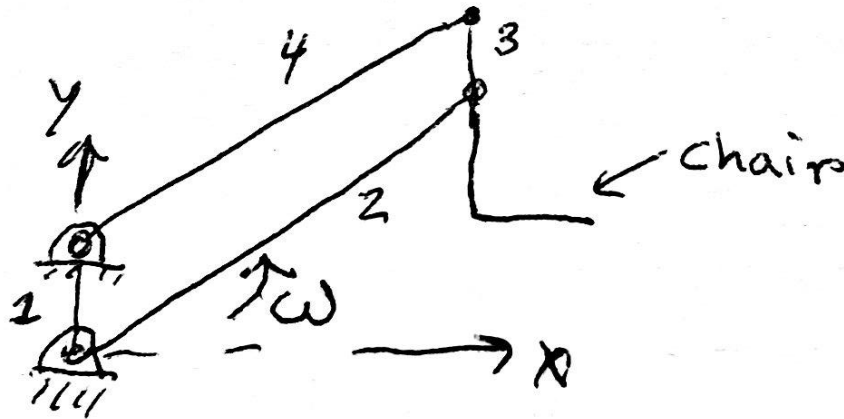


Figure 10: Initial Sketches of the Four Bar Mechanism

3.2 Counterweight Designs

Discussed within this subsection are the early details and hand sketches of each of our ideas for the counterweight of this device. In order to meet ADA Standards, the chair must lift a maximum weight of 300 pounds, requiring the counterweight of the device to weigh more than this maximum weight of the user. There are two different styles of counterweight: one that secures a mast into the concrete platform of the pool deck, and one that features a portable design with some type of added weight (sand bags, metal, water, etc.) mounted on the base to weigh it down.

3.2.1 Fixed to Deck (Anchor)

Our research led us to a few different design options for counter-weighting the device including mounting a mast style support either permanently or semi-permanently installed in the pool deck as shown in Figure 11. There is a hole drilled into the deck of the pool to a predetermined depth. Within this hole, a hollow collar is permanently installed that is made of

some type of plastic or metal material. The lowest portion of the anchor style counterweight is lowered into the hole and secured for use of the device. If the device is only semi-permanently mounted, the device can then be removed from the deck of the pool and a temporary cover can be placed on the hole to avoid injury.

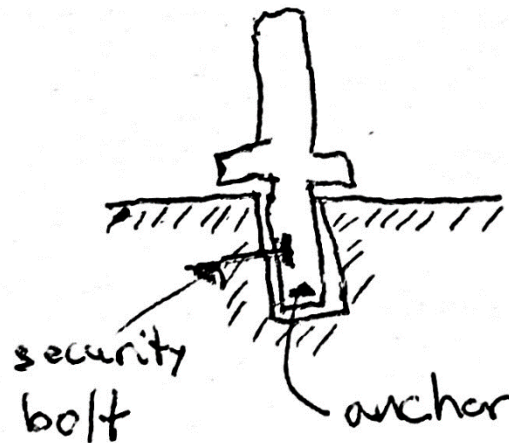


Figure 11: Anchor Design of Counterweight Mechanism

3.2.2 Movable Base with Removable Weights

Our next design concept focused on a design that would be independent of the pool and its surroundings, specifically a movable base on wheels with a storage bin type structure for housing removable counterweight materials which can be seen in Figure 12. For example, the chairlift that currently resides at the WPI facility has a two foot by two foot metal frame, inside of which about eight 1 inch-thick pieces of steel are stacked on top of one another. In our design, we would like to make these weights smaller and easier to remove, since the existing lift does not have this feature.

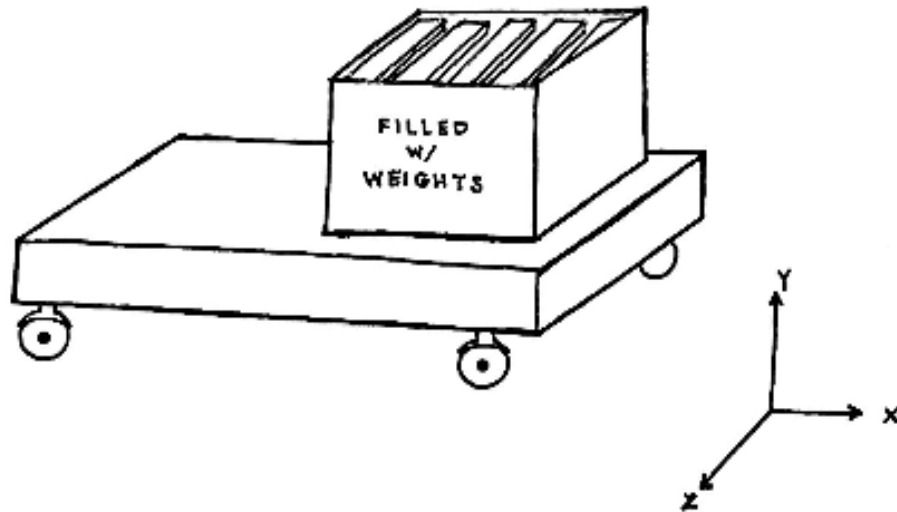


Figure 12: Counterweight Design with Removable Weights

3.2.3 Movable Base with Water Tank

After the initial thought of having some sort of counterweight material that would be removed from the device, we delved into thought into which materials would have the most substantial weight while also being easy enough for one person to remove by themselves. Initial thoughts went to steel, aluminum and sand, however there was another idea that posed an interesting design concept: a water tank as seen in Figure 13. This design idea features the same movable base on wheels as mentioned previously, yet features a tank to hold water rather than a structure to hold metal counterweights. The device would be transported to the side of the pool while empty, a pump would fill the tank to capacity, the user would use the device, the tank would be drained after use and the lift would be easily moved from the poolside to its storage location.

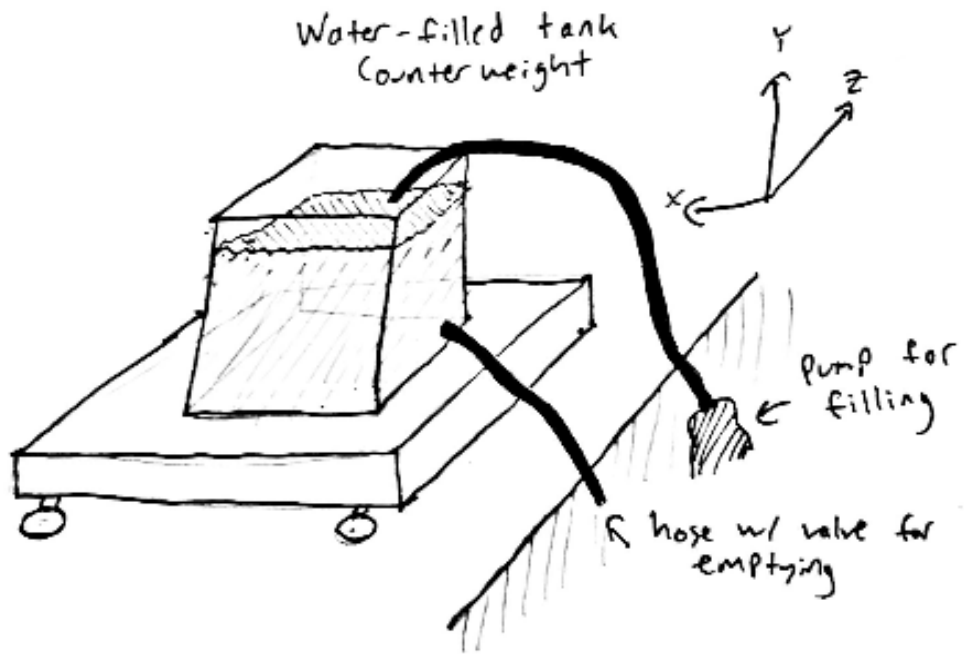


Figure 13: Detailed Design Concept of Water Tank Counterweight

4. Design Selection

A complex design such as this chairlift has many possibilities and many different permutations of those possibilities. Our team was able to narrow down the amount of decisions we had to make to two major design choices. The team needed to select a lifting mechanism to move the user up and down, and in addition the team needed to select a method to balance the weight of the user.

The use of the decision matrices shown below made helped to guide our design choices. We narrowed down our needs to four distinct and measurable quantities: price, safety, performance, and manufacturability. These factors would help our decision the most, as well as the factors to which we could most easily quantify. The decision matrix also allows for weighting of each factor, meaning if we felt one factor is more important than another, we still could use said factors in making one decision by putting them at different weights. We set our scores for each factor as an integer of one to five, and our weights as decimals adding up to one, meaning that the “winning” choice would have the highest total weighted score out of five.

4.1 Lifting Mechanism Design Selection

Lifting Mechanism					
	Price (0.20)	Safety (0.35)	Performance (0.25)	Manufacturability (0.20)	Total
Four bar	4	4	4	4	4.00
Three bar (inverted slider)	3	4	3	3	3.35
Pulley	5	2	3	5	3.45

Table 1: Lifting Mechanism Decision Matrix

The lifting mechanism came down to three designs as stated in section three. The four-bar mechanism, the three-bar inverted slider-crank mechanism, and the pulley.

Four bar linkages are a common style of mechanism used in kinematic designs, so there was a sufficient amount of research available for us to peruse. After looking at the existing designs and their use of this mechanism, we were able to identify its strengths and weaknesses. Four bar mechanisms are the simplest to build and provide a good deal of control based on our inputs and offers an ideal mechanical advantage; however, four bar mechanisms also offer the least freedom of motion, and does not offer any mechanical advantage, meaning the team would need a significant amount of force to move the linkage. Overall, the design is pretty commonly used for this application. The mechanism lifts a large weight, so the bars within the mechanism must be very sturdy, posing a threat to our already limited budget. A depiction of this style can be seen in Figure 10.

Strengths and weaknesses of the four bar design were also conceptualized, and we gravitated towards a more robust design as compared to the pulley system. We noted that a majority of existing chair lifts that used this style of lifting mechanism were either permanently or semi-permanently mounted into the deck of the pool. This offered the chance to either improve on existing technology with our new design by making it portable, or posed a potential conflict if we found we were unable to make any changes to the deck around the pool at WPI.

As with any design, there were positives and negatives associated with the pulley idea. The idea was cost effective; however, ADA Standards require that the device lift a minimum of 300 pounds. A pulley system would need to be extremely robust within a small amount of space in order to support this weight and remain open to the possibility of a portable design being mounted on some type of moving base.

Overall, as can be seen by the score in the table above, the best decision was the four bar mechanism, as it was the easiest to both design and manufacture while still accomplishing our functional requirements.

Counterweight Design					
	Price (0.20)	Safety (0.35)	Performance (0.25)	Manufacturability (0.20)	Total
Fixed to deck	1	4	2	1	2.05
Moveable base (Water Tank)	5	2	4	3	3.3
Moveable base (Weights)	2	3	3	4	3.0

Table 2: Counterweight Design Decision Matrix

4.2 Counterweight Design Selection

The other major decision for our team to make was the method in which the weight of the user is balanced, termed as the counterweight. As the user sits in the chair, the center of gravity is moved significantly towards the chair. As will be mentioned in the tipping analysis in Section 5.1, we needed to ensure the center of gravity does not go outside the rectangle formed by the caster wheels on the outside of the base.

After investigating conditions at the Recreation Center Pool at WPI, we decided that the option to permanently anchor any object into the pool deck was unavailable since the pool is primarily used as a Division III competitive racing pool, and having a chair permanently anchored next to the pool would not allow the pool to continue to meet NCAA standards. There was still the option of drilling into the deck a certain distance such that we could mount a mast while the device needed to be used and cover the hole while not in use, so as to not pose a safety

hazard. A conversation with the WPI Facilities department led us to decide that this was not feasible either, as that significant of an edit to WPI's infrastructure would be too costly.

WPI's Recreation Center currently uses one of our design options, a very large weight system stacked on to the base. As a result, the chair lift in the Recreation Center weighs just under 800 pounds, and is only movable by three people at once, sometimes four. This easily put the score for performance very low, and in addition, we were quick to not go in this direction as we wanted to avoid simply reinventing chair lifts that are currently used.

In deciding the appropriate method to use, we hoped to improve on the portability of the existing device that is in the Recreation Center at WPI and allow it to be movable by a single person rather than two to three. This meant that one of the biggest options was to implement design that, by our research, did not exist yet. This was the option to have an external tank attached to the base that would be empty while the tank was mobile, but filled with water while it was next to the deck of the pool. Not only does this option perform well, but for the purposes of our prototype the ability to take an off-the-shelf drum and sit it on top of our base meant that the cost was almost zero to us. We initially planned on making an acrylic tank, but this idea proved to be very expensive. Instead, we came up with the alternative to use a 55 gallon drum. This brought the cost way down and increased the price and manufacturability ratings for this design. This very clearly put the drum at the top of the list in the decision matrix.

With our two matrices considered, the choice for both was clear as demonstrated by our matrices. We decided to choose the four bar linkage for the linkage mechanism, and the water-filled tank for the counterweight.

5. Synthesis and Analysis

The following section details the necessary analysis completed to ensure the design would meet our functional requirements.

5.1 Tipping Analysis

One of the challenges of any existing pool chair lift with a movable base is the danger of the whole device tipping over when in use. Suspending a person of 300 pounds a distance a few feet away creates a moment reaction that needs to be counterbalanced in order to prevent tipping of the device. As mentioned in the background chapter, this is usually accomplished by including a heavy counterweight in the base of the device that can weigh several hundred pounds. This makes moving these devices difficult, and was part of the rationale behind selecting the water tank design for the counterweight. This choice does add an extra consideration to the tipping analysis; it is now necessary to make sure the device is not off balance under its own weight. Therefore the discussion will be split into two parts; unloaded and loaded configuration of the device.

5.1.1 Unloaded Configuration

The unloaded configuration considers the device without any water in the tank or a person sitting on the chair. The center of mass of the device can vary depending on the position of the linkage. Two extreme cases were chosen to be analyzed; one where the linkage was straight in front of the base with links 2 and 4 parallel to the ground and the other case where the mast is rotated to the side of the device and the chair in its highest position for the person to board the device. The analysis was conducted in SOLIDWORKS by first assigning material

properties to all components. Once this was complete the Mass Properties tool was used to determine the center of mass for each configuration. The first case as shown in Figure 14 puts the chair as far as possible from the base in the X-direction. For the device to not tip, the center of mass must be located within the boundary created by the four wheels underneath. For this case, the center of mass is within the boundary, so the device will not tip. The second case is shown in Figure 15. As mentioned this case does not have the linkage parallel to the ground, under the assumption that the chair would be at its highest point to allow the user to board the chair at the ADA approved height. As long as the chair is over the deck of the pool, it cannot be lowered significantly without the footrest hitting the ground. This was part of the reasoning behind this assumption. While the center of mass has shifted, it still remains within the bounding box, so the device will not tip.

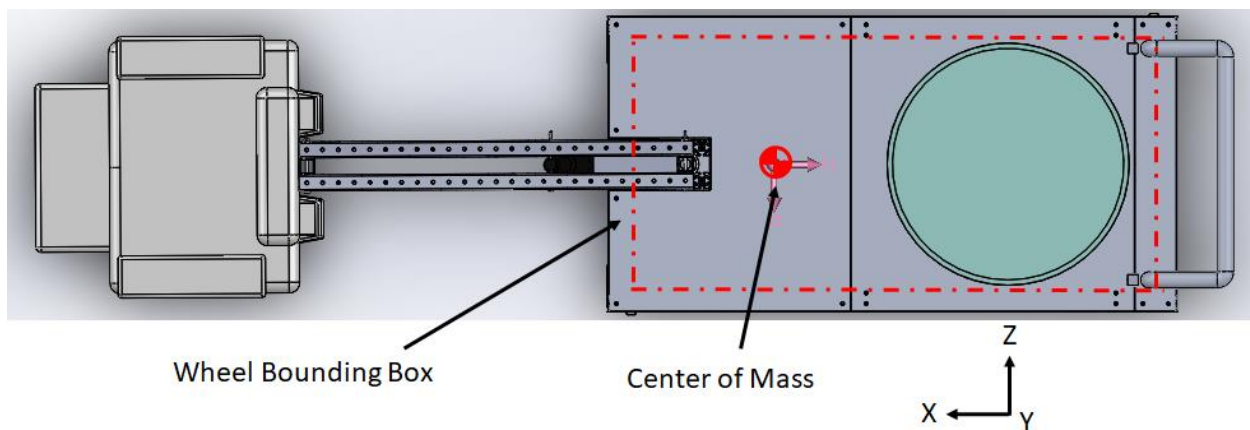


Figure 14: Unloaded Configuration Center of Mass Case 1

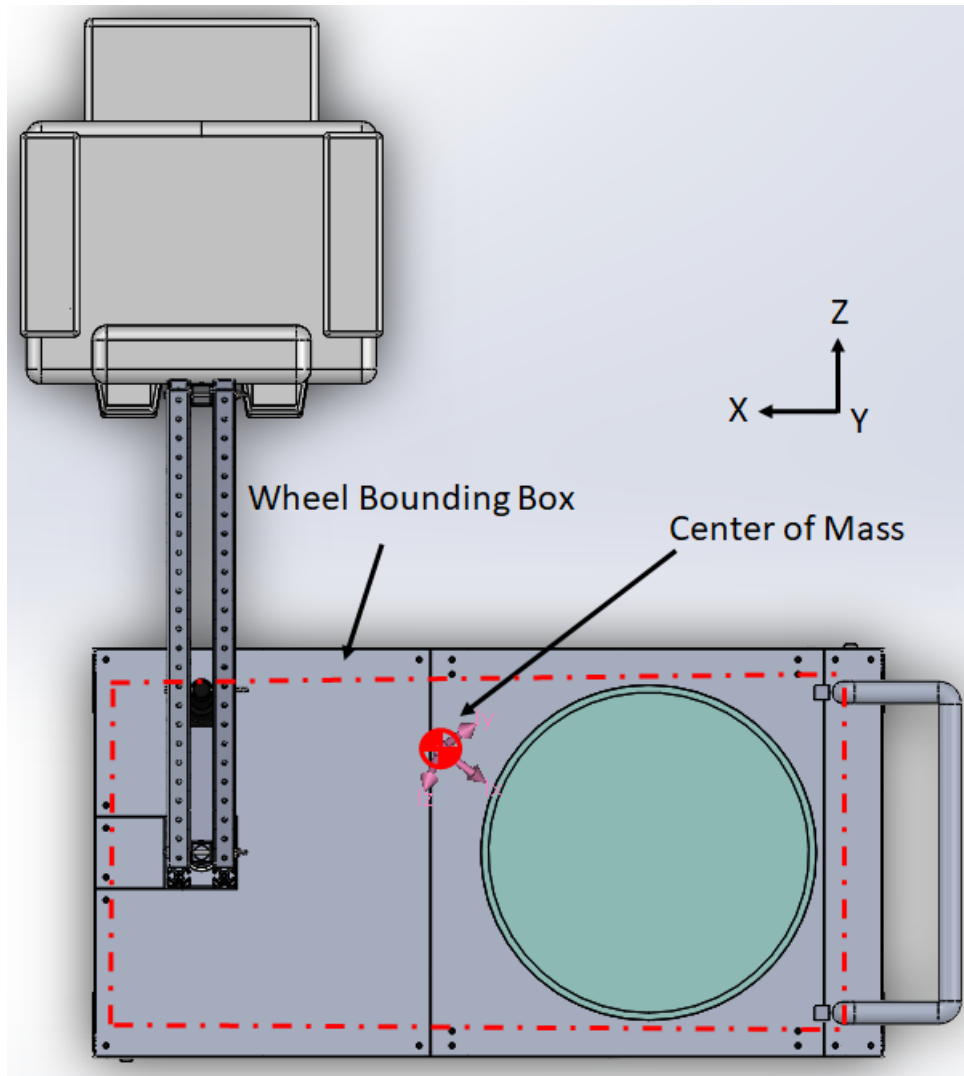


Figure 15: Unloaded Configuration Center of Mass Case 2

This analysis was made with certain assumptions regarding the position of the moving parts of the device as well as the mass of certain components. Some items that were direct purchases such as the chair and actuator could not be modeled to the exact geometry and therefore the center of mass data may be different. These items were modeled approximately and then an average density was applied to make the total mass accurate to the product we were purchasing. In the case of the chair, some extra weight was added to account for modifications we planned to make.

5.1.2 Loaded Configuration

The tipping analysis for the loaded configuration was conducted in the same manner as the unloaded configuration. The key differences are the inclusion of two additional parts in the assembly to represent the person sitting in the chair and the water held in the tank to act as a counterweight. The first case as discussed in the previous section can be seen in Figure 16. The center of gravity has shifted from the previous unloaded case, but still remains within the bounding box. The second case can be seen in Figure 17. For this case, the center of gravity has shifted outside the wheel bounding box. As the device is shown, it will tip over in this position if a person of 300 pounds were to sit in the chair. In order to prevent this, outriggers were added to the device that could be deployed when the device is in use. The outrigger acts as an extra support reaction to counteract the tipping of the device. The outrigger system is described later in section 6.5 of this report.

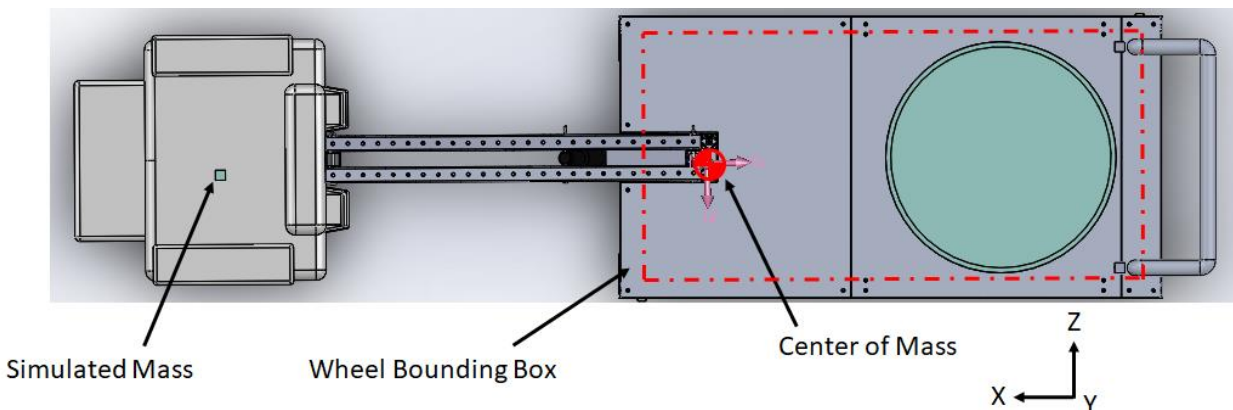


Figure 16: Loaded Configuration Center of Mass Case 1

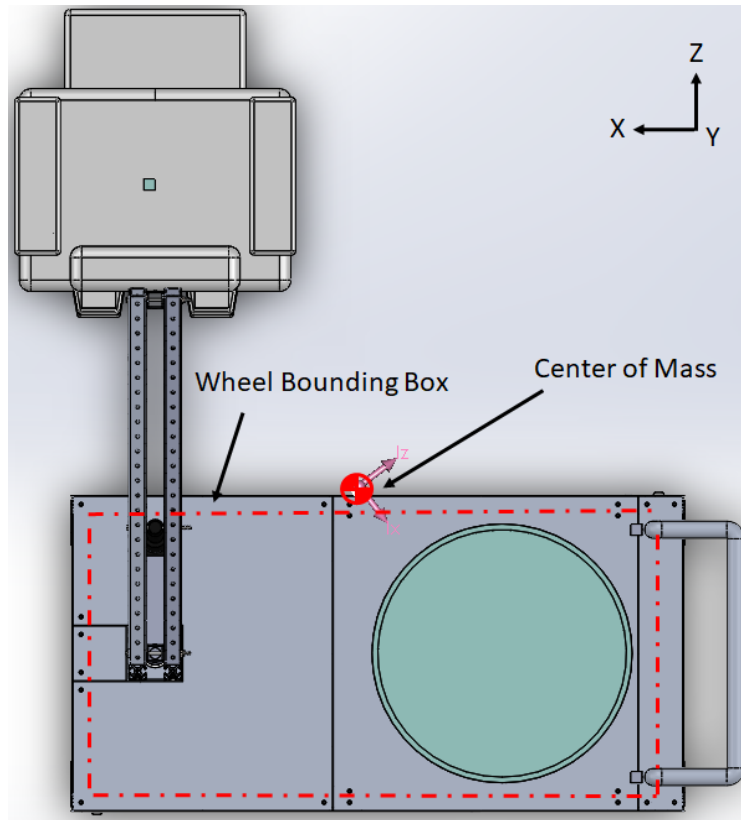


Figure 17: Loaded Configuration Center of Mass Case 2

5.2 Three Position Synthesis

One of the critical functional aspects of our design is the ability of the linkage to be able to lift a handicapped person into and out of the pool. The exact path the linkage moves through is actually quite important, as multiple ADA guidelines (and therefore functional requirements) dictate the extreme positions of the chair in the loading and unloading positions. Some of the pertinent regulations are ADA 1009.2.4 and ADA 1009.2.8 as mentioned in the background chapter which essentially dictate the start and end height of the device. In order to create a linkage able to pass through these two points, the method of three position synthesis described in Norton's Design of Machinery book was used. To perform this analysis, three desired positions of link 3 are drawn in space with the first and last positions show the extreme positions of the

linkage. As shown in Figure 18, these are the pairs of A_xB_x points for each position. From these three positions, lines are drawn connecting points A_1 to A_2 and A_2 to A_3 to each other and likewise for B. Perpendicular bisector lines are then drawn from the connecting lines A_1A_2 and A_2A_3 and where these lines intersect determine the fixed pivot O_2 of the ground link. Repeating for B_1B_2 and B_2B_3 provides the fixed pivot O_4 , see Figure 19. While these criteria provided in the ADA guidelines drove this analysis, there were other important characteristics a three position synthesis solution needed to be valid for our application.

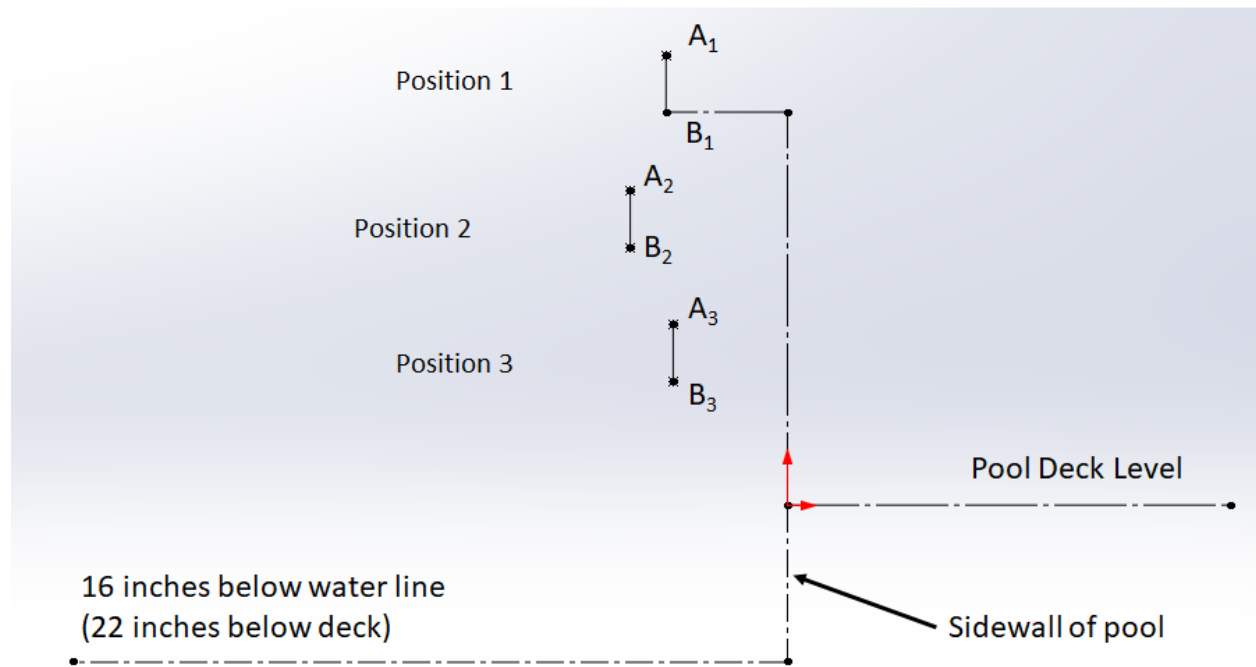


Figure 18: Three Position Synthesis with Reference Features

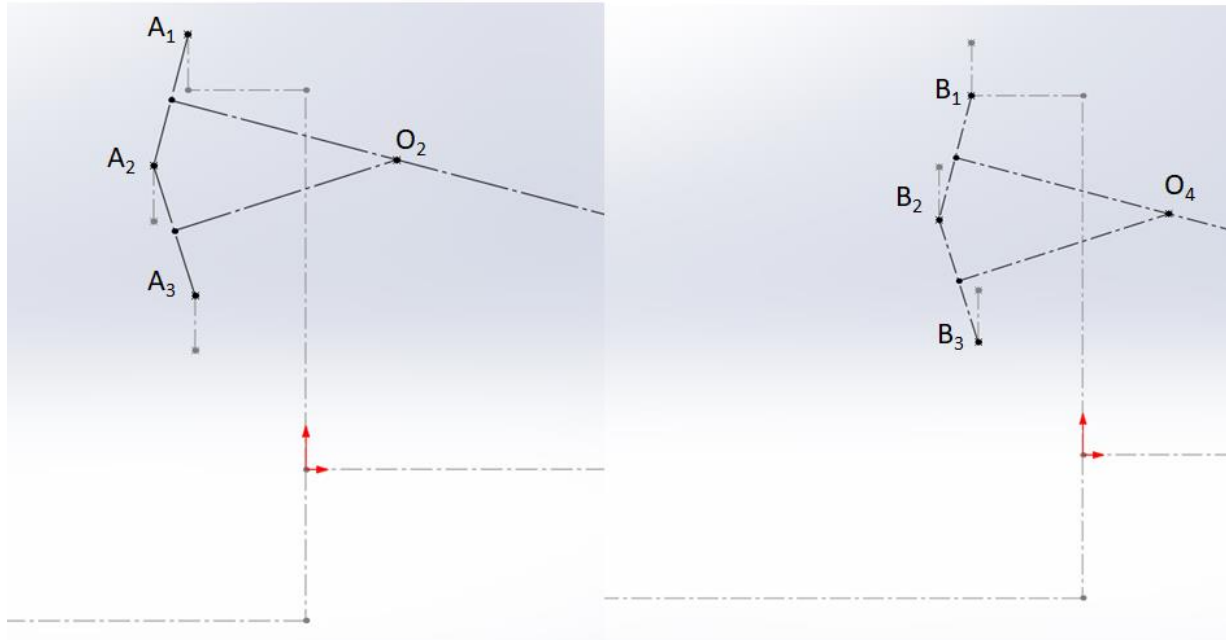


Figure 19: Determination of Fixed Pivots O_2 and O_4

While any three position synthesis solution could appear to work in arbitrary space, there are certain physical constraints that needed to be met for a solution to be valid. A series of reference features were created to represent the height of the pool deck, depth under the water level where the chair would be submerged enough to meet ADA 1009.2.8, and the sidewall of the pool. The three position synthesis would not be valid unless the location of the ground link was above the deck of the pool so the device would not be floating over the water or partially buried underground in a real application sense. The depth of the water was taken to be 18 inches plus an additional four inches from the level of the pool deck, since the water level is never even with the pool deck. Using these reference features, a length that the chair had to hang down from link 3 was determined by taking the distance from the floor to extreme upwards position and subtracting the 16 inches minimum distance between the seat and the floor. A subsequent criteria was to ensure the linkage solution allowed for this fixed distance of the chair to reach the 22 inches below the deck of the pool for the extreme downwards position. A completed figure of the

three position synthesis with the added chair feature can be seen in Figure 20. A solution was deemed satisfactory once it satisfied all the physical constraints detailed above. A final decision was made based on other factors such as the lengths of the links to make sure material costs were not unnecessarily high. The next step in the three position synthesis was to determine the location of a driver dyad.

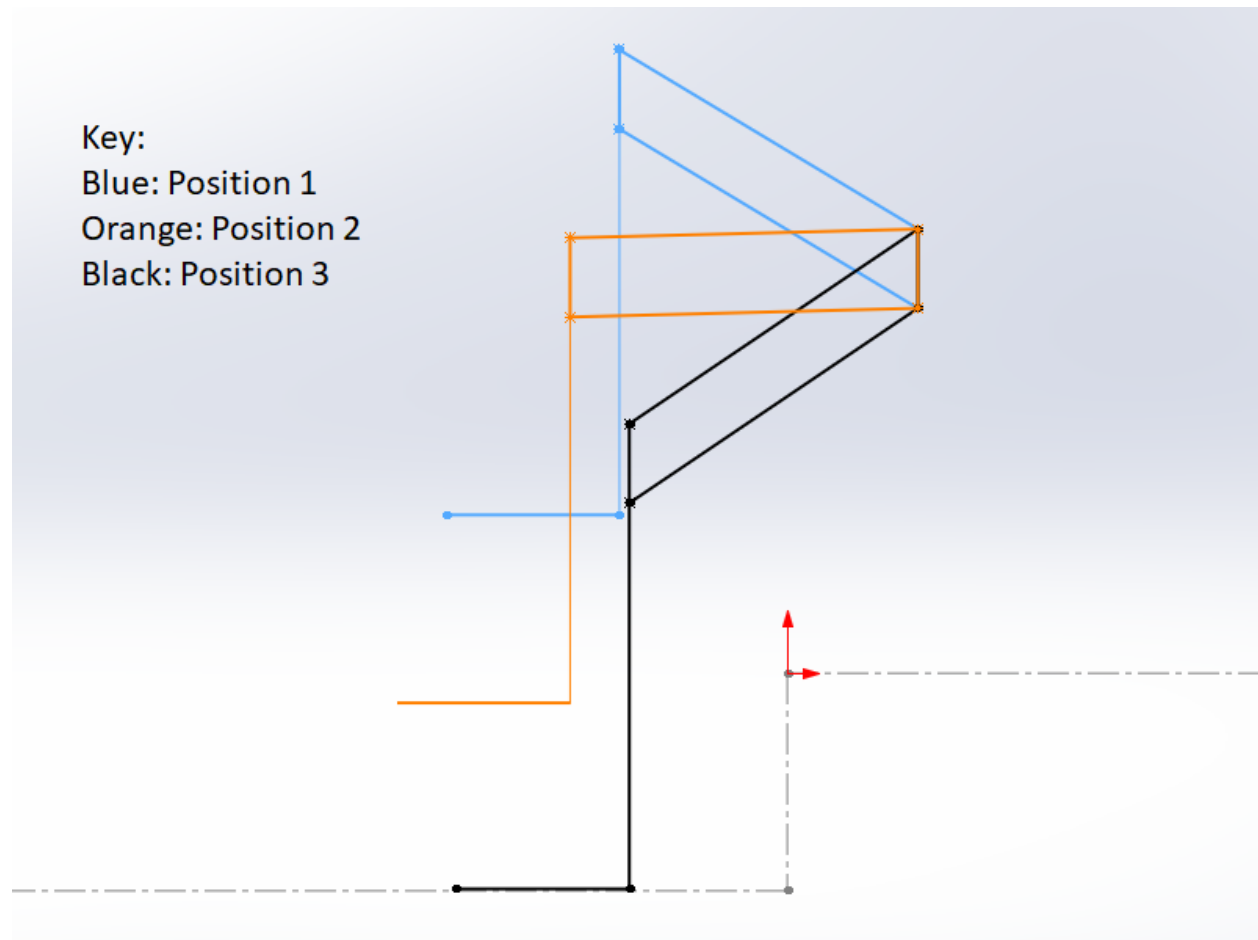


Figure 20: Completed Three Position Synthesis Showing Linkage in Each Position

A driver dyad is created by adding additional links to a four bar or other linkage that allow for the linkage to be driven from the dyad location rather than the fixed pivot of the input link. In the case of this device the driver dyad is the linear actuator that attaches to link 2 in order to drive the mechanism. The dyad should not be placed arbitrarily, since the ability of the dyad to

transmit force to the rest of the linkage is determined by the transmission angle. In order to place the dyad in a position where the transmission angle is optimal, the procedure shown on pg. 107 example 3-4 of Norton's Design of Machinery book was used. First a position on the input link must be chosen. The input link is then drawn in its two extreme positions. A line is drawn connecting the selected point C_1 and C_2 together. Draw the perpendicular bisector of this line. Since the driver dyad of this mechanism is a linear actuator rather than a crank and coupler, the length of the line between C_1C_2 is the stroke of the actuator. Extend the line between the points C_1 and C_2 to an arbitrary location where the actuator fixed pivot O_6 can be placed. The location for the actual device will be controlled by the retracted length of the actuator. Another important consideration at this point in the analysis is the distance O_6 from the mast of the mechanism. The further this point is from the mast, larger cantilevered forces will be imposed on the bracket supporting the actuator.

5.3 Kinematic and Dynamic Analysis

Once the desired motion of mechanism was achieved using the three position synthesis, the next step was to understand the kinematic and dynamic characteristics of the motion. The complete mechanism is a six bar mechanism: a four bar driven by a driver dyad (linear actuator) which is actually two more links. While in certain special cases a mechanism of more than four bars can be analyzed in one step, for this application the mechanism must be split into two four bar loops so one may be solved in order to apply the results to the other and complete the mechanism. Figure 21 is a kinematic diagram of the entire mechanism. The actuator attaches to link 2 at point C. The mechanism is driven by the driver dyad, so the first four bar loop should include this part of the mechanism. An inverted crank slider loop can be created by taking O_2C

as link 2 and O_2O_6 the ground link as shown in Figure 22. A new coordinate system is required for this four bar loop, and is given by rotating the global coordinate system a fixed angle. The second four bar loop is shown in Figure 22 and is the functional portion of the mechanism used to move the user from the deck of the pool into the water. Once the mechanism was divided into separate four bar loops, the analysis becomes far simpler.

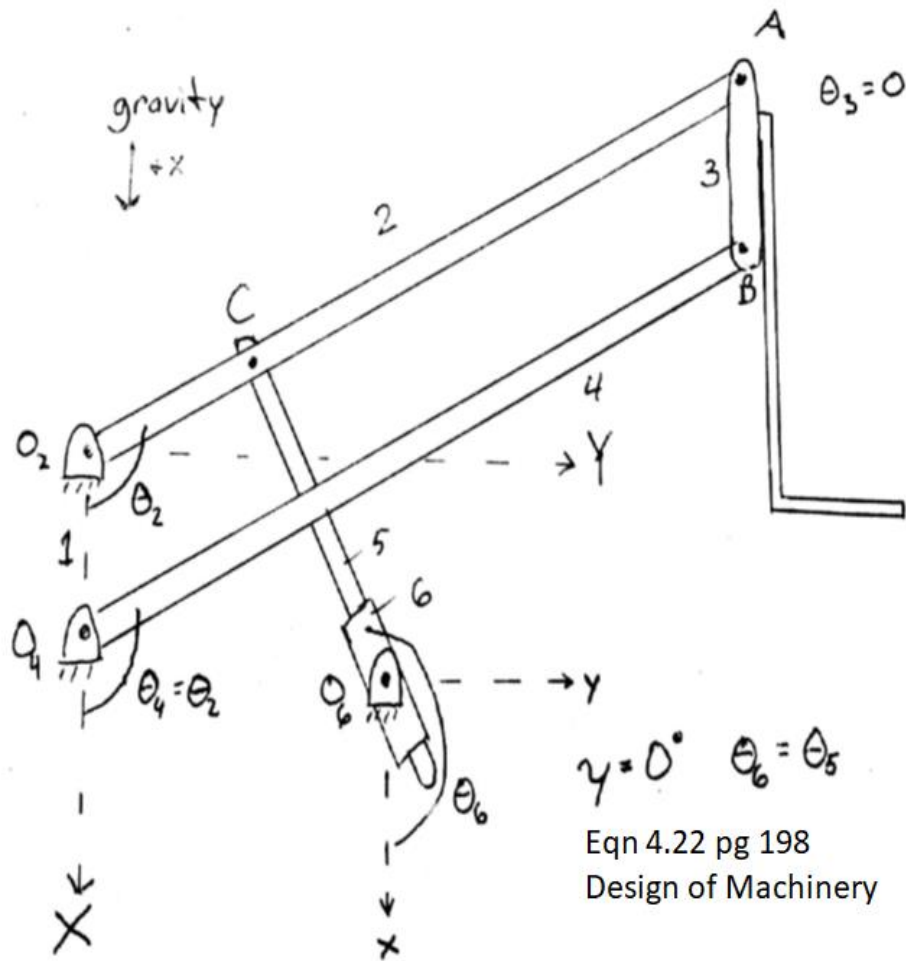


Figure 21: Kinematic Diagram of Full Mechanism

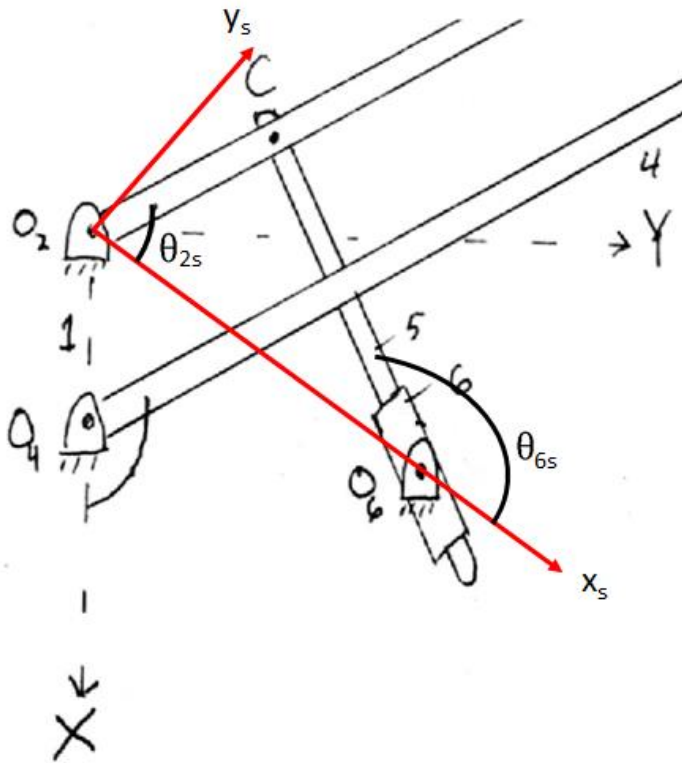


Figure 22: Inverted Crank-Slider Four Bar Loop

The kinematic analysis of the mechanism begins by relating the positions of the links of the mechanism using the known lengths of the links and input information such as the angle of one of the links. Norton's Design of Machinery book describes the process for determining the positions of an inverted crank slider given the angle of the crank. This process is not ideal for this application, since the mechanism is really being driven by the linear actuator rather than a motor at O_2 . Therefore, the equations are instead rearranged to be in terms of the length CO_6 , or how far the actuator has extended. A complete Mathcad document is available in Appendix D: Mathcad Calculations detailing this and all related calculations for the kinematic and dynamic analysis. The positions of the second four bar loop can be determined after applying a coordinate system rotation to adjust the angles found from the inverted crank-slider loop. Similar steps are

taken to determine the velocity and acceleration of the links. The linear velocity of the actuator, or b , is specified as a known value and used to find the remaining angular velocities of the inverted crank-slider four bar loop. These results are then applied to the second four bar loop to find the remaining velocity data. The acceleration uses the assumption that the linear acceleration of the actuator will be zero. Once again the results of the inverted crank-slider are determined and then applied to the second four bar loop to complete the kinematic analysis. With this information known, the next step was to calculate the dynamics of the mechanism.

The dynamic properties of the mechanism are important for several reasons. The forces caused by the motion of the mechanism are important to know in order to conduct the necessary stress analysis. Additionally, the force required to drive the mechanism influences the selection of a linear actuator. Once the equations of motion are written for each of the links in the mechanism, it becomes evident that the pin reactions of the mechanism cannot be solved by using the matrix method with the available information. The number of unknowns (including the reaction forces and the force applied by the actuator) are greater than the number of equations available. Instead, the approach used was an iterative one where the force of the actuator was assumed and then the standard procedure for calculating the forces and torque of a four bar mechanism detailed in Norton's Design of Machinery was used. Only links 1 through 4 were included, since the actuator was assumed to be an applied force. The calculations were repeated until the selected value for the force applied by the actuator resulted in approximately 0 required torque, meaning there would be no need for a motor at the fixed pivot O_2 to move the mechanism.

There are some considerations that impact the validity of these results. The reaction forces and torque is only calculated at one position of the linkage at a time, and therefore the

calculations had to be done for multiple points of the linkage to ensure the the required torque was zero at the position where the mechanism required the most torque. As a result, at the other positions the torque is not zero, and is instead a negative number. Through the process of iteration, it was observed that in order to approach a torque of zero when the torque was a negative number the force had to be decreased. When it was desired to approach a torque of zero when the torque was a positive number the force had to be increased. The way this can be understood in reality where there is no motor providing torque is that a negative value means the mechanism will move, however, the links will have higher values for velocity and acceleration since the applied force by the actuator is more than what is required to move the mechanism at the velocity and acceleration determined previously in the kinematic analysis. The exact change in velocity and acceleration is not known, but is taken to be small. The situation of negative torque is preferred to positive torque, which would mean force provided by the actuator is insufficient to move the mechanism at the given velocity and acceleration. It may be possible that the mechanism would move albeit slower, but there is the potential for the actuator to be unable to move the 300 pound load.

6. Detailed Design Description

This section will cover the description of the final design. Detail drawings of any non-standard parts are included in the referenced appendices. Key functional parts of the device including the lifting mechanism, movable base, pump and tank system, outrigger system, and electrical system will be described in particular.

6.1 Lifting Mechanism

The links for the lifting mechanism were made with 80/20 ReadyTube bars. The ReadyTube bars were favorable for the ease of assembly since each came with several pre-drilled holes. The ReadyTube links were connected with threaded rods and secured with nuts on each end. Link 3 extends downward and suspends the chair on which the user sits during operation. The ground link is a piece of ready tube that bolts to a circular shaft which in turn mates with the bearing system.

A linear actuator attaches to link 2 and drives the mechanism. The actuator was selected by first determining the force required to move the mechanism when a 300 pound person uses the device, which is the maximum required as per the ADA guidelines. Through iterative calculations the force required by the actuator while the device was moving the 300 pound load was determined to be approximately 850 pounds. From this point an actuator with appropriate load capabilities and stroke length was selected. A mounting fixture had to be created to hold the fixed pivot of the actuator in a position that would provide optimal transmission angles. The lifting mechanism can be seen in Figure 23.

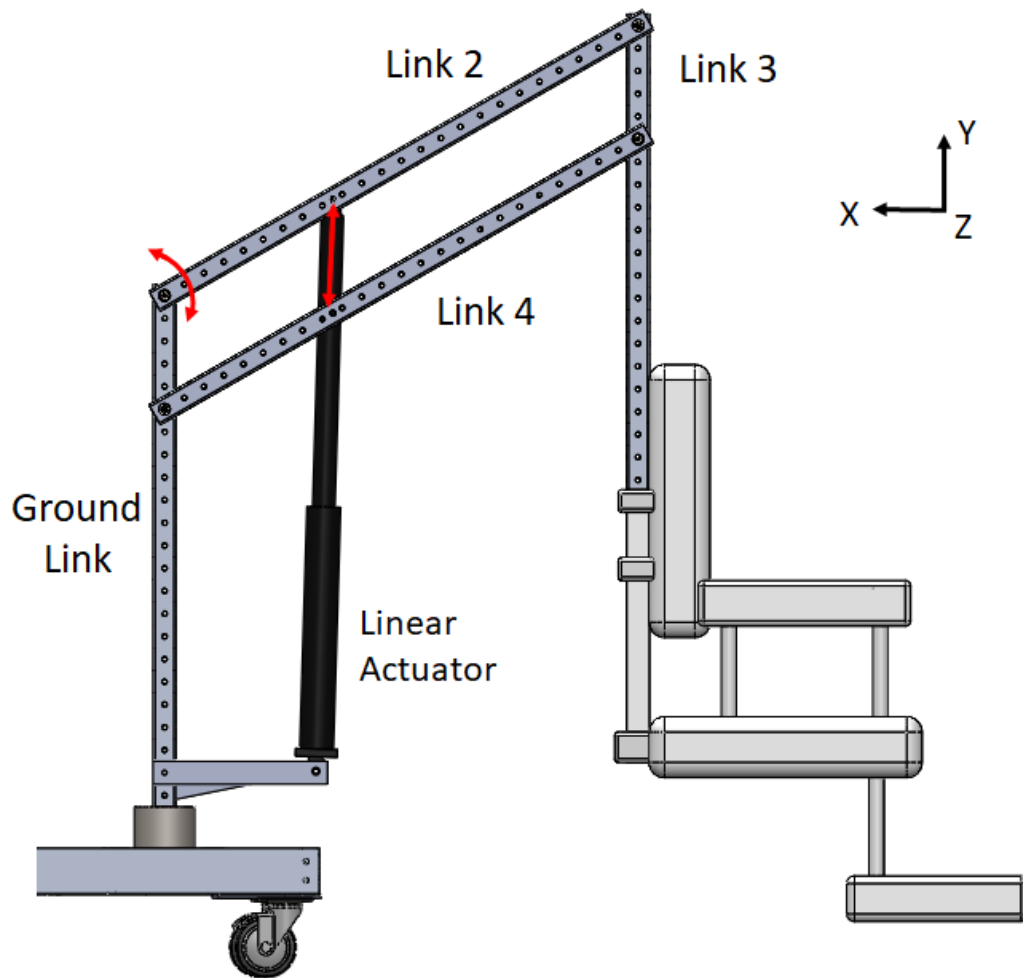


Figure 23: Lifting Mechanism

6.2 Bearing System

The lifting mechanism had to be able to rotate about the y-axis to position the chair over the pool deck. There would also be significant loading caused by the person sitting on chair at such a far distance from the mast. Therefore a bearing system was devised to allow for the mast to rotate and also handle the loading caused by using the device. Tapered roller bearings were selected since the system has a combination of axial and radial loading along the shaft. Figure 24 shows a cross-sectional view of the bearing system. An outer housing featuring shelves for each

of the two bearings was used to keep the system in place. The outer diameter of the two bearings were press fit to the housing so the outer race would remain stationary. The shaft was mated to the inner race of the bearings with a close fit. The housing itself was bolted to a plate that connected to the movable base.

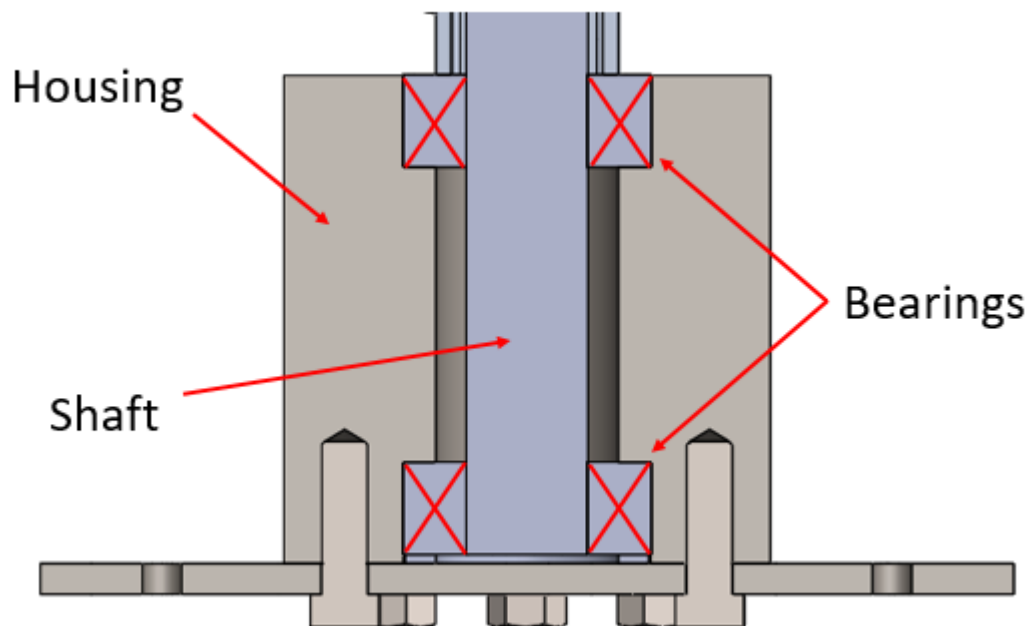


Figure 24: Cross-section View of Bearing System

6.3 Movable Base

One of the key features of our design was the mobility of the device as a whole. A movable base provided the mounting points for all the other subsystems of the device. This was made particularly easy by using 80/20 t-slotted extrusions for the framework of the base, which provided a lot of flexibility for the exact mounting locations. The base featured four caster wheels, two of which had foot brakes to allow the base to move smoothly. The exact casters

were selected based on the maximum weight of the device with a full tank and a 300 pound person sitting in the chair.

6.4 Pump and Tank System

In order to provide a counterweight to balance the system and also allow for the device to be easily movable when not in use, a fillable water tank is installed in the rear of the base. A low cost option to serve as the pump was a large 55 gallon drum. A hose connected to a portable water pump is placed in the water tank and the pump is used to siphon water from the pool temporarily. Once the use of the device is concluded, the pump can be placed in the drum and the hose in the pool, and then the water is returned to the pool. A picture of this system can be seen in Figure 25.

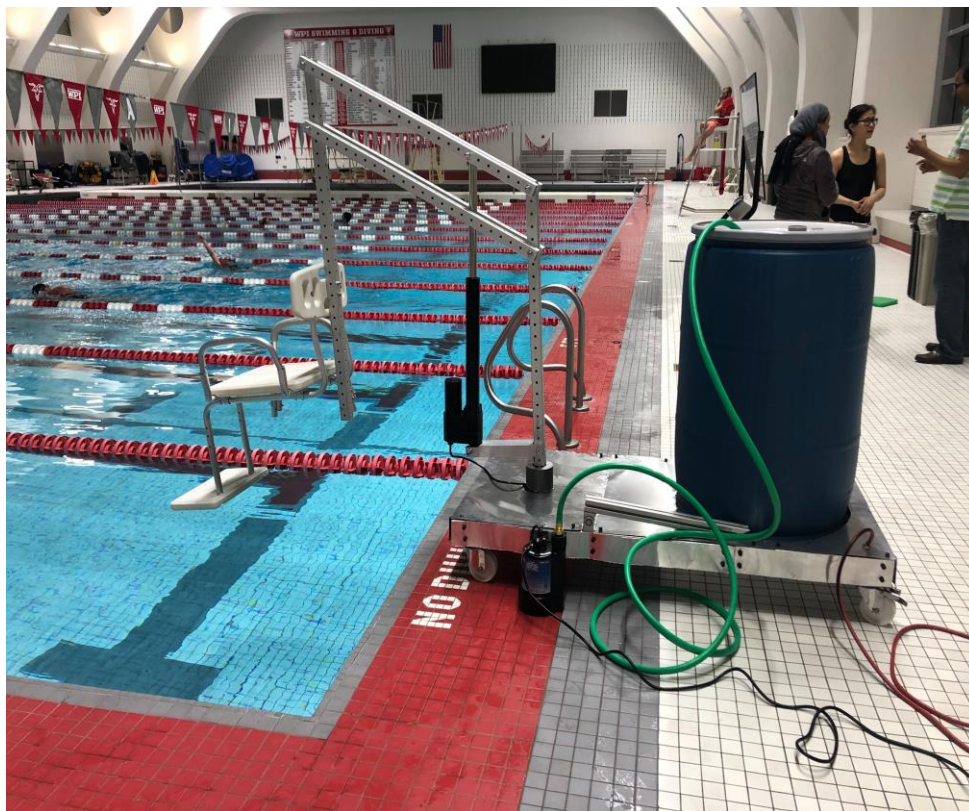


Figure 25: View of the Tank, Pump, and Base Assembly

6.5 Outrigger System

As mentioned during the discussion of the loaded configuration tipping analysis, an outrigger system had to be added to prevent the mechanism from tipping while the user boarded the device. Figure 26 shows a view of the outrigger system when deployed. This system utilizes a locking pivot joint with a piece of T-slotted 80/20 bar to engage with the ground. Rubber was applied to the end to increase traction with the ground surface. When not in use the bar can be rotated completely vertical so it will not interfere with the movement or storage of the device. A view of the outrigger in the retracted position can be seen in Figure 25.

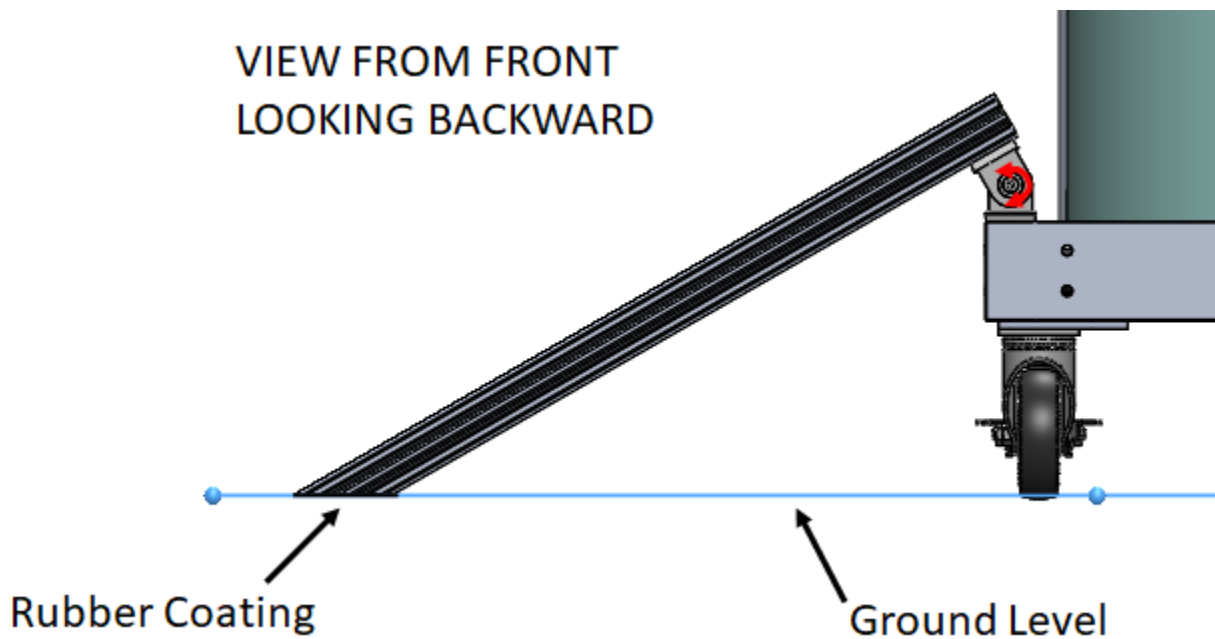


Figure 26: View of Deployed Outrigger System

6.6 Electrical System

The electrical system supplies power to and allows the control of the linear actuator in the lifting mechanism. The system was created using the wiring diagram shown in the Progressive Automations PA-31 data sheet (Figure 27). An extension cord was modified by cutting off one

end and exposing the live, neutral, and ground wires. These wires were tinned and then connected to the AC/DC converter to supply 120VAC. The control box and actuator were then wired as depicted with the accompanying wire harnesses.

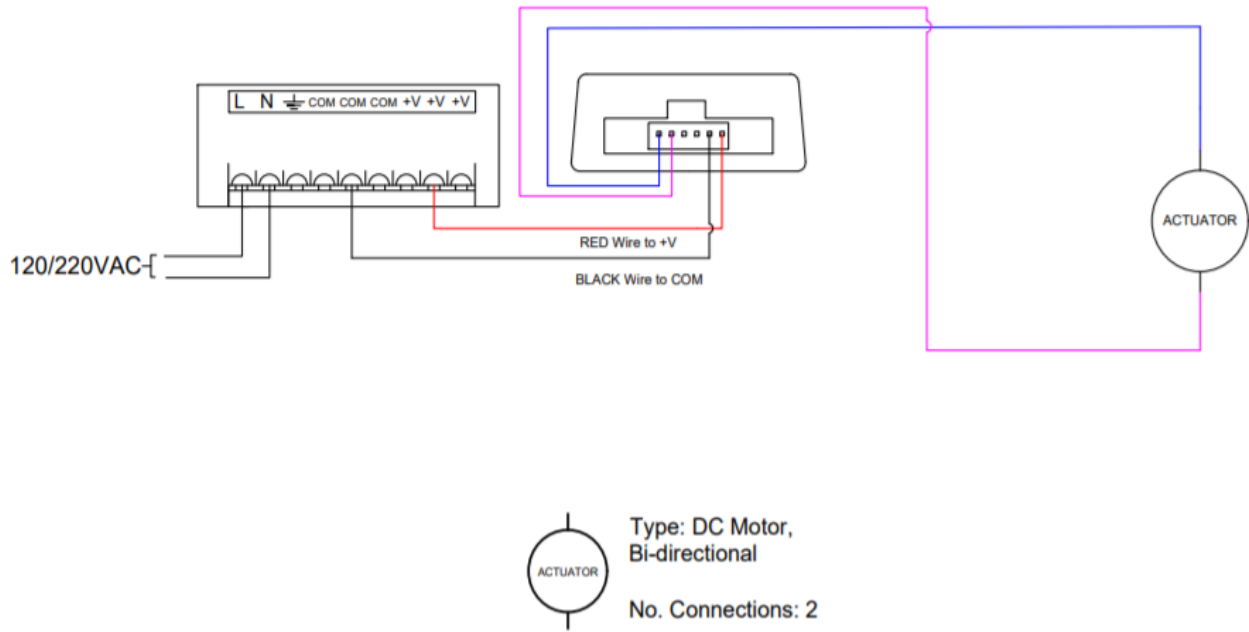


Figure 27: Progressive Automations PA-31 Wiring Diagram

7. Manufacturing

This section of the report will cover the general assembly sequence and manufacturing process when building the prototype of the device.

7.1 Movable Base

The base was the first subassembly to be built for the prototype. Two types of 80/20 t-slotted extrusions were used to make the framework of the device; the 1515 series and 1530 series. The 1515 series had maximum dimensions of 1.5 inches by 1.5 inches and features a one T-slot by one T-slot profile, while the 1530 series was 1.5 inches by 3.0 inches, and features a one T-slot by two T-slot rectangle profile. Two long 1530 series bars were placed parallel to each other so the rest of the bars could be slid in between and secured via slide in nuts and corner brackets. Once the bars were in the appropriate position the screws were tightened into place. It was important to place any slide in nuts required for later attachment into the crossbars before placing them in between the two long 1530 series bars, since the nuts could not be added once the entrance to the slot was blocked.

The next general step in the process of assembling the base was cutting the sheet metal that covered the exterior of the base. Some pieces of sheet metal were added later in the process so that holes could be cut out to allow certain parts to protrude from the top of the base, such as the water tank. Pieces were cut from three feet square sheets to match the geometry of the base. Holes were drilled in the sheet metal to allow for screws to thread in to prepared slide-in nuts. These screws ultimately secured the sheet metal to the exterior of the 80/20 bars.

Attaching the caster wheels of the base was the next step in the process. Since the minimum distance between the cross bars was restricted by the size of the brackets used, the bars

could not be placed directly under where the caster wheel bolt pattern was located. Therefore, adapter plates were used to make sure there was a sturdy surface for the caster to mount to. Screws for three of the caster wheels were placed through the caster, adapter plate, sheet metal and finally into a slide-in nut in the 80/20. The fourth screw was not over an 80/20 bar and instead a standard hex nut was threaded on from the back to secure it to the adapter plate. Two more screws attached the adapter plate to the next available 80/20 bar. In some cases, the holes between the adapter plates and sheet metal did not align with the 80/20 bars underneath. When this occurred, the 80/20 bar underneath was readjusted when possible. If this could not solve the problem, the holes in the sheet metal were widened to allow the screw to pass through.

The water tank was the next part of the base to be attached. The bottom of the barrel was traced out on a piece of sheet metal so a circular hole could be cut out to go around the barrel. The sheet was cut into two pieces to make cutting the circular hole easier. The outer diameter of the barrel was not the same at all points, and at the point where the sheet metal actually met the barrel was smaller than the bottom. As a solution, we had the two piece of sheet metal overlap so the hole was flush with the barrel. After securing the barrel, we realized that the whole device was light enough such that we could push the barrel to move the base. Therefore, we decided to remove the push bar from the design. The rest of the assembly is covered in the subsequent sections as the other major subsystems are attached to the base.

7.2 Lifting Mechanism

The first step in assembling the lifting mechanism was cutting threaded rods to act as the pins for the linkage joints. To do this correctly, a hex nut had to be threaded on prior to making the cut, then unscrewed to force the threads back into alignment after the cut. Once the rods were cut, the assembly of the linkage was straight forward as all the holes were pre-drilled in the 80/20

ready tube. The ready tube of the mast was secured with two bolts to a steel shaft. One of these bolts also secured the actuator bracket to the 80/20 ready tube. The shaft initially had a one inch diameter so the four inches of length of that were engaged with the bearings had to be turned down to a close fit with the 25 millimeter inner diameter of the bearing. The shaft was placed in the bearings and the housing was mounted to the movable base via a steel plate.

7.3 CNC Machined Parts

A few parts had complicated geometry and required more than simple drilling operations to manufacture. As a result, we utilized CNC machining to create these parts. The two parts where this method was used were the bearing housing and the actuator bracket.

7.3.1 Bearing Housing

The bearing housing was machined from a 4 inch diameter, 6 inch long cylindrical piece of low carbon steel. The final part is only 4 inches long, and therefore a facing operation was required to remove the excess material from the ends of the stock. This was performed on each side of the part in separate operations for a smoother surface finish. Next a through hole was drilled through the center of the entire piece for the shaft to go through. During the machining process we realized the design called for a through hole that was much wider than it needed to be, and also would be difficult to make much wider because the tooling was not available. The diameter of the actual part was left at 1.25 inches. Next, a pocketing operation was used to create the space for the top bearing. Afterwards the part was flipped over and a second pocket of the same diameter but slightly deeper was machined along with eight holes for a 3/8 - 24 tap which were later tapped by hand.

7.3.2 Actuator Bracket

When creating the CAM code for the actuator bracket, we realized the angled portion was difficult to machine. As a result, we made this shape rectangular instead. This would satisfy the same purpose as the triangular feature and be significantly easier to machine. The stock we used was a 3x3x12 inch³ aluminum bar. The process was completed in two operations. The first operation faced the part and machined the now rectangular support. The part was then flipped over, clamped on the rectangular support, and the two pockets were machined. During this process, the width of the part was also machined to match the desired specifications. One difference we observed in the finished product was that the pocket machined to hold the actuator had a thin layer of aluminum left over at the bottom of the pocket. We suspect this to be a combination of not extending the tool path further below the bottom of the pocket and the piece not being properly fixtured by clamping just the support feature from the first operation. The latter likely allowed the part to deflect during machining, since the other pocket was not incomplete and was machined at the same time. The two holes required in the flanges were drilled manually with a drill press after the two CNC operations were completed.

8. Testing

This section details all the testing conducted to verify performance and proof of concept for the relevant aspects of the design.

8.1 Electrical Testing

While in the process of building the device, we were able to start with some testing of the electrical components. In order to provide electricity to our device, we needed to outsource a 20 Ampere DC power supply in order to supply enough power for the actuator to lift and lower the chair into its in-use positions. After the assembly of the electrical components, we hooked up the power supply to a multimeter and measured the voltage across the power supply. We could only obtain a voltage of 10.77 volts through the multimeter, which is enough for our purposes. However, this could prove to be an issue when the device has to lift larger loads.

When the device was fully assembled, we also tested the performance of the electrical components in a dry setting where the lift was constructed, and in its actual position at the edge of the pool. During the assembly, we had to move the wiring around quite a bit to ensure the wires were in the correct position and not in the way of any vital moving components. It was important to make sure that the movement had not caused any issues with performance for the electrical system. During this testing procedure, the actuator functioned as expected, with no issues from an electrical standpoint. The actuator raised and lowered the four bar mechanism with the use of the remote and performed at a proper speed, similar to what we had predicted.

8.2 Full Device Testing

After the build of the device was completed, it was time to test the device in the Recreation Center at WPI. While all of the individual components had been tested on their own for performance and general overview, the final test within the Recreation Center was the first time we had powered up the device and completed an entire cycle.

We began by transporting the chair lift over to the WPI Recreation Center from Higgins Laboratories. The trip is roughly 100 yards, and is paved, flat ground. We kept an eye out for any potential issues along the travel path but encountered little to no obstacles along the way. Once in the athletic center, the chair lift was transported by elevator down to the pool area on the bottom floor. The chair was wheeled off into a corner of the pool area to be checked for any issues post-travel and to go over the checklist of events for the final tests.

In this corner, we had a relatively dry environment. The team decided to do a small scale test here to make sure the electronics were performing as expected, and the construction of the device was sound before we introduced water into the equation. This test proved that the components of the device were moving and functioning properly after travel. When no issues arose from the actuator and the electrical supply, we continued to the edge of the pool.

We began timing our device when we started moving it from the spot in the corner over to the edge of the pool. Once located in the correct position, we swung the chair into the correct position for the user to enter the seat, and locked the lockable caster wheels into the stationary position. The outrigger was also swung into position during this procedure. After the chair was in a safe, non-moving position, we started the process of filling the tank. We connected one end of the hose to the pump, and placed the other end within the water tank. The pump was switched on and began to fill the tank, taking approximately five minutes.

With the device in place, the water tank filled, and the wheels locked into position, the next step was to supply electricity. The extension cord was then plugged into the wall, and we continued through our required steps. The user we had originally decided to employ for this procedure only weighed roughly 150 pounds, so we expected the device to not encounter any weight-related issues during the test. However, once we assembled the device, we noticed some potential structural issues within the 80/20 frame that the bearing housing sat upon. While the housing was sound, the aluminum bars underneath were beginning to deflect under the weight being added. Activity from this point forward proceeded with extreme caution, keeping an eye out for any potential issues. The mast assembly was then swung out over the edge of the pool. Located in its proper position for movement, the chair was then lowered down into the water via the remote system that came with the control box. We decided to keep the remote on the side of the pool, as the company that provided the remote did not advertise it as being waterproof and we did not want to ruin it by exposing it to water.

Once the chair had been lowered into the pool, we let the device sit in this position for a few moments to ensure the chair was stable and behaving as expected. After a few minutes, we once again used the remote system to raise the chair out of the water and manually swiveled the mast and chair assembly back over the edge of the pool. The chair was then unplugged from the wall so as to avoid any electrical issues with water involved before moving on to emptying the tank and putting away the device.

Emptying the tank was the next step in the process, so we removed the lid from the water tank and placed the pump in the tank with the other end of the hose back over the edge of the pool to release the water. We let the pump drain the tank completely, a process that took roughly four minutes and replaced the lid. The outrigger was then loaded back into the travel position and

the caster wheels were unlocked, at which point the chair was moved back into the corner of the pool area.

At this point, we stopped the timers on our test and concluded that the entire process takes roughly eight minutes for moving the user one way (i.e. from the deck into the pool or from pool to the deck). This time could be further shortened with more experience in completing the entire procedure and still meets our functional requirements.

9. Conclusions and Recommendations

We met 14 out of 16 of our functional requirements. The two functional requirements we were unable to meet were complete ADA standard compliance and ASTM corrosion standard compliance.

One of the bigger obstacles we encountered during this project was the ability to make the device able to sustain a load of 300 pounds. With the testing that our group completed, we were able to get the device to function without a user seated on the device. Due to a limited budget, our prototype sacrificed being robust for falling within a manageable price standpoint, so the materials were unable to withstand the expected 300 pound load. As the ADA standards require the lift be able to lift a maximum weight of 300 pounds, we were unable to satisfy this functional requirement. However, we propose a few changes that would allow for this maximum weight to be achievable.

We were unable to test if the device met the ASTM standards for corrosion due to time constraints. The majority of the device was constructed with aluminum, however, some parts had to be made of low carbon steel in order to have suitable strength. Aluminum is typically corrosion resistant unless at extremes of the pH scale while non-stainless steel corrodes easily at acidic pH. The environment at a pool would be in the acidic pH range, so the steel parts would likely be at risk of corrosion.

9.1 Recommendations

For future designs of a handicapped pool chair lift, we believe that our design has highlighted some key concepts to keep in mind in order to produce a more robust prototype. The following section details the changes we could make to our device in particular in order to fully satisfy our functional requirements.

Improve material selection and robustness of parts.

To improve on the overall stability of the device, we would first redesign the mast to be more robust than our final prototype. To do this, we would change the cross section of the mast to be thicker. In its final state, the mast is only 1.5 inches wide and made out of 80/20 aluminum ReadyTube, a hollow 1/8in thick aluminum extrusion with dozens of holes pre-drilled in to all sides, severely compromising the yield strength of a part of the prototype that bears a very significant load during operation. Stainless steel would be a much stronger alternative with corrosion resistant properties. This material is much more expensive which should also be taken into consideration, as well as the location of all holes would need to be machined to more exact tolerances.

Use battery power to operate the device.

The electrical system could also be further improved in a few ways. The present design requires the device have access to external power from a wall outlet. This restriction is less than ideal due to the potential for water to cause an electrical shock or for a wall outlet to be a large distance from where the device needs to be used. The current electrical system may also fail when attempting to lift a person at the higher end of the ADA requirement of 300 pounds. During our preliminary testing of the electrical system, the maximum voltage we were able to obtain at the terminals of the AC/DC converter was 10.77 Volts instead of the advertised 12

Volts. We suspect that this is a result of a voltage drop from the long extension cord we were using. While this was not an issue to operate the actuator at lower loading, at higher loads this supplied power might not be enough. As an improvement we would recommend using an on board power supply in the form of a replaceable battery.

Conduct a more thorough analysis of the outrigger system.

The need for the outrigger system was identified late in the term of this project. A thorough stress analysis is required to understand if the current design would be able to withstand the loading caused by having a 300 pound person sit on the chair in the loading position.

Have a means of securing the hose to the tank upon use.

Currently the only challenge to operating the lift is that holding the pump in the water while also having the hose rest on the edge of the tank was difficult to manage. If the end of the hose that put water into the tank could be screwed in before use, this would make the process much easier for the user.

Improve upon the rotation system for the mast.

Our design uses a dual-bearing system to hold the mast, with a steel rod press fit into both bearings, and secured inside the vertical ReadyTube by two screws drilled through the rod. This system allows for 360 degrees of rotation which creates several undesired pinch points. A locking system for the mast would remove these pinch points, and could also ensure that the user does not freely rotate when they are desired to be stationary. An even higher technology option would be to have this rotation motorized.

Include a robust push-bar.

While our design includes a push-bar, our final prototype is moved by simply pushing the entire rig by the 55-gallon drum. This works in the short term, but would be both impractical and unprofessional from a final design standpoint. A sturdy push bar attached to the base would allow for the base to be moved and controlled easily by the user.

Include a method to secure the pump.

Our final prototype involves the user filling the tank with water by having to hold a pump a foot below the surface of the water in the pool while also plugging and unplugging the pump to turn the pump on or off. Having some sort of detachable shelf on the side of the pool would eliminate the need for an operator to hold the pump for a full five minutes while the tank is being filled.

As the MQP is designed to teach students to be innovative and to learn to accommodate real world scenarios involving price limitations and time constraints, we realize that the shortcomings of our device are realistic but could also be improved upon with a higher budget. As our device is so large and must sustain quite a heavy load, most of the materials needed to have higher strength and the assembly of these materials should be done with corrosion and other environmental factors kept in mind. With an overall budget of just over \$1,000, our decisions had to be made with manufacturability as a forefront priority and robustness of the design as a close second.

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Appendix A: Authorship

Chapter	Section	Author
Abstract		Alves
Acknowledgements		Alves
Introduction		All
Background		
	ADA Guidelines	Alves
	Current Pool Lift Designs	Alves
	Safety Guidelines and Practices	-
	Human Factors	Cadilek
	OSHA Regulations	Corwin
	ASTM Standards	Alves
	Functional Requirements	All
Design Concepts		
	Lifting Mechanism	Alves
	Three Bar Inverted Slider Mechanism	Alves
	Counterweight Designs	Alves
	Fixed to Deck (Anchor)	Alves
	Movable Base with Removable Weights	Alves
	Movable Base with Water Tank	Alves
Design Selection		
	Lifting Mechanism Decision	Corwin
	Counterweight Decision	Corwin
Synthesis and Analysis		

	Tipping Analysis	Cadilek
	Unloaded Configuration	Cadilek
	Loaded Configuration	Cadilek
	Three Position Synthesis	Cadilek
	Kinematic and Dynamic Analysis	Cadilek
Detailed Design Description		
	Lifting Mechanism	Cadilek
	Bearing System	Cadilek
	Movable Base	Cadilek
	Pump and Tank System	Cadilek
	Outrigger System	Cadilek
	Electrical System	Cadilek
Manufacturing		
	Movable Base	Cadilek, Corwin
	CNC Machined Parts	Cadilek, Corwin
	Bearing Housing	Cadilek, Corwin
	Actuator Bracket	Cadilek, Corwin
Testing		
	Electrical Testing	Alves
	Full Device Testing	Alves
Conclusions and Recommendations		All
Appendix B	Assembly Drawing and BOM	Corwin
Appendix C	Detail Part Drawings	Corwin
Appendix D	Mathcad Calculations	Cadilek

Appendix B: Assembly Drawing and BOM

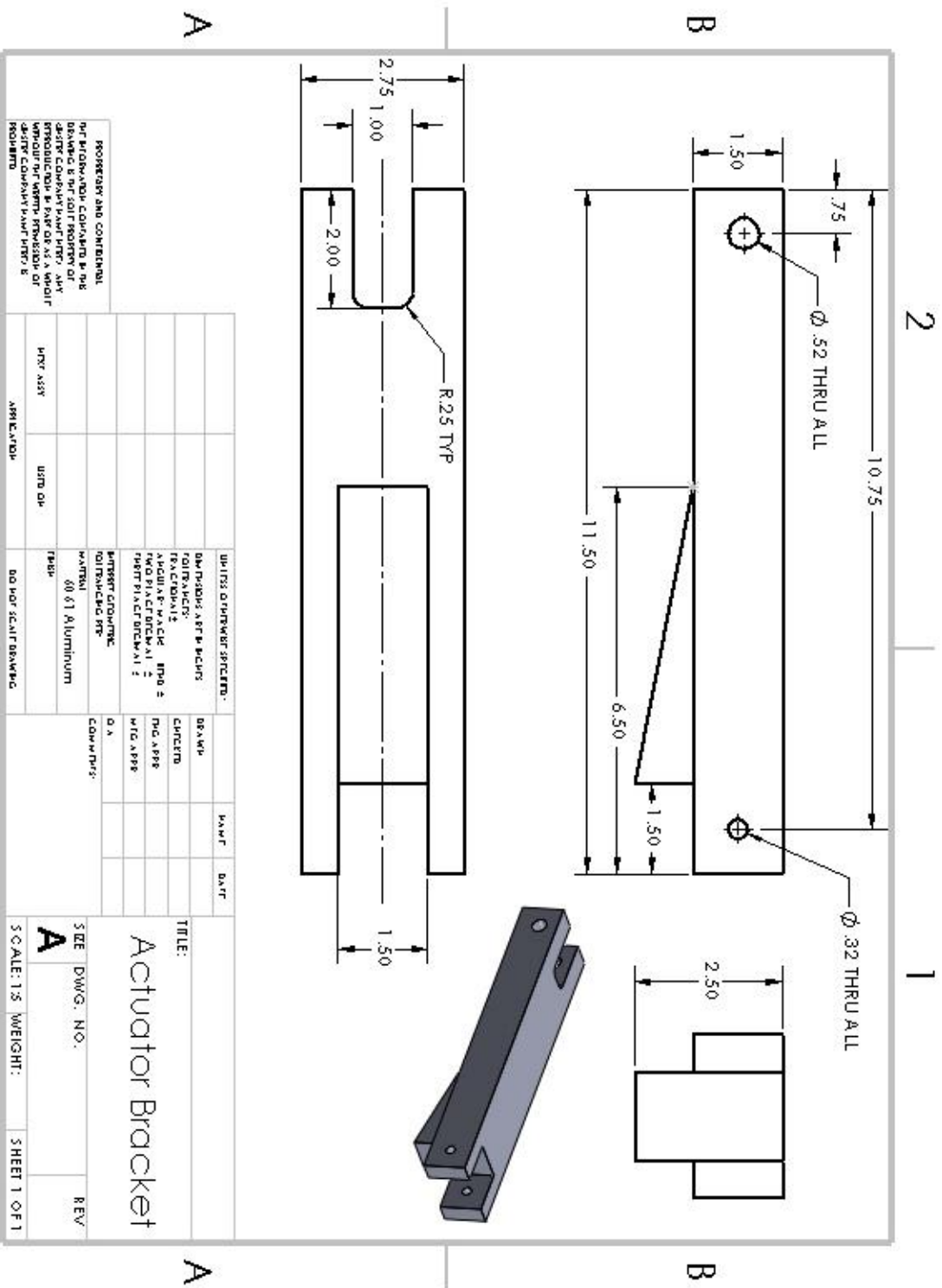
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NOTE: 2. DIMENSION 2	COMMENTS:		
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DRAWN BY: J. SMITH			
CHECKED BY: M. JONES			
APPROVED BY: K. BROWN			
DATE: 10/26/2010			
SCALE: 1:50 WEIGHT:			

TITLE:
FULL ASSEMBLY

SIZE: DWG. NO. _____ REV. _____
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	A	B	C
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3	2	2	54inch 1530
4	2	3	25inch 1530
5	8	4	25inch 1515
6	1	5	Top Sheet Metal
7	2	6	Side Sheet Metal
8	2	7	Front/Back Sheet Metal
11	81	8	Economy Slide in nut
12	40	9	4302
13	80	10	5/16-18 Screw
14	8	11	3/8-24x1.25 Hex nut
15	1	12	Actuator_Cylinder
18	1	13	36in ReadyTube
19	2	14	36in 1515
20	1	15	Chair Base
21	6	16	Armrest Dowels
22	2	17	Backrest Support
23	1	18	Backrest
24	2	19	Armrests
25	1	20	Footrest
26	5	21	5/16-18 Threaded Rods
27	16	22	Preferred Wide FW 0.25
28	4	23	1/4-20 Hex nut
29	1	24	Mast_bar
30	1	25	Mast_Rod
31	2	26	Bearing
33	1	27	Actuator Mounting Fixture
34	4	28	Caster Wheel Adapter Plates
36	1	29	Bottom Bearing Plate
38	1	30	Bearing Housing
39	1	31	Bearing washer
40	1	32	Water barrel
45	1	33	Outrigger
46	1	34	1/4-20x5.25 Hex Bolt

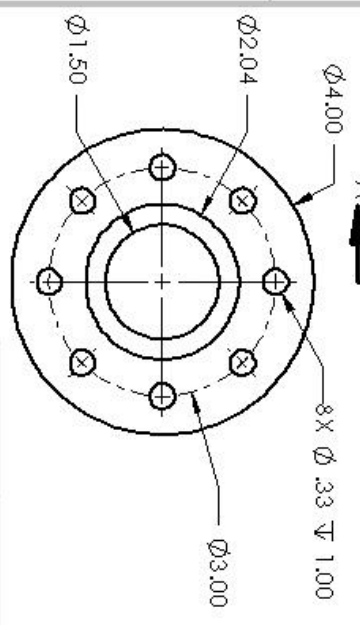
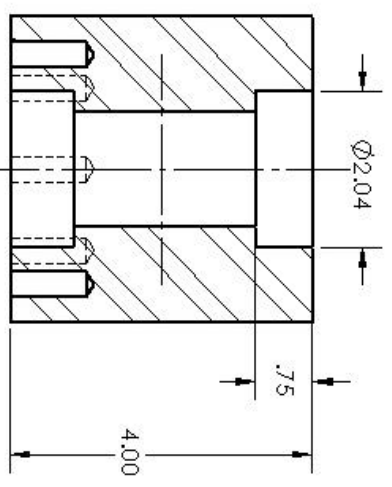
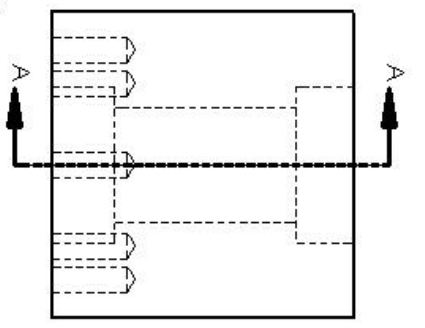


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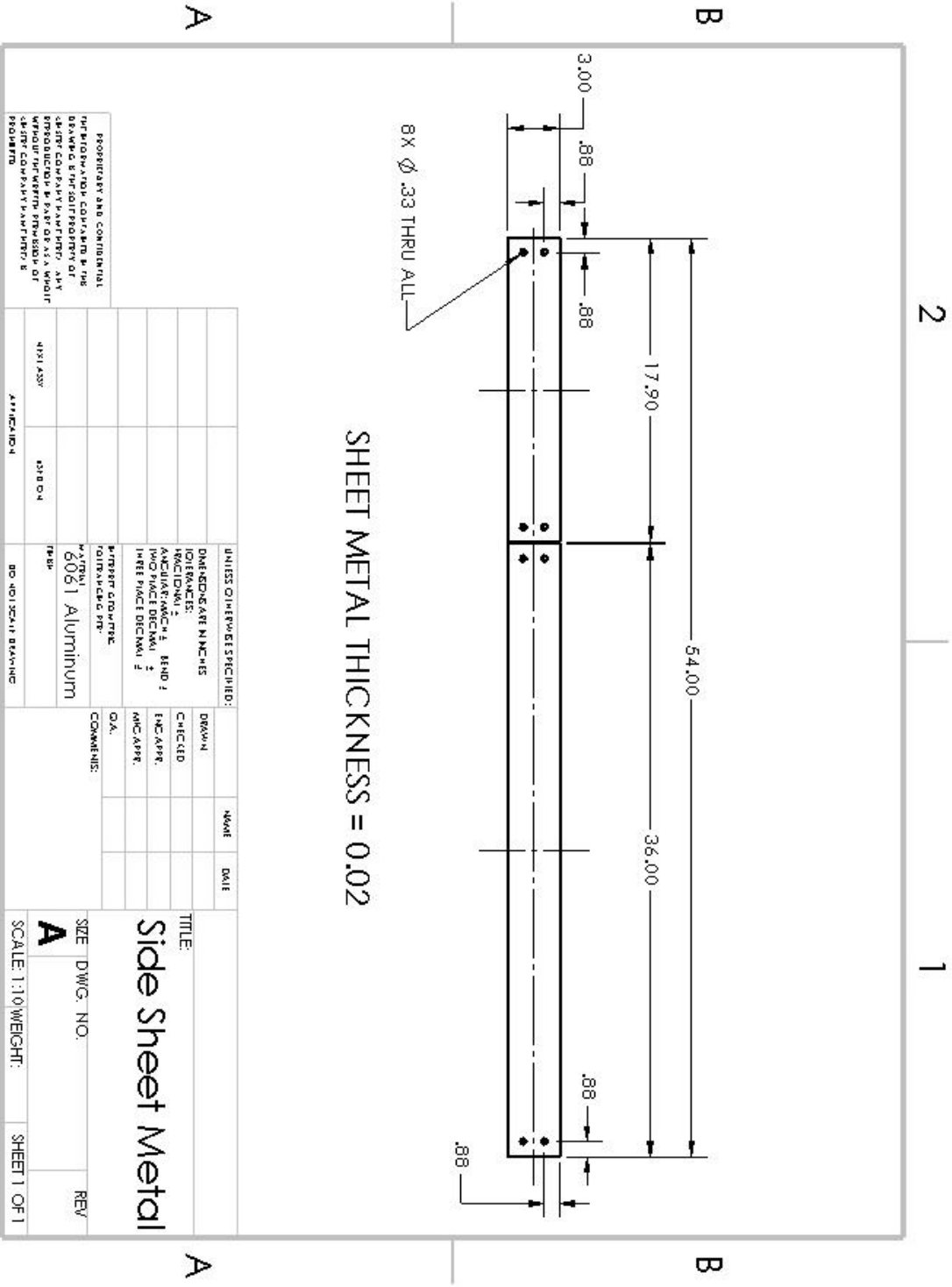
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B

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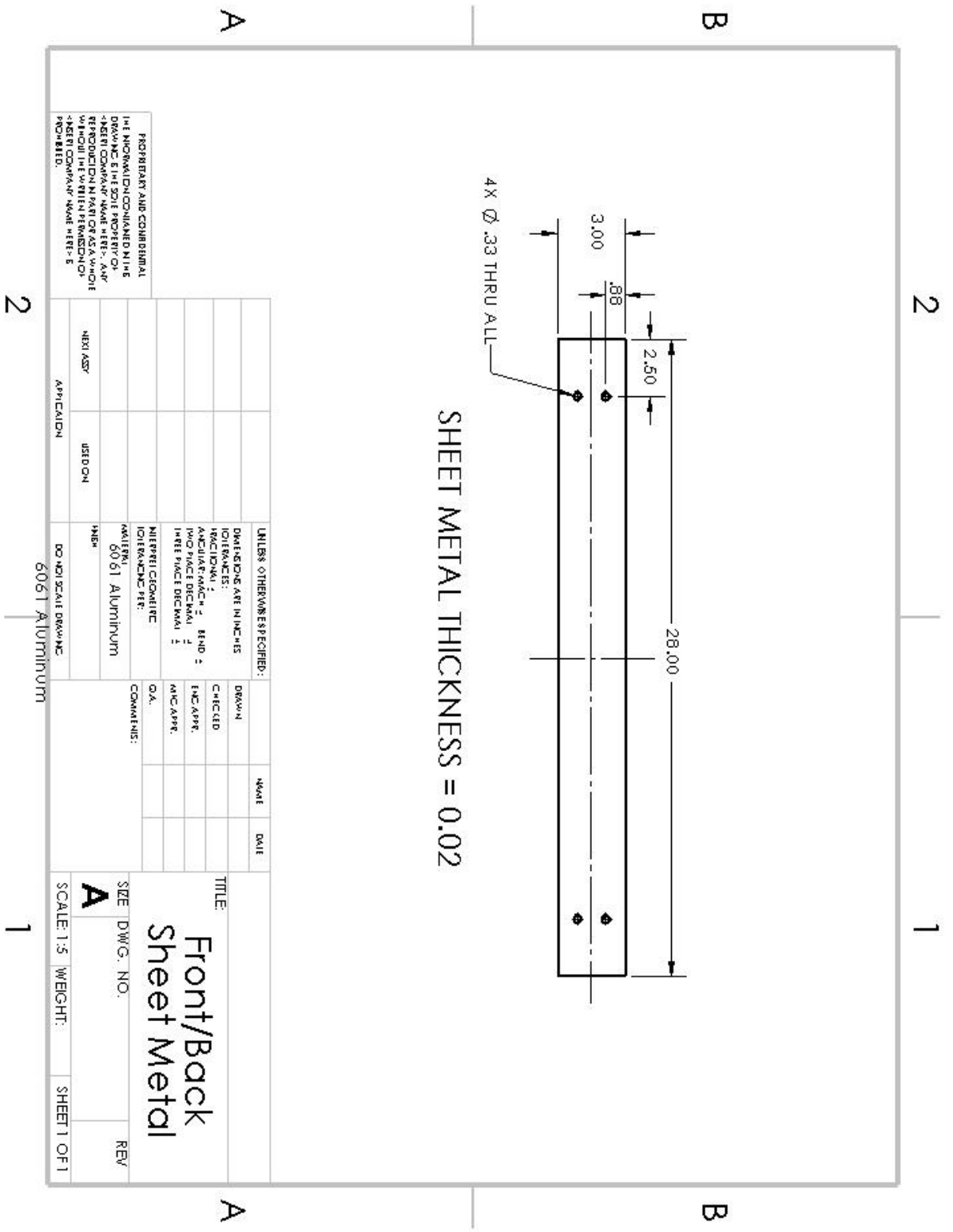
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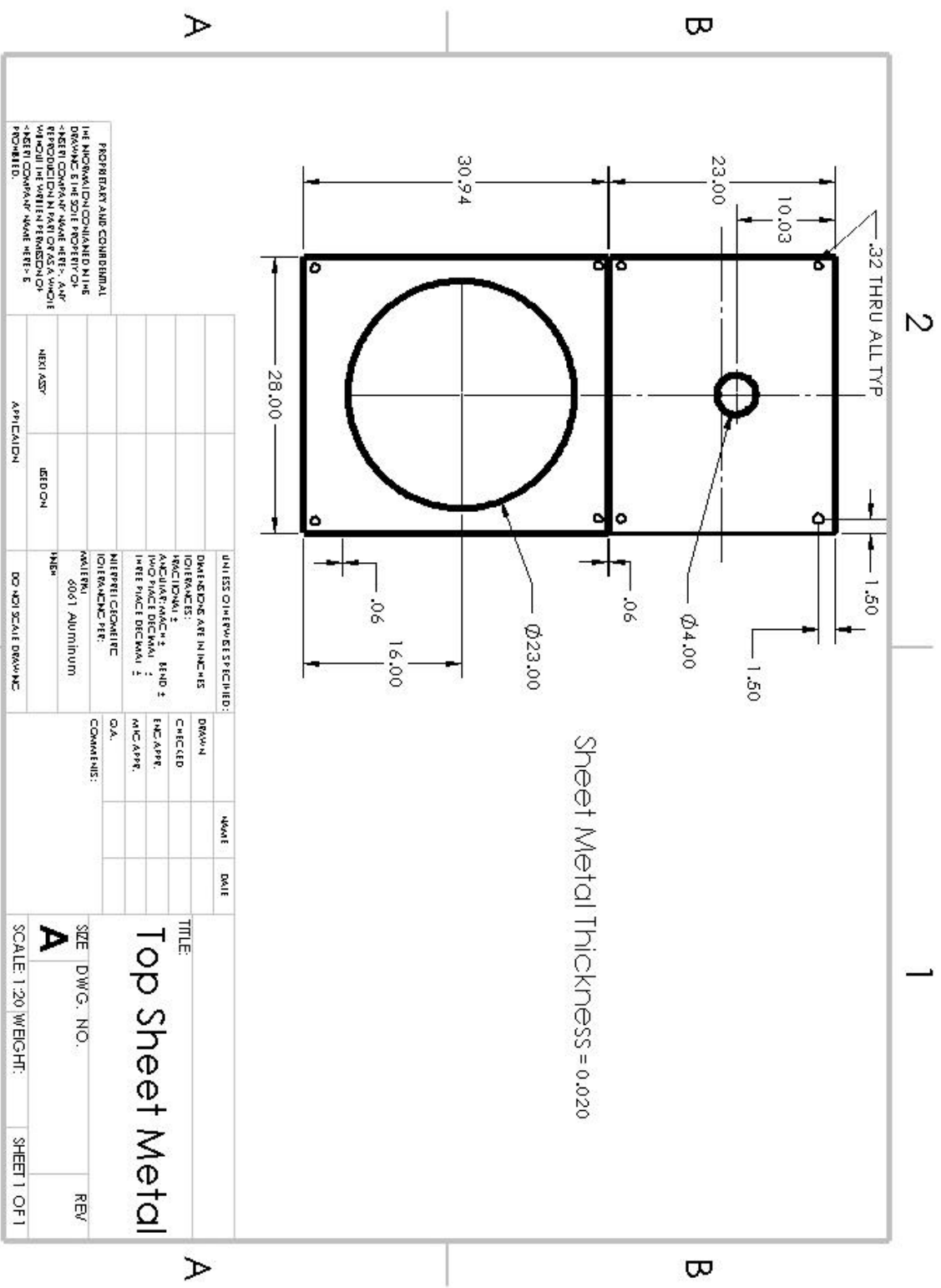
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2 1



Sheet Metal Thickness = 0.020

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FINISH				
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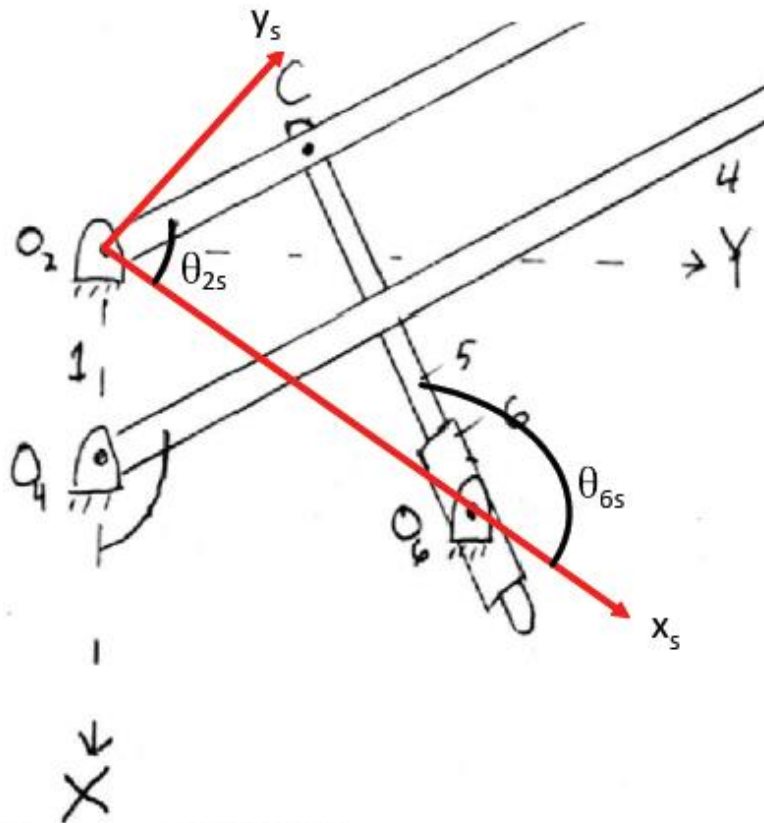
A

B

Appendix D: Mathcad Calculations

All equations reference Norton's Design of Machinery book.

Position analysis starting with inverted crank slider loop:



$a_s := 12.8\text{in}$	Length O_2 to C.
$b_s := 38.58\text{in}$	Length O_6 to C.
$c_s := 0\text{in}$	Has no length, $\gamma = 0$.
$d_s := 33.51\text{in}$	Length O_2 to O_6 .
$\gamma := 0\text{deg}$	$\Phi := 17.29\text{deg}$

values taken from Solidworks

$$a \cdot \cos \theta_2 - b \cdot \cos \theta_6 - c \cdot \overset{0}{\cos \theta_6} - d = 0$$

eqn 4.23a

$$a \cdot \sin \theta_2 - b \cdot \sin \theta_6 - c \cdot \overset{0}{\sin \theta_6} = 0$$

eqn 4.23b

using the process described on pg 195
to derive eqn 4.19:

$$a \cdot \cos \theta_2 = b \cdot \cos \theta_6 - d$$

$$a \cdot \sin \theta_2 = b \cdot \sin \theta_6$$

Square:

$$a^2 \cos^2 \theta_2 = (b \cos \theta_6 - d)^2$$

$$a^2 \sin^2 \theta_2 = b^2 \sin^2 \theta_6$$

add:

$$a^2 (\overset{1}{\cos^2 \theta_2 + \sin^2 \theta_2}) = b^2 \cos^2 \theta_6 + 2bd \cos \theta_6 + d^2 + b^2 \sin^2 \theta_6$$

$$a^2 = b^2 (\overset{1}{\cos^2 \theta_6 + \sin^2 \theta_6}) + 2bd \cos \theta_6 + d^2$$

$$\frac{a^2 - b^2 - d^2}{2bd} = \cos \theta_6$$

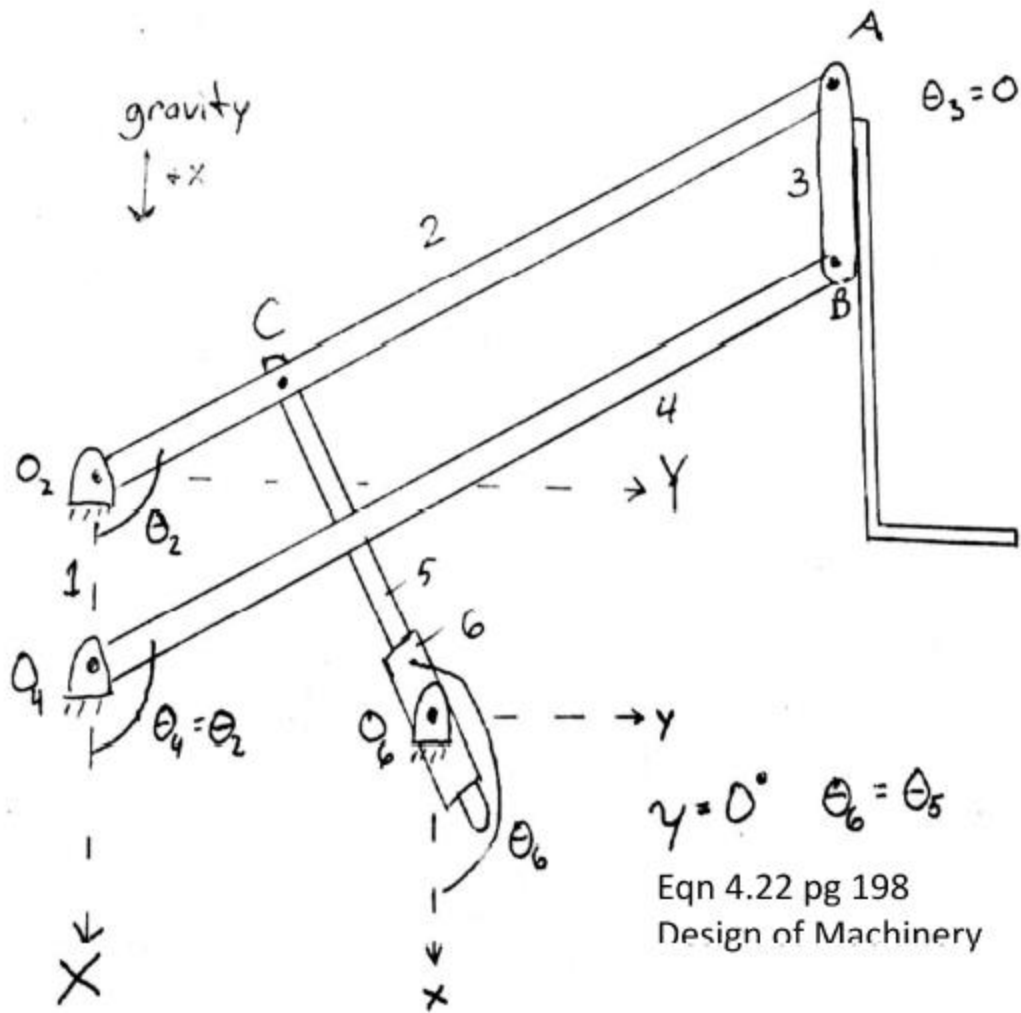
$$\cos^{-1} \left(\frac{a^2 - b^2 - d^2}{2bd} \right) = \theta_6$$

$$\theta_{6s} := \arccos \left(\frac{a_s^2 - b_s^2 - d_s^2}{2 \cdot b_s \cdot d_s} \right) = 161.187 \cdot \text{deg}$$

$$\theta_{2s} := \arcsin \left(\frac{b_s}{a_s} \cdot \sin(\theta_{6s}) \right) = 76.404 \cdot \text{deg}$$

eqn 4.24a, with substitution $\theta_5 = \theta_6$,
knowing $\gamma = 0$. (eqn 4.22)

Using solved θ_2 to complete motion of the mechanism:



$$a := 36\text{in}$$

$$b := 7.5\text{in}$$

$$c_{cc} = 36\text{in}$$

$$d := 7.5\text{in}$$

$$\theta_2 := \theta_{2s} + \Phi = 93.694 \text{ deg}$$

$$\theta_6 := \theta_{6s} + \Phi = 178.477 \text{ deg}$$

$$\theta_5 := \theta_6$$

Crank (link 2) length

Coupler (link 3) length

Rocker (link 4) length

Ground (link 1) length

Translating coordinate systems from $x_s - y_s$ to X - Y. Added angle value taken from Solidworks.

$$K_1 := \frac{d}{a} \quad \text{eqn 4.8a}$$

$$K_2 := \frac{d}{c}$$

$$K_3 := \frac{(a^2 - b^2 + c^2 + d^2)}{2 \cdot a \cdot c}$$

$$K_4 := \frac{d}{b} \quad \text{eqn 4.11b}$$

$$K_5 := \frac{(c^2 - d^2 - a^2 - b^2)}{2 \cdot a \cdot b}$$

$$\overset{\text{ww}}{A} := \cos(\theta_2) - K_1 - \cos(\theta_2) \cdot K_2 + K_3 \quad \text{eqn 4.10a}$$

$$B := -2 \cdot \sin(\theta_2)$$

$$\overset{\text{ww}}{C} := K_1 - (1 + K_2) \cos(\theta_2) + K_3$$

$$A = 0.741$$

$$B = -1.996$$

$$C = 1.286$$

$$\theta_4 := 2 \cdot \text{atan} \left[\frac{(-B - \sqrt{B^2 - 4 \cdot A \cdot C})}{2A} \right] \quad \text{eqn 4.10b}$$

$$\theta_4 = 93.694 \cdot \text{deg}$$

$$D := \cos(\theta_2) - K_1 + \cos(\theta_2) K_4 + K_5 \quad \text{eqn 4.12}$$

$$E := -2 \cdot \sin(\theta_2)$$

$$\overset{\text{ww}}{F} := K_1 + (-1 + K_4) \cos(\theta_2) + K_5$$

$$\theta_3 := 2 \cdot \text{atan} \left[\frac{(-E - \sqrt{E^2 - 4 \cdot D \cdot F})}{2D} \right] \quad \text{eqn 4.13}$$

$$\theta_3 = 2.332 \times 10^{-14} \cdot \text{deg} \sim 0$$

Using solved ω_2 to complete motion of the mechanism:

Coupler Point $R_{pa} := 47.751\text{in}$ $\delta_p := 13.146\text{deg}$ This point represents where the person sits in the chair.

$$\omega_3 := \frac{a}{b} \cdot \omega_2 \cdot \left(\frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \right) \quad \text{eqn 6.18a}$$

$$\omega_3 = 0 \frac{1}{s}$$

$$\omega_4 := \frac{a}{c} \cdot \omega_2 \cdot \left(\frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \right) \quad \text{eqn 6.18b}$$

$$\omega_4 = -0.052 \frac{1}{s}$$

For velocity and acceleration answers the "real" component of the answer is the x-component and the "imaginary" component is the y-component.

$$v_A := a \cdot \omega_2 \cdot (j \cos(\theta_2) - \sin(\theta_2)) \quad \text{eqn 6.19a}$$

$$v_A = (1.86 + 0.12i) \cdot \frac{\text{in}}{s}$$

$$v_{BA} := b \cdot \omega_3 \cdot (j \cos(\theta_3) - \sin(\theta_3)) \quad \text{eqn 6.19b}$$

$$v_{BA} = 0 \cdot \frac{\text{in}}{s}$$

$$v_B := c \cdot \omega_4 \cdot (j \cos(\theta_4) - \sin(\theta_4)) \quad \text{eqn 6.19c}$$

$$v_B = (1.86 + 0.12i) \cdot \frac{\text{in}}{s}$$

$$v_{PA} := R_{pa} \cdot \omega_3 \cdot (-\sin(\theta_3 + \delta_p) + j \cdot \cos(\theta_3 + \delta_p)) \quad \text{eqn 6.19b}$$

$$v_{PA} = -2.571i \times 10^{-15} \cdot \frac{\text{in}}{s}$$

$$v_P := v_A + v_{PA} \quad \text{eqn 6.19c}$$

$$v_P = (1.86 + 0.12i) \cdot \frac{\text{in}}{s} \quad v_{px} := \text{Re}(v_P) = 1.86 \cdot \frac{\text{in}}{s}$$

Acceleration analysis starting with inverted slider four bar loop:

$$b_{2\dot{\theta}} := 0$$

eqn 7.25a with $\theta_5 = \theta_6$, $c = 0$

$$-a \cdot \alpha_2 \sin \theta_2 - a \omega_2^2 \cos \theta_2 + b \cdot \alpha_6 \sin \theta_6 + b \omega_6^2 \cos \theta_6 + 2 \dot{b} \omega_6 \sin \theta_6 - \ddot{b} \cos \theta_6 = 0$$

$$a \cdot \alpha_2 \sin \theta_2 = -a \omega_2^2 \cos \theta_2 + b \cdot \alpha_6 \sin \theta_6 + b \omega_6^2 \cos \theta_6 + 2 \dot{b} \omega_6 \sin \theta_6$$

eqn 7.26b with $\theta_5 = \theta_6$, $c = 0$

$$a \cdot \alpha_2 \cos \theta_2 - a \omega_2^2 \sin \theta_2 - b \alpha_6 \cos \theta_6 + b \omega_6^2 \sin \theta_6 - 2 \dot{b} \omega_6 \cos \theta_6 - \ddot{b} \sin \theta_6 = 0$$

$$\alpha_6 = \frac{a \alpha_2 \cos \theta_2 - a \omega_2^2 \sin \theta_2 + b \omega_6^2 \sin \theta_6 - 2 \dot{b} \omega_6 \cos \theta_6}{b \cos \theta_6}$$

$$7.25b \rightarrow 7.25a$$

$$a \alpha_2 \sin \theta_2 = -a \omega_2^2 \cos \theta_2 + b \sin \theta_6 \left[\frac{a \alpha_2 \cos \theta_2 - a \omega_2^2 \sin \theta_2 + b \omega_6^2 \sin \theta_6 - 2 \dot{b} \omega_6 \cos \theta_6}{b \cos \theta_6} \right] + b \omega_6^2 \cos \theta_6 + 2 \dot{b} \omega_6 \sin \theta_6$$

$$a (\sin \theta_2 - \tan \theta_6 \cos \theta_2) \alpha_2 = -a \omega_2^2 \cos \theta_2 + \tan \theta_6 [-a \omega_2^2 \sin \theta_2 + b \omega_6^2 \sin \theta_6 - 2 \dot{b} \omega_6 \cos \theta_6] + b \omega_6^2 \cos \theta_6 + 2 \dot{b} \omega_6 \sin \theta_6$$

$$\alpha_2 = \frac{-a \omega_2^2 \cos \theta_2 + \tan \theta_6 [-a \omega_2^2 \sin \theta_2 + b \omega_6^2 \sin \theta_6 - 2 \dot{b} \omega_6 \cos \theta_6] + b \omega_6^2 \cos \theta_6 + 2 \dot{b} \omega_6 \sin \theta_6}{a (\sin \theta_2 - \tan \theta_6 \cos \theta_2)}$$

The formulation for α_2 is quite long, and has to be broken into two parts and summed together to fit the width of the page.

$$\alpha_{2\text{part1}} := \frac{\tan(\theta_{6s}) (-a_s \cdot \omega_2^2 \cdot \sin(\theta_{2s}) + b_s \cdot \omega_6^2 \cdot \sin(\theta_{6s}) - 2 \cdot b_{\dot{\theta}} \cdot \omega_6 \cdot \cos(\theta_{6s}))}{a_s \cdot (\sin(\theta_{2s}) - \tan(\theta_{6s}) \cdot \cos(\theta_{2s}))}$$

$$\alpha_{2\text{part2}} := \frac{-a_s \cdot \omega_2^2 \cdot \cos(\theta_{2s}) + b_s \cdot \omega_6^2 \cdot \cos(\theta_{6s}) + 2 \cdot b_{\dot{\theta}} \cdot \omega_6 \cdot \sin(\theta_{6s})}{a_s \cdot (\sin(\theta_{2s}) - \tan(\theta_{6s}) \cdot \cos(\theta_{2s}))}$$

$$\alpha_2 := \alpha_{2\text{part1}} + \alpha_{2\text{part2}} = 2.374 \times 10^{-4} \frac{\text{rad}}{\text{s}^2}$$

$$\alpha_6 := \frac{a_s \cdot (\alpha_2 \cdot \cos(\theta_{6s} - \theta_{2s}) + \omega_2^2 \cdot \sin(\theta_{6s} - \theta_{2s})) - 2 \cdot b_{\dot{\theta}} \cdot \omega_6}{b_s} = 8.395 \times 10^{-4} \frac{\text{rad}}{\text{s}^2} \quad \text{eqn 7.26a}$$

$$\alpha_5 := \alpha_6$$

$$A_C := a_s \cdot \alpha_2 \cdot (-\sin(\theta_{2s}) + j \cdot \cos(\theta_{2s})) - a_s \cdot \omega_2^2 \cdot (\cos(\theta_{2s}) + j \cdot \sin(\theta_{2s})) \quad \text{eqn 7.27a}$$

$$A_C = (-0.011 - 0.033i) \cdot \frac{\text{in}}{\text{s}^2}$$

$$A_{O6CPart1} := b_s \cdot \alpha_6 \cdot (\sin(\theta_{6s}) - j \cdot \cos(\theta_{6s})) + b_s \cdot \omega_6^2 \cdot (\cos(\theta_{6s}) + j \cdot \sin(\theta_{6s})) \quad \text{eqn 7.27b}$$

$$A_{O6CPart2} := 2b_{dot} \cdot \omega_6 \cdot (\sin(\theta_{6s}) - j \cdot \cos(\theta_{6s})) - b_{2dot} \cdot (\cos(\theta_{6s}) + j \cdot \sin(\theta_{6s}))$$

$$A_{O6C} := A_{O6CPart1} + A_{O6CPart2} = (0.011 + 0.033i) \cdot \frac{\text{in}}{\text{s}^2}$$

$$A_{O6} := -c_s \cdot \alpha_6 \cdot (\sin(\theta_{6s}) - j \cdot \cos(\theta_{6s})) - c_s \cdot \omega_6^2 \cdot (\cos(\theta_{6s}) + j \cdot \sin(\theta_{6s})) = 0 \quad \text{eqn 7.27c}$$

Using solved α_2 to complete motion of the mechanism:

$$\underline{\underline{A}} := c \cdot \sin(\theta_4) \quad \text{eqn 7.12c}$$

$$\underline{\underline{B}} := b \cdot \sin(\theta_3)$$

$$\underline{\underline{C}} := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_4)$$

$$\underline{\underline{D}} := c \cdot \cos(\theta_4)$$

$$\underline{\underline{E}} := b \cdot \cos(\theta_3)$$

$$\underline{\underline{F}} := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_4)$$

$$\alpha_3 := \frac{(C \cdot D - A \cdot F)}{(A \cdot E - B \cdot D)} = 0 \cdot \frac{\text{rad}}{\text{s}^2} \quad \text{eqn 7.12a}$$

$$\alpha_4 := \frac{(C \cdot E - B \cdot F)}{(A \cdot E - B \cdot D)} = 2.374 \times 10^{-4} \cdot \frac{\text{rad}}{\text{s}^2} \quad \text{eqn 7.12c}$$

$$A_{A,x} := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^2 \cdot \cos(\theta_2) = -2.311 \times 10^{-3} \cdot \frac{\text{in}}{\text{s}^2} \quad \text{eqn 7.13a}$$

$$A_{A,y} := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) = -0.097 \cdot \frac{\text{in}}{\text{s}^2}$$

$$A_{BA,x} := -b \cdot \alpha_3 \cdot \sin(\theta_3) - b \cdot \omega_3^2 \cdot \cos(\theta_3) = 0 \frac{\text{m}}{\text{s}^2} \quad \text{eqn 7.13b}$$

$$A_{BA,y} := b \cdot \alpha_3 \cdot \cos(\theta_3) - b \cdot \omega_3^2 \cdot \sin(\theta_3) = 0 \frac{\text{m}}{\text{s}^2}$$

$$A_{B,x} := -c \cdot \alpha_4 \cdot \sin(\theta_4) - c \cdot \omega_4^2 \cdot \cos(\theta_4) = -2.311 \times 10^{-3} \frac{\text{in}}{\text{s}^2} \quad \text{eqn 7.13c}$$

$$A_{B,y} := c \cdot \alpha_4 \cdot \cos(\theta_4) - c \cdot \omega_4^2 \cdot \sin(\theta_4) = -0.097 \frac{\text{in}}{\text{s}^2}$$

$$A_{PA,x} := -R_{pa} \cdot \alpha_3 \cdot \sin(\theta_3 + \delta_p) - R_{pa} \cdot \omega_3^2 \cdot \cos(\theta_3 + \delta_p) = 0 \frac{\text{in}}{\text{s}^2} \quad \text{eqn 7.13b}$$

$$A_{PA,y} := R_{pa} \cdot \alpha_3 \cdot \cos(\theta_3 + \delta_p) - R_{pa} \cdot \omega_3^2 \cdot \sin(\theta_3 + \delta_p) = 0 \frac{\text{in}}{\text{s}^2}$$

$$A_{Amagnitude} := \sqrt{A_{A,x}^2 + A_{A,y}^2} = 0.097 \frac{\text{in}}{\text{s}^2}$$

Magnitude and direction of the acceleration at point A.

$$A_{A,angle} := \text{atan2}(A_{A,x}, A_{A,y}) = -91.366 \text{ deg}$$

$$A_{Bmagnitude} := \sqrt{A_{B,x}^2 + A_{B,y}^2} = 0.097 \frac{\text{in}}{\text{s}^2}$$

Magnitude and direction of the acceleration at point B.

$$A_{B,angle} := \text{atan2}(A_{B,x}, A_{B,y}) = -91.366 \text{ deg}$$

$$A_{P,x} := A_{PA,x} + A_{A,x} = -2.31051 \times 10^{-3} \frac{\text{in}}{\text{s}^2}$$

Acceleration of point P in x- and y-components.

$$A_{P,y} := A_{PA,y} + A_{A,y} = -0.097 \frac{\text{in}}{\text{s}^2}$$

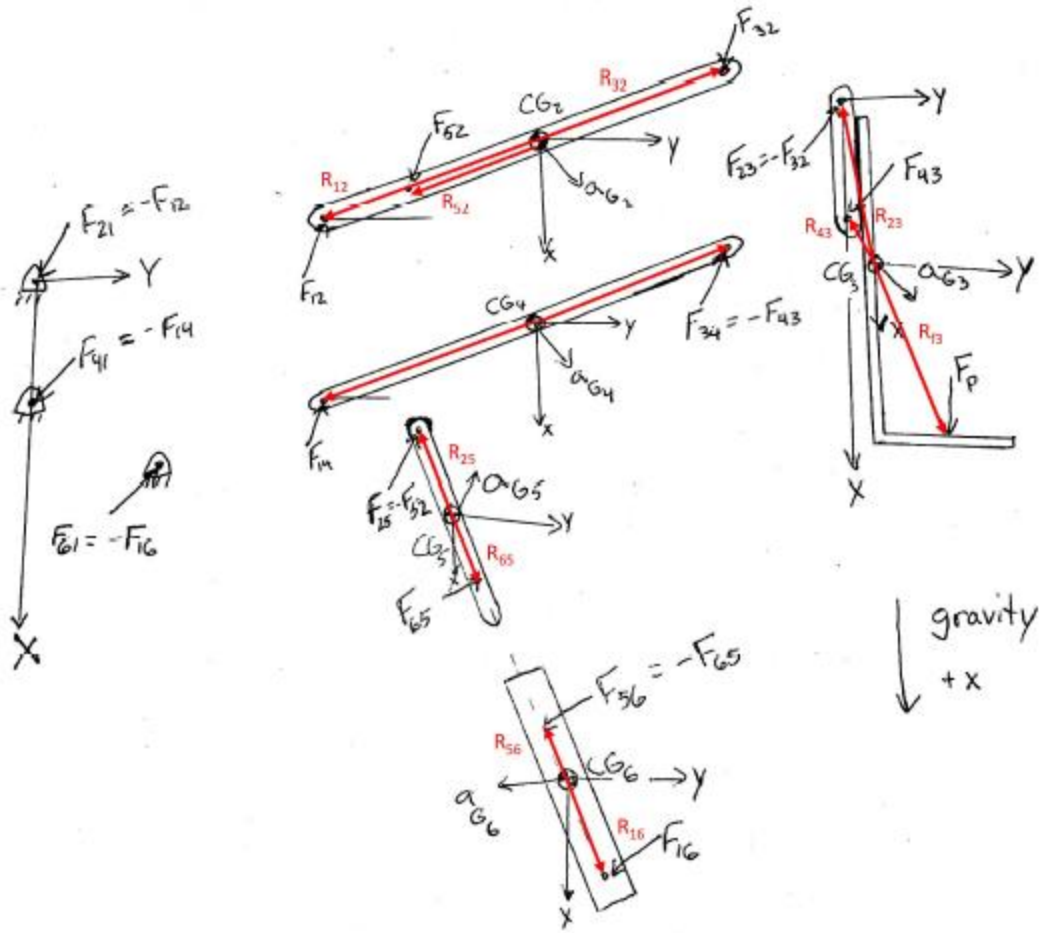
$$A_{Pmagnitude} := \sqrt{A_{P,x}^2 + A_{P,y}^2} = 0.097 \frac{\text{in}}{\text{s}^2}$$

Magnitude and direction of the acceleration at point P.

$$A_{P,angle} := \text{atan2}(A_{P,x}, A_{P,y}) = -91.366 \text{ deg}$$

Dynamic analysis of the mechanism:

blob = 12 slug



Mass of each link:

$m_2 := 0.012 \text{ blob}$

$m_3 := 0.042 \text{ blob}$

$m_4 := 0.012 \text{ blob}$

$m_5 := 0.001891 \text{ blob}$

$m_6 := 0.0077 \text{ blob}$

Values taken from Solidworks

Location of center of mass of each link:

$R_{cg2} := 19.25\text{in}$	$\delta_2 := 0\text{deg}$	from O2 to CG	
$R_{cg3} := 42.285\text{in}$	$\delta_3 := 11.472\text{deg}$	from A to CG	
$R_{cg4} := 19.25\text{in}$	$\delta_4 := 0\text{deg}$	from O4 to CG	Values taken from Solidworks
$R_{cg5} := 4.64\text{in}$	$\delta_5 := 0\text{deg}$	from C to CG	
$R_{cg6} := 5.49\text{in}$	$\delta_6 := 0\text{deg}$	from O6 to CG	

Mass moments of inertia about the CG of each link:

$I_{g2} := 1.446\text{blob}\cdot\text{in}^2$	
$I_{g3} := 9.912\text{blob}\cdot\text{in}^2$	
$I_{g4} := 1.446\text{blob}\cdot\text{in}^2$	Values taken from Solidworks
$I_{g5} := 0.026\text{blob}\cdot\text{in}^2$	
$I_{g6} := 0.112\text{blob}\cdot\text{in}^2$	

Defining position vectors of forces on each link:

$$R_{12x} := R_{cg2} \cdot \cos(\theta_2 + \delta_2 + 180\text{deg}) = 1.24 \cdot \text{in}$$

$$R_{12y} := R_{cg2} \cdot \sin(\theta_2 + \delta_2 + 180\text{deg}) = -19.21 \cdot \text{in}$$

$$R_{32} := \sqrt{(R_{cg2} \cdot \sin(\delta_2))^2 + (a - R_{cg2} \cdot \cos(\delta_2))^2} = 16.75 \cdot \text{in}$$

$$\delta_{32} := \text{atan2}[a - (R_{cg2}) \cdot \cos(\delta_2), R_{cg2} \cdot \sin(\delta_2)] = 0 \cdot \text{deg}$$

$$R_{32x} := R_{32} \cdot \cos(\theta_2 - \delta_{32}) = -1.079 \cdot \text{in}$$

$$R_{32y} := R_{32} \cdot \sin(\theta_2 - \delta_{32}) = 16.715 \cdot \text{in}$$

$$R_{23x} := R_{cg3} \cdot \cos(\theta_3 + \delta_3 + 180\text{deg}) = -41.44 \cdot \text{in}$$

$$R_{23y} := R_{cg3} \cdot \sin(\theta_3 + \delta_3 + 180\text{deg}) = -8.41 \cdot \text{in}$$

$$R_{43} := \sqrt{(R_{cg3} \cdot \sin(\delta_3))^2 + (b - R_{cg3} \cdot \cos(\delta_3))^2} = 34.967 \cdot \text{in}$$

$$\delta_{43} := \text{atan2}(b - R_{cg3} \cdot \cos(\delta_3), R_{cg3} \cdot \sin(\delta_3)) = 166.083 \cdot \text{deg}$$

$$R_{43x} := R_{43} \cdot \cos(\theta_3 - \delta_{43}) = -33.94 \cdot \text{in}$$

$$R_{43y} := R_{43} \cdot \sin(\theta_3 - \delta_{43}) = -8.41 \cdot \text{in}$$

$$R_{14x} := R_{cg4} \cdot \cos(\theta_4 + \delta_4 + 180\text{deg}) = 1.24 \cdot \text{in}$$

$$R_{14y} := R_{cg4} \cdot \sin(\theta_4 + \delta_4 + 180\text{deg}) = -19.21 \cdot \text{in}$$

$$R_{34} := \sqrt{(R_{cg4} \cdot \sin(\delta_4))^2 + (c - R_{cg4} \cdot \cos(\delta_4))^2} = 16.75 \cdot \text{in}$$

$$\delta_{34} := \text{atan2}(c - R_{cg4} \cdot \cos(\delta_4), R_{cg4} \cdot \sin(\delta_4)) = 0 \cdot \text{deg}$$

$$R_{34x} := R_{34} \cdot \cos(\theta_4 - \delta_{34}) = -1.079 \cdot \text{in}$$

$$R_{34y} := R_{34} \cdot \sin(\theta_4 - \delta_{34}) = 16.715 \cdot \text{in}$$

$$R_{pa} = 47.751 \cdot \text{in} \quad \delta_p = 13.146 \cdot \text{deg} \quad \text{Location where person sits in the chair from point B.}$$

$$R_{F3x} := R_{pa} \cdot \cos(\theta_3 + \delta_p) - R_{cg3} \cdot \cos(\theta_3 + \delta_3) = 5.059 \cdot \text{in}$$

$$R_{F3y} := R_{pa} \cdot \sin(\theta_3 + \delta_p) - R_{cg3} \cdot \sin(\theta_3 + \delta_3) = 2.45 \cdot \text{in}$$

$$F_{px} := 300 \text{ lbf} \quad \text{Force applied by occupant, all in the +x direction.}$$

Definition of inverted slider joint contact area:

Force can only be transmitted perpendicular to the axis of slip in the inverted slider joint. On page 599 of Norton's Design of Machinery text book, the position vector from the CG to the point on link 3 and link 4 where the links apply force to each other is taken to be point B in figure 11-5 since the slider length is arbitrary. However, this analysis will instead use the real length of the sliding joint to define the position vectors R_{56} and R_{65} .

$$\text{plunger} := 11.31 \text{ in} \quad \text{Length from C to the end of link 5.}$$

$$\text{cylinder} := 12.25 \text{ in} \quad \text{Length from } O_6 \text{ to the end of link 6.}$$

$$\text{contact_length} := \text{plunger} + \text{cylinder} - b_s = -15.02 \cdot \text{in}$$

The force of links 5 and 6 on each other is distributed over this length, so we will take the resultant point force location as the contact length/2.

$$\text{resultant_length} := \frac{\text{contact_length}}{2} = -7.51 \cdot \text{in}$$

This value is the distance from the free end of each of the links, a more useful distance if from the points C and O_6 , respectively.

$$\text{plunger_resultant} := \text{plunger} - \text{resultant_length} = 18.82 \cdot \text{in}$$

$$\text{cylinder_resultant} := \text{cylinder} - \text{resultant_length} = 19.76 \cdot \text{in}$$

$$R_{25x} := R_{cg5} \cdot \cos(\theta_6 + \delta_5) = -4.638 \text{ in}$$

$$R_{25y} := R_{cg5} \cdot \sin(\theta_6 + \delta_5) = 0.123 \text{ in}$$

$$R_{65x} := \text{plunger_resultant} \cdot \cos(\theta_6 + \delta_5 + 180\text{deg}) - R_{cg5} \cdot \cos(\theta_6 + \delta_5 + 180\text{deg}) = 14.175 \text{ in}$$

$$R_{65y} := \text{plunger_resultant} \cdot \sin(\theta_6 + \delta_5 + 180\text{deg}) - R_{cg5} \cdot \sin(\theta_6 + \delta_5 + 180\text{deg}) = -0.377 \text{ in}$$

$$R_{16x} := R_{cg6} \cdot \cos(\theta_6 + \delta_6) = -5.488 \text{ in}$$

$$R_{16y} := R_{cg6} \cdot \sin(\theta_6 + \delta_6) = 0.146 \text{ in}$$

$$R_{56x} := \text{cylinder_resultant} \cdot \cos(\theta_6 + \delta_6 + 180\text{deg}) - R_{cg6} \cdot \cos(\theta_6 + \delta_6 + 180\text{deg}) = 14.265 \text{ in}$$

$$R_{56y} := \text{cylinder_resultant} \cdot \sin(\theta_6 + \delta_6 + 180\text{deg}) - R_{cg6} \cdot \sin(\theta_6 + \delta_6 + 180\text{deg}) = -0.379 \text{ in}$$

$$R_{52x} := a_s \cdot \cos(\theta_2 + \delta_2) - R_{cg2} \cdot \cos(\theta_2 + \delta_2) = 0.416 \text{ in}$$

$$R_{52y} := a_s \cdot \sin(\theta_2 + \delta_2) - R_{cg2} \cdot \sin(\theta_2 + \delta_2) = -6.437 \text{ in}$$

Accelerations of each link center of mass:

$$a_{g2x} := -R_{cg2} \cdot \alpha_2 \cdot \sin(\theta_2 + \delta_2) - R_{cg2} \cdot \omega_2^2 \cdot \cos(\theta_2 + \delta_2) = -1.235 \times 10^{-3} \cdot \frac{\text{in}}{\text{s}^2}$$

eqn 7.13

$$a_{g2y} := R_{cg2} \cdot \alpha_2 \cdot \cos(\theta_2 + \delta_2) - R_{cg2} \cdot \omega_2^2 \cdot \sin(\theta_2 + \delta_2) = -0.052 \cdot \frac{\text{in}}{\text{s}^2}$$

$$a_{g3x} := -R_{cg3} \cdot \alpha_3 \cdot \sin(\theta_3 + \delta_3) - R_{cg3} \cdot \omega_3^2 \cdot \cos(\theta_3 + \delta_3) = 0 \cdot \frac{\text{in}}{\text{s}^2}$$

$$a_{g3y} := R_{cg3} \cdot \alpha_3 \cdot \cos(\theta_3 + \delta_3) - R_{cg3} \cdot \omega_3^2 \cdot \sin(\theta_3 + \delta_3) = 0 \cdot \frac{\text{in}}{\text{s}^2}$$

$$a_{g4x} := R_{cg4} \cdot \alpha_4 \cdot \sin(\theta_4 + \delta_4) - R_{cg4} \cdot \omega_4^2 \cdot \cos(\theta_4 + \delta_4) = 7.886 \times 10^{-3} \cdot \frac{\text{in}}{\text{s}^2}$$

$$a_{g4y} := R_{cg4} \cdot \alpha_4 \cdot \cos(\theta_4 + \delta_4) - R_{cg4} \cdot \omega_4^2 \cdot \sin(\theta_4 + \delta_4) = -0.052 \cdot \frac{\text{in}}{\text{s}^2}$$

$$a_{g5\text{Part1}} := R_{cg5} \cdot \alpha_6 \cdot (\sin(\theta_6) - j \cdot \cos(\theta_6)) + R_{cg5} \cdot \omega_6^2 \cdot (\cos(\theta_6) + j \cdot \sin(\theta_6))$$

$$a_{g5\text{Part2}} := 2b_{\text{dot}} \cdot \omega_6 \cdot (\sin(\theta_6) - j \cdot \cos(\theta_6)) - b_{2\text{dot}} \cdot (\cos(\theta_6) + j \cdot \sin(\theta_6))$$

$$a_{g5} := a_{g5\text{Part1}} + a_{g5\text{Part2}} = (1.47 \times 10^{-4} + 5.955i \times 10^{-3}) \cdot \frac{\text{in}}{\text{s}^2}$$

$$a_{g5x} := \operatorname{Re}(a_{g5}) = 1.47 \times 10^{-4} \cdot \frac{\text{in}}{\text{s}^2}$$

$$a_{g5y} := \operatorname{Im}(a_{g5}) = 5.955 \times 10^{-3} \cdot \frac{\text{in}}{\text{s}^2}$$

$$a_{g6} := -R_{cg6} \cdot \alpha_6 (\sin(\theta_6) - j \cdot \cos(\theta_6)) - R_{cg6} \cdot \omega_6^2 \cdot (\cos(\theta_6) + j \cdot \sin(\theta_6))$$

$$a_{g6x} := \operatorname{Re}(a_{g6}) = -1.091 \times 10^{-4} \cdot \frac{\text{in}}{\text{s}^2}$$

$$a_{g6y} := \operatorname{Im}(a_{g6}) = -4.607 \times 10^{-3} \cdot \frac{\text{in}}{\text{s}^2}$$

From this point a force applied by the actuator (F_{52}) is assumed. This value was iterated until it resulted in a required torque of approximately 0.

Some general trends during the iteration were if the torque was a positive number the actuator force needed to be increased, and if the torque was a negative number the actuator force needed to be decreased.

$$F_{52_test} := 850 \cdot \text{lbf}$$

$$F_{52x_test} := F_{52_test} \cdot \cos(\theta_6) = -849.7 \cdot \text{lbf}$$

$$F_{52y_test} := F_{52_test} \cdot \sin(\theta_6) = 22.592 \cdot \text{lbf}$$

$$\text{Matrix_A} := \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{-R_{12y}}{\text{in}} & \frac{R_{12x}}{\text{in}} & \frac{-R_{32y}}{\text{in}} & \frac{R_{32x}}{\text{in}} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_{23y}}{\text{in}} & \frac{-R_{23x}}{\text{in}} & \frac{-R_{43y}}{\text{in}} & \frac{R_{43x}}{\text{in}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_{34y}}{\text{in}} & \frac{-R_{34x}}{\text{in}} & \frac{-R_{14y}}{\text{in}} & \frac{R_{14x}}{\text{in}} & 0 \end{pmatrix}$$

$$\text{Matrix_C} := \begin{bmatrix} (m_2 \cdot a_{g2x} - F_{52x_test}) \cdot \frac{1}{\text{lbf}} \\ (m_2 \cdot a_{g2y} - F_{52y_test}) \cdot \frac{1}{\text{lbf}} \\ (I_{g2} \cdot \alpha_2 + R_{52y} \cdot F_{52x_test} - R_{52x} \cdot F_{52y_test}) \cdot \frac{1}{\text{in} \cdot \text{lbf}} \\ (m_3 \cdot a_{g3x} - F_{px}) \cdot \frac{1}{\text{lbf}} \\ (m_3 \cdot a_{g3y}) \cdot \frac{1}{\text{lbf}} \\ (I_{g3} \cdot \alpha_3 + R_{f3y} \cdot F_{px}) \cdot \frac{1}{\text{in} \cdot \text{lbf}} \\ (m_4 \cdot a_{g4x}) \cdot \frac{1}{\text{lbf}} \\ (m_4 \cdot a_{g4y}) \cdot \frac{1}{\text{lbf}} \\ (I_{g4} \cdot \alpha_4) \cdot \frac{1}{\text{in} \cdot \text{lbf}} \end{bmatrix}$$

$$\text{Matrix_B} := \text{Matrix_A}^{-1} \cdot \text{Matrix_C}$$

$$F_{12x} := \text{Matrix_B}_0 \cdot \text{lbf} = 577.748 \cdot \text{lbf}$$

$$F_{12y} := \text{Matrix_B}_1 \cdot \text{lbf} = -456.999 \cdot \text{lbf}$$

$$F_{32x} := \text{Matrix_B}_2 \cdot \text{lbf} = 271.952 \cdot \text{lbf}$$

$$F_{32y} := \text{Matrix_B}_3 \cdot \text{lbf} = 434.406 \cdot \text{lbf}$$

$$F_{43x} := \text{Matrix_B}_4 \cdot \text{lbf} = -28.048 \cdot \text{lbf}$$

$$F_{43y} := \text{Matrix_B}_5 \cdot \text{lbf} = 434.406 \cdot \text{lbf}$$

$$F_{14x} := \text{Matrix_B}_6 \cdot \text{lbf} = -28.048 \cdot \text{lbf}$$

$$F_{14y} := \text{Matrix_B}_7 \cdot \text{lbf} = 434.406 \cdot \text{lbf}$$

$$T_{12} := \text{Matrix_B}_8 \cdot \text{lbf} \cdot \text{in} = -57.365 \cdot \text{lbf} \cdot \text{in}$$

To verify this result the required torque was recalculated using the method of virtual work, which has the same result. Some velocity values needed to be calculated first, as seen below.

$$v_{g2} := R_{cg2} \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$v_{g3A} := R_{cg3} \cdot \omega_3 \cdot (-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3))$$

Velocity of each link's center of gravity.

$$v_{g3} := v_A - v_{g3A}$$

$$v_{g4} := R_{cg4} \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4))$$

$$v_{g2x} := \text{Re}(v_{g2}) \quad v_{g2y} := \text{Im}(v_{g2})$$

$$v_{g3x} := \text{Re}(v_{g3}) \quad v_{g3y} := \text{Im}(v_{g3})$$

$$v_{g4x} := \text{Re}(v_{g4}) \quad v_{g4y} := \text{Im}(v_{g4})$$

$$v_c := a_s [\omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))] \quad \text{Velocity of point C.}$$

$$v_{cx} := \text{Re}(v_c) \quad v_{cy} := \text{Im}(v_c)$$

$$T_{12_virtual_part_1} := \frac{1}{\omega_2} \cdot [m_2 \cdot (a_{g2x} \cdot v_{g2x} + a_{g2y} \cdot v_{g2y}) + m_3 \cdot (a_{g3x} \cdot v_{g3x} + a_{g3y} \cdot v_{g3y})] \quad \text{eqn 11.16}$$

$$T_{12_virtual_part_2} := \frac{1}{\omega_2} \cdot [m_4 \cdot (a_{g4x} \cdot v_{g4x} + a_{g4y} \cdot v_{g4y}) + (I_{g2} \cdot \alpha_2 \cdot \omega_2 + I_{g3} \cdot \alpha_3 \cdot \omega_3 + I_{g4} \cdot \alpha_4 \cdot \omega_4)]$$

$$T_{12_virtual_part_3} := \frac{1}{\omega_2} \cdot (-F_{px} \cdot v_{px} - F_{52x_test} \cdot v_{cx} - F_{52y_test} \cdot v_{cy})$$

$$T_{12_virtual} := T_{12_virtual_part_1} + T_{12_virtual_part_2} + T_{12_virtual_part_3} = -57.365 \text{ lbf} \cdot \text{in}$$