

Tutorial Dialog in an Equation Solving Intelligent Tutoring System

by

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A Thesis

Submitted to the Faculty of the

WORCESTER POLYTECHNIC INSTITUTE

In partial fulfillment of the requirements for the

Degree of Master of Science

in

Computer Science

by

December 2003

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Keywords: Intelligent tutoring system, model-tracing, cognitive model, tutoring, mathematical education, artificial intelligence.

Abstract

This thesis makes a contribution to Intelligent Tutoring Systems (ITS) architectures. A new intelligent tutoring system is presented for the domain of solving linear equations. This system is novel, because it is the first intelligent equation-solving tutor that combines a cognitive model of the domain with a model of dialog-based tutoring. The tutorial model is novel because it is based on the observation of an experienced human tutor and captures tutorial strategies specific to the domain of equation solving. In this context, a tutorial dialog is the equivalent of breaking down problems into simpler steps and then asking new questions to the student before proceeding to the next navigational step. The resulting system, named E-tutor, was compared, via a randomized controlled experiment, to an algebra ITS similar to the “Cognitive Tutor” by Carnegie Learning, Inc[®]. The Cognitive Tutor can provide traditional model-tracing feedback and buggy messages to students, but does not engage students in dialog. Preliminary results using a very small sample size, i.e., teaching equation solving to 15 high school students, showed that E-Tutor with dialog capabilities performed better than E-tutor without dialog. This result showed an effect size of 0.4 standard deviations for overall learning by condition. This set of preliminary results, though not statistically significant, shows promising opportunities to improve learning performance by adding tutorial dialog capabilities to ITSs. However, significant further validation is required, specifically, adding greater numbers and variations of the work to our sample size, before this approach can be deemed successful. The system is available at www.wpi.edu/~leenaar/E-tutor.

Acknowledgements

Over the course of working on this thesis, Professor Neil Heffernan contributed a significant amount of time advising me and I want to thank him for his generous support and patience. I am also indebted to Renah for lending her expertise as a math tutor; her help throughout this thesis was invaluable. Thanks to all of the family and friends who helped test E-tutor and gave constructive suggestions throughout: especially Reema, Deema, Monjid, Misa, and Sarab. Fatima, thank you for lending me your friends, I hope I didn't annoy them too much. Finally, a special note of thanks to Muttasem, Fatima and Zane for their support during my time at WPI.

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1 Introduction

This thesis makes a contribution to the field of Intelligent Tutoring Systems (ITS) architectures. This research focused on building a better algebra tutor for the task of solving equations by suggesting that replacing traditional model-tracing feedback in an ITS with a dialog-based interactive feedback mechanism will improve the performance of students significantly. This system is novel, because it is the first intelligent equation-solving tutor that combines a cognitive model of the domain with a model of dialog-based tutoring. The tutorial model is novel because it is based on the observation of an experienced human tutor and captures tutorial strategies specific to the domain of equation-solving. In this context, a tutorial dialog is the equivalent of breaking down problems into simpler steps and then asking new questions to the student before proceeding to the next step.

Over the last two decades computer-based instruction has increased in popularity and its use has expanded in academia and industry. At the beginning of the 1990's, early ITSs focused their efforts on lesson navigation, or a kind of electronic page-turner presenting frames of text or graphics. The student's response to the current page determined the frame to display next. This type of ITS is often referred to as a first generation ITS. Second generation ITSs use the model-tracing algorithm (Anderson and Pelletier, 1991) to create a model of the student and trace student thinking. Second generation ITSs have been shown by Corbett and Trask (2000) to be about half as effective as human tutors. Although Ohlsson (1986) observed that teaching strategies and tactics should be one of the guiding principles in the development of ITSs, incorporating such principles in ITSs have remained greatly unexplored (McArthur, Stasz & Zmuidzinas, 1990). Hence, ITSs where a model of both the student and the tutor are created in an effort to improve performance was the natural extension to second generation systems. Several researchers have developed third generation ITSs that model the tutor as well as the student, such as Graesser, Person & Harter (2001) with AutoTutor, Heffernan (2000) with Ms. Lindquist and Rosé, Jordan, Ringenberg, Siler, VanLehn et al. (2001) with Atlas-Andes.

Some of the research in third generation ITSs has paid special attention to teaching tactics and strategies gleaned from experienced human tutors (Heffernan, 2001). This requires extra work in dialog planning and management. It also requires an investigation into what strategies a human tutor may use in a specific subject domain. Preliminary results in this research of dialog-based ITSs show them to be effective in the domains studied.

This thesis studied the effect of adding tutorial dialog to an intelligent equation-solving tutor by implementing an ITS for solving linear equations that incorporates dialog between the student and the ITS. When students need help, problems are broken down into simpler steps and new questions are asked. This research involved studying techniques used by an experienced math tutor and imitating them in an ITS. The resulting system, called E-tutor, extended model-tracing to make a tutor that is able to engage in a tutorial dialog with the student. Thus, in addition to modeling student behavior in solving linear equations, E-tutor modeled a human tutor's responses to the student's behavior.

For the purpose of evaluating the system, an ITS similar to the “Cognitive Tutor” by Carnegie Learning, Inc.® was used as a control condition in this research. The Cognitive Tutor can indicate to students when they have made a mistake and provide hint messages, as well as provide canned explanations to some errors. When a student keeps asking for help, the hint messages get more explicit until the tutor is telling the student exactly what to do. By our definition of dialog, which involves breaking down problems into simpler steps and then asking new questions, the Cognitive Tutor does not engage the student in dialog. The Cognitive Tutor (Koedinger, Anderson, Hadley & Mark, 1995) was used by over 100,000 students in over 1% of American high schools in 2002-2003. The software has been well-tested and shown to produce larger learning gains than traditional mathematics instruction (www.carnegielearning.com). Using an ITS like the Cognitive Tutor provided a strong control condition for evaluating E-tutor.

An outline for the rest of this thesis is as follows. Chapter 2 contains the background information for completing this research. Topics on research in human tutoring, intelligent tutoring systems, model-tracing and dialog-based ITSs are covered.

To make the proposed ITS behave more like an experienced human tutor, an hour and a half of one-on-one human tutoring was observed. In Chapter 3, the procedure for collecting the human tutor dialogs and the results of the human tutoring sessions are covered. Some generalizations are made about the dialog used by the tutor. Tutorial strategies that were implemented in the system described in Chapter 4 are identified and discussed.

Chapter 4 covers the implementation of the dialog-based ITS, called E-tutor. The architecture of E-tutor is described and compared to that of the control condition, a traditional model-tracing tutor that provides hints and buggy messages but does not use dialog. The different tutorial dialogs that were implemented in E-tutor are described and compared to the feedback in the control condition.

In Chapter 5, the evaluation of E-tutor is described. E-tutor was evaluated in a randomized controlled experiment with 15 sophomores in high school. The effectiveness of E-tutor was measured with a pre- and post-test. Results are analyzed by overall learning and learning by condition. A problem analysis is done to determine what kinds of problems showed the most learning. An analysis of the kinds of errors students made was also performed to determine areas of difficulty and progress. Student log files for the control and experimental condition are also compared.

In Chapter 6, conclusions and future work are discussed. The results of the evaluation of E-tutor are recapped and suggestions for further research are made.

2 Previous Work and Background

In this chapter, the background for this thesis is presented. In section 2.1, previous research on human tutoring is discussed. Section 2.2 discusses intelligent tutoring systems in general and section 2.3 covers model-tracing ITSs. Finally, Section 2.4 discusses some of the research done with dialog-based ITSs and the results.

2.1 Previous research on tutoring

Studies indicate that human tutors provide the most effective form of instruction known (Bloom, 1984; Cohen, Kulik, & Kulik, 1982). They raise the mean performance about two standard deviations compared to students taught in classrooms. Intelligent tutoring systems offer excellent instruction, but not quite as good as human tutors. The best ones raise performance about one standard deviation above classroom instruction (e.g., Anderson, Corbett, Koedinger, & Pelletier, 1995). In other words, a human tutor can raise the student's grade by about two letter grades (e.g., from C to A) while a tutoring system can raise it by about one letter grade (e.g., from C to B).

The challenge is to create tutoring systems that are as effective as human tutors. One of the biggest remaining differences between human tutors and intelligent tutoring systems is that human tutors use natural language and most tutoring systems do not. Although tutoring systems print natural language and sometimes speak it, they usually do not let the student type or speak in natural language. We hypothesize that if tutoring systems could participate in natural language dialogs with students, then the systems would become much more effective, and possibly rival human tutors.

In studying what makes a tutoring session successful, VanLehn, Siler and Murray (1998) identified two principles for effective teaching:

- First, tutors should not offer strong hints or apply rules to problems themselves when students make mistakes. Students miss the opportunity to learn how to solve a problem when they are given an answer and are not allowed to reason for themselves.
- Secondly, a tutor may need to emphasize the rules needed to solve a problem differently according to what is causing the student to have difficulty. For instance, a tutor may need to generalize a rule or explain why a rule applies but on other occasions an explanation of why an error was made does not help with the learning (VanLehn et al., 1998). Furthermore, in 2003 Van Lehn, Siler, Murray, Yamauchi and Baggett did further research on this topic and suggested that students were more likely to learn when they reached an impasse than if the tutor had applied the correct action for them.

Merrill, Reiser, Ranney and Trafton (1992) compared the effectiveness of human tutors and intelligent tutoring systems. Their study indicated that a major reason that human tutors are more effective is that they let the students do most of the work in overcoming impasses, while at the same time provided as much assistance as necessary. Merrill, D., Reiser, Merrill, S., and Landes (1995) argue that the main thing human tutors do is to keep students on track and prevent them from following "garden paths" of reasoning that are unproductive and unlikely to lead to learning. Merrill et al. (1995) pointed to the large number of remarks made by tutors that helped keep students on track

while learning Lisp programming. They argue that this was especially important in this domain because of the hierarchical nature of computer programming where if a student does one step wrong then everything after that step is going to be wrong.

Modeling, coaching, and scaffolding are described by Collins, Brown and Hollum (1991) as the heart of cognitive apprenticeship, which they claim “help students acquire an integrated set of skills through processes of observation and guided practice.” Tutors use modeling when they do a task in front of students to demonstrate how it is done, allowing the student to observe the processes needed. When students attempt problems themselves, tutors may coach them by giving some feedback designed to remind, hint or clarify. Scaffolding is when tutors do parts of a problem for students that they are having trouble with and involves being able to assess a student’s proficiency at that point. An important part of scaffolding is fading, which entails progressively removing the support of scaffolding as the student demonstrates proficiency (Collins et al., 1991).

2.2 What is an intelligent tutoring system?

An early outline of ITS requirements was presented by Hartley and Sleeman (1973). They argued that ITSs must have knowledge of the domain, also called the expert model, knowledge of the learner, or student model, and knowledge of teaching strategies, or the tutor model. The type of ITSs discussed here are of the “coached practice” variety where both the student and the tutor work together to solve the problem but the tutor does not let the student wander too far from the correct solution path (Merrill et al., 1992). This is used rather than a “guided discovery” ITS which allows students to make mistakes and does not interfere as much, (Burton and Brown, 1982).

A student learns from an ITS primarily by solving problems that serve as good learning experiences for that student. That is, he/she learns by doing. In an ITS, the problems are selected in such a way that they reinforce learning of the fundamental principles of the subject matter. The system starts by assessing the student knowledge of the subject (also known as a student model) or what the student already knows. The system must consider the curriculum or what the student needs to know. Finally, the system must decide what curriculum element, or unit of instruction should be instructed next, and how it should be presented. From all of these considerations, the system selects, or generates, a problem, then either works out a solution to the problem (via the domain expert), or retrieves a prepared solution. The ITS then compares its solution to the one the student has prepared and performs a diagnosis based on differences between the two. Feedback is offered by the ITS based on the student-advisor considerations such as how long it's been since feedback was last provided, whether the student already received some particular advice, and so on.

In Kulik’s (1994) meta-analysis of 97 studies that compared classroom instruction to computer-based instruction, he found an average effect size¹ of 0.32 in favor of

¹ Effect size is defined as the difference between two groups divided by the standard deviation of the control group. This specifies how much the total variance is explained by the independent variable. For example the .32 effect size Kulic reported means that 63% of the students in the experimental group outperformed the average student in the control group. This is a dramatically smaller effect size than Bloom's effect size of 2.0 comparing tutoring versus classroom instruction; Bloom's effect size of 2.0 means that 98% of the students who received tutoring outperformed the average student who got traditional classroom instruction in the control group.

computer-based instruction. Kulik claimed that students learned more and learned faster in courses that involved computer-based instruction.

Ohlsson (1986) identified four principles for building intelligent tutoring systems: cognitive diagnosis, subject-matter analysis, teaching tactics and teaching strategies. McArthur et al., (1990) claims “of these four, teaching tactics and strategies are the least well-developed components of most intelligent tutors.” This study attempts to address McArthur’s concern.

2.3 Model-tracing ITS

The algorithm called “model-tracing” (Corbett, Koedinger and Pelletier, 1995) can be used to understand a student’s input. The model-tracing algorithm is given three inputs:

1. The state of working memory: represented by a group of working memory elements. Working memory elements, or WMEs, are elements of declarative knowledge and are meant to symbolize “chunks” of knowledge students are expected to have.
2. A set of productions, each representing a cognitive step, which may or may not have observable actions.
3. The student’s input to be “traced”.

The first two inputs are known as the student model.

The model-tracing algorithm has two outputs:

1. A Boolean value indicating if the student’s input was traced.
2. If the input was traced, the algorithm’s output is a set of interpretations. Each interpretation is a list of productions that can be chained together to produce the student’s action.

If the model-tracing algorithm finds such an interpretation, then the system “traced” or understood the student’s input. If the algorithm fails to find a sequence of productions that could have generated the student’s response, then the student’s response is said to be “un-interpretable”. An un-interpretable student response may get no response from the ITS, or may receive some limited form of feedback. For example, the Cognitive Tutor deals with un-interpretable responses by assuming they are wrong and marking the step orange.

Model-tracing feedback traditionally is provided in two ways, hints and buggy messages. These are described in Sections 2.3.1 and 2.3.2.

2.3.1 Buggy Rules

Buggy messages are based on an erroneous student action and can be used to model common mistakes with buggy productions. For instance, the Cognitive Tutor provides a buggy message when a student tries to divide both sides of an equation by a positive coefficient when it should be negative. For an equation such as $-7 = -5Y$, when students try to divide both sides by 5 rather than -5, they are presented with a buggy message. Table 2.1 illustrates this process.

| Equation | Student Action | Cognitive Tutor's Buggy Message |
|------------|--|---|
| $-7 = -5Y$ | Student divides both sides by 5 rather than the optimal action of dividing both sides by -5. | In this equation, y is multiplied by -5. Dividing by 5 leaves -y, so you still need to remove the negative sign. Dividing by -5 is better, since that would leave positive y. |

Table 2.1 A buggy message in the Cognitive Tutor.

Common errors in solving equations are traced with buggy rules in the Cognitive Tutor, such as the following. (Examples of these are illustrated and discussed further in Section 4.1.1)

- Trying to combine like terms before multiplying (e.g. $3+9*8 = Y$)
- Trying to transform an equation by adding instead of subtracting (e.g. $3x+4 = 2$)
- Trying to transform an equation by subtracting instead of adding (e.g. $3x-4 = 2$)
- Trying to transform an equation by multiplying instead of dividing (e.g. $3x = 9$)
- Trying to transform an equation by dividing instead of multiplying (e.g. $x/3 = 9$)

2.3.2 Hint Messages

Hints are usually provided when the student requests help. The tutor runs the production system to find a list of productions that represent the cognitive step the student took. A hint sequence is a chain of hints that the tutor can offer the student for a particular question. The Cognitive Tutor supplies hint chains that become more explicit if the student continues to ask for help, culminating in the exact step that the student should take. For instance, if a student is stuck on an equation such as $4(0+4) = Y$ and asks for help he/she will get hint #1. If the student still needs help, he/she can ask for hint #2. Finally, if the student requests another hint, he/she will get the final hint in the sequence. The final hint gives the student the correct action to choose from the menu (combine like terms), as well as the correct input (on the left side of the equation). Table 2.2 illustrates the hint sequence available in the Cognitive Tutor for this kind of problem.

| Problem: $4(0 + 4) = Y$ |
|--|
| Hint #1: Put the equation in its simplest form Be sure to do the operation inside the parentheses first: $(0 + 4)$ |
| Hint #2: Combine 0 and 4 on the left side. |
| Hint #3: Combine like terms on the left |

Table 2.2 A hint sequence in the Cognitive Tutor

A disadvantage of hint sequences, such as those used in the Cognitive Tutor, is that the first principle for effective teaching identified by VanLehn et al. (1998) may be violated. That is, tutors should not offer strong hints or apply rules to problems themselves when students make mistakes. Students may miss the opportunity to learn how to solve a problem when they are given an answer and are not allowed to reason for

themselves. The Cognitive Tutor always provides a strong hint at the end of every hint sequence. This research hypothesizes that it would be better to ask the students a new question and expect them to answer it, instead of telling them information. Sometimes the hints in the Cognitive Tutor are posed as rhetorical questions; students are not expected to answer them, thereby not maximizing the opportunity for the students to reason for themselves. The hint sequences and bug messages of the Cognitive Tutor are discussed in more detail in Chapter 4.

McArthur et al. (1990) argues that model-tracing ITSs do have their shortcomings. For example, they criticize Anderson's geometry tutor (Anderson, Boyler & Reiser, 1985) "because each incorrect rule is paired with a particular tutorial action (typically a stored message), every student who takes a given step gets the same message, regardless of how many times the same error has been made or how many other errors have been made (p. 200)." In this thesis, and the ITS proposed here, the limitations of such systems are addressed by incorporating an adaptive question and answer model whose output depends on the student's input and therefore is not fixed (see Chapter 4).

2.4 Previous research on dialog-based ITS

Several researchers have developed ITSs that incorporate tutorial dialog (see Sections 2.4.1, 2.4.2 and 2.4.3). While some preliminary results are promising, the effectiveness of such systems is still being studied across multiple knowledge domains.

2.4.1 AutoTutor

The Tutoring Research Group at the University of Memphis has developed AutoTutor (Graesser et al., 2001), an ITS that tutors college students in introductory computer literacy courses. AutoTutor attempts to imitate a human tutor by reproducing the dialog patterns and strategies that were likely to be used by a human tutor. AutoTutor uses the picture of a human face that simulates facial expressions and speech to give the impression of a discussion between the tutor and student. "AutoTutor presents questions and problems from a curriculum script, attempts to comprehend learner contributions that are entered by keyboard, formulates dialog moves that are sensitive to the learner's contributions ... and delivers the dialog moves with a talking head." (Graesser, Wiemer-Hastings, K., Wiemer-Hastings, P. & Kreuz, 1999) Effect sizes of 0.5 to 0.6 standard deviations have been cited in assessments of AutoTutor when comparing the use of AutoTutor to reading the subject material. (Graesser et al., 2001)

2.4.2 Ms. Lindquist

Ms. Lindquist, (Heffernan, 2002), is a tutoring system that uses dialog to help students in writing algebra expressions. Ms. Lindquist uses a "learning by doing" approach that allows students to practice symbolization in algebra. Ms. Lindquist models tutorial behavior as well as student behavior, by incorporating several tutorial strategies that were observed to be effective with human tutors. The cognitive student model has a set of production rules that model the problem solving skills needed to write algebraic expressions. The cognitive model is able to trace students' solution paths, diagnose their input and pass the diagnosis along to the tutorial model.

Ms. Lindquist asks questions that break down a problem in different ways. When she runs out of ways to break down a problem, then she offers rephrasings of the questions, rather than giving the student the answer directly. All of this is designed to make the student more active in the learning process. Heffernan (2001) compared student learning achieved by two different versions of Ms. Lindquist by controlling for the number of problems completed. In the control version, if a student got a problem incorrect they were told the correct answer to type in, and then moved on to solve more problems. In the experimental version, if a student made an error, Ms. Lindquist engaged them in a dialog related to the error they made and tried to help them fix that error. The result was that students that were in the dialog condition did better on the post-test, showing that students learned from these dialogs.

Heffernan did not at that time show that it was worth the additional time it took for the students to go through these dialogs. Heffernan (2002) later reported that in a web-based experiment, it was worth the additional time, by controlling for time on task, rather than the number of problems done. Specifically, Heffernan showed that even though students in the dialog condition did half the number of problems, they learned more than the control condition, with an effect size of 0.5 standard deviations. Heffernan also reported a possible motivation benefit to dialog. In summary, Ms. Lindquist seems like one example that supports the hypothesis that incorporating dialog into an ITS can lead to increases in student learning.

2.4.3 Atlas-Andes

Rosé et al. (2001) integrated Atlas and the Andes system to compare a model-tracing ITS with the same ITS incorporating dialog. Atlas consists of components and tools that facilitate incorporating tutorial dialog in an ITS. Atlas is domain-independent to be used with domain specific ITS. Andes (Gertner & VanLehn, 2000) is a model-tracing tutor in the domain of quantitative physics. Andes provides instant feedback to students by highlighting each step attempted in red or green, to indicate if they are wrong or right. Students can also ask Andes for help which is provided by a hint sequence.

By integrating Atlas and Andes, Rosé et al. were able to compare student learning between the original Andes and Atlas-Andes with dialog capabilities. The experiment did not control for the time it took to complete the problems. Results of the study showed that Atlas-Andes students scored higher on the post-test than Andes students, a difference of 0.9 standard deviations. Atlas-Andes students scored significantly higher on questions in the post-test that contained concepts for which they had participated in a dialog. Additionally, there was no significant difference between post-test scores on questions for which Atlas-Andes students did not engage in dialog and Andes students. Rosé et al. were able to conclude that the ITS with tutorial dialog was more effective in tutoring quantitative physics problems.

2.5 Chapter Conclusion

There seems to be a recurring factor in research on human tutoring. That is, an effective human tutor allows students to do as much work as possible on their own while providing just enough help to keep students from becoming confused or giving up. Collins et al., (1991) describes modeling, coaching and scaffolding as an appropriate

model for teachers who need to teach students a complex task. Fading, a component of scaffolding, will be described in Chapter 4 as a feature of E-tutor. McArthur et al. (1990) had some criticism for model-tracing tutors which often give the same canned message to students when they make an error regardless of how many times it occurs. As Ohlsson (1986) pointed out, two of the principles that should guide building ITS, teaching tactics and strategies, are, for the most part, undeveloped in tutoring systems.

This thesis proposed to address this lack by building and evaluating an ITS that is more effective because it incorporates the dialog elements from experienced human tutors. E-tutor does utilize the improvement of model-tracing tutors but also engages in coherent dialogs that ask the student new questions rather than simply provide canned hints. Students must answer these new questions correctly or exhaust a dialog before continuing with a problem. E-tutor does offer different dialogs on the same type of error, depending on the number of times the error occurs and dialogs do fade as the student progresses.

In Chapter 3, the procedure for collecting human tutor dialog is discussed. Four middle school students were coached on equation-solving problems that they had missed on a pre-test. Recurring techniques used by the tutor are described and compared to techniques mentioned in some of the research on human tutoring cited in this chapter. These techniques will be compared to techniques developed in E-tutor in Chapter 4.

3 Collecting Experienced Human Tutor Dialog

A tutorial dialog is a series of interactions between the tutor and the student during a tutoring session. Types of tutorial dialogs may include descriptions (of the problem, an action the student may take, a subgoal in solving the problem, or of a step in solving the problem), explanations (of a theory, of a concept, or of terms), prompts (such as questions, clarifying a concept, or giving a hint) and feedback (Frederiksen et al, 2000). For this research, an hour and a half of one-on-one tutorial dialog was collected. Section 3.1 describes the procedure for collecting an experienced human tutor dialog. Section 3.2 identifies some strategies used by the tutor and some generalizations are made about parts of the dialog. Finally, conclusions of this chapter are covered in Section 3.3.

3.1 Procedure

To incorporate human dialog in E-tutor, techniques used by a human math tutor were observed while teaching middle school students how to solve equations. The math tutor is also a math teacher at Doherty Memorial High School in Worcester. Although the effectiveness of the math tutor was not determined and may not be representative, it was known that she also had one year of experience in one-on-one math tutoring for hire before becoming a math teacher. She was videotaped tutoring four students in solving linear equations. The students were three pre-Algebra students and one Algebra I student (two seventh-graders and two eighth-graders) whose parents agreed to allow them to participate in this research. The students attended three different middle schools in two different districts. The tutoring sessions took place on December 8, so it was near the end of the second quarter of the school year. The pre-Algebra students had been introduced to solving equations, but they did not have a lot of experience solving them. The Algebra I student had more experience in solving linear equations.

A test of 20 algebra problems was given to the students to work on. The tests were checked and marked and given to the tutor. There was a gap of approximately 30 minutes between test-taking and the tutoring of the first student. The students were tutored for 23 minutes on average. The tutor was instructed to tutor the students on problems they got wrong on the test.

This phase resulted in one and a half hours of one-on-one videotaped human tutoring. The human tutor helped the students on a total of 26 problems that the students had missed on the test. The tutoring sessions were also transcribed (see Appendix A).

The problems that were presented to the students consisted of linear equations that required one to six operations to solve the equations. Similar problems were presented in the pre- and post-tests of the experiment (see Chapter 5). The twenty problems were as follows:

Solving Equations by Adding or Subtracting

1. $c + 32 = 124$
2. $\underline{\quad} - 13 = -4$
3. $x - 7 = -23$
4. $14 + \underline{\quad} = -32$
5. $r + 30 = -132$

Solving Equations by multiplying and dividing

1. $12r = -132$

2. $-7c = -112$

3. $\frac{k}{-6} = 12$

4. $-64 = 16y$

5. $\frac{m}{4} = -8$

Solving Two-Step Equations

1. $8n - 19 = 37$

2. $x/3 - 6 = -4$

3. $4 + 2p = 6$

4. $3c - 7 = 2$

5. $-3t/2 = 27$

Solving Equations with Variables on Each Side

1. $(2a - 5)6 = 15a$

2. $7z = 2 + 5(z - 2)$

3. $9(x - 3) = 10 - 18x$

4. $20 - 2x = 4(5x + 3)$

5. $3x + 5x - 22 = 15x + 30 - 5$

3.2 Strategies used by the human tutor

It was observed that there were several strategies that the tutor relied on. Often the tutor would introduce a subgoal in English to explain the next step when a student was stuck. Using subgoals can be helpful to students as described by Catrambone (1996):

“A particular subgoal will have a set of steps, or method, associated with it, and if the learner discovers that those steps cannot be used to achieve that subgoal on a particular problem, he or she will have a reduced search space to consider when trying to adapt the method. That is the learner knows on which steps to focus for changing the procedure. (p. 1020)”

This can be helpful for learning to solve equations and deciding what kind of transforms may be needed to solve a problem, or if one is needed at all. The human tutor often presented subgoals at the beginning of a problem based on how a variable could be isolated, such as in lines 291 – 294 of the tutor transcript (see Appendix A). The tutor also used a real example to illustrate a problem when a student seemed to have trouble with variables. This is consistent with Catrambone’s (1996) claim that “people typically prefer to study and use worked examples, or problems they have previously solved, when attempting to solve new problems (p. 1021).” Another common strategy that the math tutor used was to repeat subgoals that the student had just answered correctly. Table 3.1 shows some examples of these strategies. It was decided that these strategies could be incorporated into the ITS.

| Strategy | Example problem | Occurrences in dialogs | Example dialog |
|----------------|-----------------|--|--|
| Introduce goal | $-k/6=12$ | 22 times in 84% of the problems tutored (see lines 1, 26, 60, 105 in Appendix A) | 60. Tutor: Negative k over 6 equals twelve. A lot of us are fraction-phobic, we hate fractions right? poor fractions. So I want to get rid of the fraction so I don’t have to deal with it. So I want to get rid of the fraction and fractions are really division. So what’s the opposite of division? |

| | | | |
|--------------|--------------|--|---|
| Real example | $-k/6=12$ | 9 times in 34% of the problems tutored (see lines 64, 181, 189, 201, 301, 411 in Appendix A) | <p>301. Tutor: don't look at the sign, the sign stays the same. The opposite of division is multiplication. You'll deal with positive and negative numbers in another problem. [writes $1/4(4)$] Look, if I had 1 over 4 and I times it by 4, what's going to happen?</p> <p>302. Student: um, it's going to turn into a whole number. It's going to turn into 1.</p> <p>303. Tutor: 1, because it cancels out right? [crosses out the 4s, writes $x/4(4)$] If I had x over 4 times 4, what happens?</p> <p>304. Student: [crosses out the 4s, writes x] it cancels out and there's just x.</p> <p>305. Tutor: is there a fraction anymore?</p> <p>306. Student: no</p> <p>307. Tutor: [writes $7/6$] and if I had 7 over 6 and I want a whole number what should I multiply by?</p> <p>308. Student: [writes 6] 6</p> <p>309. Tutor: and what am I left with?</p> <p>310. Student: [crosses out the 6s] 7</p> <p>311. Tutor: so use that knowledge that you have and ...</p> <p>312. Student: [writes $m/4(4)$, crosses out the 4s]</p> |
| Repeat goals | $-3t/2 = 27$ | 22 times in 84% of the problems tutored (see lines 62, 189, 220, 675 in Appendix A) | <p>189. Tutor: that's why we've been doing that. [writes $2x = 8$] Like if we had $2x$ equals 8, and you want to solve for x. What do you divide it by?</p> <p>190. Student: 2</p> <p>191. Tutor: [writes $2x/2$ on the left side] 2. because 2 over 2 is one and you're left with just the x.</p> |

Table 3.1 Tutorial strategies observed in the human tutor.

Analysis of the human tutor dialog showed several areas where certain types of dialog recurred with the students. It was clear that dealing with fractions was a major area of contention for the students. For example, the following dialog in Table 3.2 was typical in problems involving fractions. The line numbers in this portion of dialog are correlated to the full human tutor dialog transcripts. (To see the full human tutor dialog transcripts see Appendix A) In Table 3.2, a dialog is broken down to illustrate the kinds of questions the tutor typically asked while working on a problem with a student. The tutor starts out by introducing the problem and breaks the problem down by concentrating on the left side of the equation. This is broken down further by first asking the student what operation is needed and then concentrating on how to use that operation (i.e. what number to use). In line 64, the tutor helps the student figure this out by introducing some examples that do not have a variable and asking the student to apply the operation to the examples. Once the student has applied the operation with the right number, the tutor moves on to the right side of the equation. It is apparent that the tutor's method agrees with Catrambone (1996) that breaking down a problem into subgoals is effective.

| | Strategies | Human tutor dialog | Comments |
|--|---------------------------------------|---|--|
| What operation to remove fraction? | Introduce subgoal | Problem: $-k/6=12$ | The tutor reminds the student of the subgoal of removing the fraction by the opposite operation. |
| | | 60. Tutor: Negative k over 6 equals twelve. A lot of us are fraction-phobic, we hate fractions right? poor fractions. So I want to get rid of the fraction so I don't have to deal with it. So I want to get rid of the fraction and fractions are really division. So what's the opposite of division? | |
| | 61. Student: multiplication | | |
| How to achieve goal (figure out what number to multiply) | Repeat goal | 62. Tutor: right. Multiplication cancels out division right? Is there a number I can multiply by that will get rid of the fraction? | The tutor repeats reason for opposite operation; asks student how to use it. |
| | | 63. Student: ... | |
| | Real example | 64. Tutor: ok, let me ask you a question. If I had one fifth and I wanted to multiply by a number that's going to cancel the 5. [writes $1/5$] What's the smallest number you can think of? | The tutor presents real examples without variables. |
| | | 65. Student: 5 | |
| | | 66. Tutor: [writes 5 and crosses them out, then writes $1/4$] if I had one fourth? | |
| | | 67. Student: 4 | |
| | | 68. Tutor: [writes 4, then writes $1/9$] if I had one ninth? | |
| | | 69. Student: 9 | |
| | | 70. Tutor: [writes 9] 9. So here is k over -6 . What number should we multiply by? | |
| | | 71. Student: 6 | |
| 72. Tutor: 6 or negative 6? | | | |
| 73. Student: negative 6 | | | |
| Work on right hand side | Extend to right hand side of equation | 74. Tutor: right. [writes (-6) on the left side] Now in algebra, what I do to one side of the equation, I have to ... | |
| | | 75. Student: do to the other side | |
| | | 76. Tutor: right, it has to balance out. So if I multiply -6 on one side ... | |
| | | 77. Student: ... | |
| | | 78. Tutor: we multiplied this side by -6 to get rid of the fraction right? But what I do to one side I have to do to the other. Make that an automatic thing. So what do you need to do? | |

| | | | |
|-----------------|----------------|---|--|
| Finish Prob. | | 79. Student: twelve times negative six [writes $.-6$ on the right side] | |
| | | 80. Tutor: right. So -6 cancels out and k equals? | |
| | What sign? | 81. Student: multiply 6 by 12? | |
| | | 82. Tutor: yeah. We'll think about the negative sign later. | |
| | | 83. Student: [writes $12 * 6 =$, then 72] ... 72 | |
| | | 84. Tutor: right. But the sign is? | |
| | | 85. Student: negative [writes $-$ for 72] | |
| | | 86. Tutor: so what is k ? | |
| | | 87. Student: -72 [writes $k = -72$] | |
| | Repeat subgoal | 88. Tutor: Right. See, we got rid of the fraction in the first step. It's key to do that. | |

Table 3.2 human tutorial strategies

Both immediate and delayed feedbacks have been shown to be helpful to students (Mathan and Koedinger, 2003). On most occasions, the tutor provided immediate feedback to student errors, keeping students on the correct solution path. There was one out of 26 problems where the tutor gave delayed feedback to a student error. In this instance, the tutor allowed the student to continue where she had made an error and then let her check her work and find the error (lines 351-377, Appendix A), promoting evaluative skills. This happened with the student who was taking Algebra I and was a little more advanced than the others. However, for the other 25 problems, the tutor had a remedial response, providing immediate feedback to promote the development of generative skills. This agrees with McArthur et al. (1990) in their examination of tutoring techniques in algebra where they collected one and a half hours of videotaped one-on-one tutoring sessions.

“In fact, for every student error we recorded, there was a remedial response. At least the tutors we observed were apparently not willing to let students explore on their own and perhaps discover their own errors... teachers may believe that such explorations too frequently lead to unprofitable confusion for the student (p. 209).”

In Chapter 2, teaching methods such as modeling, coaching, scaffolding, and fading were mentioned as presented by Collins et al. (1991). Our human tutor employed techniques such as these. For example, the tutor modeled a step herself when she may have thought that the student needed to observe and build on her example such as in line 561 of the dialog (see Appendix A). The tutor also used scaffolding extensively by offering hints, suggestions and by executing parts of steps, such as the dialog starting on line 393 of the tutor transcripts. Fading is evident throughout the tutoring sessions where students were presented with problems that were similar to previous problems. The tutor provided less feedback on these occasions, avoiding repetition and allowing the students to apply what they had done on a similar problem. For instance, notice the difference between the dialog for a similar problem in lines 60-88 and 96-104 of Appendix A.

3.3 Chapter conclusions

A human tutor and four middle school children were used to collect data on human tutoring. An hour and a half of tutoring in equation-solving was recorded and transcribed. Examination of the transcriptions showed that the tutor used tactics that are common to tutors according to previous research on human tutoring.

In Chapter 4, the implementation of E-tutor is described. The control condition and the experimental condition of E-tutor are explained separately. The control condition, E-tutor without dialog provided feedback with hints and bug messages. The experimental condition, E-tutor with dialog, provided feedback by breaking down problems and asking new questions about the simpler steps. The tutorial dialogs implemented in E-tutor are described in detail.

4 Implementation

E-tutor was implemented using a production rule system, the Tutor Development Kit (TDK; Anderson and Pelletier, 1991). Production rules are if-then rules that operate on a set of working memory elements. A production rule system consists of components for working memory, rule memory, and a mechanism for repeating the “*match, conflict resolution, act*” cycle. The patterns contained in working memory are matched against the conditions of the production rules. This produces a subset of rules known as the conflict set, whose conditions match the contents of working memory. One or more of the rules in the conflict set is selected (conflict resolution) and fired, which means its action is performed. The process terminates when no rules match the contents of working memory. E-tutor was implemented with 89 production rules, including 12 buggy productions.

The following is an example of a production rule used in E-tutor. This production, in pseudo code, is a rule for dividing both sides of an equation to get rid of a coefficient (e.g., “ $5x = 10$ ”). The variables used are LHS and RHS for “left hand side” and “right hand side” respectively, EXP1 and EXP2 for the expressions and OP1 for the operator. (See Appendix E for the actual code of this production rule.)

```

If goal is to solve an equation
    And the LHS is EXP1 OP1 EXP2
    And EXP1 is a number
    And EXP2 is a variable
    And OP1 is *
Then
    Set LHS to EXP2
    Set RHS to RHS divided by EXP1

```

In this chapter, the implementation of E-tutor is discussed. The control condition, E-tutor without dialog, which is modeled after the Cognitive Tutor, is presented in Section 4.1. The interfaces, hint sequences and bug messages in the Cognitive Tutor are compared to those implemented in E-tutor without dialog. The experimental condition, E-tutor with the tutorial dialog system, is covered in Section 4.2. The architectures of the control and experimental versions of E-tutor are compared. The tutorial dialogs implemented in E-tutor are described and discussed.

4.1 The control condition

The equation-solving section of the Cognitive Tutor by Carnegie Learning, Inc.[®] was used as a model for the control condition of this thesis. Researchers at Carnegie Mellon University (Koedinger et al., 1995) built the software to teach various skills in algebra, such as graphing, word problems and solving equations. This thesis was only concerned with the equation-solving section. The Cognitive Tutor has been used by over 100,000 students in over 1% of American high schools. The software has been well-tested and shown to be more effective than traditional classroom instruction alone. Evaluation studies of the Cognitive Tutor Algebra I showed an average of 85% better performance on post-tests compared to students in traditional mathematics classes. (Koedinger, Corbett, Ritter and Shapiro, 2000) Using the Cognitive Tutor as a model provided a strong control condition for evaluating E-tutor with dialog.

The interface of the equation-solving section of the Cognitive Tutor consists of a problem window, a skillometer and a message window. The problem window presents the equation to be solved, shows the steps chosen by the student and changes to the problem as the student works. Negative feedback is indicated by coloring incorrect steps orange. The message window presents hint messages when the student presses a help button or bug messages when the system recognizes an error that the student made. Skills that the student is expected to complete are shown in the skillometer. Skills are checked off as the student shows mastery.

In Figure 4.1, a snapshot of the interface is shown. The snapshot shows that the student tried to combine like terms on the left before multiplying 7 and 2. In the problem window, the action was colored in orange to show that it was incorrect. After multiplying the 7 and 2, the student asks for a hint and the first hint in the sequence is displayed in the message window. The hint, “Put the equation in its simplest form”, presents the general goal of the next step. The Skillometer shows that the student has not yet mastered the skills for selecting to multiply or divide before add or subtract and selecting to eliminate parentheses.

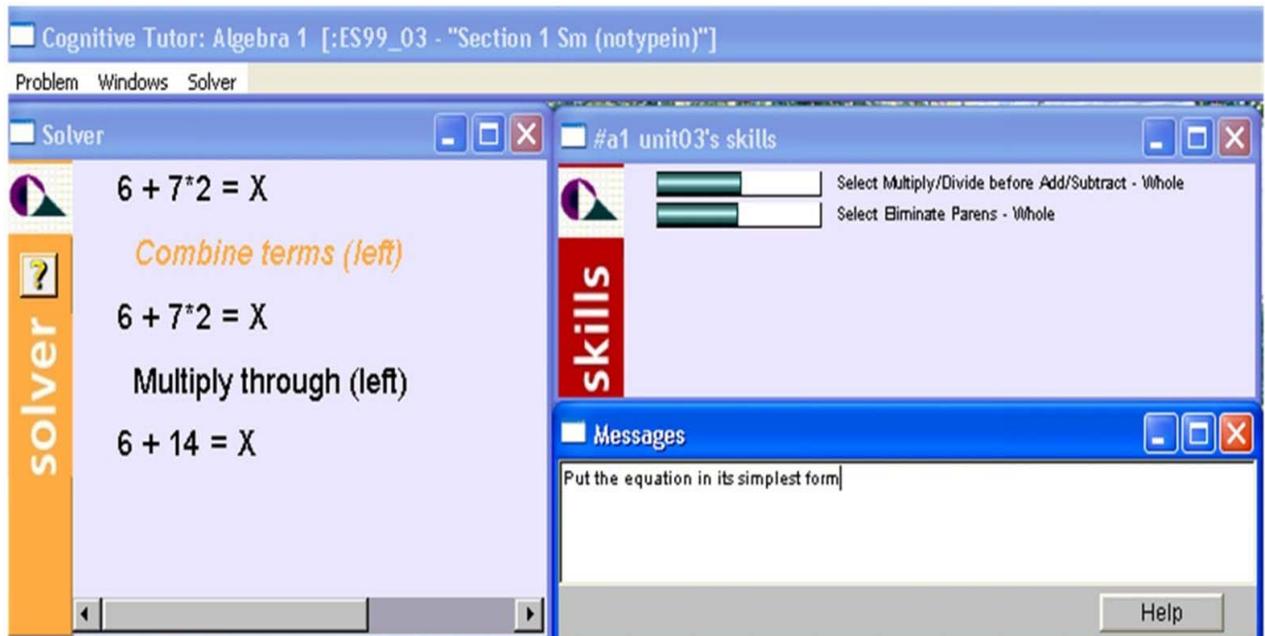


Figure 4.1 Interface of the Cognitive Tutor®

The Cognitive Tutor uses model-tracing to understand a student’s input (see Chapter 2 for more about model-tracing). The Cognitive Tutor can indicate to students when they have made a mistake and provide hint messages, as well as provide canned explanations to some errors with bug messages. When a student continues asking for help, the hint messages get more explicit until the tutor is telling the student exactly what to do. The Cognitive Tutor does not engage the student in dialog by asking them new questions.

The architecture of the equation-solving section of the Cognitive Tutor is described as follows. The tutor presents the equation to be solved in the problem window. The student must choose the next action from the “Solver” menu or ask for help. If the student selects an action, the tutor decides if the action is correct. If the action is correct, positive feedback is given in the problem window, and the student may continue by selecting the

next action. If the action was incorrect the negative feedback is given in the problem window and the tutor presents a bug message in the message window if one is available. The student may continue by choosing the next action or asking for a hint. When a student asks for a hint, the hint is displayed in the message window and the student may continue to choose the next action or ask for another hint. Figure 4.2 shows the architecture of a traditional model-tracing tutor with hints such as that of the Cognitive Tutor.

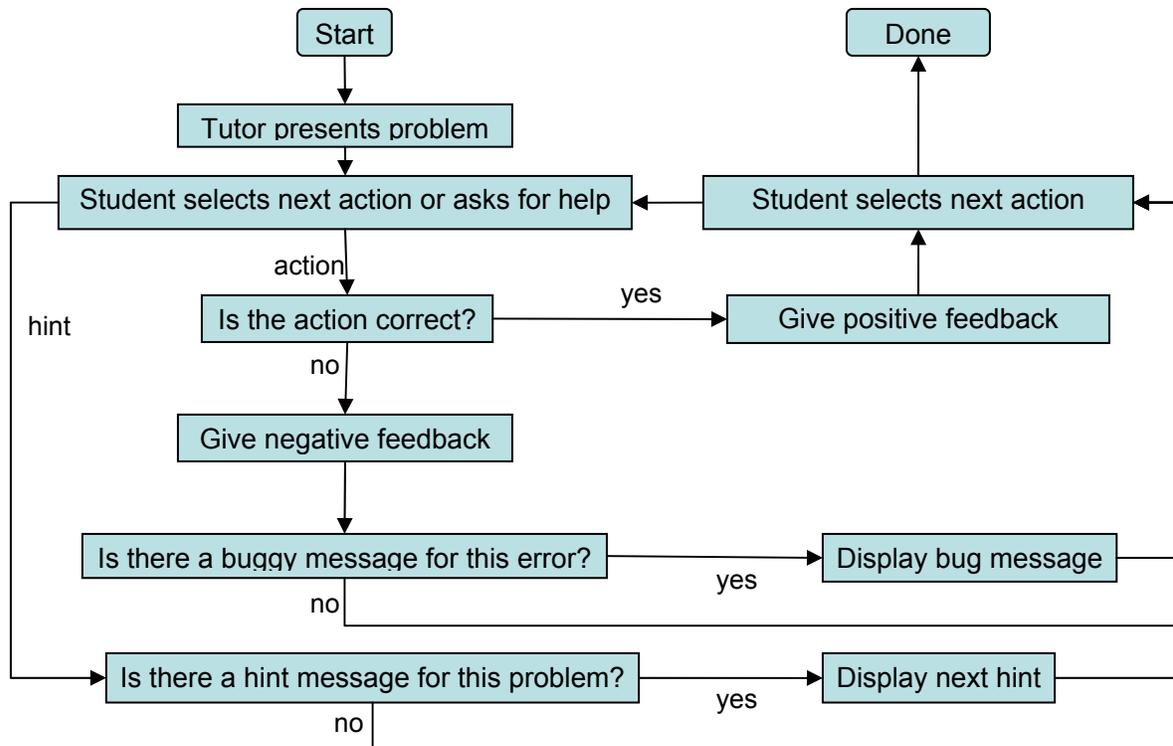


Figure 4.2 Architecture of a traditional model-tracing tutor

The control condition was implemented for this research but was designed to behave and look similar to the Cognitive Tutor. E-tutor's interface consists of a problem window, a message window and a skillometer (shown in Figures 4.3 and 4.4), as does the interface of the Cognitive Tutor. E-tutor menu items in the student action menu also mimic those in the Cognitive Tutor. Figure 4.3 shows that the student has asked for a hint before selecting any action. The hint is displayed in the message window.

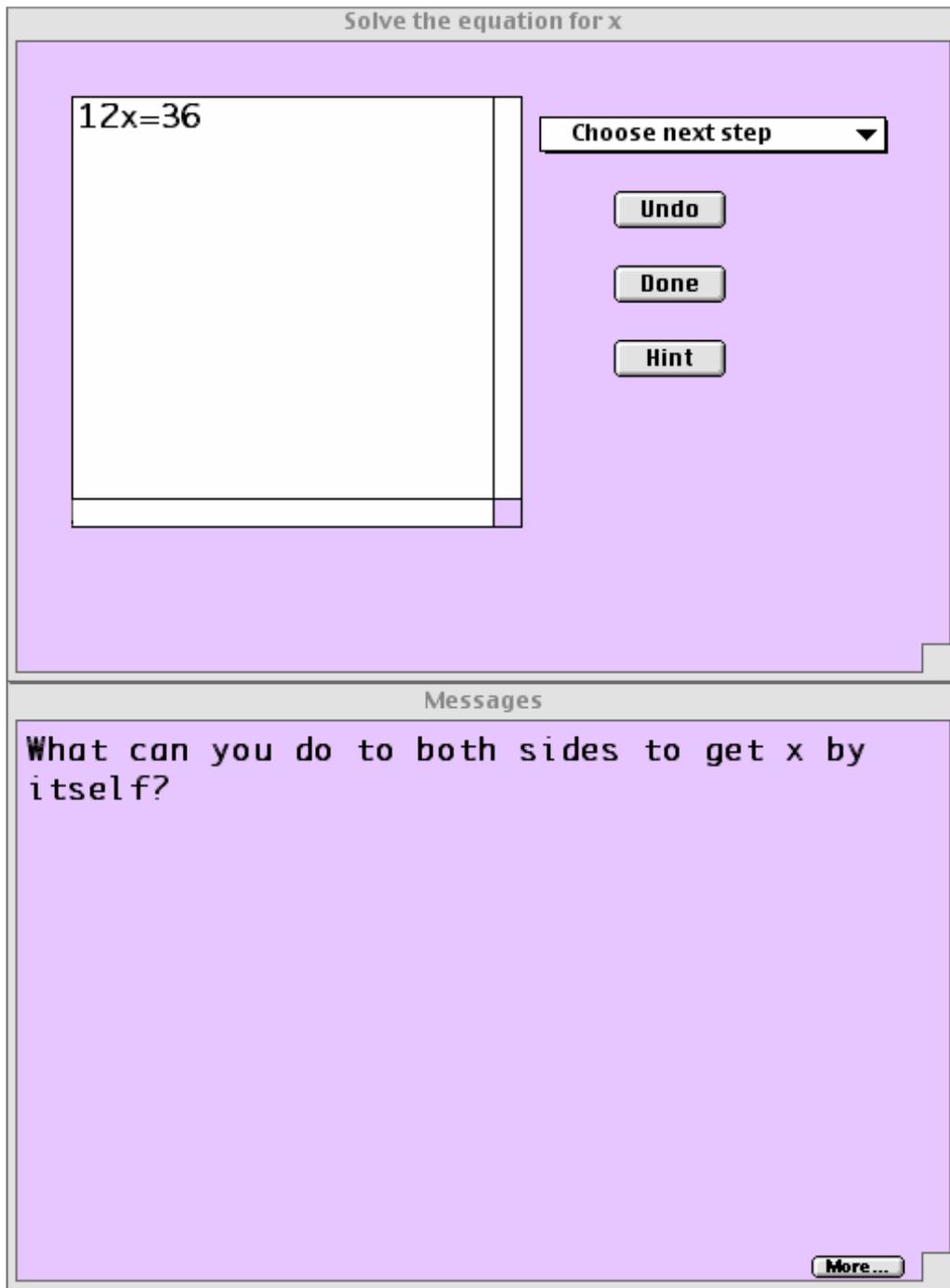


Figure 4.3 Interface of the control condition of E-tutor showing a hint message

The skillometer allowed students to see their progress as they worked. Sets of production rules are represented by the bar graph signifying skills that the student has learned. Shading in each bar represents the probability that students know each rule and check marks to the left of each bar show mastery of a skill. The skillometer can be used to present remedial problems to students when they have not mastered skills; however this feature was not used in the experiment.

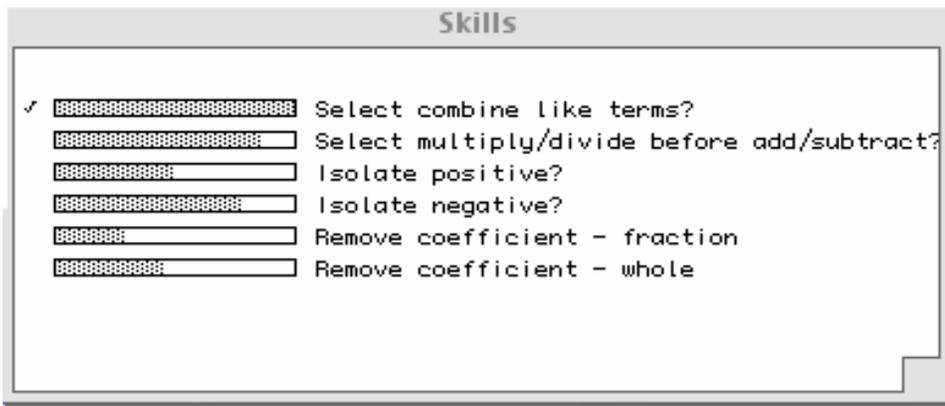


Figure 4.4 E-tutor's skillometer

4.1.1 Hint sequences and Buggy Messages

As mentioned earlier in this chapter, hints are one form of feedback that model-tracing tutors often provide. Specifically, the Cognitive Tutor gives hints when the student requests them. When a student asks for a hint at a particular point in a problem, the tutor runs the production system to generate the correct answer. See Table 4.1 for a comparison of the hint sequences in the Cognitive Tutor and the control condition of E-tutor.

In the control condition, students can ask for canned hints and may receive bug messages to specific errors. The hint and bug messages are similar to and often identical to those in the Cognitive Tutor. While bug messages are presented to students when they make specific errors, the presentation of hints depends on student initiative, that is, only when a student asks for one. Hint sequences in the Cognitive Tutor typically contain three hints culminating with a very explicit hint that tells the student the correct action and input for the next step in the problem. These hint sequences are mimicked in the control condition of E-tutor.

| Correct action | Example problem | Hint sequences in the Cognitive Tutor | Hint sequences in the control E-Tutor |
|--|-----------------|---|--|
| need to multiply before adding | $(3+9*8 = Y)$ | <ol style="list-style-type: none"> 1. Put the equation in its simplest form Be sure to do the multiplication before adding: $9*8$ 2. Multiply 9 and 8 on the left side. 3. Multiply on the left | <ol style="list-style-type: none"> 1. Put the equation in its simplest form Be sure to do the multiplication before adding. 2. Multiply 9 and 8 on the left side. 3. Multiply on the left |
| need to combine numbers. if there are numbers to combine on both sides, hints for left side first: | $(3+72 = Y)$ | <ol style="list-style-type: none"> 1. Put the equation in its simplest form 2. Combine 3 and 72 on the left side. 3. Combine like terms on the left | <ol style="list-style-type: none"> 1. Put the equation in its simplest form 2. Combine 3 and 72 on the left side. 3. Combine like terms on the left |

| | | | |
|--|------------------|--|---|
| need to do an operation within parentheses first | $(4(0+4) = Y)$: | <ol style="list-style-type: none"> 1. Put the equation in its simplest form Be sure to do the operation inside the parentheses first: $(0 + 4)$ 2. Combine 0 and 4 on the left side. 3. Combine like terms on the left | <ol style="list-style-type: none"> 1. Put the equation in its simplest form 2. Combine 0 and 4 on the left side. 3. Combine like terms on the left |
| need to multiply 2 numbers | $(4*4 = Y)$ | <ol style="list-style-type: none"> 1. Put the equation in its simplest form 2. Multiply 4 and 4 on the left side. 3. Multiply on the left | <ol style="list-style-type: none"> 1. Put the equation in its simplest form 2. Multiply 4 and 4 on the left side. 3. Multiply on the left |
| need to isolate a negative | $(7 = -8 + X)$ | <ol style="list-style-type: none"> 1. What can you do to both sides to get x by itself? 2. The -8 in $-8 + x$ is negative. How can you remove a negative? 3. Add 8 to both sides | <ol style="list-style-type: none"> 1. What can you do to both sides to get x by itself? 2. The -8 in $-8 + x$ is negative. How can you remove a negative? 3. Add 8 to both sides |
| need to isolate a positive | $(X + 4 = -9)$ | <ol style="list-style-type: none"> 1. What can you do to both sides to get x by itself? 2. The 4 in $x + 4$ is positive. How can you remove a positive? 3. Subtract 4 from both sides | <ol style="list-style-type: none"> 1. What can you do to both sides to get x by itself? 2. The 4 in $x + 4$ is positive. How can you remove a positive? 3. Subtract 4 from both sides |
| need to transform by multiplying | $(1 = X/8)$ | <ol style="list-style-type: none"> 1. What can you do to both sides to get x by itself? 2. Change the $1/8$ into a 1. 3. Multiply both sides by 8 | <ol style="list-style-type: none"> 1. What can you do to both sides to get x by itself? 2. Change the $1/8$ into a 1. 3. Multiply both sides by 8 |
| need to transform by dividing | $(-7 = -5y)$ | <ol style="list-style-type: none"> 1. What can you do to both sides to get y by itself? 2. $-5y$ is -5 times y. How do you undo multiplication? 3. Divide by -5 | <ol style="list-style-type: none"> 1. What can you do to both sides to get y by itself? 2. $-5y$ is -5 times y. How do you undo multiplication? 3. Divide by -5 |

| | | | |
|----------------------------------|--------------|---|---|
| need to make a variable positive | $(-Y = 5)$: | <ol style="list-style-type: none"> 1. What can you do to both sides to remove the negative sign from $-y$? 2. $-Y$ means -1 times y. How do you undo a number multiplied by the variable? 3. Divide by -1 | <ol style="list-style-type: none"> 1. What can you do to both sides to remove the negative sign from $-y$? 2. $-Y$ means -1 times y. How do you undo a number multiplied by the variable? 3. Divide by -1 |
| done with a problem | $(75 = Y)$ | 1. Choose DONE from the menu | 1. Click the DONE button |

Table 4.1 Hint sequences in the Cognitive Tutor and control condition of E-tutor

There are several bug messages available in the Cognitive Tutor. As mentioned in Chapter 2, buggy messages are based on an incorrect student action and can be used to model common mistakes that students make with buggy productions. The common errors that the Cognitive Tutor has bug rules for are:

- Trying to combine like terms before multiplying (e.g. $3+9*8 = Y$)
- Trying to transform an equation by adding instead of subtracting (e.g. $3x+4 = 2$)
- Trying to transform an equation by subtracting instead of adding (e.g. $3x-4 = 2$)
- Trying to transform an equation by multiplying instead of dividing (e.g. $3x = 9$)
- Trying to transform an equation by dividing instead of multiplying (e.g. $x/3 = 9$)

E-tutor's buggy messages are similar to those found in the Cognitive Tutor. Table 4.2 compares buggy messages in the Cognitive Tutor to those implemented in E-tutor.

| Incorrect action | Example problem | Bug message in the Cognitive Tutor[®] | Bug message in E-Tutor without dialog |
|--|------------------------|--|---|
| A problem is done and the student tries to choose another step. | $75 = Y$ | You already solved the equation. Y is 75. Erase your last step and pick done. | You are done with this problem. |
| Student tries to combine like terms before multiplying terms. | $3+9*8 = Y$ | When there are no parentheses, all multiplications and divisions must be done before any additions or subtractions | You can't combine like terms here |
| Student tries to transform by adding a negative number where it should be positive | $7 = -8 + X$ | To remove -8 from the right side, you need to add a positive number to it. Erase your last step and then add 8 to both sides | To remove -8 from the right side, you need to add a positive 8 to it. |
| Student tries to transform by subtracting a | $X+4 = -9$ | To remove 4 from the left side, you need to subtract a positive | To remove 4 from the left side, you need to subtract a positive 4 from |

| | | | |
|---|------------|--|---|
| negative number where it should be positive | | number from it. Erase your last step and then subtract 4 from both sides. | it. |
| Student tries to transform by adding when they should subtract | $X+4 = -9$ | Since 4 is positive, you should subtract to remove it from the left side. Erase your last step and then subtract 4 from both sides. | Since 4 is positive, you should subtract to remove it from the left side. |
| Student tries to transform by multiplying both sides of an equation instead of dividing both sides. | $5X = 15$ | To remove the coefficient of $5X$, you need to divide both sides by 5. | To remove the coefficient of $5X$, you need to divide both sides by 5. |
| Student tries to transform by dividing both sides of an equation instead of multiplying. | $X/5 = 10$ | To remove the coefficient of $X/5$ you should multiply by 5. | To remove the coefficient of $X/5$ you should multiply by 5. |
| student tries to divide both sides by a positive coefficient when it should be negative | $-7 = -5Y$ | In this equation, y is multiplied by -5 . Dividing by 5 leaves $-y$, so you still need to remove the negative sign. Dividing by -5 is better, since that would leave positive y . | In this equation Y is multiplied by -5 . Dividing by 5 leaves $-y$, so you still need to remove the negative sign. Dividing by -5 is better, since that would leave positive y . |
| Student chooses done and the variable is negative | $-X = 3$ | There is a negative sign in front of x To make it positive, multiply or divide by -1 | There is a negative sign in front of x To make it positive multiply or divide by -1 |

Table 4.2 Bug messages in the Cognitive Tutor and E-tutor

4.2 The experimental condition

E-tutor with dialog is able to carry on a coherent dialog that consists of breaking down problems into smaller steps and asking new questions about those steps, rather than simply giving hints. Several tutorial dialogs were chosen from the transcripts of the human tutoring sessions to be incorporated in the ITS. The dialogs were designed to take the place of the hints that are available in the control condition. The experimental condition of E-tutor did not have the hint button. When students make errors they are presented with a tutorial dialog if one is available. The student must respond to the dialog to exit it and return to solving the problem in the problem window. Students stay in the

loop until they respond correctly or the tutor has run out of dialog. When the tutor has run out of dialog, the last tutorial response presents the student with the correct action and input similar to the last hint in a hint sequence.

A close mapping between the human tutor dialog (see Chapter 3) and the ITS was attempted. Tutorial dialogs implemented in the ITS included the following dialogs:

- **“Add-Transform” Dialog:** tutorial dialog for transforming an equation by adding to or subtracting from both sides
- **“Multiply-Transform” Dialog:** tutorial dialog for transforming an equation by multiplying to both sides
- **“Divide-Transform” Dialog:** tutorial dialog for transforming an equation by dividing both sides
- **“Combine Like” Dialog:** tutorial dialog for combining like terms
- **“Combine Unlike” Dialog:** tutorial dialog for combining unlike terms error
- **“Distribute” Dialog:** tutorial dialog for distribution

These dialogs typically have several “sub-dialogs” that are presented depending on the number of times a particular error is made and depending on factors such as whether the error is from a wrong student action or wrong student input. There are several different paths a dialog can take depending on student responses to the tutor’s questions. Students will also get a slightly different dialog when they make the same mistake more than one time. Typically, a dialog is presented differently the first three times an error is made. If an error is made more than three times, the third dialog is presented again. Examples of some of the dialogs implemented are presented in Table 4.3.

| Simulated E-tutor Dialogs |
|---|
| <p>“Add-Transform” Dialog Example problem: $3x+4=6$ Example of one of several paths for this tutor dialog:</p> <p>Tutor: We need to get $3x$ by itself, so we need to get rid of the 4. The 4 is positive. How can you remove a positive? (Student is offered choices of addition, subtraction, multiplication and division.) Student: (Student chooses addition) Tutor: No, the opposite of addition is subtraction. Choose subtraction. Student: (Student chooses subtraction) Tutor: Right, the opposite of addition is subtraction. What should you subtract from both sides? Student: (Student types in 6) Tutor: No, you need to subtract both sides by a number that will cancel the 4 on the left side. Student: (Student types in 4) Tutor: Right, subtracting 4 from both sides leaves $3x$ by itself on the left. You should go back to the problem and subtract 4 from both sides.</p> |

“Divide-Transform” Dialog**Example problem: $5x=10$** **Example of one of several paths for this tutor dialog:**

Tutor: You need to find out what x is by itself, not what $5x$ is. $5x$ means 5 times x . What is the opposite of multiplication? (Student is offered choices of addition, subtraction, multiplication and division.)

Student: (Student chooses division)

Tutor: Right the opposite of multiplication is division. What should you divide both sides by?

Student: (Student types in 10)

Tutor: No, you need to divide both sides by a number that will leave only 1 x on the left side. 5 divided by what equals 1?

Student: (Student types in 1)

Tutor: Well, here's how it works. Any number divided by itself equals 1. So 2 divided by 2 is 1, 3 divided by 3 is 1 and so on. 5 divided by what equals 1?

Student: (Student types in 5)

Tutor: Right, if you divide both sides by 5 you get 1 x by itself on the left. You should go back to the problem and divide both sides by 5.

“Multiply-Transform” Dialog**Example problem: $x/7=4$** **Example of one of several paths for this tutor dialog:**

Tutor: You have to get rid of the fraction to get x by itself on the left. So we need to find a way to cancel the $1/7$ on the left. Fractions are really division. What is the opposite of division?(Student is offered choices of addition, subtraction, multiplication and division.)

Student: (Student chooses multiplication)

Tutor: Right the opposite of division is multiplication. What should you multiply both sides by?

Student: (Student types in 4)

Tutor: No, you need to multiply both sides by a number that will leave only 1 x on the left side. What should you multiply $x/7$ by to cancel the $1/7$?

Student: (Student types in 1)

Tutor: Well, here's how it works. To get rid of the denominator in a fraction you need to multiply the numerator by the same number, because $4/4$ is 1, $9/9$ is 1, $1394/1394$ is also 1. $1/7$ times what equals 1?

Student: (Student types in 7)

Tutor: Yes, if you multiply both sides by 7 you get 1 x by itself on the left. You should go back to the problem and multiply both sides by 7.

“Distribute” Dialog**Example problem: $3(x+4)=8$** **Example of one of several paths for this tutor dialog:**

Tutor: You have to get rid of the parentheses on the left. There is a 3 outside of the parentheses. Choose the resulting expression after removing the parentheses. (Student is offered choices of $3+x+4$, $3x+4$, and $3x+3*4$)

Student: (Student chooses $3x+4$)

Tutor: No, the 3 outside of the parentheses means that you must multiply 3 by everything in the parentheses. So the result will be $3x+3*4$ or $3x+12$. This is called distribution. You should go back to the problem and distribute on the left.

| |
|--|
| <p>“Combine Like” Dialog Example problem: $5+4=3x$ Example of one of several paths for this tutor dialog:</p> <p>Tutor: It is important to simplify both sides of the equation as much as possible by combining like terms first. Click OK for examples of like terms or click Back to Problem to return to solving the equation. Student: (Student clicks OK) Tutor: Here are some examples of like terms: The following are like terms because they are all x with a coefficient: $3x, 0.5x, 984x, -22x$ The following are like terms because they are all constants: $15, -2, 27, 9043$ The following are like terms because they are all k with a coefficient: $0k, 155k, -0.9k, 3k$ Go back to the problem and combine the like terms on the left.</p> |
| <p>“Combine Unlike” dialog Example problem: $3x+4=6$ Example of one of several paths for this tutor dialog: Tutor: You can't combine $3x$ and 4. That would be like combining apples and oranges. You can only combine like terms. For example, do you think $5t$ and $4y$ are like terms? Student: (Student clicks Yes) Tutor: No. $5t$ and $4y$ are not like terms. $5t$ and $4t$ would be like terms, or $5y$ and $4y$ would be like terms. Right now you need to isolate the $3x$ to solve the equation. The first step would be to cancel the 4 on the left side. You should go back to the problem and find a way to do that.</p> |

Table 4.3 Examples of E-tutor dialogs

Human tutors are unlikely to repeat the same questions and comments in exactly the same way when a student continues to make a certain error. To avoid giving the same dialog over and over, E-tutor keeps track of the number of times a student makes a particular error and presents different dialogs. Dialogs fade as students progress, rather than repeating concepts that students have mastered. For instance, the first, second and third sub-dialogs of the “Add-Transform” dialog may look similar to those in Table 4.4.

| | Simulated E-tutor dialog |
|-------------------------|--|
| First occurrence | <p>Add-Transform” Dialog Example problem: $3x+4=6$ Here is an example of one of several paths for this tutor dialog: Tutor: We need to get $3x$ by itself, so we need to get rid of the 4. The 4 is positive. How can you remove a positive? (Student is offered choices of addition, subtraction, multiplication and division.) Student: (Student chooses addition) Tutor: No the opposite of addition is subtraction. Choose subtraction. Student: (Student chooses subtraction) Tutor: Right, the opposite of addition is subtraction. What should you subtract from both sides? Student: (Student types in 6) Tutor: No, you need to subtract both sides by a number that will cancel the</p> |

| | |
|--------------------------|--|
| | <p>4 on the left side.</p> <p>Student: (Student types in 4)</p> <p>Tutor: Right, subtracting 4 from both sides leaves $3x$ by itself on the left. You should go back to the problem and subtract 4 from both sides.</p> |
| Second occurrence | <p>Tutor: The opposite of addition is subtraction, so you need to use subtraction to get rid of the 4 on the left side. What should you subtract both sides by?</p> <p>Student: (Student presses OK without entering a number)</p> <p>Tutor: No, you need to subtract both sides by a number that will cancel the 4 on the left side. What do you think that would be?</p> <p>Tutor: When you subtract both sides by 4 the 4 gets cancelled on the left side. You should go back to the problem and subtract 4 from both sides."</p> |
| Third occurrence | <p>Tutor: To get $3x$ by itself on the left you need to get rid of the 4. To cancel the 4 on the left side you use the opposite of addition which is subtraction. What can you subtract from both sides that will leave $3x$ by itself on the left?</p> <p>Student: (Student types in 4)</p> <p>Tutor: Right, subtracting 4 from both sides leaves $3x$ by itself on the left. You should go back to the problem and subtract 4 from both sides.</p> |

Table 4.4 Three sub-dialogs for the Add-Transform dialog

E-tutor also recognizes when a student has chosen the correct action for the next step in the problem but has entered the wrong input. For instance, for the problem in Table 4.3, $3x + 4 = 6$, a student may choose the correct action of “subtract both sides” with the wrong input of 6. This kind of error is likely a simple slip, based on the fact that the student got the action correct. In this case, the student will get the following dialog:

Tutor: You are right that you need to subtract something from both sides to isolate the $3x$. But if you subtract 6 from both sides then the 4 will not be canceled on the left. $4 - 6$ is not zero, so you need to subtract something that will cancel the 4.

Student: (Student types in 4)

Tutor: Right, so you see subtracting 6 from both sides doesn't leave $3x$ by itself on the left, but subtracting by 4 does. You should go back to the problem and subtract 4 from both sides."

Table 4.5 shows a comparison of a portion of the human tutor dialog, collected in Chapter 3, compared to a simulated E-tutor dialog. The dialogs in the table pertain to a problem that should be transformed by multiplication to solve the problem. In this scenario, the students are stuck and do not know what to do to solve the problem. Both dialogs start out by explaining that an opposite operation is needed and asking the student what the opposite of division is. In line 2, both students answer correctly that it is multiplication. In line 3, both tutors give some positive reinforcement by repeating that yes, multiplication is indeed the opposite of division. Now that the students know the operation, the tutors both ask the student what number is needed. When the students respond incorrectly in line 4, the tutors present some concrete examples. When the students get the correct number, the tutors, in line 9, give positive reinforcement by repeating the main concepts that the dialog dealt with.

| Problem: $k/6 = 2$ | |
|---|---|
| Experienced human tutor dialog | Real E-tutor dialog with simulated inputs |
| 60. Tutor: Fractions are really division, right? What's the opposite of division? | 1. Tutor says: You have to get rid of the fraction to get f by itself on the left. So we need to find a way to cancel the 1/6 on the left. Fractions are really division. What is the opposite of division? |
| 61. Student: multiplication | 2. Student chooses: multiplication |
| 62. Tutor: yeah, multiplication's gonna cancel out division, right? Is there a number I can multiply this fraction by that's gonna get rid of that fraction? | 3. Tutor says: Right, the opposite of division is multiplication What should you multiply both sides by? |
| 63. Student: 12? | 4. Student types in: 12 |
| 64. Tutor: 12 or even a smaller number ... Let me ask you a question. If I had 1/5 and I wanted to multiply by a number that's going to cancel the 5. What's the smallest number you can think of? | 5. Tutor says: Well, here's how it works. To get rid of the denominator in a fraction you need to multiply the numerator by the same number, because 4/4 is 1, 7/7 is 1, 1394/1394 is also 1. 1/6 times what equals 1? |
| 65. Student: 5 | 6. Student types in: 5 |
| 66. Tutor: if I had 1/6, what number? | 7. Tutor says: you need to multiply both sides by a number that will cancel the 1/6 on the left side. What should you multiply k/6 by to cancel the 1/6? |
| 67. Student: 6 | 8. Student types in: 6 |
| 68. Tutor: right. See if we multiply by 6 it cancels the 6 in the denominator. | 9. Tutor says: Right, if you multiply both sides by 6 you get 1 k by itself on the left. You should go back to the problem and multiply both sides by 6. |

Table 4.5 Comparison of human tutorial dialog and a simulated E-tutor dialog

The interface of the experimental condition of E-tutor was designed to be as similar to the control condition as possible (Figure 4.5). Both versions provide implicit feedback by using colors in the problem window. The experimental version provided a skillometer such as provided in the control condition (see Section 4.1). The experimental E-tutor did not have a hint button and dialogs appeared in the message window instead of hints. Dialogs often include a text input box, ok button or radio buttons for student input. It was decided to keep the bug messages in place in the experimental condition and to concentrate on replacing hint sequences with tutorial dialog.

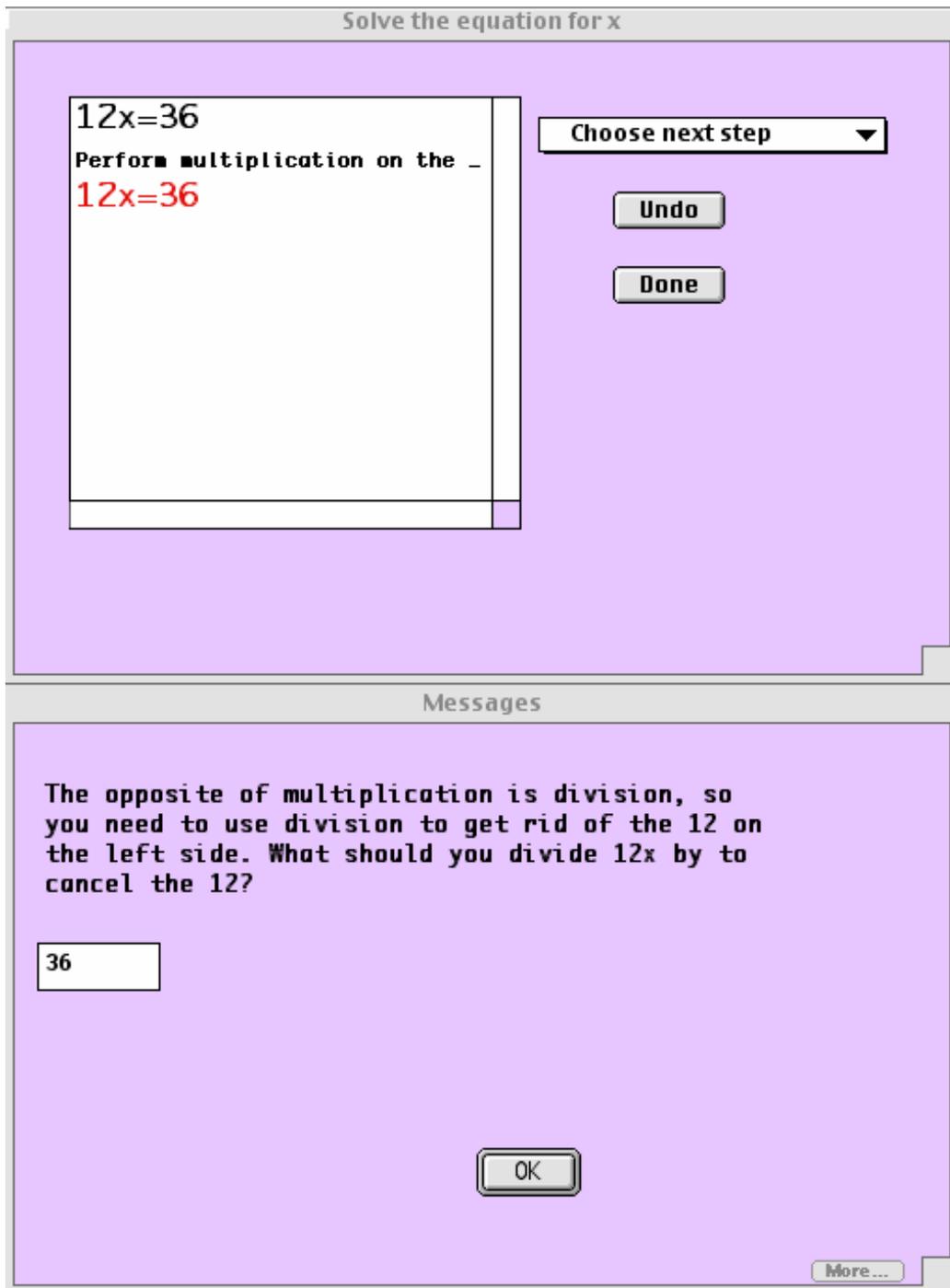
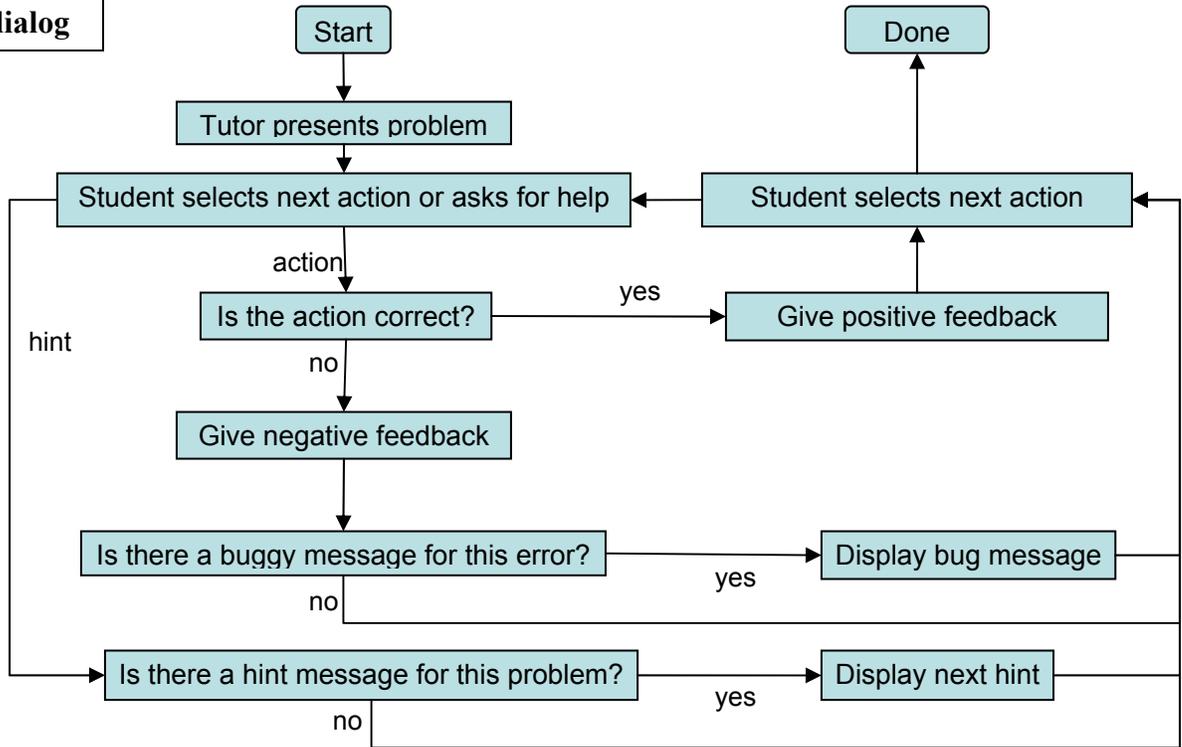


Figure 4.5 The interface of E-tutor showing a portion of the “Divide-Transform” dialog.

An important feature of the dialogs was keeping the student in the dialog until the tutor was satisfied. That meant that upon entering a dialog, the message window became modal. A student must respond correctly to the tutor’s new questions or the tutor must run out of comments and questions for the student to be allowed to work in the problem window. This forces the student to participate actively in the dialog. It is this loop that we hypothesize will do better at teaching equation-solving than hint sequences do. A look at Figure 4.6 shows the difference between the architectures of the control and experimental conditions. The red arrows show the loop that a student must actively participate in in order to continue selecting the next action.

Control E-tutor without dialog



Control E-tutor with dialog

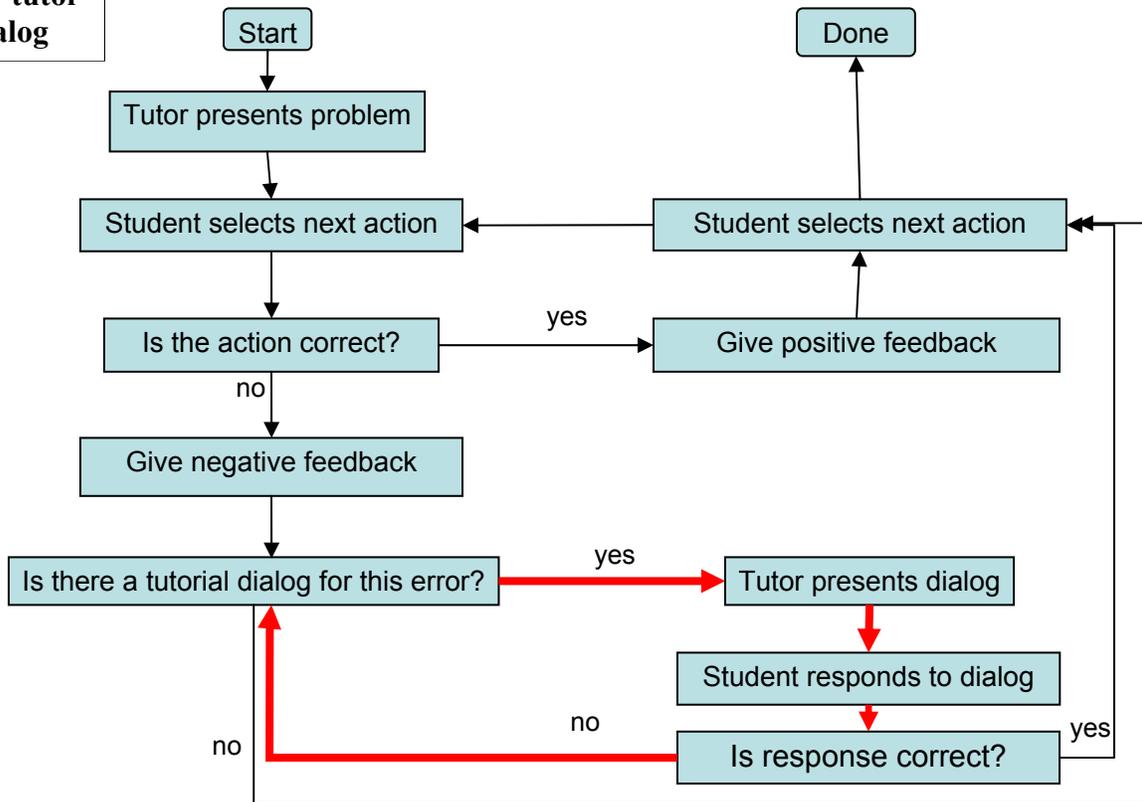


Figure 4.6 Traditional Model-Tracing Tutor Architecture compared to E-tutor’s Architecture

4.3 Chapter Conclusions

E-tutor was designed to engage students in dialog by asking new questions of students and expecting them to answer the questions. The kinds of dialogs incorporated in E-tutor were intended to resemble a human tutor. This differed from the control condition, which offered hints or buggy messages and did not ask students new questions that it expected them to answer. The control condition depended on student initiative when offering hints, but the experimental condition started tutorial dialogs when students made errors. This chapter described all of the hints and buggy messages available to students in the control condition, as well as the tutorial dialogs incorporated in the experimental condition.

Chapter 5 describes the results of this research. The experimental setup is described in detail in Section 5.1. The results of the experiment are analyzed by overall learning and by learning by condition. A problem analysis is performed to identify any interesting outcomes based on the kinds of problems. An error analysis focuses on the kinds of errors students made.

5 Analysis and Results

In this chapter, the design and analysis of a randomized controlled experiment is presented. The fundamental hypothesis that the experiment set out to validate is as follows:

Hypothesis: Dialog-based interactions as a feedback mechanism of ITSs perform better as measured by student learning, than equivalent ITS systems without it, or those that make use of strong hints.

The hypothesis will be validated in a controlled environment using a fixed number of problems without consideration for actual times taken to complete the sessions. The measure of learning used will be pre- and post-test gains.

There were two conditions in the experiment. The experimental condition used E-tutor which employed tutorial dialog to mimic a human tutor when teaching students to solve equations. The control condition was an intelligent tutor that was modeled after the “Cognitive Tutor” By Carnegie Learning, Inc. Section 5.1 covers the design and setup of the experiment done to evaluate E-tutor. Results of the experiment are covered in Section 5.2. Finally, chapter conclusions are dealt with in Section 5.3.

5.1 Experimental Design and Setup

This section describes the design and setup of the experiment. Materials for the experiment are described, followed by a description of the subjects, the design and the procedure followed in the experiment.

5.1.1 Materials/Apparatus

The experiment was run at Clark University in Worcester, Massachusetts. The Jonas Clark building has a computer lab in room 101, which contains 6 Macintosh computers running OSX. The 6 Macintosh computers were used to run the intelligent tutors. The software was installed so that every other computer was running the experimental version of E-tutor and the control version of E-tutor was running on the remaining computers.

5.1.2 Subjects

The experiment was ready to run during summer break; therefore summer school programs for middle and high school students were approached for subjects. A math teacher for “Upward Bound”, a summer enrichment program for high school students that focuses on college preparation, was willing to allow the experiment with her Algebra I students. The students were participating in the Upward Bound program at Clark University in Worcester, Massachusetts.

The experiment was carried out with 15 Upward Bound students. The students would be entering their sophomore year in high school in the fall. Their math teacher required the students to participate in the testing for course credit.

5.1.3 Design

See Table 5.1 for a summary of the experimental design.

Independent Variables:

The Experimental Condition

The experimental condition was a version of E-tutor that was modeled after a human math tutor. The ITS engages the student in tutorial dialogs by asking new questions to which the student must respond before continuing (see Chapter 4).

The Control Condition

Rather than engaging the student in a tutorial dialog, the control condition allows the student to use hint sequences for help. The hint sequences become explicit until finally the last hint tells students exactly what they need to do for the next step in solving the equation (see Chapter 4).

Dependent Variables:

Student learning was measured by similar 8-item pre- and post-test (see Appendices C and D) that were given to students before and after using the ITS. The students were asked to solve equations that generally required transforming the problem by adding, subtracting, multiplying or dividing a term from both sides of the equation.²

| Time | Control Condition (7 students) | Experimental Condition (8 students) |
|--|-----------------------------------|--|
| 20 minutes | Paper and pencil pre-test | |
| 5 minutes | Demonstration of both systems | |
| Control: average 47.8 minutes Experimental: average 55.6 minutes (over 2 sittings) | Used E-tutor without dialog | Used E-tutor with dialog |
| 20 minutes | Paper and pencil post-test | |

Table 5.1 Experimental design

Students were assigned randomly to a computer as they walked into the computer lab. Eight students used the experimental E-tutor, and seven students used the control E-tutor. No students complained about being assigned to a particular group.

5.1.4 Procedure

The experiment took place over two sessions, over a two-day period, of approximately one and a half hours each. The students were given 20 minutes to

² The pre-test and the post-test were intended to be as similar as possible. Problems 1-6 matched closely in the pre- and post-test with the same number and type of operations required to solve each problem. However, problem 7 in the post-test required an extra transform operation compared to problem 7 in the pre-test. Problem 8 in the post-test required distribution, while problem 8 in the pre-test did not require distribution. Therefore, the post-test was slightly more difficult than the pre-test. See Appendix C and Appendix D to view the pre- and post-tests.

complete a pre-test before using the ITS, which appeared to be sufficient judging from students' reactions. Upon completing the pre-test, the students were given a 5 minute demonstration showing all students both systems and the students were directed to solve as many problems as they could in the first session. In the second session, students were directed to complete the rest of the problems in the E-tutor lesson if they had not done so in the first session. The experiment did not control for time but did control for the number of problems completed in the E-tutor lesson. When the students had completed all of the problems in the lesson, they were given 20 minutes to complete the post-test. Table 5.1 shows the experimental setup.

5.2 Results

Four students were excluded from the analysis; three students who got perfect scores on the pre-test were excluded because I concluded that these students had already mastered the material. One student who skipped four problems on the pre-test, but attempted all four on the post-test was excluded because it was felt that she had neglected the problems on the pre-test by accident (by not noticing the problems on the back of the paper). The analysis of the results was carried out with a very small sample size of 6 students in the experimental group and 5 students in the control group. Because of the small sample size, statistical significance was not obtainable in most of the analyses done in the following sections. It should be noted that with such small sample sizes, detecting statically significant effects is less likely. A large note of caution is also called for since using such small sample sizes does make our conclusions more sensitive to a single child, thus possibly skewing our results.

5.2.1 Overall learning

To confirm that there was learning in the experiment overall, a repeated measures ANOVA was carried out between the number of correct problems in the pre- and post-tests as a factor. The dependent variable being predicted was test score. This test showed a statistically significant overall learning ($F(1,10) = 5.213$, $P\text{-value} = .0455$). The analysis showed that overall there was over half a problem gained (pre-test = 4.9 correct, post-test = 5.6 correct).

Overall Learning Gain

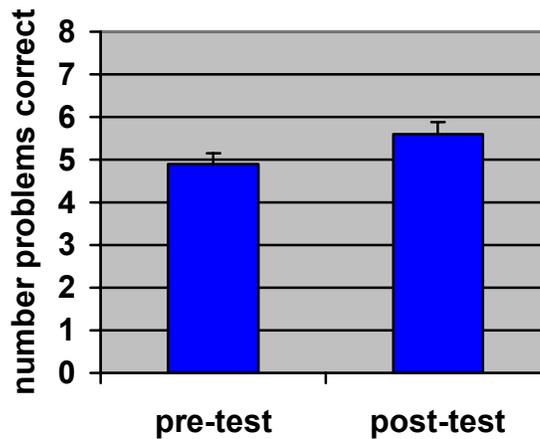


Figure 5.1 Overall learning gain

5.2.2 Learning gains by condition

To check for learning by condition, a repeated measure ANOVA was performed using experimental or control condition as a factor. The repeated measure of pre-test and post-test was a factor with prediction of test score as the dependent variable. At pre-test the experimental group outperformed the control group by an average of about 1.5 points; the difference was bordering on statistical significance ($F(1,9) = 3.9, p = 0.07$). This analysis showed that there was marginally statistically significant greater learning in the experimental condition than in the control condition ($F(1,9) = 2.3, p = 0.16$). This p-value of 0.16 indicates that there is a 16% chance the difference between scores between the two groups could have arisen by chance; so more experiments with larger numbers of students are definitely warranted to see if this effect holds up or is just a statistical fluke. The experimental condition had average pre-test score of 5.67 and post-test score of 6.67, showing a gain score of 1 problem. The control had average pre-test score of 4 problems correct and average post-test score of 4.2 problems correct. Taking the standard deviation into account, the effect size was a reasonable 0.4 standard deviations between the experimental and control conditions³, that is, an effect size for E-tutor with dialog over E-tutor without dialog.

³ The effect size was calculated by taking the difference between the average gain scores of the control and experimental conditions and dividing by the standard deviation of the control condition.

Learning Gains by Condition

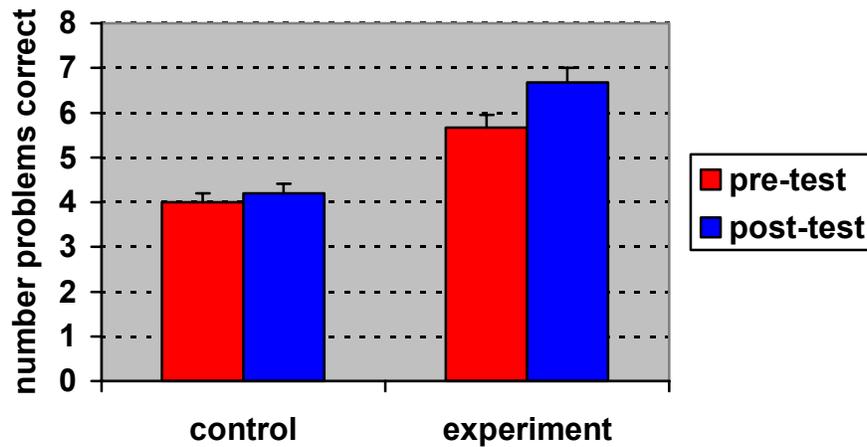


Figure 5.2 Learning gains by condition

5.2.3 Problem analysis

An analysis by problem was done to determine if there was statistically significant difference for students in the type of problem. A repeated measure ANOVA was used for these analyses with pre-test and post-test scores used as the repeated measure. An item analysis on students' mean performance on the 8 problems was performed, and a subject analysis was performed, as well.

The subject analysis showed that overall there was statistically significant learning ($F(1, 87) = 3.98$ and $p = 0.049$). However, although the experimental condition did perform better than the control condition, the repeated measure ANOVA did not reveal any statistically significant difference between the experimental condition and the control condition ($F(1, 86) = 0.38$ and $p = 0.54$). Similarly, when examined by item, learning by problem was not statistically significant overall ($F(1,7) = 1.26$, $p = 0.299$) or by condition ($F(1, 14) = 0.216$, $p = 0.65$).

An analysis was done for each problem in the pre- and post-tests. Problems 1 and 2 required one transform by adding or subtracting to solve the equation. Problem 3 required one transform by dividing to solve the equation. Problems 4 required one transform by multiplication to solve the equation. Problem 5 required 2 operations, one addition transform and one division transform. Problem six also required two transforms, one addition or division transform and one multiplication transform. Problems 7 and 8 required at least 4 operations each: distribution, two transforms by adding and/or subtracting, and one transform by division. Problems 7 and 8 in the post-test required one extra operation each when compared with the pre-test: distribution and combining like terms. Table 5.2 shows the problems for the pre-test and post-test.

| Problem | Pre-test | Post-test |
|---------|----------------|-----------------|
| 1 | $c + 32 = 124$ | $z - 13 = -4$ |
| 2 | $x - 7 = -23$ | $r + 30 = -132$ |
| 3 | $-12r = -132$ | $-7c = -112$ |
| 4 | $-k/6 = 12$ | $m/4 = -8$ |
| 5 | $8n - 19 = 37$ | $3c - 7 = 2$ |

| | | |
|---|-------------------------------|---------------------------------|
| 6 | $x/3 - 6 = -4$ | $-3t/2 = 33$ |
| 7 | $62 - 5x = 4(5x + 3)$ | $7z - 12 = 2 + 5(z - 2)$ |
| 8 | $3y + 5y - 22 = 15y + 30 - 3$ | $9(x - 3) + 10 = 10 - 26x + 8x$ |

Table 5.2 Pre-test and Post-test problems

Repeated measure ANOVA analyses were done by problem. These did not show any statistically significant differences between conditions; again the small sample size should be taken into account when considering these results. Although there were not statistically significant differences, overall the control condition had a greater number of incorrect problems in the post-test than the experimental condition. The experimental condition had a total of 15 incorrect problems in the pre-test and 8 incorrect problems in the post-test, whereas the control condition had a total of 20 incorrect problems in the pre-test and 19 incorrect problems in the post-test. See Table 5.3 for more details.

| Problem Numbers | Experimental condition (N = 6 students) | | | | | Control condition (N = 5 students) | | | | |
|-----------------|---|-------------------------|--------------------------|--------------------------|--------|------------------------------------|-------------------------|--------------------------|--------------------------|------|
| | # Incorrect in pre-test | % incorrect in pre-test | # Incorrect in post-test | % incorrect in post-test | Gain | # incorrect in pre-test | % incorrect in pre-test | # incorrect in post-test | % incorrect in post-test | Gain |
| 1 | 0 | 0% | 0 | 0% | 0% | 1 | 20% | 1 | 20% | 0% |
| 2 | 1 | 16.7% | 1 | 16.7% | 0% | 2 | 40% | 1 | 20% | 20% |
| 3 | 3 | 50% | 1 | 16.7% | 33.3% | 2 | 40% | 4 | 80% | -40% |
| 4 | 4 | 66.7% | 0 | 0% | 66.7% | 2 | 40% | 1 | 20% | 20% |
| 5 | 0 | 0% | 0 | 0% | 0% | 2 | 40% | 2 | 40% | 0% |
| 6 | 4 | 66.7% | 1 | 16.7% | 50% | 4 | 80% | 3 | 60% | 20% |
| 7 | 0 | 0% | 0 | 0% | 0% | 3 | 60% | 4 | 80% | -20% |
| 8 | 3 | 50% | 5 | 83.3% | -33.3% | 4 | 80% | 3 | 60% | 20% |
| totals | 15 | 31.3% | 8 | 16.7% | 14.6% | 20 | 50% | 19 | 47.5% | 2.5% |

Table 5.3 Problem Analysis

5.2.4 Error Analysis

An error analysis was carried out to determine any differences between errors that students made by condition. See Table 5.4 for more detail on the error analysis.

To ensure that the error coding was reliable, our error coding was compared to codings carried out independently by two math teachers. Skipped problems were obvious; however, other kinds of errors could be interpreted differently by different scorers. The math teachers were asked to code students' errors (not including skipped problems) according to the following criteria⁴:

1. Did the error happen at or during a transform?
2. Did the student do the transform correctly?
 - a. If no, what transform should have been done?
3. Was the error a sign error?
4. Was the error an additive or multiplicative error?
5. Did the student fail to simplify?
6. Was the error an order of operations error?
 - a. If yes, what operation should have been done first?
 - b. If yes, what operation did the student choose to do first?

⁴ The teachers were asked to look for other errors which are not reported here.

The correlation between the teachers' error codings was calculated. One of the math teachers' codings was adopted where correlation factors between them were over 0.7; this was chosen by alternating between the codings of the two teachers. Where correlation factors between the two teachers were under 0.7, the coding that was more closely correlated to the author's coding was used.

| Error coding | | Experimental condition (N = 6 students) | | Control condition (N = 5 students) | |
|---|---|--|------------------------------|---------------------------------------|--------------------------|
| | | # errors in pre-test | # errors in post- test | # errors in pre-test | # errors in post-test |
| Performed a transform step incorrectly | | 7 (14.6%) | 0 | 4 (10%) | 4 (10%) |
| | Correct transform was addition | 1 (2%) | 0 | 0 | 1 (2.5%) |
| | Correct transform was subtraction | 1 (2%) | 0 | 1 (2.5%) | 2 (5%) |
| | Correct transform was multiplication | 3 (6.25%) | 0 | 1 (2.5%) | 1 (2.5%) |
| | Correct transform was division | 2 (4.2%) | 0 | 2 (5%) | 0 |
| Sign errors | | 2 (4.2%) | 2 (4.2%) | 0 | 2 (5%) |
| Order of operations error | | 0 | 0 | 1 (2.5%) | 0 |
| skipped | | 5 (10.4%) | 2 (4.2%) | 14 (35%) | 12 (30%) |
| total | | 14 (29.2%) | 4 (8.3%) | 19 (47.5%) | 18 (45%) |

Table 5.4 Analysis of student errors

The error analysis did not show a statistically significant difference between the experimental and control condition. However, more errors were made in the post-test of the control condition than in the experimental condition. The experimental condition had 14 errors in the pre-test and 4 errors in the post-test, whereas the control condition had 19 errors in the pre-test and 18 errors in the post-test.

Students in the experimental condition did better on transformations in the post-test, going from 14.6% errors on the pre-test at a transformation to 0 on the post-test. However, the control condition stayed steady at 10% of this kind of error on both the pre-test and the post-test. Since most of the tutorial dialogs involved transforming an equation, this is a good indication that the dialogs were helpful to students.

5.2.5 A look at student log files

In this section, students' log files are compared when they worked on problems in E-tutor. E-tutor presented 25 equation-solving problems separated into 5 sections:

- Section #1: Combine like terms (e.g. $k=4+6*5$)
- Section #2: Transform by adding or subtracting (e.g. $x+4=6$)
- Section #3: Transform by multiplying or dividing (e.g. $12x=36$)

- Section #4: Need 2 transforms, add/subtract and multiply/divide (e.g. $6y-15=21$)
- Section #5: Equations with variables on both sides of the equation (e.g. $44k-3=17k+14$)

The experimental group did better than the control group while using E-tutor. This should be expected since the experimental group performed slightly better on the pre-test as well. Overall, the students had 39.4% correct problems, or problems with no errors, while using E-tutor. The experimental group had 45.6% correct problems, while the control group had 32% correct. Table 5.5 shows averages, overall and broken out by condition, for the number of errors per curriculum section. It is again noted that the control group usually makes more errors than the experimental group.

| | Average number of errors per problem | | |
|------------|--------------------------------------|-----------------------------------|--|
| | Overall (N = 11 students) | Control group (N = 5 students) | Experimental group (N = 6 students) |
| Section #1 | 1.27 | 1.8 | 0.83 |
| Section #2 | 1.3 | 1.7 | 0.94 |
| Section #3 | 1.43 | 1.87 | 1.06 |
| Section #4 | 0.93 | 1.05 | 0.83 |
| Section #5 | 1.9 | 1.88 | 1.95 |

Table 5.5 Average number of errors per problem, broken out by section

Table 5.6 reports the average number of times hints and buggy messages are used per problem by the control group versus the average number of times tutorial dialogs are used by the experimental group. The Add-Transform dialog was presented to students in the experimental group the most by E-tutor with dialog, 29% of the dialogs presented during the experiment. The Multiply-Transform dialog represented 22% of the dialogs used, while 19% of the dialogs presented were the Divide-Transform dialog. Combine-Like, Combine-Unlike and the Distribute Dialogs represented 16%, 11% and 3% of the dialogs used respectively. The percentages may represent the frequency of problems needing the types of skills discussed in the dialogs as well as the level of difficulty experienced by the students in these areas.

| | Average number of hints, buggy messages or dialogs per problem | | |
|------------|--|------|--|
| | Control group (N = 5 students) | | Experimental group (N = 6 students) |
| | Hints | Bugs | Dialogs |
| Section #1 | 2.3 | 0 | 2.58 |
| Section #2 | 3.8 | 0.4 | 2.16 |
| Section #3 | 4.07 | 0.33 | 3.23 |
| Section #4 | 1.95 | 0.2 | 2 |
| Section #5 | 3.9 | 0.6 | 3.7 |

Table 5.6 Average number of hints, buggy messages or dialogs per problem.

This thesis hypothesized that E-tutor with dialog would produce less “slumming” than an ITS which provided hint sequences. Slumming is defined as an instance where a

student goes straight to the bottom-out hint in a hint sequence to reach the instruction for the next step in a problem. Slumming can be detected by noting the time difference between hint requests. If the time difference between hint requests is less than 2 seconds, we surmise that the student was not reading each hint and trying to understand what to do next, but attempting to reach the bottom-out hint for the explicit instruction contained there. For E-tutor with dialog, it would be possible to slum by not answering the tutor's questions and pressing the OK button after every utterance. The student would eventually reach the last comment in the dialog that would give an explicit instruction.

During this experiment, slumming was observed several times. The control condition had 3 out of 5 students who slummed at least once during the experiment. The experimental group had 2 students out of 6 who slummed at least once during the experiment. The difference in slumming between conditions was not statistically significant and did not affect post-test scores in a statistically significant way, probably due to the small sample size.

Next, two students' performance on two specific problems are compared. These students had difficulty with problems containing fractions. Student A was in the control condition and ran into some trouble on two problems containing fractions. Student B, in the experimental condition, also had some trouble on these two problems. Although we do not know exactly what the students were thinking, some guesses are made in Tables 5.7 and 5.8. We can hypothesize about the differences between the control and experimental conditions by analyzing the log files as is done here.

| | Student A in control condition | Comments |
|-------------------------------------|---|---|
| Problem: $f/9=15$ | time to complete problem: 2 minutes, 46 seconds Problem: $f/9=15$ | |
| | 1. Student chose COMBINE on the left side for next step | The student chose actions that were incorrect for the problem. The tutor finds these actions to be un-interpretable and colors the equation red. |
| | 2. Student chose DIVIDE on the left side for next step | |
| | 3. Student asks for hint. | The student decides to ask for help. |
| | 4. Tutor says: What can you do to both sides to get f by itself? | The first hint is to state the goal in a rhetorical question. |
| | 5. Student chose DIVIDE on the right side for next step | Despite the first hint, the student chooses another incorrect action. |
| | 6. Student asks for hint. | |
| | 7. Tutor says: What can you do to both sides to get f by itself? | When the student requests another hint, she gets the first hint in the sequence again, which is to state the goal of isolating the variable. |
| | 8. Student asks for next level of hint. | |
| | 9. Tutor says: Change the $1/9$ to a 1. | The second hint suggests that the student figure out how to get rid of the fraction. |
| | 10. Student asks for "bottom-out" hint. | |
| | 11. Tutor says: Multiply both sides by 9. | The third and final hint, or the "bottom-out" hint, gives the student the exact action and input required for this step. Every hint sequence ends with an explicit instruction. |
| | 12. student chose Multiply both sides by... 9 for next step | The student uses the instruction in the last hint and solves the rest of the problem without assistance. |
| | 13. Problem: $9f/9=15*9$ | |
| | 14. Student chose MULTIPLY on the right side for next step | |
| 15. Problem: $9f/9=135$ | | |

| | | |
|--|--|--|
| | 16. Student chose DIVIDE on the left side for next step | |
| | 17. Problem: $f=135$ | |
| Problem: $z/8=22$ | time to complete problem: 1 minute, 42 seconds Problem: $z/8=22$ | 2 nd occurrence of problem of this type (involving a fraction and requiring a multiply transform) |
| | 1. Student chose DIVIDE on the left side for next step | Student makes same mistake as in line 2 of the previous problem. |
| | 2. Student asks for hint. | The student asks for all 3 hints in the sequence. The hints are the same ones offered in the previous problem. This may indicate that the student still does not understand how to solve the problem but knows that the last hint in the sequence would tell her what to do. |
| | 3. Tutor says: What can you do to both sides to get z by itself? | |
| | 4. Student asks for next level of hint. | |
| | 5. Tutor says: Change the $1/8$ to a 1. | |
| | 6. Student asks for "bottom-out" hint. | |
| | 7. Tutor says: Multiply both sides by 8. | |
| | 8. student chose Multiply both sides by... 8 for next step | The student uses the instruction in the last hint and solves the problem. |
| | 9. Problem: $8z/8=22*8$ | |
| | 10. Student chose DIVIDE on the left side for next step | |
| | 11. Problem: $z=22*8$ | |
| 12. Student chose MULTIPLY on the right side for next step | | |
| 13. Problem: $z=176$ | | |

Table 5.7 A portion of a student log file in the control condition

The first hint in Table 5.7 is to state the question Student A should answer ("What can you do to both sides to get f by itself?"). This is a rhetorical question that the student is not expected to answer. The second hint answers the question in the first hint by stating that the student needs to change the $1/9$ to a 1. Finally, the third hint bottoms out and gives the student an explicit direction in the statement: "Multiply both sides by 9." This thesis hypothesized that an ITS such as the Cognitive Tutor could be improved by replacing hint sequences and buggy messages with a dialog component. By asking new questions that the student must answer, the student is participating more actively in learning to solve the problem. The ITS should avoid giving explicit hints rather than requiring students to work out solutions on their own. In Table 5.8, Student B makes similar errors as Student A, but is engaged in tutorial dialog.

| | Student B in experimental condition | Comments |
|-------------------------------------|---|---|
| Problem: $f/9=15$ | Multiply-transform dialog Time to complete problem: 3 min 13 sec Problem: $f/9=15$ | |
| | 1. Student chose DIVIDE on the left side for next step | Student B makes the same error at Student A in line 2. |
| | 2. Tutor says: You have to get rid of the fraction to get f by itself on the left. So we need to find a way to cancel the $1/9$ on the left. Fractions are really division. What is the opposite of division? | The tutor starts a tutorial dialog that begins with clarifying the goal and asking the student what kind of operation is needed to transform the equation. |
| | 3. Student chooses: multiplication | The student chooses the correct inverse operation. |
| | 4. Tutor says: Right, the opposite of division is multiplication What should you multiply both sides by? | |
| | 5. Student types in: 15 | Although the student knew that the opposite of division is multiplication, he doesn't understand how multiplying both sides will help. |

| | | |
|-------------------------------------|---|---|
| | 6. Tutor says: No, $f/9$ is really f times $1/9$. you need to multiply both sides by a number that will cancel the $1/9$ on the left side. | The tutor begins explaining the goal and asks the student again for a value that will cancel the fraction. |
| | 7. Student types in: f | The student may be guessing here. |
| | 8. Tutor says: No, you need to multiply both sides by a number that will leave only $1 f$ on the left. What should you multiply $f/9$ by to cancel the $1/9$? | The tutor rewords line 6. |
| | 9. Student types in: 15 | |
| | 10. Tutor says: Well, here's how it works. To get rid of the denominator in a fraction you need to multiply the numerator by the same number, because $4/4$ is 1, $7/7$ is 1, $1394/1394$ is also 1. $1/9$ times what equals 1? | The tutor presents some real examples to help drive home the concept. |
| | 11. Student types in: 9 | The student seems to get it, or he could still be guessing. |
| | 12. Tutor says: Right, if you multiply both sides by 9 you get $1 f$ by itself on the left. You should go back to the problem and multiply both sides by 9. | The tutor gives some positive feedback and reiterates the goal and solution. |
| | 13. Tutor dialog done, student goes back to problem. | The student continues solving the problem without help. |
| | 14. student chose Multiply both sides by... 9 for next step | |
| | 15. Problem: $9f/9=15*9$ | |
| | 16. Student chose MULTIPLY on the right side for next step | |
| | 17. Problem: $9f/9=135$ | |
| | 18. Student chose DIVIDE on the left side for next step | |
| | 19. Problem: $f = 135$ | |
| Problem: $z/8=22$ | Multiply-transform dialog Time to complete problem: 1 minute, 48 seconds Problem: $z/8=22$ | 2nd occurrence of problem of this type (involving a fraction and requiring a multiply transform) |
| | 1. Student chose DIVIDE on the left side for next step | Student makes the same error as in the previous problem. |
| | 2. Tutor says: The opposite of division is multiplication, so you need to use multiplication to get rid of the fraction on the left side. What should you multiply $z/8$ by to cancel the $1/8$? | The tutor dialog begins to fade since the student has been through this type of dialog before. |
| | 3. Student types in: 8 | The student answers the new question posed in the dialog in the first try. In the previous problem, the student answered a similar question on the 4th try. |
| | 4. Tutor says: Yes, if you multiply both sides by 8 you get $1 z$ by itself on the left. You should go back to the problem and multiply both sides by 8. | The tutor gives positive feedback and reiterates the goal and solution. |
| | 5. Tutor dialog done, student goes back to problem. | The student finishes solving the problem without further trouble. |
| | 6. student chose Multiply both sides by... 8 for next step | |
| | 7. Problem: $8z/8=22*8$ | |
| | 8. Student chose DIVIDE on the left side for next step | |
| | 9. Problem: $z=22*8$ | |
| | 10. Student chose MULTIPLY on the right side for next step | |
| 11. Problem: $z=176$ | | |

Table 5.8 A portion of a student log file in the experimental condition

It is difficult to tell if Student B learned more than Student A because of the tutorial dialog in the experimental condition. However, Student B did do slightly better on problems with fractions (problems 4 and 6) in the post-test than Student A did. Student A skipped problems 4 and 6 in both the pre-test and the post-test. Student B

skipped problem 4 in the pre-test but solved it correctly in the post-test. However, Student B got problem 6 wrong on the pre-test and skipped it on the post-test. Student A did not seem to have the confidence to even try the two problems on the post-test, but Student B did improve. This looks promising, although Student B may have benefited from more practice on this type of problem.

5.3 Chapter Conclusions

An experiment was run to evaluate the effectiveness of E-tutor with dialog. The randomized controlled experiment was carried out with 15 high school sophomores who were randomly assigned to use E-tutor with dialog or E-tutor without dialog. The results are taken with a grain of salt because of the small sample size. After excluding 4 students from the study, there were only 6 students in the experimental condition and 5 students in the control condition. Also to be taken into consideration is that the experimental group was outperforming the control group at pre-test, so the results could be skewed.

Learning gains were measured by comparing scores on a pre-test and a post-test with a repeated measures ANOVA. Performance by problem was also examined and an error analysis was carried out. This chapter showed evidence that incorporating dialog in an equation-solving tutor seemed helpful to students. When controlling for number of problems, E-tutor with dialog performed better than E-tutor without dialog.

6 Conclusions and Future Work

This thesis studied the effect of adding tutorial dialog to an intelligent tutoring system for solving equations. The goal of this research was to implement an ITS for solving linear equations that incorporates dialog between the student and the ITS and to evaluate its effectiveness for algebra students. The ITS, named E-tutor, was used to tutor high school sophomores in solving linear equations.

6.1 Thesis Summary

There seems to be a recurring factor in research on human tutoring, namely, that an effective human tutor allows students to do as much work as possible on their own while providing just enough help to keep students from becoming confused or giving up. Collins et al., (1991) describes modeling, coaching and scaffolding as an appropriate model for teachers who need to teach students a complex task. Fading, a component of scaffolding, was described in Chapter 4 as a feature of E-tutor. McArthur et al. (1990) had some criticism for model-tracing tutors, which often give the same canned message to students when they make an error no matter how many times it occurs. As Ohlsson (1986) pointed out, two of the principles that should guide building ITS, teaching tactics and strategies, are, for the most part, undeveloped in tutoring systems. Previous work on ITS that attempt to behave more like a human tutor by incorporating capabilities to carry on dialog show promising results with statistically significant learning gains.

ITSs that model student thinking with model-tracing algorithms have been found to be effective (Anderson, Corbett, Koedinger & Pelletier, 1995). An ITS that uses model-tracing works out each problem step-by-step with the student and gives help when the student needs it. Several researchers have developed ITS that model the tutor as well, such as Graesser, Person & Harter, (2001) with AutoTutor, Heffernan (2000) with Ms. Lindquist and Rosé, Jordan, Ringenberg, Siler, VanLehn et al. (2001) with Atlas-Andes. This requires extra work in dialog planning and management. It also requires an understanding of what makes a human tutor effective and what strategies a tutor may use in a specific subject domain. Preliminary results in this research of dialog-based ITS show them to be effective in the domains studied.

An ITS similar to the “Cognitive Tutor” by Carnegie Learning, Inc. was used as a control condition in this research. The Cognitive Tutor can indicate to students when they have made a mistake and provide hint messages, as well as provide canned explanations to some errors. When a student keeps asking for help, the hint messages get more explicit until the tutor is telling the student exactly what to do. The Cognitive Tutor does not engage the student in dialog by asking them new questions.

This thesis proposed to make a better ITS by building and evaluating an intelligent tutoring system that is more effective because it incorporates the dialog elements from experienced human tutors. E-tutor does utilize the improvement of model-tracing tutors but also engages in coherent dialogs that ask the student new questions rather than simply provide canned hints. Students must answer these new questions correctly or exhaust a dialog before continuing with a problem. E-tutor does offer different dialogs on the same type of error, depending on the number of times the error occurs and dialogs do fade as the student progresses.

A human tutor and four middle school children were used to collect data on human tutoring. An hour and a half of tutoring in equation-solving was recorded and transcribed.

Examination of the transcriptions showed that the tutor used tactics that are common to tutors such as modeling, scaffolding and fading. Some of the tutor's strategies were identified and generalized.

E-tutor was designed to engage students in dialog by asking new questions of students and expecting them to answer the questions. The kinds of dialogs incorporated in E-tutor were intended to resemble a human tutor. Techniques within the dialogs included modeling, scaffolding, and fading. This differed from the control condition, which offered hints or buggy messages and did not expect students to respond to them. This chapter described all of the hints and buggy messages available to students in the control condition. The tutorial dialogs incorporated in the experimental condition were also covered.

An experiment was run to evaluate the effectiveness of E-tutor with dialog. The randomized controlled experiment was carried out with 15 high school sophomores who were randomly assigned to use E-tutor with dialog or E-tutor without dialog. Learning gains were measured by comparing scores on a pre-test and a post-test. The experiment showed evidence that suggested incorporating dialog in an equation-solving tutor is helpful to students. Although the sample size was very small, there were some results in the analyses that suggest that, when controlling for number of problems, E-tutor with dialog performed better than E-tutor without dialog with an effect size of 0.4 standard deviations for overall learning by condition.

6.2 Limitations and Future Work

There were some limitations in this research that may have affected the results of the experiment. E-tutor with dialog presented tutorial dialogs to students when they made certain errors. However, E-tutor without dialog depended on student initiative for the appearance of hints. That is, the students had to press the Hint button if they wanted a hint. Although students in the control group were told that they could request hints whenever they wanted, the results may have been confounded by this dependence on student initiative in the control group. We may also be skeptical about the results because the sample size was very small. Additionally, the experimental group performed better on the pre-test than the control group, so they were already better at solving equations than the control group.

In the future, an experiment could be run with a larger and more balanced sample of students. The confound with student initiative could be removed for a better evaluation of the two conditions. Another improvement would be to employ more tutorial strategies. A possible enhancement would be to allow the tutor to choose strategies dynamically according to students' learning styles or what types of errors they make. A question that could be answered with another experiment is whether tutorial dialogs are more motivating for students, or does dialog keep students' attention longer? Another experiment that controls for time rather than for the number of problems would examine whether E-tutor with dialog was worth the extra time.

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Appendix A – Human Tutor Transcripts

Sunday, December 8, 2002

Dialogs begin after students have attempted a 20 problem pre-test.

Student #1, 8th grade, pre-Algebra student

Problem: $__ - 13 = -4$

[Student skipped the problem]

1. Tutor: Something minus 13 is going to give me negative four, so how are we going to figure that out? So we're just gonna put a variable there, a number, an unknown, right? So let's just put "x". Rewrite this, it's easier to deal with an "x" than a blank right? [writes $x - 13 = -4$] x minus 13 equals negative 4. So we have to find out what x is, if we know what x is we know the answer. So our first step is to get all the numbers on one side and x by itself. So 4 is already on the other side, right? but we have to get rid of this number so x is by itself. Do you know what to do?
2. Student: no
3. Tutor: well you do the opposite operation, what's the opposite of negative 13?
4. Student: positive 13
5. Tutor: positive 13, [writes +13 on the left side of the equation] so we're gonna add 13 to both, in algebra what you do to one side you do to the other and it's the same with all of these. So the opposite of negative 13 is positive 13. So you're going to add 13 to both sides. So go ahead and add 13.
6. Student: so 13 and 4? [writes $-13 + 4$ on the right side of the equation]
7. Tutor: yeah
8. Student: so you subtract 4 from 13
9. Tutor: yeah so there's a reason why we do that. What is negative 13 and positive 13?
10. Student: ...zero
11. Tutor: yeah, zero. [crosses out -13 on the left side of the equation] So that cancels out, and mission accomplished. I wanted x by itself and I got x by itself. now we have minus 4 plus 13, or 13 minus 4, which is?
12. Student: 9
13. Tutor: so we have x equals 9. [writes $x = 9$] so try this, plug in 9 there to see if it works. Does 9 minus 13 give you negative 4?
14. Student: 13 minus 9?
15. Tutor: no, 9 minus 13. do you know how to add positive and negative numbers?
16. Student: no
17. Tutor: I'll show you. Here's 9 minus thirteen, the signs are different. 9 is really positive 9. when the signs are different we subtract. 13 minus 9 is what?
18. Student: 13 minus 9? 4
19. Tutor: okay now take the sign of the larger number. What's the larger number here?
20. Student: 13
21. Tutor: and what's the sign?
22. Student: positive
23. Tutor: what's the sign in front of the thirteen?
24. Student: negative
25. Tutor: negative, so put the negative in front of the answer.

Problem: $14 + \underline{\quad} = -32$.

[Student dropped the negative sign from 32 when subtracting on the right]

26. Tutor: Okay let's do another one. 14 plus blank is going to give me negative 32. So let's put an x instead of the blank. [writes $14 + x = -32$] We need to find out what x is. If we find x, we have the answer. So the trick is, we have to get x on one side and everything else on the other. So you have to get rid of that 14. Do you remember how to do that?

27. Student: subtract 14?

28. Tutor: yeah, subtract 14 from both sides. In algebra what you do to one side you do to the other, always, always, always.

29. Student: so negative 14 ... [writes $14 - 14 = 0$ on the left side and $-32 - 14$ on the right side]

30. Tutor: you're going to subtract 14 from this side and subtract 14 from that side. So it's negative 32 minus 14. How do we do that?

31. Student: add

32. Tutor: right

33. Student: 46 [writes 46 on the right side]

34. Tutor: right 46, but the sign of the larger number is?

35. Student: negative

36. Tutor: right

37. Student: so negative 46 [writes a negative sign for 46]

38. Tutor: so your next line... the trick with math is you really have to organize your thoughts, or even if you have the answer, it's going to be jumbled up and you won't see it. So next step, we cancelled the 14, so what's left on this side?

39. Student: zero

40. Tutor: only x right? so x equals?

41. Student: 46, negative 46

42. Tutor: right, negative 46, that's your answer. So why don't you write that down so it's clear? [Student writes $x = -46$]

Problem: $r+30=-132$

[Student made a sign error when subtracting on the right]

43. Tutor: Okay here's another problem. r plus 30 equals negative 132 so we have to find out what r is.

44. Student: subtract 30

45. Tutor: very good, why don't you do that over here?

46. Student: so do you add 132 or subtract?

47. Tutor: well we want to get r by itself, so what number are we trying to get rid of?

48. Student: 30

49. Tutor: so we need the opposite of 30

50. Student: minus 30

51. Tutor: right so subtract 30 from both sides.

52. Student: [writes $-132 - 30 =$] so do you add 30 and 132?

53. Tutor: yeah, the signs are the same, so we add and take the sign of the larger number, which is ...

54. Student: negative

55. Tutor: now if they're both negative, why are we adding them?

56. Student: ...
57. Tutor: because the signs are the same. That's why we're adding them. So r is what?
58. Student: negative 162 [writes -162]
59. Tutor: right. with some of these problems, you got them right but you never said the final answer. So at the end of each of these problems you should put r is, or x is whatever. Just put it step by step so it's organized if you ever want to recheck your work. So why don't you write what r equals here? [Student writes $r = -162$] Okay, you got that right and you got that right too, but you should put c equals 16. what's this one?

Problem: $-k/6=12$ **[Student skipped problem]**

60. Tutor: Negative k over 6 equals twelve. A lot of us are fraction-phobic, we hate fractions right? poor fractions. So I want to get rid of the fraction so I don't have to deal with it. So I want to get rid of the fraction and fractions are really division. So what's the opposite of division?
61. Student: multiplication
62. Tutor: right. Multiplication cancels out division right? Is there a number I can multiply by that will get rid of the fraction?
63. Student: ...
64. Tutor: ok, let me ask you a question. If I had one fifth and I wanted to multiply by a number that's going to cancel the 5. [writes $1/5$] What's the smallest number you can think of?
65. Student: 5
66. Tutor: [writes 5 and crosses them out, then writes $1/4$] if I had one fourth?
67. Student: 4
68. Tutor: [writes 4, then writes $1/9$] if I had one ninth?
69. Student: 9
70. Tutor: [writes 9] 9. So here is k over -6 . What number should we multiply by?
71. Student: 6
72. Tutor: 6 or negative 6?
73. Student: negative 6
74. Tutor: right. [writes (-6) on the left side] Now in algebra, what I do to one side of the equation, I have to ...
75. Student: do to the other side
76. Tutor: right, it has to balance out. So if I multiply -6 on one side ...
77. Student: ...
78. Tutor: we multiplied this side by -6 to get rid of the fraction right? But what I do to one side I have to do to the other. Make that an automatic thing. So what do you need to do?
79. Student: twelve times negative six [writes -6 on the right side]
80. Tutor: right. So -6 cancels out and k equals?
81. Student: multiply 6 by 12?
82. Tutor: yeah. We'll think about the negative sign later.
83. Student: [writes $12 * 6 =$, then 72] ... 72
84. Tutor: right. But the sign is?
85. Student: negative [writes $-$ for 72]
86. Tutor: so what is k?
87. Student: -72 [writes $k = -72$]

88. Tutor: Right. See, we got rid of the fraction in the first step. It's key to do that.

Problem: $-64=16y$

[Student doesn't complete problem by writing last step ($y = -4$)]

89. Tutor: alright let's do the next one. Negative 64 equals 16y what do we want to figure out here?

90. Student: what y equals?

91. Tutor: right, so what do we need to do?

92. Student: divide 64 by 16

93. Tutor: right, we have to get rid of that 16 so it's going to cancel out. Go ahead and do that.

94. Student: [writes $64/16$] y equals negative 4 [writes $y = -4$]

95. Tutor: right. Oh you got the right but you didn't finish. Here's another fraction. Let's get rid of the fraction. What do you think we need to do?

Problem: $m/4=-8$

[Student skipped problem]

96. Student: this is gonna be 4?

97. Tutor: yeah, multiply by 4. instead of doing it all in your head, write it down. Sometimes it's easier.

98. Student: [writes $4 * 4 = 16$]

99. Tutor: okay you're trying to multiply this by four, so you're just left with m, right?

100. Student: right

101. Tutor: and now you multiply 4 by negative 8.

102. Student: 32, negative 32

103. Tutor: right so what's the answer?

104. Student: [writes $m = -32$]

Problem: $8n-19=37$

[Student skipped problem]

105. Tutor: very good. Let's see, okay let's do the next one. You're on the right track here. The first step is we have to get what? We're solving for n here and we have to get n by itself right?

106. Student: yeah

107. Tutor: so what's the first thing you have to do?

108. Student: add 37

109. Tutor: well the 37 is already by itself so we don't have to worry about the 37. there's something else we can get rid of.

110. Student: 19?

111. Tutor: mmhmm

112. Student: ...

113. Tutor: how do we get rid of the 19?

114. Student: with negative 19

115. Tutor: well that's already negative, so what's the opposite of negative?

116. Student: positive

117. Tutor: so we're adding 19

118. Student: [writes $37 + 19 = 56$] 37 plus 19 is 56
 119. Tutor: now we have $8n$ equals what?
 120. Student: 56
 121. Tutor: okay write that down
 122. Student: [writes $8n = 56$]
 123. Tutor: okay we're almost there. I don't really care what $8n$ is, I want to know what n is. So what do I need to do to find out what n is?
 124. Student: 56 divided by 8
 125. Tutor: very good
 126. Student: [write $56/8 = 7$] 7
 127. Tutor: right, so what's the answer?
 128. Student: [writes $n = 7$]

Problem: $x/3 - 6 = -4$

[Student skipped problem]

129. Tutor: okay next one. X over 3 minus 6 equals negative 4. where's the fraction here?
 130. Student: [points to $x/3$]
 131. Tutor: what do we need to multiply by?
 132. Student: 3
 133. Tutor: right
 134. Student: so...
 135. Tutor: multiply everything by 3, that means this by 3, that by 3 and that by 3.
 136. Student: ...
 137. Tutor: rewrite the equation after you do that. What are you left with?
 138. Student: [writes $x/3 * 3$ and crosses out the 3s] x . and multiply this [points to -6] by 3?
 139. Tutor: everything
 140. Student: [writes $x - 18 = -12$]
 141. Tutor: okay now we have to get x by itself. What do we do?
 146. Student: get rid of the 18.
 147. Tutor: get rid of the 18, very good.
 148. Student: so minus 18
 149. Tutor: there's already a minus there.
 150. Student: addition
 151. Tutor: very good
 152. Student: so 18 plus negative?
 153. Tutor: negative 12 plus 18. because you're adding 18 here, you're gonna add 18 here.
 154. Student: okay
 156. Tutor: signs are opposite so am I adding or subtracting?
 157. Student: subtracting. [writes $18 - 12 = 6$] 18 minus 12 is 6
 158. Tutor: what's the sign of the larger number?
 159. Student: positive
 160. Tutor: so what's the answer?
 161. Student: positive 6 [writes $x = 6$]
 162. Tutor: very good. When you're adding positive and negative numbers, I've noticed you're doing the right thing but when it comes to adding positive and negative numbers you make mistakes. Here's the trick, if the signs are different you subtract, and

take the sign of larger number. If they signs are the same, add and take the sign of the larger number. Maybe that's something that will help you. Okay let's do another one.

Problem: $-3t/2 = 27$

[Student skipped problem]

167. Tutor: well we want to get rid of the fraction and there's a 2 in the denominator. So...

168. Student: multiply by 2

169. Tutor: right

170. Student: so 3 times 2

171. Tutor: the whole thing, $3t$ over 2 times 2, it will cancel

172. Student: multiply everything by 2?

173. Tutor: everything by 2, [writes $(2) \cdot 3t/2$] so what happens to this side?

174. Student: get $6t$

175. Tutor: or we can just cancel these 2's out, what are we left with on this side?

176. Student: [crosses out the 2s] $3t$

177. Tutor: $3t$, negative $3t$, maybe we should have multiplied by negative 2 so the negatives cancel out. [Student writes a negative sign for the 2] Alright so we're left with $3t$ and multiply by negative 2 on that side.

178. Student: [writes $27 * -2 = -54$] negative 54

179. Tutor: okay now solve for t , I don't want to know what $3t$ is, I want to know what t is. So what's my last step?

180. Student: 27 divided by 3?

181. Tutor: No, $3t$ equals negative 54, why don't you write that down. [student writes $3t = -54$] divide both sides by 3 right? here let me show you something on the side. If I had [writes $7/7$], what is that?

182. Student: 7 divided by 7.

183. Tutor: which is?

184. Student: one

185. Tutor: [writes $900/900$] what is $900/900$?

186. Student: one

187. Tutor: [writes $3t/3$] so if I had $3t$ over 3. what do I have? That cancels out [crosses out the 3s] and I'm just left with, what am I left with all by itself?

188. Student: t

189. Tutor: that's why we've been doing that. [writes $2x = 8$] Like if we had $2x$ equals 8, and you want to solve for x . What do you divide it by?

190. Student: 2

191. Tutor: [divides by 2 on the left side] 2. because 2 over 2 is one and you're left with just the x . and what you do to one side, you have to do to the other. [divides by 2 on the right side] So if I have $3t$ here and divide by 3, what happens?

192. Student: one

193. Tutor: so there's a one here and what am I left with?

194. Student: t

195. Tutor: t . and what you do to one side, you have to do to the other, right?

196. Student: so I divide this by 3?

197. Tutor: right

198. Student: [writes $54/3 = 18$] 18

199. Tutor: good, but 54 was negative so what's the answer going to be?

200. Student: negative [writes a negative sign for 18, then $t = -18$]

201. Tutor: very good. Let's go back for a second. You agree with me that 7 over 7 is one and three over three is one and anything over itself is one. You get confused when there is a t or x there. Would you agree with me that 1t is really just t and the one is understood?

202. Student: no

203. Tutor: no. when we see a t all by itself that's just like saying 1t. so when we cancel that out we're left with 1, 1t is really t. In math, the one is understood. 1x is really x, 1y is really y. when we cancel the 3 out and we're left with one, that one doesn't just disappear. The one is really there, we just don't say 1t, we just say t.

Problem: $(2a-5)6=15a$

[Student tried guessing that a was 3, then quit]

204. Tutor: there's something we have to do before we can solve for a. what do you think that is?

205. Student: do what's inside the parentheses.

206. Tutor: yeah, but they're not like terms. This is a 2a, so we can't really do anything there. "Please excuse my dear Aunt Sally", have you learned that?

207. Student: yes

208. Tutor: what does the p stand for?

209. Student: parentheses

210. Tutor: yes, we have to get rid of the parentheses. How do we do that?

211. Student: ...

212. Tutor: there's something outside the parentheses. What it is it?

213. Student: 6

214. Tutor: yeah what does that mean when there's a 6 outside the parentheses?

215. Student: multiply

216. Tutor: multiply by what?

217. Student: 5

218. Tutor: and by what?

219. Student: 2a

220. Tutor: right you're actually, it's called distribution right? You're distributing to the 5 and you're distributing to the 2a.

221. Student: so you multiply 6 by 5 and 6 by 2a

222. Tutor: right and it doesn't matter which one you do first.

223. Student: ...

224. Tutor: there's 6 times 2a. 6 times 2 is 12, 6 times 2a is ...

225. Student: 12a [writes $12a - 30$]

226. Tutor: so 12a minus 30 equals what?

227. Student: [writes $= 15a$]

228. Tutor: so we got rid of the parentheses. Now we want to get a by itself. we want to get all the a's together and all the numbers together. So we have to do some switching, some adding and subtracting. So what do you want to do first?

229. Student: I don't know

230. Tutor: let's see, do you want to get rid of the 30?

231. Student: ok

232. Tutor: we have minus 30, what's the opposite of minus 30?

233. Student: positive

234. Tutor: so what are we doing?

235. Student: subtracting

236. Tutor: no we're adding, what number are we adding?
237. Student: 30. here?
238. Tutor: to both sides
239. Student: that's zero [crosses out -30 on the left side]
240. Tutor: right. they cancel out. Don't forget to add 30 to this side. Whatever you do to one side you have to do to the other.
241. Student: [writes $+30$ on the right side]
242. Tutor: now can we add those? Are they like terms?
243. Student: ...
244. Tutor: does 30 have an a?
245. Student: no
246. Tutor: does 15 have an a?
247. Student: yes
248. Tutor: so we can't do anything with them, they're not like terms. Do you know what like terms means?
249. Student: yes
250. Tutor: ok, now we have to write what we have so far. We got rid of the 30.
251. Student: [writes $12a=15a+30$]
252. Tutor: ok, this is what we have so far. You should always write it horizontally so it's easier to know what to move around. We're almost done. We still need to get all the numbers on one side and all of the a's together. So we have to get the $12a$ and $15a$ together.
253. Student: add 15 to 12
254. Tutor: yes but I need to get the $15a$ on this side of the equal sign. How do we do that?
255. Student: I don't know
256. Tutor: well we just do what we did before. $15a$, is that positive or negative?
257. Student: positive
258. Tutor: what's the opposite of positive?
259. Student: negative
260. Tutor: right, I'm going to subtract $15a$ from both sides. [writes $-15a$ on the right side] What's $15a$ minus $15a$?
261. Student: zero
262. Tutor: [crosses out $15a$ on the right, writes $-15a$ on the left] right. now I have $12a$ minus $15a$. what is $12a$ minus $15a$?
263. Student: 3?
264. Tutor: [writes 3] 3. positive or negative 3?
265. Student: negative
266. Tutor: negative. [writes a negative sign for the 3] Why?
267. Student: because 15 is bigger
268. Tutor: good. Negative 3 or negative $3a$?
269. Student: $3a$
270. Tutor: [writes an a next to the 3] minus $3a$ equals? There's something here that we can't forget.
271. Student: 30
272. Tutor: [writes $= 30$] negative $3a$ equals 30. ok we're almost done. We don't care what negative $3a$ is, we want to know what a is. So we have to get rid of that. How do we do that?
- Student: 30 divided by ...
273. Tutor: by what?

274. Student: 3
 275. Tutor: by 3 or ... you're close
 276. Student: mmm
 277. Tutor: this poor negative sign is neglected. By 3?
 278. Student: by negative 3
 279. Tutor: by negative 3. write that down
 280. Student: so it would be negative 10
 281. Tutor: very good. Write in your last line horizontally, a ...
 282. Student: [writes $a = -10$]
 283. Tutor: so a is negative 10, very good.

Student #2, 8th grade, algebra I student

Problem: $m/4 = -8$

[Student skipped problem]

284. Tutor: so problem set 1 they're all correct. Problem set 2, let's work on number 3. here we weren't sure, was that a negative sign?
 285. Student: yeah [darkens negative sign]
 286. Tutor: ok, you just have to be clear. Problem set 2, number 5 you didn't do. Probably because it was a fraction and you didn't want to deal with it.
 287. Student: yeah
 289. Tutor: fractions are lonely people, nobody likes them. Well we know how to get rid of fractions right? what am I multiplying by?
 290. Student: by negative 4?
 291. Tutor: well look, a fraction is really division, so what's the opposite of division?
 292. Student: multiplying
 293. Tutor: multiplying. So when you do an opposite operation they cancel out. So what's there?
 294. Student: 4
 295. Tutor: 4. positive or negative?
 296. Student: positive
 297. Tutor: positive, so what should I multiply by?
 298. Student: negative 4
 299. Tutor: no positive.
 300. Student: oh wait, isn't it positive?
 301. Tutor: don't look at the sign, the sign stays the same. The opposite of division is multiplication. You'll deal with positive and negative numbers in another problem. [writes $1/4(4)$] Look, if I had 1 over 4 and I times it by 4, what's going to happen?
 302. Student: um, it's going to turn into a whole number. It's going to turn into 1.
 303. Tutor: 1, because it cancels out right? [crosses out the 4s, writes $x/4(4)$] If I had x over 4 times 4, what happens?
 304. Student: [crosses out the 4s, writes x] it cancels out and there's just x.
 305. Tutor: is there a fraction anymore?
 306. Student: no
 307. Tutor: [writes $7/6$] and if I had 7 over 6 and I want a whole number what should I multiply by?
 308. Student: [writes 6] 6
 309. Tutor: and what am I left with?
 310. Student: [crosses out the 6s] 7
 311. Tutor: so use that knowledge that you have and ...

312. Student: [writes $m/4(4)$, crosses out the 4s]
 313. Tutor: but what you do to one side, it's like a scale you have to balance, what you do to one side of the equal sign you have to do to the other. So what did we just do? We multiplied by?
 314. Student: 4
 315. Tutor: so we did that to one side we have to do it to the other. So what else?
 316. Student: multiply by 4?
 317. Tutor: yeah
 318. Student: [writes $m = -32$] negative 32
 319. Tutor: and that's the answer, now how long did that take you?
 320. Student: a minute

Problem: $(2a - 5)6 = 15a$

[Student made a sign error]

321. Tutor: did you do please excuse my dear aunt sally?
 322. Student: no
 323. Tutor: do you know what I'm talking about, order of operations?
 324. Student: yeah
 325. Tutor: so please, what does p stand for?
 326. Student: parentheses
 327. Tutor: let's get rid of the parentheses. How do I get rid of them?
 328. Student: multiply everything by 5. oh are you talking about this one or this one?
 329. Tutor: this one
 330. Student: okay, multiply everything by 6. [writes $12a$] ... is that negative?
 331. Tutor: yeah
 332. Student: [writes $-30 = 15a$]
 333. Tutor: ok, next step, we have to get the a's together and the numbers together. So what do you want to do?
 334. Student: um switch the sides?
 335. Tutor: ok ... so what happens when you switch sides?
 336. Student: um you have to change the signs.
 337. Tutor: good.
 338. Student: [writes $30 = 15a - 12a$]
 339. Tutor: was 30 negative or positive?
 340. Student: positive
 341. Tutor: was it?
 342. Student: well there were 2 negatives so I changed it to positive.
 343. Tutor: why were there 2 negatives?
 344. Student: because minus negative 30 so I changed it to positive
 345. Tutor: you had $12a$ minus 30, because that's what we had here. But you didn't move 30 to the other side so why did you minus a minus?
 346. Student: I don't know. My teacher said when you have 2 negatives ...
 347. Tutor: she's right, but there is no minus negative, it's just minus 30
 348. Student: oh [adds a negative sign to 30]
 349. Tutor: so negative 30 equals whatever that will be.
 350. Student: [writes $-3a = 30$, then $a = -10$]

Problem: $7z = 2 + 5(z - 2)$

[Student added 2 to 5z and got 7z]

351. Tutor: first thing we should do is get rid of the parentheses. Are there parentheses on this side?
352. Student: yes
353. Tutor: go ahead and distribute
354. Student: [writes $7z = 2 + 5z - 10$]
355. Tutor: alright we want to get all the numbers on one side and all the z's on the other.
356. Student: oh wait can I ...
357. Tutor: do it anyway you want because there might be 10 ways to do it.
358. Student: [writes $7z - 5z = -12$, then $2z = -12$, then $z = -6$] *student made a mistake when combining 2 and -10.*
359. Tutor: alright now, let's see if we're right. z equals negative 6, if I plug in negative 6 wherever there's a z, this should work out. [writes $7(-6) = 2(-6 - 2)$] 7 times negative 6 equals 2 plus 5 times negative 6 minus 2. so what's 7 times negative 6.
360. Student: negative 42
361. Tutor: [writes -42] negative 42 should equal this and that's how we'll know you're right. what is negative 6 minus 2? Signs are the same, so we're adding.
362. Student: negative 4
363. Tutor: Signs are the same, so we're adding.
364. Student: oh. 8
365. Tutor: take the sign of the larger number
366. Student: negative 8
367. Tutor: Negative 8 times 2 ... we did something wrong here. I think you tried to do 2 steps at once and you missed something along the way. Let's see here, [writes $7z = 2 + 5z - 10$] $7z = 2 + 5z - 10$, you had that. What was the next thing you did?
368. Student: added 2 to 10
369. Tutor: 2 minus 10 is what? Signs are different so were just subtracting. What's 10 minus 2?
370. Student: 8
371. Tutor: what's the sign of the larger number?
372. Student: negative 8 [writes $7z = 5z - 8$, then $2z = -8$, then $z = -4$]
373. Tutor: okay let's see if it works. If I plug in negative 4, this equation this statement is going to be true. [writes $7(-4) = 2 + 5(-4 - 2)$] Negative 28 equals 2 plus 5 times, what's minus 4 minus 2?
374. Student: negative 2
375. Tutor: no, the signs are the same so we add.
376. Student: negative 6
377. Tutor: [writes $-28 = 2 + 5(-6)$] okay negative 28 equals negative 30 plus 2 which is negative 28, [writes $-28 = -28$] so you were right.

Student #3, 7th grade, pre-Algebra student

Problem: $14 + \underline{\quad} = -32$

[Student made a sign error by doing $32 - 14$ instead of $-32 - 14$]

378. Tutor: so we have 14 plus blank equals negative 32. maybe we should put a variable there, an x there. Maybe that will be easier to figure out in our head what the answer's going to be. So pick a letter
379. Student: l

380. Tutor: [writes $14 + l = -32$] okay 14 plus l equals negative 32. so if we figure out what l is we'll have the answer. So in algebra we want to get l on one side and everything else on the other side. So our goal is to get l all by itself
381. Student: okay
382. Tutor: so I'm going to try to get rid of this 14. [writes -14 on left side] what's 14 minus 14?
383. Student: zero
384. Tutor: Well, there you go, that cancels out right? But in algebra what you do to one side you have to do to the other. So if I subtract 14 here, I have to subtract 14 there. [writes -14 on the right side] So 14 minus 14 cancels and we're left with l equals whatever this comes out to be.
385. Student: so if there's 2 negative numbers it's positive right?
386. Tutor: no that's only when you multiply. Here we're just adding them together. When you multiply or divide two negatives make a positive, not when you're adding or subtracting. When you're adding or subtracting, they are what they are. You have negative 32 minus 14, the signs are the same so I want you to add them.
387. Student: just add them?
388. Tutor: just add them
389. Student: 46
390. Tutor: [writes $l = 46$] 46, what's the sign of the larger number?
391. Student: negative
392. Tutor: [adds negative sign] so you're answer's going to be negative. That's your answer.

Problem: $r + 30 = -132$

[Student made a sign error by doing $132 - 30$ instead of $-132 - 30$]

393. Tutor: Okay, let's do number 5. I want to get r by itself. when I get r by itself, I'll have my answer. So I have to get rid of that 30. what do I do to make that a zero?
394. Student: minus 30
395. Tutor: very good
396. Student: and then you do it over here
397. Tutor: right, what you do to one side you have to do to the other.
398. Student: you minus or add it?
399. Tutor: no you minus. If I minus 30 on one side, I minus 30 on the other.
400. Student: ...
401. Tutor: [writes -30 on right side] so I have minus 132 here minus 30, the signs are the same, so what operation am I doing?
402. Student: um
403. Tutor: I'm adding, the signs are the same so I'm adding
404. Student: oh so you're adding
405. Tutor: well you have minus 132 minus 30. what do you do when you have 2 negative numbers? The trick is just add them and take the sign of the larger number.
406. Student: ok. 162 and negative
407. Tutor: so what's r ?
408. Student: um ...
409. Tutor: you just said it
410. Student: 162
411. Tutor: right, so write it down. [Student writes $r = -162$] Now let me tell you why we did that. When you're adding negative and positive numbers, now I'm not talking

about multiplying and dividing, if the signs are the same we add them, take the sign of the larger number. If the signs are different, subtract and take the sign of the larger number. For example, [writes $-5 - 7$] if I had minus 5 minus 7, the signs are the same so what am I doing?

412. Student: you add them

413. Tutor: add them, take the sign of the larger number. What's 7 plus 5?

414. Student: 13, no 12

415. Tutor: [writes $= -12$] right. If 7 is positive and 5 is positive, the signs are the same so we add them. The same works for negative numbers. Let's do another one, [writes $-6 - 2 =$] what is negative 6 minus 2? Signs are the same ...

416. Student: how are the signs the same? When --

417. Tutor: well the sign in front of the number

418. Student: oh, negative 4

419. Tutor: no we're adding

420. Student: oh yeah, 8

421. Tutor: and the sign is

422. Student: negative

423. Tutor: [writes -8] here, same sign we add, different sign we subtract. [writes $-10 + 8 =$] So negative 10 + 8. the signs are different so we

424. Student: subtract

425. Tutor: so 10 minus 8 is

426. Student: 2

427. Tutor: sign of the larger number?

428. Student: negative 2 [tutor writes -2]

Problem: $-7c = -112$

[Student subtracted 7 from both sides instead of dividing]

429. Tutor: Okay, let's do this one. I really don't care what negative 7c is, I want to know what c is. If I find c then I have my answer. Negative 7 is really multiplied by c right?

430. Student: yeah

431. Tutor: how would I cancel that out?

432. Student: subtract it?

433. Tutor: well no, because it's not being added, it's being multiplied by. What's the opposite of multiplication?

434. Student: division

435. Tutor: division, so I'm going to divide both sides by negative 7. [divides by -7 on both sides] negative 7 cancels out and I'm left with?

436. Student: c

437. Tutor: [crosses out 7s on left side] c, what's negative divided by negative?

438. Student: negative

439. Tutor: no now we're dividing

440. Student: oh yeah positive

441. Tutor: so c equals 112 over 7

442. Student: that's the answer?

443. Tutor: that's the answer. That's it. But do you think we can reduce that fraction? Because that's an improper fraction, does 7 go into 112 evenly? Let's find out.

444. Student: [writes $112/7 = 16$] so the answer is 16?

445. Tutor: yep that's the answer.

Problem: $8n - 19 = 37$
[Student skipped problem]

446. Tutor: so we want to get $8n$ by itself. 37 is already by itself, but there's something else I want to get rid of.
447. Student: so you add?
448. Tutor: add what?
449. Student: 8 and 37 ?
450. Tutor: no
451. Student: wait ...
452. Tutor: ultimately I want to get everything with n on one side, everything that doesn't have an n next to it on the other side. 8 has an n next to it, how about that? [points to the 19]
453. Student: no
454. Tutor: so I have to get rid of it, I have to put it on the other side. What do I do?
455. Student: subtract this and this? [points to $8n - 19$]
456. Tutor: no those are not like terms. Do you know what like terms are?
457. Student: no
458. Tutor: that means the only way you can subtract anything from $8n$ is if that had an n too. It's like saying 5 apples minus 4 apples is 1 apple, or 6 apples minus 2 apples is 4 apples. But if I said what is 5 apples minus 2 oranges, can I do that?
459. Student: no
460. Tutor: no, because they're not like terms, one's apples and one's oranges. So I can't subtract this 19 from $8n$ because n is a variable, it's like this is apples and this is oranges.
461. Student: so you add them?
462. Tutor: no. I just want to get this $8n$ by itself, so I need to cancel the 19 . how do I make it so this is zero?
463. Student: you subtract it from itself.
464. Tutor: yeah so what am I going to do?
465. Student: minus 19
466. Tutor: not minus, what's the opposite of subtraction?
467. Student: addition
468. Tutor: yeah, I'm going to add 19 . [writes $+19$ on left] what's 19 minus 19 ?
469. Student: zero
470. Tutor: [crosses out the 19 s on the left] right, but I'm not finished. What I do to one side --
471. Student: you have to do to the other.
472. Tutor: right, so I'm going to add 19 there. [writes $+19$ on the right]
473. Student: [writes $37 + 19 = 56$]
474. Tutor: we're almost done. I don't care what $8n$ is, I want to know what n is. That 8 is in the way, so what do I do?
475. Student: you minus?
476. Tutor: well no, $8n$ is really one number. It's 8 times n , what's the opposite of multiplication?
477. Student: division
478. Tutor: so I'm going to divide both sides by what number?
479. Student: 8
480. Tutor: right

481. Student: so I divide 56 by 8?
 482. Tutor: well both sides. $8n$ divided by 8 is going to give me --
 483. Student: [writes $8n/8 = 56/8$, crosses out the 8s on the left] n
 484. Tutor: okay write that down, n equals
 485. Student: 56 divided by 8, wait I know this, 7 [writes $n = 7$]
 486. Tutor: right you're done, n equals 7

**Problem: $(2a - 5)6 = 15a$
 [Student skipped problem]**

487. Tutor: have you done please excuse my dear aunt sally?
 488. Student: no
 489. Tutor: it's parentheses, exponents, multiplication, division, addition, subtraction. Remember that, that's order of operations. In other words, when you're doing a math problem, there's a certain order that you do things. The first thing you have to do is get rid of the --
 490. Student: parentheses
 491. Tutor: parentheses, which we have here. How do I get rid of parentheses?
 492. Student: um
 493. Tutor: this 6 on the outside of the parentheses means I'm distributing it. In other words, in plain English, I multiply 6 by this and I multiply 6 by that. [draws arrows from the 6 to 2a and then to 5] I multiply 6 by everything in there. Let's do that step by step. What 6 times negative 5?
 494. Student: um wait
 495. Tutor: well what's 6 times 2a?
 496. Student: 12
 497. Tutor: 12 what?
 498. Student: 12a
 499. Tutor: [writes $12a -$] 12a minus, what's 6 times 5?
 500. Student: 30
 501. Tutor: [writes $-30 = 15a$] okay 12a minus 30, we got rid of the parentheses, equals 15a.
 502. Student: so that's the answer?
 503. Tutor: no, I want to know what the answer is, what is a? we need to figure it out. There's a few more steps left. I want to get all the a's on one side of the equation and everything without an a on the other side. I don't have that yet. The 15a is here, the 12a is there, the 30 is in the middle, it's all mixed up. I have to organize it now. Get all the a's on one side of the equal sign and everything without an a on the other side.
 504. Student: ok
 505. Tutor: there are many ways we can do this, it's up to you. How do you want to tackle this?
 506. Student: [inaudible]
 507. Tutor: so you want to move the 12a over there?
 508. Student: yeah
 509. Tutor: how would I cancel that out so that it's zero?
 510. Student: subtract by itself?
 511. Tutor: yeah, exactly subtract it by itself. [writes -12 on the left] so 12a minus 12a
 512. Student: is zero
 513. Tutor: [crosses out 12a] right you're subtracting on this side, it's just zero. But you have to subtract it on the --

514. Student: other side.
515. Tutor: good. So you're just left with what on this side?
516. Student: 30
517. Tutor: negative 30, the subtraction sign and negative are the same
518. Student: oh
519. Tutor: [writes $-12a$ on the right] okay and what's $15a$ minus $12a$? what's $15a$ minus $12a$ apples?
520. Student: 13
521. Tutor: 15 minus 12
522. Student: oh, 3
523. Tutor: and $15a$ minus $12a$?
524. Student: $3a$?
525. Tutor: yeah, so negative 30 equals?
526. Student: um
527. Tutor: you just did it.
528. Student: $3a$?
529. Tutor: yeah, well you cancelled this out right? so you have negative 30 equals, every time you do a step rewrite the entire equation, $15a$ minus $12a$. and you just told me that that is $3a$ because they are like terms and we can add them or subtract them. So negative 30 equals what?
530. Student: $3a$
531. Tutor: [writes $-30 = 3a$] $3a$. we're almost done, I really don't care what $3a$ is, I want to know what a is. How do we get rid of that 3? Now remember, $3a$ is one number, it's really 3 times a .
532. Student: subtract it by itself to make it zero?
533. Tutor: no because then you'll make the whole thing zero. We just want to get rid of the 3. we had another problem like that, let's look at that one. Like here I had $8n$ equals 56 and I wanted to know just what n was. What did I do?
534. Student: you divided it.
535. Tutor: mmhmm, because it's 8 times n , the opposite of multiplication is --
536. Student: division
537. Tutor: right, this is 3 times a . opposite of multiplication is --
538. Student: division
539. Tutor: what am I dividing by?
540. Student: a ?
541. Tutor: no, because I want to know what a is, I'm trying to cancel out something.
542. Student: 3?
543. Tutor: [writes $/3$ on the right] yes. What's 3 over 3?
544. Student: 1
545. Tutor: 1. what's 4 over 4?
546. Student: 1
547. Tutor: what's a billion over a billion?
548. Student: 1
549. Tutor: 1. so 3 over 3 is 1 right? [crosses out the 3s on the right] we're just left with $1a$, or just a . $1a$ and a are the same thing. And how many times does 3 go into negative 30?
550. Student: um
551. Tutor: forget about the negative for a second. How many times does 3 go into 30?
552. Student: um 10.

553. Tutor: and negative divided by positive is?
 554. Student: negative
 555. Tutor: so negative 10 equals a, okay?
 556. Student: yeah.

Student #4, seventh grade, pre-Algebra student

Problem: $c + 32 = 124$

[Student skipped problem]

557. Tutor: okay let's do number 1. We want to know what c is right? once we know what c is we'll have the answer. So c plus 32 equals 124. So the trick is to get c all by itself. In other words c is by itself, it's not added to anything, it's not subtracted by anything. So how do we get rid of the 32?
 558. Student: subtract it
 559. Tutor: by itself, right. [writes -32 on the left] So 32 minus 32 is what?
 560. Student: zero
 561. Tutor: I'm going to write the first one and you'll do the next one all right? So you said we want c by itself so we're going to subtract by 32. [writes -32 on the right] Do you see what I'm doing? Okay. In algebra it's like a scale, what you do to one side you have to do to the other. If I subtract one side by 32, I have to subtract the other side by 32. [crosses out the 32 on the left] So 32 minus 32 they cancel out and we're left with c equals 124 minus 32, and I'm going to let you do all the hard work.
 562. Student: [writes $124 - 32 = 92$]
 563. Tutor: okay so c equals 92, we're done. That's the answer, you want to do another one? Of course you do. So just write c equals 92 so we know. [student writes $c = 92$]

Problem: $x - 7 = -23$

[Student combined incorrectly]

564. Tutor: why don't you rewrite the equation over here so we're not confused.
 Student: x minus 7 equals negative 23 [writes $x - 7 = -23$]
 565. Tutor: alright. Okay, I want to get x by itself, once I get x by itself I have my answer. So there's a minus 7 there. What's the opposite of minus 7?
 566. Student: positive 7
 567. Tutor: in other words, what are we doing? We're adding 7.
 568. Student: yeah
 569. Tutor: so we're adding 7 to this side and we have to --
 570. Student: subtract
 571. Tutor: add 7 to the other side. So go ahead and do that.
 572. Student: [writes +7 on both sides] okay 7 plus 7 is
 573. Tutor: zero
 574. Student: oh, 7 minus 7?
 575. Tutor: yeah, negative 7 plus 7 is zero
 576. Student: oh yeah that's zero, and then you add 7 to this which is um...
 577. Tutor: so it's negative 23 plus 7.
 578. Student: you go higher or lower than 23?
 579. Tutor: you're on the number line, negative 23 you're going to go higher.
 580. Student: oh yeah ... 16
 581. Tutor: am I negative or positive?

582. Student: negative
 583. Tutor: negative. So what's my next line?
 584. Student: uh ...
 585. Tutor: what's x equal?
 586. Student: negative 16?
 587. Tutor: yeah
 588. Student: ohh.
 589. Tutor: now if I was your teacher, I would say you're wrong even though you're right. You know why? Everyone has their own style but I can't tell that you know that x equals negative 16. write it out horizontally step by step so I know. [student writes $x = -16$]

Problem: $14 + \underline{\quad} = -32$
[Student skipped problem]

590. Tutor: there's a blank there, I'm going to put a letter there to make it easier. 14 plus, let's use x. [writes $14 + x = -32$]
 591. Student: okay
 592. Tutor: once we know what x is we can fill in the blank and we're done. So we want to get x by itself. so what should we do?
 593. Student: okay well we have to ... add this together with the same number.
 594. Tutor: yeah
 595. Student: and that's probably going to be zero.
 596. Tutor: what's going to make it zero?
 597. Student: 14 minus 14
 598. Tutor: very good.
 599. Student: and that's going to be zero right? [writes $x =$] and then you have to do it with this too, you have to add it. So I'm be going this way on the number line [gestures to the right]
 600. Tutor: well let's see I'm subtracting 14 so I'm going this way [gestures to the left]
 601. Student: [counts]... 46
 602. Tutor: negative or positive?
 603. Student: nnegative
 604. Tutor: so what's x then?
 605. Student: x is negative 46
 606. Tutor: very good. So just write that down. [student writes -46]

Problem: $r + 30 = -132$
[Student made a sign error by doing $132 - 30$ instead of $-132 - 30$]

607. Tutor: r plus 30 is negative 132. what can you do here?
 608. Student: you can um subtract 30. [writes $132 - 30 =$]
 609. Tutor: very good. So it's minus 132 minus 30, right? Which way am I going on the number line?
 610. Student: [points to the right]
 611. Tutor: I'm subtracting so I'm going that way [points left]. So just add 30
 612. Student: 142, 152, 162
 613. Tutor: positive or negative?
 614. Student: negative

615. Tutor: so r is?
 616. Student: r is negative 162 [writes $r = -162$]
 617. Tutor: very good.

Problem: $-7c = -112$ **[Student skipped problem]**

618. Tutor: so negative $7c$ equals minus 112. the first thing we can do, the minus signs here, the negative signs they cancel out. The next line we can write, well if negative $7c$ equals negative 112 then $7c$ equals 112.
 619. Student: yeah
 620. Tutor: we don't have to deal with the negative signs so let's not. They just add confusion. So maybe your next line you can write that.
 621. Student: so it will be 7
 622. Tutor: $7c$
 623. Student: right, $7c$ equals 112 [writes $7c = 112$]
 624. Tutor: good, if you have negatives on both sides and you only have 2 terms ...
 625. Student: it would be equal
 626. Tutor: just like if negative 4 equals negative 4, then 4 equals 4 right?
 627. Student: right
 628. Tutor: okay I don't really care what $7c$ equals, I want to know what c equals, so what do I do?
 629. Student: well doesn't the space here mean multiply?
 630. Tutor: yes
 631. Student: okay you have to ...
 632. Tutor: opposite of multiplication is?
 633. Student: division
 634. Tutor: mmhmm
 635. Student: so you divide 112 into ... 7?
 636. Tutor: mmhmm
 637. Student: I hate division
 638. Tutor: just do it step by step, [student writes $112/7$, starts long division by dividing 11 by 7] so it's 112 by 7 ... 7 goes into 42 how many times?
 639. Student: ... oh 6 times
 640. Tutor: mmhmm, evenly right?
 641. Student: right
 642. Tutor: so there's no remainder. So what's my answer?
 643. Student: c equals 16 [writes $c = 16$]
 644. Tutor: very good.

Problem: $k/-6 = 12$ **[Student skipped problem]**

645. Tutor: alright, here's a fraction. A lot of us are fraction phobic. Are you one of them?
 646. Student: yes.
 647. Tutor: okay we're going to get rid of this fraction before we even tackle the problem. we don't want to deal with fractions. [writes $\frac{1}{4}(4)$] If I had one fourth times 4 what am I left with?
 648. Student: one fourth divided by 4?

649. Tutor: one fourth times 4, what is that?
650. Student: I hate fractions
651. Tutor: don't these 4's cancel out?
652. Student: oh, 1?
653. Tutor: 1. [writes $1/9(9)$] so if I had one ninth times it by 9
654. Student: 1
655. Tutor: 1, [writes $2/9(9)$] if I had 2 ninths and I multiply by 9
656. Student: 2
657. Tutor: so it looks like if I take what's in the denominator and multiply it I'm left with what's upstairs. What do you think if I want to get rid of this, what should I multiply by?
658. Student: oh um ... say that again
659. Tutor: what should I multiply this fraction by to get rid of this denominator? Like here I multiplied one fourth by 4, one ninth by 9, 2 ninths by 9.
660. Student: oh so in this you divide, I mean multiply by 2 or something?
661. Tutor: no. here I took the nine, the denominator, and I multiplied the whole number by 9.
662. Student: then you got the 1 right?
663. Tutor: yeah I got whatever was upstairs. What do you think I should multiply by?
664. Student: the x?
665. Tutor: negative 6 [writes (-6) on left]
666. Student: oh okay.
667. Tutor: what am I left with?
668. Student: x
669. Tutor: [crosses out -6s on left] x equals, what I do to one side I have to do to the other. What did I do here? I multiplied by?
670. Student: 12
671. Tutor: by negative 6, so I have to do that on this side
672. Student: oh okay. So 12 times 6, [writes $12 \times 6 =$] ok ...72 [writes $x = 72$]
673. Tutor: it's actually by negative 6 so x is
674. Student: negative 72
675. Tutor: so when you see a fraction just multiply by the denominator. So if you have x over 5 just multiply everything by 5. that gets rid of the fraction. But what you do to one side you have to do to the other.

Problem: $-64 = 16y$
[Student skipped problem]

676. Tutor: okay this one. I don't care what 16y is, I want to know what y is. 16y is like 16 times y.
677. Student: yeah
678. Tutor: so what do you think?
679. Student: ok, you have to multiply it, I mean divide it right?
680. Tutor: yeah, by what?
681. Student: the negative 64 by 16
682. Tutor: mmhmm, and once we do that we have the answer.
683. Student: [tries multiplying 16 by 3 and gets 48, then by 4 and gets 64]... 4
684. Tutor: so what does y equal?
685. Student: 4 [writes $y = 4$]

Problem: $m/4 = -8$

[Student skipped problem]

686. Tutor: another fraction, but we know how to get rid of the fraction. What do we do?

687. Student: multiply the 4 by negative 8 right?

688. Tutor: right, multiply 4 by both sides

689. Student: and it just leaves you with ...

690. Tutor: [writes $m =$] m, m equals ...

691. Student: um

692. Tutor: m equals whatever negative 8 times 4 is.

693. Student: oh 8 times 4 is ... 32

694. Tutor: negative 8 times 4

695. Student: oh, negative 32. [writes -32] that's it?

696. Tutor: that's it.

End of transcript

Appendix B – Assessment given to students for human tutoring session**Solving Equations by Adding or Subtracting**

1. $c + 32 = 124$
2. $\underline{\hspace{1cm}} - 13 = -4$
3. $x - 7 = -23$
4. $14 + \underline{\hspace{1cm}} = -32$
5. $r + 30 = -132$

Solving Equations by multiplying and dividing

1. $12r = -132$
2. $-7c = -112$
3. $\frac{k}{-6} = 12$
4. $-64 = 16y$
5. $\frac{m}{4} = -8$

Solving Two-Step Equations

1. $8n - 19 = 37$
2. $x/3 - 6 = -4$
3. $4 + 2p = 6$
4. $3c - 7 = 2$
5. $-3t/2 = 27$

Solving Equations with Variables on Each Side

1. $(2a - 5)6 = 15a$
2. $7z = 2 + 5(z - 2)$
3. $9(x - 3) = 10 - 18x$
4. $20 - 2x = 4(5x + 3)$
5. $3x + 5x - 22 = 15x + 30 - 5$

Appendix C – pre-test given to students in experiment

Name _____

1. *Solve the equation for c:*

$$c + 32 = 124$$

2. *Solve the equation for x:*

$$x - 7 = -23$$

3. *Solve the equation for r:*

$$-12r = -132$$

4. *Solve the equation for k:*

$$\frac{k}{-6} = 12$$

5. *Solve the equation for n:*
 $8n - 19 = 37$

6. *Solve the equation for x:*
 $x/3 - 6 = -4$

7. *Solve the equation for x:*
 $62 - 5x = 4(5x + 3)$

8. *Solve the equation for y:*
 $3y + 5y - 22 = 15y + 30 - 3$

Appendix D – post-test given to students in experiment**Name** _____

1. *Solve the equation for z:*

$$z - 13 = -4$$

2. *Solve the equation for r:*

$$r + 30 = -132$$

3. *Solve the equation for c:*

$$-7c = -112$$

4. *Solve the equation for m:*

$$\frac{m}{4} = -8$$

5. *Solve the equation for c:*
 $3c - 7 = 2$
6. *Solve the equation for t:*
 $-3t/2 = 33$
7. *Solve the equation for z:*
 $7z - 12 = 2 + 5(z - 2)$
8. *Solve the equation for x:*
 $9(x - 3) + 10 = 10 - 26x + 8x$

Appendix E – Actual production rule in E-tutor

```

;;DIVIDE-NUMBER-FROM-TERM-LEFT
;;divide a number from both sides if the left side has
;;a variable times a number and the right side has a
;;number
;;example: 2x=4 --> x=4/2
(defproduction divide-number-from-term-left equation (=problem)
  =problem>
  isa equation-problem
  lhs-exp =lhs
  rhs-exp =rhs
  quantities-list ($front $back)
  =lhs>
  isa expression-op
  first-exp =first-exp
  op =operator1
  second-exp =second-exp
  side =side1
  =first-exp>
  isa expression-number
  value-t =num1
  =second-exp>
  isa expression-literal
  value-t =var1
  =rhs>
  isa expression-number
  value-t =num2
  side =side2
  !eval! (or (char= =operator1 #\*) (char= =operator1 #\&))
  ==>
  !eval! =new-num2 (/ =num2 =num1)
  =new-lhs>
  isa expression-op
  first-exp =lhs
  op #/
  second-exp =first-exp
  side =side1
  =new-rhs>
  isa expression-op
  first-exp =rhs
  op #/
  second-exp =exp1
  side =side2
  =exp1>
  isa expression-number
  value-t =num1
  side =side2
  =problem>
  lhs-exp =new-lhs
  rhs-exp =new-rhs
  quantities-list ($front =new-lhs =new-rhs $back)
  :action 'divide-both-sides
  :input =num1 #\equal-value-p
  :messages (help
    `(What can you do to both sides to get #\space ,=var1 by itself?)
    `(,=num1,=var1 is ,=num1 times #\space ,=var1
      ~n How do you undo multiplication?)
    `(Divide both sides by ,=num1)))

```