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This report represents the work of one or more WPI undergraduate students submitted to the faculty as evidence of completion of a degree requirement. WPI routinely publishes these reports on the web without editorial or peer review.

Abstract:

Destructive testing has been a staple in the manufacturing world for decades, and while the technology being manufactured has progressed to meet the demands of a modern world, the methods of testing have remained stationary. Using a non-destructive method of testing called Holographic Metrology, the user can save time, money, and materials while receiving accurate results. This project covers the validation of Holographic Metrology, while using these tools to study vibrations and dynamics of blade and rotor models.

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Motivation:

The current method of testing for the stress of an engine or turbine blade is outdated and should be updated from the current destructive method of testing to a non-destructive method of testing. The current method of testing requires a large amount of strain gauges to be fixed to the blade, after which the blade will be shaken using a piezoelectric shaker to simulate the motion of the blade in the engine chamber. This method is time consuming and takes many hours or even days to attach and run trials on a single collection of blades. Through Holographic Metrology the process of testing the manufactured parts can be modernized to the quality of the part.

Nomenclature:

a	width of the beams
b	height of the rectangle cross-section
f_n	natural frequency
h	height of the triangle cross-section
i	imaginary part of complex number, dimensionless
t	time, sec.
A	area of cross section
A_r	amplitude of the reference field
A_o	amplitude of the object field
A_{t_o}	amplitude of the time varying object field
AF_j	amplitude of harmonic function
ρA	beam mass per unit length, kg/m
C	center of gravity
C	torsional constant
E	Young's modulus, Pa
F_o	complex object light field
F_v	complex light field of vibrating object
F_r	complex reference light field
F_h	complex light field on hologram
G	shear factor
I	area moment of inertia about a neutral axis
I_{im}	intensity of reconstructed image
I_p	polar area moment of inertia
I_r	irradiance of the reference field

I_{t_o}	irradiance of the time varying object field
I_{t_n}	time varying irradiance of the n-th frame recorded by the CCD camera
I_{Xc}	area moment of inertia
I_{Yc}	area moment of inertia
J_o	zero order Bessel function
K	sensitivity vector
K_1	illumination vector
K_2	observation vector
L	length of the beam
L_t	time varying displacement vector
L_{tz}	recorded displacement of object in z-direction of utilized coordinate system in hologram interferometry
L_z	displacement of object in z-direction, same coordinate system as for L_{tz}
L_{zrr}	displacement of object in z-direction for retro-reflective case
M	bending moment
$M(\Omega_t)$	characteristic function
R_p	position vector of object point
R_1	position vector of illumination point
R_2	position vector of recording plane
V	shear moment
γ	Poisson ratio
$\Delta\theta_n$	finite phase step imposed on the reference beam
λ	wavelength of laser beam, μm
π	constant
ϕ_o	time independent phase of light field modulated by interaction with the object

ϕ_r	phase of illuminating light field
$\Delta\phi_t$	time varying phase difference between two fields
ω	radian frequency of vibration, rad/sec
Ω_t	time varying fringe locus function
ρ	beam mass per unit length

Background:

Objective and goals:

Use non-destructive holographic testing to find the modes and resonance frequencies for turbine blades proving as the study of vibrations and dynamics. Provide evidence to support non-destructive testing as a proof of concept. Optimize current and future protocols of non-destructive holographic testing.

Sponsor/Client: Pratt and Whitney:

Pratt and Whitney is a subsidiary of Raytheon Technologies and is an aerospace manufacturing company that produces both commercial and military products. Pratt and Whitney is one of the three largest aerospace manufacturers in the world, alongside General Electric, and Rolls-Royce. Pratt and Whitney also produces gas-powered energy turbines and industrial turbines.

F-35 Jet:

Perhaps the most commonly known modern fighter jet, the F-35 is a staple in the United States military fleet. Pratt and Whitney creates and produces the F-35 Engine and received 4.4 billion dollars from the Department of Defense to create these engines for the future fleet of fighter jets engines. As of 2022, 13 different nations have the F-35 fighter jet as part of their Air Force and Defense Fleet.

Turbine and Engine Blades:

The internal blades are a key part in creating a successful engine or turbine, and these blades are produced with indredially tight specifications and tolerances. The turbine and engine blades need to be able to withstand incredible levels of heat and a larger amount of vibration while the engine or turbine is in use. The greatest concern to

the longevity of a turbine blade is the exposure to repeated vibrations. The waves of the vibration can interact with other waves and create modes in the blades. The frequencies that this phenomenon occurs can be measured through Holographic Metrology.

Properties of Light:

1. Wavelength: Wavelength is the period of the wave of light, or the distance from where the wave shape repeats. The wavelength of a beam of light is inversely proportional to the frequency of the beam of light. Assuming the wave of light is traveling at a constant speed, then the longer the wavelength, the shorter the frequency and vice versa.
2. Frequency: Frequency is the time at which a beam of light takes to repeat the shape of its wave. As stated above the frequency of the beam of light is inversely proportional to the wavelength. Frequency will also be used when oscillating a sample through a piezoelectric shaker.
3. Intensity: Intensity is the optical power per unit area in a beam of light and it is important due to the fact that optical lasers are used at a specific intensity. The intensity of the light also comes into effect when it is diverted and when it reflects off the sample object.

Overall Concept:

The basic principle of holographic interferometry is the superposition of two or more coherent light waves. This interference is holographically recorded and subsequently reconstructed. The holograms themselves are visually identified as interference fringe patterns that are quantifiable and related to the optical path difference (OPD) and the sensitivity vector (SV) of the optical setup. The OPD is the difference in travel length between the interfering light waves, while the SV is the vector

difference between the incident and scattered light (that observed by the image sensor) at the testing sample. A change to these parameters is made in the measurable known fringe pattern. Since the SV marks the direction of the measured displacement, when the sample moves, this parameter changes and therefore, the fringe pattern changes as well. The changes in fringe pattern allows the change in position of the engine blades to be measured accurately without contact.

History of Holographic Testing:

Holographic Testing was created by recording a reference beam and subtracting the reflection of an additional beam of light from the original reference beam. This technique was created by Karl Stetson and later improved by Charles Vest. Holographic Testing is an important tool in Non Destructive Testing, (NDT). NDT consists of a variety of non-invasive inspection techniques used to evaluate material properties, components, or the entire process. The techniques can also be utilized to detect, characterize, or measure the presence of damage mechanisms (corrosion or cracks). NDT is also commonly referred to as nondestructive examination (NDE), nondestructive evaluation (NDE), and nondestructive inspection (NDI). Many NDT techniques are capable of locating defects and determining the features of the defects such as size, shape, and orientation. The purpose of this form of testing is to inspect a component in a safe, reliable, and cost effective manner without causing damage to the equipment or shutting down operations. This is in contrast to destructive testing where the part being tested is damaged or destroyed during the inspection process.

NDT can be performed during or after the part or piece is manufactured, or even on equipment that is in service. In manufacturing, NDT inspections can determine if parts are fit for a desired function. Parts are inspected to ensure they will last a certain amount of time or cycles before failure. During operation, NDT inspections can be used to figure out the current damage state of equipment, monitor damage mechanisms, and make informed decisions for remaining equipment life evaluations.

Fringe Projection:

The digital fringe projection is a 3-D measurement for the non-contact digitalization of component surfaces.

With the help of a projector different light-dark fringe patterns are produced by the measuring part. A CCD camera missed the deflection of the fringe pattern. Then a digital 3D profile of the target is calculated via a mathematical calculation method.

With fringe projection, the measurement area can be scaled over a wide range, from less than a millimeter up to more than one meter, whereby the resolution scales accordingly. The method suits small samples due to its high resolution and accuracy for precise detail measurements, as well as large area surveys due to its speed.

Fringe Locus Functions:

These fringes are the wrapped phase of the cosine fringe function, and relate directly to the object deformation. A robust phase-unwrap program converts the wrapped phase fringes to numerical data for comparison with finite element analyses models.

Software:

MatLab:

MatLab is a coding language and online program that is developed by Mathworks. MatLab is used to assist in the post processing of images and is a needed tool when taking third derivatives while unwrapping videos of modes and fringes. MatLab also has the ability to create masks of an image, where only the useful data is shown in the sample. An example of the MatLab code used to process videos is shown below.

HoloStudio:

HoloStudio is another tool that is used while post processing images. This software is made with Holographic Metrology as the main focus. Using Holostudio, the user can take the 2D image and run an unwrapping feature allowing the image to be rendered in three dimensions. An example of the 2D to 3D render process is shown below.

LaserView4.4:

LaserView 4.4 is the imaging software used to capture accurate images of the mode and fringes in a sample. This software has many capabilities including the ability to change exposure, capture videos, calculate aspect and beam ratios, adjust applied frequencies, and export the captured images into usable files that can then be post processed by either HoloStudio or MatLab.

Vimba Viewer: Vimba Viewer is a software that allows the user to view the camera setting prior to capturing the images in the LaserView4.4 software. This software allows the user to set the resolution of the camera and make sure the sample is in focus prior to applying the stimuli.

Current Problem:

Pratt and Whitney have been using Destructive Testing since the company was founded. This method of testing is not inherently wrong or incorrect, but it does lead to an increase in spending and a large time gap in the manufacturing process as a newly machined part will be destroyed while testing the minimum and maximum stress and strain of the part at each step of manufacturing. The current method for testing a completed F-35 engine is to hang the engine from the ceiling in a warehouse and move the engine in a measurable sign wave pattern. In addition to the large amount of money and energy needed to complete this process, over 5,000 strain gauges need to be placed on the engine and tracked by hand throughout the process. NDT is a needed step into the future and can increase safety, decrease production cost and save many hours per part and days of work over the course of an engine build.

Methodology:

Mathematical Theory and Fringe Locus Functions:

To determine the equations that will support the analytical study the forces and the moments of the beam must be taken into account. Set V as the shear moment of the beam and set M as the bending moments of the beam. $p(x)$ will represent the loading per unit of the length on the beam. The forces in the y -direction of the beam is

$$dV - p(x)dx = 0$$

The force in the x -direction of the beam is

$$dM - Vdx - 1/2p(x)(dx)^2 = 0$$

The following properties are true in these equations

$$dV/dx = p(x) \text{ and } dM/dx = V$$

By combining the above equation, it can be derived as

$$d^2M/dx^2 = dV/dx = p(x)$$

The bending moment of the beam can be related by the equation

$$M = EI * d^2y/dx^2$$

Then combining the following two equations, it can be assumed that

$$\frac{d^2}{dx^2} (EI \frac{d^2y}{dx^2}) = p(x)$$

Assuming harmonic motion and the equation, $p(x) = \rho A \omega^2 y$, it can be integrated into the above equation

$$\frac{d^2}{dx^2} (EI \frac{d^2 y}{dx^2}) - \rho A \omega^2 y = 0$$

It can be assumed to be a unique case where (EI) is a constant and equation above can now be described as

$$EI \frac{d^4 y}{dx^4} - \rho A \omega^2 y = 0$$

From the equation above the following fourth-order differential equation can be derived

$$\frac{d^4 y}{dx^4} - \beta^4 y = 0$$

This fourth-order differential equation can be solved by the general solution of

$$y = A \cosh \beta x + B \sinh \beta x + C \cos \beta x + D \sin \beta x.$$

Natural frequencies of vibrations can be modeled as

$$\omega_n = \beta_n^2 \sqrt{\frac{EI}{\rho A}} = (\beta_n L)^2 \sqrt{\frac{EI}{\rho A L^4}}$$

For this equation to be true, there must be boundary conditions are

Where $x=0$ then $y=0$ and $dy/dx=0$, and $\beta(B+D)=0$ and therefore $B=-D$. By then combining the general solution and the boundary conditions, the equation as followed is formed.

$x=L$ where L is the length of the beam, $M=0$ or $\frac{d^2y}{dx^2} = 0$ and $V=0$ or $\frac{d^3y}{dx^3} = 0$.

This condition is also based on the condition where $(y)_{x=0} = A + C = 0$, and therefore $A=-C$.

The fourth order differential equation is then solved by the equation

$$A(\sinh\beta L - \sin\beta L) + B(\cosh\beta L + \cos\beta L) = 0$$

This final equation is then reduced to solve into the equation

$$\cosh\beta L \cos\beta L + 1 = 0.$$

The values of $\beta_n L$ for each mode shape are as follows:

Mode One: 1.87510407

Mode Two: 4.69409113

Mode Three: 7.85474744

Mode Four: 10.99554073

Mode Five: 14.13716839

Using these values for each Mode Number the natural frequency for Mode One would be as follows

$$\omega_1 = \frac{1.8751^2}{L^2} \sqrt{\frac{EI}{\rho A}} = \frac{3.515}{L^2} \sqrt{\frac{EI}{\rho A}}.$$

The bending moments and the moments of Inertia will change based on the geometries of the cross-sections of the cantilever beams. The two cross sections that will be used in this experiment are the rectangular cross section and the triangular cross section.

The following equations represent the rectangular cross section then followed by the triangular cross section.

$$I_{x_c} = \frac{ab^3}{12} \text{ and } I_{y_c} = \frac{a^3b}{12}$$

Bending \Rightarrow

$$f_n = \frac{\omega_n}{2\pi} = \frac{(\beta_n L)^2}{4\pi} \sqrt{\frac{E}{3\rho} \frac{b}{L^2}}$$

Triangular Cross Sections

$$I_{x_c} = \frac{ah^3}{36} \text{ and } I_{y_c} = \frac{a^3h}{48}$$

Bending \Rightarrow

$$f_n = \frac{\omega_n}{2\pi} = \frac{(\beta_n L)^2}{12\pi} \sqrt{\frac{2E}{\rho} \frac{h}{L^2}}$$

The beams that were used in this experiment were Beam 1 and Beam 2. Beam 1 has a rectangular cross section and Beam 2 has a equilateral triangle. The dimensions of the two beams are as follows.

Beam 1: Cross-Sectional Area: a=3.81cm, b=0.85cm, L=15.24cm

Beam 2: Cross-Sectional Area: a=2.54cm, h=2.20cm, L=10.16cm.

Electro-Optical Holography

The equations of the irradiance of the n-th frame of the holographic system can be measured by the equation

$$I_{t_n} = I_{t_o} + I_r + 2A_{t_o} A_r \cos(\Delta\phi_t + \Delta\theta_n) M(\Omega_t)$$

The parameters of the irradiance equation is as follows

$$I_{t_o} = A_{t_o}^2 \text{ and } I_r = A_r^2$$

The image is then represented by the equation

$$[(I_{t1} - I_{t3})^2 + (I_{t2} + I_{t4})^2]^{0.5} = 4A_{t_o}A_r|M(\Omega_t)|$$

In this equation each of the irrancements is represented by the following equations

$$I_{t1} = I_{t_o} + I_r + 2A_{t_o}A_r \cos(\Delta\phi_t)M(\Omega_t)$$

$$I_{t2} = I_{t_o} + I_r + 2A_{t_o}A_r \sin(\Delta\phi_t)M(\Omega_t)$$

$$I_{t3} = I_{t_o} + I_r - 2A_{t_o}A_r \cos(\Delta\phi_t)M(\Omega_t)$$

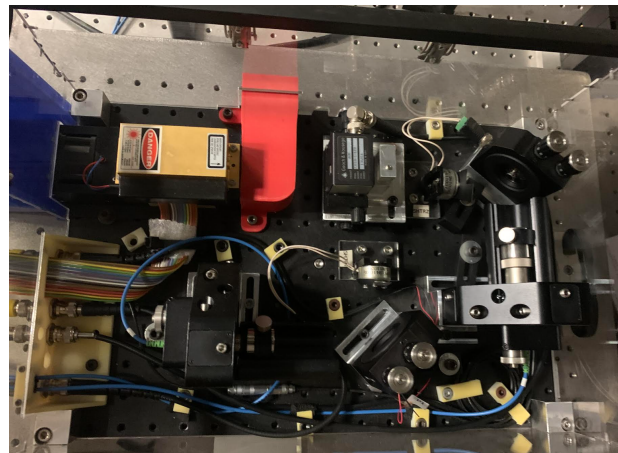
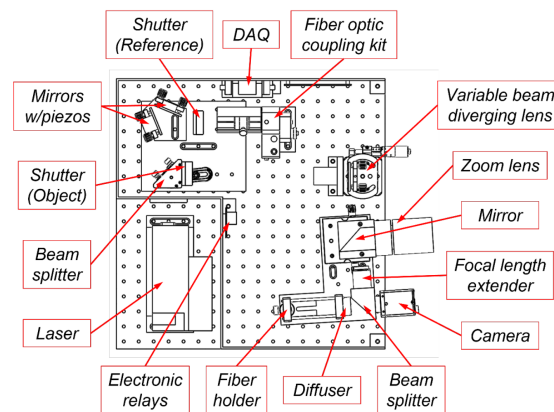
$$I_{t4} = I_{t_o} + I_r - 2A_{t_o}A_r \sin(\Delta\phi_t)M(\Omega_t)$$

Where the points $J_o(\Omega_t)$ equals zero then the center of the fringes are located at these different points of the object's surface. This can be represented by the equation

$$(I_1 - I_3)^2 + (I_2 - I_4)^2 = 16I_oI_rJ_o^2(\Omega_t)$$

Setup and Housing:

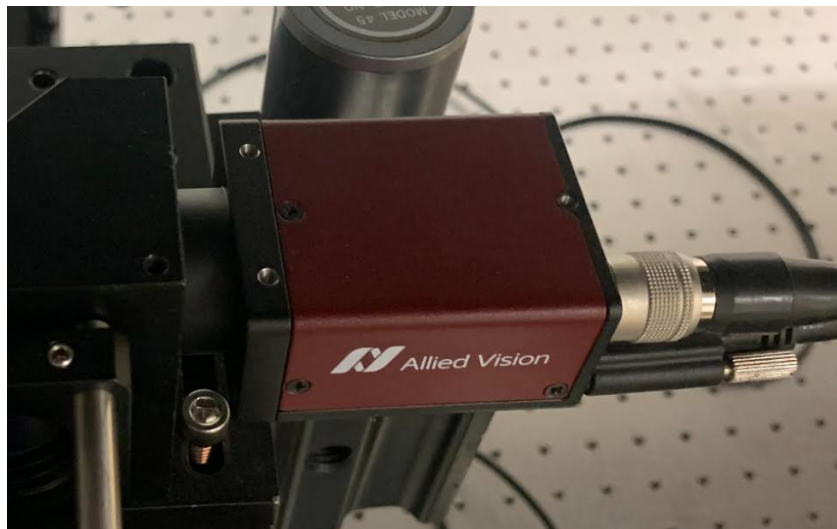
The housing mechanics and motherboard of the holographic system is shown below. The main unit houses the subsystems, which are made up by the DAQ unit and the components listed below. The stimulation subsystem is composed of the arbitrary function generator. The main unit is covered and the laser is housed outside of the main housing to limit stray light that could affect the optical results.



The components of highest importance are detailed below:

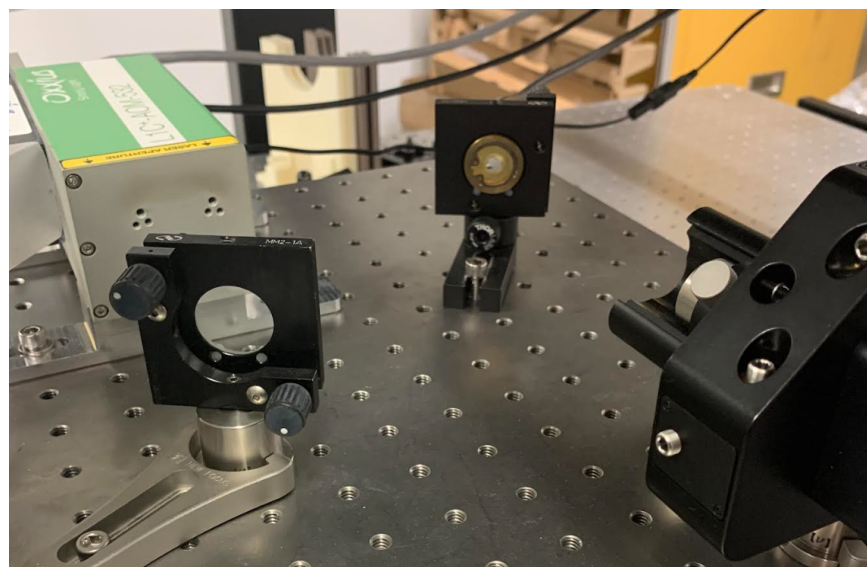
Camera Setup and Senors:

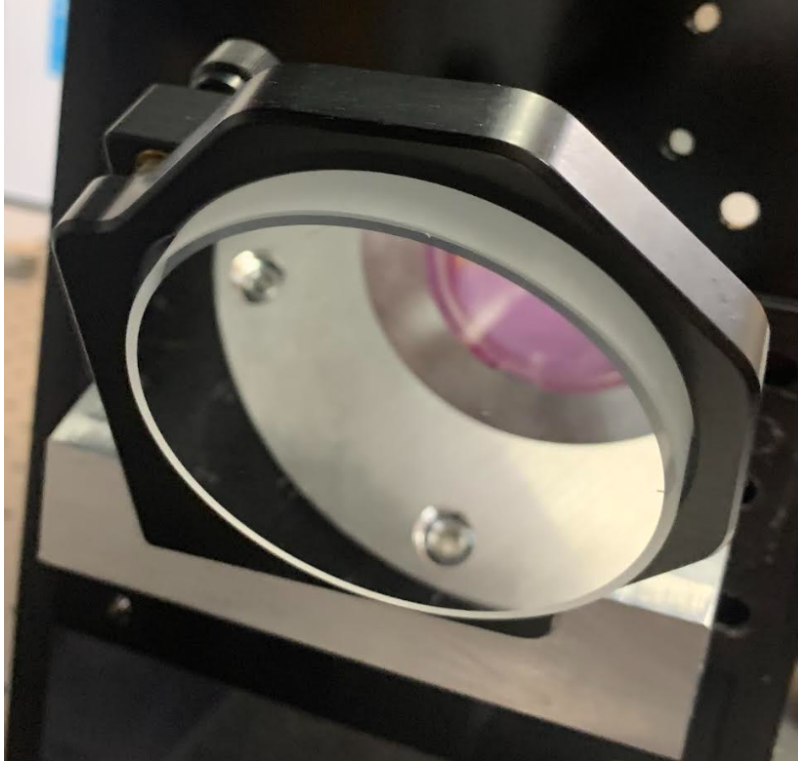
The four parameters to describe this setting are X, Y, W and H, meaning x-position, y-position, width and height, respectively. These will determine a region in the image detector for which to use and display.



Laser and AOM connection:

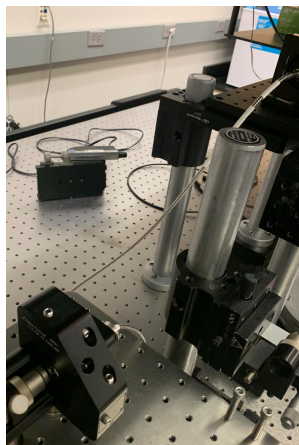
The AOM module is integrated inside of the laser module. There are two cables exiting this composed module, one for power (laser emission) and the other for operating the AOM. Both cables, one of which uses serial-style ports and the other SMA connections, are connected to the benchtop controller. The controller must be connected to a plug for power.





DAQ connection:

The DAQ box, which is screwed onto the inner wall of the holographic setup, uses a USB type 2 connection. The connections inside of the setup must not be modified. The DAQ box is connected to a wall connector designed to be used for connecting to the computer from the outside.



Function generator connection:

The function generator must be connected to a plug for power, and to the computer using a USB cable. This instrument is composed of two output channels which are both used for controlling the stimulus and stroboscopic features of the system. Channel 1 must be connected to the sample test piece either directly or through an amplifier. Channel 2 must be connected to the Analog Input port.



Model Experiment:

Verifying the Holographic System through a series of images conducted on cantilever beams as a proof of concept. To prove the Holographic System is capturing accurate results and images, the final results of the cantilever experiment will be compared to a computer simulated Finite Element Analysis of the cantilever using COMSOL. The physical test of the cantilever beams and the COMSOL analysis should match. The cantilever beam will be attached to a piezoelectric shaker that will cause vibrations throughout the beam. The cantilever beam will be imaged under these conditions and the fringes, modes, and displacements will be calculated. The physical results of the cantilever beam fringes, modes and displacement will be discussed in the results of methodology section.

Comsol Finite Element Analysis:

Finite Element Analysis is a method of predicting how a physical product reacts to real world simulations such as vibration, heat, and fluid flow. Finite Element Analysis is important to the overall process as it can predict the expected results of an experiment. Finite Element Analysis can be conducted using various computer programs and during the course of the project COMSOL was used to conduct these computer simulations. The COMSOL analysis of the cantilever beam will be compared in the proof of methodology section.

Conducting Image Capture and Overall Setup of Holographic System:

Pathing and Use of the Holographic Metrology System:

1. The Laser is connected using a 9-pin cable. The Laser will emit green light into the system. The SMA cable will be connected into the benchtop controller.

2. The IEEE Fly Wire capture will be inserted into the computer. This will connect the Stingray camera capture to the computer.
3. The USB cable from the main breadbox will be connected into the main computer system along with the IEEE Fly Wire capture.
4. The function generator will be connected into the computer and uses the wall power as the power source. The function generator will be connected to the piezoelectric shaker through the BNC output port. The function generator will relay the signal at which the piezoelectric shaker will oscillate. The rate of oscillation is key due to the fact that the shaking is what will simulate the rotation of the engine and turbine blades.
5. The lasers, cameras, and computer are then connected and ready to begin the testing phase.

Testing Laser Beam Path:

1. The Laser must be initiated and the power source must be connected in the correct order for the laser to shine throughout the system. This process may take up to 5 minutes for the laser to appear.
2. The Laser beam will shine through a diverter that will split the beam into two parts. One beam will travel through the system in a straight line and reflect onto the testing sample. The other beam will be reflected back at a 45 degree angle and it will be directed into a fiber optic that will ultimately serve as the reference beam.
3. It is important that the reference beam is calibrated correctly before the reference beam is attached to the Fly Wire capture.

4. The main beam is focused through an apparatus and then reflected onto the sample. The laser is then reflected off the sample and is captured by the main sample camera.
5. The image captured from the sample camera and the image captured from the reference beam will be subtracted from each other to create the Holographic image.

Image Processing and Capture: The image will be viewed and captured through LaserView 4.4 and saved into a file where it can then be processed using HoloStudio or Matlab.

1. The Beam Ratio must be calculated before the images are captured on LaserView.
2. The Beam Ratio is the ratio of intensity of the object beam that is reflecting off of the sample, and the reference beam. The Beam Ratio should remain at least at a level of 1.0 for minimum quality. For optimal quality the Beam Ratio should remain at a range between 2.0 and 2.7.
3. The Beam Ratio is automatically calculated through the software and should refresh when the shutter is reloaded.
4. To change the Beam Ratio from one section of the image to another a Region of Interest (ROI) must be reset. This is completed by selecting the ROI button and selecting a different region of the image with a green square as shown below.
5. In addition, the exposure of the image must be selected accurately. The exposure is automatically calculated but there is a manual option for specific cases.

6. When manually selecting the exposure the number of saturated pixels and the shape of the exposure function must be balanced.
7. There are two main options of exposure when capturing an image, time-average and double exposure. The option used in this project is time-average mode.
8. Once time average mode is enabled the multiplier must be adjusted for visual purposes. This allows the image to have the correct contrast while taking the image.
9. Apply the frequency through the frequency generator to view the modes and fringes of the turbine and engine blades as they experience the known frequencies. The frequencies can be controlled through the LaserView 4.4 menu.
10. Note: Double Exposure mode can be used to collect the same image. When the user is saving the images it must be saved through the corresponding Time-Average or Double Exposure files.

Post Processing:

1. The images, once captured will then need to be processed so that data and results can be gathered.
2. The two main methods of Post Processing are by computer software, such as MatLab and Holostudio.
3. While using either method the images must be uploaded correctly through the proper files or else the software will not be able to process the necessary data.

Results:

The results of the COMSOL FEA are shown below in sequence. The actual averages are taken directly from the resulting data from HoloStudio. Then the Percent Difference is taken by the formula $(x1-x2)/x1$.

Validation and Results:

Rectangular Cross-Section

Mode Shape	FEA	Experiment	% Deviation
1 Bending	310	297	4.193548387
2 Bending	1905	1909	-0.2099737533
1 Torsion	2225	2450	-10.11235955
3 Bending	5223	5430	-3.963239518

Triangular Cross-Section

Mode Shape	FEA	Experiment	% Deviation
1 Bending	1423	1429	-0.4216444132
2 Bending	7787	7643	1.849235906
1 Torsion	5973	5643	5.524861878
3 Bending	18651	16845	9.68312691

The computer aided simulation and calculation of modes and bendings are listed above. The result of the methodology section proves the Holographic Metrology is confirmed for the simple cantilever beam sample. With this confirmation, the Holographic Metrology system and setup will be taken and applied to the actual engine

and turbine blade beam. The largest percent difference in the sample size was 10.9 percent. Sources for this percent difference are due to real world added vibrations including the stand of the cantilever beam being elevated from the air supported table. The more the gap between the cantilever beam setup and the air table is, the larger the possible outside vibrations can be. The larger percent deviations were observed for the mode shapes at higher frequencies and these mode shapes were first torsion and 3rd bending moments for both the rectangular and triangular cross sections.

The following procedure should be conducted on the official turbine and engine blades provided by Pratt and Whitney.

Conclusion and Recommendations

Two cantilever beams with a rectangular and triangular cross sectional area were investigated throughout this report. The overall connection between the Finite Element Analysis and the experiment results showed a strong correlation between the first and second modes only. The process of Electro-Optical Holography was used to gather successful results for the rectangular and triangular cross sectional area. This system can be used in double exposure or time-averaged mode to collect accurate and rapid results. The overall study proved an accurate connection between the first and second bending modes, whereas the first torsion and third bending modes were not statistically significant when compared to the Finite Element Analysis.

This study should be continued where the excitement method is a sound frequency produced by a function generator to examine the effect of these sound waves. The piezoelectric shaker, while effective, had to be clamped onto support structures and thus when examining the first torsion and third bending the Finite Element Analysis and actual collected results were statistically insignificant.

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