

DESIGN AND CONTROL OF AN AUTONOMOUS HELICOPTER

A Major Qualifying Project Report

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## **Abstract**

Through mathematical modeling, control scheme development, and extensive testing, this project has taken the first steps in rendering a miniature helicopter autonomous. This report contains the relevant equations of motion, the associated control schemes, as well as the steps taken to create a sensor-driven flight computer allowing autonomous operation. Further development and testing of the control board is necessary to apply the completed control scheme which allows autonomous flight of a remote control helicopter to a preselected position.

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## 1.0 Introduction

The helicopter plays an invaluable role in many fields, incurring countless uses; from military combat to scenic tourism and everything in between. Helicopters today undertake a myriad of different and highly specialized missions; such as search and rescue, seek and destroy, and simple transportation. The purpose of this project is to take the first steps in creating a helicopter capable of completing such a mission entirely on its own. The goal is to create an autonomous helicopter.

While the implementation of an autonomously operated helicopter has obvious benefits in almost every conceivable mission, it is the surveillance based ones that make the most use of this technology. Surveillance, with its demand for covertness and inherent danger in the case of enemy and unfriendly observation profits from autonomization in two ways. It does not put the pilot in danger, as it does in any mission; but it also allows the helicopter to be built on the scale of an R/C toy, something a transportation mission could never accommodate. It is this platform that the team has chosen to use to develop a computer controlled guidance system for.

For the purposes of this project, the final goal will be to design an R/C helicopter capable of following a lead ground vehicle at a predetermined height with a payload of a video capture device relaying images back to base. The real world corollaries to this demonstration are clear. A direct comparison would be a network of autonomous helicopters following a mission sensitive ground unit, such as a military transportation vehicle, at a predetermined radius and height alerting the vehicle of any present danger in its vicinity well before it is ever encountered.

The project entails technical work from many disciplines. The helicopter has been optimized to operate efficiently and for long periods of time. This requires improvements in the aeronautical and electronic mechanisms. The system of circuits of sensors and processors required to allow such operation had to be designed and constructed. Lastly, the computer program, based on algorithms derived from aeronautical laws and equations was written. For this project, these subjects came together to create a helicopter that will pave the way for future autonomous vehicles.

## 2.0 Background

### 2.1 Autonomization

Autonomous robots are gaining recognition as an increasingly effective method in manufacturing, surveillance, and space exploration. As the technology to create such automates becomes more cost effective, the practicality of such devices in everyday situations increases. Giving the computer more control over flight is not a new concept. The US-Iraq war saw the advent of UAV, or unmanned aerial vehicles, come into play as surveillance vehicles as well as mobile weapons platforms. The UAV designed for this project is based around a T-Rex 450SE electric hobby helicopter. The signals to and from the multiple servos and the motor are to be intercepted and controlled via a computer which monitors the helicopter's stability using a network of gyroscopes and accelerometers. The ultimate goal of this project is to create a heterogeneous network of UAVs which can perform a variety of tasks.

Why is it important to make the technological jump from human operated vehicles to autonomous vehicles? There are three main reasons for the advent of autonomous wartime vehicles: economics, public opinion, and technological advantage. The economic reasons are very clear, the basic pay of a new soldier out of training costs \$15,282 per year. This number does not include other military expenses such as housing, food, travel, and insurance for the new soldier. In comparison, the preliminary budget for the project's autonomous helicopter is under \$2,000. Another importance reason for the use of UAVs is public opinion. Another US name on the casualty list is another son or daughter to an American family. It is hard to support a war when your son or daughter may die fighting. However, if a UAV gets shot down, there is no one to grieve, no one to write home to, and nothing to report except a loss of hardware. This can be very important when considering the effects of imbedded media showing live action from the front lines. Another consideration to the important of UAVs is the technological advantage given to the soldiers in a combat zone. US military technology is based on the theory of kill ratio, which is how many enemy soldiers does a US soldier kill on average before being killed. In military operations in Afghanistan, the US military reached a kill ratio of 50:1. The use of UAVs will only increase this number, allowing for larger military operations with fewer soldiers.

Autonomous flight requires a number of sensors to maintain equilibrium during flight as well as continuously track its position relative to a certain location. In order to keep the helicopter in a hovering mode a three axis accelerometer and a three axis gyroscope are needed to track shifts away from an equilibrium state. As the sensors pick up changes in acceleration and angles from the stable hover state, a computer uses the data from the sensors to adjust the current to the motors and servos to compensate for the changes and thus rebalancing the helicopter in a stable hover position. In addition to stability control, an autonomous robot must be able to travel to locations based on a fixed coordinate plane to conduct missions. Current UAVs use GPS navigation as well as other methods to ascertain its positioning. More detail will follow on the helicopter guidance system.

There are limiting factors to the effectiveness of UAVs in production today, most importantly is flight time. Power requirements for UAVs can be significant with a fleet of

sensors, production of thrust, computation, and mission requirements. Current UAVs under research and development are utilizing new methods in order to ascertain longer flight times including hybrid technology, power harvesting from thrusters, and solar power. The power source selected for this project is a Lithium Polymer battery with an estimated flight time of fifteen minutes. However, it is hoped that an automatic landing and recharge sequence can be programmed into the flight parameters.

## 2.2 Helicopter Components

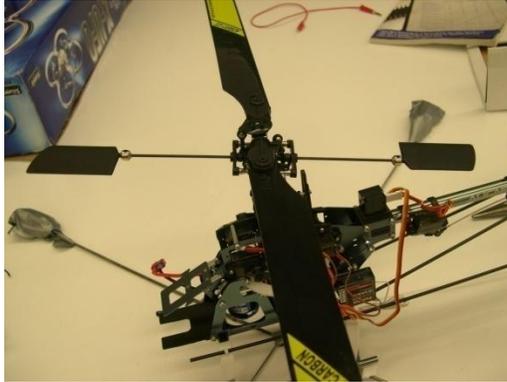
The laws and equations governing the dynamics of helicopter flight certainly are extensive and intricate; it comes with no surprise then that the components controlling the helicopter reflect that complexity. A modern full scale helicopter contains thousands of moving parts, most machined to very tight tolerances. While this platform, an R/C helicopter, does not match its full size cousin in number of components, it certainly does in regards to their complexity.

A typical helicopter uses its single main rotor to produce both lift and planar velocity vectors. This rotor typically is an assembly of four symmetric aerofoil blades (see Fig. 2.1). This is where the first and most striking difference between a full scale and R/C helicopter comes into play.



Figure 2.1: Apache Helicopter showcasing symmetric main rotor

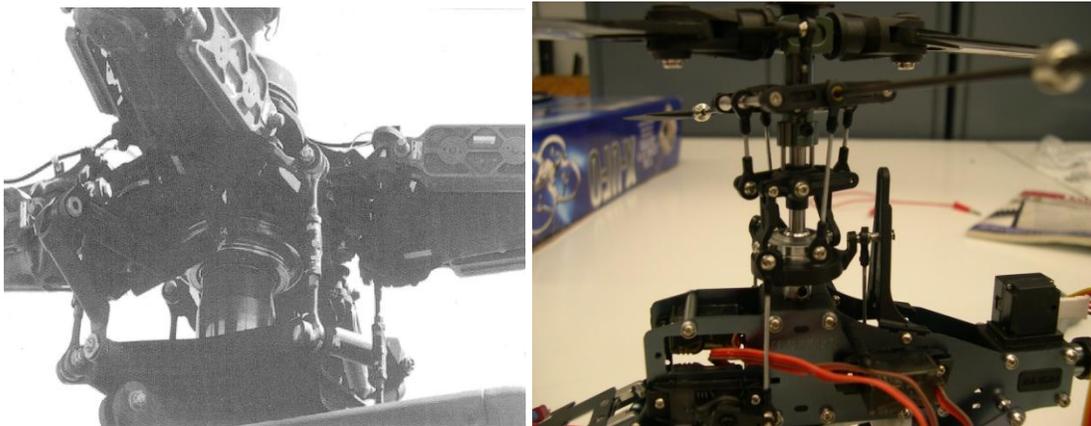
Unlike the full scale helicopter, the R/C uses two horizontally opposed miniature stabilizer blades in conjunction with two horizontally opposed main blades to control planar motion while the main blades control lift characteristics. Figure 2.2 depicts the set up in the T-Rex 450.



**Figure 2.2: T-Rex 450S showcasing main/stabilizer blade configuration**

During ascent and descent, a typical helicopter will increase and decrease the angle of attack of its blades respectively; this is known as collective pitch, each blade angles up or down an identical amount. To pitch and roll, the helicopter will tilt the entire rotor assembly in the direction of the desired force vector; this is known as cyclic pitch. Whereas the R/C helicopter will produce lift in the same fashion, by increasing the angle of attack of its two main blades, it pitches and rolls by modulating the pitch of the auxiliary stabilizer blades, known as fly bars.

The device that controls all this blade activity is called the swash plate. It allows the main rotor shaft to spin while simultaneously controlling the collective pitch of all of the blades and the angle of each individual blade to produce cyclic pitch. Figure 2.3 depicts the swash plate from a T-Rex 450SE alongside a swash plate from a full scale helicopter, the AH-64. Although the linkages differ between the R/C and full scale helicopters, their function is identical.



**Figure 2.3: Comparison of AH-64 [3] and T-Rex 450S swashplates**

Power to the main and tail rotors in a typical standard full size helicopter is provided via a gas powered internal combustion engine. In the case of the R/C helicopter, power is provided via an electric motor run off of a battery. For the purposes of this project the details of the propulsive system that are relevant are efficiency and power requirements. On this scale, an electric motor excels in both.

The final critical component on a helicopter is its tail rotor. The purpose of this part is twofold. It allows the helicopter to yaw, to change angle of orientation with respect to its central z axis. It also cancels out the torque produced by the main rotor which allows the helicopter to hover in position without changing its heading and rotating about itself. The operation of the R/C tail rotor is identical to the full scale helicopter. Changing the angle attack of the tail rotor blades changes the lift they produce which, since the rotor is mounted horizontally, produces a thrust vector normal to the aircraft body in a direction opposite the torque produced by the main rotor.



**Figure 2.4: R/C helicopter tail rotor [5]**

## **3.0 Mission Specifications**

### **3.1 Objective**

While the implications of this project are as broad and expansive as they come, the final objective of this MQP is in fact very defined. It is true the goal is quite simply to create an autonomous helicopter; there are however, many details to that goal. Many attempts, some successful, others not, have been made at achieving autonomous flight, what separates this project from others is both the specific objective, and the path towards it.

The success of this project relied on creating a control scheme which allows an R/C hobby helicopter to autonomously maneuver to a predetermined location. A simple goal, yet there were many variables inhibiting its achievement.

In the following chapters the team will outline the precise steps and approach taken to get the helicopter to operate in this fashion. Component selection, hardware arrangement, the guidance and navigation strategy, as well as the inhibiting variables are described in detail.

### **3.2 Component Selection**

The selection of proper components was a vital first step in the completion of this project. Components had to meet strenuous regulations set forth by the team before they were discussed officially, presented, and ultimately acquired. Due to the high cost of precision aero and electrical parts required for the success of this project, it was imperative that the right components were attained. An untimely failure would not only set the project back, it would cripple it.

Different parts had different requirements, with a project of this caliber, borrowing from many disciplines; the various components had to meet specialized and particular requirements. Essential properties of the mechanical components included weight and strength. Desirable sensor properties were precision, accuracy, and data transmission rate. Cost transcended all categories. The following sections outline in detail the individual components purchased to construct the helicopter, how they work, and what part they play in the final product.

### 3.2.1 Chassis

The remote control helicopter model ultimately chosen was the T-Rex 450SE. Made by Align, a popular and highly recommended Taiwanese electronics company, the T-Rex 450SE is a mid-size R/C helicopter relative to others on the market.



**Figure 3.1: Basic T-Rex 450S model [4]**

Figure 3.1 shows the T-Rex 450SE base model without certain mechanisms required for flight, such as the servos, gyro and battery. The T-Rex's dimensions are 650mm (from tail to tip of main rotor blade) by 228mm tall with a ready flight mass of about 680g. The main frame components are made from high strength carbon fiber and aluminum alloy. As a result, this model is capable of both high stability in all reasonable weather conditions and precision flying.

The main rotor blades that came with the helicopter kit are Align 325mm Carbon blades, which are strong and stable in moderate flying conditions. However, carbon blades are not as efficient as wooden blades for hovering and low-speed maneuvers. Since this is how the helicopter will be behaving, the team has chosen to replace the current blades with Align PRO Wooden blades. These 335mm wooden blades will provide the most efficiency in terms of flight time.

### 3.2.2 Motor



**Figure 3.2: Align 430L 3550KV 300W Brushless Motor [4]**

The motor is the Align model number: 430L 3550KV 300W Brushless Motor. This is a highly efficient (91%) electric motor that is designed to be light weight, power-saving, and powerful. The motor is the main power source of the helicopter, providing power that turns both the main and tail rotor blades. The dimensions of this particular motor are 3.17x27.5x33mm; it weighs 58g and has a maximum continuous current of 28A.

### 3.2.3 Electronic Speed Controller



**Figure 3.3: Align 35A Brushless ESC [4]**

An electronic speed controller (ESC) serves to vary the motors speed based on the inputs it receives. The ESC chosen for this project is the Align 35A Brushless ESC. This particular ESC plugs directly into the receiver's throttle control and interprets control information from the transmitter. It supports a continuous current of 35A, is 45x23x12mm and has a mass of 25g.

### 3.2.4 Servos

A servomechanism (servo) is a device that provides control of a desired operation using feedback loops. An RC servo is made up of a DC motor that is linked to a potentiometer, which serves to send pulse signals that ultimately translate into position

commands. Servos are powered by the onboard battery and are plugged into the helicopters receiver.



Figure 3.4: Maxwell-MX56BB Ball-Bearing Servo [4]

Figure 3.4 shows one of the servos that are being used in the helicopter. Three of these control all pitch and roll functions of the main blades, while a fourth controls the pitch of the tail rotor.

### 3.2.5 Battery

Due to the mission requirements of this project, the selection of an efficient battery was of utmost importance. The most efficient type of battery that can be used on a RC model is the Lithium-Polymer pack. These batteries are split into multiple cells with the same standard voltage. Most Li-Po battery packs are 2-5 cells with a standard voltage of 3.7V per cell. Instead of the lithium-salt electrolyte being held in an organic solvent as in the Li-Ion battery, it is held in a solid polymer composite which provides a higher efficiency. The power/weight ratio of a Li-Po battery is  $\sim 2800$  W/kg compared to 1800 W/kg of Li-Ion.



Figure 3.5: APlus 2200mAh-25C 3S1P [4]

The battery chosen to mate with the helicopter was the APlus 3S1P, a 2200mAh Li-Po battery in a 3-cell series configuration. Each cell has a nominal voltage of 3.7V, giving it a total voltage of 11.1V. The battery is connected to the ESC, which connects to

the motor and the receiver. This battery is 100 x 34 x 23 mm, weighs 179g and has a standard discharge rate of 25C.

### 3.2.6 Receiver

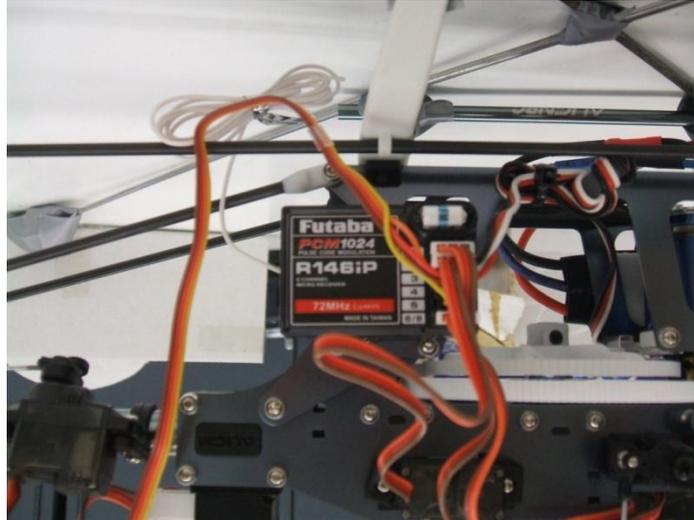


Figure 3.6: Futaba PCM R146ip Micro Receiver with connected components

Figure 3.6 shows the 6-channel receiver mounted, and connected to the various servos, gyro, and ESC. It receives radio signals from the remote control transmitter that it interprets and digitizes to enable a user to manually control all aspects of motion of the helicopter. This receiver is 28.7x42.7x20mm, weighs 16.5g, operates at 72 MHz frequency, and has a receiving range of up to 300m.

### 3.2.7 Transmitter



Figure 3.7: Futaba 6EXH Computer Transmitter 72MHz

Figure 3.7 shows the radio transmitter (remote control) that will be used to test fly the helicopter. The Futaba 6EXH Computer Transmitter is a 6-channel transmitter has all the controls necessary for manual helicopter flight. The right stick is used to control the throttle of the motor and the angle of attack of the tail rotor. The left stick is used to control the angles of attack of the main rotor blades and stabilizers. A convenient feature that this transmitter has is a simulator cable port, allowing one to connect it to a computer and use a simulator program to learn to fly.

### 3.2.8 Summary and Cost of Helicopter Components

In addition to the frame and blades commonly associated with helicopters, such a vehicle requires a battery, motor, ESC, servos, gyroscope, receiver and transmitter. All of these components need to work together in synchronization and without interference for flight to be successful. The choice to purchase a ready-to-fly (RTF) T-Rex 450SE kit that was professionally assembled and calibrated to was made to ensure that there were no mistakes in the construction and assembly of the helicopter. Additionally, purchasing a kit was considerably cheaper than obtaining parts individually.

Part	Cost	Qty	Total
Base kit (+motor/ESC/T-Rex/Blades)	\$200-250	1	\$225.00
Battery	\$55.00	1	\$55.00
Servos	\$22.00	4	\$88.00
Gyro	\$55.00	1	\$55.00
Motor	\$47.00	0	\$0.00
ESC	\$55.00	0	\$0.00
Tx/Receiver	\$160.00	1	\$160.00
Blades	\$20.00	0	\$0.00
<b>Total</b>			<b>\$583.00</b>

Table 3.1: Cost of individual parts of helicopter

Table 3.1 shows the individual cost of each part that would have needed to be purchase in order to build a comparable helicopter. The package purchased was the RTF T-Rex 450S from [www.Flying-Hobby.com](http://www.Flying-Hobby.com) for \$379 and included all of these parts 100% pre-assembled and ready to fly. This saved the project a considerable amount of time and money.

### 3.2.9 Processor

In order to setup communication between the various sensors, servos, and motor, the helicopter is fitted with a processor programmed with the necessary algorithms to attain flight. A microcontroller fits all the features of a computer: inputs, outputs, processing, and memory onto a single chip. It is the platform used to achieve this communication. The typical microcontroller only takes up one chip; as a result it is very small and uses very little power.

An alternative to a microcontroller is a programmable logic controller, or PLC. PLCs are microprocessors with additional hardware, used to simplify the user interface. They are often uses special-purpose programming languages that can be simpler than generic languages, such as C, for people with little programming experience. This however, is its only positive attribute. PLCs are sufficiently slower than other microcontrollers that one would not be capable of doing the necessary calculations fast enough to control our helicopter.

The microcontroller that was ultimately selected and fitted to the protoboard was the MSP430f1611 (Fig. 3.8). This particular model comes with 48 KB of flash memory, 10 KB of RAM, an 8 MHz processor, a 16-bit RISC architecture, and a 12 bit A/D converter, all running off of just 330  $\mu$ A at 3.3 V.



Figure 3.8: MSP430 microprocessor [7]

### 3.2.10 Sensors

#### 3.2.10.1 Accelerometers / Gyroscopes

The purpose of the accelerometers is to read the acceleration in each of the Cartesian axes. The accelerometers use the distortion of crystals to generate an electric current. The resulting voltage corresponds directly with the amount of acceleration it experiences. The gyroscopes are used to measure the angular velocity as well as filter out the angular acceleration from the accelerometers. By taking the derivative of the angular velocity over an increment of time and subtracting it from the same axis's accelerometer's reading, the translational acceleration is found.

The team has selected to use a five degree of freedom chip that combines three accelerometers (ADXL330) with two gyroscopes (IDG300) in their respective axis. A third gyroscope of the same type was also selected in order to complete the needed number of sensors. Since the third gyroscope will be used to stabilize the yaw, it is not a problem to have it separate from the five degree of freedom board as the control equation for the yaw is not coupled to any other sensors.

### 3.2.10.2 Sonar

The purpose of the sonar is to corroborate the interpreted data from the Z-axis accelerometer and reduce any error. The sonar selected for these Z displacement measurements is the LV-MaxSonar EZ1 (see Fig. 3.9).



Figure 3.9: LV-MaxSonar EZ1 [6]

Selected for its compact size, this particular sonar comes with the added benefit of minimal power consumption, requiring only 2mA. In addition, this sensor could also detect objects from 0-254 inches with a 1 inch error, falling perfectly within the mission requirements of this project. Figure 3.10 depicts the sonar beam, dispersed over a 12” grid, used to measure a 3.25” dowel. The red dots represent a beam supplied with 3.3 volts and the solid line represents a beam supplied with 5 volts.

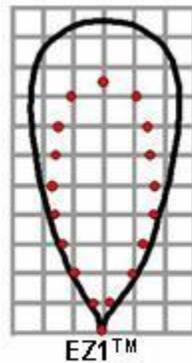


Figure 3.10: Sonar beam dispersion [6]

## 4.0 Helicopter Dynamics

For the purposes of controlling a helicopter, it is necessary to understand the fundamental equations governing the forces in play, how they develop and how they produce motion in the helicopter. The following sections describe in detail the various forces produced by the helicopter blades as well as their derivation, how these forces can be manipulated to produce desired results in hovering, and how these forces translate to producing actual motion.

### 4.1 Blade Theory

Blade theory for helicopters differs from that of a fixed wing aircraft primarily in that the velocity across a helicopter's blade is not constant across its length. Since it is a rotating assembly, the further a point is from the center, the higher its rotational velocity. For a fixed wing aircraft, lift is a product of atmospheric conditions, shape of its wing, and velocity.

$$dL = \frac{1}{2} \rho V^2 c C_l dy \quad (4.1)$$

As the velocity is constant across the wing equation (4.1) can be written:

$$L = \frac{1}{2} \rho V^2 A C_l \quad (4.2)$$

For a helicopter, the velocity increases as distance ( $y$ ) increases. The velocity is equal to the rotational speed ( $\Omega$ ) multiplied by the distance from the center ( $y$ ). With this in mind equation (4.1) becomes:

$$dL = \frac{1}{2} \rho (\Omega y)^2 c C_l dy \quad (4.3)$$

By integrating across the entire length of the blade ( $R$ ) we get (for a single blade):

$$dL = \frac{1}{6} \rho (\Omega R)^2 c R C_l \quad (4.4)$$

In vertical flight, the thrust is equal the lift. Therefore, if:

$$T = \frac{1}{2} \rho (\Omega R)^2 \pi R^2 C_T \quad (4.5)$$

Then:

$$C_T = \frac{1}{6} C_l \sigma \quad (4.6)$$

Where ( $\sigma$ ) is the solidity factor  $\sigma = \frac{Nc}{\pi R}$ . When we combine equations (4.5) and (4.6) we find the basic equation for thrust in vertical flight:

$$T = \frac{1}{12} \rho (\Omega R)^2 N C_T R \quad (4.7)$$

At this time it is also important to note that:

$$T = 2\rho A V_i^2 \quad (4.8)$$

Where ( $V_i$ ) is the velocity of the air pulled through the blades and ( $A$ ) is disk area. Therefore:

$$V_i = \sqrt{\frac{T}{2\rho\pi R^2}} \quad (4.9)$$

This will play an important role when calculating the climb and decent, as well as the power requirements later on.

## 4.2 Optimizing for Hover

Optimizing hover is a simple concept of minimizing the power while maintaining the minimum force required to hover. Finding this value, however, is not quite as simple. The independent variables that can be changed in an attempt to minimize power are blade size, cross section, number of blades, mass, and rotation speed.

The first task in optimizing for hover is choosing a shape for the cross section. The shape will yield a relationship between its angle of incidence ( $\alpha$ ) and the coefficient of lift. For the purposes of this project, the cross section is assumed to be symmetric across its chord, as most available blades are. A symmetric airfoil's lift coefficient will increase linearly with the angle of incidence until it approaches its stall angle.

$$C_L = a\alpha \quad (4.10)$$

This is a deceptively complicated relationship because the angle of incidence in hover is directly dependent on the rotational speed and angle of attack ( $\theta$ ). The angle of incidence is found by subtracting the induced angle ( $\phi$ ) from the angle of attack. The induced angle is found by the equation:

$$\phi = \tan^{-1} \left[ \frac{V_c + V_i}{\Omega R} \right] \quad (4.11)$$

Where  $V_c$  is the speed of the helicopter and the following for hover where the velocity is zero:

$$\phi = \tan^{-1} \left[ \frac{V_i}{\Omega R} \right] \quad (4.12)$$

The induced angle is the result of the relative air velocity across the blade moving in two directions. The first,  $\Omega R$ , is the speed of the air relative to the plane the blades are moving in; the second,  $V_i$ , is the velocity of the air being pulled perpendicularly through the blade.

Considering that the thrust must equal the weight of the aircraft in hover, the angle of attack must be balanced with the rotational speed while in flight. While this is true with the other values of the blade, angle of attack is the easiest to change and implement. Consider equations (4.7), (4.11), and (4.12). The relationship between these variables is:

$$\theta = \frac{12W}{(\Omega R)^2 a N c R C_T \rho} + \frac{V_i}{\Omega R} \quad (4.13)$$

With the angle of attack known with respect to the rotational velocity, it can be eliminated as a variable with only rotational speed left as the independent term.

### 4.3 Equations of Motion

Rotational motion of a helicopter can be described in three terms: pitch, roll, and yaw which coincide with  $p$ ,  $q$ , and  $r$  respectively. These  $p$ ,  $q$ , and  $r$  represent angular velocities in the  $x$ ,  $y$ , and  $z$  direction respectively. These angular velocities are directly measured using three analog gyroscopes; and three accelerometers all of which are found on the inertial board. Solving for the rotational movement of the helicopter establishes three 2<sup>nd</sup> order differential equations:

$$\dot{p} = \frac{l_{xz}\dot{r} - [l_{zz} - l_{yy}]qr + l_{xz}pq + M_1}{I_{xx}} \quad (4.14)$$

$$\dot{q} = \frac{-[l_{xx} - l_{yy}]rp - l_{xz}[p^2 - r^2] + M_2}{I_{yy}} \quad (4.15)$$

$$\dot{r} = \frac{l_{xz}\dot{p} - [l_{yy} - l_{xx}]pq - l_{xz}qr + M_3}{I_{zz}} \quad (4.16)$$

Solving these equations for  $M_1$ ,  $M_2$ ,  $M_3$ , the  $x$ ,  $y$ , and  $z$  direction torques, we find the forces on the helicopter which must be countered by inputs to the servo motors. These inputs will control the rotors of the helicopter and will provide an equal and opposite torque to counter the random perturbations associated with a stable hover position. In order to obtain more manageable results, the equations were linearized to obtain the following rotational velocities:

$$\dot{p} = \frac{l_{xz}\dot{r} + M_1}{I_{xx}} \quad (4.17)$$

$$\dot{q} = \frac{M_2}{I_{yy}} \quad (4.18)$$

$$\dot{r} = \frac{l_{xz}\dot{p} + M_3}{I_{zz}} \quad (4.19)$$

$$\dot{\phi} = p \quad (4.20)$$

$$\dot{\theta} = q \quad (4.21)$$

For the purposes of locating the helicopter in the inertial frame it became necessary to model the flight path of the helicopter during operation. Unlike fixed wing aircraft, the translational movement of the helicopter is necessarily coupled to the rotational motion of the helicopter. This led to three 2<sup>nd</sup> order differential equations consisting of terms for inertial matrix tensors and two Euler angle equations.

$$\dot{u} = rv - qw + \frac{x}{m} - g \sin \theta \quad (4.22)$$

$$\dot{v} = pw - ru + \frac{y}{m} + g \cos \theta \sin \phi \quad (4.23)$$

$$\dot{w} = qu - pv + \frac{T}{m} + g \cos \theta \cos \phi \quad (4.24)$$

$$\dot{\phi} = p + q \sin \phi \tan \theta + r \cos \phi \tan \theta \quad (4.25)$$

$$\dot{\theta} = q \cos \phi - r \sin \phi \quad (4.26)$$

The linearized equations of translational motion are as follows:

$$\dot{u} = \frac{x}{m} - g\theta \quad (4.27)$$

$$\dot{v} = \frac{y}{m} + g\phi \quad (4.28)$$

$$\dot{w} = \frac{T}{m} + g \quad (4.29)$$

In the body frame of the helicopter, the only force it will be generating is thrust, which is always in the Z direction. For this reason, the variable Z has been replaced with T, for thrust. In the body frame there are no X and Y direction forces produced by the helicopter, the only source of these forces is outside the control of the helicopter itself, for this reason they can be neglected.

#### 4.4 Thrust Approximation

To simplify controlling the thrust of the helicopter, the rotation speed should be set at a high constant value. Unfortunately it is difficult to see the effects on the thrust due to a complex equation that results from balancing equations (4.1) through (4.9). The solution lies in approximating the equation for the coefficient of thrust.

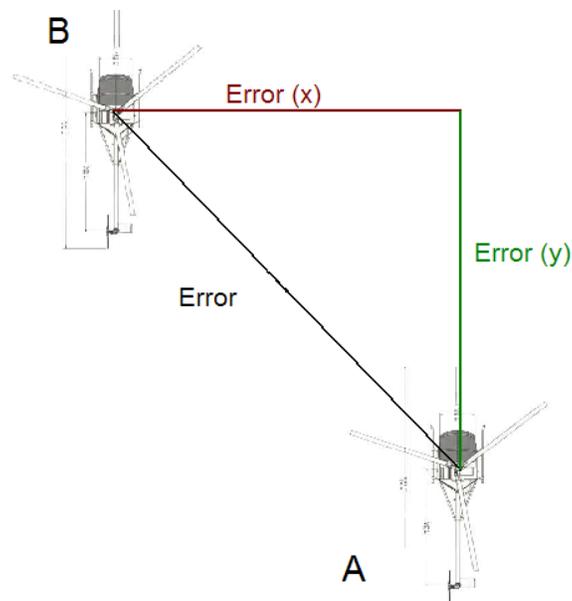
$$C_t = \frac{\sigma a}{2} * \left( \frac{1}{3} * \theta - \frac{\lambda}{2} \right) \quad (4.30)$$

$$\lambda = \frac{\sigma a}{1.15 * 16} * \left[ \sqrt{1 + \frac{32 * \theta}{(1.15^2 * \sigma a)}} - 1 \right] \quad (4.31)$$

## 5.0 Flight

### 5.1 Flight System Overview

The flight system chosen is, in principle, a simple one. Essentially, there are two positions in the helicopters reference frame, where it is, and where it needs to be. Figure 5.1 demonstrates this, position A is where the helicopter needs to be, this can be its start location in the case of hovering flight, or this can be where it needs to travel to in the case of guided flight. Position B, is where the helicopter is in relation to the inertial frame. The quantity “Error” is the distance between these two points. It is the job of the helicopters on board computer to constantly calculate and update this quantity and compensating for it with corrections and inputs.



**Figure 5.1: Flight system diagram**

The processor relies on integrating the inputs from the accelerometers and gyroscopes to obtain the helicopters position as described in Section 4.3. These values are then translated into the Error quantity, signifying how far its current position is from position A, its destination. Depending on the magnitude of this Error figure, the computer will output a signal to the various servos and motor to obtain a force vector with the proper magnitude and direction to reduce the error quantity. The magnitude of this force vector is chosen based on the equations of motion outlined in the preceding section.

To understand what exact signal the microprocessor needs to output to the servos and motor in order to get the desired reaction in depth analysis and testing of these mechanisms became necessary. The following sections outline in detail the tests done and the results that were obtained.

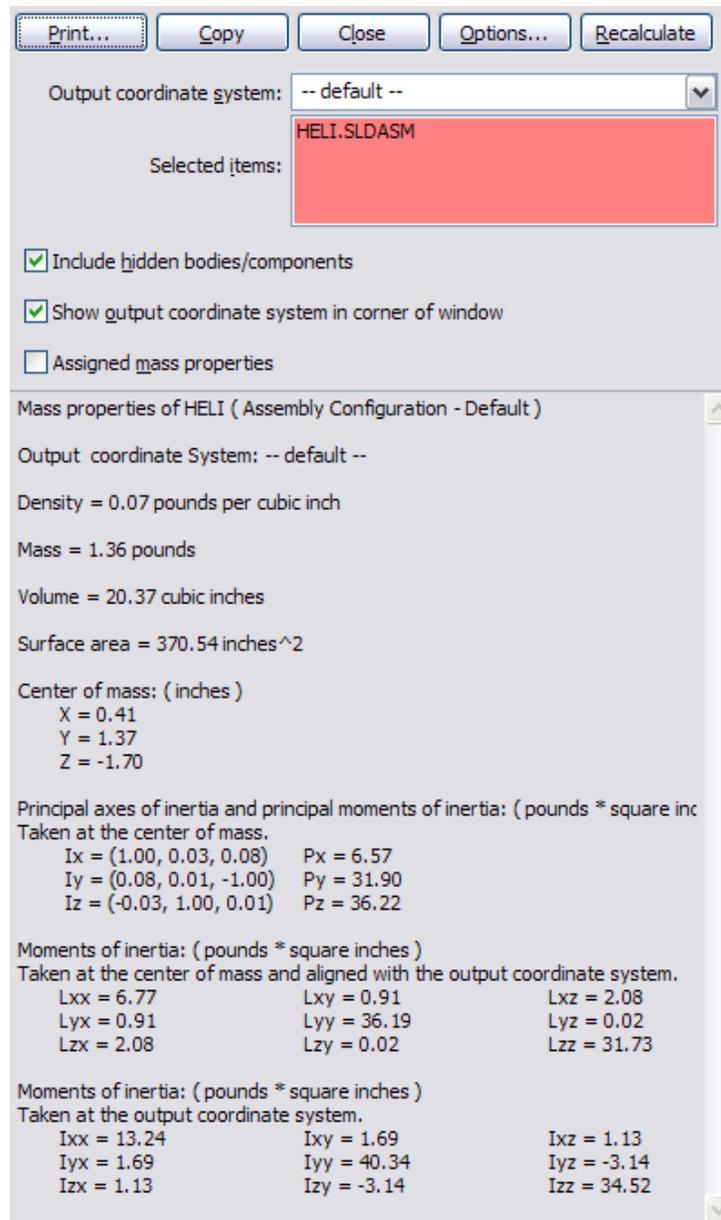
## 5.2 Moment of Inertia Calculations

As outlined in the Section 4.3, among the variables that need calculating are the moments of inertia of the helicopter. The team set about acquiring these moments by creating a scale model using the CAD program SolidWorks<sup>®</sup>. Figure 5.2 shows the completed model.



**Figure 5.2: SolidWorks model of helicopter**

Each component of the model has been sized and massed to the specifications of the real helicopter. As a result, the SolidWorks<sup>®</sup> model is not only built to scale, but it contains all the same properties as the actual model. The properties of interest for the purposes of this project are center of mass, and moments of inertia. Figure 5.3 shows these properties which were calculated using the SolidWorks<sup>®</sup> mass properties plug-in.



**Figure 5.3: Helicopter moments of inertia**

The values of interest are the moments taken at the output coordinate system. These values, which remain constant, were taken and used in the angular acceleration and velocity equations of motion.

## 5.3 Servo Control

### 5.3.1 Motor RPM Analysis

In order to obtain the necessary thrust of the helicopter, there must be a known relationship between the control signals and the rotational speed of the rotor. Without an encoder to monitor the rotation speed and create a closed loop control, it becomes necessary to make an equation that changes the signal depending on the angle. This can be done through the use of the data gathered through the following rpm tests coupled with the power equations.

In order to find this relationship, a circuit was constructed containing a photoelectric resistor and a resistor of equal impedance. The circuit was placed directly underneath the rotating blades with a laser pointer positioned directly over the blades aimed at the photoelectric resistor. As the rotor spins, the blades break the laser beam thereby changing the resistance of the photoelectric resistor (Fig. 5.4). This change in resistance was monitored using Simulink<sup>®</sup> through dSPACE<sup>®</sup>, which measured the frequency of these resistance changes to obtain the rpm of the rotor.

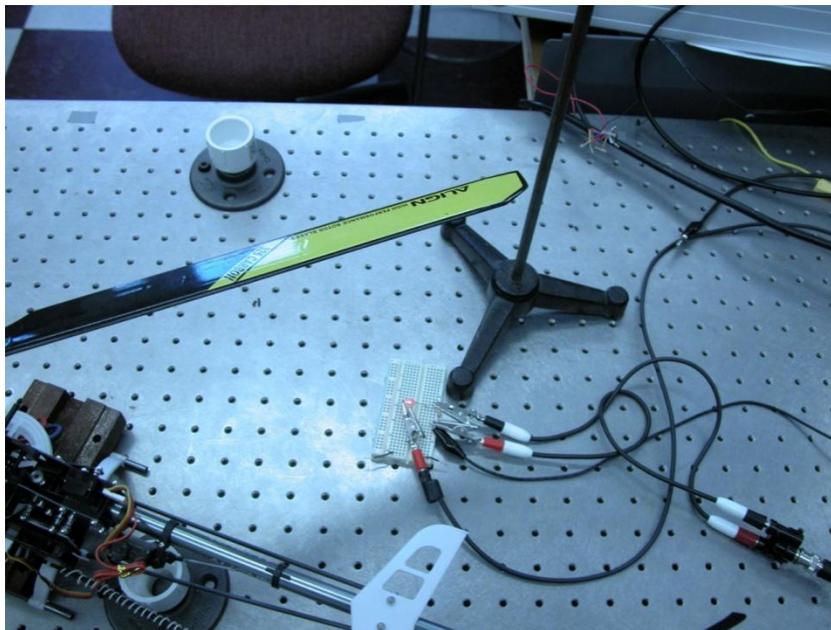


Figure 5.4: Photo resistor tachometer setup

Fig. 5.5 shows rpm data as a function of controller throttle collected at zero angle of attack.

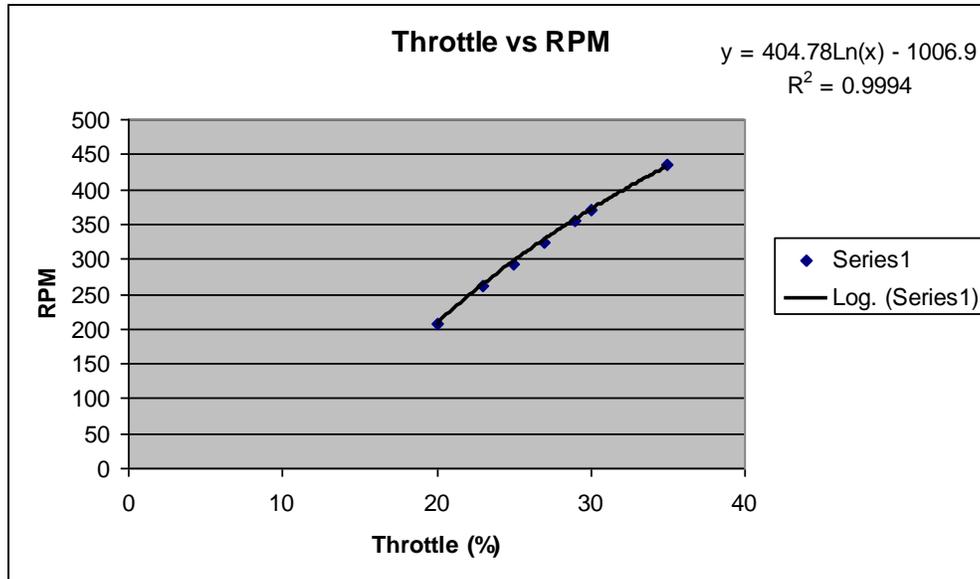
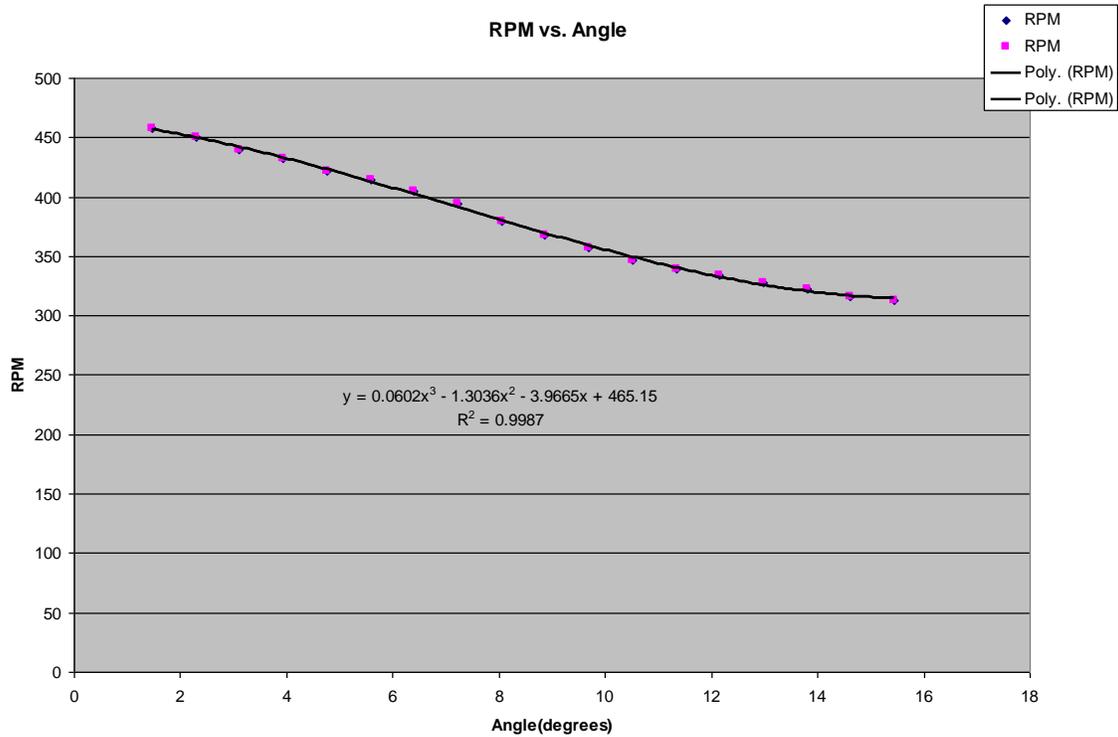


Figure 5.5: Throttle vs. RPM chart

The next step is linking rpm with motor input while accounting for variations in rpm due to different load conditions. These different load conditions arise due to the increase in drag as angle of attack is increased. As such, rpm is not only a function of voltage input, but of blade angle of attack as well.

Fig. 5.6 shows this relation between rpm and angle of attack. In this test, the throttle was held constant at 20% while angle of attack was varied from its minimum to its maximum setting. The tests for this recorded data for 1.5 seconds with a sampling rate of 50 microseconds using dSPACE<sup>®</sup>. The period was approximated using Microsoft Excel<sup>®</sup> to calculate the number of samples between a significant change in the voltage across the resistor. When analyzing the data it was taken into account that the beam of light was broken twice per revolution due to there being two blades in the main rotor. As such the voltage changed across the photoelectric resistor twice per revolution.

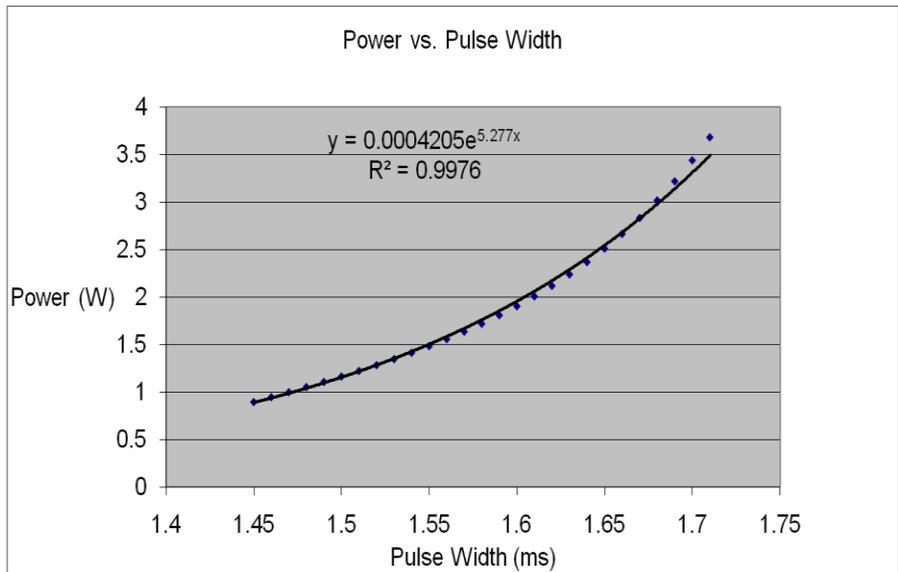


**Figure 5.6: RPM vs. Angle of attack depicting drag effect on blade rotation**

The next step was to correlate power with motor input. The pulse-width modulation (PWM) signal to the speed controller was measured and recorded using Simulink<sup>®</sup> and dSPACE<sup>®</sup>. This signal was relate to power produced using the equations below. The relation is shown in Fig. 5.7.

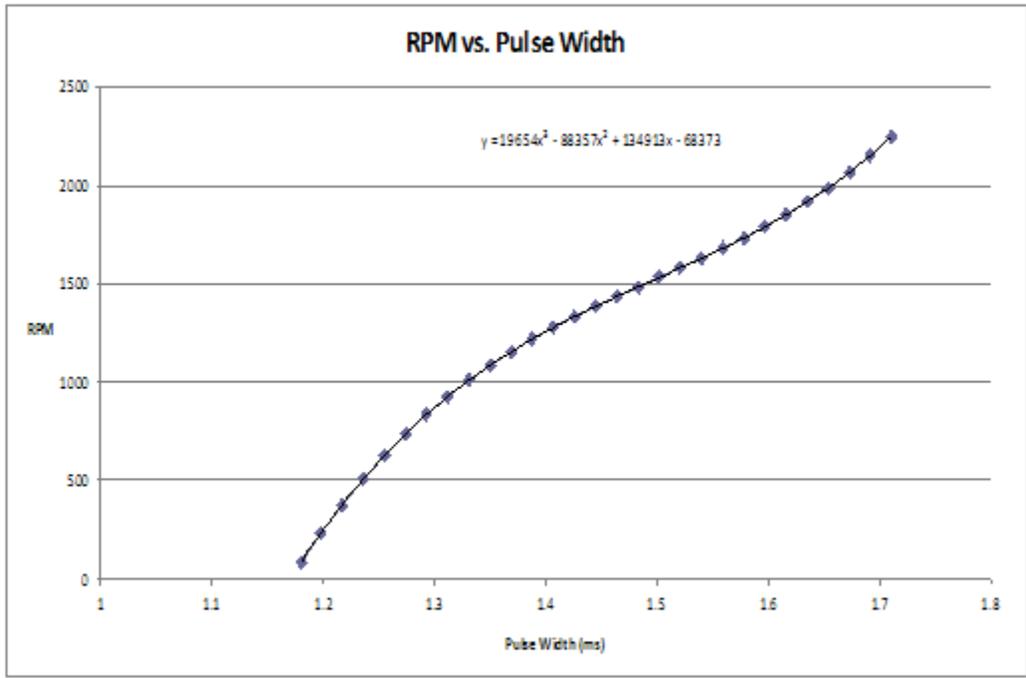
$$P = \rho A (\Omega R)^3 C_p \quad (5.1)$$

$$C_p = \sqrt{0.5 C_t^3 + \frac{C_d}{8}} \quad (5.2)$$



**Figure 5.7: Power vs. Pulse width to Speed Controller**

Using the data from the rpm vs. pitch, these equations are used calculate an approximation for  $C_d$  vs. pitch by assuming  $P$  is constant. Using data from rpm vs. throttle a good approximation for  $P$  vs. throttle can be found. These two approximations use the approximated  $C_t$  equation. In order to find the throttle needed to keep rotation constant, reapply equation (5.1) by replacing  $P$  with the throttle equation and the rotation speed with the desired value. To simplify the calculation for the required pitch to get the desired thrust, using the exact rotor dynamic equations will reduce the complexity significantly. The final relation, pulse width to rpm is shown below in Fig. 5.8.



**Figure 5.8: RPM vs. Pulse Width to Speed Controller**

### 5.3.2 Servo Angle Analysis

With main blade angle of attack acting as a major variable in the equations for thrust, the team set about studying the relationship between servo input and angle of attack, their net output. Measurements of the pulse width delivered to each of the three servo motors controlling the inclination of the swashplate were taken using dSPACE® (Fig. 5.9). These were compared with the resultant angle using an inclinometer and dual level setup ensuring the straightness of the fly bar assembly (Fig. 5.10).

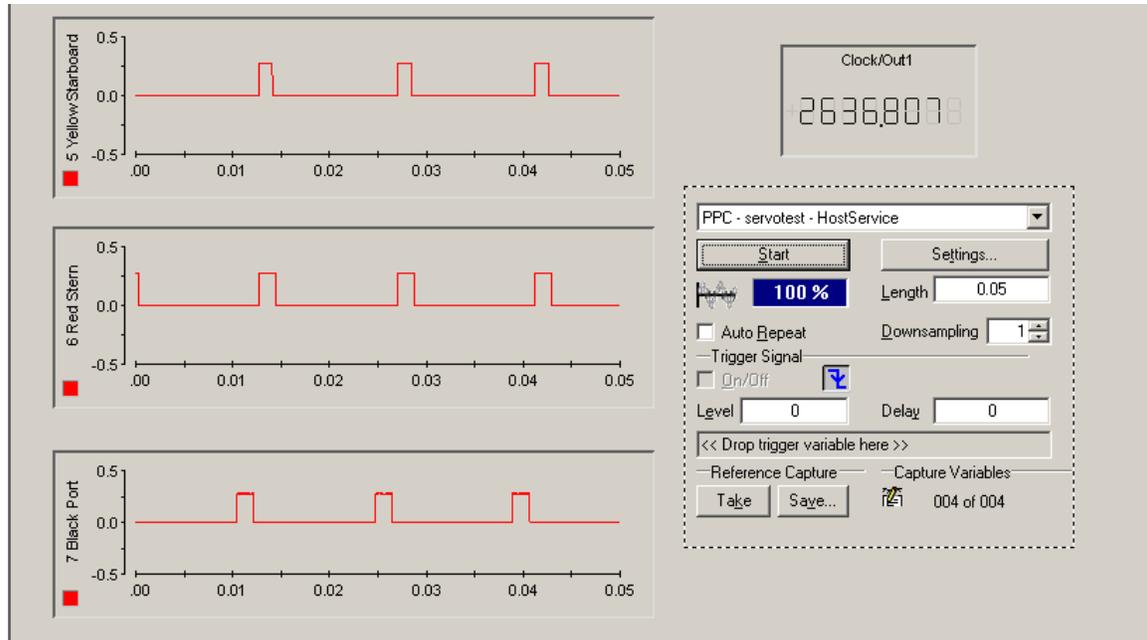
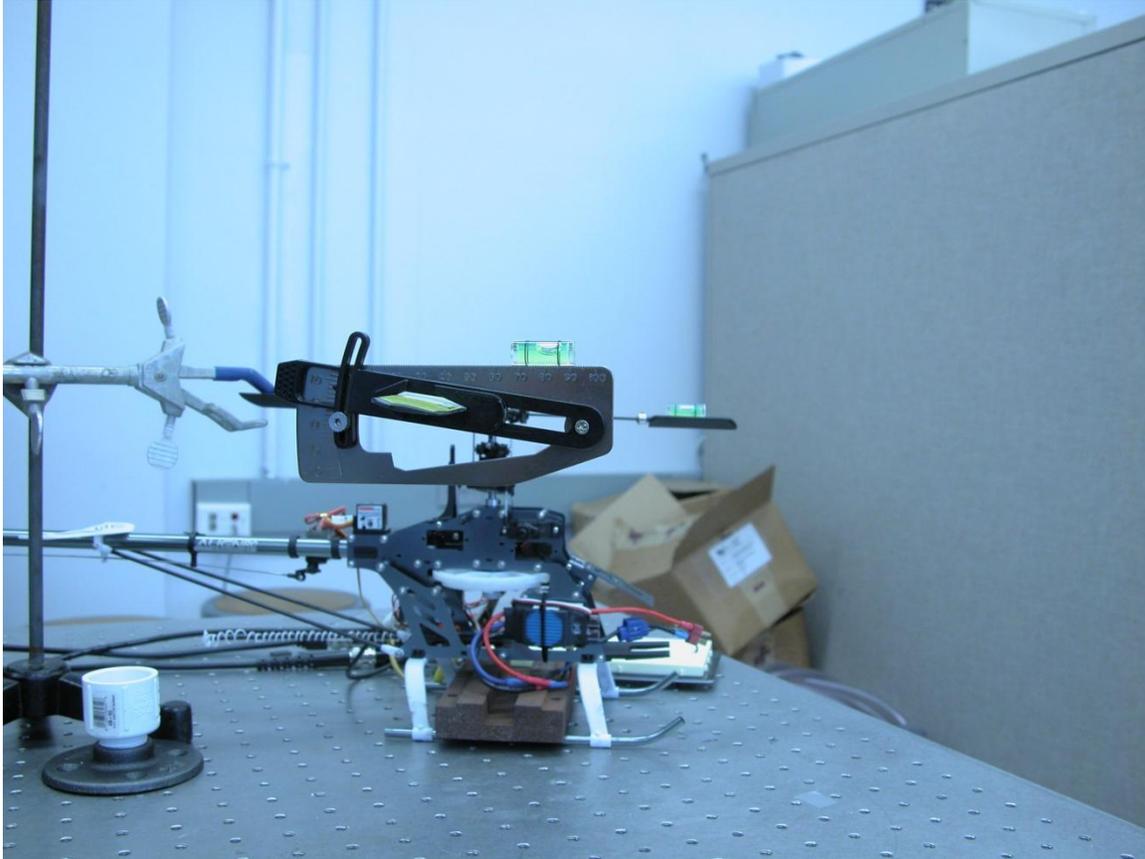
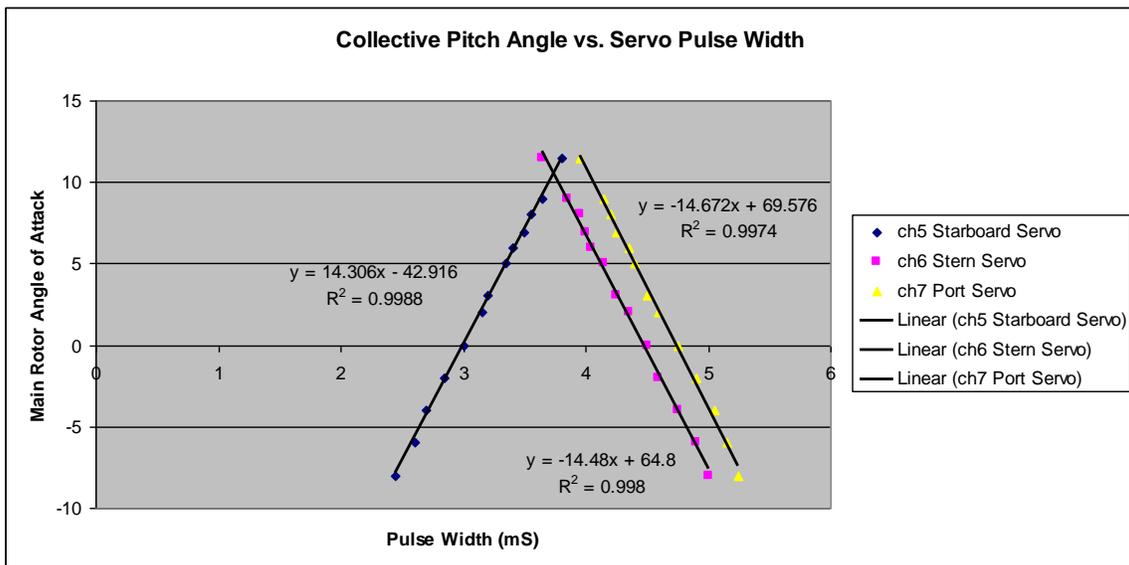


Figure 5.9: dSPACE® layout used to collect servo pulse width



**Figure 5.10: Measuring blade angle of attack**

Figure 5.11 shows the resultant relationship between the pulse width and blade angle. With the data for each servo motor showing higher than 99% correlation between data points, it is safe to use the resultant linear relationships in the control program algorithms.



**Figure 5.11: Collective Pitch Angle vs. Servo Pulse Width**

It should be noted that inducing cyclical pitch, motion in either the X or Y direction changes the position of the servo motors and therefore the pulse they require. This behavior is further analyzed in Section 5.5.

## 5.4 Control

The following section illustrates the behavior of the craft induced by changes in certain parameters. It shows the implementation of the previously developed equations of motion and simulates helicopter motion using MATLAB®.

In order to effectively control a vehicle, all of the states in its equations of motion must converge to their desired values, and, by extension, their derivatives must approach zero. In the case of the project's helicopter, the only variables that can be controlled are the torques and the thrust. The most computationally efficient way to use these control variables is to assign one control to one state. Looking back at the non-linearized equations, this cannot be done by algebraically finding which variable controls which state since they are all coupled.

The most effective method for controlling the position of the helicopter is to separate the longitudinal controls from the lateral controls. The only control that can be applied to the lateral equations is the thrust. In the longitudinal equations, the most effective use of the third torque ( $M_3$ ) is to keep the yaw stable, leaving us free to control longitudinal position through the first two torques ( $M_1$  and  $M_2$ ). To decide what torque applies to what translation, the rotational equations must be ignored. The linearized translational equations show that the only state that causes acceleration in the longitude is the angular orientation. Manipulating equations (4.25) and (4.26) into a three by three matrix (matrix G) allows the connection between  $M_1$  and  $M_2$  to the angular orientation in the inertial frame. From all this we find that at small angles,  $M_1$  and  $M_2$  control the motion in the y and x axes, respectfully.

In order to get each state to approach its desired value, the difference between the current states and the desired states (sub d) are treated as errors. For the purposes of this project, four error equations are used to govern each control (all in inertial frame).

$$e_1 = k_1(P - P_d) \quad e_1 = k_1(P - P_d) \quad (5.3)$$

$$e_2 = k_2(\dot{P} + e_1) \quad e_2 = k_2(\dot{P} + e_1) \quad (5.4)$$

$$e_3 = k_3(\xi - \xi_d) \quad (5.5)$$

$$e_4 = k_4(\dot{\xi} + e_3) \quad e_4 = k_4(\dot{\xi} + e_3) \quad (5.6)$$

$$M_1 = -e_1 - e_2 - e_3 - e_4 \quad (5.7)$$

$$M_2 = -e_1 - e_2 - e_3 - e_4 \quad (5.8)$$

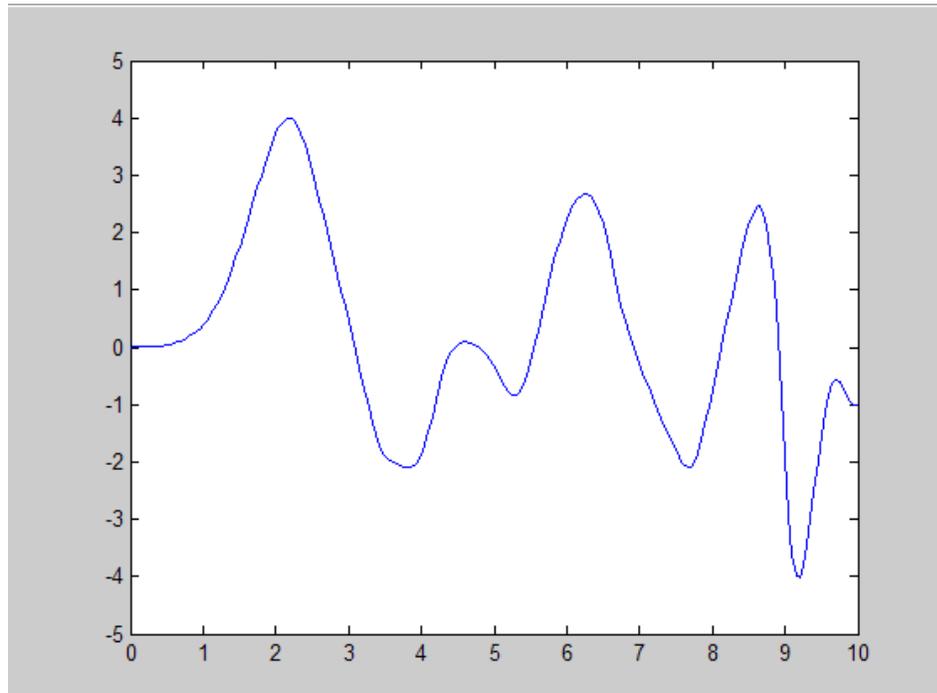
$$M_3 = -e_3 - e_4 \quad (5.9)$$

$$T = 9.8 * m(-e_1 - e_2 + 1) \quad (5.10)$$

These equations are then used to make the states move toward their desired values. In the case of  $M_1$ , P is y and  $\xi$  is roll, while  $M_2$  uses x and pitch for P and  $\xi$  respectfully. Since we are only using  $M_3$  to keep the yaw stable,  $\xi$  will be yaw and we will ignore  $e_1$  and  $e_2$ . Similarly for the thrust, we will be using z for P, and will ignore  $e_3$  and  $e_4$ . The thrust also needs to compensate for the effects of gravity. It should also be

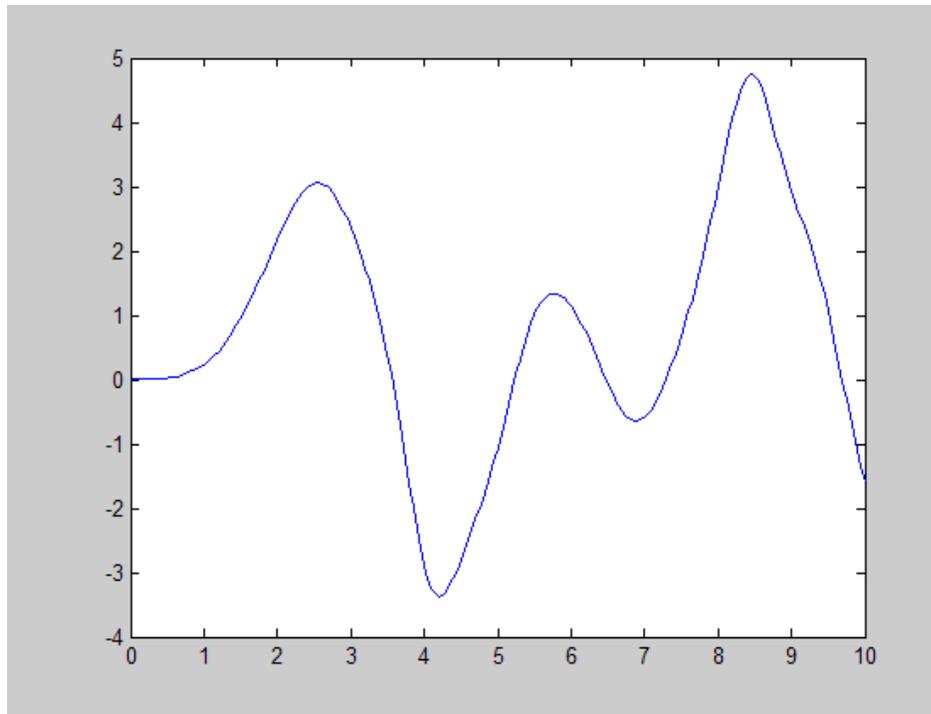
noted that  $k$  values may differ between moments due to the difference in moments of inertia. The value of  $k_1$  and  $k_3$  determine how fast the helicopter will move to the desired state while  $k_2$  and  $k_4$  will determine how little overshoot there will be. In order for dampening to occur,  $k_2$  should be larger than  $k_1$  and  $k_4$  should be larger than  $k_3$ . In most cases  $k_3$  should be larger than  $k_1$  to ensure that the helicopter does not flip upside-down.

To demonstrate the process of choosing the  $k$  values, let us look at the MATLAB<sup>®</sup> simulation from Appendix D. It should be noted that the  $k$  values for  $M_1$  are multiplied by a factor of three to make up for the different inertial moments. Let us start with all the  $k$  values at 1 and examine its effect on the  $x$  position over time when the desired state is  $x=1$  and pitch is zero. (Initial values are all set at zero.)



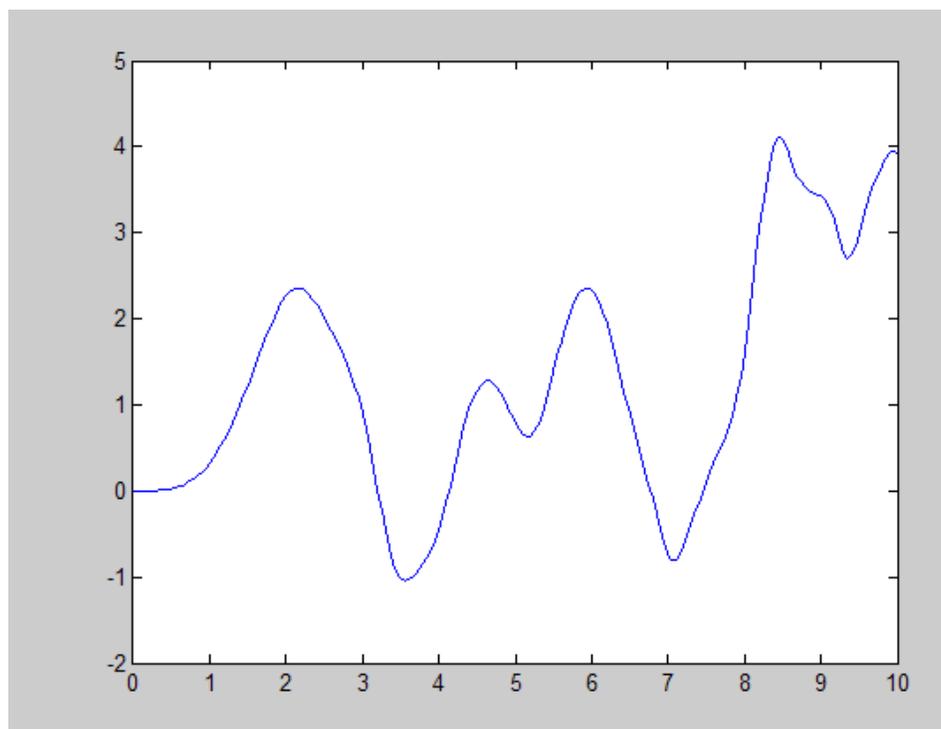
**Figure 5.12: All  $k = 1$ ; effect on  $x$  position over time**

As mentioned before, it is necessary to have a smaller  $k_1$  value than the other  $k$  values. So this is the effect of setting  $k_1$  to 0.5.



**Figure 5.13:  $k_1 = 0.5$ ; effect on  $x$  position over time**

This causes the controls to focus less on the distance from the desired position and more on the other desired states. If the value of  $k_1$  continued to decrease it would take longer and longer for the state to converge. So now it is time to increase  $k_2$ .



**Figure 5.14: Increase  $k_2$ ; effect on  $x$  position over time**

Under these constants the helicopter is trying to the desired x position but can't equalize the angular velocity well enough to cause convergence. The adjustment for this is the increase of  $k_4$  to 2.

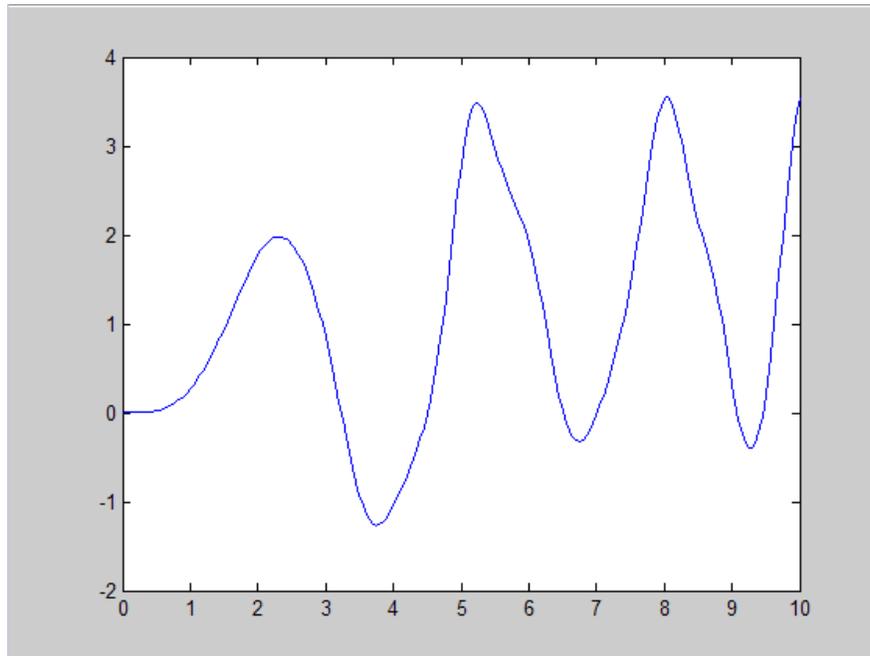


Figure 5.15:  $k_4 = 2$ ; effect on x position over time

When this is done the results are beginning to reflect the desire to move all state to their desired values. Increasing the values of  $k_2$  and  $k_4$  will make the goal seeking behavior more and more apparent. Continuing this process we eventually see what the effect of  $k_2$  and  $k_4$  equaling 32.

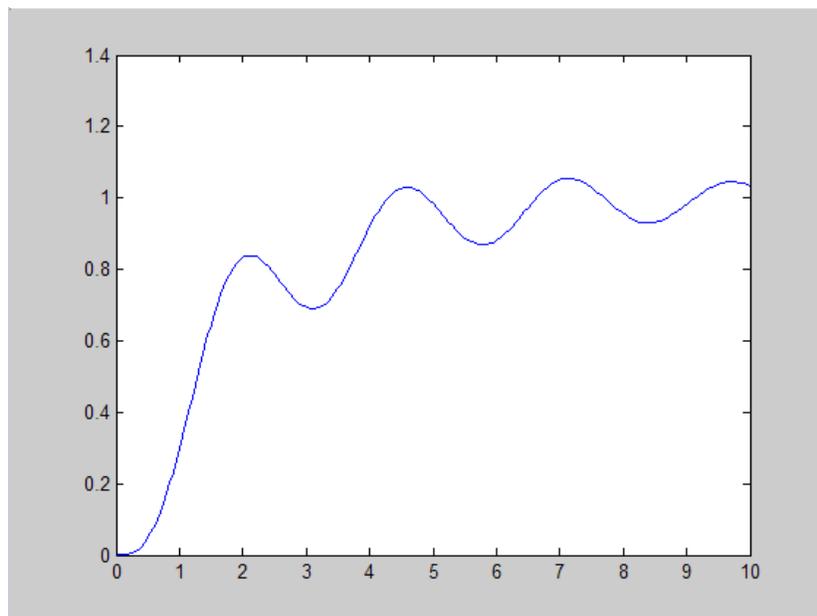
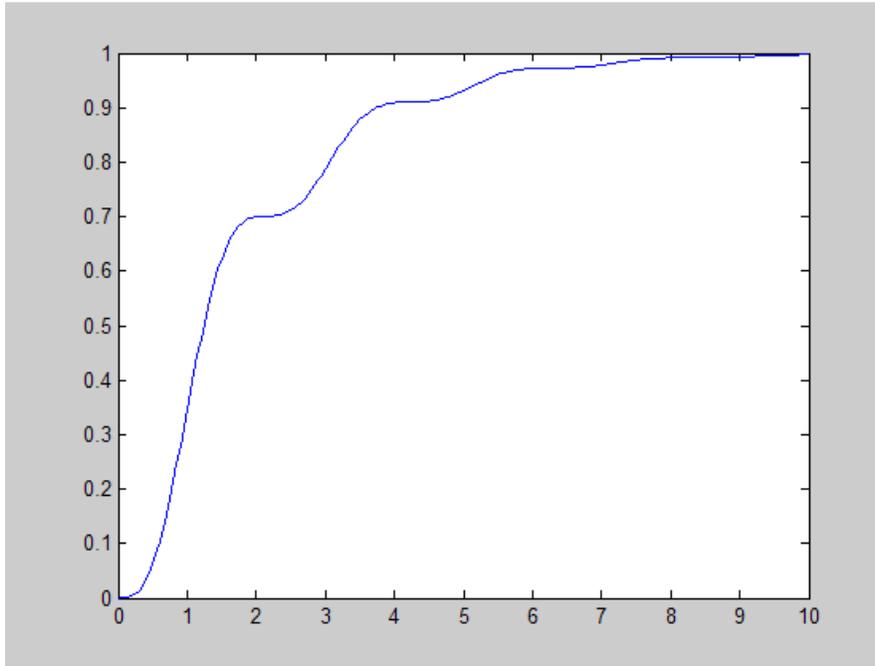


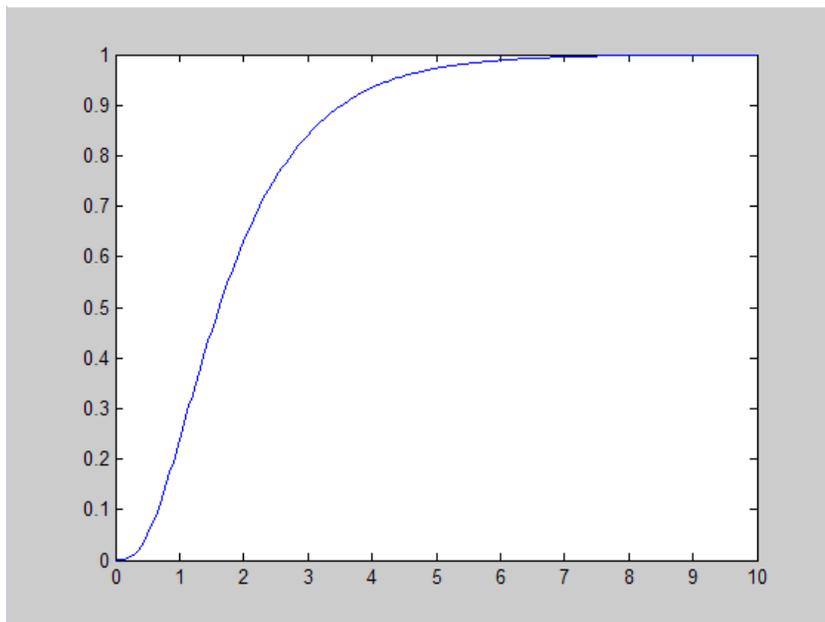
Figure 5.16:  $k_2 = k_4 = 32$ ; effect on x position over time

Now the dampening is apparent and the states are converging to their desired values. The problem now is that the process is not smooth enough. To smooth out the convergence, decrease  $k_4$  and slightly raise  $k_3$ . This will increase the rate at which the helicopter attempts to pitch to zero and decrease the resistance of keeping it there. When  $k_3=2$  and  $k_4=20$  the following results occur.



**Figure 5.17:  $k_3 = 2$ ,  $k_4 = 20$ ; effect on x position over time**

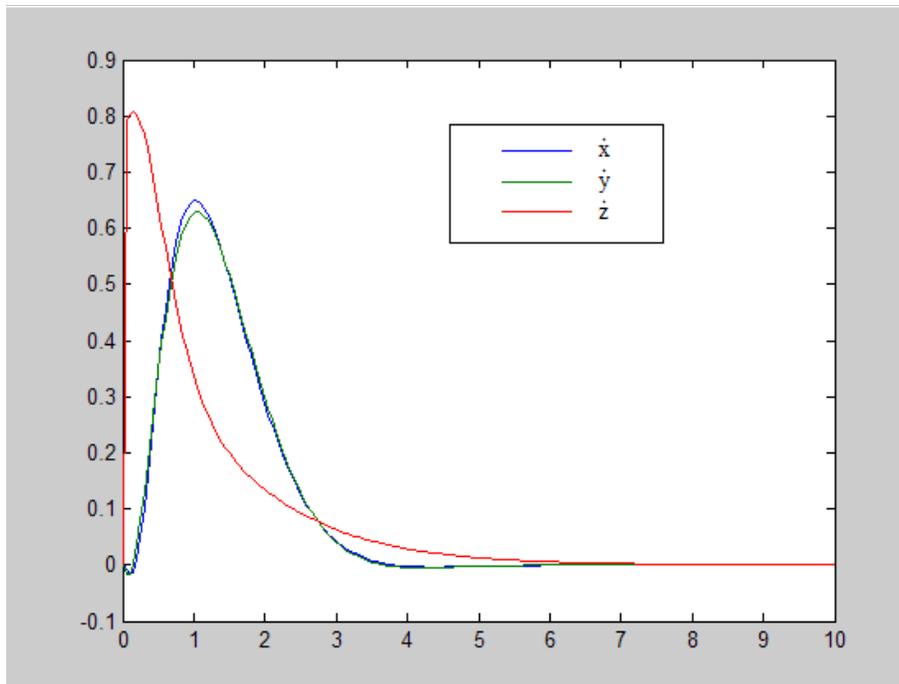
This change clearly results in a smoother convergence to the desired state. In order to get a near perfect curve the  $k_3$  constant is increased to 5.



**Figure 5.18:  $k_3 = 5$ ; effect on x position over time**

The results of this are a stable transition to the desired value. Now that the ratios between  $k_1$  and  $k_2$ , as well as  $k_3$  and  $k_4$ , are visible, the adjustments are made in order to determine how fast the helicopter should converge. In the previous settings it took approximately seven seconds to reach their desired state. I should be noted that this is the time it will take to reach any desired state given that the helicopter is capable of the producing the necessary accelerations. This is where the balancing act begins. The speed at which the helicopter converges is essentially how much it can recover from outside forces. One solution is to add another set of errors or another layer of control in order to regulate the response to outside forces. This is not practical for this project as the current control is already very taxing on the microprocessor. The other solution is to reach a balance where the states converge quickly without being outside the helicopters ability. Another consideration is the error between the desired controls and the generated controls. The faster the simulation converges, the more precise both the sensors and generated forces must be.

When considered that this helicopter is meant to fly indoors without major air disturbances, acceptable values for the k constants are  $k_1=.75$ ,  $k_2=35$ ,  $k_3=5$ ,  $k_4=20$ .



**Figure 5.19: Translational velocities vs. time**

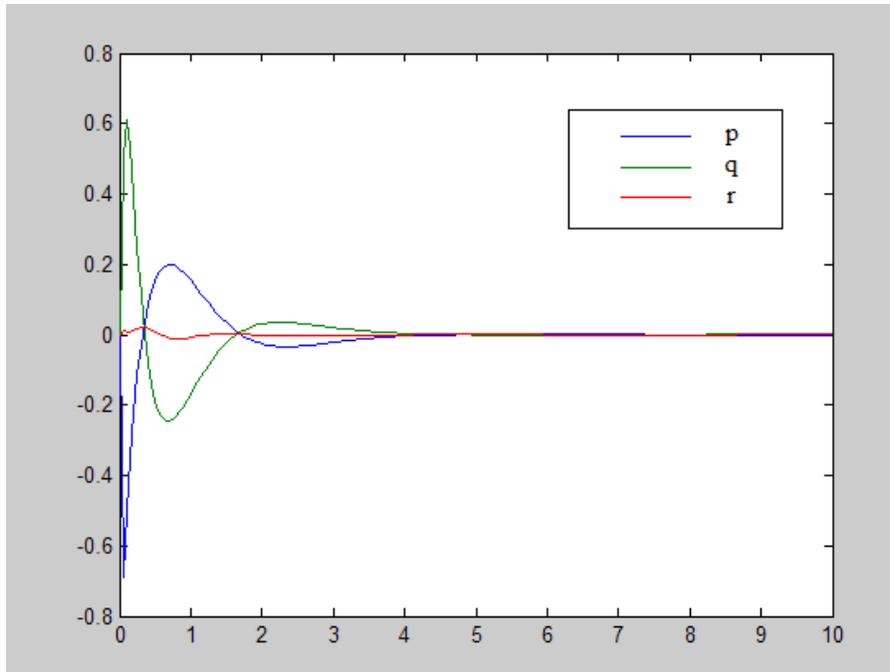


Figure 5.20: Rotational velocities vs. time

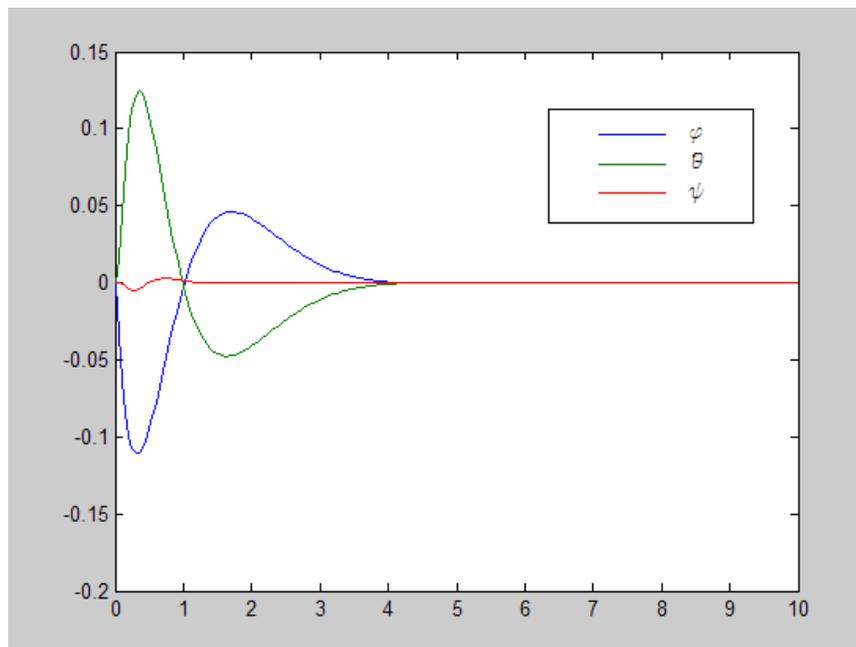
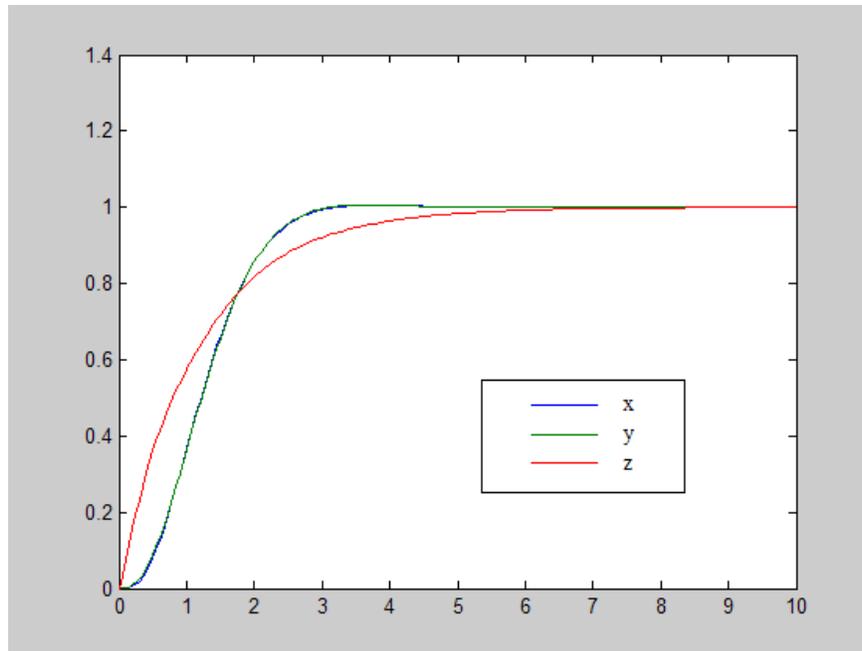


Figure 5.21: Orientation angles vs. time



**Figure 5.22: Position vs. time**

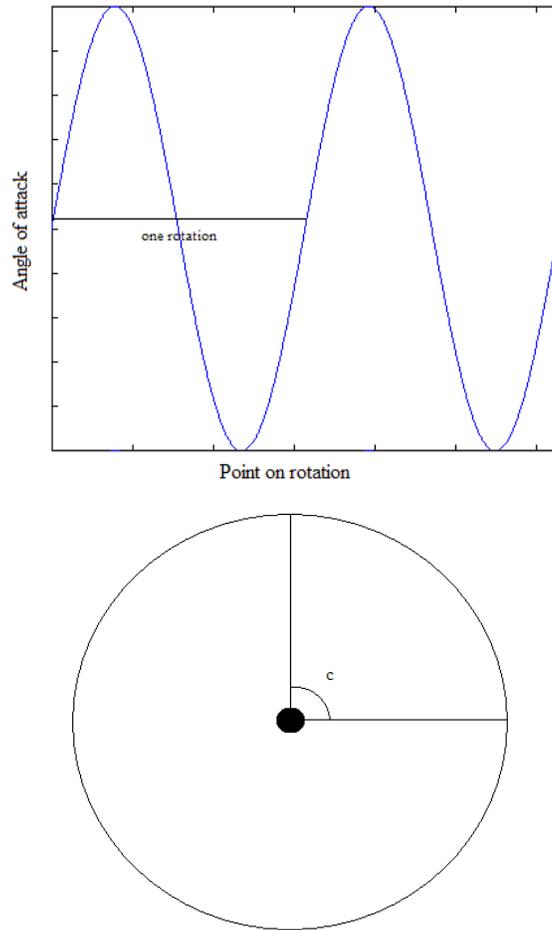
These graphs show the behavior of the helicopter over a period of ten seconds. The chosen constants have little problem bringing the helicopter to its chosen position and orientation. The longitudinal position is reached in three seconds while the lateral position is reached in about seven. If we consider that the largest source of error in controls will be the thrust, this is a safer path for the helicopter to take. Given constraints in processing and modeling, this is more than acceptable performance.

## 5.5 Control Signals

In order for the control board to execute the control equations it must know what signals it needs to send to the servos. This section puts together the data collected regarding the helicopters servos and the previously developed control scheme.

The model helicopter has four servos and a speed controller for the motor that use the square waves to move to a specific setting. On a traditional model helicopter, one signal would control one function. This is known as a 1-S swashplate. The helicopter used in this project uses a different system where the blade pitch,  $M_1$ , and  $M_2$  are all controlled collectively. This is known as a 3-S swashplate. The result of this is a complicated system of control that is not easily equated to traditional control.

In order to use the 3-S it is important to understand how control works with a 1-S. The longitudinal moments are generated differently depending on the model of helicopter used. The most common method is the use of a flybar with paddles. The flybar is linked to the main rotor shaft in such a way that, when the servos move the swashplate, the geometric angle of attack of the paddles forms a sine wave (Fig. 5.23).



**Figure 5.23: Angle of attack of flybar vs. rotation of main rotor**

We could also think of it in terms of the following equation, where (a) is the maximum angle of attack, (b) is the point on the rotation disk, and (c) is the offset caused by the swashplate to control the direction.

$$aoa = a * \sin(b + c) \quad (5.11)$$

The result of this is that the moment being generated by the paddles would be in the direction of the trough of the resulting wave. This is caused by the fact that half of the disk is being pushed upward around the peak and downward at the trough. However, this is not enough to calculate the moment due to the fact that the flybar is not rigidly attached to the main rotor's axis. The main blades are also attached to the motion of the flybars in such a way as to change the collective pitch as well as the paddles angle of attack. The effect of this is that the main rotor's geometric angle of attack will also change with respect to its point on the rotation plane creating a second torque.

While it would be possible to create an analytical model of this process to get the combined torque, it would be impractical given the constraints on the project. To

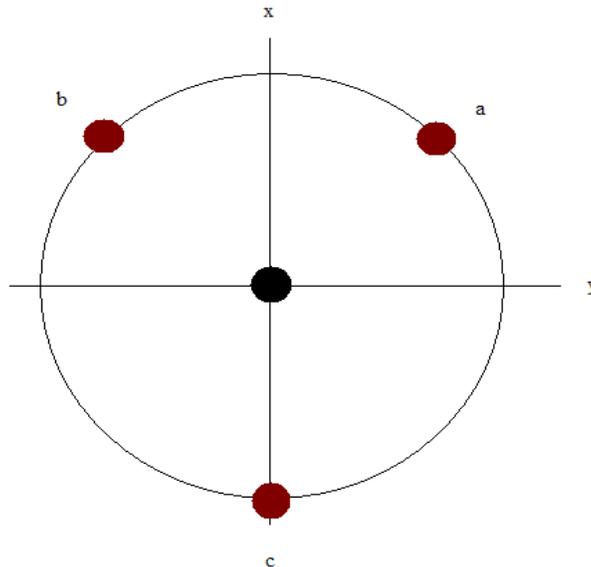
overcome this, the most practical solution is to treat the model helicopter as though it were a real helicopter. The torques ( $M_1, M_2$ ) on a real helicopter are generated through a change in the plane of rotation for the main rotor. The resulting moment is calculated by the following equation, where  $d$  is the distance from the center of mass,  $T$  is the thrust, and  $\tau$  is angular shift.

$$M_i = d * T * \sin \tau_i \tag{5.12}$$

The difficulty with making this approximation is in relating and finding the values to use for  $\tau$ . The best way to find these values would be to build a testing apparatus that limits the planes of rotation and measure the resulting torques. However, the design and construction of an effective apparatus was beyond the resources available to the team. The next best solution would be to attach the completed circuit board with all the sensors mounted and with a modification to record the pulse to the servos. If the data gathered from manual flying were to be compared to the equations of motion, the angular shifts could be approximated from the data. Unfortunately, this method could not be implemented in time for the deadline of the project.

The solution for approximating the angular shift of the plane in a time and cost effective manor was to approximate the shifts to those of the swashplate. It is well known to model helicopter pilots that the helicopter will fly in the direction of the swashplate. While it is unlikely that the angular shifts are exactly equal, there should be enough room for error in the control equations to allow stable flight.

If the swashplate were a 1-S type, finding the relationship between the signals to the servo and the angle would be a simple task as each angle would be controlled by one servo. For the 3-S swashplate, the angles must be found analytically.



**Figure 5.24: Sketch of servo pin positions on swashplate**

The servos are connected to pins that are attached to the swashplate as seen in the figure above. If we relate the height ( $h$ ) of each pin relative to the base, we can find the

angle of the swashplate. If  $\tau_1$  is the angle in the y-direction and r is the distance from a to b, the following equations results.

$$A = \frac{h_b - h_a}{r} \quad (5.13)$$

$$\tau_1 = \sin^{-1} A \quad (5.14)$$

If we enter this into the moment equation we find that the sine functions disappear resulting in a less taxing equation.

$$M_1 = d * T * A \quad (5.15)$$

To find  $M_2$  we can assume that the difference between the height of pin c and the average of pins a and b will give us a similar relation.

$$B = \frac{2h_c - h_b - h_a}{2r} \quad (5.16)$$

$$M_2 = d * T * B \quad (5.17)$$

The final relation we need to make is that of the collective pitch. Although it has been stated earlier that the moments result in a changing angle of attack for the main rotor blades, it can be assumed that the collective pitch is independent of the moments for calculation of thrust. The results of signal vs. angle experiment in Appendix B can also be related to the average heights of all three pins.

Once the experimental results for the relation between signal and height are used with the previous equations, we get values for what signal (w) is needed depending on the result from the control equations.

$$w_a = \left( 7.6897 + 8.2873 * \frac{M_1}{d*T} + .83897 * \vartheta - 16.574 * \frac{M_2}{d*T} \right) * 10^{-4} \quad (5.18)$$

$$w_b = \left( 23.169 + 8.07944 * \frac{M_1}{d*T} + .81778 * \vartheta - 16.1589 * \frac{M_2}{d*T} \right) * 10^{-4} \quad (5.19)$$

$$w_c = \left( 13.6418 - 8.18658 * \frac{M_1}{d*T} - 16.37315 * \frac{M_2}{d*T} \right) * 10^{-4} \quad (5.20)$$

If we are trying to keep the rotational speed constant, the signal to the speed controller is a function of the angle. Details for this are in Section 5.3.1. The last control is for the rudder. This is the easiest to test provided you have a method to calculate the rotation speed. The process is a simple matter of experimentally measuring the rotational speed and the change in the thrust at the tail for various signals. Then it is a simple matter of using equation (equation of T vs.  $C_t$ ) to make an approximated equation for  $C_t$  as a function of the signal. Appendix A2 describes a test deemed inconclusive.

## 6.0 Recommendations / Conclusion

This project set about to take the first steps in creating a helicopter capable of operating autonomously. Having thoroughly analyzed servo behavior, established equations of motion, and created a control scheme capable of allowing an R/C helicopter to move to any given destination in a smooth and controlled manner, the team deems this project a success.

As with any technical undertaking, there is still work that can be done to achieve autonomous flight. The team has done all it can in the development of an autonomous helicopter, what is left to do is implement the control scheme onto a protoboard and begin testing and optimizing the craft. The team has created a control board integrating a single axis gyroscope and a 5 degree of freedom sensor board which utilizes 2 gyroscopes and 3 accelerometers; these sensors, once fitted to the board serve as the inputs in the control scheme. Further development on the software side needs to occur in order to employ the control code on the board itself. Once completed, autonomous flight will be a matter of mounting the board followed by thorough testing and refining of control constants developed in Section 5.4.

In regards to a guidance system wherein the helicopter is made to track and follow an object there is considerable room for development. In order to further develop such a system, the desired position and orientation for the control loops must be derived from the output of the chosen environment imaging system. One such environment imaging system is presented in Appendix C.

Using the same control system presented in Section 5.4, it is possible to control the behavior by simply modifying the values of the desired position and orientation into functions. Simple movements such as circling can be achieved by making the desired position a function of time. More complicated patterns can be achieved by making the desired values into if/then statements. It is even hypothetically possible to make a program that allows the desired states to be changed wirelessly and have it follow any number of paths.

The most difficult part of making the helicopter fully autonomous will be integrating the environment imaging system. The system in Appendix C is limited in range and function since it requires line of sight with the ground vehicle. This system's other limitation is that it can't react to anything but the LED light and therefore is likely to collide with other obstacles.

In order to integrate the imaging system from Appendix C with the control scheme presented in Section 5.4 some changes may need to be made. The most likely change would have to be made in the strength of error correction for yaw. In other words their k-values may need to increase in order to keep the vehicle in sight at all times. Another solution would be to buy or design a three dimensional pivot for the goniometer in order to allow a larger range of sight.

The last suggestion would be to acquire better components than those used in this project. A more powerful microcomputer would be the first choice as the calculations for dynamics and controls tax the limit of the current model without the environment imaging component. The next priority would be more sensors to provide redundancy, and thus, more accurate readings.

Having made these suggestions, we are reminded of the many potential applications previously been mentioned in this report. Serving in everything from patrols to rescues, helicopters are the foundation and tool of choice for many agencies operating in a wide variety of conditions and settings. Linking all these operations is the explicit need for precision and safety; it is in this realm that autonomous flight truly excels. It is for this reason that the team has gone to such lengths in achieving this goal.

## Bibliography

1. *Afghan Fighting Kills 200 Taliban*. <http://www.msnbc.msn.com/id/14653529/> (accessed October 08, 2007).
2. Army, US. *GoArmy.com > Benefits > Pay*. [http://www.goarmy.com/benefits/money\\_basic\\_pay.jsp](http://www.goarmy.com/benefits/money_basic_pay.jsp) (accessed October 08, 2007).
3. Leishman, Gordon J. Principles of Helicopter Aerodynamics, Second Edition, Cambridge University Press, New York, New York, 2006.
4. Flying Hobby. *T-Rex 450S Ready to Fly*. <http://www.flying-hobby.com> (accessed October, 2007)
5. Synergy RC. *Characteristics*. August 19, 2006. [http://www.synergyrc.com/es/index.php?option=com\\_content&task=view&id=8&Itemid=78](http://www.synergyrc.com/es/index.php?option=com_content&task=view&id=8&Itemid=78)
6. MaxBotix, Inc. *The LV-MaxSonar High Performance Ultrasonic Range Finders*. 2007. <http://www.maxbotix.com>.
7. Texas Instruments. *MSP430 Ultra-Low Power Microcontrollers*. 2007. <http://focus.ti.com/mcu/docs/mcuhome.tsp?sectionId=101>.
8. Seddon, J. Basic Helicopter Aerodynamics. Washington, DC, American Institute of Aeronautics and Astronautics; London, UK. Blackwell Scientific Publications, 1990.

## Appendix A: Results from Wind Tunnel Tests of Main Rotor Blades

Wind tunnel testing was performed to determine the lift curve slope of the blade for the equation for thrust as well as the drag resulting from changes in the angle of attack. While an approximation of the lift curve slope was simple to obtain, there was simply not enough drag to be read by the instruments in the wind tunnel. An alternate method was used to calculate drag by measuring the change in rotational speed as the angle of attack increases as discussed in Section 5.3.1.

The experimental setup was a rotor blade clamped to a force balance set atop two scales. The force balance is a mechanism that allows an airfoil to change its angle of attack and uses the scales to measure lift. The experimental procedure was as follows;

1. Set Blade Angle
2. Set tunnels speed
3. Zero scales
4. Take readings off the scale and add together
5. Turn off tunnel and repeat

Table 2 shows the results of this test for the 325 mm blades used on the final setup of the helicopter. The test concluded the slope of the lift-curve to be .00384.

angle	lift	lift coefficient
0.4	-0.018	-0.005438066
1.4	-0.004	-0.001208459
2.3	0.006	0.001812689
3.2	0.011	0.003323263
4.1	0.019	0.005740181
5.8	0.049	0.014803625
6.6	0.068	0.020543807
7.8	0.086	0.025981873
9	0.091	0.027492447
9.8	0.09	0.027190332
4.9	0.026	0.007854985
5.7	0.037	0.011178248
6.5	0.06	0.018126888
4.7	0.0283	0.008549849
slope		0.003844119

**Table A.1: Wind tunnel test results**

## Appendix B: Z-Axis Moment Test

As described in Section 5.5, it is necessary to include in the control scheme a relation between moment about the z-axis and the pulse delivered to the tail servo. A simple test apparatus was constructed by mounting a bearing on the helicopter and placing the assembly in a vice next to a vertically mounted scale (Fig. B.1). Upon modulating the signal sent to the servo, a change in the force exerted on the scale was recorded, this force multiplied by the distance to the center of mass of the helicopter gives us the torque,  $M_3$ .

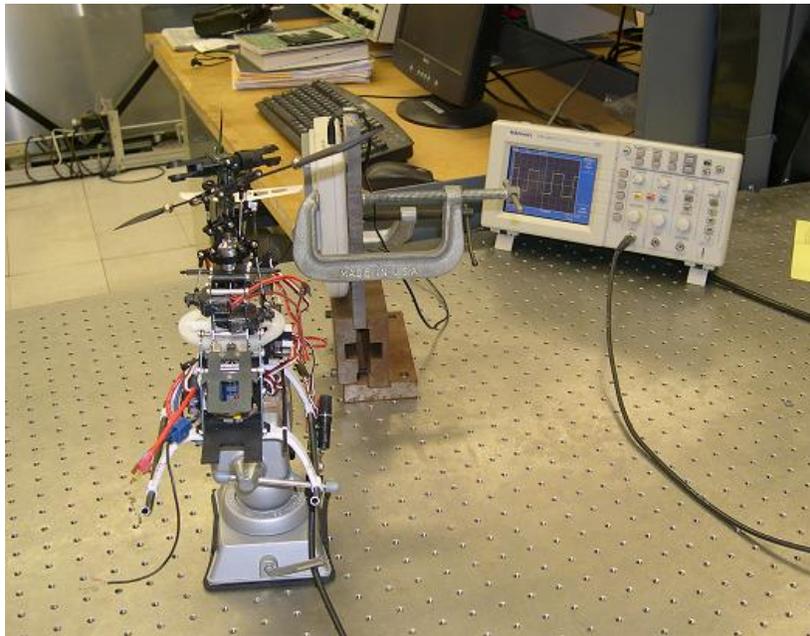


Figure B.1: Z-Axis Moment Testing

The data collected is shown in figure B.2; the test was carried out at 20% and 30% throttle.

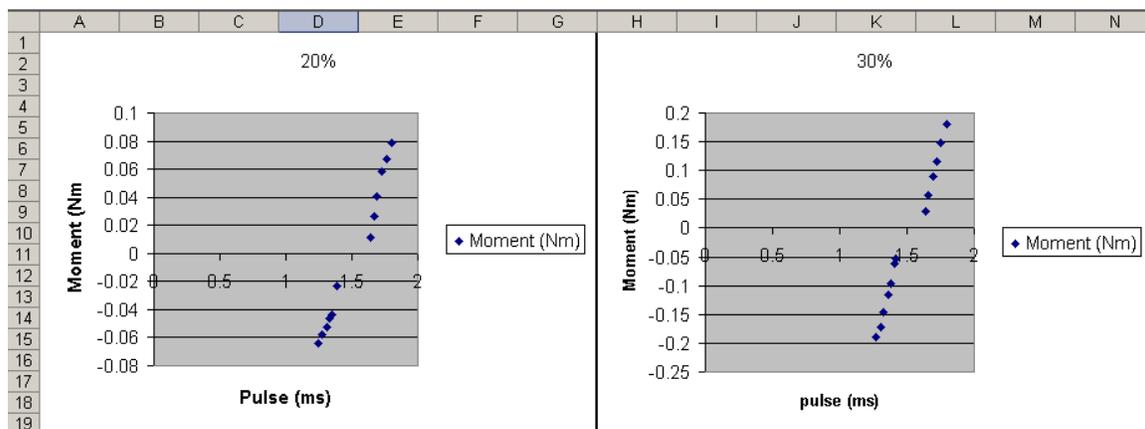


Figure B.2: Z-Axis Moment vs. Pulse Width at 20% and 30% Throttle

## Appendix C: Considerations for Guided Flight

In preparation for guided flight, the team has developed a guidance and navigation strategy outlined as follow. In order to find the position of the helicopter relative to the ground vehicle as well as its orientation, six values must be found. The values for the position in terms of  $x$ ,  $y$ , and  $z$  will be found by making use of a distance sensor to find the value of  $z$ , and an optical goniometer capable of measuring two angles.

A cluster of LED lights will be placed on the ground vehicle while the optical goniometer will be programmed to pick up only that frequency of light. The optical goniometer will see the light if it is within the cone of its vision, and then map it on to a grid.

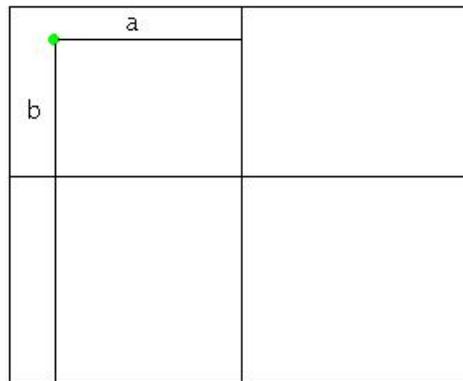


Figure C.1: Mapped data point from goniometer

This position of the light on the grid corresponds to the angle between the center of the goniometer's cone of vision. From these angles  $a$ ,  $b$ , and distance  $z$ , we can calculate the values of  $x$  and  $y$ .

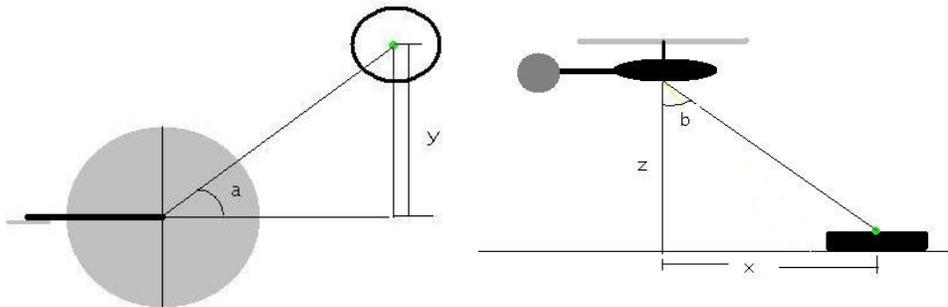


Figure C.2: Angle / Distance relationships

To find and control the orientation of the vehicle, a gyroscope and a dual axis accelerometer will be used. The gyroscope will find and balance the angular acceleration of the yaw and use double integration to find current yaw. The purpose of the accelerometer will be to balance roll and control the pitch in order to follow the ground vehicle.

## Appendix D: MATLAB<sup>®</sup> Source Code

```
function Xdot = mqp2(t,x)

%The x matrix values are as follows x=[trans velocities in body
(x,y,z);
%ang body velocities (p,q,r); euler angles inertial; Inertial position
(x,y,z);

%Moments of Inertia

%Rotation Matrix Angles
G = [1 sphi*tan(x(8)) cphi*tan(x(8));...
      0 cphi -sphi;...
      0 sphi/ctheta cphi/ctheta];

%Rotation Matrix Translation
sphi = sin(x(7));
cphi = cos(x(7));
ctheta = cos(x(8));

pit = [cos(x(8)) 0 -sin(x(8)); 0 1 0; sin(x(8)) 0 cos(x(8))];

rol = [1 0 0; 0 cos(x(7)) sin(x(7)); 0 -sin(x(7)) cos(x(7))];

yaw = [cos(x(9)) sin(x(9)) 0; -sin(x(9)) cos(x(9)) 0; 0 0 1];

DCM = rol*pit*yaw;

%Gravity in the inertial
ge = [0; 0; -9.8];
%Gravity in the body
gb = DCM*ge;
%Mass and moments of inertia
m =; Ixx =; Ixz =; Iyy =; Izz =;
%Desired coordinates
xd=; yd=; zd=;
%Dampening constant
k1=; k2=; k3=; k4=;
%Error correction control
d1=([x(10);x(11);x(12)]-[xd;yd;zd]);
d2=(m*([x(1);x(2);x(3)]+k1*DCM*d1));
d3=[x(7);x(8);x(9)];
d4=(( [x(4);x(5);x(6)]+k3*G'*d3));
Vv1=-k1*d1-k2*d2;
Vv2=-k3*d3-k4*d4;
Vv=[Vv1(1)+Vv2(2);-3*Vv1(2)+3*Vv2(1);9.8*Vv1(3)];
X = 0;
Y = 0;
Z = m*Vv(3)+9.8*m;

M1=Vv(2);
M2=Vv(1);
M3=Vv2(3);
```

```
%x_dot = x(1); y_dot = x(2); z_dot = x(3); p = x(4) q = x(5); r = x(6);
phi = x(7); theta = x(8); psi = x(9);
```

```
%Translational Motion (body)
```

```
x_double_dot = x(6)*x(2)-x(5)*x(3)+X/m+gb(1);
y_double_dot = x(4)*x(3)-x(6)*x(1)+Y/m+gb(2);
z_double_dot = x(5)*x(1)-x(4)*x(2)+Z/m+gb(3);
Vb_dot=[x_double_dot; y_double_dot; z_double_dot];
```

```
%Rotational Motion (body)
```

```
p_dot = [(Ixx+(Ixz*(Ixx-Iyy))/Izz)*x(4)*x(5)+(Iyy-Izz-
(Ixz*Ixz/Izz))*x(5)*x(6)+M1+(Ixz/Izz)*M3]/(Ixx+(Ixz*Ixz/Izz));
```

```
q_dot = (- (Ixx-Iyy)*x(6)*x(4)-Ixz*(x(4)*x(4)-x(6)*x(6))+M2)/Iyy;
```

```
r_dot = [(-Izz+(Ixz*(-Ixx+Iyy))/Izz)*x(6)*x(5)+((Ixz*Ixz/Ixx)+Ixx-
Iyy)*x(5)*x(4)+M3+(Ixz/Izz)*M1]/(Izz-(Ixz*Ixz/Ixx));
```

```
omb_dot = [p_dot; q_dot; r_dot];
```

```
%Inertial frame
```

```
Vb=[x(1);x(2);x(3)];
```

```
PHIdot = G*[x(4);x(5);x(6)];
```

```
Ve = DCM'*Vb; %velocity in the inertial frame
```

```
Xdot = [Vb_dot; omb_dot; PHIdot; Ve];
```

Notes:

The parameters we used for simulation were:

```
m = .615; Ixx = .3876; Ixz = -.0919; Iyy = 1.0101; Izz = 1.1804;
```

The resulting k values were:

```
k1=.75; k2=35; k3=5; k4=20;
```