

# AMPLIFIED SPONTANEOUS EMISSIONS IN NEODYMIUM-DOPED FIBER LASERS

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## CONTENTS

List of Figures	1
1. Introduction	2
2. Background	4
2.1. Total Internal Reflection	4
2.2. Fiber Structure	4
2.3. Radiative Transitions	6
2.4. Energy Levels of the Nd <sup>3+</sup> Ion	6
2.5. Gain in Optical Amplifiers	8
3. Modeling ASE in single mode fibers	11
3.1. Intensity Distributions	11
3.2. Transition Rates and Upper State Population	11
3.3. Calculating Power	13
3.4. Discretization Techniques	13
4. Experimental Procedure	16
5. Results and Analysis	18
5.1. Theoretical Results	18
5.2. Experimental Results	20
6. Conclusions	21
6.1. Future work	21
References	22
Appendix A. MATLAB code	23

## LIST OF FIGURES

1 Condition for total internal reflection	4
2 Light is accepted into an optical fiber in a cone of half-angle $\alpha$	5
3 Radiative Transitions	6
4 Energy Levels of the Nd:YAG laser	7
5 Transition cross sections for Nd-doped silica glass	8
6 Single Ion Fluorescence in Nd-Doped Silica Glass	9
7 Several double-clad fiber designs	10
8 Intensity profiles	12
9 The emission spectrum was simplified into a series of bins as shown above.	14

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10	In the 2009 script, the forward and backward ASE powers are calculated in two passes up and down the length of the fiber.	14
11	Volume elements within the fiber core	15
12	Processes calculated within a single cell	15
13	Experimental setup for measuring the ASE spectrum of the Nd-doped fibers	17
14	Effect of bending a multimode fiber on the ASE spectrum	17
15	Calculated ASE spectrum at various pumping powers	18
16	Minimum fraction of excited state ions required to achieve gain	19
17	Calculated spectrum at the end of fiber of varying length	20
18	Measured pump absorption levels with exponential fit curve.	21

## 1. INTRODUCTION

Since its development in the 1970's, fiber optic communications has become ubiquitous in the modern world. Doped fiber amplifiers have proven to be an effective way to provide signal gain. There are many dopants which may be used to produce gain at a variety of wavelengths. This report will focus on the neodymium-doped fiber amplifier (NDFA). Most communications signals are transmitted at infrared or radio wavelengths, but these wavelength ranges are not suitable for some applications. Underwater communication, for example, can be very difficult due to the strong absorption of radio waves in salt water. Extremely long wavelengths transmit better, but data rates are seriously limited [1]. An alternative option is to use a blue-green wavelength ( $\sim 450$  nm) for underwater communications, allowing for low-loss transmission with high data rates [2]. Neodymium-doped silica glass fibers can produce lasers around 900 nm which can be frequency doubled to produce blue-green lasers for this application.

The purpose of a doped fiber amplifier is to absorb energy from a short wavelength "pump" laser and transfer as much of that energy as possible to a desired signal. The absorption of the pump energy causes a fraction of the electrons in the fiber core to become excited. This state can only be maintained for a brief period, however (about  $300 \mu\text{s}$  in the case of the NDFA), before dropping back to the ground state if it is not struck by a signal photon in this time. This causes a spontaneous emission in a random direction, which does not contribute to the desired signal. A fraction of these spontaneous emissions may be trapped in the fiber core, and then they may experience gain in a similar manner to the signal. Each time one of these spontaneous photons causes a stimulated emission, it depletes an excited electron which could otherwise have imparted additional energy to the main signal. This amplified spontaneous emission (ASE) can be a major source of noise in this type of laser. Under certain conditions, ASE may even grow large enough to significantly limit signal gain at the desired wavelength [3].

Fiber amplifiers are an efficient way to make high-powered lasers for a variety of applications. In particular, the 900 nm transition of the Nd-doped fiber laser is a good candidate for frequency doubling to produce blue lasers. One problem

that arises when trying to excite the 900 nm transition in a Nd laser is ASE from the 1060 nm transition, which can limit the possible length of the fiber amplifier [4].

This report will discuss the development of a model for simulating ASE in an optical fiber, building upon the work of a MQP completed in 2009 [5]. The goal of this project is to refine that model to provide a more precise prediction of the loss due to ASE in Nd-doped fibers. The model will be compared against measurements of ASE in single-mode Nd-doped step index fiber. The previous model assumed a uniform distribution of excited states in the core of the fiber. The new model will improve upon this approximation by assuming a Gaussian intensity distribution. This is a more accurate representation of the fundamental  $LP_{01}$  mode. The numerical model will be built upon the the MATLAB script created by the previous MQP group and will facilitate the computation of the ASE spectrum of the NDFA, and an examination of the effects of ASE on signal gain.

## 2. BACKGROUND

The study of ASE in optical fibers demands an understanding of several optical phenomena.

**2.1. Total Internal Reflection.** Light can be guided through an optical fiber by means of total internal reflection (TIR). From a ray optics perspective, TIR occurs when the angle of incidence of a beam of light on an interface between two media is greater than the critical angle  $\theta_c$  as shown in Figure 1. The critical angle can be found by using Snell's Law

$$(1) \quad n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

In Equation 1,  $n_1$  and  $n_2$  are the refractive indices of the core and cladding, respectively, and  $\theta_1$  and  $\theta_2$  are the angles of a beam of light crossing the interface with respect to the normal. By setting the angle of refraction in the cladding to 90 degrees, so that "transmitted" light does not actually escape the fiber core, we can obtain an equation for the necessary angle of incidence from inside the core.

$$(2) \quad \theta_c = \arcsin \frac{n_2}{n_1}$$

Note that this equation requires that  $n_1 > n_2$ , since  $\arcsin x$  is only defined for  $x$  between -1 and 1. It is clear from Eq. (2) that the critical angle will decrease as  $n_2/n_1$  decreases which will increase the acceptance angle  $\alpha$ , as shown in Figure 2. Light can be coupled more easily into fibers with larger acceptance angles.

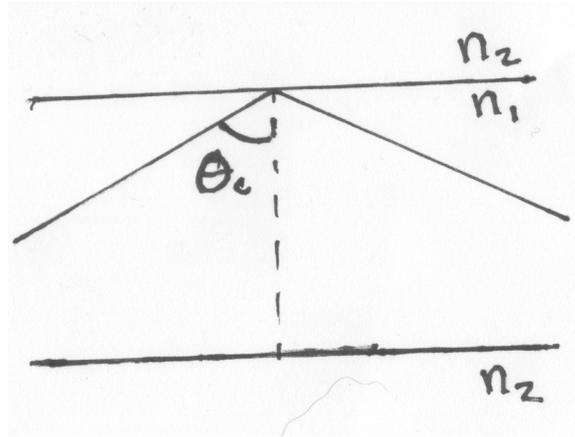


FIGURE 1. Total internal reflection occurs when  $n_1 > n_2$  and a beam of light is incident on the edge of the core at an angle  $\theta \geq \theta_c$

**2.2. Fiber Structure.** A typical optical fiber consists of three major parts: the core, the cladding, and a protective outer coating. The core and cladding have carefully adjusted refractive indices such that  $n_{clad}/n_{core} < 1$ , which allows light to be reflected back into the core by TIR. Light in a cone of half angle  $\alpha$  may enter the fiber and become trapped in the core, as shown in Figure 2, and light incident on the end of the fiber at greater angles will be partially transmitted through the

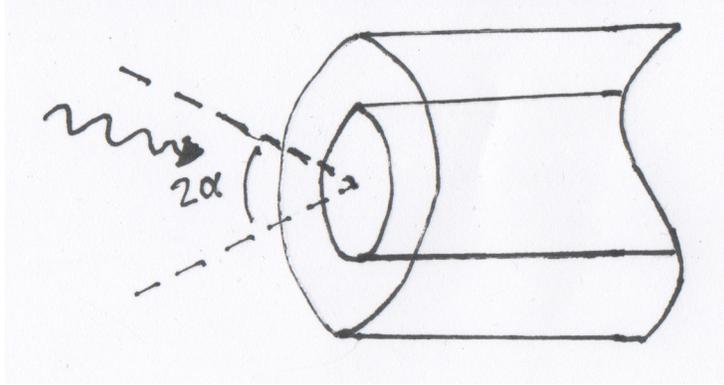


FIGURE 2. Light is accepted into an optical fiber in a cone of half-angle  $\alpha$

core-cladding interface. Unfortunately, fiber manufacturers do not advertise these values directly, but instead give the numerical aperture (NA) of their fiber. The NA can still be used to approximate the refractive indices of the core and cladding. NA is defined by the relation

$$(3) \quad \text{NA} = \sqrt{n_1^2 - n_2^2}$$

Since the refractive indices of the core and cladding are typically very close, we can write the above equation in an approximate form [6]

$$(4) \quad \text{NA} \simeq n\sqrt{2\Delta}$$

where  $n_1 \approx n_2 \approx n$  and  $\Delta = (n_1 - n_2)/n_1$

We may also use the NA to calculate the acceptance angle of the fiber by applying Snell's Law once again at the fiber end [6]. If  $n_0$  is the refractive index of the air then

$$(5) \quad n_0 \sin \alpha = n_1 \sin(90 - \theta_c)$$

$$(6) \quad = n_1 \cos(\theta_c)$$

$$(7) \quad = n_1 \sqrt{1 - \sin^2 \theta_c}$$

$$(8) \quad = n_1 \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

$$(9) \quad = \text{NA}$$

Since the refractive index of air is extremely close to 1, the acceptance angle can be written quite simply as  $\alpha = \arcsin \text{NA}$ .

Light that has become trapped by TIR is said to be in a guided mode. Only specific modes may actually propagate in the fiber because they must satisfy the TIR condition. The transverse (perpendicular to the core/cladding interface)

component of the light wave must also be such that an integer number of wavelengths “fit” inside the core. Otherwise, the mode will leak into the cladding over time. Optical fibers with small core areas often permit only a single guided mode for the wavelengths used in communications. A plot of the electric field amplitude of this light versus the fiber radius has an approximately gaussian distribution [7]. Thus, the intensity of the mode is the square of this distribution.

**2.3. Radiative Transitions.** There are three radiative transitions that all play a part in the generation of ASE within a NDFA. The first is absorption, in which an electron absorbs a photon and moves to a higher energy level, as shown in Figure 3(a). Absorptions in a laser can be divided into two categories: ground state absorption (GSA) and excited state absorption (ESA). GSA occurs when an electron is excited from the ground state to the upper state desired for lasing ( ${}^4F_{3/2}$  for the 1060 nm transition of the NDFA), while ESA occurs when an electron is excited from this state to one of even higher energy.

Another basic radiative transition is spontaneous emission, which occurs when an electron in an excited state decays to a lower state and emits a photon, as shown in Figure 3(b).

The third transition is stimulated emission, which occurs when a photon is incident on an electron in an excited state and the electron moves down to a state with lower energy and emits another photon identical in wavelength and phase to the first. This process, which is illustrated in Figure 3(c), is responsible for the signal gain of a fiber amplifier or any other laser. When multiple photons have the same wavelength and their electric and magnetic fields oscillate in phase with one another, they are said to form a beam of coherent light, which is desirable in many applications.

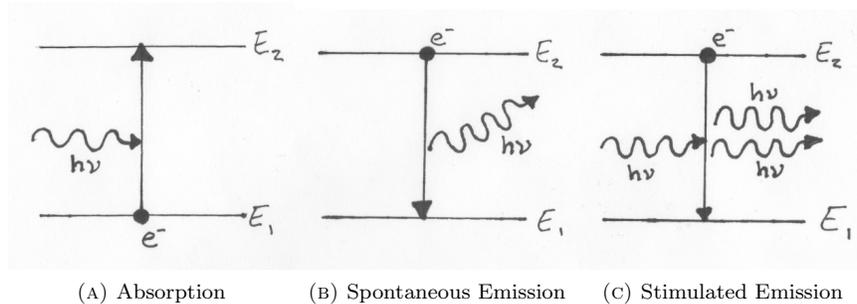


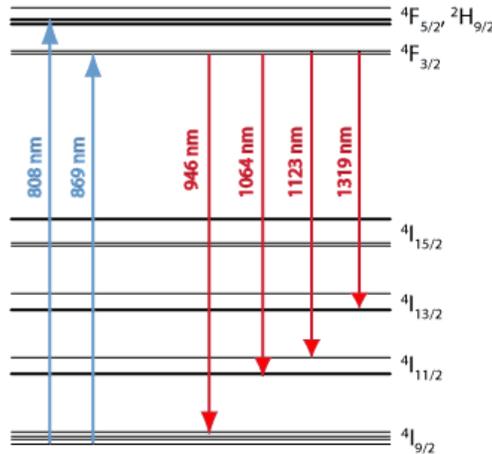
FIGURE 3. Radiative Transitions

ASE is the result of spontaneously emitted photons that interact with excited electrons to cause stimulated emissions at wavelengths other than the desired signal wavelength. This adds some noise to the output signal and, more significantly, moves an electron from the excited state to the ground state, decreasing the total amount of signal gain that could be achieved.

**2.4. Energy Levels of the  $\text{Nd}^{3+}$  Ion.** Quantum Mechanics dictates that electrons may orbit the nucleus of an atom only in states with specific energies. This means that there is a discrete set of possible energies for electrons orbiting an

atomic nucleus. Furthermore, the angular momenta of the electrons are also discretized. Electrons also have their own inherent angular momentum, known as spin, which must be  $\pm\hbar/2$ . The Pauli Exclusion Principle states that only one electron may occupy an available state at a time, so only one electron may have each possible combination of energy, orbital angular momentum, and spin [8]. In spectroscopic notation, energy levels are labeled by  $^{2S+1}L_J$ , where  $S$  is the total spin of the electrons,  $L$  refers to the orbital angular momentum number  $l$ , written as  $0 = S, 1 = P, 2 = D$ , etc. and  $J$  is the total angular momentum of the state. The electrons in neighboring atoms create an electric field which pushes on the atom very slightly, altering the energy of each state by a small amount. Thus, each energy level is in fact a multiplet of  $2J + 1$  discrete sub-levels known as Stark energy levels. This small separation and the corresponding spread of each energy level is apparent in Figure 4. The end result is that the discrete energy levels form quasi-continuous bands with some larger gaps, such as the gap between  $^4F_{3/2}$  and  $^4I_{15/2}$  in the  $\text{Nd}^{3+}$  ion.

The energy level structure of the  $\text{Nd}^{3+}$  ion allows for multiple types of laser transitions. For example, imagine an electron in the  $^4I_{9/2}$  state that absorbs a photon of wavelength 808 nm, as in Figure 4. The electron now occupies the  $^4F_{5/2}$  state. This electron will almost immediately emit a phonon (lattice vibration) and decay to the nearby  $^4F_{3/2}$  state if it is available. Let's say that this electron encounters an incoming photon of wavelength 1064 nm. This interaction will induce the stimulated emission of an identical 1064 nm photon, and the electron now occupies the lower energy  $^4I_{11/2}$  state. Once again, the electron will quickly decay non-radiatively back to the nearby ground state,  $^4I_{9/2}$  where it began. This entire process is known as a four-level laser transition [9].



Source: [http://www.rp-photonics.com/neodymium\\_doped\\_gain\\_media.html](http://www.rp-photonics.com/neodymium_doped_gain_media.html)

FIGURE 4. Energy Levels of the Nd:YAG laser (Nd-doped yttrium aluminum garnett;  $\text{Nd:Y}_3\text{Al}_5\text{O}_{12}$ )

There are also three-level laser transitions. This is the case when the photon that causes the stimulated emission has enough energy to bring the excited electron back down to the ground state directly. In the example above, this would be a 946

nm photon instead of a 1064 nm photon. It is more difficult to achieve gain in a three level laser because the laser beam can be reabsorbed via GSA, which is not the case for four-level lasers due to the low population of states above the ground state.

The probability of an absorption or stimulated emission occurring in a medium containing many ions is often characterized by a transition cross section. The cross section can be thought of as the apparent area of the ion as “seen” by a photon of wavelength  $\lambda$ . The stimulated emission and absorption cross sections for Nd-doped glass, measured by R. Martin [10], can be found in Figure 5.

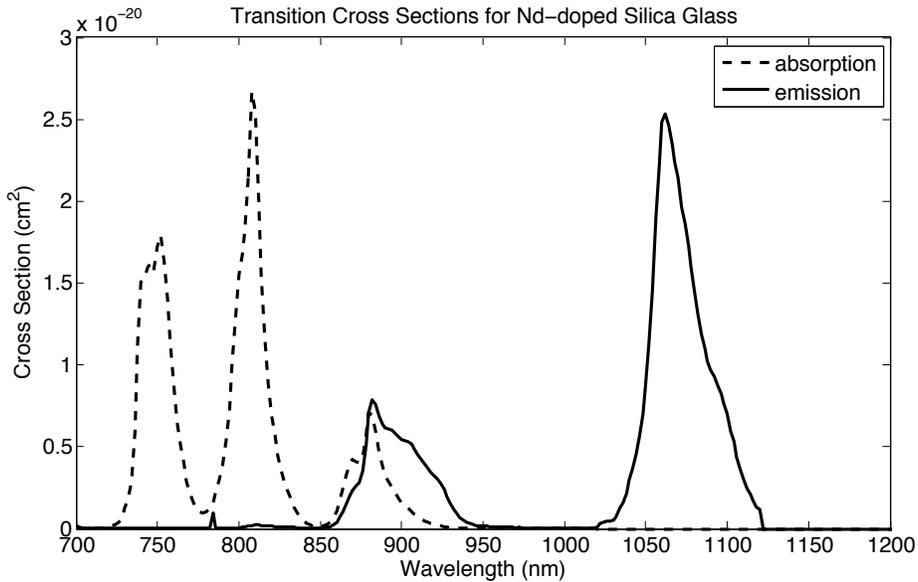


FIGURE 5. Transition cross sections for Nd-doped silica glass

If no photon triggers a stimulated emission, the electron will spontaneously emit a photon after a certain lifetime, which is about  $300 \mu\text{s}$  for Nd ions in silica glass. The number of spontaneous emissions at a given wavelength are related to the stimulated emission cross section, with a preference for shorter wavelengths. This is characterized by the single ion fluorescence spectrum, the power emitted at each wavelength by a single excited ion [11]. Figure 6 shows the single ion fluorescence spectrum for an Nd ion in silica glass. This spontaneous emission is the source of ASE.

It is easy to see from Figures 4 and 5 that the 1060 nm peak in the Nd emission cross section corresponds to the  ${}^4F_{3/2} \rightarrow {}^4I_{11/2}$  transition. The absence of any absorption cross section indicates that this is a four-level laser transition. The  $\sim 900 \text{ nm}$  transition is more difficult to excite because the laser may be reabsorbed by electrons in the ground state. Moreover, ASE at 1060 nm can consume some of the upper state population necessary for signal gain [4].

**2.5. Gain in Optical Amplifiers.** In any kind of amplifier energy must be added to the system before any signal gain can occur. For optical fiber amplifiers, this is

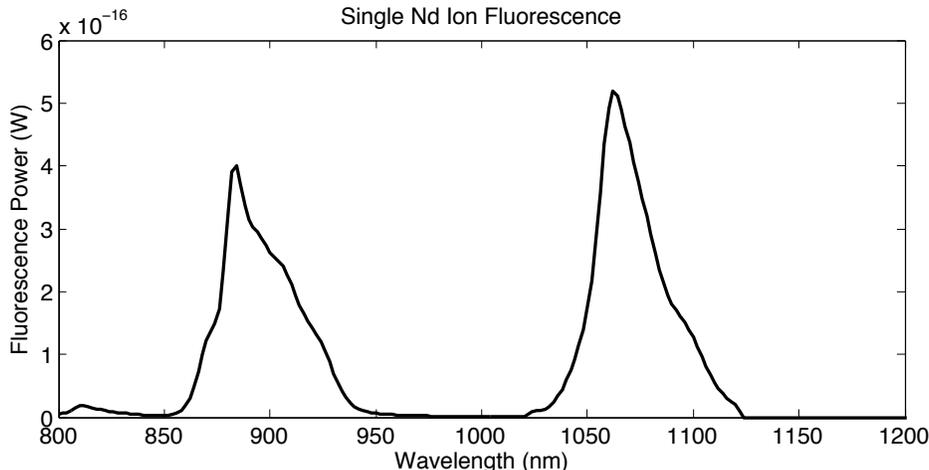


FIGURE 6. Fluorescence spectrum for a single Nd ion in silica glass, calculated according to the method described by D. E. McCumber in [11].

typically accomplished by saturating the fiber with “pump” light to excite as many electrons as possible. The goal is to increase the population of the upper state such that the signal beam will cause stimulated emissions and experience gain.

In the NDFA a typical pumping wavelength is 808 nm, but other wavelengths are also possible, as there is a semi-continuous band of unoccupied states above the  ${}^4F_{3/2}$  state. Any wavelength at which there is significant absorption can be used for the pump light, since electrons that are pumped to a higher energy than the desired lasing level will decay non-radiatively to the bottom of the conduction band. The pump light can be coupled into the core, or simply into the inner cladding, since many fibers are doubly clad. A doubly clad fiber is simply one that has an additional layer of inner cladding with a large area to allow pump light to be coupled into the fiber more easily than in a single-clad fiber. The inner cladding may take on many different shapes to improve core absorption, as shown in Figure 7, but in this model only fibers with circular cladding and a centered core are considered, for the sake of simplicity. It is easier to focus the pump light into the cladding than into the core, but longer fibers are necessary to ensure that a significant amount of the pump light will be absorbed, due to poor overlap between the core’s small cross section and the guided modes in the inner cladding. The pump light may be coupled into one or both ends of the fiber.

The gain and loss of photons from the beam can be characterized by the gain coefficient  $\gamma$ , which is the fractional gain or loss of power per unit length

$$(10) \quad \frac{dP}{dz} = \gamma P(z) \quad \text{or} \quad dP(z) = P(z)\gamma dz$$

Equation (10) describes the absorption of any light propagating in the fiber core. Since  $\gamma$  represents the amount of absorption or stimulated emission occurring



Source: <http://http://www.rp-photonics.com/double-clad-fibers.html>

FIGURE 7. Several possible geometries for a double-clad fiber are shown, with the core in blue, the inner cladding in light gray, and the outer cladding in dark gray. The fraction of pump light actually inside the core is equal to the ratio of the core area to the inner cladding area  $\eta = A_{core}/A_{clad}$

in the fiber, it is clear that it depends on the population of excited electrons. As mentioned above, the probability of an absorption or stimulated emission occurring is related to the transition cross sections,  $\sigma_{abs}$  and  $\sigma_{se}$ . If  $N_1$  and  $N_2$  are taken to be the densities of ions in the ground and excited states, respectively, then we can write

$$(11) \quad \gamma = -N_1\sigma_{abs} + N_2\sigma_{se}$$

In a real optical fiber the upper state population will vary in different parts of the fiber, such that  $N_2 = N_2(\vec{r})$  and  $\gamma = \gamma(\vec{r})$ . In this case, the only way to find the gain is to integrate Eq. (10) over the cross sectional area of the fiber. The equation then becomes

$$(12) \quad dP(z) = \int_r \int_\theta I(\vec{r})\gamma(\vec{r})dz r dr d\theta$$

There are also other sources of loss that can become important in longer fibers, such as Rayleigh and Raman scattering. Any bending in the fiber will also cause loss. All of these can be accounted for in a single attenuation coefficient  $\alpha$ , with units of inverse length. The total loss in a section of fiber of length  $L$  can be written

$$(13) \quad P_{out} = P_{in}e^{-\alpha L} \quad (\text{Beer's Law})$$

The effect of these additional sources of loss is extremely small for fibers below  $\sim 50\text{m}$  in length [5]. All fibers used in this project were less than 1m in length, so the effect of these losses of this type is not significant.

### 3. MODELING ASE IN SINGLE MODE FIBERS

This section outlines the methods used to approximate the Nd-doped fiber laser in a MATLAB simulation. First, the amplifier will be described in terms of continuous variables, and then the equations will be discretized to allow for numerical computation in MATLAB.

**3.1. Intensity Distributions.** In the MATLAB script, the intensities of the different wavelengths are calculated as follows. The pump laser operates at a wavelength shorter than the single-mode cutoff wavelength for this fiber. Thus, the pump light may propagate in a few different modes at the same time. The maxima of these modes are not all at the center of the core, and are instead more evenly distributed. It is therefore reasonable to assume a uniform intensity distribution in the core, as plotted in Figure 8a.

$$(14) \quad I_p(r, z) = P_p(z)\eta/A_{core}$$

where  $P_p(z)$  is the pump power propagating through the fiber at  $z$  and the factor  $\eta$  is an overlap factor, used in the case of cladding pumping. Cladding pumping is a pumping method used with double-clad fibers in which the pump light is guided in the inner cladding rather than the core. This allows a much greater fraction of the available pump light to be coupled into the fiber.

The electric fields propagating through the fiber have an approximately gaussian mode profile [7], so the intensity distribution can be written as

$$(15) \quad I(r, z, \lambda) = \frac{2P(z, \lambda)}{\pi w^2(\lambda)} e^{-2r^2/w^2(\lambda)}$$

where  $w(\lambda)$  is the beam waist for the various ASE wavelengths [6]. A plot is included in Figure 8b ASE may propagate in either direction in the fiber, so I will denote the forward and backward propagating intensities as  $I_f$  and  $I_b$ , respectively.

**3.2. Transition Rates and Upper State Population.** The transition rates for to the pump light wavelength are the easiest to obtain, because the pump light is uniformly distributed throughout the core. The stimulated emission and absorption rates for the pump light are

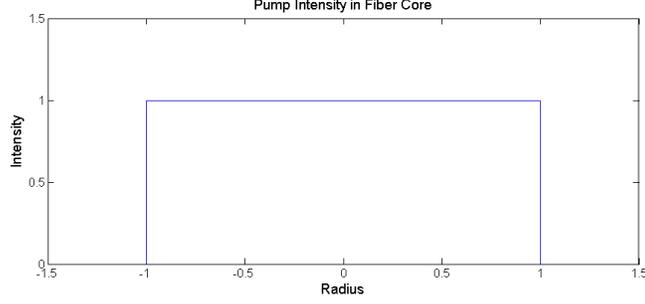
$$(16) \quad R_{21}(r, z) = \frac{I_p(z)}{E_p} \sigma_{se}(\lambda_p)$$

$$(17) \quad R_{12}(r, z) = \frac{I_p(z)}{E_p} \sigma_{abs}(\lambda_p)$$

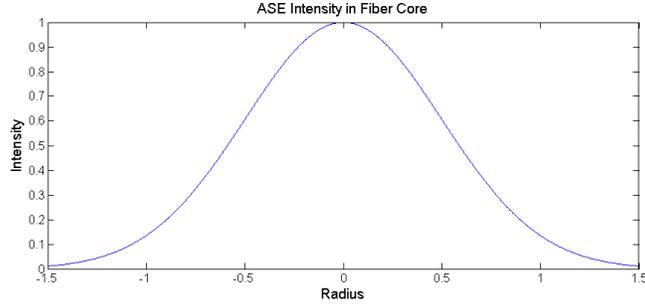
where  $\sigma_{se(abs)}$  are the transition cross sections, and  $E_p$  is the energy of a pump photon.

The transition rates due to ASE will vary with wavelength. Both forward- and backward-propagating ASE will contribute to the transition rate in the same way, so they may be added together.

$$(18) \quad Q_{21}(r, z, \lambda) = \frac{I_f(r, z, \lambda) + I_b(r, z, \lambda)}{E(\lambda)} \sigma_{se}(\lambda)$$



(A) Pump intensity



(B) Single mode intensity

FIGURE 8. The uniform distribution of the pump light approximates multiple modes, while the Gaussian distribution of the signal and ASE light approximates a single mode. In both diagrams, the space between -1 and 1 is the fiber core, and the maximum intensity is normalized to unity. In a double clad fiber, the pump power would be spread across the entire inner cladding rather than just the core, and the single mode intensity would be unaffected.

$$(19) \quad Q_{12}(r, z, \lambda) = \frac{I_f(r, z, \lambda) + I_b(r, z, \lambda)}{E(\lambda)} \sigma_{abs}(\lambda)$$

Finally, the excited state  $N_2$  will decay after a lifetime  $\tau$ , so there is one more emission rate

$$(20) \quad A_{21} = \frac{1}{\tau}$$

These transition rates can be combined to form a differential equation for the upper state population

$$(21) \quad \frac{dN_2}{dt} = N_1(R_{12} + Q_{12}) - N_2(R_{21} + Q_{21} + A_{21})$$

Since  $N_1 = N - N_2$ , this can be rearranged to produce

$$(22) \quad dN_2 = [N(R_{12} + Q_{12}) - N_2(R_{21} + Q_{21} + A_{21} + R_{12} + Q_{12})]dt$$

**3.3. Calculating Power.** The gain coefficient is given at each  $(r, z)$  by

$$(23) \quad \gamma = N_2(\sigma_{se} + \sigma_{abs}) - N\sigma_{abs}$$

where the change in power from  $z$  to  $z + dz$  is  $dP = \gamma dz P(z)$ . Thus, the gain in an infinitesimal length element  $dz$  can be calculated as follows,

$$\begin{aligned} dP_p(z) &= 2\pi \int_{r=0}^{r=\infty} I_p(r, z) \gamma_p(r, z) dz r dr \\ dP_f(r, z, \lambda) &= 2\pi \int_{r=0}^{r=\infty} [I_f(r, z, \lambda) \gamma_{ASE}(r, \lambda) dz + I_{source}(r, z, \lambda)] r dr \end{aligned}$$

The last term represents the new ASE that will be spontaneously generated throughout the fiber. It is equal to

$$I_{source}(r, z, \lambda) = E_{ASE}(\lambda) g(\nu) \frac{1}{\tau} \frac{\Delta\Omega}{4\pi} N_2(r, z) \delta z$$

where

$$g(\nu) = \frac{8\pi n^2 \tau}{\lambda^2} \sigma_{se}(\nu)$$

is the line shape function, which characterizes the probability of a photon being spontaneously emitted with frequency  $\nu$ .  $\Delta\Omega = 2\pi(1 - \cos(\frac{\pi}{2} - \theta_c))$  is the solid angle from which light may enter a guided mode in the fiber, and  $E_{ASE}(\lambda) = \frac{hc}{\lambda}$  is the energy of a photon of wavelength  $\lambda$ . A detailed derivation of the ASE source term can be found in the previous MQP report [5].

The change in backward propagating ASE power has the same form,  $dP_b$  is the change between  $z$  and  $z - dz$ .

**3.4. Discretization Techniques.** The equations above have all been written for continuous  $r$  and  $z$ . In order to perform these calculations on a computer, all of the equations must of course be broken up into discrete pieces and solved numerically. This is where my script makes its greatest departure from the 2009 MQP. In both cases, the fiber is broken up into segments of length  $dz$ . In the current version, the fiber is also divided into rings of width  $dr$  to accommodate the Gaussian mode profile. Both versions also approximate the emission and absorption spectra as step functions, as shown in Figure 9.

In the 2009 script,  $dP_f(z, \lambda)$  is calculated in each length segment sequentially from  $z = 0$  to  $z = L$ , with the output of each element serving as the input for the next element. After the forward ASE has been calculated all of the way down the fiber, the upper state population predicted by the forward pass is used to calculate  $dP_b(z, \lambda)$  from  $z = L$  to  $z = 0$ . This process is illustrated in Figure 10 This method does not accommodate a time dependence, so we must assume a steady upper state population, setting  $\frac{dN_2}{dt} = 0$  in Equation (21). This yields the steady state rate equation

$$(24) \quad N_2 = N \frac{R_{12} + Q_{12}}{(R_{21} + Q_{21} + A_{21} + R_{12} + Q_{12})}$$

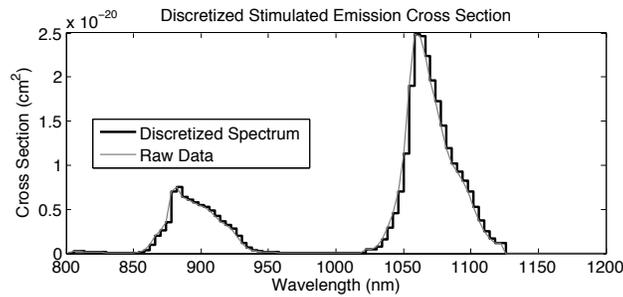


FIGURE 9. The emission spectrum was simplified into a series of bins as shown above.

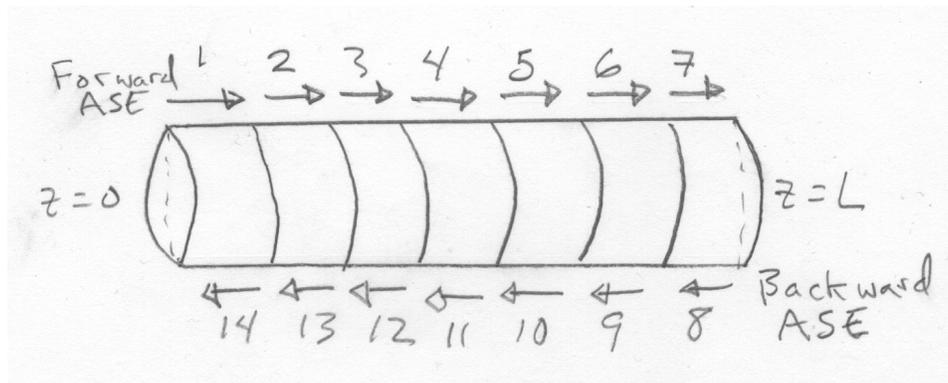


FIGURE 10. In the 2009 script, the forward and backward ASE powers are calculated in two passes up and down the length of the fiber.

This calculation is iterated using the upper state population from the previous calculation as an initial condition until the change in the upper state population from one iteration to the next drops below a very small threshold.

As demonstrated in the 2009 project, this approximation method is reasonably accurate and fairly quick to compute. In a real fiber however, the absorption of pump light and generation of ASE would of course occur at all points in the fiber simultaneously, not in a wave propagating up and down the fiber as described above. Thus, this method could be made more physically accurate by reducing the fiber length to a very small size.

In the current version of the script, this is achieved by using matrix operations to calculate the ASE in many small elements simultaneously. Lengthwise, the fiber is broken up into segments of width  $dz$  and radially, the fiber core is divided into rings of thickness  $dr$ , as shown in Figure 11. Functions of  $(r, z)$  can be approximated by a matrix with a column for each length segment and a row for each ring.

The rate equation is solved at all points in the fiber (all elements of the matrix) simultaneously. Consider the above procedure applied to a single volume

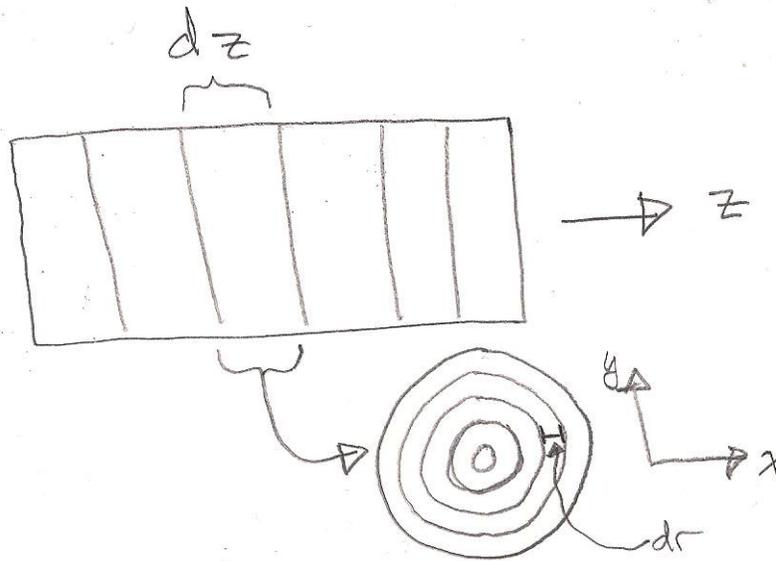


FIGURE 11. The fiber core is divided into a series of “slices in the longitudinal direction and rings in the radial direction.

element of the fiber, yielding the forward and backward ASE powers leaving the fiber element. These powers can then be used as inputs for the elements to the left and right. The processes involved in this calculation are illustrated in Figure 12

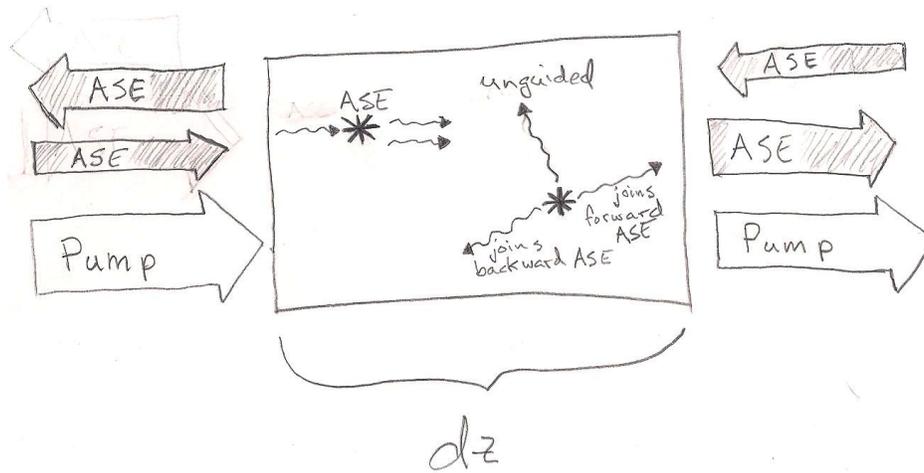


FIGURE 12. In this diagram, \* represents excited ions within a volume element engaged in stimulated emission (left) and spontaneous emission (right).

This method of calculation lends itself well to a more accurate approximation of the rate equation as well. Rather than assume a steady state population of excited ions, we can keep the  $dt$  in Equation 22 by assuming that all of the light propagates at the same speed down the fiber. By making this assumption, we can substitute  $dt = dz/c$ . This is a reasonable assumption for the short fiber lengths and relatively narrow wavelength range encountered in a NDFA. This substitution makes it so that the light in one length segment of length  $dz$  will be in an adjacent segment after a time  $dt$  has passed. Thus, simulating the propagation of light in the fiber is as simple as shifting all of the values in the intensity matrix one column to the right or left, depending on whether the beam is propagating in the  $+z$  or  $-z$  direction. The time evolution of the laser can be simulated in this way. The calculation is iterated until the change in the ASE intensity from one iteration to the next drops below a very small threshold for a sufficient number of iterations.

This matrix method has some distinct advantages over the sequential method. When using a large number of elements, the matrix method is significantly faster to compute, because it takes advantage of MATLABs ability to manipulate matrices very efficiently. Also, the sequential method would sometimes oscillate between two fairly different upper state populations with the population taking one shape on the forward pass and another on the backward pass. The ASE power would still converge to a single value, but this behavior is struck me as undesirable, and has been eliminated by performing the forward and backward calculations simultaneously, without the need to recalculate the upper state population in between the two.

#### 4. EXPERIMENTAL PROCEDURE

After the MATLAB script was completed I took some measurements of actual ASE spectra from Nd-doped fiber amplifiers. Figure 13 illustrates the basic setup used in the lab. The laser source operated at 532 nm, with a maximum power output of 50 mW. The polarizing filter was a simple camera filter mounted on a stand to allow it to be rotated to various angles. This filter served to adjust the initial pump power, because the laser source produced polarized light. The pump beam was focused into the Nd-doped fiber by a microscope objective. The light was coupled once more from the doped fiber into an undoped fiber which was connected to the input of the optical spectrum analyzer (OSA). This free space coupling between the two fibers was aligned using a precision translation stage.

There were a few different fibers available for coupling the light into the OSA; two multimode fibers and one single mode fiber. There is a tradeoff between the two types of fiber, as both have drawbacks. The multimode fibers are much easier to couple light into, but when a broadband signal is coupled from a single mode fiber into a multimode fiber, signals at different wavelengths can interfere with one another [12]. Figure 14 shows two spectra both taken at full power with the multimode fiber plugged into the OSA. The oscillating peaks which do not line up with from one trial to the next are indicative of this type of interference. On the other hand, the single-mode fiber has a much smaller core and it is therefore difficult to couple a large signal into it. This results in a large signal to noise ratio, requiring more measurements to be averaged together to obtain accurate results. It can take quite a long time to take a sufficient number of measurements at the

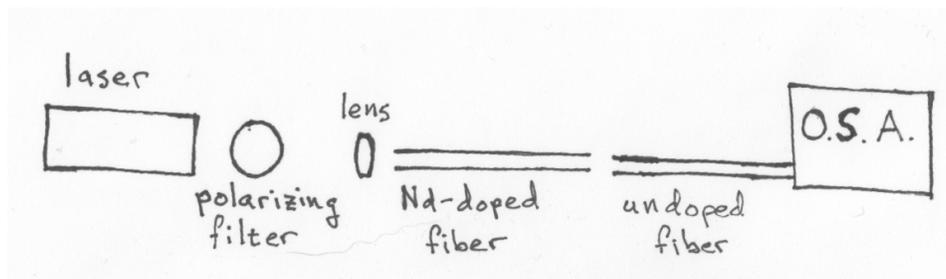


FIGURE 13. Experimental setup for measuring the ASE spectrum of the Nd-doped fibers

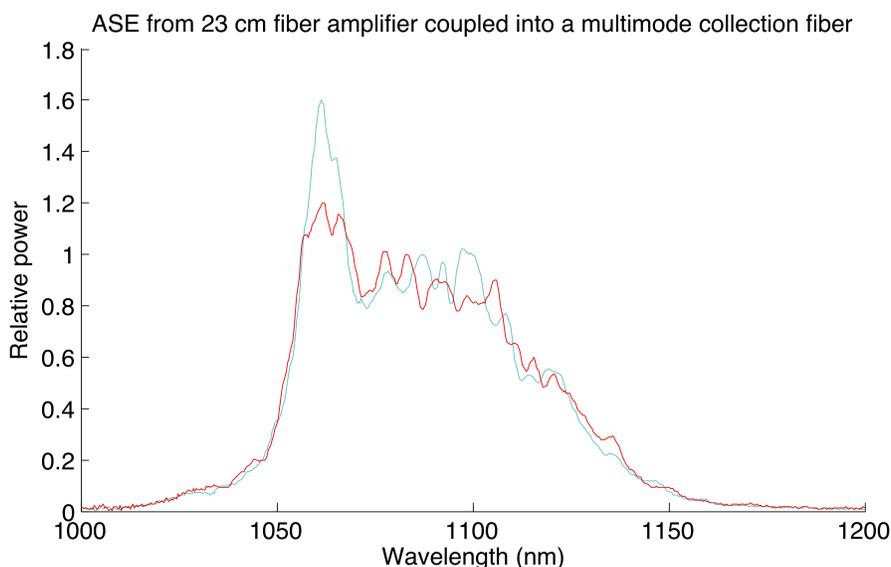


FIGURE 14. Both of the spectra plotted above were taken with the same pumping power and then coupled into a multimode fiber.

highest sensitivity setting, so the single-mode fiber is not necessarily a better choice than the multimode fiber.

Measurements were taken for two Nd-doped fibers of different lengths and a variety of pumping powers. One fiber was quite short at only 23 cm and the other was a bit longer at 93 cm. Doping concentration was calculated to be approximately  $7.7 \times 10^{17} \text{ cm}^{-3}$  based on the attenuation reported by the manufacturer [13].

Since the pumping wavelength of 532 nm is not common for communications purposes, the absorption coefficient of the fiber was not quoted by the manufacturer for this wavelength. Therefore, this detail had to be determined experimentally. This was accomplished by wrapping the fiber around a post three times to strip off cladding modes, then the remainder of the fibers length was placed in a narrow channel created by placing two rulers about 1 cm apart. The

light emitted from the side of the fiber was then measured at regular intervals along the fiber using a photodetector.

The power actually coupled into the fiber is difficult to find without damaging the fiber. I tried to determine it by measuring the light emerging from the end of the fiber and using the absorption coefficient to find the coupled power.

## 5. RESULTS AND ANALYSIS

**5.1. Theoretical Results.** If the MATLAB script is working properly, then the gain will be proportional to the pumping power at low power, above the transparency point (the fiber length must not be so long that the signal is simply reabsorbed.) We can determine whether the ASE has undergone significant gain by comparing the spectrum at the end of the fiber to the single ion fluorescence spectrum. At low pumping powers, the upper state population should be too low for significant gain to occur, producing an ASE spectrum nearly identical in shape to the single ion fluorescence. At higher powers gain will occur, creating tall spikes at local maxima in the fluorescence spectrum, since the gain coefficient is proportional to the emission cross section (see Equation (11).) This means we can qualitatively see the amount of gain in the fiber by looking at the slope of the peaks in the ASE spectrum. Specifically, the four level transition at 1060 nm should show high levels of gain because there is no risk of reabsorption by the ground state. Figure 15 shows that the spike centered at 1060 nm does indeed become narrower as the pumping power is increases, indicating higher levels of ASE. The spectra in the diagram may be compared to the single ion fluorescence spectrum. If the calculated spectrum is very similar in shape to the fluorescence spectrum, as is the case for the 10 mW line, this indicates minimal ASE.

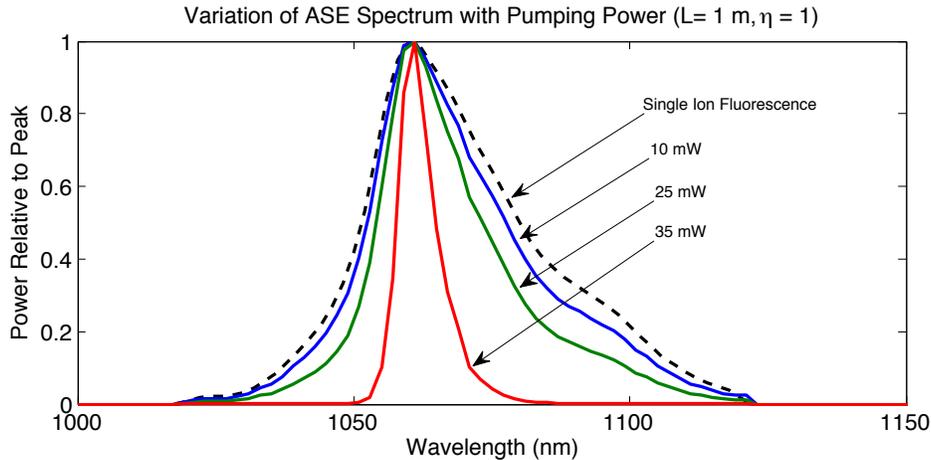


FIGURE 15. Calculated ASE spectrum at various pumping powers in a 1 meter fiber, normalized to unity at the peak.

It is quite straightforward to excite the 1060 nm transition, as it is a four level transition and therefore immune to the effects of ground state absorption. In order to produce blue lasers via frequency doubling however we must excite a

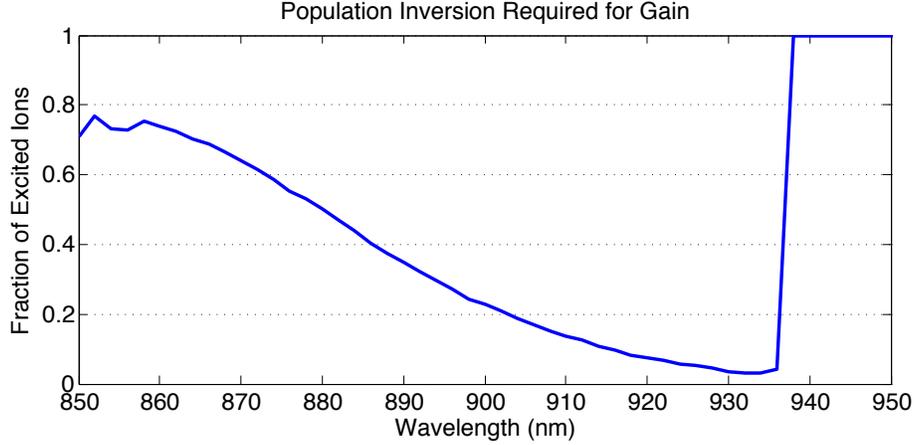


FIGURE 16. Minimum fraction of excited state ions required to achieve gain

transition in the 900–950 nm range, where GSA can be a serious obstacle. In order to produce gain in this range, the fraction of Nd ions in the excited state must exceed a certain threshold, as determined by Equation (11), with  $\gamma = 0$ . Figure 16 shows that gain can be achieved most easily at 932 nm, where only about 3% of the Nd ions must be excited. At shorter wavelengths, this fraction increases, with a 900 nm signal requiring that 23% of ions be excited.

This minimum fraction of excited ions can be used to calculate the minimum pumping power required to achieve signal gain at a given wavelength. For low pumping powers, the fraction of excited ions is approximately equal to the product of the pump absorption rate and the lifetime of the excited state.

$$(25) \quad \frac{N_2}{N} \approx R_{12}\tau = \frac{P_p}{A_{core}} \frac{\lambda_p^2}{hc} \sigma_p \tau$$

In Nd-doped silica glass,  $\tau \approx 300 \mu s$ . For the fibre I used in the lab,  $\sigma_p = 6.36 \times 10^{-21} cm^2$  at the pump wavelength of 532 nm. The method for obtaining this value is explained in Section 5.2. For a fiber with a 1.6  $\mu m$  core radius, this is equal to approximately  $58 W^{-1} P_p$ . Therefore, coupling 10 mW of pump power into the fiber core should excite approximately 58% of the Nd ions. The MATLAB simulation shows that with 10 mW of pumping power, the fraction of excited ions remains nearly constant at this level throughout the fiber core.

One way to improve the fraction of excited ions is to simply increase the pumping power, but there are practical limits to doing so, and in shorter fibers only a small fraction of pump light is absorbed, making a very inefficient laser. The other way to improve the fraction of ions in the excited state is to increase the fiber length, allowing a greater fraction of the pump to be absorbed. Increasing the fiber length also increases the amount of GSA, making reabsorption of the signal a possibility. Furthermore, ASE at 1060 nm has a tendency to consume a large number of the excited states since it is not subject to GSA. The effects of GSA

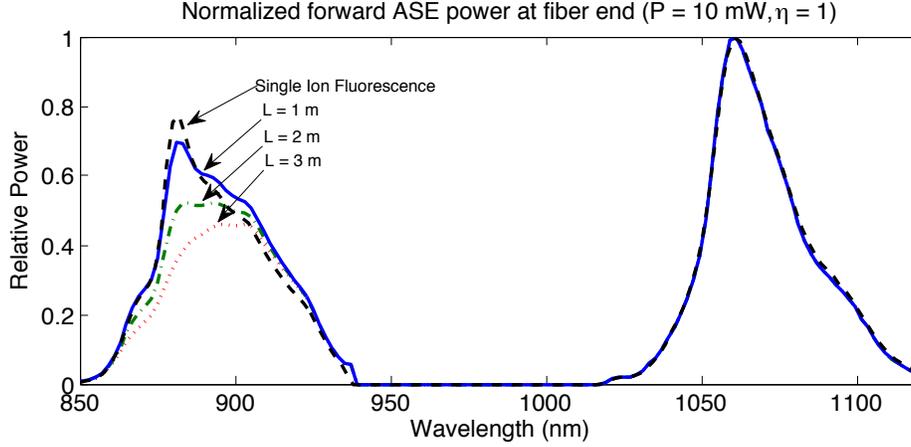


FIGURE 17. Calculated spectrum at the end of fiber of varying length with low pumping power, normalized to unity at the 1060 nm peak.

on the spectrum shape are shown in Figure 17. As the fiber length is increased from 1 m to 3 m the size of the 900 nm peak drops in comparison to the 1060 nm peak. The peak also shifts to longer wavelengths because the shorter wavelengths are absorbed more readily. Without GSA, one would expect the spectrum to grow narrower as the length is increased, allowing greater pump absorption and higher population inversion.

**5.2. Experimental Results.** One of the unknowns in the experiment was the absorption coefficient of the fiber for the pump wavelength, because the pump was outside of the fiber's suggested operating range. By measuring the light escaping from the side of the fiber, I was able to determine the absorption coefficient ( $\alpha$ ) using a curve fitting program (See Figure 18). The curve in the plot is

$$P = 0.001922 \exp(-0.0049z).$$

From this it is also easy to determine the absorption cross section, which is simply

$$(26) \quad \sigma_{abs} = \alpha/N = 0.0049 \text{ cm}^{-1}/7.7 \times 10^{17} \text{ cm}^{-3}$$

$$(27) \quad = 6.36 \times 10^{-21} \text{ cm}^2$$

After obtaining the absorption coefficient, I measured the power exiting the end of a 93 cm NDFA in order to determine the approximate amount of power coupled into the beginning of the fiber. The power exiting the fiber end was measured to be 2.73 mW. Using the absorption coefficient calculated above, I found

$$\begin{aligned}
 (28) \quad P_{coupled} &= P_{measured} \exp(\alpha z) \\
 (29) \quad &= 0.00273 \exp(0.0049 * 93) \\
 (30) \quad &= 4.3 \text{ mW}
 \end{aligned}$$

This is likely to be an overestimate of the coupled power, since there was a fair amount of light visibly escaping the sides of the fiber, which may indicate cladding modes. By referring back to Figure 15, we can see that even at 10 mW of coupled pumping power in a 100 cm fiber, there is little deviation from the single ion fluorescence spectrum. It is not until the power is considerably higher than this that noticeable ASE begins to occur. This could explain why no convincing evidence of ASE could be found in the experimental data.

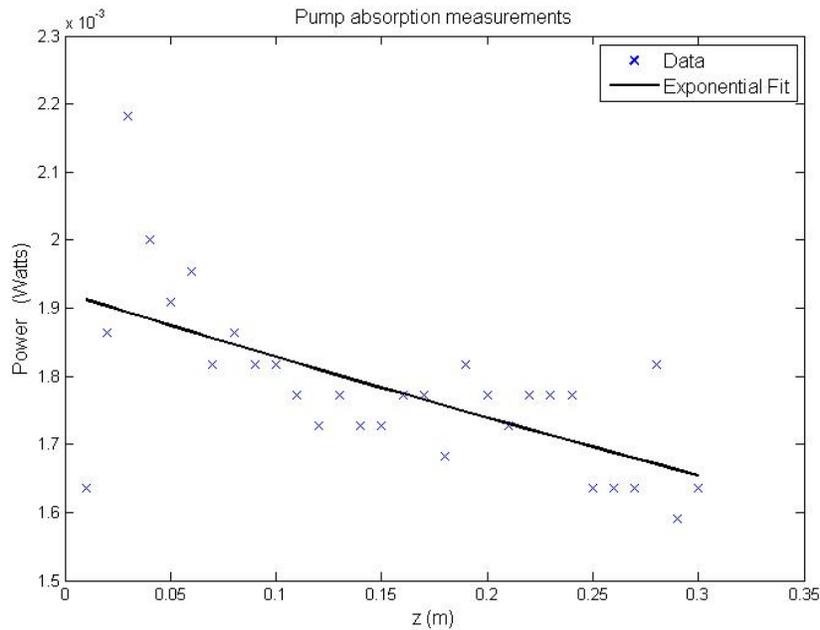


FIGURE 18. Measured pump absorption levels with exponential fit curve.

## 6. CONCLUSIONS

**6.1. Future work.** One factor that can contribute to ASE is the reabsorption of the laser light by unexcited electrons, which is known as ground-state absorption (GSA). The 900 nm transition is susceptible to GSA because it is a three-level transition, while the undesired 1060 nm ASE is a four-level transition, which does not experience GSA due to the low population of states with the right amount of energy to reabsorb it. Thus, in a longer fiber the four-level gain may become very large while the three-level signal is limited by GSA. By reducing GSA in the fiber, the 1060 nm ASE can be also be reduced [4]. The GSA may be reduced by shortening the fiber, but the effective mode area must then be increased to allow the

same pump power to be used. Increasing the mode area will eventually produce a multimode fiber, which is typically avoided due to higher levels of dispersion. However, a large mode area (LMA) fiber would permit higher pump power. Thus, the secondary goal of this project was originally to extend the model of ASE in single-mode fibers to LMA fibers as well, investigating the feasibility of using a W-fiber geometry to adjust the mode cutoff in the same manner used for single mode fibers, but time constraints did not allow me to undertake that challenge. This would of course require the computation of Bessel functions, since the Gaussian approximation is not very good for higher-order modes. The model I developed could be modified to use an approximate analytic solution similar similar to that found in [14] with different boundary conditions to reflect the double-clad fiber geometry. My MATLAB script may be able to accommodate this kind of modification without too much hassle. Rather than one general ASE power array, the new code would need to calculate a separate array for each mode that is to be considered in the calculation. The filtering effect of the W-fiber geometry would also be fairly simple to implement. Perhaps this can be the topic of a future MQP.

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## APPENDIX A. MATLAB CODE

---

```

function gain = asecalc(pump,len)
% Main program. It may be adjusted to fit other pump and signal powers,
% dopants and fiber sizes by editing input_parameters.m
clc;

tic;
StartTime = now;

set(0,'DefaultFigureWindowStyle','docked'); %Docking figures

addpath Resources

global z_bins r_bins lambda_bins min_wavelength max_wavelength threshold
global pumpPower initialSignal lambda_p lambda_s
global length_0 a NA n
global lifetime N A eta SolidAngle

input_parameters

length_0 = len;
pumpPower = pump;

% Physical constants
c = 2.99792*10^8; % m/s
h = 6.62606*10^-34; % Js

dz = length_0/z_bins; % incremental step along fiber
z = zeros(z_bins+1,1);
for i = 1:z_bins+1 %sets up z as a column vector
    z(i,1) = dz*(i-1);
end

dr = a/(r_bins);
r = zeros(1,r_bins+1);
for i = 2:r_bins+1 %sets up r as a row vector
    r(1,i) = dr*(i-1);
end

ring_area = zeros(length(r)-1,1);
for i = 1:length(r)-1 % sets up the area of each ring to be used in the Riemann sum later
    ring_area(i,1) = pi*(r(1,i+1)^2 - r(1,i)^2);
end
ring_area(length(r),1) = 0;

rawSpectrum = getSpectrum(); %Defines the Spectrum from the call function getSpectrum

lambda_step = floor((max_wavelength - min_wavelength)/lambda_bins);
lambda_ase = zeros(1,1,lambda_bins);
sigma_ase = zeros(1,1,lambda_bins);
sigma_aseabs = zeros(1,1,lambda_bins);
delta_nu_ase = zeros(1,1,lambda_bins);

```

```

eqspec = zeros(size(rawSpectrum));
stop_index = 1;

for k = 1:lambda_bins
    wavelength1 = min_wavelength+lambda_step*(k-1);
    wavelength2 = min_wavelength+lambda_step*k;
    max_sigma_se = 0;
    lambda_ase(k) = round(wavelength1 + lambda_step/2);
    for m = stop_index:length(rawSpectrum(:,1))
        if rawSpectrum(m,1) > wavelength1
            start_index = m-1;
            break
        end
    end
    for m = stop_index:length(rawSpectrum)
        if rawSpectrum(m,1) > wavelength2
            stop_index = m-1;
            break
        end
    end
    for m = start_index:stop_index
        if rawSpectrum(m,3) > max_sigma_se
            max_sigma_ase = rawSpectrum(m,3);
            sigma_aseabs(k) = rawSpectrum(m,2);
            sigma_ase(k) = rawSpectrum(m,3);
        end
    end
    eqspec(:,1) = rawSpectrum(:,1); % eqspec is used to show the actual and
        % equivalent spectra later on

    for m = start_index:stop_index
        eqspec(m,2) = sigma_aseabs(k);
        eqspec(m,3) = sigma_ase(k);
    end

    %Convert from wavelength to frequency: nu = c/lambda.
    %dnu = c/lambda^2*dlambda
    delta_nu_ase(k) = c*lambda_step*10^-9/(lambda_ase(k)*10^-9)^2;
end

lineshape = zeros(1,1,length(lambda_ase));
for k = 1:length(lambda_ase)
    lineshape(k) = 8*pi*n^2*lifetime*sigma_ase(k)/((lambda_ase(k)*10^-9)^2);
end

V_s = 2*pi*a/lambda_s*NA; %V-parameter

w_s = a*(0.65 + 1.619/V_s^1.5 + 2.879/V_s^6); % m beam waist

```

```

V_ase = zeros(1,1,length(lambda_ase));
w_ase = zeros(1,1,length(lambda_ase));
for k = 1:length(lambda_ase)
    V_ase(k) = 2*pi*a/(lambda_ase(k)*10^-9)*NA;
    w_ase(k) = a*(0.65 + 1.619/V_ase(k)^1.5 + 2.879/V_ase(k)^6);
end

E_s = h*c/lambda_s; %J energy of signal photon
E_p = h*c/lambda_p; %J energy of pump photon
E_ase = zeros(1,1,length(lambda_ase));
for k = 1:length(lambda_ase)
    E_ase(k) = h*c/(lambda_ase(k)*10^-9); %J, energy of ASE photon
end

% sigma_pabs = getAbs(lambda_p); %cross section for pump absorption
sigma_sse = getEms(lambda_s); %cross section for signal emission
% sigma_pse = getEms(lambda_p); %cross section for pump stimulated emission
sigma_sabs = getAbs(lambda_s); %cross section for signal absorption
sigma_pse = 0;
sigma_pabs = 8e-21;

%In this loop we define the initial pump power.
%This will allow us to calculate the signal power for a variety of pump
%powers, and plot them simultaneously.

signal_gain = 0; %Priming the next loop
iteration = 0;
deltagain = 1;
loop = 1;
test_pass = 0;
checkpoint = length(z);

%absorption coeff:
alpha = 8.5; % dB/m

% Preallocating vectors for speed:
N_2 = zeros(length(z),length(r));
% N_2 = N*(pumpPower*eta*exp(-alpha*z(:))*sigma_pabs)/A/E_p./...
%( (pumpPower*eta*exp(-alpha*z(:))*sigma_pabs)/A/E_p + 1/lifetime )*ones(1,length(r));
P_asef = zeros(length(z),1,length(lambda_ase));
P_aseb = zeros(length(z),1,length(lambda_ase));
P_s = zeros(length(z),1);
P_p = zeros(length(z),1);
% P_p = pumpPower*exp(-alpha*z(:));
I_s = zeros(length(z),length(r));
I_p = zeros(length(z),length(r));
I_asef = zeros(length(z),length(r),length(lambda_ase));
I_aseb = zeros(length(z),length(r),length(lambda_ase));
I_ase = zeros(length(z),length(r),length(lambda_ase));
Q_12 = zeros(length(z),length(r),length(lambda_ase));
Q_21 = Q_12;
gamma_ase = Q_12;

```

```

ASE_source = Q_12;
dP_asef = P_asef;
dP_aseb = P_asef;

P_p(1) = pumpPower;
P_s(1) = initialSignal;

% Below is the main loop of the script. Each iteration represents a time
% step of dz/c seconds. The loop must run until at least z_bins + 1
% iterations. Then it checks periodically for convergence of the signal,
% pump, and ASE until it is below the threshold at all points in the
% fiber.

figure(1);

while loop == 1

    iteration = iteration + 1;

    % calculate intensities
    I_p = P_p*eta/A*ones(1,length(r));
    I_s = P_s*2/(pi*w_s^2)*exp(-(r + dr/2).^2/w_s^2);
    I_asef = 2/pi*multiprod( multiprod( P_asef , 1./(w_ase.^2) ) , ...
        exp(-multiprod( (r + dr/2).^2 , 1./w_ase.^2) ) );
    I_aseb = 2/pi*multiprod( multiprod( P_aseb , 1./(w_ase.^2) ) , ...
        exp(-multiprod( (r + dr/2).^2 , 1./w_ase.^2) ) );
    I_ase = I_asef + I_aseb;

    % calculate transition rates
    R_12 = I_p/E_p*sigma_pabs; % pump absorption
    R_21 = I_p/E_p*sigma_pse; % pump stimulated emission
    W_12 = I_s/E_s*sigma_sabs; % signal absorption
    W_21 = I_s/E_s*sigma_sse; % signal stimulated emission
    Q_12 = multiprod( I_ase , 1./E_ase.*sigma_aseabs );
    Q_21 = multiprod( I_ase , 1./E_ase.*sigma_ase );
    A_21 = 1/lifetime; % spontaneous emission rate

    dN_2 = ( N.*(R_12 + W_12 + sum(Q_12,3)) - N_2.*(R_12 + R_21 + W_12+ W_21 + ...
        sum(Q_12 + Q_21,3) + A_21) )*dz*n/c;
    N_2 = N_2 + dN_2;

    % calculate gain coefficients
    gamma_p = N_2*(sigma_pse + sigma_pabs) - N*sigma_pabs;
    gamma_s = N_2*(sigma_sse + sigma_sabs) - N*sigma_sabs;
    gamma_ase = multiprod( N_2 , sigma_ase + sigma_aseabs ) - ...
        multiprod( N*sigma_aseabs , ones(size(N_2)) ) ;
    % gamma_s = 0;
    % gamma_ase = zeros(size(I_asef));
    N_2_mode = multiprod(N_2 , ones(size(lambda_ase))).*...
        multiprod( ones(size(N_2)) , 2./w_ase.^2 ) .*...
        multiprod( ones(size(z)) , exp( multiprod( -r.^2 , 1./w_ase.^2 ) ) ).*...

```

```

        multiprod( ones(length(z),1,length(lambda_ase)) , ring_area' );
ASE_source = multiprod( E_ase.*lineshape.*delta_nu_ase , N_2_mode )/ ...
lifetime*SolidAngle*dz;

% calculate dP
dP_p = I_p.*gamma_p*ring_area*dz;
dP_s = I_s.*gamma_s*ring_area*dz;
dP_asef = multiprod( (I_asef.*gamma_ase*dz + ASE_source), ring_area );
dP_aseb = multiprod( (I_aseb.*gamma_ase*dz + ASE_source) , ring_area );

% calculate new powers and shift in the appropriate direction
P_p = circshift(P_p + dP_p,[1 0]);
P_s = circshift(P_s + dP_s,[1 0]);
P_asef = circshift(P_asef + dP_asef,[1 0]);
P_aseb = circshift(P_aseb + dP_aseb,[-1 0]); % shifted backward

% reset boundary conditions
P_p(1) = pumpPower;
P_s(1) = initialSignal;
P_asef(1, :, :) = 0;
P_aseb(end, :, :) = 0;

%remove negative values that can appear when N_2 is very small
P_asef( P_asef < 0 ) = 0;
P_aseb( P_aseb < 0 ) = 0;
%   P_s(P_s<0) = 0; % these lines aren't typically necessary
%   P_p(P_p<0) = 0;

if iteration == checkpoint
    P_s_prev = P_s;
    P_asef_prev = P_asef;
    P_aseb_prev = P_aseb;
end

%% This section will plot the evolution of the system in time as the
% calculation progresses, but this is very slow.
% not recommended if you actually want results, but may be helpful to see.
%   if floor(iteration/z_bins) == iteration/z_bins
%       figure(1);plot(lambda_ase(:),P_asef(end,:));hold on;
%       figure(2);plot(z,P_asef(:,65));hold on;
%   end
%
%   if iteration > 100000 - 100
%       figure(4);plot(z,P_asef(:,65));hold on;
%       figure(3);plot(lambda_ase(:),P_asef(end,:));hold on;
%   end

% figure(1);subplot(2,2,1);surf(r,z,N_2/N);title('N_2');shading interp
% subplot(2,2,2);surf(lambda_ase(:),z,squeeze(dP_asef));
% title('dP_asef');shading interp;set(gca,'zsc','log');
% subplot(2,2,3);surf(lambda_ase(:),z,squeeze(ASE_source(:,1,:)));

```

```

%title('Source term');shading interp;set(gca,'zsc','log');
% subplot(2,2,4);surf(lambda_ase(:),z,squeeze(P_asef + P_aseb));
%title('ASE Power');shading interp;set(gca,'zsc','log');
%%

% convergence criteria
if iteration > checkpoint % wait for signal to propagate to end of fiber
    % (checkpoint is initialized to length(z)

    loop = 0;
    q1 = abs(P_s./P_s_prev - 1) > threshold; % signal convergence criterion
    q2 = abs(P_asef./P_asef_prev - 1) > threshold; % forward ase convergence criterion
    q3 = abs(P_aseb./P_aseb_prev - 1) > threshold; % backward ase convergence criterion

    fprintf('Iteration # %4d, %5d of %g points above convergence threshold \n', ...
        iteration, sum(q1(:))+sum(q2(:))+sum(q3(:)),numel(z)+2*numel(z)*numel(lambda_ase) )
    if sum( q1(:) ) + sum( q2(:) ) + sum( q3(:) ) > 100
        loop = 1;
        checkpoint = checkpoint + z_bins;
    elseif any(q1)
        loop = 1;
    elseif any( q2(:) )
        loop = 1;
    elseif any( q3(:) )
        loop = 1;
    elseif test_pass < 1
        test_pass = test_pass + 1;
        loop = 1;
        checkpoint = checkpoint + z_bins;
    end
    P_s_prev = P_s;
    P_asef_prev = P_asef;
    P_aseb_prev = P_aseb;
end

end %End iteration

uStatePop = N_2/N; % Fraction of excited ions for plotting

EndTime = now;
['Start Time: ' datestr(StartTime,13)]%#ok<NOPTS>
['End Time: ' datestr(EndTime,13)] %ok<NOPTS>

gain = P_s(length(P_s))/initialSignal; %gain of signal power
dB = 10*log10(gain); %gain in decibels
SignalPower = P_s(length(P_s));

fname = sprintf('P_%2.2g_bins%gx%g.mat',pumpPower,z_bins,r_bins);
fname = strrep(strrep(strrep(strrep(...
    fname,'e-0','e-'),'e-0','e-'),'e+0','e+'),'e+0','e+');

```

```
fname = ['C:\Users\ggeddes\Desktop\',fname];  
% I had to save here because I began getting errors when saving to filer.  
  
save(fname)  
  
beep on; beep; beep off  
  
hold all; generate_plots  
toc;
```

---