

Credibility Theory

A Major Qualifying Project
submitted to the Faculty of
WORCESTER POLYTECHNIC INSTITUTE
in partial fulfillment of the requirements for the
degree of Bachelor of Science



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Date:
December 18, 2014

Report Submitted to:

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Abstract

In insurance, actuaries need to decide how much they trust each policyholder's experience rate. Credibility theory is the study of how much merit to give a policyholder's experience. Using data provided by Unum, the Buhlmann-Straub approach to credibility was found to raise loss ratios overall, while still being under the Tolerable Loss Ratio. Thus, Unum's customers may be overcharged for their coverage and competitors may price their insurance lower than Unum, while meeting their own Tolerable Loss Ratios.

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Introduction

Credibility theory is an important part of actuarial science, yet there is no uniform procedure that actuaries use to assign credibility. In the actuarial world, it appears that each insurance company creates their own method for assigning credibility. In a survey conducted by the Society of Actuaries, insurance companies were asked what factors they considered in their long-term disability credibility formulas. While ten insurance companies answered life years of exposure, others responded that they considered actual claims, elimination period and even average age. The differences in calculating credibility may stem from the fact that much of the theory and formulas for calculating credibility were developed long ago. One of the pioneers of credibility theory, A.W. Whitney, published his formula for credibility in 1918. While advances have been made since Whitney's formula was created, it is still a crucial formula in calculating credibility. In fact, Unum's credibility formula is of the form $Z = \frac{n}{n+k}$ that Whitney suggested.

Similar to many of the companies surveyed by the Society of Actuaries, Unum's credibility formula uses life years of exposure and claim amounts as factors. But despite having similarities to other insurance company's credibility formulas, the credibility formula that Unum uses does have some differences compared to other credibility calculation methods. For instance, Unum's credibility formula does not have any variance component. This component is of particular importance in long-term disability insurance. In this type of insurance, reserve amounts are constantly changing because of changing expectations of the total size of a claim. So, an appropriate credibility calculation should take into account how information changes over time.

This paper explores different options for calculating credibility. An emphasis is placed on the Bühlmann-Straub method, but the full credibility method is also explored. This paper also

explains the weaknesses of the Unum credibility formula and analyzes how a change in credibility calculation would affect Unum's assigned credibility factors and loss ratios.

Background

Long Term Disability

Long term disability insurance is a type of insurance that assists the insured in the event that they become disabled and unable to work for an extended period of time. When healthy and working, the insured will have a portion of their paycheck taken out and deposited as premium for the insurance. Should they become ill or injured for a few months or years, long term disability coverage will pay a portion of their salary in the form of a monthly benefit. The monthly benefit is typically between 50 to 66% of the insured's annual salary.

Long term disability is often sold as a group product, meaning the employer will reach out to insurance companies and buy coverage for their employees (should they choose to enroll in coverage). Groups can range from less than 100 lives to more than 10,000 lives. The number of lives will affect the amount of trust given to the experience (or recent history of claims) for the group.

Disability pricing is based on the present value of the dollar amount held aside to pay off a given claim. This amount is known as the reserve and is calculated based on a multitude of factors, such as amount paid per month to the insured and the expected duration and severity of a disability. Having to value claims based on all of the factors makes long term disability a tricky product to price. But, a group's history and previous experience can assist in accurately pricing.

Credibility

In actuarial science, credibility theory is the study of adjusting premium rates based on the previous experience of a group. The adjustment of premium rates based on past experience is a delicate balance. A group with consistent experience should have a high credibility. But, if the group is assigned too high of a credibility, the insurer could experience large losses if the group

has an above normal number of claims. On the other hand, insurers do not want to overreact to past experience and continually change premium rates. This would not only be costly for the insurance company, but the fluctuating rates might also upset the insured. Clearly, credibility is a complex topic that is still being explored.

Credibility theory was first explored by A.W. Whitney in 1918.¹ In his work, Whitney created two important formulas in credibility theory. The first important formula is

$$\text{Case Rate} = Z * \text{Experience Rate} + (1 - Z) * \text{Manual Rate}$$

In this formula, the Case Rate is the blended rate of the experience rate and the manual rate and Z is the credibility factor. The credibility factor has a number of properties. First, the credibility factor is between zero and one. When the credibility factor is equal to zero, the case rate is equal to the manual rate. When the credibility factor is equal to one, the case rate is equal to the experience rate. This special case is referred to as full credibility. In many cases, requirements for full credibility may be independent of the credibility factor calculation and instead may be based solely on the frequency of claims.² When a credibility factor is between zero and one, this indicates that a group has enough experience to receive some credibility, but not enough to receive full credibility. As explained by L.H. Longley-Cook in a report for the Casualty Actuary Society, “credibility theory is concerned with establishing measures of credibility and standards of full credibility.”

The second important formula proposed by Whitney is

$$Z = \frac{n}{n+k}$$

In this formula, n represents earned premiums and k needs to be determined. Whitney believed that a credibility factor needed to be of this form because this formula will always produce a credibility factor within the desired range of zero and one.

Approaches to Credibility

Limited Fluctuation Approach

The limited fluctuation approach to credibility is one of the oldest approaches going back to the work of Mowbray in 1914. Albert Mowbray was an American mathematician who was extremely influential in the development of actuarial methods. Mowbray served as a professor at the University of California for over 30 years while making strides in the actuarial field.³ The limited fluctuation approach to full credibility, also known as “American credibility”, uses frequentist models to determine the number of expected claims required for full credibility. Frequentist models determine the probability of an event based on its frequency. These models ignore prior information and only look at the observed data. When using this method for full credibility, insureds want their premium to be determined based upon their own experience and nothing else. This method involves assuming that annual claims are independent and identically distributed and calculating the number of claims needed to get actual claims minus expected claims within a small probability.

Under the limited fluctuation approach, full credibility will be assigned to the estimator of aggregate claims based solely on observed data such that S is within $100c\%$ of the true value s with probability $1 - \alpha$.

$$P[-cs < S - s < cs] = 1 - \alpha$$

or

$$P\left[\frac{-cs}{\sqrt{\text{Var}(S)}} < \frac{S - s}{\sqrt{\text{Var}(S)}} < \frac{cs}{\sqrt{\text{Var}(S)}}\right]$$

The expected value and variance of claim amounts, X_i , are assumed constant.

$$E[x_i] = m$$

And

$$Var[x_i] = \sigma^2$$

There is a general case obtained from this where we calculate λ_F , the minimum number of expected claims over the next period needed to obtain full credibility:

$$\lambda_F = \left(\frac{\chi_\alpha^2}{c^2} \right) \left[1 + \left(\frac{\sigma}{m} \right)^2 \right]$$

There are a few assumptions necessary under this approach. We assume the number of claims has a Poisson distribution, the mean and variance of loss severity are constant throughout for all values and that the Central Limit Theorem applies. The parameter c is called the range parameter, χ_α is the point on the normal curve where the area between $-\chi_\alpha$ and χ_α is equal to which is the probability level. The standard deviation divided by the mean is known as the coefficient of variation.⁴

To determine the minimum number of expected claims needed over the next period to obtain full credibility you would start by choosing a range parameter, c , the probability level, $1 - \alpha$, and estimating the coefficient of variation. If the assumptions stated above are true, then calculate λ_F using the previously stated equation.

This approach to full credibility has its strengths and weaknesses. The main strength of the limited fluctuation approach is its simplicity to use which leads to the general acceptance and use of this approach. This approach is good for the experience rating where there is a default premium. There are many weaknesses in this approach as well. To start, the limited fluctuation approach uses frequentist paradigm which means that when using this approach prior data is ignored in the

calculations and confidence intervals are applied as well. Another weakness is the fact that there are arbitrary assumptions involved in calculating full credibility with finding c and $1 - \alpha$. Finally, the assumption that the formula is based on a Poisson distribution is not applicable in many situations.^{5 6}

Bühlmann-Straub Method

The Bühlmann-Straub Method is a variation of the Bühlmann method that was first introduced in 1967 by Prof. Hans Bühlmann. Bühlmann is a Swiss mathematician who has a career working on the applications of actuarial methods. Bühlmann is a pioneer in the credibility field.⁷ As previously stated, the Bühlmann approach is part of the greatest accuracy theory. This method was created in 1967 using the writing of a fellow mathematician Bailey, who published work on the greatest accuracy theory in 1942 and 1943, to help shape the new method. This approach consists of using prior data to construct a predictive distribution in order to project future aggregate claims. The Bühlmann-Straub method is a more generalized approach to the Bühlmann method that was created in 1972 in a joint collaboration between Bühlmann and Straub. The Bühlmann-Straub method allows for the size of groups to change over time, and takes into consideration the number of lives and severity of the claims.⁸

Herzog explains in the Introduction to Credibility Theory that the Bühlmann-Straub approach employs a point estimator C , where the experience rate is R and the manual rate is H ,

$$C = ZR + (1 - Z)H$$

The credibility factor Z is defined as,

$$Z = \frac{n}{n+k}$$

This formula uses claim dollars per life to find credibilities. X_{if} is the claim dollars per life in the

j^{th} year for the i^{th} policy. The variable n is the number of exposure units and k is defined as,

$$k = \frac{\text{expected value of process variance}}{\text{variance of hypothetical means}} .$$

In the Buhlmann-Straub method, k is the expected value of the process variance divided by the variance of the hypothetical means. Simply put, the size of k depends on the magnitude of the variance of claim dollars per life for different policies. But to understand k , the numerator and the denominator should be studied separately. If each policyholder is thought of as a process, then the expected value of the process variance is the variance in claim dollars per life that a randomly selected policy would be expected to have. As for the denominator, it is the variance of the hypothetical means. Each policyholder has a mean claim dollars per life value over all years of experience. The variance of the hypothetical means is the variance of the means of the policies. So, k is the ratio of how much the claim dollars per life of any given policy is expected to vary from year to year and the amount of claim dollars per life that all policies vary compared to one another.

Hypothetical mean refers to the average severity or average frequency, while process variance refers to the variance of severity or frequency. The variance of hypothetical means measures the variance of overall group means. On the other hand, the expected value of the process variance is the expected value of the variances of each group. So for instance, as the variance of each group decreases, k decreases and the credibility approaches one. Also, as the mean of each group moves further away from the overall mean, the variance of hypothetical means gets larger, k gets smaller, and the credibility approaches one. The logic behind this is that if each group's data is drastically different from each other group, it is easier to identify which group a set of data comes from. Therefore, that data should be more credible to predict future claims from the identified group. The formula to calculate the expected value of the process variance, v , is

$$\frac{\sum_{i=1}^r \sum_{j=1}^{n_i} m_{ij} (X_{ij} - X_i)^2}{\sum_{i=1}^r (n_i - 1)}$$

For the variance of hypothetical means, a , the formula is

$$\frac{\sum_{i=1}^r m_i (X_i - X)^2 - v(r - 1)}{m - \frac{1}{m} \sum_{i=1}^r (m_i^2)}$$

Where X_i is

$$\frac{\sum_{j=1}^{n_i} m_{ij} X_{ij}}{\sum_{j=1}^{n_i} (m_{ij})}$$

And $X_{..}$ is

$$\frac{\sum_{i=1}^n m_i X_i}{m_{..}}$$

The variable m represents the number of exposure units.

With this method, once the number of policy holders, periods of observations, and exposure measures are determined, the average claim amount, X_i , for each policy holder over all observation periods can be calculated. Next, the average claim amount, $X_{..}$, can be calculated over all observation periods and over each policy holder. Using the previously mentioned formulas, the expected process variance, v , and the variance of the hypothetical means, a , can be calculated. These values can then be used to calculate k and the credibility factor Z . Once the credibility factor is established you can compute the compromise estimate of the case rate, C .⁹

The Bühlmann-Straub method has its strengths and weaknesses. This method is the most practical to use due to the fact that it addresses changes in group sizes over time. It allows k to be adjusted each year in order to reflect trends in the data. This method is strong because it is a relative concept, it is based on relative variances of the data.¹⁰ This method has its weaknesses as well. This method is more difficult to apply than that of the limited fluctuation approach. Variances need

to be able to be identified and computed, which makes this method more complicated. A remark is that there is only one k for all policies. A policy with good experience gets the same credibility as a policy with bad experience if they have the same amount of lives.

About Unum

Unum is renowned throughout the insurance industry as the leading disability insurance company in the nation. Unum, a Fortune 500 company, has been around for over one-hundred and fifty years. Originally known as Union Mutual, the company was founded in Portland, Maine and headquartered in Boston, Massachusetts. Throughout the years, other insurance companies like Provident Life in Chattanooga, Tennessee or Paul Revere in Worcester, Massachusetts developed and began insuring groups across the nation. In the 1980s, Union Mutual eventually changed its name to a portmanteau of itself, Unum, as it is known today. In the following decade, Unum acquired and merged with Paul Revere and Provident Life, thus creating UnumProvident, the largest disability provider in the nation.¹¹ Since then, Unum has dropped the Provident from its name, but remains known as Unum Group.

Today, Unum has flourished in the insurance market, setting standards and leading the way in disability insurance. In addition to Long and Short Term Disability Insurance, Unum Group provides many other types of insurance. These include but are not limited to Life, Individual Disability, and Accidental Death and Dismemberment Insurance, as well as maintaining a closed block of Long Term Care Insurance.¹² Unum can be found throughout the United States, as well as throughout the globe. The headquarters had long since been in Portland, Maine, but now resides in Chattanooga. Unum also acquired Colonial Life, located in Columbia, South Carolina. Overseas, offices can be found in Ireland and England.

Working with the Data

Unum's Credibility Formula

The Unum credibility formula for Long Term Disability insurance is shown in Figure 1. This formula takes into consideration claim count and also life years exposure. Life years exposure is the total amount of lives for a policy over the last three years. In addition, this formula resembles the classic, $Z = \frac{n}{n+k}$, credibility formula first mentioned by Whitney. In the Unum formula, k equals $35 - \frac{LYE}{1000}$. This affects the credibility formula in many ways. First of all, a credibility factor should be between zero and one. A credibility factor of zero means that a group's experience should be given no weight in the case rate. A credibility factor of one means that a group's experience should be given full weight. But, the Unum credibility formula can have values less than zero and greater than one. So, the formula must be floored at zero and capped at one to keep this property. Another property of the Unum credibility formula is that it grants full credibility for any case with greater than 35,000 life years exposure. This amount of life years exposure was determined to grant full credibility by a team of actuaries at Unum.

$$\frac{\text{Max}(\text{Actual Claim Count}, \text{Expected Claim Count})}{\text{Max}(\text{Actual Claim Count}, \text{Expected Claim Count}) + 35 - \frac{\text{Life Years Experience}}{1000}}$$

Figure 1 Unum's Credibility Formula

There are some components of the Unum credibility formula that are apparent weaknesses. First, there is no severity component to the credibility formula. The formula only considers claim count, but not the size of the claims. This is an important component of Long Term Disability because the total amount that a disability claim will cost is not known at the time of the disability and the estimate of the total cost of the claim gets more accurate over time. Adding a severity

component to the credibility formula allows for a more robust credibility calculation as new estimates for the cost of a claim are added each year. One example of a credibility calculation that takes the variance of data over time into consideration is the Bühlmann-Straub method.

Smoothing the Data

In order to employ the Bühlmann-Straub method, the Unum data needed to be manipulated into a form that was useful. The data needed to be organized so that there were total claim dollars for each group in each year of observation. In order to accomplish this, each claim needed to be assigned an incurred year. This is simply the year of the disability date. Next, each claim needed to have a total cost associated with it. There were a few different ways that this could have been accomplished. The first option was that the cost of each claim be the initial reserve for that claim. The second option was to assign the total cost of the claim as the sum of the amount already paid and the remaining reserve. Ultimately, the second method was chosen. This method was chosen because it incorporates more information than the first method. For instance, imagine a claim with an initial reserve of \$200,000, which after two years of payments totaling \$100,000 is closed. The first option would assign a cost of \$200,000 to that claim, where the second option would assign a cost of \$100,000 to the claim. Clearly, the second option is the more accurate one. Now, consider the same claim, but instead of the claim closing after two years, the reserve is adjusted to \$40,000 after two years. In this case, the first option would still result in an estimate of \$200,000 for the cost of the claim, but the second option would estimate the cost of the claim as \$140,000. Once again, the second option is more accurate because it uses more information that is observed. Therefore, the second option was used when estimating the total cost of a claim.

With these two new fields for each claim, the next step was to create a pivot table from the claim spreadsheet. For this pivot, the row label was the policy number, the column label was the

disability year and the data chosen was the sum of the total estimate claim amount. The end result is a table where each policy is assigned a total claim dollars amount for each year of observation. Lastly, a vlookup is performed to find the number of lives for each policy for each year of observation. Once the data was reorganized in this manner, the Bühlmann-Straub method could be performed.

Application of the Bühlmann Method

When starting to work with the reorganized Unum data, a number of problems became clear. First, when organizing the claims by year of disability, there was an obvious lack of claims that were incurred in 2013. This is probably due to the fact that claims that were incurred in 2013 were not reported by the year's end. This type of delay is typical in insurance and typically an incurred but not reported (IBNR) estimate is established to account for missing claims. In order for the Bühlmann-Straub method to be used, actual total claim amounts are needed for each group in each year used in the calculation. So, since not all claims incurred in 2013 have been reported, it is not appropriate to use the claim totals in 2013 for the Bühlmann-Straub method. Using the claim totals for 2013 would ignore the fact that there is missing information in the data not accounted for. Therefore when using the Bühlmann-Straub method, the most recent year of data is not used in the calculation. Instead, the three years of data before the most recent year should be used. For instance, when calculating credibility factors for the beginning of 2014, data from 2010 to 2012 is used. Three years was decided because it gives enough information to use the Bühlmann-Straub method. But using more than three years may not improve the calculation because group characteristics can change significantly in that amount of time. Using three years of data is also convenient because it follows the theory behind Unum's credibility calculation and allows for the use of information provided, such as life year's exposure (LYE).

There were also other inconsistencies encountered in working with the data. The Bühlmann-Straub method involves finding the amount of claims per life in each year of observation. However in many cases, groups incurred claims in a year where that group did not have any lives. This would imply that a group incurred claims before it was actually covered. It is not possible to find claims per life for a year with claims but no lives and there is no way to justify attributing those claims to a different year. Thus, claims incurred in a year with no lives were not included in the calculation of the Bühlmann-Straub method.

The Bühlmann-Straub method involves finding the ratio of the expected value of the process variance and the variance of the hypothetical means. First, the expected value of the hypothetical means, or ν , is calculated.

$$\nu = \frac{\sum_{i=1}^r \sum_{j=1}^{n_i} m_{ij} (X_{ij} - X_i)^2}{\sum_{i=1}^r (n_i - 1)}$$

In this equation, each i represents a group and j represents a year of coverage. Also, m_{ij} is the number of lives the i^{th} policy has in the j^{th} year of observation, X_{ij} represents the claims per life for the i^{th} policy in the j^{th} year and n_i represents the number of years of observation for the i^{th} policy. Lastly, X_i represents the average claims per life over the total period of observation for the i^{th} policy.

In the Bühlmann-Straub spreadsheet, each row contains the information for one policy. So, the columns for total claim amounts can be divided by the lives column for the corresponding year to get the X_{ij} 's. The only problem, as previously mentioned, is when there are no lives for a policy in a given year. To avoid an error in these cases, the `iferror` command is used. Next, X_i needs to be calculated for each policy. This is simply $X_i = \frac{\sum_{j=1}^{n_i} m_{ij} X_{ij}}{\sum_{j=1}^{n_i} m_{ij}}$, for each policy. From here, calculating and summing all the $m_{ij}(X_{ij}-X_i)^2$ terms is easily done. Lastly, the denominator needs to

be calculated. Since n_i is the number of years of observation for the i^{th} policy, the (n_i-1) can take a value of 0, 1 or 2 for each policy.

The next part of the Bühlmann-Straub method is calculating a , the variance of the hypothetical means.

$$a = \frac{\sum_{i=1}^r m_i (X_i - X_{..})^2 - v(r-1)}{m_{..} - \frac{1}{m_{..}} \sum_{i=1}^r m_i^2}$$

In this equation, m_i is the number of lives in the year of observation for the i^{th} policy. In other words, $m_i = \sum_{j=1}^{n_i} m_{ij}$ for the i^{th} policy. Also, $m_{..}$ is the total number of lives for all policies in all years of observation. Numerically, $m_{..} = \sum_{i=1}^r m_i$. Lastly, $X_{..}$ is the dollars of claims per life over all policies and years of observation. Numerically, $X_{..} = \frac{\sum_{i=1}^n m_i X_i}{m_{..}}$. In practice, all of these components are easily calculated and a is obtained. Then to find k , divide v by a .

Once k is calculated, the Bühlmann-Straub method uses the formula $Z = \frac{m}{m+k}$, where m represents the number of lives over all periods of observation for a policy. However, when the Bühlmann-Straub method was used, the most recent three years of data were not used because the claim amounts were not accurate. So the total number of lives used in the Bühlmann-Straub method is one year behind the most recent three years of data. Since there is an accurate estimate of lives for the last three years in the spreadsheet of policies that were provided (i.e. life years exposure), it made sense to use this estimate in the final calculation of the credibility factor. So, the calculation of the credibility factor became $Z = \frac{LYE}{LYE+k}$.

Bühlmann-Straub Example

A simple example will be used to illustrate how the Bühlmann-Straub method can be

executed. The tables referenced in this example can be found in Table 1. In this example, there are four policy holders and two years of data. The method starts with lives data in columns B through D and claim dollars data in columns E through G. Next, the claim dollars per life need to be calculated for each policy in each year in columns H and I. However, since group 3 does not have any history in year 1, it does not have a claim dollars per life amount in year 1. Then, an overall claim dollars per life value needs to be calculated for each policy. This is done by taking the total claim dollars for a policy over all years of experience and dividing it by the sum of the lives for that policy over all years of experience. Next in columns K and L, the calculation $m_{ij}(X_{ij}-X_i)^2$ needs to be performed. These values are used in the calculation of v as part of the expected value of the process variance. In this calculation, m_{ij} is the number of lives in j^{th} year for the i^{th} policy, X_{ij} is the claim dollars per life in the j^{th} year for the i^{th} policy and X_i is the overall claim dollars per life of the i^{th} policy. Again, there is no value in column K, row 5, since group three has no history in year 1. The next step is to find the total years of experience. Thus, there are ones in all cells in columns M and N, except in year one of group 3. Column O is the sum of M and N minus one. The reason behind this is that the calculation of v is a variance calculation. So, if a policy has only one year of experience, it will not have any variance. Thus, that year of experience will not affect the expected value of the process variance. Finally, a value can be calculated for v . To find v , divide the sum of columns K and L and divide it by the sum of column O. In this example, we have a value for v of 3,115,857. Next to calculate a , a claim dollars per life value must be calculated for all policies over all year. This is done by summing all the claims over all policies and all years and dividing that sum by the sum of all the lives over all policies and all years. This calculation is found in cell J8. In column P, a calculation of how far each overall claim dollars per life value is from the total claim dollars per life value. Clearly, column P is a component of a variance

calculation, which is appropriate because a is the variance of the hypothetical means. Column Q is simply the total lives over all years for a policy squared, which is used for the calculation in cell R3. In that calculation, m is the total lives over all policies and years of experience. Once all these values are obtained, a can be calculated. To calculate a , take the sum of column P, rows 3 through 6, subtract v times the total number of policies minus 1 and divide this number by the number calculated in cell R3. In this example, the value for a is 7,413. Finally, k is calculated by dividing v by a to obtain a value of 420. From this point, credibility factors can be calculated and used to create case rates for these fictitious policies.

Using the Bühlmann-Straub method on the Unum data resulted in similar k values for 2012 and for 2013. The k value for 2013 was 6,336, while the k value for 2012 was 6,450. It is important to understand why k behaves this way. First, k is the ratio of the expected value of the process variance and the variance of the hypothetical means. In 2012, the expected value of the process variance v was 131,439,435 and the variance of the hypothetical means a is 20,377. In 2013, the expected value of the process variance v is 116,465,537 and the variance of the hypothetical means a is 18,380. So from 2012 to 2013, the expected value of the process variance, the variance of the hypothetical means and k all decreased. The process variance decreases because claim amounts per life get more consistent. Even though more policies were added, the policies with many years of exposure get more consistent levels of claims per year with the addition of the 2013 data. This may not always be the case, but it is in the case of Unum's data. The variance of the hypothetical means also decreases. This would decrease if the average claims per life for each policy came closer together. Despite more policies being added in 2013, the variance of the hypothetical means still decreases. But, since the expected value of the process variance decreases by 11% and the variance of the hypothetical means decreases by 10%, k decreases in 2013. The Unum data

demonstrates how k is a dynamic measure that changes as trends in the data change.

A	B	C	D	E	F	G	H	I	J
	Lives			Claim Dollars			Claim Dollars per Life		
Group	Year 1	Year 2	Overall	Year 1	Year 2	Overall	Year 1	Year 2	Overall
1	1000	1200	2200	\$ 100,000.00	\$ 143,000.00	\$ 243,000.00	100	119	110
2	2400	2000	4400	\$ 125,000.00	\$ 125,000.00	\$ 250,000.00	52	63	57
3	0	800	800	\$ -	\$ 40,000.00	\$ 40,000.00		50	50
4	1500	1300	2800	\$ 200,000.00	\$ 150,000.00	\$ 350,000.00	133	115	125
Totals									
4			10200			\$ 883,000.00			87

K	L	M	N	O	P	Q	R
$m_{ij}(X_{ij}-X_i)^2$		Count Years of Experience					
Year 1	Year 2	Year 1	Year 2	n_i-1	$m_i(X_i-X_{..})^2$	m_i^2	$m_{..} - \frac{1}{m_{..}} \sum_{i=1}^r m_i^2$
109,298	91,081	1	1	1	1,255,182	4,840,000	6,996
8,177,276	64,566	1	1	1	14,204,545	19,360,000	
	-	-	1	-	2,000,000	640,000	
785,158	120,192	1	1	1	43,750,000	7,840,000	
				3	61,209,727		
	v:	3,115,857	a:	7,413	k:	420	

Table 1 Buhlmann-Straub Example Walkthrough

From k to Loss Ratios

In order to analyze how the Buhlmann-Straub method may improve the calculation of credibility factors, projected loss ratios needed to be calculated for 2013 and 2014. In order to perform this analysis, the new credibility factors needed to be combined with Unum data to predict the amount of inforce premium that would have been collected using these new credibility factors. The formulas that were used in this process can be found in Table 2. First, the new credibility factors were used to calculate the new estimated case rate. This case rate is the rate that an actuary would calculate for a policy. However, this is not the rate that would be used in practice. In the insurance industry, the case rate is changed by discounts and other incentives that are used to make the sale. The resulting rate is called the inforce rate and it is the rate that is charged to the policyholder. Unfortunately, there is no one-to-one relationship between the case rate and the inforce rate. So to estimate the inforce rate under the Buhlmann-Straub method, the Unum inforce rate needs to be multiplied by the ratio of the new case rate and Unum's case rate. The same method was used to estimate the new inforce premium. Lastly, projected loss ratios were calculated by taking a third of the expected claims for the year under review and dividing that number by the inforce premium for that year. Once these loss ratios were obtained, they could be analyzed to see the effects that a change in credibility calculations would have on Unum's data.

Value	Formula
New Credibility	$Credibility_{New} = \frac{N}{N + K(t)}$
New In-Force Rate	$InfRate_{New} = InfRate * \left(\frac{CaseRate_{New}}{CaseRate_{Est}} \right)$
New In-Force Premium	$InfPrem_{New} = InfPrem * \left(\frac{InfRate_{New}}{InfRate} \right)$
Projected Loss Ratio	$PLR_{New} = \frac{\frac{Expected\ Claims_{2014}}{3}}{InfPrem_{New}}$
Unum Projected Loss Ratio	$PLR_{Unum} = \frac{\frac{Expected\ Claims_{2014}}{3}}{InfPrem}$

Table 2 Formulas Used to Calculate Loss Ratios

Results

Observing the Data

After applying the Buhlmann-Straub method to the data, the new credibility factors had to be tested to see how they compare and draw conclusions. Many different approaches to organizing and comparing the data were considered. The first part of the analysis came down to how the new credibility factors compared one to one to the credibility factors of Unum. The next part was to see the effects on the loss ratios, which are a measure of losses incurred over a set period to the amount of collected premium for that period. The loss ratios were tested in a variety of ways to observe the effects on accuracy and the effects on business. In this section, the results will be diagrammed and discussed to draw conclusions on the new method of calculating credibility.

Comparison of Credibility Factors

All policies had new credibility factors calculated under the Buhlmann-Straub Method. The new credibility factors were taken and compared to the old credibility factors. The table below shows the aggregate results of the comparison.

Group Size	New H/L Unum	Count	Avg. Unum Credibility	Avg. New Credibility
<500	Higher	140	0.097	0.134
	Lower	153	0.290	0.147
	Total	293	0.198	0.141
500-1999	Higher	351	0.173	0.227
	Lower	488	0.427	0.264
	Total	839	0.321	0.249
2000+	Higher	39	0.398	0.475
	Lower	164	0.798	0.646
	Total	203	0.721	0.613
All	Grand Total	1335	0.355	0.280

Table 3 Average Credibility Factors by Group Size 2013

In Table 3, the average credibility factors for both Unum and the New Method are broken down into group size, as well as whether the New factor is higher or lower than the Unum factor. The data is all for $k = 6,450$, as these are the factors to be used in policy year 2013. For example, the .097 in Row 1, Unum column, means that the average Unum credibility factor, given that the New recalculated factor is larger, will be .097. As expected, any average in the Lower row will have a higher average in the Unum column than in the New column.

There are some key takeaways that we find in Table 3. The simplest and most prominent is that the Credibility factors are lower in the New method than in Unum's method. This is in large part due to policies that previously had credibility factors equal to 1. Under Unum's formula this can occur when the Life Years Exposure is greater than 35,000. However, even with a k as small as 2,000, an LYE of 35,000 would produce a credibility factor of about .95. In the large group rows of the table, the effect of this jump off is seen clearly. The average credibility drops by over .1 overall, and 80.7% of the claims have lower New credibility factors. Another takeaway that is seen in the table is that the spread of the averages for the New column is much smaller when compared to the overall averages by group size. That is, Unum's lows are lower, and their highs are higher. The rationale behind this is due to the group sizes more directly affecting the credibility factors for the Buhlmann-Straub Method. Unum's formula concerns itself more with claim counts, whereas the New Method ignores this on an individual policy level, and instead focuses on group size.

The same analysis was performed for the credibility factors to be used in policy year 2014. Similar results are observed. The changes in credibility factors are slightly less drastic than in 2013.

Group Size	New H/L Unum	Count	Unum Credibility	New Credibility
<500	Higher	151	0.095	0.132
	Lower	151	0.283	0.156
	Total	302	0.189	0.144
500-1999	Higher	375	0.175	0.231
	Lower	495	0.412	0.270
	Total	870	0.310	0.253
2000+	Higher	50	0.424	0.519
	Lower	205	0.796	0.659
	Total	255	0.723	0.632
All	Grand Total	1427	0.358	0.298

Table 4 Average Credibility Factors by Group Size 2013

Loss Ratio Analysis

Introduction to Loss Ratios

Loss Ratios are a measure of how well insurance premiums are covering losses. A loss ratio is simply Losses:In-Force Premium. A basic and intuitive property is that if the LR is equal to 1, the premium charged covers the losses incurred. It follows that a $LR > 1$ means losses exceed premium, and vice versa for $LR < 1$. Insurance companies set loss ratios to measure the success of a policy. A loss ratio that is consistently too high (above a set amount that is less than 1) will be concerning to a company, and they will likely charge a higher premium based on experience to make up for the losses. Loss ratios are a great way to look into how credibility is affecting profit and losses.

Tolerable Loss Ratios

A company will set a benchmark loss ratio with they define as “Tolerable.” Referred to as TLR in this study, the Tolerable Loss Ratios are, in a sense, a goal that the company tries to meet with their premium and expenses. TLR’s take into account costs of business to still obtain a profit. For example, a TLR of .8 implies that if there is \$800 in losses, and \$1,000 in charged premium,

there will be upwards of \$200 that will be tied up in labor expenses, broker's fees, and other fixed/variable costs. TLR's vary depending on the size of the business. In Unum's case, TLR's are set as .52, .66, and .77 for small, medium, and large sized businesses, respectively. From a business perspective, it would be most ideal for the recalculated loss ratios to be below the TLR. However, it is also worth considering whether the accuracy to the TLR improves under the Buhlmann-Straub method.

Comparing Loss Ratios

The 2013 projected loss ratios (PLR's) were analyzed through a range of different forms of comparisons. Among these were the aggregate amounts and their difference from the TLR. To quantify the difference between the PLR's and TLR's, many scatterplots were drawn up to observe the trends and spread of the data points. Lastly, an octant analysis was performed to breakdown the data into different buckets and draw conclusions from there.

Aggregate Loss Ratios

The loss ratios for each life group were calculated and averaged to see what general trends were occurring.

Group Size	LR_{New}	LR_{Unum}	TLR	$ TLR - LR_{New} $	$ TLR - LR_{Unum} $
SCORE <500	0.540	0.517	0.52	0.020	0.003
MCORE 500-1999	0.616	0.581	0.66	0.044	0.079
NCG 2000+	0.644	0.657	0.77	0.126	0.113

Table 5 Aggregate Loss Ratio Analysis for 2013

From Table 5 it is clear that the aggregate loss ratio is closer for medium sized groups under the Buhlmann-Straub method. The small sized groups are projected above the TLR. The large groups

are lower than both the TLR and the Unum LR. The numbers for 2014 show a slightly better picture in terms of accuracy.

Group Size	PLR_{New}	PLR_{Unum}	TLR	$ TLR - PLR_{New} $	$ TLR - PLR_{Unum} $
SCORE <500	0.528	0.511	0.52	0.008	0.009
MCORE 500-1999	0.588	0.569	0.66	0.072	0.091
NCG 2000+	0.635	0.639	0.77	0.135	0.131

Table 6 Aggregate Loss Ratio Analysis for 2014

The smallest group ends up with a higher loss ratio, above the TLR, however, it is closer to the TLR than the Unum loss ratio. The medium line comes in higher than Unum, but is closer to the TLR as well and also without exceeding the TLR. The largest group comes in less in both the New and Unum cases, with Unum's being more accurate to the TLR. It is worth noting that in both 2013 and 2014, the medium group holds the most number of policies. The following two tables are in support of the above two. They show policy counts for each year, broken down by group size and distance from the TLR.

Distance to TLR	Group Size			Total
	SCORE <500	MCORE 500-1999	NCG 2000+	
Closer	165	517	110	792
Further	128	321	93	542
No Difference	0	1	0	1
Total	293	839	203	1335

Table 7 Loss Ratio Comparison on a Count Basis 2013

Distance to <i>TLR</i>	Group Size			Total
	SCORE <500	MCORE 500-1999	NCG 2000+	
Closer	169	526	129	824
Further	133	343	126	602
No Difference	0	1	0	1
Total	302	870	255	1427

Table 8 Loss Ratio Comparison on a Count Basis 2014

“Closer” is defined mathematically as,

$$|TLR - PLR_{Unum}| > |TLR - PLR_{New}|$$

And “Further” follows from this,

$$|TLR - PLR_{Unum}| < |TLR - PLR_{New}|$$

As is easily seen in both, the count of policies that fall “closer” to the TLR under the new method is greater than that which does not. This still leads to the question of how far away the policies are, and whether or not they are moving drastically or minimally.

Scatterplots

To get a clearer picture of the data, a scatterplot analysis was performed. The projected loss ratios for Unum were plotted against the projected loss ratios for both 2013 and 2014. The scatterplot revealed a very strong linear relationship, with slope close to 1, and an R² value close to 1 as well.

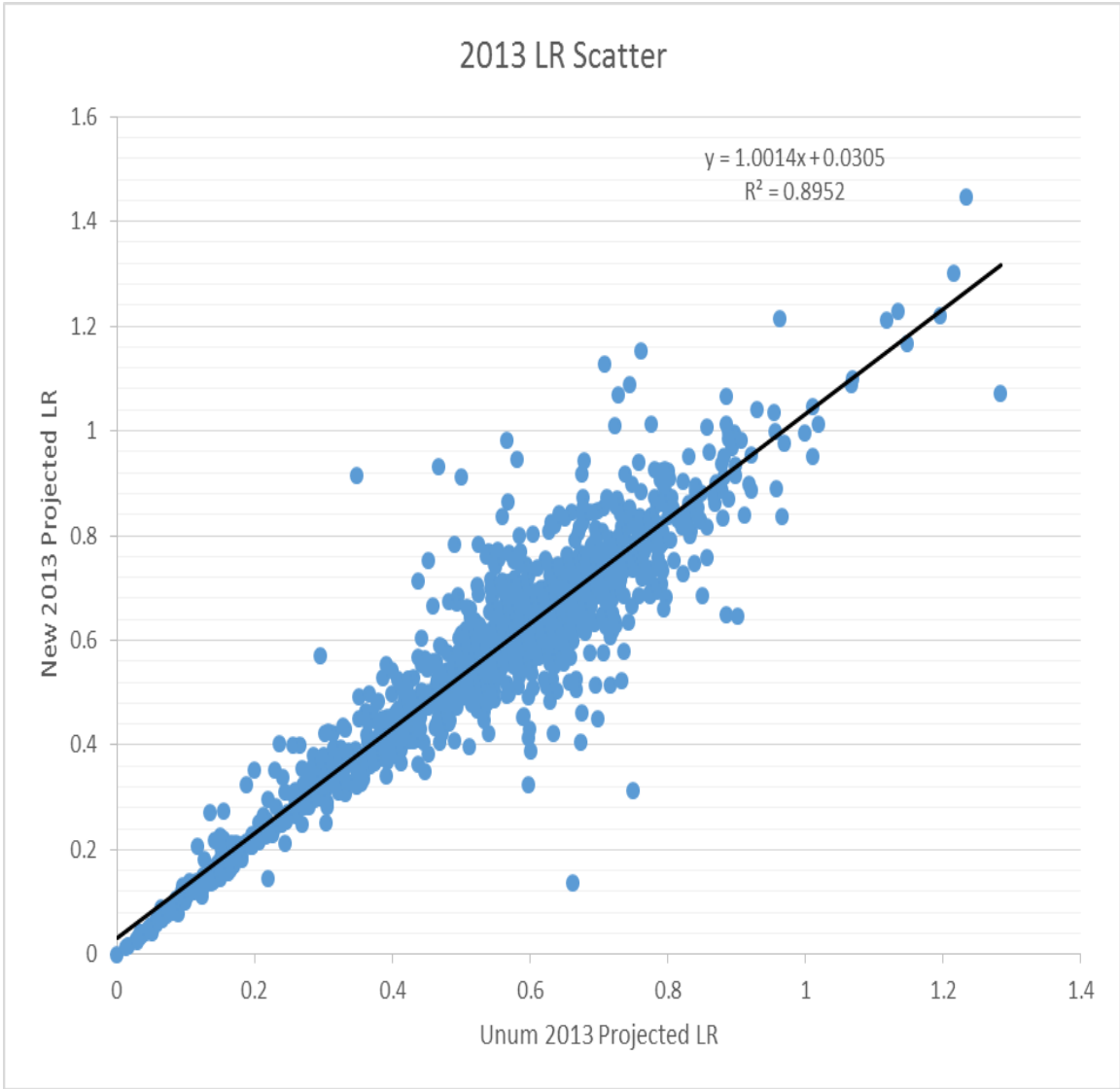


Figure 2 Scatterplot Unum vs. New Projected Loss Ratio 2013

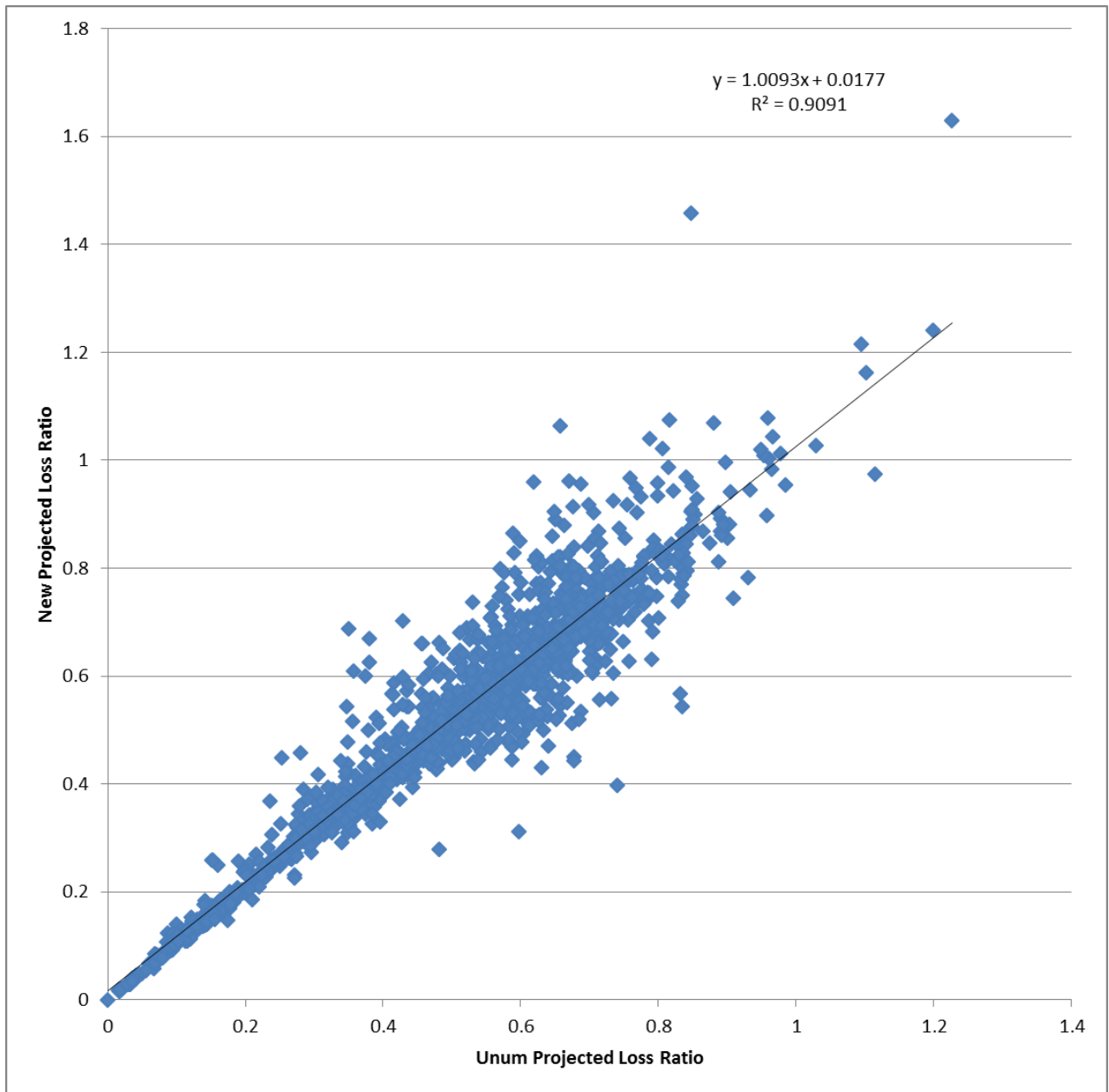


Figure 3 Scatterplot Unum vs. New Projected Loss Ratio 2014

Both scatters do not distinguish group size, as was previously separated in other figures. The next step was to look at the different group sizes. Each group size has their own unique TLR, so plotting the TLR with the data can assist in seeing patterns and trends.

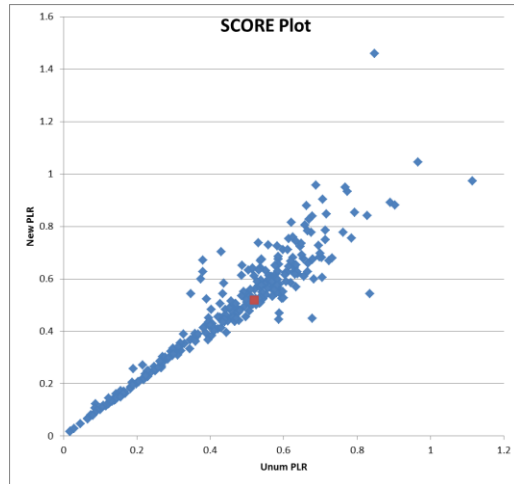


Figure 4 Scatter 2014 <500 Lives

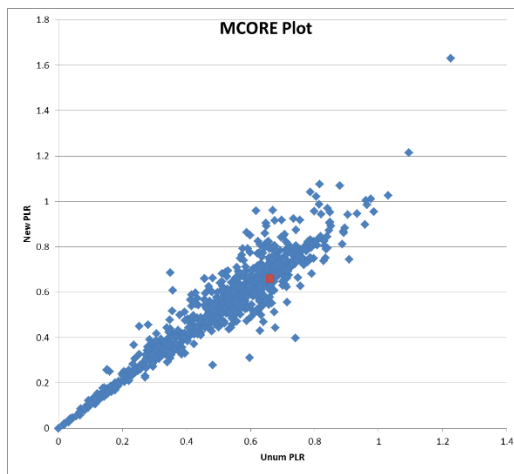


Figure 5 Scatter 2014 500-1999 lives

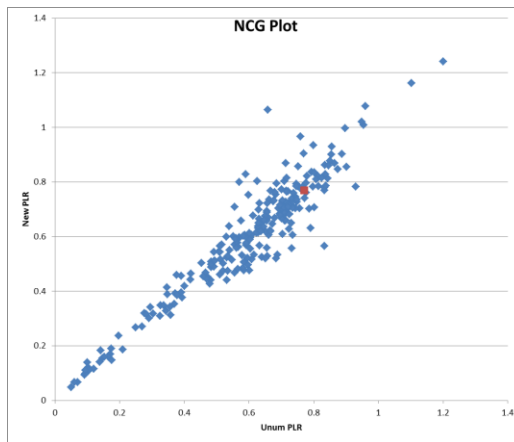


Figure 6 Scatter 2014 2000+ lives

The points that have large residuals are now clearly hovering around the TLR for each group. Otherwise, they fall right back around the line of best fit. The points near the line of best fit imply that the new credibility calculations did not affect the amount of premium collected significantly. The points near the TLR that are farther away, were indeed affected by the change in credibility factor. The next step is to look at whether these changes are adverse, in both an accuracy sense, as well as a business sense.

Accuracy to the Tolerable Loss Ratio was explored in the Aggregate Loss Ratio section's tables. Keeping the same criteria of what defines "Closer" and "Further" from before, the PLR data points in the scatter were sorted and filtered onto their own plots. While not intuitive, the picture created shows the areas of the lot that clearly define where the "Closer" and "Further" points will always fall.

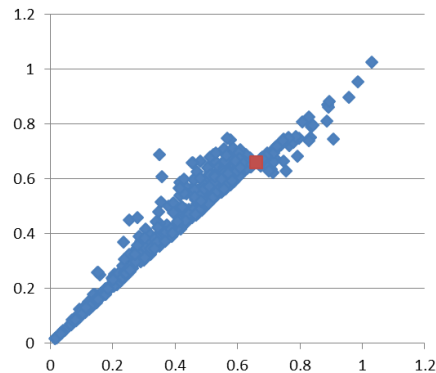


Figure 7 "Closer" Scatterplot for MCORE 2014

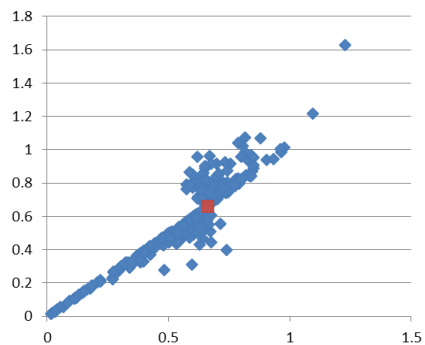


Figure 8 "Further" Scatterplot for MCORE 2014

Both figures are derived from policies in Projection Year 2014, for the medium sized policies. Thus, the red point in the graphs are at (.66,.66). An X shape characterized by an absence of points are clear in both graphs. The X shape is composed of two lines of slopes 1 and -1, intersecting the TLR point. The clear distinction in areas of the graphs led to an octant analysis described in the following section.

Octant Analysis

PLR's for both the Unum method and the New method can be classified into different areas of the scatter, eight in total. The criteria for "Closer" and "Further" having such convenient diagonal axes allows the eight different sections to be outlined on the scatter, centered at the TLR for each group size. The octants that the points fall in tell a story about how accurate or how beneficial the change in credibility factor for a policy is.

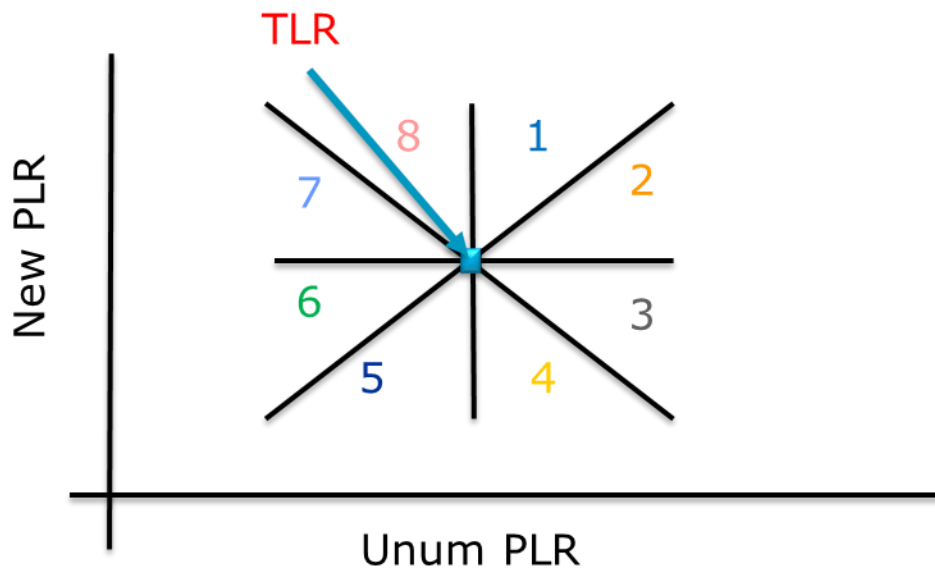


Figure 9 Octant Outline

Octants to the right of the vertical axis will contain only points where Unum's PLR is greater than the TLR, and to the left will have points where Unum's PLR was less than the TLR. Octants above the horizontal axis will have New PLR's greater than the TLR, and below will have

New PLR's less than the TLR. This means that the quadrant containing octants 1 and 2 will have both Unum's and the New PLR above the TLR. Breaking it down further, the areas that the "Closer" figure had points will be the octants that are considered more accurate. Octants 2, 3, 6, and 7 all contain points that have New PLR's closer to the TLR than Unum's PLR. In short, the difference between octants 1 and 2 is that in octant 2, the New PLR is now closer to the TLR than Unum's loss ratio, whereas the New PLR is further away from the TLR than Unum's loss ratio in octant 1.

The data points for the scattered were assigned an octant based on this criteria and plotted again to get a better understanding of where the data is moving. Below are the plots for each size of business, with the octants overlaid.

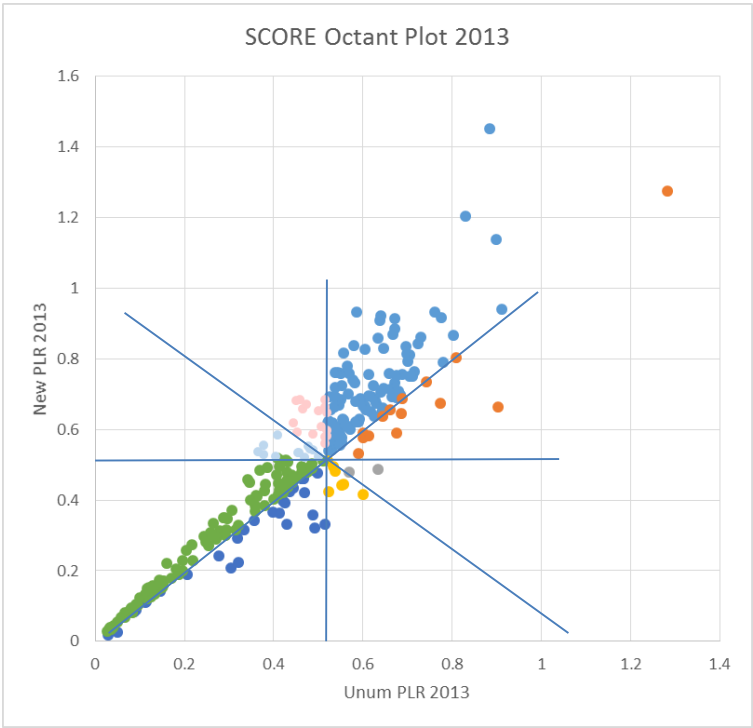


Figure 10 SCORE Octant Plot 2013

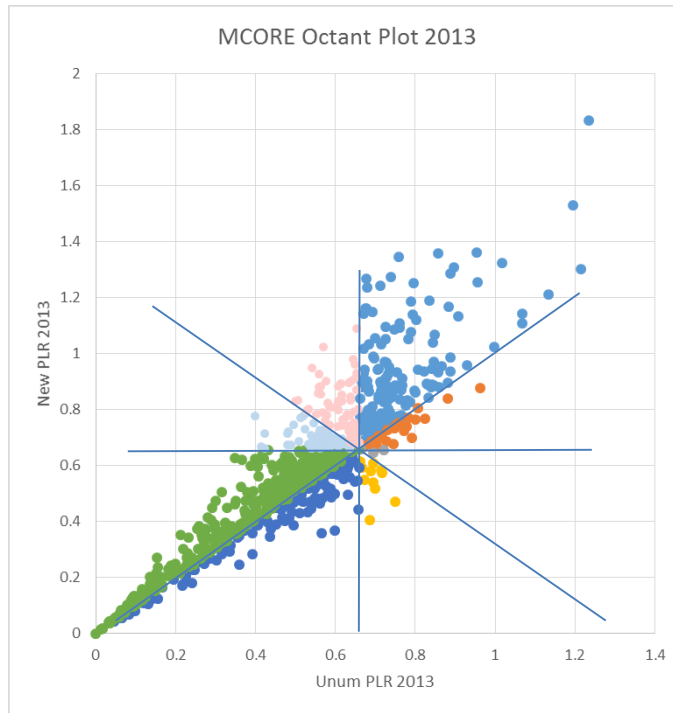


Figure 11 MCORE Octant Plot 2013

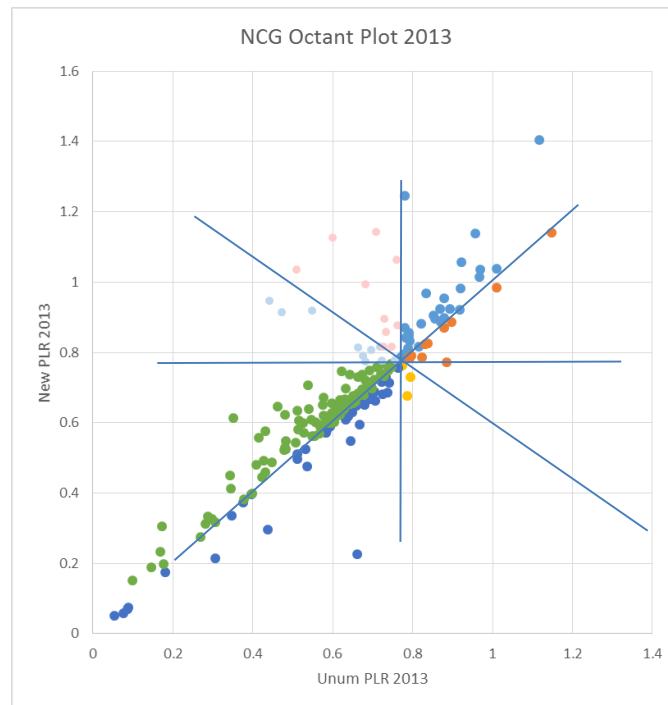


Figure 12 NCG Octant Plot 2013

Octant	Count	% of Total	Relative Positions	Increase/Decrease in PLR	Accuracy to TLR	Meaning
1	269	20.2%	TLR<Unum<New	+	-	Unum's PLR is higher than TLR, and New is even further
2	67	5.0%	TLR<New<Unum	-	+	Unum's PLR is higher than TLR, and New is in between
3	12	0.9%	New<TLR<<Unum	-	+	Unum's PLR is higher than TLR, New is lower than TLR, and closer
4	27	2.0%	New<<TLR<Unum	-	-	Unum's PLR is higher than TLR, New is Lower and further away
5	197	14.8%	New<Unum<TLR	-	-	Unum's PLR is lower than TLR, New is even lower than both
6	656	49.1%	Unum<New<TLR	+	+	Unum's PLR is lower than TLR, New is closer to TLR, but still below
7	58	4.3%	Unum<<TLR<New	+	+	Unum's PLR is lower than TLR, New is closer, but now higher than TLR
8	49	3.7%	Unum<TLR<<New	+	-	Unum's PLR is lower than TLR, and New is not only higher, but further away
Total	1335	100.0%				

Table 9 Count of Policies by Octant 2013

Upon analysis of the table, it is clear that the most claims fall within the 6th octant. This also reflects in Table 9, which shows that many loss ratios increase, but stay under the tolerable loss ratio. This increase in accuracy can be regarded as a positive for the most part despite the forfeited profit that comes from it. It demonstrates that another company could sell insurance for a cheaper price and still achieve their tolerable loss ratios under the Bulmann-Straub Method. A drawback from the method is the amount of policies that lie in the 1st quadrant. These are policies that need more weight to their experience rate to account for poor experience, but do not receive it due to the nature of the method. The smaller groups are most adversely affected by this. The low

LYE's for small groups causes the credibility factors to come in low, so bad experience groups will suffer the worst changes in loss ratios under the New Method.

Similar results can be seen for the plots with 2014 data.

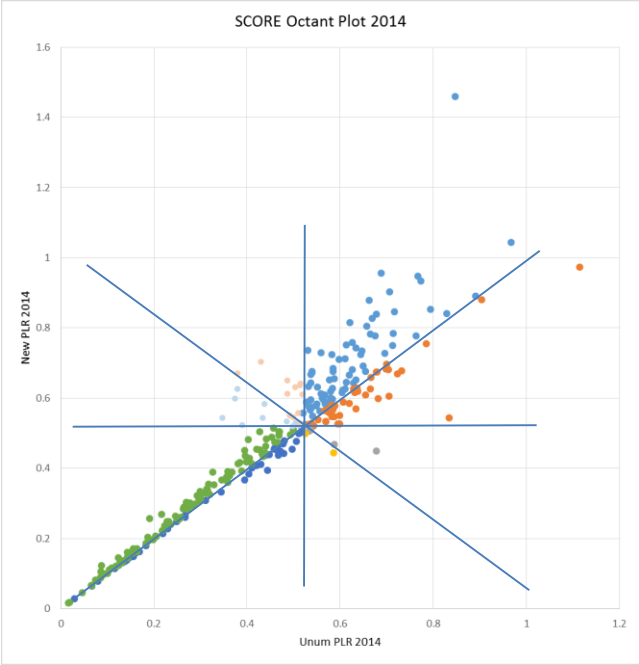


Figure 13 SCORE Octant Plot 2014

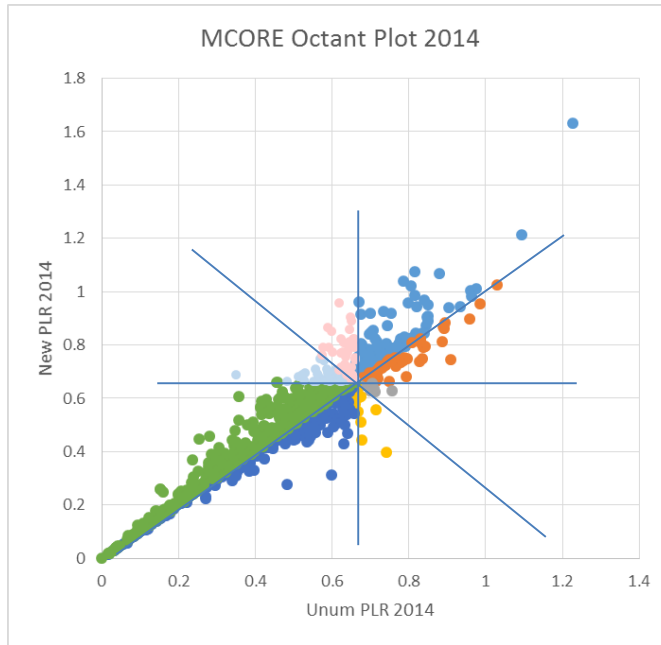


Figure 14 MCORE Octant Plot 2014

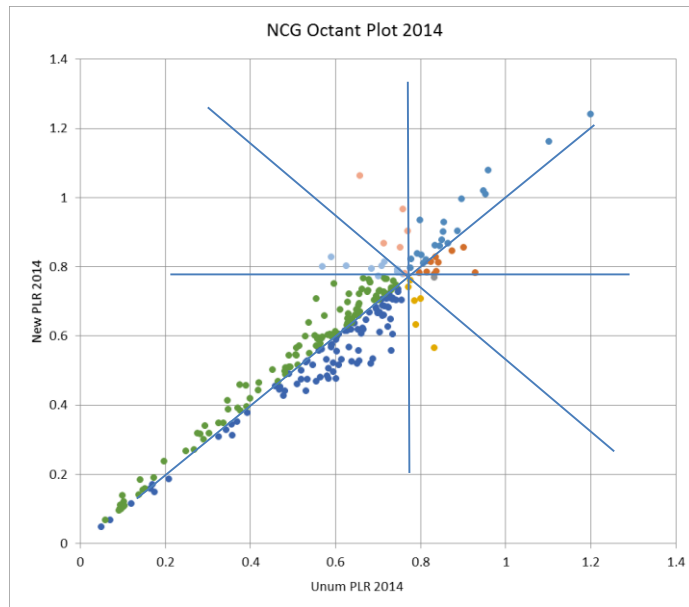


Figure 15 NCG Octant Plot 2014

SIC Analysis

The Buhlmann-Straub Method improved the accuracy of projected loss ratios for the Unum data, but the method left room for improvement. In an attempt to achieve more favorable results, the data was divided into groups. Since Buhlmann-Straub method takes the claim dollars per life into account, grouping policies by some criteria may promote accuracy and uniformity in the calculations. One grouping that was used was grouping by SIC divisions.

Standard Industrial Classification (SIC) codes are four-digit numerical codes assigned by the U.S. government to business establishments to identify the primary business of the establishment. The classification was developed to facilitate the collection, presentation and analysis of data; and to promote uniformity and comparability in the presentation of statistical data collected by various agencies of the federal government, state agencies and private organizations.¹³ The classification covers all economic activities. The data given by Unum can be divided into 10 divisions by the first two digits of the code which could identify the major industry groups. Figure 17 lists the 10 divisions based on SIC codes and shows the percentages of Unum's business that are in each grouping.

First Two Digits of SIC Code	Industry Group
01-09	Agriculture, Forestry, Fishing
10-14	Mining
15-17	Construction
20-39	Manufacturing
40-49	Transportation and Public Utilities
50-51	Wholesale Trade
52-59	Retail Trade
60-67	Finance, Insurance, Real Estate
70-89	Services
91-99	Public Administration

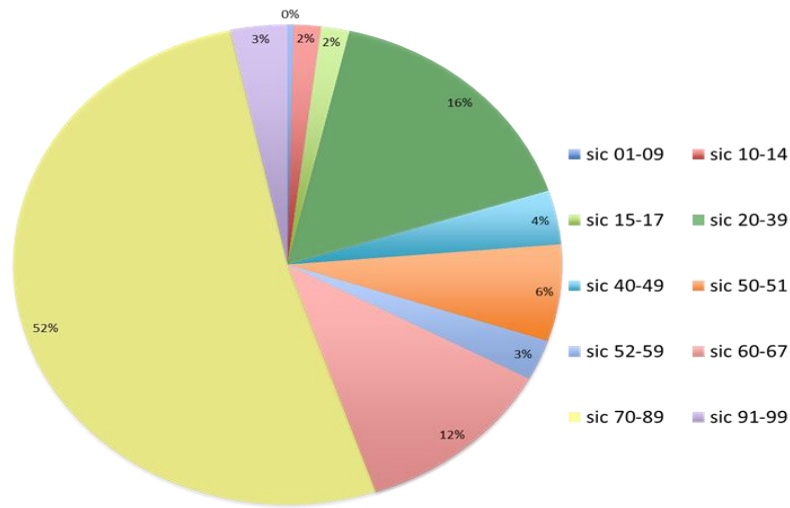


Figure 16 Standard Industry Classification Code Breakdown

After these divisions were made, the Buhlmann-Straub method was performed on each one. Table 10 lists the k for individual SIC divisions.

SIC division	01-09	10-14	15-17	20-39	40-49	50-51	52-59	60-67	70-89	91-99
k	2,144	265	-25,024	2,805	955	28,961	891	10,741	8,825	564

Table 10 SIC specific k values using the Buhlmann-Straub Method

In the table, there is a negative number appeared as k. In Buhlmann-Straub method, the estimate of the variance of the hypothetical means could be negative. In this situation, zero would be used as the variance of the hypothetical means. As shown in the table, the k for small groups varies from -25,024 to 28,961. This result is because of the inconsistency and bias in given data. Thus, the k values generated by the Buhlmann-Straub method for small groups should not be considered credible. In this case, only three subgroups with a number of policies greater than 100 were taken into consideration.

The loss ratios for each division were calculated and compared with the loss ratio for overall k, the loss ratio for Unum and the tolerable loss ratio. The table below shows the projected loss ratio calculated for SIC 20-39, which has a k of 2,805.

Group Size	PLR_{20-39}	PLR_{New}	PLR_{Unum}	TLR
<500	0.479	0.458	0.450	0.52
500-1999	0.590	0.510	0.550	0.66
2000+	0.685	0.593	0.650	0.77

Table 11 Projected Loss Ratios for SIC 20-39

From Table 11 it is obvious that the SIC specific loss ratios for this division are all closer to Tolerable Loss Ratio than the Unum loss ratios and the loss ratios for overall k. This can

especially be seen for medium sized groups, where the SIC specific loss ratio is 0.040 closer than Unum loss ratio and 0.080 closer than the new loss ratio calculated for overall k. Thus, this division shows a good picture of accuracy. The scatterplots for this division can be seen in Figure 18.

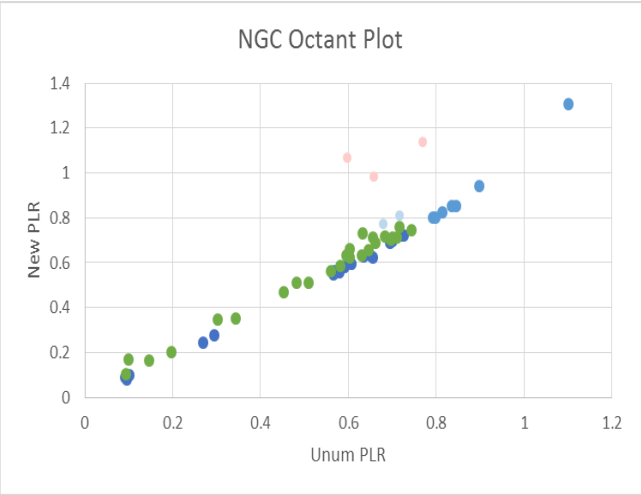
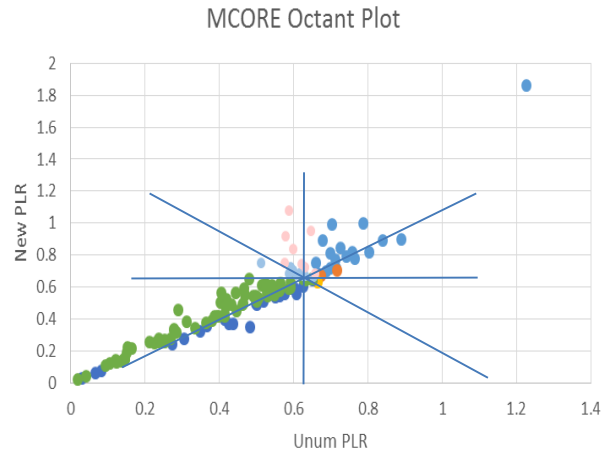
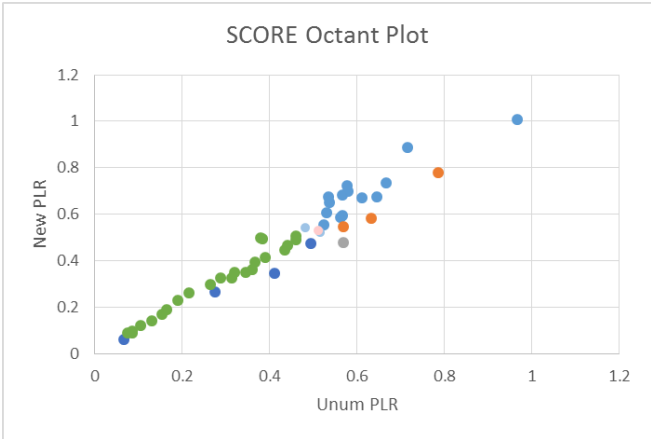


Figure 17 Scatterplots for SIC 20-39

As shown in the plots, it is clear that for SIC 20- 39, most policies fall within the 6th octant. These scatter plots also shows that loss ratios for all group sizes increase, but in many cases without going over the tolerable loss ratio. This will result in an increase in accuracy and competitiveness despite the forfeited profit.

A similar analysis was done for the SIC 60-67 grouping. Table 12 shows the Loss Ratio comparison for SIC 60-67, with k = 10,741.

Group Size	PLR_{60-67}	PLR_{New}	PLR_{Unum}	TLR
<500	0.459	0.442	0.459	0.52
500-1999	0.564	0.515	0.543	0.66
2000+	0.626	0.633	0.621	0.77

Table 12 Projected Loss Ratios for SIC 60-67

In this case, the SIC specific loss ratio is equal to the Unum loss ratio for small sized groups. But in the medium and large sized groups, the SIC specific loss ratios were closer to the tolerable loss ratio without exceeding it. Thus, an improvement in credibility calculation has been achieved in this division as well. The scatterplots for this division can be seen in Figure 19. These scatters follow a similar pattern as the previous ones, where a large portion of the points can be found in the green, or 6th octant.

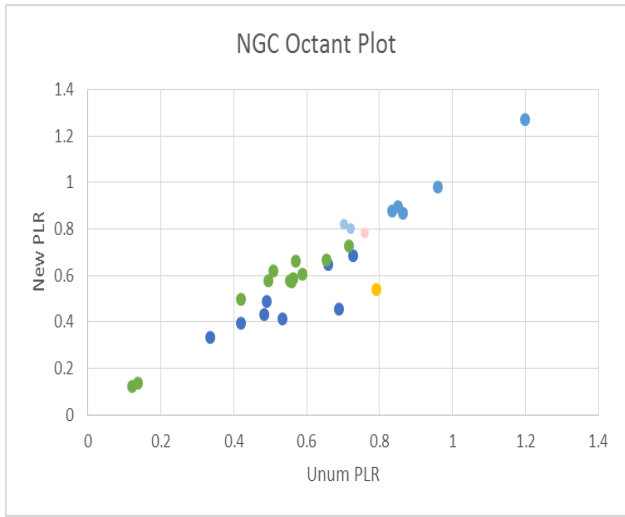
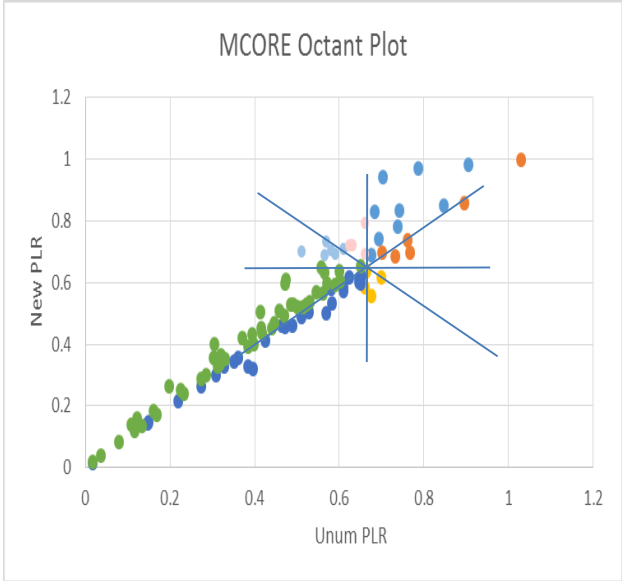
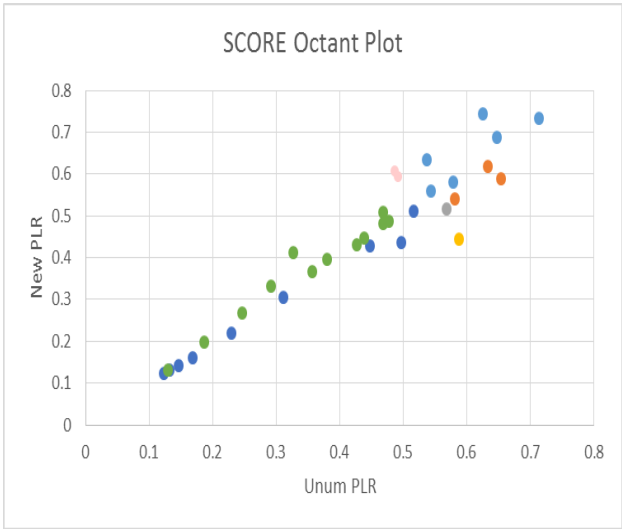


Figure 18 Scatterplots for SIC 60-67

Lastly, the same loss ratio analysis was done on the SIC 70-89 division. This division contains 52% of Unum's total policies. In this division, $k = 8,825$.

Group Size	PLR_{70-89}	PLR_{New}	PLR_{Unum}	TLR
<500	0.540	0.427	0.537	0.52
500-1999	0.600	0.532	0.597	0.66
2000+	0.615	0.601	0.639	0.77

Table 13 Projected Loss Ratios for SIC 70-89

From Table 13, it is clear that the SIC specific projected loss ratio for small sized groups is greater than the TLR. So, for small sized groups, the overall k method is the only method that produced a projected loss ratio under the TLR. Once again, the SIC specific loss ratios for medium sized groups are more accurate to the TLR than the other two methods. For the large policies, the SIC specific loss ratios is between the Unum loss ratio and the loss ratio for overall k. Thus, the SIC specific k does not improve accuracy in large sized groups in this case. Once again, scatterplots have been provided for this grouping. The scatters follow the same pattern as before with a large number of points found in the 6th octant. This octant is where the SIC specific loss ratio is closer to the TLR than Unum's loss ratio without the SIC specific loss ratio going over the TLR. The concentration of points in the 6th octant is the most pronounced in the graph of medium sized groups.

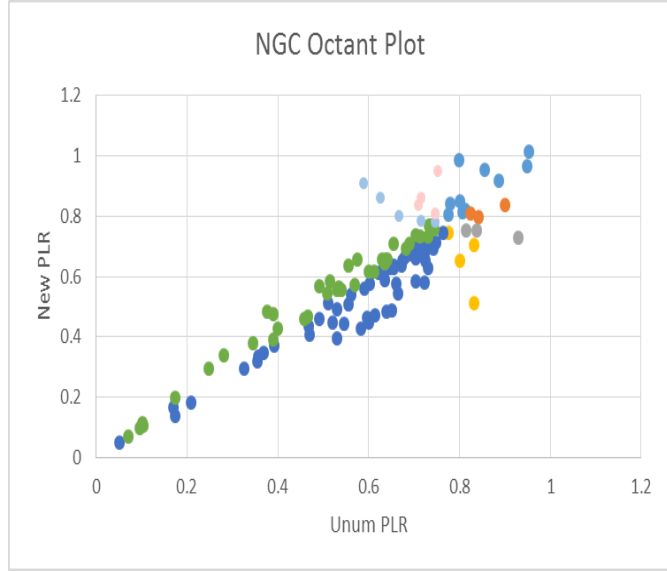
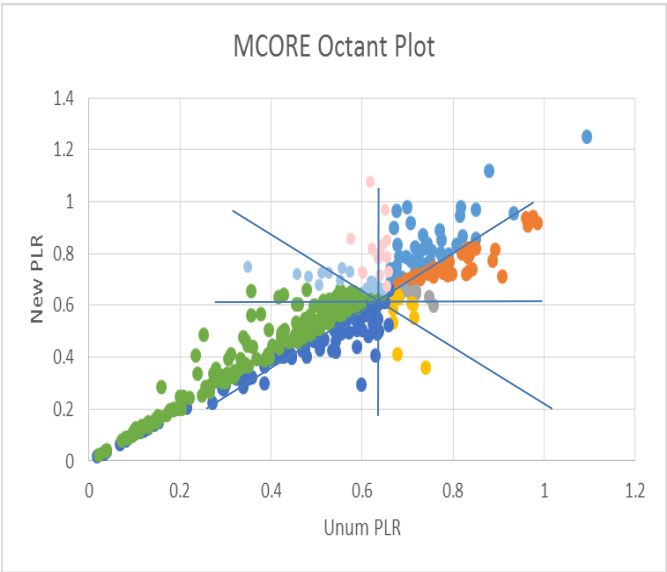
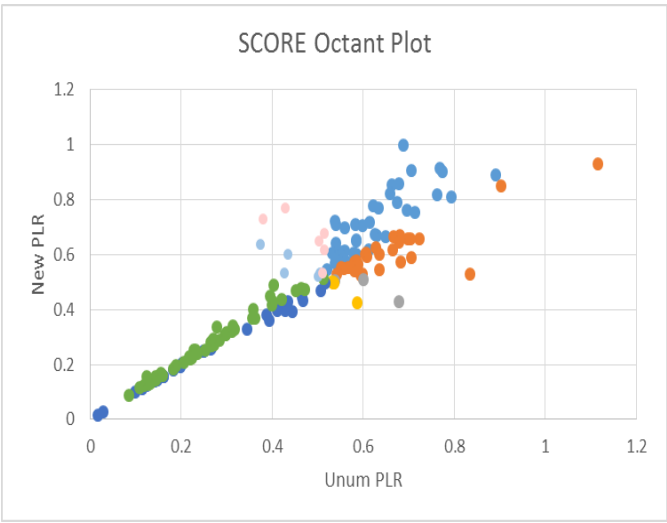


Figure 19 Scatterplots for SIC 70-89

Analysis and Conclusion

Accuracy and Profitability

Using the Buhlmann-Straub method for calculating credibility, the projected loss ratios for Unum's data mostly got closer to the tolerable loss ratios as compared to using Unum's credibility formula. This result was obtained not only for projected loss ratios for 2013, but for projected loss ratios for 2014 as well. As mentioned, this increase in accuracy comes at the cost of higher loss ratios overall. In this study, it was assumed that having a loss ratio that was closer to the tolerable loss ratio was a favorable result. In the real world, a decision to change credibility methods would take into consideration the benefits of giving fairer premiums to policyholders in comparison to the cost of the loss in premium collected. The size of the premium sacrificed is a large part of the decision to switch credibility factors.

Table 14 illustrates the amount of the changes in premium collected that Unum would experience if the Buhlmann-Straub method was used to calculate credibility. As would be expected, Unum's credibility method collected more premium in the octants where its loss ratio was lower than the loss ratio under the Buhlmann-Straub method and vice versa. In total, \$8.5 million less premium would have been collected using the Buhlmann-Straub method in 2013 and \$6.4 million less premium would have been collected in 2014. While this was only a small portion of the total premium collected in these years, a change in premium calculation that has this large of an effect on collected premium is an important business decision. Influencing this decision is the idea that loss ratios can still meet the tolerable level at a lower premium amount. This could potentially mean that other companies could undercut Unum's premiums and still cover their losses. With such a large portion of policies falling in the 6th octant, there could be a lot of potential business being missed due to inaccurate credibility calculations.

Year	2013			2014		
Octant	Sum of New Premium	Sum of Unum Premium	Difference In Premium	Sum of New Premium	Sum of Unum Premium	Difference in Premium
1	104,121,530	113,590,884	(9,469,354)	92,134,900	98,932,144	(6,797,244)
2	39,208,165	37,552,924	1,655,240	52,286,022	49,751,385	2,534,637
3	11,889,253	10,535,423	1,353,830	13,763,823	12,339,262	1,424,561
4	22,886,047	19,176,929	3,709,118	19,403,050	16,444,563	2,958,488
5	185,656,068	169,105,014	16,551,054	269,462,433	251,251,728	18,210,705
6	209,180,802	226,037,742	(16,856,940)	270,971,002	286,089,868	(15,118,866)
7	18,188,569	21,216,729	(3,028,160)	25,480,696	28,580,014	(3,099,319)
8	15,623,834	18,037,568	(2,413,734)	20,036,416	26,518,170	(6,481,754)
Total	606,754,267	615,253,214	(8,498,947)	763,538,342	769,907,134	(6,368,792)

Table 14 Change in Premium Under the Buhlmann-Straub Method

SIC Breakdown

In many cases, dividing all the policies into subgroups based on SIC code increased the accuracy of loss ratios without exceeding the tolerable loss ratio. The magnitude of the change in premium collected can be seen in Table 15. While the strategy of making subgroups based on SIC code works well in certain cases, it fails in cases where there are too few lives. In these subgroups, k varies wildly and this leads to inaccurate amounts of premium collected. Thus, one possible improvement to the Buhlmann-Straub method is to subgroup by SIC code for subgroups with a large number of lives.

SIC	Count	Sum of Unum Premium	k	Sum of New Premium	Difference In Premium
20-39	234	125,096,130	6,450	119,650,494	(5,445,636)
			2,805	118,177,392	(6,918,738)
60-67	170	77,497,108	6,450	75,607,622	(1,889,486)
			10,741	76,225,313	(1,271,795)
70-89	741	432,157,732	6,450	432,274,421	116,689
			8,825	441,042,723	8,884,991

Table 15 Change in Premium Using the SIC Groupings

Potential Improvements

A possible improvement that could be implemented is adding a full credibility criterion. The Buhlmann-Straub method never assigns a credibility factor of one, but credibility factors do approach one as lives increase. In Unum’s current credibility calculation, any LYE that exceeds 35,000 lives grants full credibility. If this criteria has been successful for Unum, it could override the Buhlmann-Straub for cases where the LYE exceeds 35,000 lives. This could address issues where the Buhlmann-Straub assigns credibility factors as low as .85 to policies that were formerly fully credible. The lack of weight given to the experience rate can drive loss ratios up since the premiums are cut.

Another possibility to explore is similar to the idea of breaking the policies into subgroups based on SIC. Instead of SIC, k could be calculated based on group size. This has the potential to address the issue of LYE’s being too low in the credibility calculation. For example, if a claim with bad experience has a low number of lives, a large k will drive the credibility factors down, thus

not accounting for experience where it should. The idea behind splitting k 's calculation up by group size is that policies with thousands of life years exposure maybe should not be grouped with groups that have very low LYE. Breaking the groups down by their size yielded lower k values of 1579 for small sized groups, 1494 for medium sized groups and 1406 for large sized groups. Since k is calculated using claim dollars per life, it is not clear as to why the k values would decrease when separated by group.

Improvements When Working With the Data

A number of other improvements could be made to improve the accuracy and credibility of the Buhlmann-Straub method. The first of these improvements comes from a data entry and extract level. Having accurate and updated claims data is very important when calculating k in the method. Gaps in data can significantly affect the calculations of k . The variance of the claim dollars per life is essentially what drives the calculation, and if it were to be missing or inconsistent, the k value will be inaccurate.

The most obvious improvement is to test the Buhlmann-Straub method on more sets of data. With more years of data, loss ratios could be calculated using the Buhlmann-Straub method using actually claims for the years 2012 and back. This would be an interesting comparison to how the Buhlmann-Straub method affected projected loss ratios. It would be good to be able to look at its effectiveness over time to measure how it affects accuracy and profitability.

In addition to more backtesting, Buhlmann-Straub credibility could be calculated side-by-side with Unum's existing credibility for comparisons in the future. Perhaps if competitors are winning cases, Unum could compare those cases and see what the premium would be under the Buhlmann-Straub approach. In the end, the decision to change credibility methods is a business decision that weighs customer satisfaction and losses in profit.

References

- ¹ Norberg, Ragnar. "Credibility Theory." *Department of Statistics*. London School of Economics. Web. 10 Oct. 2014. <<http://isfaserveur.univ-lyon1.fr/~norberg/links/papers/CRED-eas.pdf>>.
- ² Herzog, Thomas N. *Introduction to Credibility Theory*. 4th ed. Winsted, CT: ACTEX Publications, 2010. 1-136. Print.
- ³ "Albert Henry Mowbray." *Insurance Hall of Fame Laureate Profile*. Insurance Hall of Fame. Web. 16 Dec. 2014. <<http://www.insurancehalloffame.org/laureateprofile.php?laureate=24>>.
- ⁴ Herzog, Thomas N.
- ⁵ Brehm, Paul. "Basic Concepts in Credibility." The Casualty Actuary Society, 6 Mar. 2006. Web. 22 Oct. 2014. <casact.org>.
- ⁶ Doss, Ken. "Introduction to Credibility." *The Casualty Actuarial Society*. 1 Mar. 2011. Web. 23 Nov. 2014. <<https://www.casact.org/education/rpm/2011/handouts/WS1-Doss.pdf>>.
- ⁷ "Prof. Hans Buhlmann." *Insurance Hall of Fame Laureate Profile*. Web. 16 Nov. 2014. <<http://www.insurancehalloffame.org/laureateprofile.php?laureate=78>>.
- ⁸ Goulet, Vincent. "Principles and Application of Credibility Theory." *Journal of Actuarial Practice*, 1 Jan. 1998. Web. 14 Nov. 2014.
- ⁹ Herzog, Thomas N.
- ¹⁰ Brehm, Paul.
- ¹¹ "Our History." *History*. Unum Group. Web. 3 Dec. 2014.
- ¹² "UNUM GROUP Company Information." *UNUM GROUP*. Hoovers. Web. 24 Dec. 2014. <http://www.hoovers.com/company-information/cs/company-profile.Unum_Group.eb0bfbc804952094.html>.
- ¹³ The Leader in SIC and NAICS Codes. Web. 21 Dec. 2014. <<http://siccode.com/en>>