

# Developing Methods And Graph Representations Towards Automated Design Of Gear Trains

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Submitted By

Penelope Belliard

Ruizhe Chen

Yinjie Liu

Advisor

Prof. Pradeep Radhakrishnan  
Mechanical Engineering Department

Co-advisor

Prof. David C. Brown  
Computer Science Department

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Penelope belliard, Ruizhe Chen, Yinjie Liu and  
Professor Pradeep Radhakrishnan

## **ABSTRACT**

This project focused on identifying the requirements that can be built into a software tool to automatically design gear trains. In the project's first stage, various kinematic- and stress-based gear and shaft design equations were sourced from different handbooks. In the second stage, the iterative design process was developed for implementation in the proposed software tool. The final stage involved developing graph representations for gears and shafts as well as creating basic grammar rules to generate gear trains.

## **EXECUTIVE SUMMARY**

In this Major Qualifying Project, our team focused on research an automation process that can generate different solutions as well as save time when compared to designing manually. However, automation requires identification of all the necessary steps and equations so that the designs can be correctly generated. This project focused on identifying the requirements that can be built into a software tool to automatically design gear trains. In the project's first stage, various kinematic- and stress-based gear and shaft design equations were sourced from different handbooks. Using these equations, designs for gear trains for three test problems were manually generated. In the second stage, the iterative design process was developed into a flow chart for implementation in the proposed software tool. The final stage involved developing graph representations for different types of gears and gear trains, as well as creating basic grammar rules to add gears and shafts based on input conditions.

### **A. Approach**

Our team step forward on the way of automated design and achieve several initial goals. These goals are solving three problems related to different design aspects manually. The design aspects need to include stress analysis, direction analysis, and position analysis. Refer to the tested problem section of this report, they are the gearbox, LEGO, and clock design problems. Then, the problem solving approaches are documented, and the logics are adopted into graph representations. The graph representations help our team to reorganize the problem solving logics into rules and actual computer programs.

### **B. Project Result and Achievement**

The project achievements can be classified to three categories: theory, graph and rules. The theories are the documentation of the three design problems. The graphs are the representation of the design theories. And the rules are generated based on the structure of the graph representations. The project result can be considered as a rationalized approach of automation in gear train designs by using graph representations and graph rules. These rules take design requirements, follow the documented design procedure, and produce the design solution using nodes and arcs. Our team also utilized the 3D printing technic to realize our gearbox design solution.



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## **INTRODUCTION**

Computer aided design software and technologies has been developing rapidly within the last twenty years. Designing complex mechanical systems requires extensive knowledge and training from design engineers. Countless fundamental engineering errors have led to catastrophes. These errors are usually made while on the correct engineering standard. In order to avoid engineering errors various iterations of calculation need to be made. This, we believe, can be replaced by computer aided suggestions and simulations. The simulation phase has been well-developed by different companies, such as the SolidWorks and AutoCAD. In sharp contrast, there is no existing software that provides engineers with feasible gearbox solutions that minimize design cost, weight, and power loss.

## **PROJECT GOALS AND PROBLEM STATEMENT**

This Major Qualifying Project is an approach on designing a gearbox by using computerized programs. To achieve so, our advisor and our team are going to survey various machine elements, kinematic and dynamic equations. We will also state the number of graph grammar representations for those machine elements, such as gears, shafts, bearings, and how those are all related. Such process is mostly conceptual and computational; and the GraphSynth software is used to simplify the process. The GraphSynth tool uses a series of nodes, arcs, and labels to represent the data analyzed. The final version of this graph will be used in grammar rules to develop various conditions that will assist the creation of multiple design solutions. These grammar rules, may be in integers, variables, and parameters, will be coupled with appropriate tree-search algorithms to generate topologies of gear boxes. Furthermore, gear design equations taken from Robert L. Norton's "Machine Design" book will be used to identify the stresses each gear is subjected to. These designs should include the following inputs: input and output positions, speed and torque information, and size of the bounding box. During analysis, the various criteria that can be changed such as gear parameters. Some of the other constraints include: material, safety factor, backlash, and even durability requirement. Overall, the goal is to automatically generate different gearbox designs for a single user input by applying various machine element types such as straight and step shafts, spur, bevel, and helical gears.

# LITERATURE REVIEW

There are many examples of automated design of gear trains available in the literature. We will elaborate on approaches that use graph-grammars. Swantner and Campbell [1] combine graph grammars and search and optimization techniques to generate various gear train designs. The authors developed a graph representation for different gears. Based on this representation, they developed four grammar rules (explained in detail below), one each to add a spur gear pair, a worm-gear pair and a bevel gear pair, and one to create a staged transmission system where shafts are added. Figure 1 shows a grammar rule where a gear is attached to an existing gear, thus forming a gear-pair. The various labels used to represent the features are “gear”, “contact”, “new”, “shaft” and “speed”. A directed arc connects the two nodes representing the gear and non-directed arc represents the connection between the shaft node and the gear node

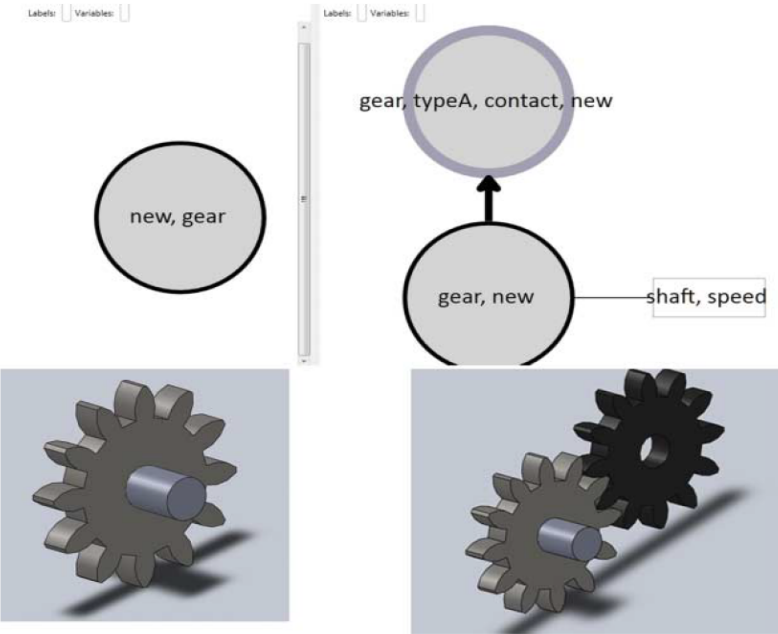


Figure 1: Generating a New Gear Pair [1]

Figure 2 shows a rule that adds two new nodes and two arcs to the existing gear pair on the left-hand side of this rule. One of the newly added nodes represents the shaft identified by the label “shaft” and the other node represents a gear. You will also see that the label “new” that is seen on one of the gear nodes on the left-hand side is removed. The “new” label now appears on the newly added gear node. This rule allows adding a gear to an existing shaft. Combining this rule with the previous allows for creating multiple stages.



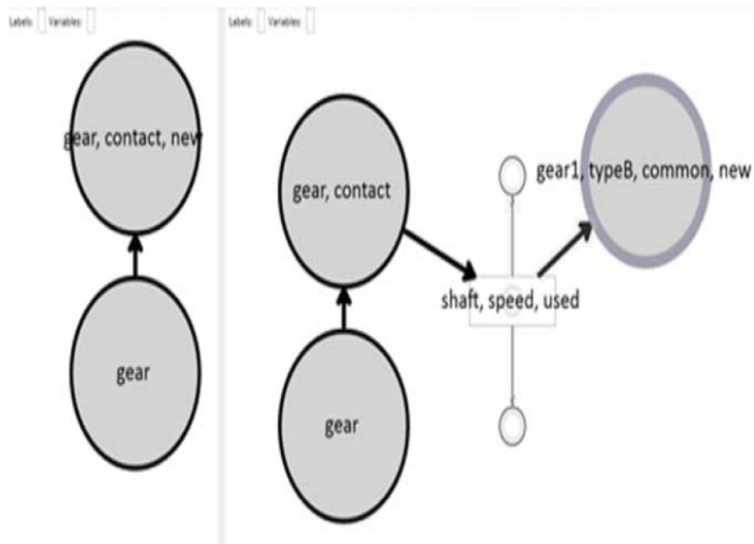


Figure 2: Creating a Staged Gear Transmission [1]

Figure 3 shown below is a rule where a worm-gear pair is added to an existing gear pair and Figure 4 shows a rule where a bevel gear pair is added.

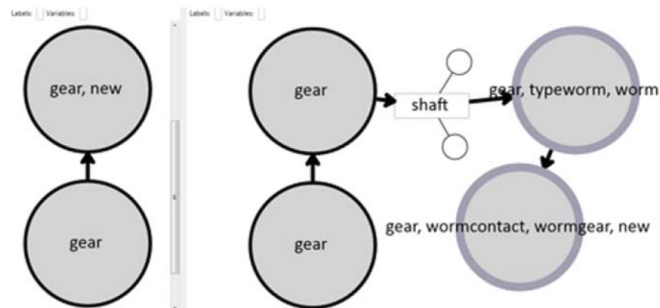


Figure 4: Adding a Bevel Gear Into a Gear Train [1]

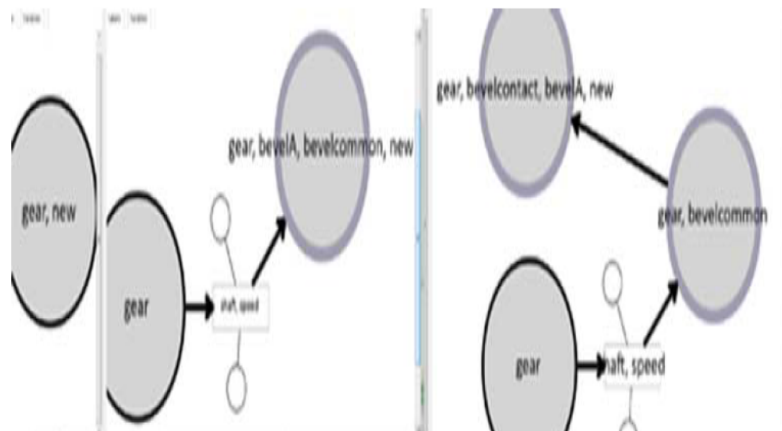


Figure 3: Adding a Worm Gear Into a Gear Train [1]

The process adopted by Swantner and Campbell [1] is illustrated in their flow chart shown in Figure 5. The first step is collecting user input. This is followed by a tree search whereby a gear train is generated using the rules described above. The generation process is influenced by the user input that is used to determine the required number of stages. One generated gear train is selected and passed to an optimization routine. This routine is used to determine optimal values for various gear parameters such as diametral pitch, number of teeth, etc. such that all the gears can meet various stress, location and bounding box constraints. If no suitable gear are available, then the process repeats using a different gear train generated in the search process until a solution is found

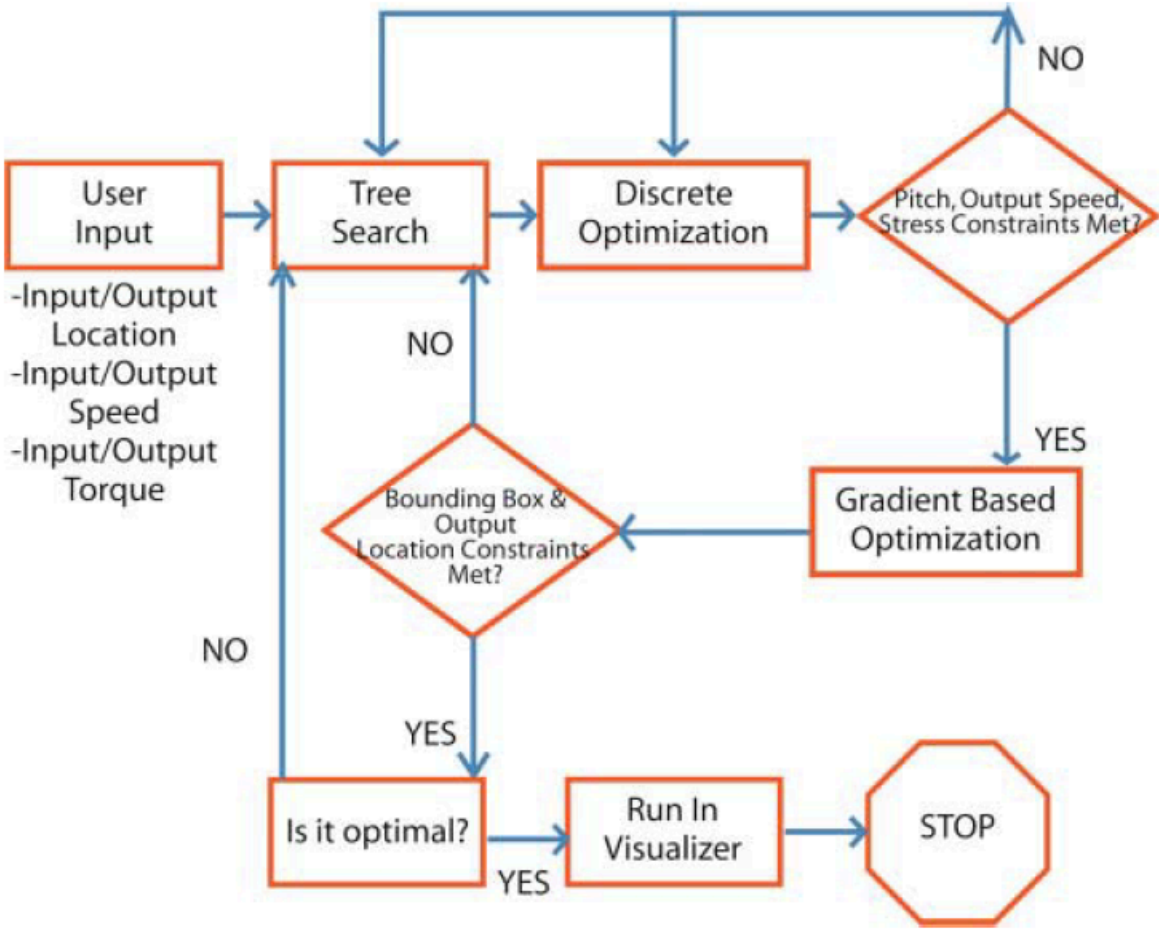


Figure 5: Solution Search Logic from Swantner and Campbell [1]

Lin, et al. [2] used a similar procedure to generate geared transmission systems. The graph representation developed by them for a geared transmission system is shown in Figure 6, where the

left-hand side figure is the actual transmission system and the right-hand side figure is the graph representation.

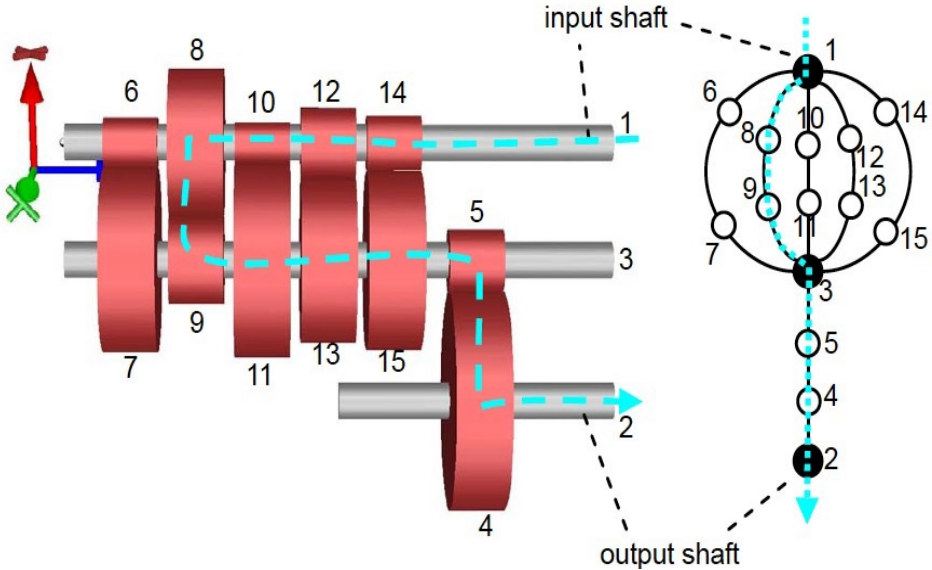


Figure 6: Graph Representation of Gear Box Solutions [2]

As you may notice, the shafts (numbered 1, 2, and 3) are represented by dark colored nodes. The gears are represented by other nodes. The dotted line shows the gears that are currently engaged in the transmission system. This representation is the basis for creating graph rules that helps in generating different solutions through a search process. There are nine rules in their approach and those are briefly explained below.

Rule 1 shown in Figure 7 involves deleting a gear set. The deletion is done in a way that the target speed ratio is not altered. Each deletion of a node may consequently eliminate one design approach. This reduces the number of output solutions by each deletion.

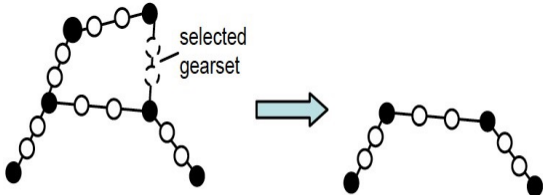


Figure 7: Deleting a Gear Set [2]

Rule 2 shown in Figure 8 adds gears to shaft. One gear is added to each shaft. Rule 3 shown in Figure 3 adds an additional stage. Whenever rules involving the addition of gears are applied,

each gear is assigned a face width.

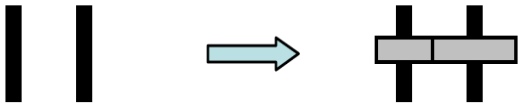


Figure 8: Creating a New Gear Pair [2]

Rule 3 represents replacing a gear pair, it is similar to creating a new gear. This rule selects an existing gear shaft and add a new shaft, making a new shaft pair. Then the rule connects two shafts using two new gear pairs, shown in Figure 9. The gear width is also selected within a margin of error of less than one inch.

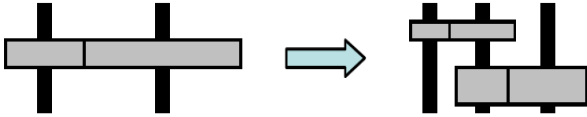


Figure 9: Add New Stage to the Gear Pair [2]

Rule 4 involves rearrangement of gear locations along shafts as shown in Figure 10. The amount of adjustment for individual gear sets is controlled by a standard deviation of normal distribution,  $g\sigma$ . It decreases the search process progresses, such that  $g\sigma = (g\sigma_0 - g\sigma_1) \cdot \exp(-i / gk_2) + g\sigma_1$  where  $g\sigma_0 = l/2$ ,  $g\sigma_1 = g\sigma_0 / 100$ , and  $gk_2 = 65/3$  where  $i$  is the iteration number. Overall, this rule calculates the average location distribution of the two new pairs and mount them in the desired order. The face width is also tuned to ensure that gear sets do not hit the side walls of the bounding box in the  $z$ -direction. This rule is part of the tuning rules during design generation.

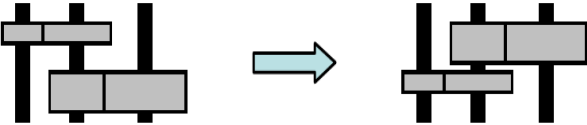


Figure 10: Relocating the Gear Sets Along the Shaft [2]

Another rule involved in tuning the generated design is rule 5 where the diameters of gears are changed. Figure 11 does not indicate what is exactly changing because this is primarily carried out in the backend code. Changing gear diameters mainly changes the pitch diameter of each gear. This will also adjust the relative gear ratio. The amount of adjustment is also controlled by a standard deviation of the location constraints. This allows more even gear sizes. Suggest in the paper, the

minimum diameter of gears is 0.030 m.

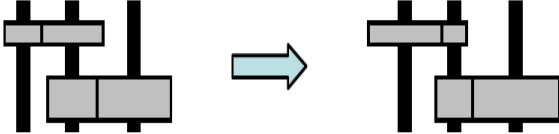


Figure 11: Changing Diameters of Gears [2]

Rule 6 shown in Figure 12 and Rule 7 shown in Figure 14 are used to delete unused gear sets and shafts. Rule 8 shown in Figure 13 is used to add two gear stages and an additional shaft. Rule 9 shown in Figure 15 is used to reposition the shafts.

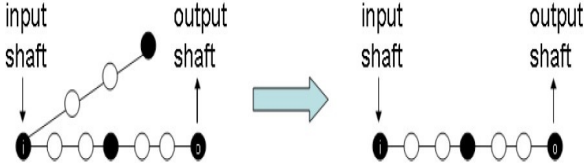


Figure 12: Deleting Unused Gear Sets and Shafts [2]

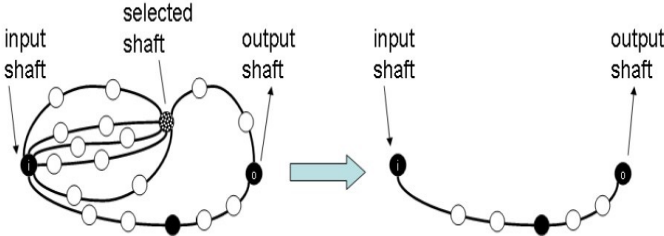


Figure 14: Deleting a Shaft in Graph Representation [2]

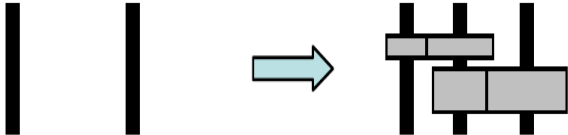


Figure 13: Creating a New Shaft [2]

The amounts of adjustment in all the directions are controlled by a standard deviation, which calculates the location normal distribution of the gear possible location in its orbit respect to its driving gear. Therefore, the shafts can be relocated into a more organized manner.

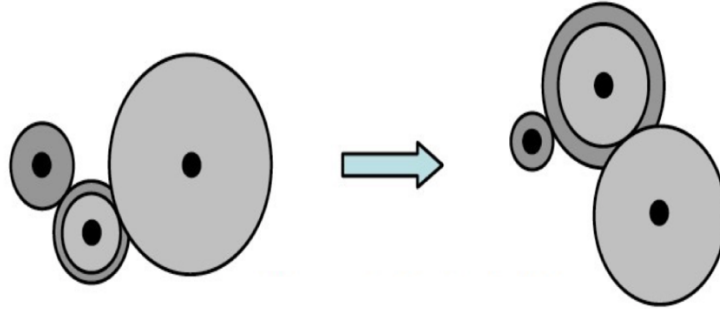


Figure 15: Reposition the Shafts [2]

The processes and rules described in [1] and [2] provide sufficient background and guidance in this major qualifying project. There are other research papers in this area as well. Lin and Schmidt [3] presented their work on a grammar-based assistance tool for gear train designers with focus on epicyclic gear trains used in automobile transmissions, and various machine tool gearboxes. Since our MQP focuses on spur, helical and bevel gears in this initial phase, we are not elaborating further on this paper.

Tudose, et al. [4] presents the specific process and analysis required while designing a two-stage gear train. This paper details all the design requirements. These requirements involve gear life span, gear prosperities, clearances, and machine elements attachments. This paper mentions various design requirements that are significant to gear transmission designing process. For instance, the requirement of gear and shafts' position, size, and transmission ratio. We concurred that the number of these requirements are applicable to our design procedures which will be mentioned in the next chapter.

While designing a gear train, a large number of assumptions must be made based on the given input information. For instance, the input information may include input/output positions, speed or torque ratio, and gearbox service span. Constrains such as the relative difference between the required and the actual gearing ratio must be within the range of  $\pm 2.5\%$  on both stages need to be considered. This decision is made based on the common engineering error which is below 5%. This paper also mentions constrains to guide the design accuracy, that gear teeth on each gear must not be undercut and the normal addendum coefficients on both stages should be in the range of  $-0.5$  to 1. These two constrains ensures the accuracy of gear transmission. Stress analysis is also essential to gear train design, therefore the authors states the Hertzian contact pressure on the teeth of both stages must not exceed a specified value to ensure the design efficacy, the bending stress on the teeth of gears 1 through 4 must not exceed a specified value, the bending strains on the three shafts in key locations must be below certain threshold values to enable the correct functioning of the

gearings and the bearings, and the torsional strains in the three shafts must be below a threshold value. Additionally, other environmental and manufacturability are mentioned such that the fatigue life safety factors on the three shafts must not fall below a specified value, and the operating temperature of the gear train must not exceed a specified value. These assumptions and constrains, will not only enhance the result of our manual design solutions but also improve our design solutions.

## **METHODOLOGY**

To arrive at the logic for automating the design of gear trains, the team decided that it was important to manually design gears trains and analyze the gears and shafts. Multiple design iterations were carried out manually and timed. These manual calculations allow us to understand and determine all the necessary equations and conditions that are used in gear design [5]. Besides, the goal is to decrease the number of iterations that need to be made, which in turn decreases the time it takes to generate a solution. The eventual goal is to implement all the equations in a program to generate different gear train solutions. The equations listed in this section and the flow chart shown in *Figure 20* explains our approach.

### **Gear Analysis:**

Every gear must be analyzed for various stress related to tooth bending and surface wear. Bending stress calculations are based on Lewis' equation while surface stress calculations are based on Buckingham equations [5]. The bending stress,  $\sigma_b$ , and surface stress,  $\sigma_{cb}$ , equations shown below for a given gearbox design. These equations differ depending on the gear type being used and the units used (English or SI) [6].

$$\sigma_b = \frac{W_t P_d K_a K_m}{F J K_v} K_s K_B K_I$$
$$\sigma_c = C_p \sqrt{\frac{W_t C_a C_m}{F I D_p C_v} C_s C_f}$$

Where application factors  $K_a$  and  $C_a$ , load distribution factors  $K_m$  and  $C_m$ , dynamic factors  $K_v$  and  $C_v$ , size factor  $K_s$  and  $C_s$ , idler factor  $K_I$ , elastic coefficient  $C_p$ , surface finish factor  $C_f$ , bending geometry factor  $J$ , surface geometry factor  $I$ , and face width  $F$ , are constants or constraints that define the overall operating condition of a system. The values for these constraints are chosen based on operating condition or application (see table 2). Using the torque  $T$  on the gear, the tangential force



$W_t$  of a gear pair can be determined. See figure X taken from Norton et al. Machine Design [5].

The equation below lists the relationship between tangential force  $W_t$  at each gear and the pitch

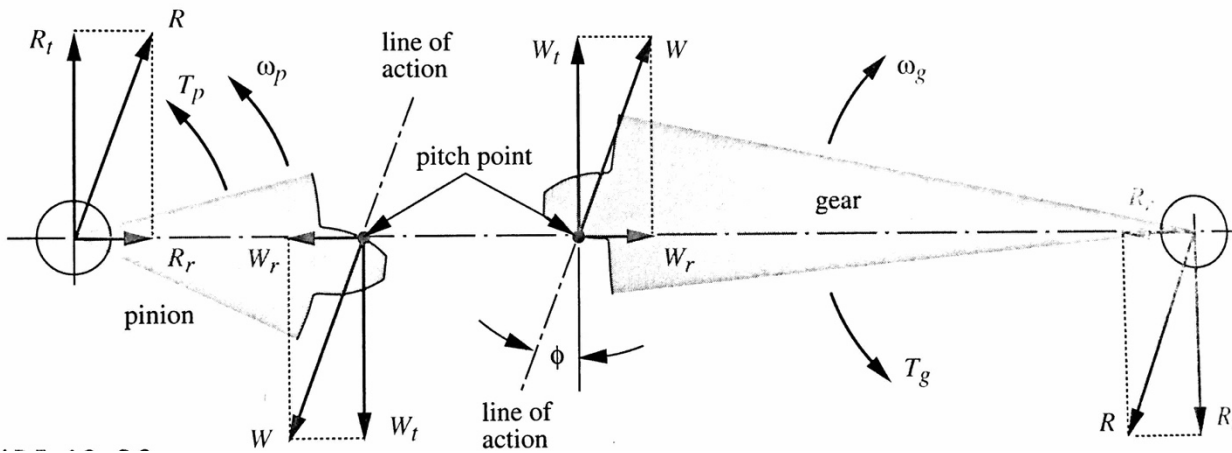


Figure 16: Meshed Gear Teeth Properties

$$W_t = \frac{T}{\frac{1}{2}D_{p,g}}$$

Since the surface and bending stresses of various systems are dependent on diametral pitch  $P_d$  and pitch diameter  $D_p$ , in our approach, we chose to set the diametral pitch and pitch diameter of a set of gears as unknown variables. Because most gear manufacturers databases are categorized by  $P_d$  Doing so will allow us to directly look up gears online based on  $P_d$  values found. For simplification purposes, the bending and surface safety factors  $N_b$  and  $N_c$ , respectively, is assumed to be 1.5, according to AGMA standards [6], for both pinion and gear. Diametral pitch, pitch diameter and the number of teeth are related as shown below. The variable  $N$  listed in this equation, is the number of teeth within a gear; not to be confused with the safety factors.

$$P_d = \frac{N}{D_p}$$

$J$  is the bending geometry factor and is based on number of teeth on gear and pinion.  $J$  is found using table below for loading at the highest point of single-tooth contact (HPSTC) [6].

Table 1: I and J factors based on the number of teeth on gear and pinion.

**I AND J FACTORS FOR:<sup>1</sup>**

20.0 DEG. PRESSURE ANGLE  
 0.0 DEG. HELIX ANGLE  
 0.250 TOOL EDGE RADIUS  
 EQUAL ADDENDUM ( $x_1 = x_2 = 0$ )

2.250 WHOLE DEPTH FACTOR  
 0.024 TOOTH THINNING FOR BACKLASH  
 LOADED AT HIGHEST POINT OF SINGLE TOOTH CONTACT

GEAR TEETH	PINION TEETH															
	12		14		17		21		26		35		55		135	
	P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12 I																
J	U	U														
14 I																
J	U	U	U	U												
17 I																
J	U	U	U	U	U	U										
21 I								0.078								
J	U	U	U	U	U	U	0.33	0.33								
26 I								0.084		0.079						
J	U	U	U	U	U	U	0.33	0.35	0.35	0.35						
35 I								0.091		0.088		0.080				
J	U	U	U	U	U	U	0.34	0.37	0.36	0.38	0.39	0.39				
55 I								0.102		0.101		0.095		0.080		
J	U	U	U	U	U	U	0.34	0.40	0.37	0.41	0.40	0.42	0.43	0.43		
135 I								0.118		0.121		0.120		0.112		0.080
J	U	U	U	U	U	U	0.35	0.43	0.38	0.44	0.41	0.45	0.45	0.47	0.49	0.49

Ta

<sup>1</sup> The letter "U" indicates a gear tooth combination which produces an undercut tooth form in one or both components and should be avoided. See Section 7 and Fig 7-1.

In the absence of test data that define the level of transmission error to be expected in a design, a design must estimate the dynamic factor  $K_v$  (or  $C_v$ ). The dynamic accounts for internally generated gear tooth loads which are induced by non-uniform meshing action (transmission error) of gear teeth and can be calculated based on the assumption of quality index  $Q_v$  and the pitch line velocity  $V_t$ .

$$K_v = C_v = \left( \frac{A}{A + \sqrt{V_t}} \right)^B$$

Where  $V_t = \frac{D_p}{2} \omega_p$ ;  $B = \frac{(12-Q_v)^{2/3}}{4}$  and  $A = 50 + 56(1 - B)$  for  $Q_v$  values greater than five [5].

Using Figure 17 taken from Norton's Machine Design,  $V_t$  and  $Q_v$  values are found.

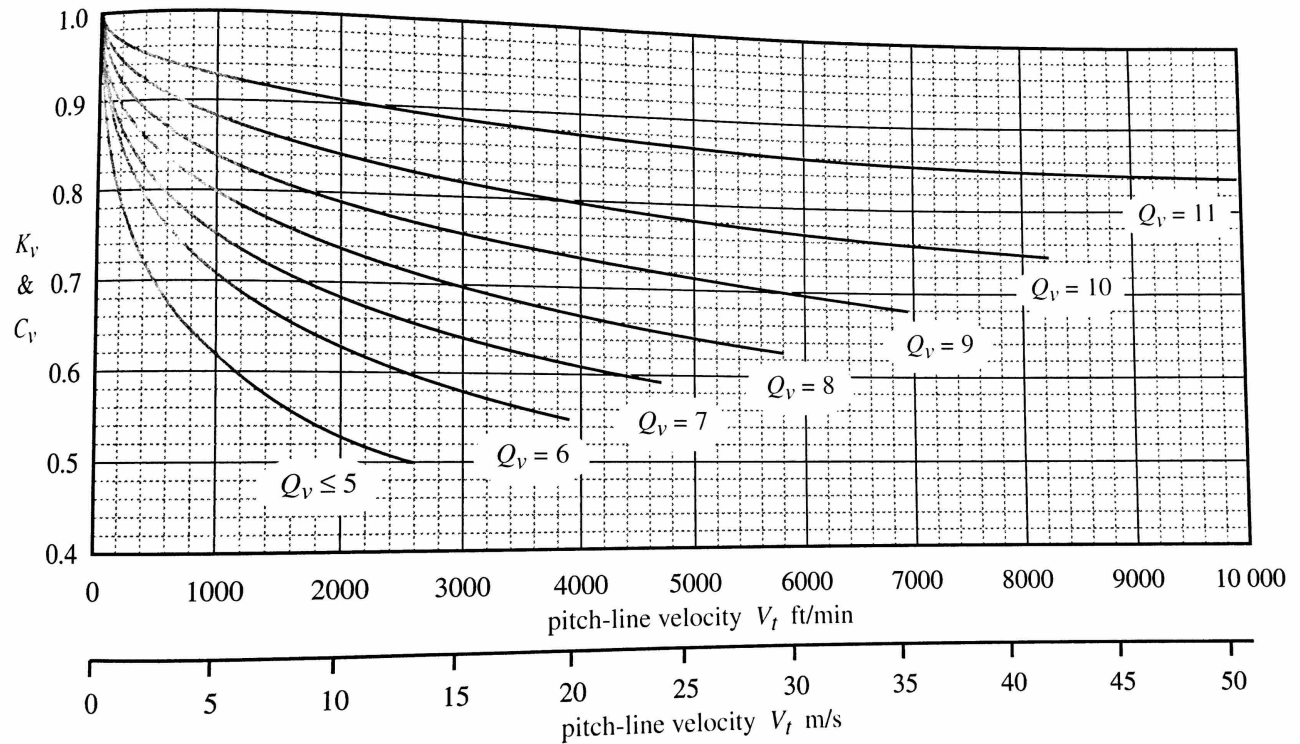


Figure 17: Chart used for determining dynamic factor  $K_v$  and  $C_v$  and pitch-line velocity  $V_t$

The quality index  $Q_v$  is an empirical value and is used to relate tolerances to performance. The AGMA recommendation is to round the value of  $Q_v$  to the next lower integer [6]. Solutions will therefore be approximate. This value is identified using Table 2 or Table 2 below if application purpose is identified.

Table 2: Table Of Quality Index

Application		Quality index, $Q_v$
Cement mixer drum driver		3-5
Cement kiln		5-6
Steel mill drives		5-6
Corn pickers		5-7
Punch press		5-7
Mining conveyor		5-7
Clothes washing machine		8-10
Printing press		9-11
Automotive transmission		10-11
Marine propulsion drive		10-12
Aircraft engine drive		10-13
Gyroscope		12-14

Pitch velocity		Quality index, $Q_v$
ft/min	m/s	
0-800	0-4	6-8
800-2000	4-10	8-10
2000-4000	10-20	10-12
> 4000	> 20	12-14

The maximum allowable pitch-line velocity  $V_{t,max}$  must be checked to ensure it is greater than the actual pitch-line velocity previously calculated. This value should not exceed the recommended values.

$$V_{t,max} = [A + (Q_v - 3)]^2$$

The AGMA recommendation for face width of a gear is given below.

$$\frac{8}{P_d} < F < \frac{12}{P_d}$$

Since  $P_d$  is still not known at this stage in the calculation, a theoretical approximation is made for the face width of each gear. Doing so will simplify the calculation for the load distribution factor,  $K_m$  and  $C_m$ .

Table 4: Table Of Load Distribution Factors

**Table 12-16**

**Load Distribution Factors  $K_m$**

Face Width in (mm)	$K_m$
<2 (50)	1.6
6 (150)	1.7
9 (250)	1.8
≥20 (500)	2.0

Table 5: Table Of Reliability Factor

**Table 12-19**

**Reliability Factor  $K_R$**

Reliability %	$K_R$
90	0.85
99	1.00
99.9	1.25
99.99	1.50

The application factor  $K_a$ , intended to account for the weight  $n$ —the driving and driven machinery, can be assumed as one under uniform operating condition, otherwise it can be calculated. When there are no specific requirements for a gears internal structure, the size factor  $K_s$  or  $C_s$  and the rim bending factor  $K_B$  are the same for all gears within a gear train.

Table 6: Table of Application Factors

**Table 12-17 Application Factors  $K_a$**

Driving Machine	Driven Machine		
	Uniform	Moderate Shock	Heavy Shock
<b>Uniform</b> (Electric motor, turbine)	1.00	1.25	1.75 or higher
<b>Light Shock</b> (Multicylinder engine)	1.25	1.50	2.00 or higher
<b>Medium Shock</b> (Single-cylinder engine)	1.50	1.75	2.25 or higher

At this stage in our calculations, all factors for the governing equations are identified and the stress equations calculated, leaving  $P_d$  as the unknown variable to be input in the bending and surface safety factors  $N_{b,c}$  shown below.

$$N_b = \frac{S_{fb}}{\sigma_b}$$

$$N_c = \left(\frac{S_{fc}}{\sigma_c}\right)^2$$

$S_{fb}$  and  $S_{fc}$  are the corrected bending and surface fatigue strength, respectively.

$$S_{fb} = \frac{K_L}{K_T K_R} S_{fb'}$$

$$S_{fc} = \frac{C_L C_H}{C_T C_R} S_{fc'}$$

Before the  $N_{b,c}$  and  $S_{fb,fc}$  are calculated, life factor  $K_L$  and  $C_L$ , temperature factor  $K_T$  and  $C_T$ , reliability factor  $K_R$  and  $C_R$ , and uncorrected bending and surface fatigue strength,  $S_{fb'}$  and  $S_{fc'}$  respectively, need to be identified. The factors  $K_T$  and  $C_T$  and  $K_R$  and  $C_R$  are assumed based on working environment of the system. However, the uncorrected bending fatigue strength needs to be calculated based on material hardness using the Brinell Hardness  $HB$  material assumption taken from Table 26 [5].

$$S_{fb'} = -274 + 167HB - 0.152HB^2$$

The life factor  $K_L$  can therefore be calculated using the equation below.

$$K_L = 1.3558 N_{cyc}^{-0.0178}$$

In this case,  $N_{cyc}$ , is the number of cycles of operation a gear pair can withstand. These are determined using the table below [5].

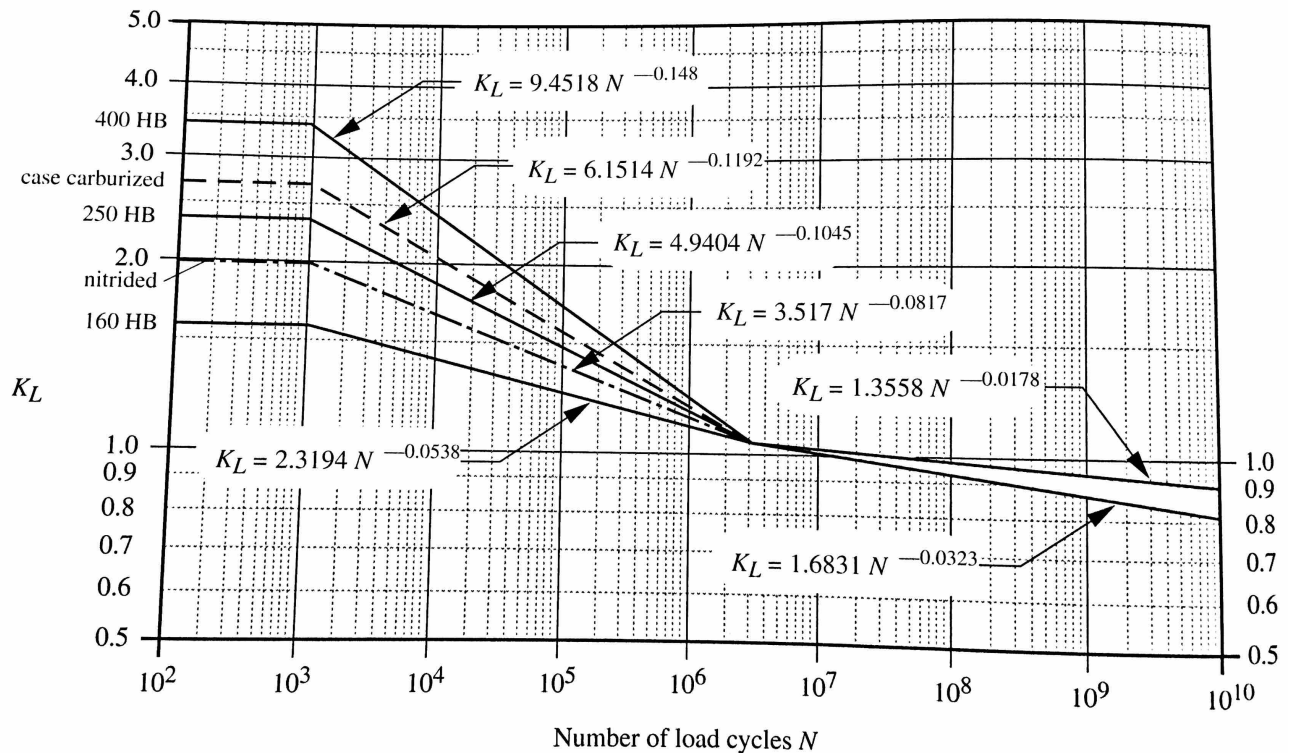


Table 7: Table used in identifying Life Factor,  $K_L$ , calculation

The uncorrected surface fatigue strength shown below is obtained using the lower curve of Table 7, where  $HB$  here is the same material value chosen before.

$$S_{fc'} = 26000 + 327HB$$

The life factor for surface fatigue strength  $C_L$  can be calculated using Table X and the equation below.

$$C_L = 1.448N^{-0.023}$$

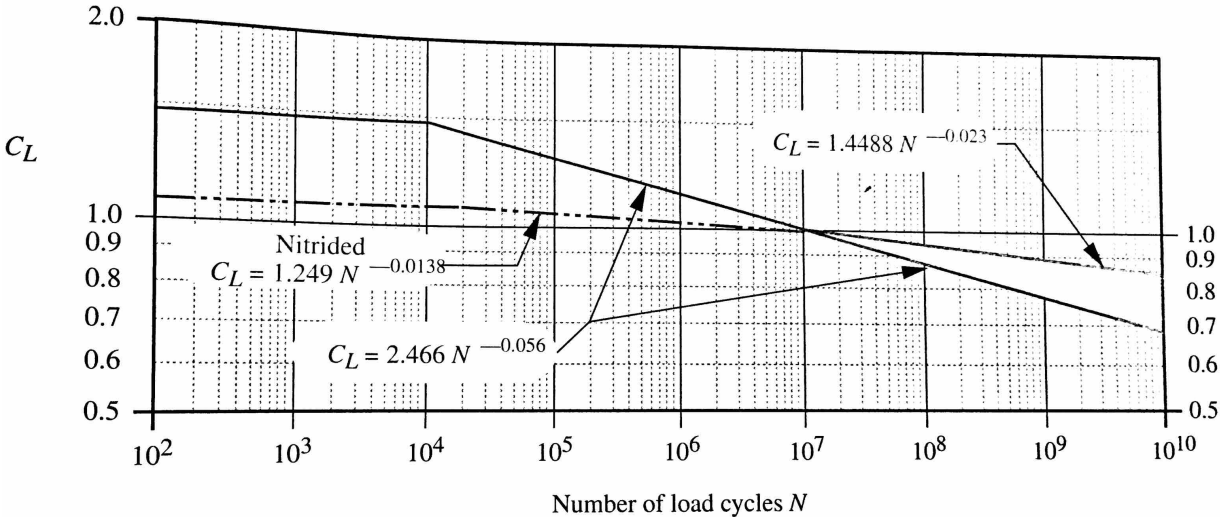


Figure 18: Chart used in identifying surface Life Factor  $C_L$ .

By solving the equations above, and comparing and contrasting the size of gears calculated (see Gear Design Process), the final result is presented. The solution found is expressed by the diametral pitch  $P_d$  of a given set of gears. As such, this approach can be applied to a wide range of complex systems involving a gear transmission. Setting the diametral pitch and pitch diameters as unknowns simplifies the governing bending and surface stress equations. While setting the safety factor to 1.5 satisfies AGMA standards [6] and solidifies the accuracy of a potential solution.

**Shaft Analysis:**

Shafts are integral to gear trains. Assumptions for the shafts length, material, type, connection, factor of safety  $N_{sf}$ , and the other factors that define the modified Goodman line approach defined by the shaft diameter are made based on application. All equation listed for this purpose can be found in Norton et. al Machine Design [5].

$$D = \left[ \frac{32N_{sf}}{\pi} \left( \frac{\sqrt{(k_{f,bending}M_a)^2 + \frac{3}{4}(K_{f,torsion}T_a)^2}}{S_f} + \frac{\sqrt{(k_{fm}M_m)^2 + \frac{3}{4}(K_{fsm}T_m)^2}}{S_{ut}} \right) \right]^{\frac{1}{3}}$$

Where  $N_{sf}$  is the safety factor,  $k_f$ ,  $k_{fm}$ , and  $k_{fsm}$  are the bending fatigue stress concentration factor and torsional fatigue stress-concentration for both alternating and mean components, respectively,  $T_a$  and  $T_m$  are the mean and alternating components of torque,  $M_m$  and  $M_a$  are the bending alternating and mean moments,  $S_{ut}$  is the ultimate tensile strength of the material, and  $S_f$  is the corrected fatigue strength at the selected cycle of life.

Since input force and gear ratios are found in the gear calculation previously mentioned, the maximum and minimum tangential forces  $F_{t,n}$  can be calculated.

$$F_{t,n} = \frac{T_n}{r_g} \quad n = \text{maximum or minimum}$$

Using both the pressure angle assumed and  $F_{t,n}$ , the maximum resultant force  $F_{max}$  and minimum resultant normal force  $F_{min}$  of a shaft are found using the equation below. The pressure angle,  $\phi$ , see Figure 19, is the angle between the line of centers and another line perpendicular to the line of action and going through the center of the gear or pinion [5].

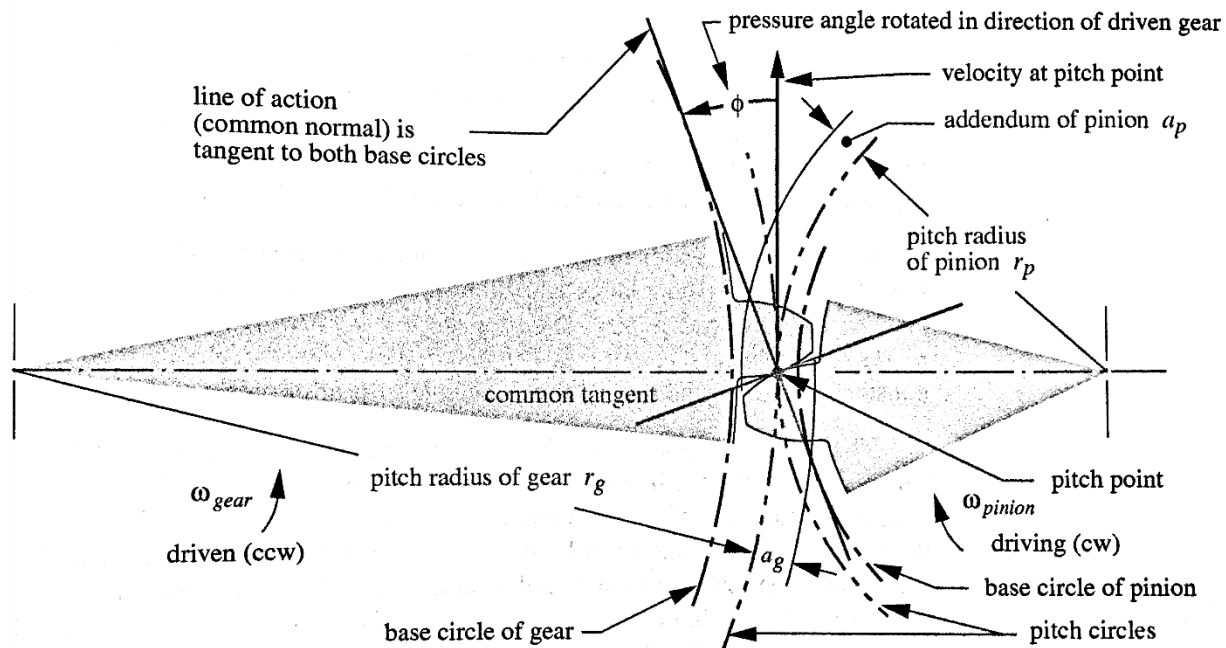


Figure 19: Free Body Diagram Of Gear Tooth Connection

$$F_n = \frac{F_{t,n}}{\cos\phi} \quad \phi = \text{pressure angle}$$



The maximum and minimum moments  $M_n$  on the shaft can now be found, using the equation below, by assuming the gears are centered between two supported bearings that are an assumed distance  $s$  apart.

$$M_n = F_n \frac{l}{s_{dist.}}$$

Based on the assertion made for the connection method an assumption can be made for the stress concentration factor  $K_t$  for both bending and torsion at the critical locations. The critical locations are where both moment and torque components are the largest. Now that the maximum and minimum moment(s) and torque(s) are known, the mean and alternating moment  $M_m$  and  $M_a$  and torque  $T_m$  and  $T_a$ , respectively, of the shaft can be determined using the equations below.

$$M_m = \frac{M_{max} + M_{min}}{2}$$

$$M_a = \frac{M_{max} - M_{min}}{2}$$

$$T_m = \frac{T_{max} - T_{min}}{2}$$

$$T_a = \frac{T_{max} + T_{min}}{2}$$

Then, the uncorrected endurance strength limit  $S_e$  is calculated and the equation is chosen based on material type. The equation below is used for low-carbon steel.

$$S_e = 0.5S_{ut}$$

Here, an ideal and inexpensive trial material (I.e: low-carbon or cold rolled steel) is chosen and material property  $S_{ut}$  is found based on the chosen material. If this material is proven to be less durable for application, then another material is to be selected.

Based on other assumptions about the operating conditions and environment the endurance strength limit can be corrected. These assumptions Where the corrected strength endurance limit is expressed as  $S_f$  in the equation below.

$$S_f = C_{load}C_{size}C_{surf}C_{temp}C_{reliab}S_e$$

The consecutive c-variables listed are factors assumed based on loading, size, temperature, and reliability of the systems application. However, the surface constant  $C_{surf}$ , is found using **Error! Reference source not found.**. Before calculating the shafts' diameter, the notch sensitivity  $q$ —a measure of how sensitive a material is to notches or geometric discontinuities—of the chosen material for both bending and torsion needs be determined as well as the bending and torsional

fatigue stress concentration factor  $K_f$  of each shaft. The bending fatigue stress concentration factor for the mean component  $K_{fm}$  is said to be one for all cases [5].

$$k_f = 1 + q(K_t - 1)$$

Once all factors are identified, the shaft diameter can be calculated by using the modified Goodman line approach mentioned above. The final theoretical diameter is determined and the actual diameter is found through an online industrial supplier. Further analysis and details on constants listed can be found in Robert L. Norton’s Machine Design book.

**Gear Design Process:**

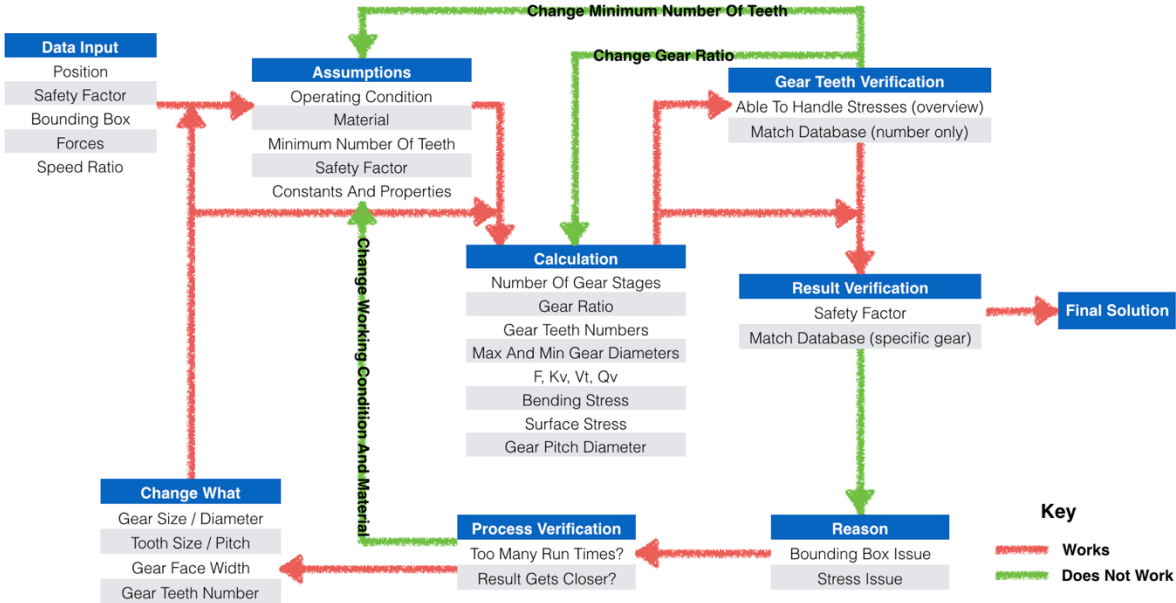


Figure 20: Flow Chart Of Gear Train Design Logic

After determining the gear and shaft analysis equations, the next step is to arrive at a flow that not only helps in fine-tuning the gear train design process but will also help in implementing the same for automation purposes. Shown in Figure 1 is the flow chart of the gear train design process. This flow chart specifically deals only with gears and not shafts.

As shown in Figure 1 above, the first step is to collect all inputs and requirements from the user, shown in the flow chart under “Data Input”. These may include input and output positions, size of the bounding box that houses the gear train, speed ratio, safety factor against bending and surface stresses and input torque or force, depending on whether the input is angular or translational. The input and output positions, speed ratio, and the input torque or force are required information while bounding box, safety factor, and position are optional information. These optional data can be

handled in the next step under “assumptions”, which is where a few assumptions will be made to constrain the design process.

The reason for making assumptions is to reduce the set of possible solutions. The various assumptions made include the operating condition of the gear train, materials used, minimum number of teeth and safety factor among the gear design process. The value of minimum number of teeth is determined based on the minimum number of teeth correlated with the normal ratio range of the chosen gear type. For optimal feasibility of a solution, when a solution to the manual design problems are found, the team compares them to the gear property charts found in online professional gear suppliers such as McMaster-Carr. After determining the stage ratio, the next step is to use all the assumptions derived previously and determine the diametral pitch of every gear in the system. In case the resulting values are very high, then the minimum number of teeth or the stage ratio is changed and the process repeated. When the minimum number of teeth is changed, some of the assumptions (such as  $P_d$ ) might require revisiting. There could be instances where the gears may not fit inside a specified bounding box. In that case, the assumptions related to face width or number of teeth will have to be revisited and the process continued.

Table 8: Table Of Maximum Gear Transmission Speed Ratio

Type	Normal Ratio Range	Pitch Line Velocity (m/s)	Efficiency Range
Spur	1:1 to 6:1	25	98-99%
Helical	1:1 to 10:1	50	98-99%
Double Helical	1:1 to 15:1	150	98-99%
Bevel	1:1 to 4:1	20	98-99%
Worm	5:1 to 75:1	30	20-98%
Crossed Helical	1:1 to 6:1	30	70-98%

Because the safety factor of one machine element can vary from 1.1 to 6.0 depending on its application safety requirements, it can also be assumed when information related to safety factor is not available. In our analysis, we assumed a safety factor of 1.5. We also assumed the life span of our design solutions. The most common one-shift life span of a machine element is 5 years. So we assume that each machine element will need to handle a single shift life span of 5 years with 2000 operating hours per year. All other constraints (i.e:  $K_n$ ,  $C_n$ ,  $I$ ,  $J$ , etc. see Gear Analysis) that need to be identified would be done so in this step before the stress analysis is executed for each design

process.

Once all input data is collected and assumptions realized, the next step is to determine the number of stages of a gear train under “Calculations.” All gear teeth are assumed based on the speed ratio of a given design; the specific calculation procedure is shown in *Appendix 6*. Since we have information about the overall ratio as well as, input and output values, a simple decomposition of the speed ratio can be used to determine the mounting type or minimum number of teeth. The solutions found are compared with Table 8 for accuracy. If the overall speed ratio required and the values in table 1 are not equal, then more than one stage is required. About multiple stages, if equal transmission ratio in each stage is desired, depending on the number of the stages required, a root of the number of the stages would give the ratio for each stage. The first step is to check if a two-stage gear-train would be sufficient. This is compared with the values in Table 1 and if found to be within the specified ranges, then the next step in the gear-train design is continued. It is to be noted that the maximum range per stage depends on the type of gear as well. A detailed explanation of the stage ratio determination is presented in the section below.

### **Determining Stage Ratio:**

Determining a meaningful and rational logic for calculating the desired gear transmission ratio based on the gearbox design problem is important. Some of the assumptions considered include that the overall accuracy of the final design solution should be within 1%. And the gears selected must according to gear teeth numbers provided by professional gear suppliers such as McMaster-Carr and MSC.

We will explain this section using an overall transmission ratio of 30:1. This is the simplest way to present the reduction ratio; therefore, this information can be applied to the next step. In the contrast, if the given transmission ratio is specified as 5:2. Then, this ratio needs to be recalculated to 1 and in this case, will be 2.5:1.

Assuming that spur gears are being used for this gear train, the maximum gear reduction is 6. as specified in Table 1 above. Therefore, by comparing the extracted number 30, to this recommended transmission ratio, it is obvious that designing a one-stage gearbox is not suitable.

In order to design a multistage gear transmission, it is necessary to determine the transmission ratio for each gear pair. We will first use the simplest and most convenient method by taking the square root of the extracted number. This divides the transmission into two stages and also determines the gear deduction ratio for each gear pair. This this case, we will take the square root of

30, which equals to 5.4772. Then, this number need again be compared with the recommended maximum spur gear transmission ratio, 6. Hence 5.4772 is smaller than 6, this transmission ratio fits the required standard.

By taking the backwards calculation to determine the accuracy of the system, which falls under one percent. This means a two-stage gear transmission with a ratio of 5.4772 to 1 can be used to achieve the final deduction ratio of 30 to 1. After the transmission ratio is determined, specific gear teeth number can therefore be calculated. By various assumptions of the driving gear teeth number such as 8, 10, and 12. The correlated driven gear teeth numbers are rounded as 44, 55, 66, shown in Table 9. After checking with online gear suppliers, based on previous logic, they offer gear pair with 8 teeth and 44 teeth. By again doing the inverse calculation to calculate the accuracy of the transmission, the error is about 0.37%. Therefore, the gearbox can be designed by two stages with that same transmission ratio, which the driving gears have 8 teeth and the driven gears have 44 teeth.

Table 9: Gearbox Problem Design Logic Solution 1

Ratio	Arrangement	Status	Final Status
5.4772	8, 44	Selected	Eliminated
5.4772	10, 55	Eliminated	N/A
5.4772	12, 66	Selected	Eliminated

Hence the fist possible solution is presented in the previous step, this solution can now be compared with the theoretical solution from the gearbox problem. It is obvious that even the driving gears only have 8 teeth, they are not able to handle the required torque. Therefore, the solution presented is not suitable for the gearbox design requirement. A two stage gearbox does not meet the gearbox design requirements; a three-stage gearbox needs to be considered. By following the same procedure, and using the same extracted number from, cube root will be used to determine transmission ratio for the three-stage gearbox. Specifically, the cube root of 30 equals to 3.1072, and this number is smaller then the recommended maximum spur transmission ratio 6. So, we can conclude this is a possible transmission ratio being applied to the gearbox.

After the transmission ratio is determined, specific gear teeth number can now be calculated. Based on the assumption the driving gear teeth numbers can be 8, 10, 12, and 14. By going through simple calculation using the transmission ratio determined, the correlated driven gear teeth numbers are rounded as 25, 32, 37, 44, shown in Table 10. Because professional gear suppliers only provide gears with even number of teeth, the gear pair with teeth number 8 to 25 and 12 to 37 are eliminated.

Table 10: Gearbox Problem Design Logic Solution 2

Ratio	Arrangement	Status	Final Status
3.1072	8, 25	Eliminated	N/A
3.1072	10, 32	Selected	Eliminated
3.1072	12, 37	Eliminated	N/A
3.1072	14, 44	Selected	Eliminated

By doing the inverse calculation, the output accuracy for the two possible transmission ratio are 3% and 1.38%; therefore, these two solutions do not fit the design requirement which the accuracy should fall under only one percent. Furthermore, since the two possible solutions presented in *step 9* are not suitable based on the design requirement, it is not necessary to verify these solutions with complex stress analysis.

Based on the result from step one through ten, it is obvious that the root calculation does not provide an accurate solution to the gearbox design problem. Overall, a different approach needs be taken to make sure the gearbox has three stages and also enhance the output accuracy compare to the previous possible solutions. Therefore, our team decide to use decomposition to determine the exact transmission ratio. The method of decomposition means decomposes the extracted number from *step 2* into various factors. Since we are calculating a three-stage gearbox, there will be three factors generated in the process of decomposition. Specifically, the extracted number 30 can be decomposed as  $1 \times 1 \times 30$ . Following the same logic, other decomposition of 30 can be  $1 \times 2 \times 15$ , and  $2 \times 3 \times 5$ . Now comes the crucial step that since it is shown in *step 7* that a transmission ratio of 1 to 5 does not properly handle the required torque, the decomposition needs be executed to a more even arrangement.

Continue from the previous step, one decimal variables are first considered, such as .5. So that the extracted number 30 can be decomposed as  $2 \times 4 \times 2.5$ . Hence all decomposed variables are smaller then 5, therefore the transmission ratio  $2 \times 4 \times 2.5$  can be considered as a possible solution. Additionally, since the method of decomposition is used, the transmission accuracy is 100%. The next step is to determine the accurate gear teeth numbers, shown in Table 11. There many solutions can be chosen since the transmission ratio is low. All solutions are plugged into the final equation of the gearbox design problem shown in *Appendix XX*. Then, a reasonable gear teeth arrangement will be selected. Specifically, the gear pair chosen for the gearbox are 16 to 48, 10 to 40 and 16 to 40.

Table 11: Gearbox Problem Design Logic Final Solution

Ratio	Arrangement	Status	Final Status
3	8, 24	Selected	Possible Solution
3	10, 30	Selected	Possible Solution
3	12, 36	Selected	Possible Solution
3	14, 42	Selected	Possible Solution
3	16, 48	Selected	Selected
3	18, 54	Selected	Possible Solution
3	20, 60	Selected	Possible Solution
4	8, 32	Selected	Possible Solution
4	10, 40	Selected	Selected
4	12, 48	Selected	Possible Solution
4	14, 56	Selected	Possible Solution
2.5	8, 20	Selected	Possible Solution
2.5	10, 25	Eliminated	N/A
2.5	12, 30	Selected	Possible Solution
2.5	14, 35	Eliminated	N/A
2.5	16, 40	Selected	Selected
2.5	18, 45	Eliminated	N/A
2.5	20, 50	Selected	Possible Solution

## TEST PROBLEMS

The governing equations and process flow described in the methodology section are tested on three different gear train design problems. These designs are generated manually and are used to refine the process derived earlier.

### **Problem 1:**

The first problem is sourced from Swanter and Campbell [1] and involves designing a gear train with the inputs listed in *Table 12* below.

Table 12: Gearbox Design Problem Data

<b>Inputs</b>
<b>Input Position:</b> (0, 0, 0)
<b>Output Position:</b> (3, 1, 4)
<b>Input Torque:</b> 1.5 ft-lb
<b>Input Speed (<math>\omega_{in}</math>):</b> 3000 rpm
<b>Output Speed (<math>\omega_{out}</math>):</b> 100 rpm
<b>Speed ratio</b> ( $\omega_{in} : \omega_{out}$ ): 30 : 1

In order to solve the problem, various assumptions were made to generate an accurate and rational result. These assumptions are listed in *Table 13* below.

Table 13: Gearbox Design problem Assumptions

<b>Assumptions</b>
<b>Material:</b> AGMA Grade 1 Steel, 250HB, $Q_v = 10$
<b>Gear Ratio:</b> 1/4, 1/3, 1/2.5
<b>Gear Teeth No.:</b> 10, 16, 40, 48, 64
<b>Gear Type:</b> Spur
<b>Mounting Method:</b> 10/40, 16/48, 16/40
<b>Facewidth factor:</b> 12
$N_{b,s} = 1.5$
$T_{oper} < 200^\circ F$
$K_a = 1$



Based on the problem statement, English units are used. There are many assumptions made to satisfy the operating environment such as operating temperature  $T_{oper}$ , safety factor, application factor  $K_a$ , and material used. We also assumed the calculated gear box will have an operating life span of 10 years with 2000 steady operation hours each year. Other assumptions include that the gear pressure angle is assumed to be 20 degrees, and the face width factor is 12. *Table 14* below details all factors found in our calculations and used in finding the gears online. Detailed calculation procedures are presented in *Appendix 1*.

### Problem 2

Table 14: Gearbox Design Solution

Gear Name	N	Pd	Dp(in)	F(in)	Pressure Angle	Position
A	10	12	0.83	1	20	(0,0,0)
B	40	12	3.33	1	20	(0.5, -2.04, 0)
C	16	10	1.6	1.2	20	(0.5, -2.04, 2.2)
D	48	10	4.8	1.2	20	(3.65, -2.14, 2.2)
E	16	8	2	1.5	20	(3.65, -2.14, 3.7)
F	40	8	5	1.5	20	(2.99, 1, 3.7)

Inputs
Input Position: N/A
Output Position: N/A
Input Torque: 0.3 Nm/ hand
Input Speed ( $\omega_i$ ): 250rpm
Output Speed ( $\omega_{out}$ ): 60rpm
Speed ratio ( $\omega_{in}$ : $\omega_{out}$ ): 250 : 60

Figure 21: Change caption

The second problem involves designing a gear train to mimic the hour-, minute- and second-hands of a clock. The key difference between the two being that for this design problem the focus is on machine element position analysis instead of forces analysis. The gear train has to fit in an enclosure that has a diameter of 2.5cm, and thickness of 0.5cm. The watch must have at least two

pointers; one indicates hour, the other indicates minute. It is also powered by a steady rotating motor which has rotational speed of 250 rpm. We also assumed that there is only input, fourteen gears and nine shafts to save material.

Additional assumptions were made for motor input. The motor is expected provide a steady rotation, and must have one-shift service life of five years with 8760 hours of operation each year. The failure rate of the designed watch was also assumed to be below 1% for the entire system.

Table 15: Clock Problem Design Solution

Assumptions
<b>Material:</b> AGMA Grade 2 Steel, 250HB, $Q_v = 10$
<b>Gear Ratio:</b> 17/34, 17/85, 17/102
<b>Gear Teeth No.:</b> 17, 34, 85, 102
<b>Gear Type:</b> Spur
<b>Mounting Method:</b> see table 10
<b>Facewidth factor:</b> 12
$N_{b,s} = 1.5$
$T_{oper} < 200^\circ\text{F}$
$K_a = C_a = K_s = C_s = C_f = C_H = C_R = C_T = C_{xc} = K_x = 1$
$K_m = C_m = 1.6; C_{md} = 1.5; C_p = 2276$

Gear Name	Number of Teeth	Diametral Pitch	Pitch Dia.	Face Width (m)	Pressure Angle	Position
A	17	17	2	0.7	20	(0, -9, 2.2)
B	34	17	4	0.7	20	(0, -6, 2.2)
C	17	17	2	0.7	20	(0, -6, 3)
D	85	17	10	0.7	20	(0, -12, 3)
E	17	17	2	0.7	20	(0, -12, 3.8)
F	85	17	10	0.7	20	(0, -6, 3.8)
G	17	17	2	0.7	20	(0, -6, 4.6)
H	85	17	10	0.7	20	(0, 0, 4.6)
I	17	17	2	0.7	20	(0, 0, 3.8)
J	102	17	12	0.7	20	(-7, 0, 3.8)
K	17	17	2	0.7	20	(-7, 0, 3)
L	34	17	4	0.7	20	(-5.43, 2.56, 3)
M	17	17	2	0.7	20	(-5.43, 2.56, 2.2)
N	85	17	10	0.7	20	(0, 0, 2.2)
O	17	17	2	0.7	20	(0, 0, 1.4)
P	102	17	12	0.7	20	(7, 0, 1.4)
Q	17	17	2	0.7	20	(7, 0, 2.2)
R	34	17	4	0.7	20	(5.43, 2.56, 2.2)
S	17	17	2	0.7	20	(5.43, 2.56, 3)
T	85	17	10	0.7	20	(0, 0, 3)

Once assumptions were made and equations derived, a theoretical solution for the gears in the gear train was found, shown in *Table 15*. Detailed calculations are available in . . . and the actual *Appendix 5*.

### Problem 3:

The third design problem was to create a gear train to help move the head and arm of a Lego-C3PO simultaneously. The inputs given and assumptions made for this calculation can be seen in *Table 16*.

Table 16: Lego Design Problem Assumptions

Lego Design Problem Design Assumptions	
Rotational Speed	N/A
Purpose	Museum display (large in size)
Operation Temperature	Below 200 degree Fahrenheit.
Service Life	5 years with 2000 operating hours per year.
Input Speed	15 rpm
Input Power	0.1 hp
Material	AGMA Grade 2 Steel (250HB)
Safety Factor	1.5

Additionally, there are no specific requirements about the rotational speed and torque provided; so, we assumed the source of power input is able to provide 15 rpm rotational speed and 0.1 horsepower of torque. Other assumptions about the material and machine element properties are similar to the gearbox and clock problem, where it needs to handle one-shift service life of five years with 2000 operations hours each year.

Moreover, like the spur design procedures, bevel gear design process also requires determining the surface and bending fatigue stresses in order to establish the diametral pitch of the

Table 17: LEGO Design Problem Solution

Gear Number	Demential Pitch	Number of Teeth	Pressure Angle	Pitch Cone Angle	Diameter(in)	Face Width (in)	Shaft Diameter(in)
A(input)	6	20	20	60.26	3.3	1.12	1
B(left arm)	6	35	20	29.47	5.83	1.12	1
C(right arm)	6	35	20	29.47	5.83	1.12	1

entire gear train. However, the pitch cone angle must be defined in such a way that the transmission can deliver forces evenly and smoothly. To be more specific, the pitch cone angle is applied to determine the pitch cone length and depends on the gear teeth number. The pitch cone length defined, is equivalent to the face width of spur gear designs. The solutions found for this case are listed in Table 17 and detailed calculations are presented in Appendix 2.

**Shaft Design:**

Once the basic gear properties are defined and surveyed, using the shaft calculations procedure mentioned, various shaft diameters are determined. To design shafts, various assumptions were made. Straight shafts were considered for all cases and calculated for manufacturing convenience. Also, keyways are used to mount gears. Specifically, double end-milled keyways are assumed for shaft-gear connections. Additionally, since the size of the entire gearbox is under one foot, relevantly small, the length of each shaft is assumed to be four inches. Furthermore, we also assume there is no gap between two gears that are mounted on the same shaft, and the gears are mounted at the center point of each shaft. Other assumptions were made to eliminate the operation conditions and machine element properties, such as the operating temperature of the shafts being below 200 °F with a safety factor of 1.5. The shafts designed to mounted gears in Problem 1 are listed in *Table 18*.

The final theoretical and actual results for the shafts are slightly different, because the theoretical shaft diameter represents the minimum diameter that shaft should have.

Table 18: Gearbox Shaft Design Solution

Shaft Number	Shaft Length(in)	Theoretical Diameter(in)	Actual Diameter(in)	Material
<b>A</b>	4	0.44	0.5	SAE 1018
<b>B</b>	4	0.63	1	SAE 1019
<b>C</b>	4	0.61	1	SAE 1020
<b>D</b>	4	0.83	1	SAE 1021
<b>E</b>	4	0.82	1	SAE 1022
<b>F</b>	4	1.07	1.125	SAE 1023

## **GRAPH REPRESENTATION AND GRAMMAR RULES**

To automatically design gear-trains, it is important to devise a computational representation scheme. In this MQP, we are employing graph representation and graph grammar technique to represent various machine elements and the rules to automatically generate gear trains. This is based on the work of Swantner and Campbell [1] and Lin, Yi-shih, et al. [2] here. The first step is to determine the graph representation for various gears, shafts and the overall gear train. The second step is to devise grammar rules based on the representation derived previously. These grammar rules would help in generating various gear-train topologies, which then can be used for further analysis. The graph representation and grammar rules are developed using a software called GraphSynth. This section provides more details on graph representations and grammar rules.

### **Graph representation for gears:**

Consider the spur gear shown below in Figure 22. The various design parameters of a spur gear are shown in Figure 23.



*Figure 22: Spur Gear Sample*

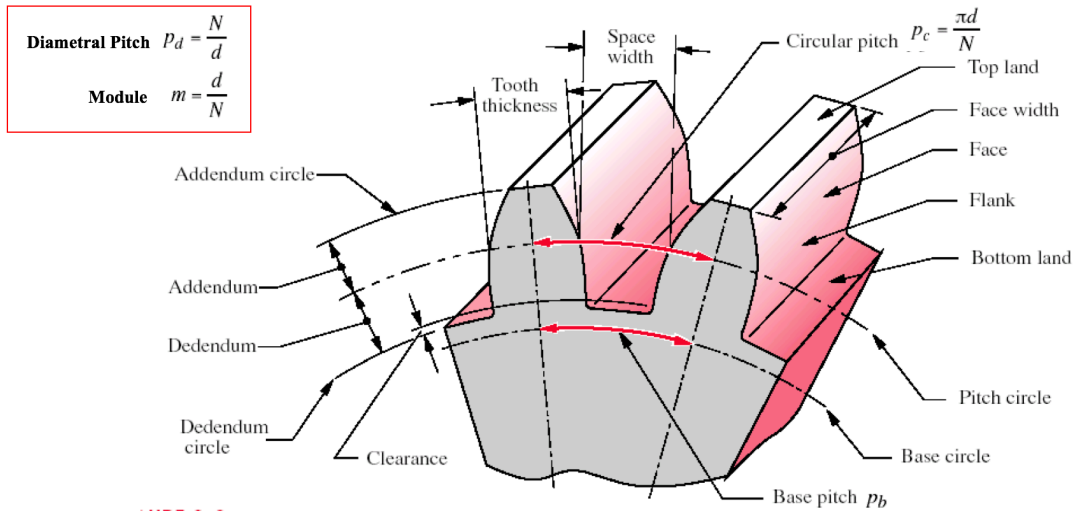


Figure 23: Spur Gear Properties

Some of the dimensions specified in the figure such as addendum, dedendum, top land, base circle, etc. are derived parameters while the parameters such as diametral pitch, pitch diameter, face width, etc. are all calculated or estimated. Our objective while developing the graph representation was that the graph should contain all necessary information that can fully specify all the design parameters of every gear and shaft. That means not only are the dimensional information included but also are all information related to the forces experienced and other assumptions considered during design included in the representation. For spur gears, we considered the following parameters to be required at a minimum: rpm, type (spur), pitch diameter, diametral pitch, position (x,y,z) in 3D space, face width, forces (normal, tangential), pressure angle, material, safety factor and limit cycles. Figure 24 shows the graph representation of a spur gear.

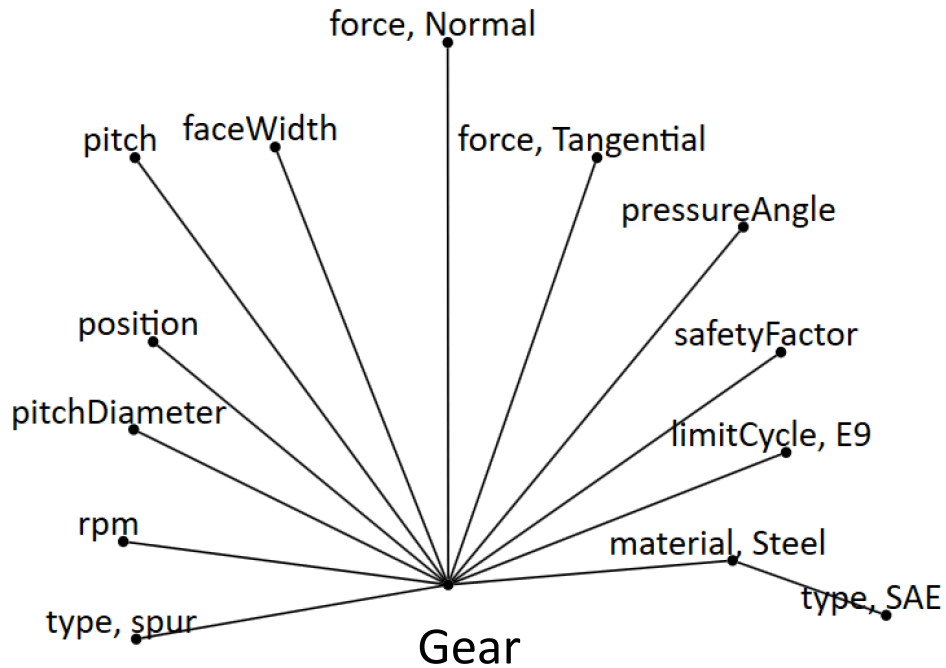


Figure 24: Spur Gear Graph Representation

In this graph representation, the small black dots are referred to as nodes and the line connecting two nodes is referred to as an arc. The text appearing next to the node is referred to as a label. The label is indicative of the type or function of that node. You may notice in this representation that the gear and all its important parameters are represented using nodes and labels. Every gear type has a similar structure whereby the base node indicates that a gear is being represented with all its parameters emanating from that base. This structure will have variations depending on the type of gear. For instance, bevel gears will include additional node to represent the pitch cone angle. GraphSynth program also provides the ability to associate numerical values to each node. That way, quantities such as rpm, pitch diameter, the coordinates for position, etc. can be specified in the graph itself. An example nomenclature of the node labels and corresponding variables associated with one of the gears in Problem 1 (Appendix 1) is presented below.

Table 19: Spur Gear Nomenclature

Sample Node Label	Variables	Variable Type	Actual Variable Meaning	Relationships With Other Nodes
rpm	3000	Element property	gear A has rotation velocity 3000 rpm	N/A
type, spur	N/A	Element property	this is a spur gear	N/A
pressureAngle	20	Element property	gear A has 20 degree pressure angle	N/A
pitch	12	Element property	gear A has diametrical pitch of 12	D=N/P with diameter and number of teeth
position	0,0,0	Element property	(0,0,0) is the position of gear A	N/A
material, steel	250	Element property	gear A is made off of steel which has hardness of 250HB	Material property
type, SAE	1018	Element property	the steel label is SAE1018	Material property
faceWidth	1	Element property	gear A has 1 inch face width	12/D with diameter
safetyFactor	1.5	Element property	the safety factor of gear A is at least 1.5	N/A
limitCycle	100000000	Element property	gear A has a limit cycle of 10^9 without failure	N/A
tangentialForce	41	Element property	tangential force between two gears is 41 lbft	N/A
normalForce	15	Element property	normal force between two gears is 15 lbft	N/A
pitchDiameter	0.83	Element property	pitch diameter is 0.83 inch	N/A

### Representing gears in Mesh:

When two gears are meshing, there are a few common parameters. For instance, the diametral pitch, faceWidth, normal and tangential forces and pressure angle are common to the gears. Therefore, the nodes representing these parameters are made common to the two gears in mesh. Not only that, when two gears are meshing, another node with label “connection” is used to indicate that these two gears are indeed meshing. Figure 25 below represents the graph showing two gears in mesh.

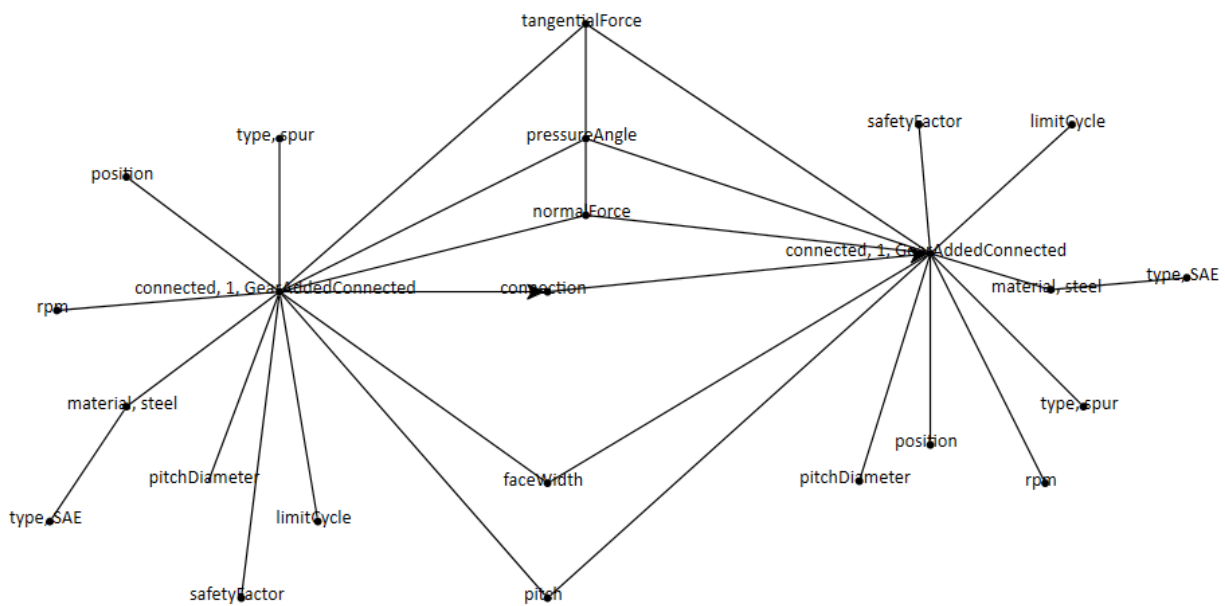


Figure 25: Mashed Gears Graph Representation



You will see that the node with label “Gear” has additional labels “connected” and “GearAddedConnected”. This is used to indicate that that the gear is meshing with another gear and is mounted on a shaft. The directed arc is used to connect the three nodes, one with label “GearAddedConnected”, the other with “connection” and the last with label “GearAddedConnected”. The arrow head is used to signify the direction of power. In order to reduce the complexity of the graph representation and the file size generated by the software, there are various common machine element properties linked to both gears. These nodes not only indicates the shared properties but also presents correlated mashed gear pair.

**Graph representation for shaft:**

Similar to the gear graph representation, the shaft is also represented in a tree from. The surrounding branches carries shaft properties such as type, shaft diameter, shaft length, and its material. All shafts have similar structure; however, this graph representation may vary based on the shaft type. Specifically, a stepped shaft may contain one additional node compare to a straight shaft, which is the step angle and length. Since we assumed all shafts applied to the shaft design solution are all straight shafts, the graph representation of the shafts is shown in Figure 26.

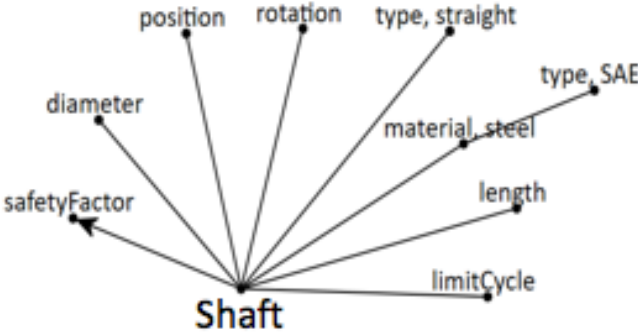


Figure 26: Shaft Graph Representation

The nomenclature of the shaft is presented in Table 20. Similar to the gear graph representation, machine element properties such as position, safetyFactor, limitCycle, and rotation included in the same format. Most of the nodes are shared with gears that mounted onto the shaft, the common properties are connected similar to the gear to gear connection.

Table 20: Shaft Nomenclature

Sample Node Label	Variables	Variable Type	Actual Variable Meaning	Relationships With Other Nodes
shaft	N/A	Machine element	machine element shaft	N/A
diameter	1	Element property	shaft diameter of shaft 1 is 1 inch	Torque and forces
position	0,0,0	Element property	(0,0,0) is the position of shaft 1	N/A
torsionForce	N/A	Element property		Gear pressure angle
rotation, Z	N/A	Element property	shaft 1 rotates about the z axis	N/A
type, straight	N/A	Element property	shaft 1 is a straight shaft	Torque and forces
material, steel	250	Element property	shaft 1 is made off of steel which has hardness of 250HB	Torque and forces
type, SAE	1018	Element property	the steel label is SAE1018	Material property
bendingForce	N/A	Element property		Material property
length	4	Element property	shaft 1 is 4 inches long	N/A
safetyFactor	1.5	Element property	the safety factor of shaft 1 is at least 1.5	N/A
limitCycle	1E+09	Element property	shaft 1 has a limit cycle of 10 <sup>9</sup> without failure	N/A
keyWay, doubleMilled	N/A	Element property	shaft 1 has a double milled keyway	Force
torque	1.5	Element property	1.5 ftlb torque applies on the shaft	N/A

**Representing input information:**

All input information is also provided with a graph representation. Note that the input requirements are inputted by the user. Which means, the structure of the input graph representation can be significantly different. Figure 27 shows the input information specifically for the gearbox design problem. The structure of the input graph is similar to the gears and shafts’, which a center node is surrounded by various branches carries specific information. You may notice there are sub-branches comes out from node “torque” and “speed”.

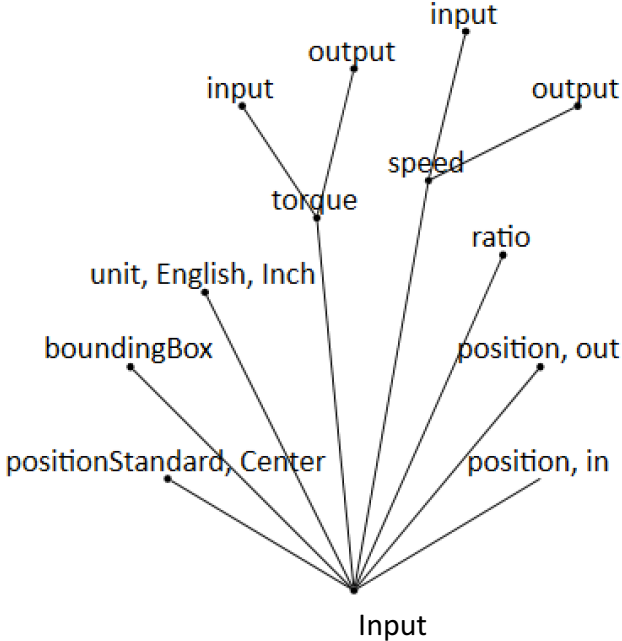


Figure 27: Data Input Graph Representation

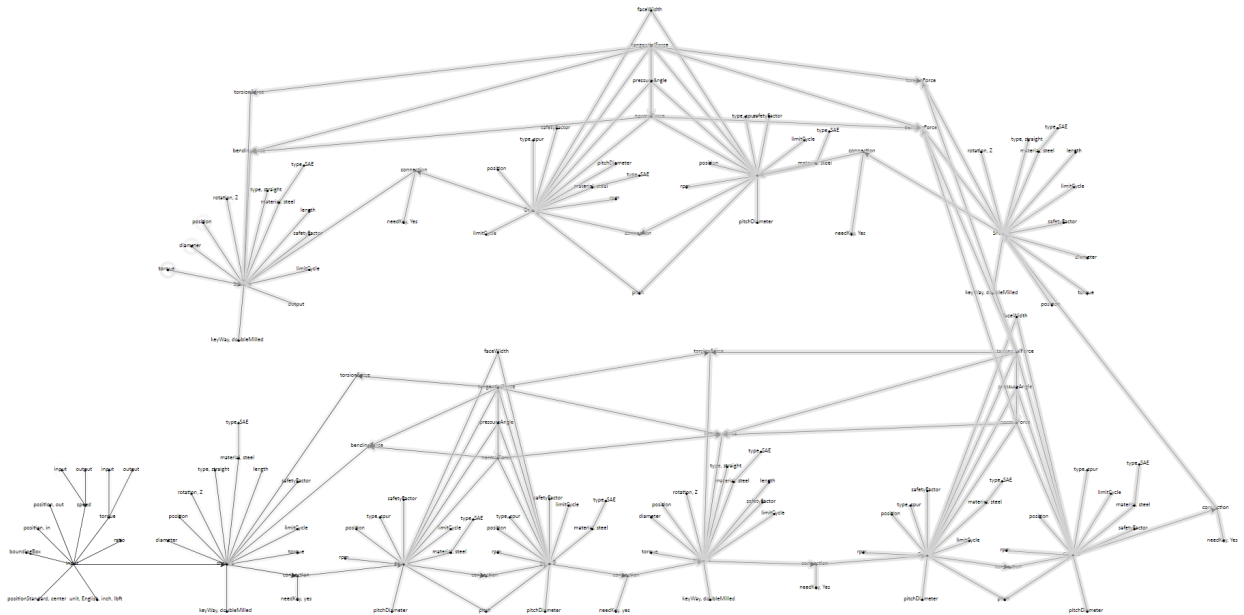
Table 21: Data Input Nomenclature

Sample Node Label	Variables	Variable Type	Actual Variable Meaning	Relationships With Other Nodes
Input	N/A	Requirement input	output	N/A
boundingBox	3,1,4	Requirement input	(3,1,4) is the bounding box	N/A
position, in	0,0,0	Requirement input	(0,0,0) is the input position	N/A
position, out	3,1,4	Requirement input	(3,1,4) is the output position	N/A
ratio	30, 1	Requirement input	30/1 is the input and output speed ratio	ratio=output/input for speed node; ratio=input/output for torque node.
speed	3000	Requirement input	connection node for speed input and output	N/A
torque	1.5	Requirement input	connection node for torque input and output	N/A
input	3000/1.5	Requirement input	3000 rpm is the speed input, 1.5 ftlb is the torque input	N/A
output	100	Requirement input	1000 rpm is the speed output	N/A
positionStandard, center	N/A	Requirement input	machine elements are located at the center	N/A
unit, English, Inch	N/A	Requirement input	english units are used	N/A

In the graph representation, “Input” labelled node is the central node, where all other nodes attach. The nodes with labels “position, in” and “position, out” indicate the input and output positions. The node with “positionStandard, center” indicates the denoted position of gears and shafts lie in the center of them. The node with “boundingBox” label is used to indicate the dimensions of the bounding box if required. The “unit, English, Inch” label illustrates the unit system of the entire process. The “ratio” means the targeted ratio of the gearbox. Notice that it is the speed reduction ratio, so a ratio of 30:1 imply that the output speed is 30 times slower than the input speed. The “torque” and “speed” nodes are further subdivided into input and output. Depending on whatever is specified, the other quantity can be easily calculated in code.

**Graph Representation of a complete gear train:**

Following our explanation of the graph representation for various gears and shaft, the following figure shows the completed representation of the gear train developed for Test Problem 1’s solution.



**Grammar Rules for Design Generation:**

Grammar rules are used to generate different gear train designs (similar to what is shown in Figure (graph for Test Problem 1)) based on the graph representation illustrated above. The starting graph is always the input conditions shown in Figure 28 and is reproduced here.

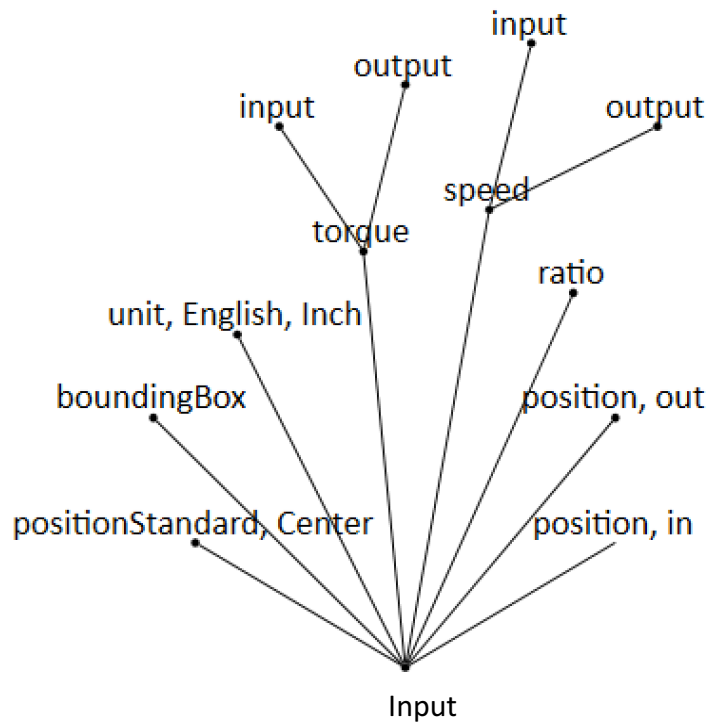


Figure 28: Graph of input conditions using GraphSynth

Two types of grammar rules are created, one that instructs the design process as to what the next steps are and the other generates different gear train topologies. Within the second type of grammar rules, there are two sub-divisions, one set that creates a skeletal structure and the other adds more details.

Rule 1:

The first rule shown in Figure 29, identifies the “Input” node and connects a node with label “numberofstages” . This instructs the embedded code to determine the number of stages in this gear train. The logic for determining the number of gear stages is shown in the Appendix.

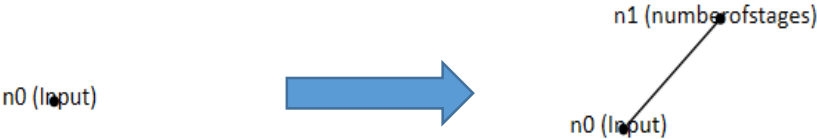


Figure 29: Rule 1, adding the number of stages.

This rule is executed once at the beginning of the search process.

Rules 2 and 3:

Rules 2 and 3, shown in Figure 30 and Figure 31, respectively, add “Shaft” and “Gear” nodes respectively to the graph depending on the number of stages. The assumption here is that a compound gear train arrangement is followed during design. If a two-stage gear-train is required, then we require four gears and three shafts. These rules are successively applied until the required number of gears and shafts have been added to the graph. The nodes are just added to the graph and not connected.



Figure 30: Rule 2, adding "Shaft" node



Figure 31: Rule 3, adding "Gear" node

Rule 4:

Rule 4 shown in Figure 32 identifies the “Input” node, “Shaft” node and the “Gear” node in the graph and connects the “Input” and “Shaft” nodes using an arc. This process also adds a label “connected” to the “Shaft” node. In addition, the rule creates a new node with label “connection” and then connects “Shaft” node and the new node with an arc and the new node with “Gear” node with another arc. The “Gear” node has another label called “connected” appended to it. The three arcs that the rule adds are directed arcs signifying the power flow from input to gear. Now, this rule is applied only once if there is only single input and single output. That way, the “Input” node is not connected to more than one shaft. The use of “connected” label helps in preventing the same node from being recognized by the rule. There is a function in GraphSynth called “Negate Labels”. This allows us to specify a label or a list of labels. Nodes with those labels will be ignored during the recognition process.

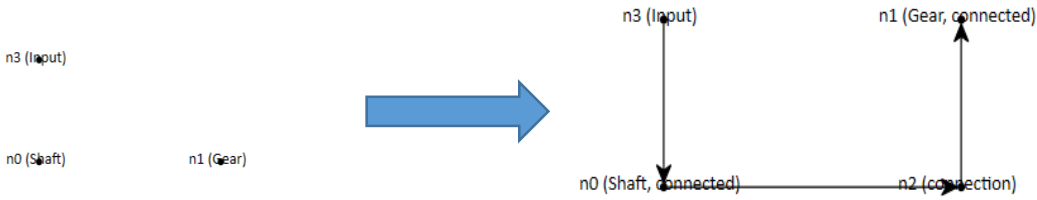


Figure 32: Rule 4, identifying "Input," "Shaft," and "Gear" nodes and connecting the "Input" and "Shaft" using an arc.

Rule 5:

The role of rule 5, shown in Figure 33, is to connect all gears to each other. Note that we have not connected shaft yet, because it would be easier to separate that process in the next rule. Like the “negate label” feature expressed in rule 4, here “negate labels” is continuously applied until all shafts and gears are correctly connected. The connection node in Rule 5 is renamed “connectionMethod”; this node defines the various gear connection method (I.e: keyway, press fitted, etc).

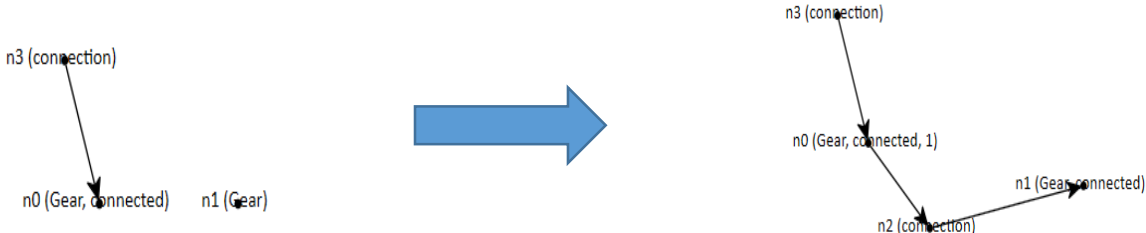


Figure 33: Rule 5, connecting all "Gear" nodes.

Rule 6:

Rule 6, shown in Figure 34, then connects the “Gear” node that has the “connected” label with another “Gear” node that does not have the “connected” label. In the process, the node with the “connected” label on the left hand side will take up placeholder label “1” just to avoid recognizing the same node again by this rule.

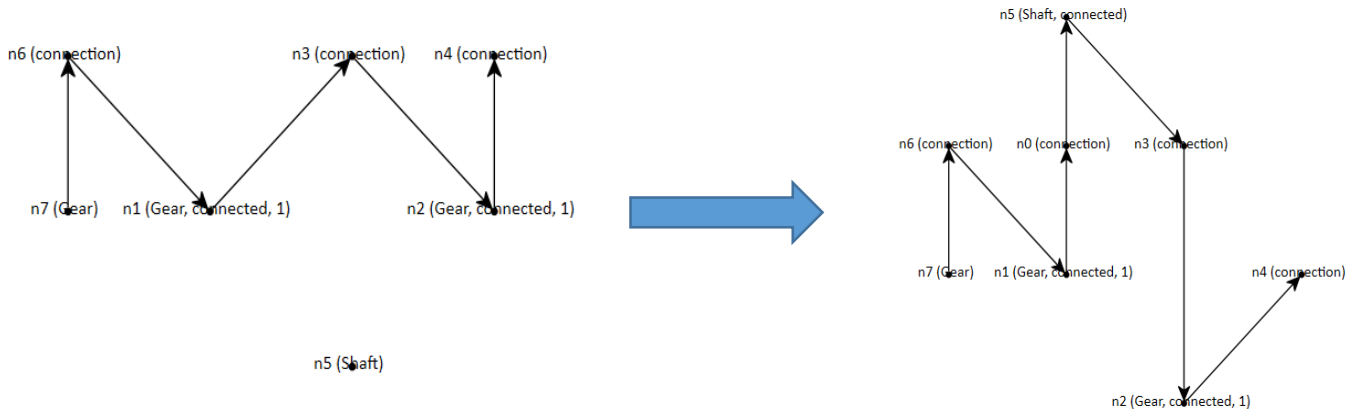


Figure 34: Rule 6, connecting the "Gear" node with "connected" label to "Gear" nodes without the “connected” label.

Moreover, when two gears are connected, a new node with label “connection” is added.

Rule 7 and 8:

What rule 7 and 8 does is this: when there are three gears connected consecutively without a shaft in between, it breaks one gear to gear connection and creates a gear-shaft-gear connection. Then rule 8 connects the shaft and gear and labels “output” on the output shaft. These rules are combined to create the graph to the right of Figure 35.

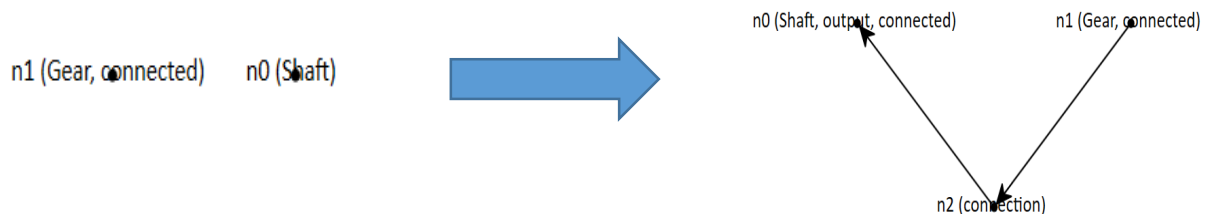


Figure 35: Rule 7 and Rule 8, breaking gear to gear connection and creating gear-shaft-gear connection, and connecting the last shaft and gear to the “Output” node, respectively.

Result of Rule Set 1:

After the first set of rules are applied, a graph identifying all gears and shafts, like the one shown below (Figure 36), is generated. Graphs optimized are based on the number of stages given in rule 1 and thus vary in size.

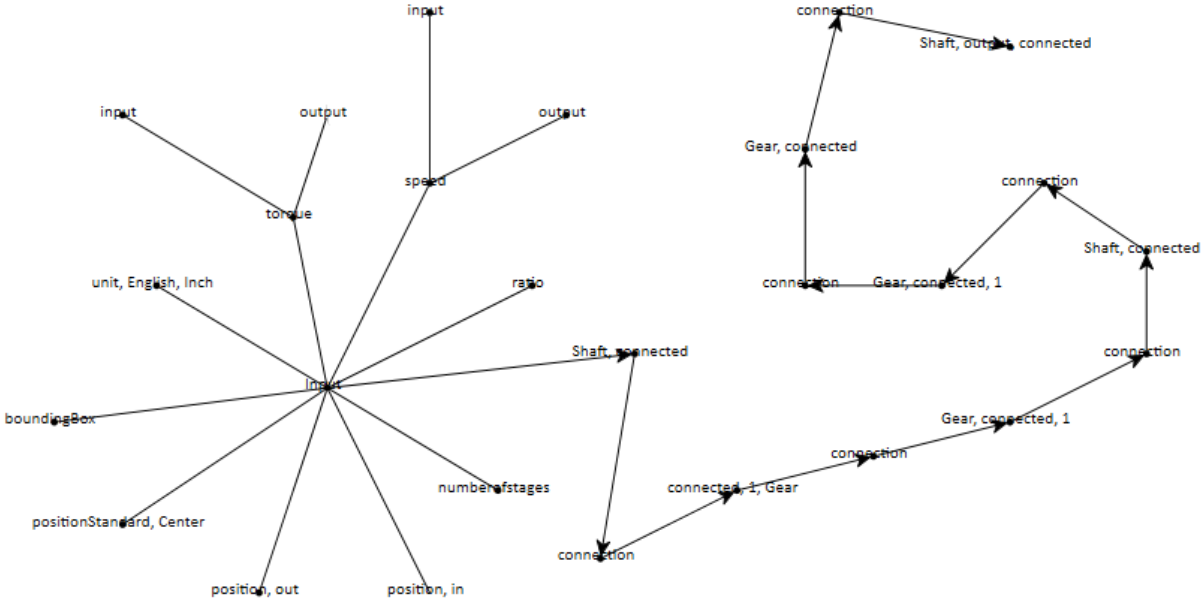


Figure 36: Graph generated after applying rule set 1.

Rule Set 2:

The second rule set is comprised of four rules. Rules defined here are used to add features (type of gear, shaft type, and connection method) to gear and shaft nodes, or identifying the relationships between them and connecting them accordingly. The input of this rule set is the output of the last rule set, so that two rules can be applied smoothly and rationally.

Rule 9:



Rule 9, of the second rule set adds all machine element properties to a single gear, shown in Figure 37. However, the added properties in this rule do not include shared features, which is added in the subsequent rules.

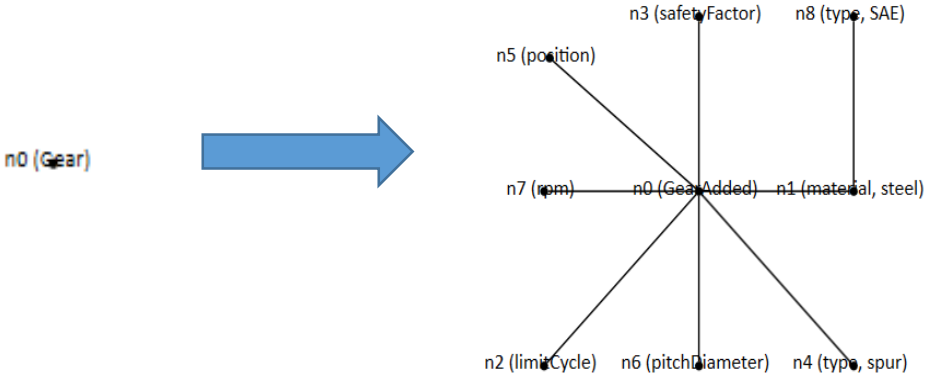


Figure 37: Adding machine element properties to a single "Gear" node.

Properties such as shaft length, gear teeth number, and material, to name a few, added by this rule are applied to a single machine element (shaft or gear), or in this case, a single gear.

Rule 10:

Rule 10, shown in Figure 38, adds features to shafts, and is similar to the first rule. Like rule 4 and 5 expressed above, the “negate label” feature is applied. Notice that the name of shaft/gear nodes of two rules above are changed, because we do not want the rule to be applied by multiple times on one node.

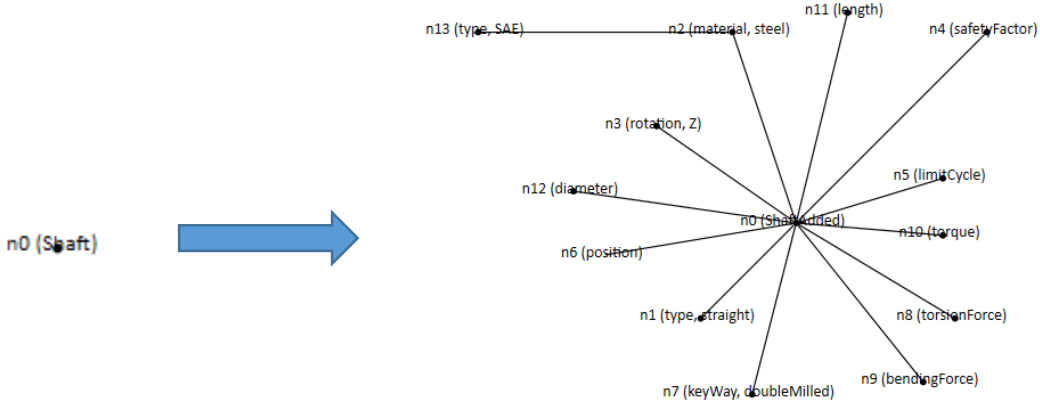


Figure 38: Rule 10, adding properties and features to shafts.

Rule 11:

Rule 11, shown in Figure 39, is used to connect two meshed gears with common features. These features include: gear element properties, such as diametral pitch  $P_d$  and pressure angle  $\varphi$ , otherwise two gears cannot mesh if these properties do not match. Because pressure angle  $\varphi$  shares a relationship between opposing forces  $F_t$  and  $F_n$  (see Gear Analysis), they are also included here and connected accordingly.

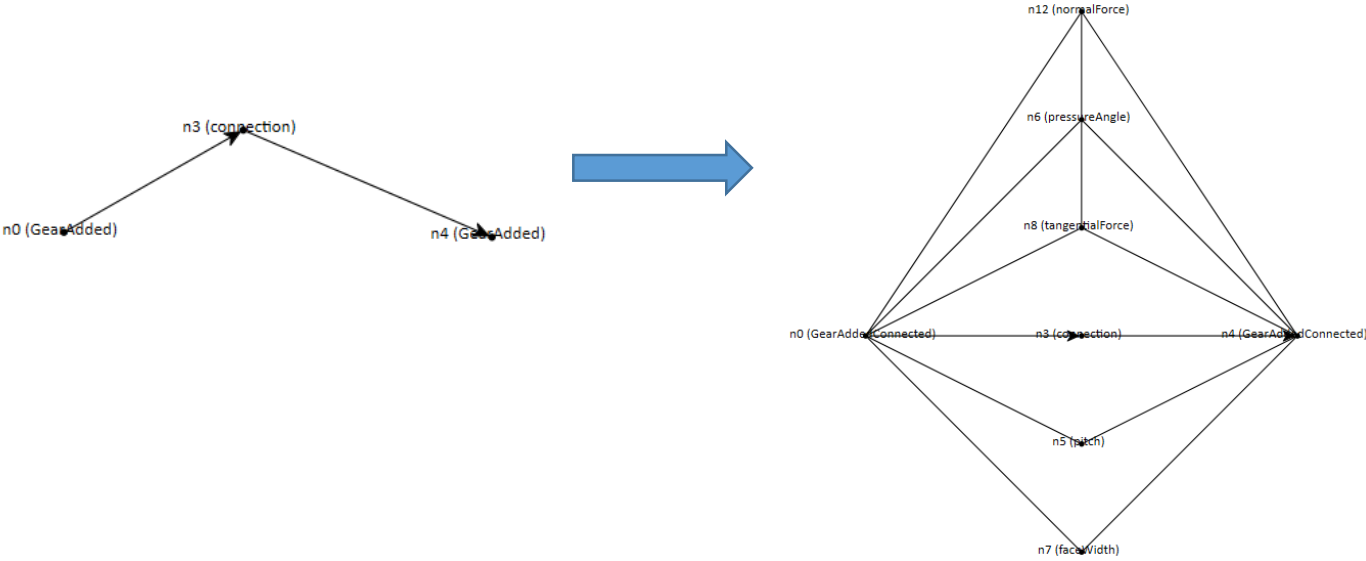


Figure 39: Rule 11, connecting machine elements with common features.

Rule 12:

While noting the relationships between properties of gears, we can also note that the forces identified on the shaft nodes also apply on gears. This is represented in the last rule shown in Figure 40.

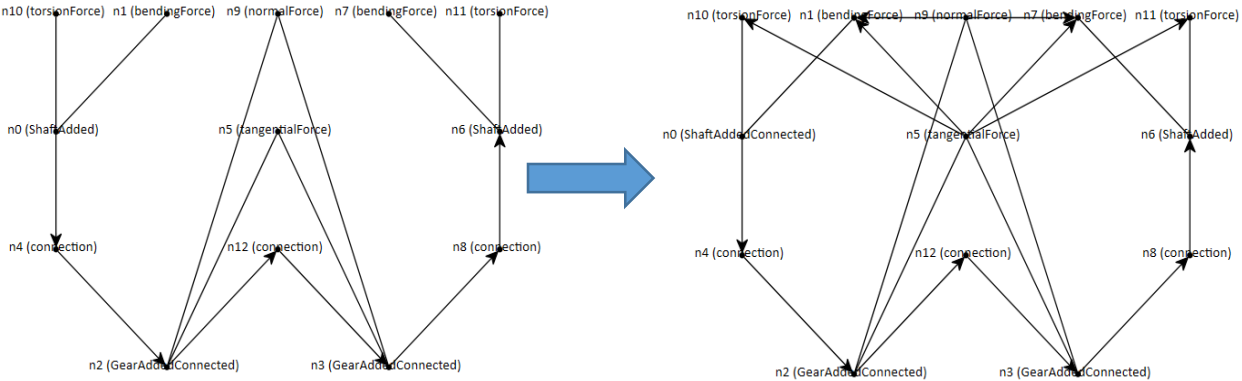


Figure 40: Rule 12, identifying the relationship between forces on shaft and gear.

## Final Graph Representation:

After all rules are successfully applied, the output is a graph representation of the given gear train shown in Figure 41. The graph shown below is for a two-stage gear train, thus the graph varies in size depending on the number of stages given in rule 1. The result expresses all machine elements, their properties, and the relationships between them, as nodes connected by arcs where necessary.

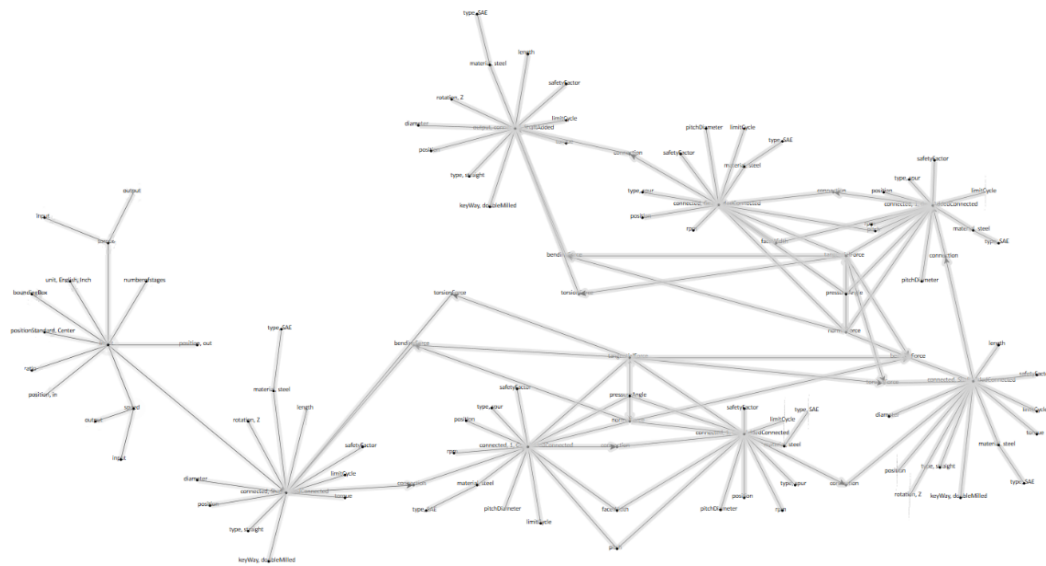


Figure 41: Graph representation of a two-stage gear train.

## **CONCLUSION AND FUTURE WORK**

### **Conclusions:**

This project identified various requirements to correctly automate the design of gear-trains. The necessary equations for gear and shaft design were identified. These equations were used to manually design gear-trains for three test problems of varying complexities. In the process, the team arrived at the process flow that can be used while implementing a program for the same. The team also developed computational representation schemes using graph grammar approach. The developed graph representations are comprehensive and encompasses a lot of gear and shaft data in them when compared to the existing graph representations. The grammar rules developed are able to generate various gear train topologies as well.

### **Future Work:**

In terms of future work, the goal is complete all the activities related to automatically designing gear trains. The various activities that will be required are:

- Integrating the design process flow, graph representation and grammar rules to generate gear-train designs
- Incorporating optimization techniques and online gear and shaft data to parametrize gears and shafts
- Integrate the graph output with the automatic SolidWorks Gear Generator

The ultimate goal of what we are doing is a new software, which can generate gear trains automatically, though we are still in the coding stage. We will come up with the software as soon as we finish all coding parts, but we should make partially functional software for testing as well, so that the software and code can be refined simultaneously when the project progresses. Integrating all codes in a software would definitely cost some efforts in the future, but it could save more if we want the final product fully functional and reach the best result.

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# **APPENDIX**

## **Appendix 1**

### **Gearbox Design Problem Manual Solution**

**Problem Statement:** Design a gear transmission where the input is located at (0,0,0), and the output location is (3,1,4). The torque input is 1.5ft-lb, with a rotational speed of 3000 rpm. The required speed reduction from input to output is 30:1.

**Design Solution:** The design solutions are listed into two tables as follows. The first potation is the general design solution, shown in *Table 22*, which displays all technical information of the design objects. The second table presents the specific design solution of the design objects, shown in *Table 23*. This shows specific machine element properties along with their positions.

*Table 22: Gearbox Design Problem General Solution*

<b>General Design Solution</b>	
Element	1 input motor, 6 gears, and 4 shafts
Gear Ratio	1/4, 1/3, 1/2.5
Gear Teeth Number	10, 16, 40, 48, 64
Pressure Angle	20 degrees
Mounting Method	Presse fitted
Additional Information	The position of each gear will be posted at the end of the solution procedure.

*Table 23: Gearbox Design Problem Specific Solution*

<b>Gear Name</b>	<b>N</b>	<b>Pd</b>	<b>Dp(in)</b>	<b>F(in)</b>	<b>Pressure Angle</b>	<b>Position</b>
A	10	12	0.83	1	20	(0,0,0)
B	40	12	3.33	1	20	(0.5, -2.04, 0)
C	16	10	1.6	1.2	20	(0.5, -2.04, 2.2)
D	48	10	4.8	1.2	20	(3.65, -2.14, 2.2)
E	16	8	2	1.5	20	(3.65, -2.14, 3.7)
F	40	8	5	1.5	20	(2.99, 1, 3.7)

**Assumption:** Position unit: inches.  
 Steady input torque 1.5ft-lb.  
 Steady input rotational speed, 3000 rpm.  
 Uniform driving and driven machine, which application factor  $K_a=1$ .  
 Operation temperature below 200°F.  
 Must handle single service life of 10 years, 2000 operating hours per year.

Failure rate is below 1%.

AGMA standard full depth teeth must be used for each gear.

Material: AGMA Grade 1 Steel (250HB, Quality Index Qv=10)

Safety Factor: Greater or equal than 1.5.

Face width factor of 12.

Assuming the best design solution by using the smallest amount of material.

Ignore the shaft diameter, connection method, and bearing calculations.

### Calculation Process:

1. Since the pitch diameter  $D_p$  is unknown, the relation equation for each gear will contain two unknown variables, which can be listed out as follows.

$$P_d = \frac{N}{D_p}$$

$$P_{d,A} = \frac{10}{D_{p,A}}$$

$$P_{d,B} = \frac{40}{D_{p,B}}$$

$$P_{d,C} = \frac{16}{D_{p,C}}$$

$$P_{d,D} = \frac{48}{D_{p,D}}$$

$$P_{d,E} = \frac{16}{D_{p,E}}$$

$$P_{d,F} = \frac{40}{D_{p,F}}$$

2. The bending geometry factor J for such combination can be found in Table 25 for loading are the highest point of single-tooth contact (HPSTC), which are approximately as follows.

$$J_A = 0.30$$

$$J_B = 0.40$$

$$J_C = 0.31$$

$$J_D = 0.42$$

$$J_E = 0.31$$

$$J_F = 0.40$$

3. The velocity factor,  $K_v$  or  $C_v$ , can be calculated based on the assumption of quality index and the pitch line velocity, which is calculated from the given speed.

$$V_t = \frac{D_p}{2} \omega_p, \text{unit : in}$$

$$V_{t,AB} = \frac{D_{p,A}}{(2)(12)} (3000 \text{rpm})(2\pi) = D_{p,A} 785 \frac{ft}{min}$$

$$V_{t,CD} = \frac{D_{p,C}}{(2)(12)} (750 \text{rpm})(2\pi) = D_{p,C} 196.25 \frac{ft}{min}$$

$$V_{t,EF} = \frac{D_{p,E}}{(2)(12)} (250 \text{rpm})(2\pi) = D_{p,E} 65.42 \frac{ft}{min}$$

$$B = \frac{(12 - Q_v)^{2/3}}{4} = \frac{(12 - 10)^{2/3}}{4} = 0.397$$

$$A = 50 + 56(1 - B) = 50 + (56)(1 - 0.397) = 83.77$$

$$K_v = C_v = \left( \frac{A}{A + \sqrt{V_t}} \right)^B = \left( \frac{83.77}{83.77 + \sqrt{V_t}} \right)^{0.397}$$

$$K_{v,AB} = C_{v,AB} = \left( \frac{83.77}{83.77 + \sqrt{D_{p,A} 785}} \right)^{0.397}$$

$$K_{v,CD} = C_{v,CD} = \left( \frac{83.77}{83.77 + \sqrt{D_{p,C} 196.25}} \right)^{0.397}$$

$$K_{v,EF} = C_{v,EF} = \left( \frac{83.77}{83.77 + \sqrt{D_{p,E} 65.42}} \right)^{0.397}$$

4.

5. The maximum allowable pitch-line velocity must be checked to ensure it is greater than the actual pitch-line velocity. The pitch line velocities calculated in *step 4* include  $D_p$  as a variable. Therefore, the  $D_p$  for each gear must not exceed  $8239/(x)$  values for each gear.

$$V_{t,max} = [A + (Q_v - 3)]^2$$

$$V_{t,max} = [83.77 + (10 - 3)]^2$$

$$V_{t,max} = 8239 \frac{ft}{min}$$

$$V_{t,max} > V_{t,AB}, V_{t,CD}, V_{t,EF}$$

6. Since the assumption is made that the face width factor is 12, the gear face width of each gear pair therefore can be determined as follows. Mention again from the assumptions, the gear face width factor is between 8 to 16, which is based on AGMA standard.



$$F \doteq \frac{12}{P_d}$$

$$F_{AB} = \frac{12}{P_{d,AB}}$$

$$F_{CD} = \frac{12}{P_{d,CD}}$$

$$F_{EF} = \frac{12}{P_{d,EF}}$$

7. An approximation for the face width of each gear is made (that it is greater than 0.78in for each gear) and this will allow for calculating the load distribution factor. The application factor  $K_a$ , which is intended to account for the mass and other operating conditions of the driving and driven machinery and our assumption being that the system is operating under uniform condition, is one. Since there are no specific requirements about gear internal structure, the size factor  $K_s$  or  $C_s$ , and the rim bending factor  $K_b$  are all unity for these gears. Additionally, since the assumption is made that the gearbox is powered by a steady rotation, there will be no noise imposing into the power transmission from the power input. On the other hand, this problem only consider the power transmission drive the gearbox in a single direction. Therefore, there will not be any external overload condition, and the peak torque is equal to the average torque. These two variables both can be used to obtain the gear transmitted load.

$$K_m = C_m \doteq 2$$

$$K_a = C_a \doteq 1$$

$$K_s = C_s \doteq 1$$

$$K_B \doteq 1$$

8. The equations below list the relationship between tangential force at each gear and the pitch diameter.

$$W_t = \frac{T_{\max}}{\frac{1}{2}D_{p,g}}$$

$$W_{t,AB} = \frac{(1.5 \text{ ft} \cdot \text{lb})(4)(12 \text{ in} / \text{ft})}{\frac{1}{2}D_{p,B}} = \frac{144}{D_{p,B}}$$

$$W_{t,CD} = \frac{(1.5 \text{ ft} \cdot \text{lb})(4)(3)(12 \text{ in} / \text{ft})}{\frac{1}{2}D_{p,D}} = \frac{432}{D_{p,D}}$$

$$W_{t,EF} = \frac{(1.5 \text{ ft} \cdot \text{lb})(4)(3)(2.5)(12 \text{ in} / \text{ft})}{\frac{1}{2}D_{p,F}} = \frac{1080}{D_{p,F}}$$

9. After obtaining constants and factors above, the bending stress in pinion and gear now can be computed.

$$\sigma_b = \frac{W_t P_d K_a K_m}{FJ K_v} K_s K_B K_I$$

$$\sigma_{b,A} = \frac{\frac{144}{D_{p,B}} P_{d,AB}}{\frac{12}{P_{d,AB}} 0.30 \left( \frac{83.77}{83.77 + \sqrt{D_{p,A} 785}} \right)^{0.397}} \frac{(1)(2)}{(1)(1)(1)}$$

$$\sigma_{b,B} = \frac{\frac{144}{D_{p,B}} P_{d,AB}}{\frac{12}{P_{d,AB}} 0.40 \left( \frac{83.77}{83.77 + \sqrt{D_{p,B} 785}} \right)^{0.397}} \frac{(1)(2)}{(1)(1)(1)}$$

$$\sigma_{b,C} = \frac{\frac{144}{D_{p,D}} P_{d,CD}}{\frac{12}{P_{d,CD}} 0.31 \left( \frac{83.77}{83.77 + \sqrt{D_{p,C} 196.25}} \right)^{0.397}} \frac{(1)(2)}{(1)(1)(1)}$$

$$\sigma_{b,D} = \frac{\frac{144}{D_{p,D}} P_{d,CD}}{\frac{12}{P_{d,CD}} 0.42 \left( \frac{83.77}{83.77 + \sqrt{D_{p,D} 196.25}} \right)^{0.397}} \frac{(1)(2)}{(1)(1)(1)}$$

$$\sigma_{b,E} = \frac{\frac{144}{D_{p,F}} P_{d,EF}}{\frac{12}{P_{d,EF}} 0.31 \left( \frac{83.77}{83.77 + \sqrt{D_{p,E} 65.42}} \right)^{0.397}} \frac{(1)(2)}{(1)(1)(1)}$$

$$\sigma_{b,F} = \frac{\frac{144}{D_{p,F}} P_{d,EF}}{\frac{12}{P_{d,EF}} 0.40 \left( \frac{83.77}{83.77 + \sqrt{D_{p,F} 65.42}} \right)^{0.397}} \frac{(1)(2)}{(1)(1)(1)}$$

10. AGMA Grade 1 Steel, which is hardened to 250HB, is chosen as the material for all gears. The approximate elastic coefficient between steel to steel,  $C_p$ , is 2300. The specific value of the elastic coefficient can be calculated as follows, while  $\nu_p$  and  $\nu_g$  are both assumed as 0.3 and the elastic modulus is 30E6 psi.

$$C_p = \sqrt{\frac{1}{\pi \left[ \left( \frac{1-\nu_p^2}{E_p} \right) + \left( \frac{1-\nu_g^2}{E_g} \right) \right]}} = \sqrt{\frac{1}{\pi \left[ \left( \frac{1-0.3^2}{30E6} \right) + \left( \frac{1-0.3^2}{30E6} \right) \right]}} = 2290$$

11. In order to ensure the accuracy of the calculation, the external surface geometry factor I is calculated from following equations instead of selecting values from a solution table. On the other hand, since the expression is too large, the external surface geometry factor can be

rounded as 0.1.

$$\rho_p = \sqrt{\left(r_p + \frac{1}{P_d}\right)^2 - (r_p \cos \phi)^2} - \frac{\pi}{P_d} \cos \phi$$

$$\rho_A = \sqrt{\left(\frac{D_{p,A}}{2} + \frac{1}{P_{d,AB}}\right)^2 - \left(\frac{D_{p,A}}{2} \cos 20^\circ\right)^2} - \frac{\pi}{P_{d,AB}} \cos 20^\circ$$

$$\rho_C = \sqrt{\left(\frac{D_{p,C}}{2} + \frac{1}{P_{d,CD}}\right)^2 - \left(\frac{D_{p,C}}{2} \cos 20^\circ\right)^2} - \frac{\pi}{P_{d,CD}} \cos 20^\circ$$

$$\rho_E = \sqrt{\left(\frac{D_{p,E}}{2} + \frac{1}{P_{d,EF}}\right)^2 - \left(\frac{D_{p,E}}{2} \cos 20^\circ\right)^2} - \frac{\pi}{P_{d,EF}} \cos 20^\circ$$

$$\rho_g = C \sin \phi \mp \rho_p = (r_p + r_g) \sin \phi - \rho_p$$

$$\rho_B = \left(\frac{D_{p,A}}{2} + \frac{D_{p,B}}{2}\right) \sin 20^\circ - \sqrt{\left(\frac{D_{p,A}}{2} + \frac{1}{P_{d,AB}}\right)^2 - \left(\frac{D_{p,A}}{2} \cos 20^\circ\right)^2} - \frac{\pi}{P_{d,AB}} \cos 20^\circ$$

$$\rho_D = \left(\frac{D_{p,C}}{2} + \frac{D_{p,D}}{2}\right) \sin 20^\circ - \sqrt{\left(\frac{D_{p,C}}{2} + \frac{1}{P_{d,CD}}\right)^2 - \left(\frac{D_{p,C}}{2} \cos 20^\circ\right)^2} - \frac{\pi}{P_{d,CD}} \cos 20^\circ$$

$$\rho_F = \left(\frac{D_{p,E}}{2} + \frac{D_{p,F}}{2}\right) \sin 20^\circ - \sqrt{\left(\frac{D_{p,E}}{2} + \frac{1}{P_{d,EF}}\right)^2 - \left(\frac{D_{p,E}}{2} \cos 20^\circ\right)^2} - \frac{\pi}{P_{d,EF}} \cos 20^\circ$$

$$I = \frac{\cos \phi}{\left(\frac{1}{\rho_p} \pm \frac{1}{\rho_g}\right) D_p} \doteq 0.10$$

$$I_{AB} = \frac{\cos 20^\circ}{\left( \frac{1}{\sqrt{\left(\frac{D_{p,A}}{2} + \frac{1}{P_{d,AB}}\right)^2 - \left(\frac{D_{p,A}}{2} \cos 20^\circ\right)^2} - \frac{\pi}{P_{d,AB}} \cos 20^\circ} + \frac{1}{\left(\frac{D_{p,A}}{2} + \frac{D_{p,B}}{2}\right) \sin 20^\circ - \sqrt{\left(\frac{D_{p,A}}{2} + \frac{1}{P_{d,AB}}\right)^2 - \left(\frac{D_{p,A}}{2} \cos 20^\circ\right)^2} - \frac{\pi}{P_{d,AB}} \cos 20^\circ} \right) D_{p,A}}$$

$$I_{CD} = \frac{\cos 20^\circ}{\left( \frac{1}{\sqrt{\left(\frac{D_{p,C}}{2} + \frac{1}{P_{d,CD}}\right)^2 - \left(\frac{D_{p,C}}{2} \cos 20^\circ\right)^2} - \frac{\pi}{P_{d,CD}} \cos 20^\circ} + \frac{1}{\left(\frac{D_{p,C}}{2} + \frac{D_{p,D}}{2}\right) \sin 20^\circ - \sqrt{\left(\frac{D_{p,C}}{2} + \frac{1}{P_{d,CD}}\right)^2 - \left(\frac{D_{p,C}}{2} \cos 20^\circ\right)^2} - \frac{\pi}{P_{d,CD}} \cos 20^\circ} \right) D_{p,C}}$$

$$I_{EF} = \frac{\cos 20^\circ}{\left( \frac{1}{\sqrt{\left(\frac{D_{p,E}}{2} + \frac{1}{P_{d,EF}}\right)^2 - \left(\frac{D_{p,E}}{2} \cos 20^\circ\right)^2} - \frac{\pi}{P_{d,EF}} \cos 20^\circ} + \frac{1}{\left(\frac{D_{p,E}}{2} + \frac{D_{p,F}}{2}\right) \sin 20^\circ - \sqrt{\left(\frac{D_{p,E}}{2} + \frac{1}{P_{d,EF}}\right)^2 - \left(\frac{D_{p,E}}{2} \cos 20^\circ\right)^2} - \frac{\pi}{P_{d,EF}} \cos 20^\circ} \right) D_{p,E}}$$

12. The surface stress in the pinion-gear mesh is now computed using the following expressions:

$$\sigma_{c,p8} = C_p \sqrt{\frac{W_t C_a C_m}{FID_p C_v} C_s C_f}$$

$$\sigma_{c,AB} = 2290 \sqrt{\frac{\left(\frac{144}{D_{p,B}}\right) (1)(2)}{\left(\frac{12}{P_{d,AB}}\right) (0.1) D_{p,A} \left(\frac{83.77}{83.77 + \sqrt{D_{p,A} 785}}\right)^{0.397}} (1)(1)} = 2290 \sqrt{\frac{0.6 P_{d,AB}^3}{\left(\frac{83.77}{83.77 + \sqrt{\frac{7850}{P_{d,AB}}}}\right)^{0.397}}}$$

$$\sigma_{c,CD} = 2290 \sqrt{\frac{\left(\frac{432}{D_{p,D}}\right) (1)(2)}{\left(\frac{12}{P_{d,CD}}\right) (0.1) D_{p,C} \left(\frac{83.77}{83.77 + \sqrt{D_{p,C} 196.25}}\right)^{0.397}} (1)(1)} = 2290 \sqrt{\frac{0.94 P_{d,CD}^3}{\left(\frac{83.77}{83.77 + \sqrt{\frac{3140}{P_{d,CD}}}}\right)^{0.397}}}$$

$$\sigma_{c,EF} = 2290 \sqrt{\frac{\left(\frac{1080}{D_{p,F}}\right) (1)(2)}{\left(\frac{12}{P_{d,EF}}\right) (0.1) D_{p,E} \left(\frac{83.77}{83.77 + \sqrt{D_{p,E} 65.42}}\right)^{0.397}} (1)(1)} = 2290 \sqrt{\frac{1.41 P_{d,EF}^3}{\left(\frac{83.77}{83.77 + \sqrt{\frac{1047}{P_{d,EF}}}}\right)^{0.397}}}$$

13. The uncorrected bending fatigue strength can be calculated based on the material assumption and Table 26.

$$S_{fb'} = -274 + 167HB - 0.152HB^2$$

$$S_{fb'} = -274 + 167(250) - 0.152(250)^2$$

$$S_{fb'} = 31967 \text{ psi}$$

14. The life cycles can be determined by the rational made assumptions mentioned below the problem statement.

$$N_{AB} = (3000 \text{ rpm}) \left(\frac{60 \text{ min}}{\text{hr}}\right) \left(\frac{2000 \text{ hr}}{\text{year}}\right) (10 \text{ years})(1 \text{ shift})$$

$$N_{AB} = 3.6E9 \text{ cycles}$$

$$N_{CD} = (750 \text{ rpm}) \left(\frac{60 \text{ min}}{\text{hr}}\right) \left(\frac{2000 \text{ hr}}{\text{year}}\right) (10 \text{ years})(1 \text{ shift})$$

$$N_{CD} = 9.0E8 \text{ cycles}$$

$$N_{EF} = (250 \text{ rpm}) \left(\frac{60 \text{ min}}{\text{hr}}\right) \left(\frac{2000 \text{ hr}}{\text{year}}\right) (10 \text{ years})(1 \text{ shift})$$

$$N_{EF} = 3.0E8 \text{ cycles}$$

15. The life factor  $K_L$  can therefore be calculated as follows.

$$K_L = 1.3558N^{-0.0178}$$

$$K_{L,AB} = 1.3558(3.6E9)^{-0.0178} = 0.9164$$

$$K_{L,CD} = 1.3558(9.0E8)^{-0.0178} = 0.9393$$

$$K_{L,EF} = 1.3558(3.0E8)^{-0.0178} = 0.9579$$

16. The operating temperature is assumed to be below 200°F; therefore, the temperature factor  $K_T$  is unity. Also, a reliability level of 99% is assumed, which makes the gear reliability  $K_R$  also unity. The corrected bending fatigue strength is then calculated as follows.

$$S_{fb} = \frac{K_L}{K_T K_R} S_{fb'}$$

$$S_{fb,AB} = \frac{0.9164}{(1)(1)} 31976 = 29303 \text{ psi}$$

$$S_{fb,CD} = \frac{0.9393}{(1)(1)} 31976 = 30035 \text{ psi}$$

$$S_{fb,EF} = \frac{0.9579}{(1)(1)} 31976 = 30629 \text{ psi}$$

17. Same, the uncorrected surface fatigue strength can be obtained from the lower curve of Table 27.

$$S_{fc'} = 26000 + 327 HB$$

$$S_{fc'} = 26000 + 327(250)$$

$$S_{fc'} = 107750 \text{ psi}$$

18. The life factor for surface fatigue strength  $C_L$  can be calculated as follows. Also, the temperature factor and reliability factor for surface fatigue strength are also equal to 1. Since the gears and pinions have the same material hardness, therefore the hardness ratio factor  $C_H$  equals to 1.

$$C_L = 1.448N^{-0.023}$$

$$C_{L,AB} = 1.448(3.6E9)^{-0.023}$$

$$C_{L,AB} = 0.8729$$

$$C_{L,CD} = 1.448(9.0E8)^{-0.023}$$

$$C_{L,CD} = 0.9012$$

$$C_{L,EF} = 1.448(3.0E8)^{-0.023}$$

$$C_{L,EF} = 0.9243$$

$$C_T = K_T = 1$$

$$C_R = K_R = 1$$

$$C_H = 1$$

19. The corrected surface fatigue strength is then calculated as follows.

$$S_{fc} = \frac{C_L C_H}{C_T C_R} S_{fc}'$$

$$S_{fc,AB} = \frac{(0.8729)(1)}{(1)(1)} 107750 = 94055 \text{ psi}$$

$$S_{fc,CD} = \frac{(0.9012)(1)}{(1)(1)} 107750 = 97104 \text{ psi}$$

$$S_{fc,EF} = \frac{(0.9243)(1)}{(1)(1)} 107750 = 99593 \text{ psi}$$

20. Our requirement is that the , the equations for each set of gear pair can be obtained and arranged as follows.

$$N_{b,pinion} = \frac{S_{fb}}{\sigma_{b,pinion}}$$

$$1.5 = \frac{29303}{2.0P_{d,AB}^3} = \frac{30035}{1.61P_{d,CD}^3} = \frac{30629}{1.94P_{d,EF}^3}$$

$$\left( \frac{83.77}{83.77 + \sqrt{\frac{7850}{P_{d,AB}}}} \right)^{0.397} \quad \left( \frac{83.77}{83.77 + \sqrt{\frac{3140}{P_{d,CD}}}} \right)^{0.397} \quad \left( \frac{83.77}{83.77 + \sqrt{\frac{1047}{P_{d,EF}}}} \right)^{0.397}$$

$$N_{b,gear} = \frac{S_{fb}}{\sigma_{b,gear}}$$

$$1.5 = \frac{29303}{1.5P_{d,AB}^3} = \frac{30035}{1.19P_{d,CD}^3} = \frac{30629}{2.0P_{d,EF}^3}$$

$$\left( \frac{83.77}{83.77 + \sqrt{\frac{31400}{P_{d,AB}}}} \right)^{0.397} \quad \left( \frac{83.77}{83.77 + \sqrt{\frac{9420}{P_{d,CD}}}} \right)^{0.397} \quad \left( \frac{83.77}{83.77 + \sqrt{\frac{2617}{P_{d,EF}}}} \right)^{0.397}$$

$$N_{c,pinion\&gear} = \left( \frac{S_{fc}}{\sigma_{c,pinion}} \right)^2$$

$$1.5 = \frac{94055}{2290 \left( \frac{0.6P_{d,AB}}{\left( \frac{83.77}{83.77 + \sqrt{\frac{7850}{P_{d,AB}}}} \right)^{0.397}} \right)^2} = \frac{97104}{2290 \left( \frac{0.94P_{d,CD}}{\left( \frac{83.77}{83.77 + \sqrt{\frac{3140}{P_{d,CD}}}} \right)^{0.397}} \right)^2} = \frac{99593}{2290 \left( \frac{1.41P_{d,EF}}{\left( \frac{83.77}{83.77 + \sqrt{\frac{1047}{P_{d,EF}}}} \right)^{0.397}} \right)^2}$$

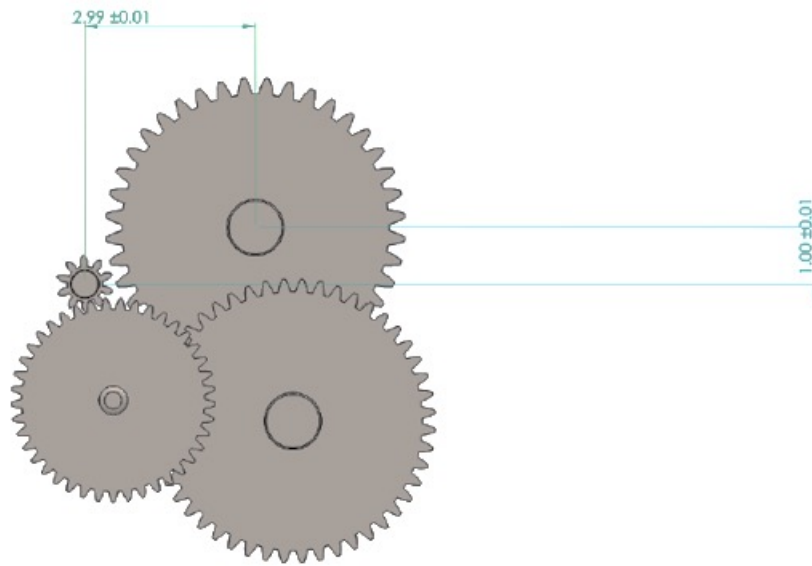
21. By solving the equations above, and compare and contrast the size of gears, the final result is presented as *Table 24*. To be more specific, the

Table 24: Gearbox Final solution Comparison

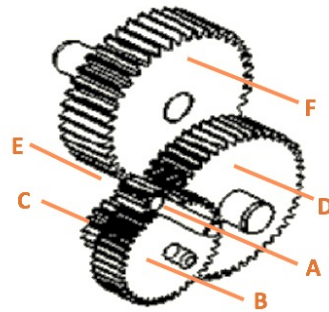
<b>Pinion Pd</b>	Gear A: 20.94	Gear C: 22.75	Gear E: 25.85
<b>Gear Pd</b>	Gear B: 22.25	Gear D: 24.90	Gear F: 21.26
<b>Gear Pinion Pd</b>	AB: 12	CD: 10.5	EF: 9.5

22. The final solution with specific gear properties are listed in the *Table 23*. The 3D modeling of

the final solution is presented in *Figure 42* and *Figure 43*.



*Figure 42: Gearbox Design Solution Graph 1*



*Figure 43: Gearbox Design Solution Graph 2*

*Table 25: I and J Factors based on the number of teeth on gear and pinion [6]*



**I AND J FACTORS FOR:<sup>1</sup>**

20.0 DEG. PRESSURE ANGLE  
 0.0 DEG. HELIX ANGLE  
 0.250 TOOL EDGE RADIUS  
 EQUAL ADDENDUM ( $x_1 = x_2 = 0$ )

2.250 WHOLE DEPTH FACTOR  
 0.024 TOOTH THINNING FOR BACKLASH  
 LOADED AT HIGHEST POINT OF SINGLE TOOTH CONTACT

GEAR TEETH	PINION TEETH															
	12		14		17		21		26		35		55		135	
	P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12 I																
J	U	U														
14 I																
J	U	U	U	U												
17 I																
J	U	U	U	U	U	U										
21 I																
J	U	U	U	U	U	U	0.33	0.33								
26 I																
J	U	U	U	U	U	U	0.33	0.35	0.35	0.35						
35 I																
J	U	U	U	U	U	U	0.34	0.37	0.36	0.38	0.39	0.39				
55 I																
J	U	U	U	U	U	U	0.34	0.40	0.37	0.41	0.40	0.42	0.43	0.43		
135 I																
J	U	U	U	U	U	U	0.35	0.43	0.38	0.44	0.41	0.45	0.45	0.47	0.49	0.49

Ta

<sup>1</sup> The letter "U" indicates a gear tooth combination which produces an undercut tooth form in one or both components and should be avoided. See Section 7 and Fig 7-1.

Table 26: Table relating the Material Strength and Hardness

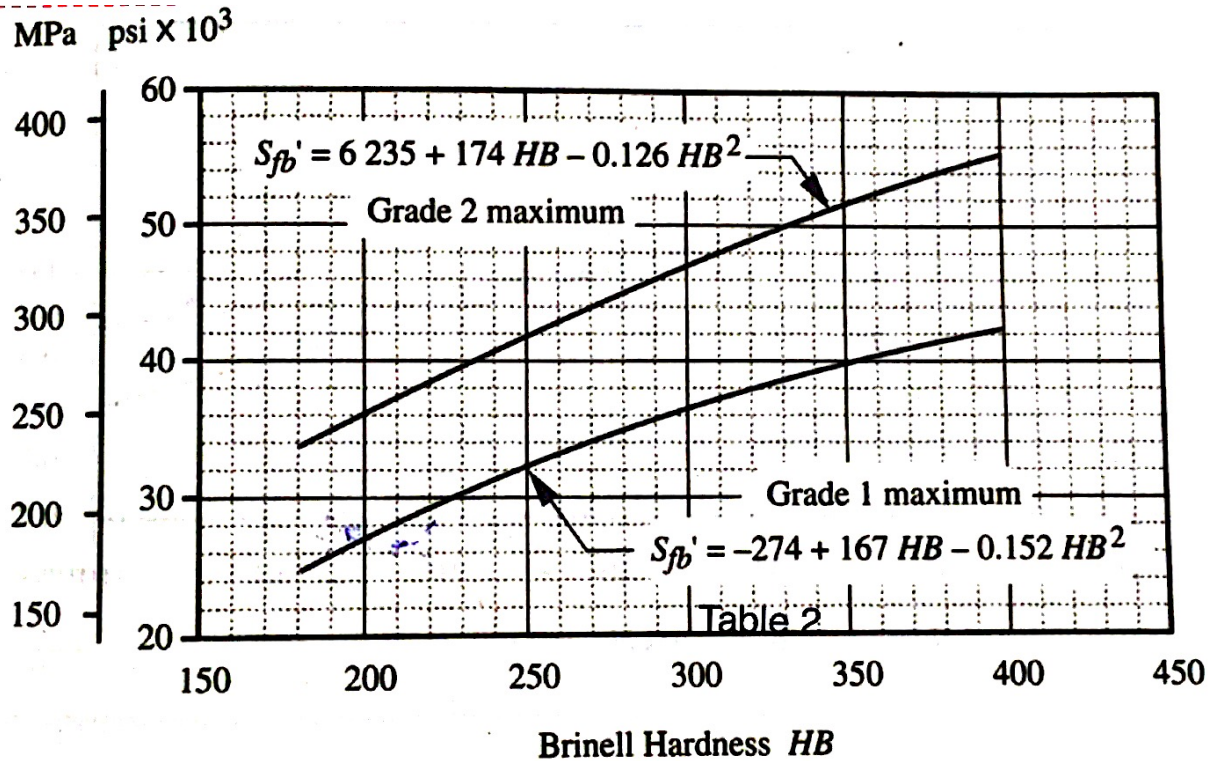
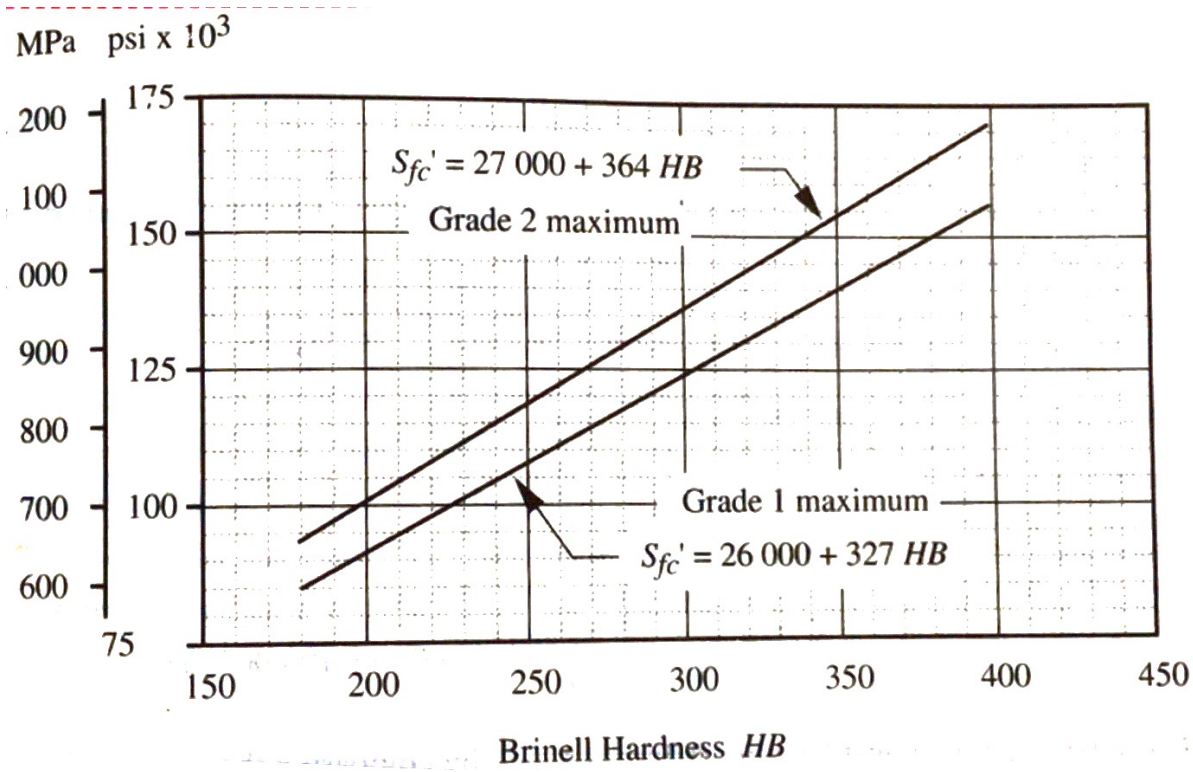


Table 27: Brinell Hardness Table 600-1200 MPa



## Appendix 2

### LEGO C3PO Design Problem Manual Solution

**Problem Statement:** Design a gear transmission for LEGO C3PO, which allows the C3PO to rotate its head and arm at the same time.

**Design Solution:** The design solutions are listed into two tables as follows. The first potation is the general design solution, shown in Table 28 which displays all technical information of the design objects. The second table presents the specific design solution of the design objects, shown in Table 29. This shows specific machine element properties along with their positions.

Table 28: LEGO C3PO Design Problem General Solution

General Design Solution	
Element	1 input motor, 3 bevel gears, and 3 shafts
Pressure Angle	20 degrees
Mounting Method	Presse fitted
Additional Information	The position of each gear will be posted at the end of the solution procedure.

Table 29: LEGO C3PO Design Problem Actual Solution

Gear Number	Demential Pitch	Number of Teeth	Pressure Angle	Pitch Cone Angle	Diameter(in)	Face Width (in)	Shaft Diameter(in)
A(input)	6	20	20	60.26	3.3	1.12	1
B(left arm)	6	35	20	29.47	5.83	1.12	1
C(right arm)	6	35	20	29.47	5.83	1.12	1

#### Gear Positions

Gear A	(0,0,0)
Gear B	(-1.65, 0, 2.92)
Gear C	(1.65, 0, 2.92)

**Assumption:**

- No specific requirements about the rotational speed needed.
- This LEGO C3PO is large, which is for museum display.
- Only focus on input and output direction changes.
- Operation temperature below 200°F.
- Must handle one-shift service life of 5 years, 2000 operating hours per year.
- Failure rate is below 1%.
- 15 rpm and 0.1hp from the power input.
- Material: AGMA Grade 2 Steal (250HB).
- Safety Factor: Greater or equal than 1.5.
- Other factors assumptions:

$$K_a = C_a = K_s = C_s = C_f = C_H = C_R = C_T = 1$$

$$K_m = C_m = 1.6$$

$$K_v = C_v = 0.652$$

$$C_L = 0.890$$

$$C_p = 2276$$

$$C_{xc} = K_x = 1$$

$$C_b = 0.634$$

$$C_{md} = 1.5$$

### Calculation Process:

1. The uncorrected bending fatigue stress can be made from the curves of Table 4. Since AGMA Grade 2 Steel is chosen, which is hardened to 250HB. The uncorrected fatigue strength in bending is found from the upper curve in the Table. The result gives 41860psi.

$$S_{fb'} = 6235 + 174HB - 0.126HB^2$$

$$S_{fb'} = 6235 + 174(250) - 0.126(250)^2$$

$$S_{fb'} = 41860 \text{ psi}$$

2. The life factor  $K_L$  is found from the equation in Table 7 based on the required number of cycles in the life of the gears. The life factor must be determined through the number of cycles,  $N$ , for the required life of 5 years. The result gives  $9E7$  cycles, and life factor of 0.9786.

$$N = (15 \text{ rpm}) \left( \frac{60 \text{ min}}{\text{hr}} \right) \left( \frac{2000 \text{ hr}}{\text{year}} \right) (5 \text{ year}) (1 \text{ shift})$$

$$N = 9.0 \times 10^7 \text{ cycles} = 9E7 \text{ cycles}$$

$$K_L = 1.3558N^{-0.0178}$$

$$K_L = 1.3558(9.0 \times 10^7)^{-0.0178}$$

$$K_L = 0.9786$$

3. Since the operation by assumption is below 200°F, the temperature factor,  $K_T$ , is 1. Also, the failure rate is below 1%, which means the reliability level of 99%. Therefore, the reliability factor,  $K_R$ , is also 1.

$$K_T = 1$$

$$K_R = 1$$

4. The corrected bending fatigue stress is then equal to 40964psi.

$$S_{fb} = \frac{K_L}{K_T K_R} S_{fb'}$$

$$S_{fb} = \frac{0.9786}{(1)(1)} 41860$$

$$S_{fb} \doteq 40964 \text{ psi}$$

5. The uncorrected surface fatigue stress can be made from the curves of Table 8. Since AGMA Grade 2 Steel is chosen. The uncorrected fatigue strength in surface is found from the upper curve in the Table. The result gives the uncorrected surface fatigue stress is 118000psi.

$$S_{fc'} = 27000 + 364 HB$$

$$S_{fc'} = 27000 + 364(250)$$

$$S_{fc'} = 118000 \text{ psi}$$

6. The uncorrected surface fatigue stress needs to be corrected by several factors. The life factor  $C_L$ , is found based on the life cycles of the gears. The life factor is 0.9507. The same temperature factor  $C_T$ , and reliability factor  $C_R$  values can be used which all equal to 1. Also, since the assumption shows the entire system uses the same material, the hardness ratio factor  $C_H$  equals to 1. Therefore, the corrected surface fatigue stress is 112182psi.

$$C_L = 1.4488 N^{-0.023}$$

$$C_L = 1.4488 (9.0 \times 10^7)^{-0.023}$$

$$C_L = 0.9507$$

$$C_T = K_T = 1$$

$$C_R = K_R = 1$$

$$C_H = 1 + A(m_G - 1) = 1$$

$$S_{fc} = \frac{C_L C_H}{C_T C_R} S_{fc'}$$

$$S_{fc} = \frac{(0.9507)(1)}{(1)(1)} 118000$$

$$S_{fc} \doteq 112182 \text{ psi}$$

7. Since the above vales have been determined, the calculation therefore can be move onto the torque transmission. To be more specific, this step calculates the pinion torque from the given power and speed, which equals to 420.38 lb-in.

$$T_p = \frac{P}{\omega_p}$$

$$T_p = \frac{(0.1hp) \left( 6600 \frac{\text{in} \cdot \text{lb}}{\text{sec}} / hp \right)}{(15rpm) \left( 2\pi / 60 \right) \frac{\text{rad}}{\text{sec}} / rpm}$$

$$T_p = 420.38 \text{ lb} \cdot \text{in}$$

8. The pitch diameter of pinions and gears can be found as follows. Since the diametral pitch is unknown, the solution contains an unknown value  $P_d$  and  $D_p$ .

$$D_p = \frac{N_p}{P_d}$$

$$D_g = \frac{N_g}{P_d}$$

$$D_p = \frac{20}{P_d}$$

$$D_g = \frac{35}{P_d}$$

9. The pitch cone angles can be found as follows.

$$\alpha_g = \tan^{-1} \left( \frac{N_g}{N_p} \right)$$

$$\alpha_g = \tan^{-1} \left( \frac{35}{20} \right)$$

$$\alpha_g \doteq 60.26^\circ$$

$$\alpha_p = 90^\circ - \alpha_g$$

$$\alpha_p = 90^\circ - 60.26^\circ$$

$$\alpha_p = 29.74^\circ$$

10. The maximum recommended pitch cone length L need to be determined.

$$L = \frac{D_p}{2 \sin \alpha_p}$$

$$L = \frac{\left(\frac{20}{P_d}\right)}{2 \sin \alpha_p}$$

$$L = \frac{10}{P_d \sin \alpha_p}$$

11. This value is used to calculate the face width F. Also, the bending geometry factors for pinion and gear can be found in Table 7.

$$F = \frac{L}{3}$$

$$F = \frac{\frac{10}{P_d \sin \alpha_p}}{3}$$

$$F = \frac{10}{3P_d \sin \alpha_p}$$

$$F = \frac{10}{3P_d \sin 29.74^\circ}$$

$$F = \frac{6.72}{P_d}$$

$$J_p = 0.237$$

12. The bending stress in the pinion can be determined.

$$\sigma_{b, \text{pinion}} = \frac{2T_p P_d K_a K_m K_s}{d F J K_v K_x}$$

$$\sigma_{b, \text{pinion}} = \frac{(2)(420.38)}{\left(\frac{20}{P_d}\right)} \frac{P_d}{\left(\frac{6.72}{P_d}\right)(0.201)} \frac{(1)(1.6)(1)}{(0.652)(1)}$$

$$\sigma_{b, \text{pinion}} = 76.37 p_d^3$$

13. Also, the bending stress in the gear can be determined.

$$\sigma_{b,gear} = \frac{2T_p P_d K_a K_m K_s}{d FJ K_v K_x}$$

$$\sigma_{b,gear} = \frac{(2)(420.38) P_d (1)(1.6)(1)}{\left(\frac{20}{P_d}\right) \left(\frac{6.72}{P_d}\right)(0.237)(0.652)(1)}$$

$$\sigma_{b,gear} = 64.77 P_d^3$$

14. The surface geometry factor of pinion and gear, I, can be found in Table 9. Therefore, the design pinion torque can be determined.

$$T_D = \frac{F}{2} \frac{IC_v}{C_s C_{md} C_f C_a C_{xc}} \left( \frac{S_{fc} d}{C_p C_b} \frac{0.774 C_H}{C_T C_R} \right)^2$$

$$T_D = \frac{\left(\frac{6.72}{P_d}\right)}{2} \frac{(0.076)(0.652)}{(1)(1.5)(1)(1)(1)} \left( \frac{(112182)(2.5)}{(2276)(0.634)} \frac{(0.774)(1)}{(1)(1)} \right)^2$$

$$T_D = \frac{2511.85}{P_d}$$

15. By assuming the operating pinion torque  $T_p$ , is smaller than the design pinion torque  $T_D$ , the z value equals to 0.667. Then, the surface stress can be determined.

$$\sigma_c = C_p C_b \sqrt{\frac{2T_D}{FId^2} \left(\frac{T_p}{T_D}\right)^z \frac{C_a C_m}{C_v} C_s C_f C_{xc}}$$

$$\sigma_c = (2276)(0.634) \sqrt{\frac{(2) \left(\frac{2511.85}{P_d}\right)}{\left(\frac{6.72}{P_d}\right) (0.076)(2.5)^2} \left(\frac{420.38}{\left(\frac{2511.85}{P_d}\right)}\right)^{0.667} \frac{(1)(1.6)}{0.652} (1)(1)(1)}$$

$$\sigma_c = 49407 P_d^3$$

16. Since the assumption is made that the safety factor must be greater or equal then 1.5, the equations can be rearranged as follows.



$$N_{b, \text{pinion}} = \frac{S_{fb}}{\sigma_{b, \text{pinion}}}$$

$$1.5 = \frac{40964}{76.37 P_d^3}$$

$$N_{b, \text{gear}} = \frac{S_{fb}}{\sigma_{b, \text{gear}}}$$

$$1.5 = \frac{40964}{64.77 P_d^3}$$

$$N_c = \left( \frac{S_{fc}}{\sigma_c} \right)^2$$

$$1.5 = \left( \frac{112182}{49407.77 P_d^{0.3335}} \right)^2$$

17. By solving the equations, values of diametrical pitch are determined.

$$P_{d, \text{pinion}, \text{min}} = 7.10 \frac{\text{teeth}}{\text{in}}$$

$$P_{d, \text{gear}, \text{min}} = 7.49 \frac{\text{teeth}}{\text{in}}$$

$$P_{d, \text{surface}(c), \text{min}} = 6.36 \frac{\text{teeth}}{\text{in}}$$

18. Compare the result, it is obviously the diametrical pitch calculated by surface stress is the smallest. To be more specific, this value represents the smallest gear size can be chosen for this mechanical system.

$$P_{d, \text{surface}(c), \text{min}} < P_{d, \text{pinion}, \text{min}} < P_{d, \text{pinion}, \text{min}}$$

$$\therefore P_d < 6.36 \frac{\text{teeth}}{\text{in}}$$

19. Overall, the final solution is presented as Figure 44: LEGO C3PO Design Solution. Also, no specific gears can be found on MSC or McMaster-Carr. Therefore, the solution is displayed without actual products. Also, the 3D model of the final solution is shown in Table 28

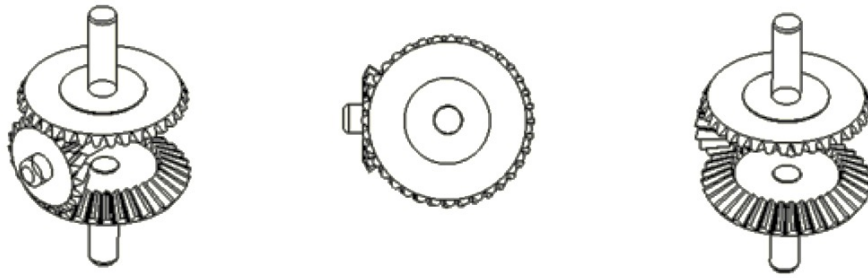


Figure 44: LEGO C3PO Design Solution

Table 30: Table used for Life Factor  $K_L$  calculation

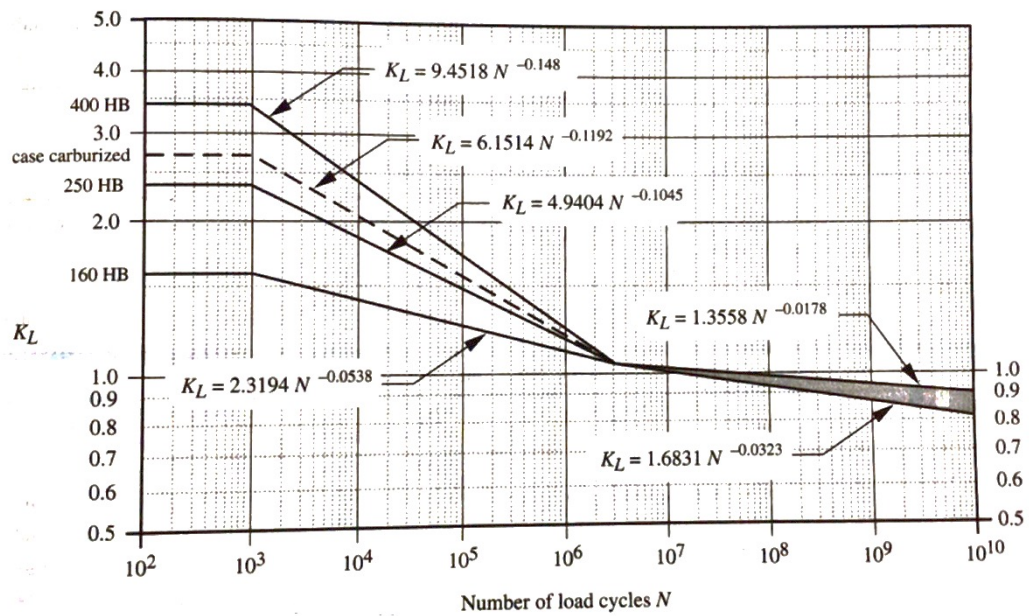


Table 31: Table used for identifying Bending Geometry Factors

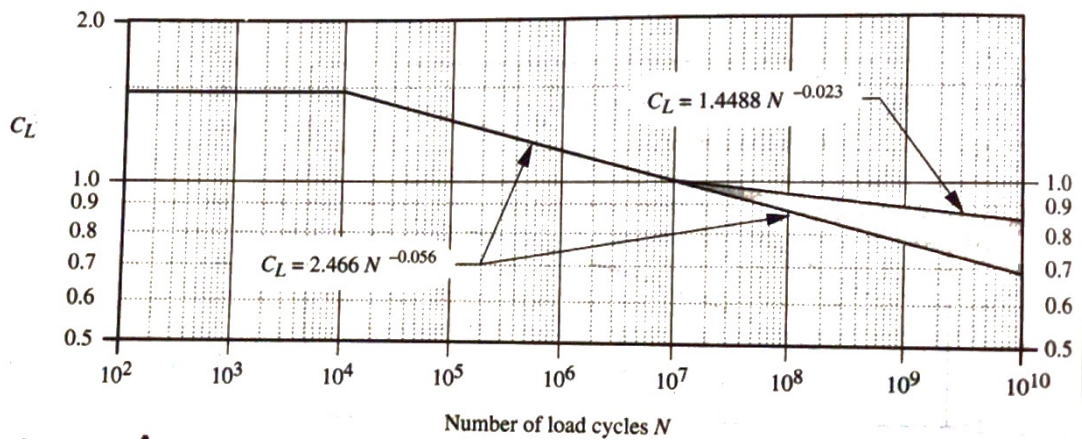
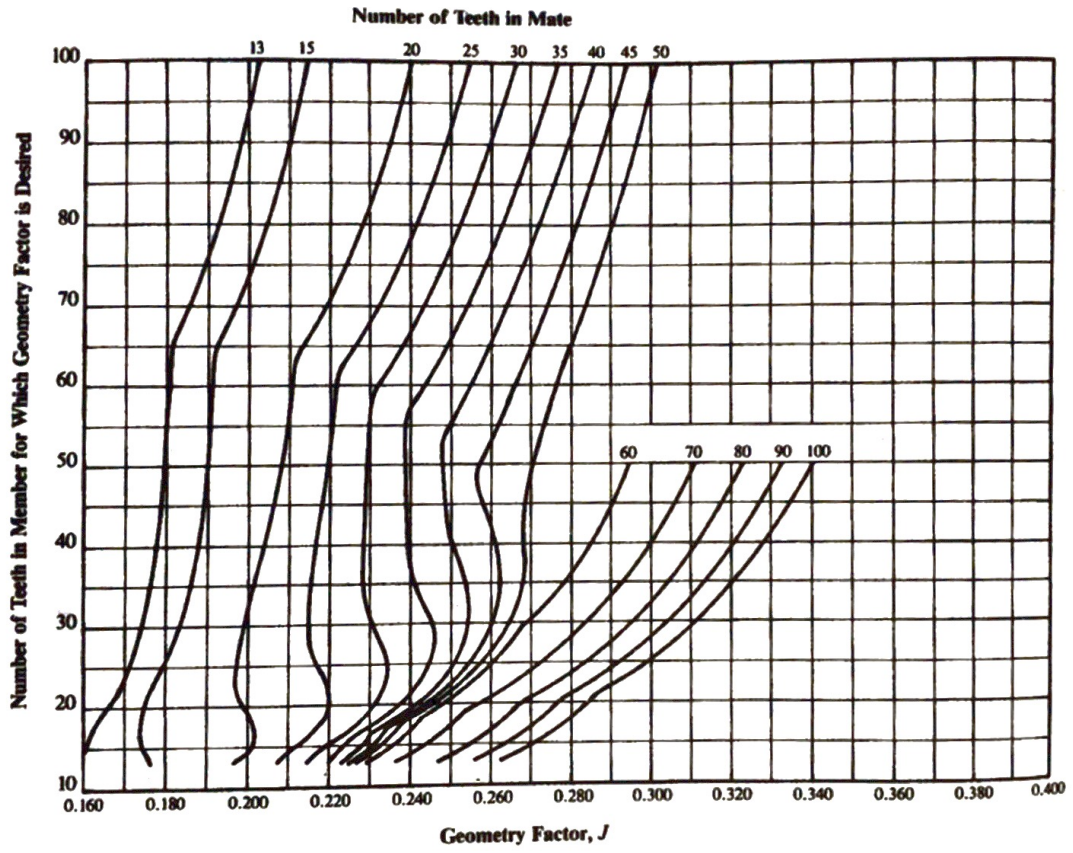


Table 32: Relationship between Geometry Factor  $J$  and Gear Teeth Number  $N$  used to identify  $J$



## Appendix 3

### Clock Design Problem Manual Solution

**Problem Statement:** Design a gear train with multiple gear stages, which can reduce the gear speed from 250rpm to three normal speed of three hands (hour hand, minute hand and secondhand) of a clock. The gear train should be able to fit in a column of 25mm diameter and 5mm thickness.

**Design Solution:** The design solutions are listed into two tables as follows. The first potation is the general design solution, shown in Table 33 which displays all technical information of the design objects. The second table presents the specific design solution of the design objects, shown in Table 34. This shows specific machine element properties along with their positions.

Table 33: Clock Design Problem General Solution

General Design Solution	
Element	1 input motor, 20 spur gears, and 11 shafts
Pressure Angle	20 degrees
Mounting Method	Presse fitted
Speed Deduction	First 8 gears fulfills deduction rate of 250/1, and the last 6 gears' is 60/1

Table 34: Clock Design Problem Actual Solution

Gear Name	Number of Teeth	Diametral Pitch	Pitch Diame ter	Face Width	Pressure Angle	Position
A	17	17	2	0.7	20	0, -9, 2.2
B	34	17	4	0.7	20	0, -6, 2.2
C	17	17	2	0.7	20	0, -6, 3
D	85	17	10	0.7	20	0, -12, 3
E	17	17	2	0.7	20	0, -12, 3.8
F	85	17	10	0.7	20	0, -6, 3.8
G	17	17	2	0.7	20	0, -6, 4.6
H	85	17	10	0.7	20	0, 0, 4.6
I	17	17	2	0.7	20	0, 0, 3.8
J	102	17	12	0.7	20	-7, 0, 3.8
K	17	17	2	0.7	20	-7, 0, 3
L	34	17	4	0.7	20	-5.43, 2.56, 3
M	17	17	2	0.7	20	-5.43, 2.56, 2.2
N	85	17	10	0.7	20	0, 0, 2.2
O	17	17	2	0.7	20	0, 0, 1.4
P	102	17	12	0.7	20	7, 0, 1.4
Q	17	17	2	0.7	20	7, 0, 2.2
R	34	17	4	0.7	20	5.43, 2.56, 2.2
S	17	17	2	0.7	20	5.43, 2.56, 3
T	85	17	10	0.7	20	0, 0, 3

**Assumption:** Each hand applies 0.3 Nmm damping torque to the system.  
 The pitch of gears and pinions is 17 /mm.  
 Must handle one-shift service life of 5 years, 2000 operating hours per year.  
 Failure rate is below 1%.  
 Material: AGMA Grade 2 Steel (250HB).  
 Safety Factor: Greater or equal than 1.5.  
 Other factors assumptions:

$$K_a = C_a = K_s = C_s = C_f = C_H = C_R = C_T = 1$$

$$K_m = C_m = 1.6$$

$$K_v = C_v = 0.652$$

$$C_L = 0.890$$

$$C_p = 2276$$

$$C_{xc} = K_x = 1$$

$$C_b = 0.634$$

$$C_{md} = 1.5$$

### Design Procedure

1. Gears in clock and watch are supposed to have minor applied torque. Thus, we firstly focus on geometric features of the other gear train. There are 3 outputs and 1 input, with ratio of 250, 60 and 60. We can calculate the number of stages we need to construct the system.

$$N_1 = \log_6 r = \log_6 250 = 3.1 \approx 4$$

$$N_2 = N_3 = \log_6 r = \log_6 60 = 2.3 \approx 3$$

2. So the numbers of stages are 4, 3 and 3. We assume the ratio of first stage is

$$1:2 \times 1:5 \times 1:5 \times 1:5$$

3. The ratio of second and third stage:

$$1:6 \times 1:2 \times 1:5$$

4. Pitch diameter of pinions:

$$\frac{25}{(1+5+1+12) \times 1.3} \times 2 \approx 2mm$$

5. Pitch diameter of gears:

$$\frac{5}{5 \times 1.3} = 0.7mm$$

6. For a clock, the three output gears must be in the middle, and the first and second output gears would have an input gear to drive the next stage. Thus, we have five gears with the same X and Y coordinate, which the final design solution is shown in Table 33.

## Appendix 4

### Gearbox Problem Shaft Design Problem Manual Solution

**Problem Statement:** Calculate and design a shaft system for the gearbox problem. Which accurate result and gear properties are provided in the Gearbox Design Manual Solution.

**Design Solution:** The design solutions are listed into two tables as follows. The first potation is the general design solution, shown in *Table 35*, which displays all technical information of the design objects. The second table presents the specific design solution of the design objects, shown in *Table 36*. This shows specific machine element properties along with their positions.

*Table 35: Gearbox Problem Shaft Design General Solution*

General Design Solution	
Element	1 input motor, 20 spur gears, and 11 shafts
Pressure Angle	20 degrees
Mounting Method	Presse fitted
Speed Deduction	First 8 gears fulfills deduction rate of 250/1, and the last 6 gears' is 60/1

*Table 36: Gearbox Problem Shaft Design Actual Solution*

Shaft Number	Shaft Length(in)	Theoretical Diameter(in)	Actual Diameter(in)	Material
A	4	0.44	0.5	SAE 1018
B	4	0.63	1	SAE 1019
C	4	0.61	1	SAE 1020
D	4	0.83	1	SAE 1021
E	4	0.82	1	SAE 1022
F	4	1.07	1.125	SAE 1023

**Assumption:** Unit: Inches  
End-milled keyway needed for such application.  
The length of the shaft is 4 inches, with safety factor of 1.5.  
Pressure angle to be assumed as 20 degrees.  
Operation temperature is bellow 200 °F.  
The notch radius is assumed to be 0.01in.  
Theoretical stress concentrate factor  $K_t= 3$ .

### Design Procedure:

1. Since the input force is given, 1.5 ft-lb, and gear ratio for each gear sets are calculated out, the maximum and minimum tangential forces can be calculated as follows.

$$F_{t,\max} = \frac{T_{\max}}{r_g}$$
$$F_{t,\max,AB} = \frac{72\text{in} \cdot \text{lb}}{\frac{3.33\text{in}}{2}} = 43.24\text{lb}$$
$$F_{t,\max,CD} = \frac{216\text{in} \cdot \text{lb}}{\frac{4.8\text{in}}{2}} = 90\text{lb}$$
$$F_{t,\max,EF} = \frac{540\text{in} \cdot \text{lb}}{\frac{5\text{in}}{2}} = 216\text{lb}$$

2. The maximum and minimum resultant forces are found as follows.

$$F_{\max} = \frac{F_{t,\max}}{\cos \phi}$$
$$F_{\max,AB} = \frac{43.24\text{lb}}{\cos 20^\circ} = 46.02\text{lb}$$
$$F_{\max,CD} = \frac{90\text{lb}}{\cos 20^\circ} = 95.78\text{lb}$$
$$F_{\max,AB} = \frac{216\text{lb}}{\cos 20^\circ} = 229.86\text{lb}$$

$$F_{\min} = \frac{F_{t,\min}}{\cos \phi}$$
$$F_{\min} = -46.02\text{lb}$$
$$F_{\min} = -95.78\text{lb}$$
$$F_{\min} = -229.86\text{lb}$$

3. The maximum and minimum moments on the shaft can now be found, by assuming the gears are centered between two supported bearings that are 4 inches apart. Since a keyway will be needed at the gear, the assumption can be made that a stress concentration factor of 3 for both bending and torsion at the critical locations. The critical locations are where both moment and torque components are the largest, in Table 12.

$$M_{\max} = F_{\max} \frac{l}{4}$$

$$M_{\max,AB} = 46.02 \frac{4}{4} = 46.02 \text{ in} \cdot \text{lb}$$

$$M_{\max,CD} = 95.78 \frac{4}{4} = 95.78 \text{ in} \cdot \text{lb}$$

$$M_{\max,EF} = 229.86 \frac{4}{4} = 229.86 \text{ in} \cdot \text{lb}$$

$$M_{\min} = F_{\min} \frac{l}{4}$$

$$M_{\min,AB} = -46.02 \text{ in} \cdot \text{lb}$$

$$M_{\min,CD} = -95.78 \text{ in} \cdot \text{lb}$$

$$M_{\min,EF} = -229.86 \text{ in} \cdot \text{lb}$$

4. The mean and alternating components of both moment and torque are needed for the stress analysis. The mean and alternating components for moment are obtained as follows.

$$M_m = \frac{M_{\max} + M_{\min}}{2}$$

$$M_{m,AB} = \frac{46.02 - 46.02}{2} = 0 \text{ in} \cdot \text{lb}$$

$$M_{m,CD} = \frac{95.78 - 95.78}{2} = 0 \text{ in} \cdot \text{lb}$$

$$M_{m,EF} = \frac{229.86 - 229.86}{2} = 0 \text{ in} \cdot \text{lb}$$

$$M_a = \frac{M_{\max} - M_{\min}}{2}$$

$$M_{a,AB} = \frac{46.02 + 46.02}{2} = 46.02 \text{ in} \cdot \text{lb}$$

$$M_{a,CD} = \frac{95.78 + 95.78}{2} = 95.78 \text{ in} \cdot \text{lb}$$

$$M_{a,EF} = \frac{229.86 + 229.86}{2} = 229.86 \text{ in} \cdot \text{lb}$$

5. The mean and alternating components for torque are obtained as follows.



$$T_m = \frac{T_{\max} - T_{\min}}{2}$$

$$T_{m,AB} = \frac{72 - 18}{2} = 27 \text{ in} \cdot \text{lb}$$

$$T_{m,CD} = \frac{216 - 72}{2} = 72 \text{ in} \cdot \text{lb}$$

$$T_{m,EF} = \frac{540 - 216}{2} = 162 \text{ in} \cdot \text{lb}$$

$$T_a = \frac{T_{\max} + T_{\min}}{2}$$

$$T_{a,AB} = \frac{72 + 18}{2} = 45 \text{ in} \cdot \text{lb}$$

$$T_{a,CD} = \frac{216 + 72}{2} = 144 \text{ in} \cdot \text{lb}$$

$$T_{a,EF} = \frac{540 + 216}{2} = 378 \text{ in} \cdot \text{lb}$$

6. A specific trial material needs to be selected for the computation progress. The most ideal choice would be most inexpensive; therefore, low-carbon and cold-rolled steel can be chosen. Specifically, SAE 1018 which has material property  $S_{ut} = 64$  kpsi and  $S_y = 54$  kpsi. If this material is proven less strong, then high-carbon steel needs to be selected. The uncorrected endurance strength limit can be calculated as follows.

$$S_{e'} = 0.5S_{ut} = (0.5)(64000) = 32000 \text{ psi}$$

7. The uncorrected endurance strength limit must be corrected by various factors and constants. The loading is assumed as bending and torsion, so the load factor is 1. Since the size of the shaft is unknown, the size factor can be temporarily assumed as 1 and adjusted later. The surface constant can be found in Table 13, which is 0.84 in this application. Also, since the operation is below 200 °F, the temperature factor is equal to 1. Additionally, the reliability factors of the shafts must be greater or equal to the reliability factor in the gears, which is 99% failure proof; therefore, the reliability factor is also 1.

$$S_e = C_{load} C_{size} C_{surf} C_{temp} C_{reliab} S_{e'}$$

$$S_e = (1)(1)(0.84)(1)(1)(32000) = 27000 \text{ psi}$$

8. The notch sensitivity of the chosen material for both bending and torsion are listed as follows, where variable  $a$  is the Neuber's Constant or can also be found in Table 14. Additionally, the notch radius  $r$  is assumed to be 0.01in.

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

$$q_{bending} = 0.50$$

$$q_{torsion} = 0.57$$

9. The fatigue stress concentration factor for both bending and torsion are calculated as follows. The theoretical stress concentrate factor  $K_t$  can be obtained by using Table 15, which resulted 3. Also, the same factor should be used on the mean stress components compare to the bending fatigue stress concentration factor.

$$K_f = 1 + q(K_t - 1)$$

$$K_t = A \left( \frac{r}{d} \right)^b$$

$$K_{f,bending} = 1 + (0.50)(3.0 - 1) = 2$$

$$K_{f,torsion} = 1 + (0.57)(3.0 - 1) = 2.15$$

$$K_{fm} = K_{f,bending} = 2$$

$$K_{fsm} = K_{f,torsion} = 2.15$$

10. Finally, the shaft diameter can be calculated by using the modified Goodman line approach.

$$D = \left\{ \frac{32N_{sf}}{\pi} \left[ \frac{\sqrt{(k_{f,bending}M_a)^2 + \frac{3}{4}(K_{f,torsion}T_a)^2}}{S_f} + \frac{\sqrt{(k_{fm}M_m)^2 + \frac{3}{4}(K_{fsm}T_m)^2}}{S_{ut}} \right] \right\}^{\frac{1}{3}}$$

$$D_A = \left\{ \frac{(32)(1.5)}{\pi} \left[ \frac{\sqrt{[(2)(46.02)]^2 + \frac{3}{4}[(2.15)(45)]^2}}{27000} + \frac{\sqrt{[(2)(0)]^2 + \frac{3}{4}[(2.15)(27)]^2}}{64000} \right] \right\}^{\frac{1}{3}} = 0.44in$$

$$D_B = \left\{ \frac{(32)(1.5)}{\pi} \left[ \frac{\sqrt{[(2)(46.02)]^2 + \frac{3}{4}[(2.15)(45)(4)]^2}}{27000} + \frac{\sqrt{[(2)(0)]^2 + \frac{3}{4}[(2.15)(27)(4)]^2}}{64000} \right] \right\}^{\frac{1}{3}} = 0.63in$$

$$D_C = \left\{ \frac{(32)(1.5)}{\pi} \left[ \frac{\sqrt{[(2)(95.78)]^2 + \frac{3}{4}[(2.15)(144)]^2}}{27000} + \frac{\sqrt{[(2)(0)]^2 + \frac{3}{4}[(2.15)(72)]^2}}{64000} \right] \right\}^{\frac{1}{3}} = 0.61in$$

$$D_D = \left\{ \frac{(32)(1.5)}{\pi} \left[ \frac{\sqrt{[(2)(95.78)]^2 + \frac{3}{4}[(2.15)(144)(3)]^2}}{27000} + \frac{\sqrt{[(2)(0)]^2 + \frac{3}{4}[(2.15)(72)(3)]^2}}{64000} \right] \right\}^{\frac{1}{3}} = 0.83in$$

$$D_E = \left\{ \frac{(32)(1.5)}{\pi} \left[ \frac{\sqrt{[(2)(229.86)]^2 + \frac{3}{4}[(2.15)(378)]^2}}{27000} + \frac{\sqrt{[(2)(0)]^2 + \frac{3}{4}[(2.15)(162)]^2}}{64000} \right] \right\}^{\frac{1}{3}} = 0.82in$$

$$D_F = \left\{ \frac{(32)(1.5)}{\pi} \left[ \frac{\sqrt{[(2)(229.86)]^2 + \frac{3}{4}[(2.15)(378)(2.5)]^2}}{27000} + \frac{\sqrt{[(2)(0)]^2 + \frac{3}{4}[(2.15)(162)(2.5)]^2}}{64000} \right] \right\}^{\frac{1}{3}} = 1.07in$$

11. The final theoretical and actual result can be obtained as *Table 36*. As a conclusion, since there is no specific price or material limit, the actual diameters are chosen to be the most convenient to purchase which are listed down below. Step shaft can be considered, but again for user's convenience straight shafts are selected.

Table 37: Critical Location of Shaft Keyways

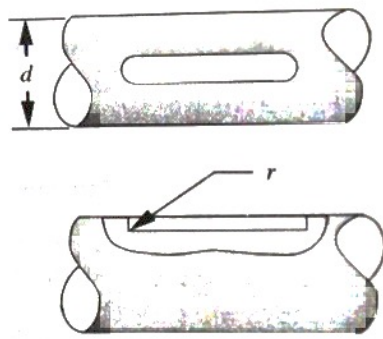
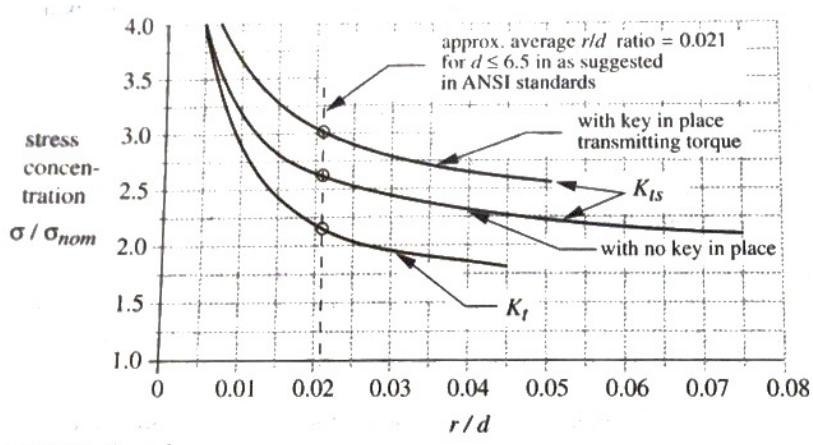


Table 38: Table of Shaft Surface Constant

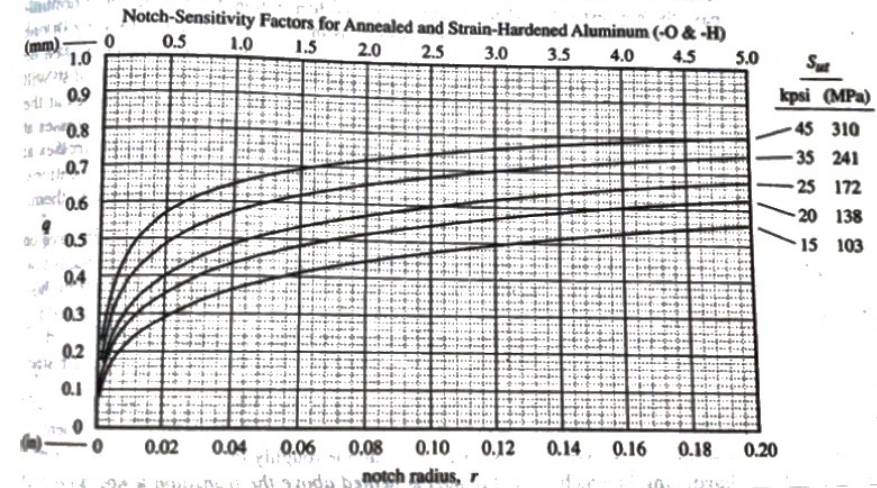
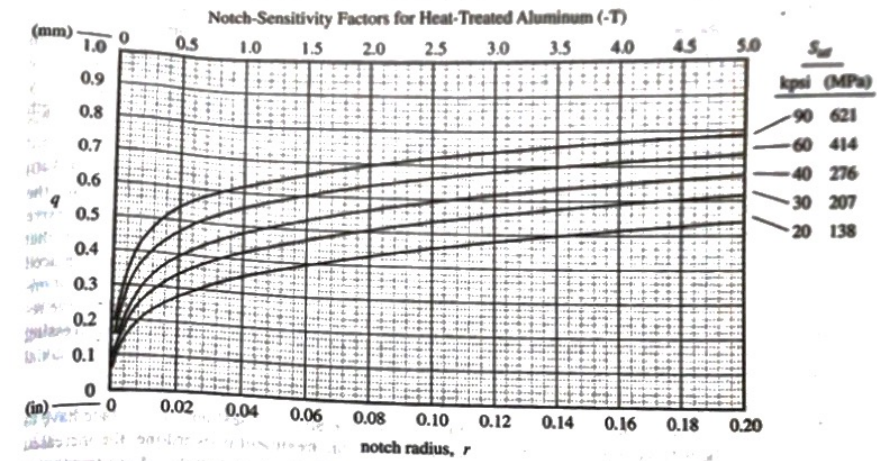
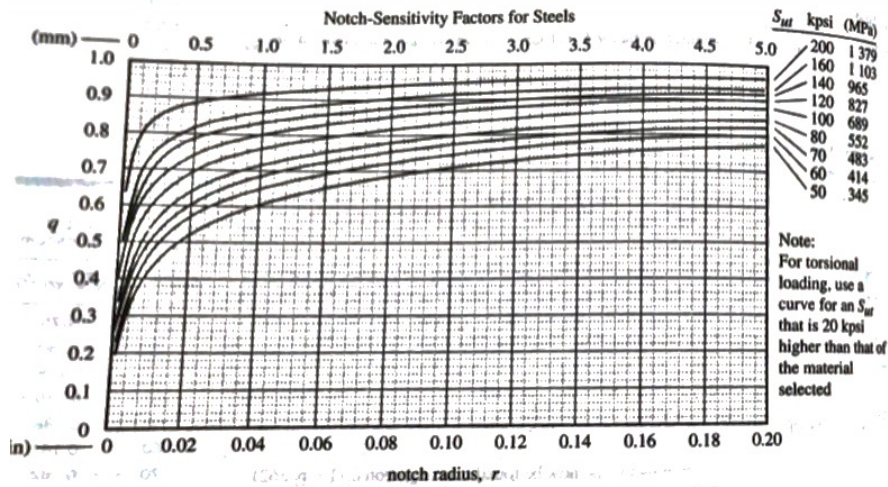


Table 39: Table of Shaft Surface Factor

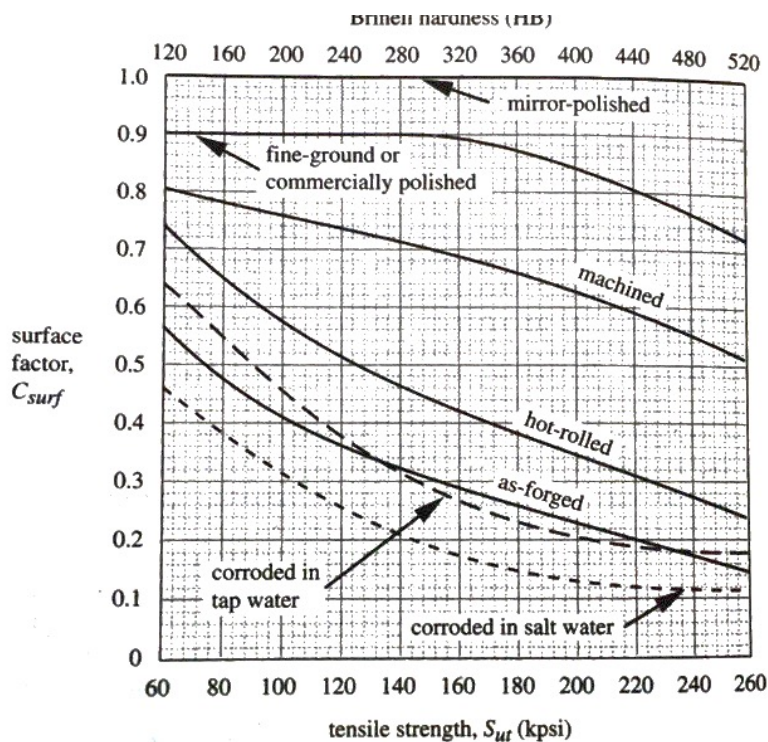
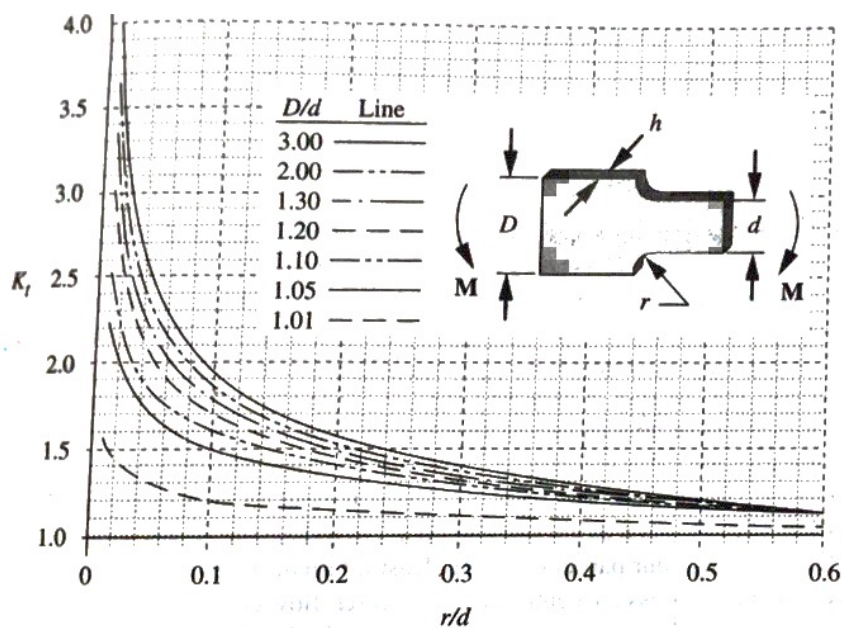


Table 40: Table of Theoretical Stress Concentration Factor



$$\sigma_{nom} = \frac{Mc}{I} = 6 \frac{M}{hd^2}$$

$$\sigma_{max} = K_t \sigma_{nom}$$

and:

$$K_t = A \left( \frac{r}{d} \right)^b$$

where:

$D/d$	$A$	$b$
3.00	0.907 20	-0.333 33
2.00	0.932 32	-0.303 04
1.30	0.958 80	-0.272 69
1.20	0.995 90	-0.238 29
1.10	1.016 50	-0.215 48
1.05	1.022 60	-0.191 56
1.01	0.966 89	-0.154 17

## **Appendix 5**

This appendix presents the specific machine elements found from online supplier HPC Gears. Based on the gearbox design solution shown in *Appendix 1*, our team searched major industrial supplies such as MAC, HPC Gears, and McMasrter-Carr. However, only one of the supplier provides gears that meets the gearbox design requirements. The result is shown in Tables 26-28.



<b>Gear Name</b>	<b>Calculated N</b>	<b>Calculated P<sub>d</sub></b>	<b>Online N</b>	<b>Online P<sub>d</sub></b>	<b>Part #</b>	<b>Manufacturer</b>	<b>Cost</b>
<b>A</b>	17	17	16	16	<a href="#">G16-16</a>	HPC Gears	13.64EUR-13.98EUR
<b>B</b>	34	17	32	16	<a href="#">G16-32</a>	HPC Gears	17.80EUR-21.91EUR
<b>C</b>	17	17	16	16	<a href="#">G16-16</a>	HPC Gears	13.64EUR-13.98EUR
<b>D</b>	85	17	84	16	<a href="#">G16-84</a>	HPC Gears	55.80EUR-60.47EUR
<b>E</b>	17	17	16	16	<a href="#">G16-16</a>	HPC Gears	13.64EUR-13.98EUR
<b>F</b>	85	17	84	16	<a href="#">G16-84</a>	HPC Gears	55.80EUR-60.47EUR
<b>G</b>	17	17	16	16	<a href="#">G16-16</a>	HPC Gears	13.64EUR-13.98EUR
<b>H</b>	85	17	84	16	<a href="#">G16-84</a>	HPC Gears	55.80EUR-60.47EUR
<b>I</b>	17	17	16	16	<a href="#">G16-16</a>	HPC Gears	13.64EUR-13.98EUR
<b>J</b>	102	17	100	16	<a href="#">G16-100</a>	HPC Gears	74.28EUR-84.41EUR
<b>K</b>	17	17	16	16	<a href="#">G16-16</a>	HPC Gears	13.64EUR-13.98EUR
<b>L</b>	34	17	32	16	<a href="#">G16-32</a>	HPC Gears	17.80EUR-21.91EUR
<b>M</b>	17	17	16	16	<a href="#">G16-16</a>	HPC Gears	13.64EUR-13.98EUR
<b>N</b>	85	17	84	16	<a href="#">G16-84</a>	HPC Gears	55.80EUR-60.47EUR
<b>O</b>	17	17	16	16	<a href="#">G16-16</a>	HPC Gears	13.64EUR-13.98EUR
<b>P</b>	102	17	100	16	<a href="#">G16-100</a>	HPC Gears	74.28EUR-84.41EUR
<b>Q</b>	17	17	16	16	<a href="#">G16-16</a>	HPC Gears	13.64EUR-13.98EUR
<b>R</b>	34	17	32	16	<a href="#">G16-32</a>	HPC Gears	17.80EUR-21.91EUR
<b>S</b>	17	17	16	16	<a href="#">G16-16</a>	HPC Gears	13.64EUR-13.98EUR
<b>T</b>	85	17	84	16	<a href="#">G16-84</a>	HPC Gears	55.80EUR-60.47EUR

Table 41: Table of Gears Found from Online Suppliers for Clock problem



<b>Gear Name</b>	<b>Calculated N</b>	<b>Calculated P<sub>d</sub></b>	<b>Online N</b>	<b>Online P<sub>d</sub></b>	<b>Part #</b>	<b>Manufacturer</b>	<b>Cost</b>
<b>A</b>	20	6	20	6	<a href="#">G6-20</a>	HPC Gears	60.34EUR-71.38EUR
<b>B</b>	35	6	35	6	<a href="#">AB6-35</a>	HPC Gears	411.66EUR
<b>C</b>	35	6	35	6	<a href="#">AB6-35</a>	HPC Gears	411.66EUR

*Table 42: Table of Gears Found from Online Suppliers for Lego design solution*

<b>Gear Name</b>	<b>Calculated N</b>	<b>Calculated P<sub>d</sub></b>	<b>Online N</b>	<b>Online P<sub>d</sub></b>	<b>Part #</b>	<b>Manufacturer</b>	<b>Cost</b>
<b>A</b>	10	12	10	12	<a href="#">G12-10</a>	HPC Gears	13.45EUR-16.21EUR
<b>B</b>	40	12	40	12	<a href="#">AB12-40</a>	HPC Gears	122.21EUR
<b>C</b>	16	10	16	10	<a href="#">G10-16</a>	HPC Gears	19.69EUR-26.78EUR
<b>D</b>	48	10	48	10	<a href="#">AB10-48</a>	HPC Gears	190.34EUR
<b>E</b>	16	8	16	8	<a href="#">G8-16</a>	HPC Gears	30.71EUR-32.34EUR
<b>F</b>	40	8	40	8	<a href="#">AB8-40</a>	HPC Gears	235.39EUR

*Table 43: Table of Gears Found from Online Suppliers for Gear Box design solution*

## Appendix 6

### Bounding Box and Intersection Analysis of Gear Train

**Problem statement:** The problem statement is to design a calculation logic for the bounding box after the gear sizes are determined. This bounding box needs to present the maximum space required for the designed gearbox.

**Assumption:** All the gears and shafts are considered as cylinders based on their maximum diameters. The bounding box is assumed to be a cuboid box instead of a specific 3D outline of a possible design solution.

### Methods and Logic:

1. Based on the assumption, while creating a general presentation method for the designed gears and shafts in a 3D space, we consider all the gears and shafts as cylinders for the calculation convenience. Since mentioned in the gearbox design problem that all the gears and shafts are rotating of the z-axis; therefore, each machine element can be represented as follows. Where R is the radius of the base, r is parameter, and L is the length of each cylinder. Specifically,  $(R \sin r)$  and  $(R \cos r)$  define a circle with radius R in x-y plane, and the circle extend above and below by  $L/2$  to form a cylinder. In this case, cylinder is along z-axis and centered at  $(0, 0, 0)$ . Also note that r is arc parameter, which means it can be any number between 0 and  $2\pi$ , so that a single vector can express a continuous shape in space.

$$\begin{bmatrix} R \sin r \\ R \cos r \\ (-\frac{L}{2}, \frac{L}{2}) \end{bmatrix}$$

2. After the volume of each machine element is defined, the location of each element needs be defined. Suppose we need a cylinder orient to

$$\begin{matrix} \theta \\ \delta \end{matrix}$$

Where  $\theta$  represents the angel between cylinder and z-axis when  $z \geq 0$ , and  $\delta$  is the angle between the projection of cylinder on x-y plane and x-axis, when  $z \geq 0$ . In order to create such cylinder, we can rotate the cylinder around y-axis by  $\theta$ , and rotate it again around z-axis by  $\delta$ .

3. Hence the volume and position of each machine element is defined, the rotational matrix based on the above steps can be calculated as following format. Each step is a rotational matrix, which can rotate vector in space by a certain angle around a certain axis.

$$\begin{matrix} \begin{bmatrix} \cos\delta & -\sin\delta & 0 \\ \sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix} & \times & \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} & \times & \begin{bmatrix} R \sin r \\ R \cos r \\ (-\frac{L}{2}, \frac{L}{2}) \end{bmatrix} \\ \text{Second step} & & \text{First step} & & \text{Original vector} \end{matrix}$$

4. Replace range  $(-\frac{L}{2}, \frac{L}{2})$  by length variable  $k$ (which can be the length of shaft or face width of gear), and we have

$$\begin{bmatrix} R \cos \theta \cos \delta \sin r + k \sin \theta \cos \delta - R \sin \delta \cos r \\ R \cos \theta \sin \delta \sin r + k \sin \theta \sin \delta + R \cos \delta \cos r \\ -R \sin \theta \sin r + k \cos \theta \end{bmatrix}$$

5. Also considering the translational motion, center of cylinder may be not at  $(0, 0, 0)$ , so we have expression as follows. Where  $x_0, y_0,$  and  $z_0$  are the center position of cylinder, and  $G/S$  is vector that represent a gear or shaft in 3d space. This presents a general cylinder in space.

$$G/S = \begin{bmatrix} R \cos \theta \cos \delta \sin r + k \sin \theta \cos \delta - R \sin \delta \cos r - x_0 \\ R \cos \theta \sin \delta \sin r + k \sin \theta \sin \delta + R \cos \delta \cos r - y_0 \\ -R \sin \theta \sin r + k \cos \theta - z_0 \end{bmatrix}$$

6. Bevel gears requires different consideration, because it can be simplified as frustum of cone instead of cylinder. The standard frustum in space, orient toward z-axis and centered at origin can be expressed as follows. Where  $a$  and  $b$  are the parameters of the defined cones. They determine the size and slope of each cone. Note that we have already replace the length variable  $L$  by  $k$ .

$$\begin{bmatrix} (ak + b) * R \sin r \\ (ak + b) * R \cos r \\ k \end{bmatrix}$$

7. Then by applying the exact same process of two rotational matrices, and we have the expression below. Where  $G_b$  is the vector presenting frustum of cone in 3d space.

$$G_b = \begin{bmatrix} (ak + b)R \cos \theta \cos \delta \sin r + k \sin \theta \cos \delta - (ak + b)R \sin \delta \cos r - x_0 \\ (ak + b)R \cos \theta \sin \delta \sin r + k \sin \theta \sin \delta + (ak + b)R \cos \delta \cos r - y_0 \\ -(ak + b)R \sin \theta \sin r + k \cos \theta - z_0 \end{bmatrix}$$

8. Now, it is necessary to check whether gear A and gear B are tangential. Note that gear A and B are any defined gear pair. Where  $G_A$  and  $G_B$  are the vectors of gear A and B.

$$G_A - G_B = 0$$

9. The solution should be the relationship between  $r_A, r_B, k_A, k_B,$  radius parameter and length variable of two gears. And for  $r_A \in [0, 2\pi) r_B \in [0, 2\pi),$  given any  $k_A$  in actual range, there is one and only one solution to  $k_B, r_A, r_B.$  Then we know that these two cylinders are tangential.
10. It is also wise to proof whether the design solution is tangential from the inside. To be more specific, for any  $k_A$  and its corresponding  $k_B,$  if there is any combination of  $r_A$  and  $r_B$  that makes the distance between vector A and B larger than  $2R_A$  and  $2R_B.$  If it is true, then two gears are not tangential from inside.

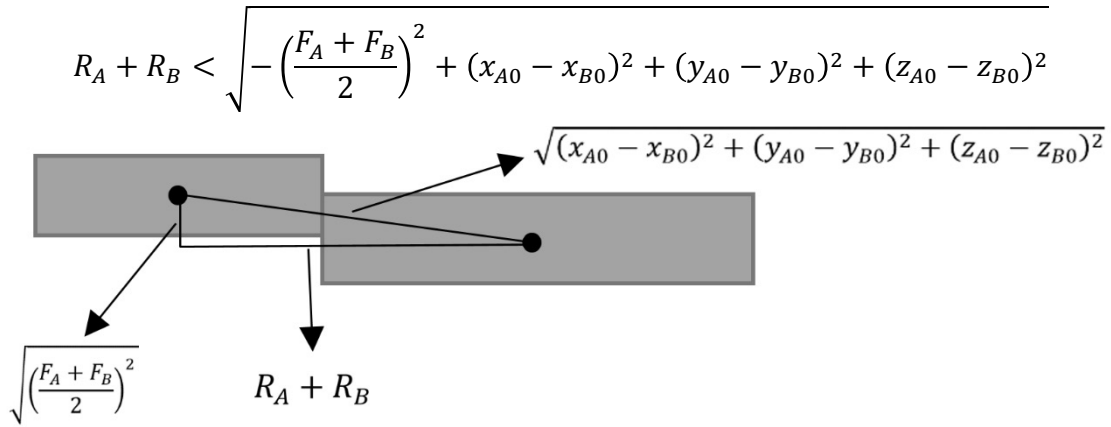
**Check if two matched gears are actually attaching.**

11. Hence we proofed all the gears are tangential, it is also needed to proof that all the gear pairs are matching properly. The initial equation is stated as follows.

$$G_A - G_B = 0$$

12. By only considering the relation between  $k_A$  and  $k_B$ . If there exist solutions of them that satisfy both of their actual range, two gears mesh with each other.

13. There is also an alternative way to do it, but it only works for gears in cylindrical shape by heck whether the following equations are true. These equations are the equations of the Pythagorean theorem, and is represented by above graph. If the inequality exist, two gears will not be too far away from each other to mesh.



14. After checking the status of matched gear pairs, whether a shaft goes through the middle of a gear is therefore being proofed. If a shaft goes along z-axis through  $(0, 0, 0)$ , it will have its own axis as stated as follows. Where  $k_s$  is the shaft length variable.

$$\begin{bmatrix} 0 \\ 0 \\ k_s \end{bmatrix}$$

15. Then apply those two steps mentioned in the step 3, we can check if the center of gear sits on the axis of shaft. Specifically,  $[x_{s0}, y_{s0}, z_{s0}]$  represents the center position of each shaft, and  $[x_{g0}, y_{g0}, z_{g0}]$  presents the center position of each gear. While solving for variable  $k_s$ . If  $k_s$  has one real solution, then there is a shaft goes through the gear.

$$\begin{bmatrix} \cos\delta & -\sin\delta & 0 \\ \sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ k_s \end{bmatrix} - \begin{bmatrix} x_{s0} \\ y_{s0} \\ z_{s0} \end{bmatrix} = \begin{bmatrix} k_s \sin\theta \cos\delta - x_0 \\ k_s \sin\theta \sin\delta - y_0 \\ k_s \cos\theta - z_0 \end{bmatrix}$$

$$\begin{bmatrix} k_s \sin\theta \cos\delta - x_{s0} \\ k_s \sin\theta \sin\delta - y_{s0} \\ k_s \cos\theta - z_{s0} \end{bmatrix} - \begin{bmatrix} x_{g0} \\ y_{g0} \\ z_{g0} \end{bmatrix} = 0$$

16. Now the gear and shaft directions must be checked. In order to attach gears and shaft, they must be rotating on the same axis. By doing so, we check the whether the following equations are true. Specifically,  $\theta_s$  and  $\delta_s$  are the angle of the shaft; and  $\theta_g$  and  $\delta_g$  are the angle of gears.

$$\theta_s = \theta_g, \delta_s = \delta_g.$$

17. While checking whether gears and shafts are attached, the process is very similar to the one used for two gears. The only difference is that the shaft is supposed to go right through the gear, so we only need to use the Pythagorean theorem once to identify the distance between centers of gear and shaft. By doing so, we need to check whether the following equation is true. Where L represents the length of shafts, and F is face width of gears.

$$\sqrt{(x_{A0} - x_{B0})^2 + (y_{A0} - y_{B0})^2 + (z_{A0} - z_{B0})^2} \leq \frac{L - F}{2}$$

18. Once the gears and shafts are confirmed to be attached, we much check whether there is any intersection exist in the transmission. To achieve so, we need to check for two defined cylinder vector A and B as follows. This will provide the relations between variable  $r_A$ ,  $r_B$ ,  $k_A$ , and  $k_B$ . Also note that the radius R is the addendum radius instead of pitch radius. Among all possible solutions, if any k is in the actual range of shaft length or gear face width, there is an intersection here.

$$A - B = 0$$

19. After all steps presented above, the bounding box of the designed gearbox can now be defined. The bounding box can be represented by 6 vectors as following matrixes. Where BB represents the vector for the bounding box, and each vector has one fixed variable. Also,  $x_i$ ,  $y_i$ , and  $z_i$  have certain range, which are the space position of bounding box.

$$BB = \begin{bmatrix} x_i \\ y_i \\ z_0 \end{bmatrix} \text{ or } \begin{bmatrix} x_i \\ y_0 \\ z_i \end{bmatrix} \text{ or } \begin{bmatrix} x_0 \\ y_i \\ z_i \end{bmatrix}$$

20. Then apply equation listed below to all gears, shafts, and the bounding box. We will have equations for  $k_{G/S}$  and  $x_i/y_i/z_i$ . If any solution which has its all variables in actual range, there is an intersection here. Also notice that this method can be used to check intersection for any cuboid in space if we apply those two rotational matrix, so that we can check for rack with the same method.

$$G/S - BB = 0$$